Effect of Non Gaussian Noises on the Stochastic Resonance-Like Phenomenon in Gated Traps

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Abstract

We exploit a simple one-dimensional trapping model introduced before, prompted by the problem of ion current across a biological membrane. The voltage-sensitive channels are open or closed depending on the value taken by an external potential that has two contributions: a deterministic periodic and a stochastic one. Here we assume that the noise source is colored and non Gaussian, with a q-dependent probability distribution (where q is a parameter indicating the departure from Gaussianity). We analyze the behavior of the oscillation amplitude as a function of both q and the noise correlation time. The main result is that in addition to the resonant-like maximum as a function of the noise intensity, there is a new resonant maximum as a function of the parameter q.

The growing interest in stochastic resonance (SR) has motivated a wealth of studies in physical, chemical and biological systems [1]. In particular, it has been found to play a relevant role in several problems in biology: mammalian sensory systems, increment of tactile capacity, visual perception, low frequency effects and low amplitude electromagnetic fields, etc [2].

In particular, there is an experiment on SR related to the measurement of the current through voltage-sensitive ion channels in a cell membrane [3]. These channels switch (randomly) between open and closed states, thus controlling the ion current. This and other related phenomena have stimulated several theoretical studies of the problem of ionic

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transport through biomembranes, using different approaches, as well as different ways of characterizing stochastic resonance in such systems [4]. It is worth remarking here that recent detailed studies on the source of fluctuations in some biological systems [5] clearly indicate that noise sources in general could be non Gaussian and that their distribution bounded.

In Ref. [6] we have studied a toy model, prompted by the work in [3], sketching the behaviour of an ion channel. This included the simultaneous action of a deterministic and a stochastic external field on the trapping rate of a gated imperfect trap. Rather than attempting a precise modeling of the behaviour of an ionic channel, we proposed there a simple model of dynamical trap behaviour. Our main result was that even such a simple model of a gated trapping process shows a SR-like behaviour. In that initial study we assumed that the stochastic external field was a Gaussian white noise.

In this work, and due to recent experimental evidences of the boundness and non Gaussian character of noise sources in biological systems [5], we analyze the same model, but using a correlated non Gaussian noise source. Here we have Gaussian or non Gaussian behaviour of the noise probability distribution (pdf) depending on whether a parameter q is $= 1$ or $\neq 1$, respectively. If $q > 1$, the pdf results more extended than a Gaussian one, while in the other case, that is $q < 1$, the pdf is bounded. This latter aspect is of overwhelming relevance for the problem of ion currents through membranes.

The study of gated trapping processes, i.e. a trapping process where the traps have some kind of internal dynamic has attracted considerable interest [7]. Many authors discussed the way to link the gated trapping processes with the measured behavior of the so called ionic pumps [4]. For example, among other factors, the ion transport depends on the membrane electric potential (which plays the role of the barrier height) and can be stimulated by both dc and ac external fields.

In Ref. [6], the study was based on the so called *stochastic model* for reactions [8–10], generalized in order to include the internal dynamics of traps. The dynamical process consists of the opening or closing of the traps according to an external field. Such a field has two contributions, one periodic with a small amplitude, and the other stochastic whose intensity will be (as usual) the tuning parameter. The starting model equation was

$$\partial_t \rho(x,t) = D\partial_x^2 \rho(x,t) - \gamma(t)\delta(x)\rho(x,t) + n_u, \tag{1}$$

where γ is a stochastic process that represents the absorption probability of the trap, ρ is the particle density (particles that have not been yet trapped); for a given realization of γ, x is the coordinate over the one-dimensional system and n_u is a source term that represents a constant flux of ions. The injection of ions can be at a trap position or at any other position. In this last case the ion can diffuse to the trap position. This diffusion coefficient would represent an effective diffusion through the volume rather a diffusion over the membrane surface.

The absorption is modelled as

$$\gamma(t) = \gamma^* \theta[B\sin(\omega t) + \xi - \xi_c], \tag{2}$$

where $\theta(x)$, the Heaviside function, determines when the trap is open or closed. The trap works as follows: if the signal, composed of the harmonic part plus ξ (the noise contribution),

reaches a threshold ξ_c the trap opens, otherwise it is closed. We are interested in the case where $\xi_c > B$, that is, without noise the trap is always closed. When the trap is open the particles are trapped with a given frequency (probability per unit time) γ^*. In other words the open trap is represented by an "imperfect trap" . Finally, in order to complete the model, we must give the statistical properties of the noise ξ. In [6] we assumed that ξ is an uncorrelated Gaussian noise of intensity ξ_0. Here we use a "colored" non Gaussian noise given by

$$\dot{\xi} = -\frac{1}{\tau}\frac{d}{d\xi}V_q(\xi) + \frac{1}{\tau}\eta(t) \tag{3}$$

where $\eta(t)$ is a Gaussian white noise of zero mean and correlation $< \eta(t)\eta(t') >= 2D\delta(t-t')$, and $V_q(\eta)$ is given by [11]

$$V_q(\xi) = \frac{1}{\beta(q-1)}\ln[1 + \beta(q-1)\frac{\xi^2}{2}], \tag{4}$$

where $\beta = \tau/D$. When $q \to 1$ we recover the limit of ξ being an Ornstein-Uhlenbeck process.

