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We study Brownian motors driven by colored non Gaussian noises, both in the overdamped regime and in the case with inertia, and analyze how the departure of the noise distribution from Gaussian behavior can affect its behavior. We analyze the problem from two alternative points of view: one oriented mainly to possible technological applications and the other more inspired in natural systems. In both cases we find an enhancement of current and efficiency due to the non-Gaussian character of the noise. We also discuss the possibility of observing an enhancement of the mass separation capability of the system when non-Gaussian noises are considered.

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## I. INTRODUCTION

The study of noise induced transport by "ratchets" has attracted in recent years the attention of an increasing number of researchers due to the biological interest and also to its potential technological applications [1, 2]. Since the pioneering works, besides the built-in ratchet-like bias and correlated fluctuations (see for instance [3]), different aspects have been studied, such as tilting [4, 5] and pulsating [6] potentials, velocity inversions [4, 7], etc. There are some relevant reviews [8, 9] where the biological and/or technological motivation for the study of ratchets can be found.

Recent studies on the role of non Gaussian noises on some noise-induced phenomena like stochastic resonance, resonant trapping, and noise-induced transitions [10, 11, 12, 13, 14, 15] have shown the possibility of strong effects on the system's response. For instance, enhancement of the signal-to-noise ratio in stochastic resonance, enhancement of the trapping current in resonant trapping, or shifts in the transition line for noise-induced transitions. These results motivate the interest in analyzing the effect of non Gaussian noises on the behavior of Brownian motors. Here we analyze the effect of a particular class of colored non Gaussian noise on the transport properties of Brownian motors. Such a noise source is based on the nonextensive statistics [16, 17] with a probability distribution that depends on  $q$ , a parameter indicating the departure from Gaussian behavior: for  $q = 1$  we have a Gaussian distribution, and different non Gaussian distributions for  $q > 1$  or  $q < 1$ .

Some of the motivations for studying the effect of non Gaussian noises are, in addition to its intrinsic interest within the realm of noise induced phenomena, the existence of experimental data indicating that for several biological problems fluctuations have a non Gaussian character. Examples are current measurements through voltage-sensitive ion channels in a cell membrane or experiments on the sensory system of rat skin [18]. Also, recent detailed studies on the source of fluctuations in different biological systems [19] clearly show that, in such a context, noise sources in general are non Gaussian. Even though the previous arguments refer to biological aspects that are not directly related to ratchets, they strongly induce to think about the possible relevance of considering non Gaussian noises in those biological situations where the ratchet transport mechanism can play a role. In addition, from the point of view of technological applications, the finding of new conditions that may lead to an enhancement of the efficiency of the devices is always desirable.

We show here that, as a consequence of the non Gaussian character of the driving noise and from two alternative points of view, we can find a kind of enhancement of the system's response. The first –direct– point of view, takes as free parameters those that could be controlled in the case of technological applications. In this case we find a remarkable increase of the current together with an enhancement of the motor efficiency when non-Gaussian noises are considered (showing an optimum for a given degree of departure from the  $q = 1$  Gaussian behavior). Moreover,

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when inertia is taken into account it is found that, again when departing from the Gaussian case, there seems to be a remarkable increment in the mass separation capability of these devices. The second point of view is the more natural one when thinking of biological systems, as it considers the non-Gaussian noise as a primary source. In this case we also find an enhancement of the current and efficiency due to departure from Gaussian behavior, which occurs for low values of noise intensity.

We begin presenting the general framework within which we will work, and the nature of the non Gaussian noise. We continue discussing the first of the two points of view, and the results showing the enhancement we can find within it. After that we discuss the second point of view where we compare Gaussian and non Gaussian behaviors but adopting a constant width criterion, and discuss the results. Finally we draw some general conclusions.

## II. FRAMEWORK

We begin considering the general system

$$m \frac{d^2 x}{dt^2} = -\gamma \frac{dx}{dt} - V'(x) - F + \xi(t) + \eta(t), \quad (1)$$

where  $m$  is the mass of the particle,  $\gamma$  the friction constant,  $V(x)$  the ratchet potential,  $F$  is a constant “load” force, and  $\xi(t)$  the thermal noise satisfying  $\langle \xi(t)\xi(t') \rangle = 2\gamma T \delta(t-t')$ . Finally,  $\eta(t)$  is the time correlated forcing (with zero mean) that allows the rectification of the motion, keeping the system out of thermal equilibrium even for  $F = 0$ . For this type of ratchet model several different kind of time correlated forcing have been considered in the literature [8, 9]. In almost all studies authors have used Gaussian noises. The few exceptions which considered non Gaussian processes correspond mainly to the case of dichotomic noises [2, 4, 9].

The main characteristic introduced by the non Gaussian form of the forcing we consider here, is the appearance of arbitrary strong “kicks” with relatively high probability when compared, for example, with the Gaussian Ornstein–Uhlenbeck (OU) noise and, of course, with the dichotomic non Gaussian process.

We will consider the dynamics of  $\eta(t)$  as described by the following Langevin equation [10]

$$\frac{d\eta}{dt} = -\frac{1}{\tau} \frac{d}{d\eta} V_q(\eta) + \frac{1}{\tau} \zeta(t), \quad (2)$$

with  $\langle \zeta(t) \rangle = 0$  and  $\langle \zeta(t)\zeta(t') \rangle = 2D\delta(t-t')$ , and

$$V_q(\eta) = \frac{D}{\tau(q-1)} \ln \left[ 1 + \frac{\tau}{D} (q-1) \frac{\eta^2}{2} \right].$$

Previous studies of such processes in connection with stochastic resonance problems [10, 11] and dynamical trapping