We define the current through the trap as $J(t) = \langle\gamma_j(t)\rho(jl,t)\rangle$. The brackets mean averages over all realizations of the noise. In [6], that is in the case of $\xi(t)$ being a Gaussian white noise, we have obtained some analytical results and solved the equation numerically. However, here we should resort only to Monte Carlo simulations.

As in [6], we choose to quantify the SR-like phenomenon by computing the amplitude of the oscillating part of the absorption current given by $\Delta J = J|_{\sin(\omega t)=1} - J|_{\sin(\omega t)=-1}$. The qualitative behaviour of the system can be explained as follows. For small noise intensities the current is low (remember that $\xi_c > B$), hence ΔJ is small too. For a large noise intensity the deterministic (harmonic) part of the signal becomes irrelevant and the ΔJ is also small. Therefore, there must be a maximum at some intermediate value of the noise.

The simulations were performed on a one dimensional lattice of L sites with periodic boundary conditions. Initially there are no particles on the lattice. The particles are injected randomly every $1/(n_u L)$ units of time, with uniform distribution over the lattice, and are allowed to perform a continuous time random walk (characterized by the jump frequency at each neighbor site). There is no restriction on the number of particles at each site. A particle can be removed from the system with a given probability distribution characterized by γ when it reaches the trap site. A detailed description of the algorithm used can be found in [10]. The reaction times were generated according to the following probability density function

$$p(t) = \exp\left(-\int_0^t \langle\gamma(t')\rangle dt'\right). \tag{5}$$

All simulations shown in the figures correspond to averages over 1000 realizations.

We have plotted all results as functions of the non Gaussian noise intensity ξ_0. It is related to the (generating) white noise intensity through $\xi_0 = 2D/(5 - 3q)$ [12].

In what follows we show the results corresponding to the case when the noise source is non Gaussian. In Fig. 1 we show the amplitude of the absorption current $\Delta J(t)$ as a function of the noise intensity ξ_0 for: (a) different $q's$ and fixed τ and observational time (t), (b) for three different τ and fixed values of q and t. The results are in agreement with

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those found in the case of Gaussian white noise. In the first case we see that the system response increases when $q < 1$, and there is a shift of the maximum of $\Delta J(t)$ to larger noise values for increasing q. In the second case the curves also show a shift of the maximum to larger noise intensities as τ increases. Figure 2 shows the resonant intensity noise values as a function of τ and for different values of q. For $q < 1$ the maximum arises for lower values of the noise intensity than for $q > 1$. In Fig. 3 we depict the maximum of $\Delta J(t)$ as a function of q, for two different observational times. Here we obtain one of the main results of this work: the existence of new resonant-like maximum as a function of the parameter q. This implies that we can find an optimal value of q (~ 0.5, *corresponding to a bounded and non Gaussian pdf*) yielding the largest system response.

Finally, in Fig. 4 we show the value of $\Delta J(t)$ as a function of τ for fixed t and a large value of ξ_0. The behaviour of ΔJ is in agreement with the shift of the curves shown in Fig. 1b. The inset shows the phase-shift between the input and output signals, a result that is in agreement with the main figure.

The present results show that the use of non Gaussian noises in the simple trapping process defined in Eq. (1) produces significant changes in the system response when compared with the Gaussian case. In particular we want to emphasize that we have found a double resonance-like phenomenon indicating that, in addition to an optimal noise intensity, there is an optimal q value which yields the larger enhancement of the system response. The remarkable fact is that it corresponds to $q < 1$ indicating that this enhancement occurs for a non Gaussian and **bounded** distribution. Due to the evidences found in Ref. [5], such a result is of great relevance in a biological context. Clearly, the present study corresponds to the analysis of a toy model, more realistic ones will be the subject of further work.

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FIGURES

FIG. 1. Value of ΔJ (amplitude of the oscillating part of the absorption current) as a function of ξ_0 for a given observational time ($t = 1140$). (a) different values of q (triangles $q = 0.5$, crosses $q = 1.0$, squares $q = 1.5$) and a fixed value of τ ($\tau = 0.1$). (b) different values of τ (triangles $\tau = 0.01$, circles $\tau = 0.1$, squares $\tau = 1.0$) and a fixed value of q ($q = 0.5$).

FIG. 2. Values of ξ_0^{max}, the noise intensity corresponding to the maximum of ΔJ, as a function of τ for fixed observational time ($t = 1140$) and different values of q: circles $q = 0.5$, triangles $q = 1.5$.

FIG. 3. Dependence of ΔJ_{max}, the value of ΔJ at the maximum, as a function of q for fixed τ ($\tau = 0.1$) and different observational times: circles $t = 633$, crosses $t = 1140$.

FIG. 4. Dependence of ΔJ as a function of τ for fixed values of ξ_0 ($\xi_0 = 4.7$), q ($q = 0.5$) and t ($t = 1140$). The inset shows the phase shift ϕ as a function of ξ_0 for different values of τ: squares $\tau = 0.01$, circles $\tau = 0.1$, triangles $\tau = 1.0$.

The main plot shows ΔJ (arb. units) versus τ (arb. units). The inset shows ϕ (rad.) versus ξ_0 (arb. units).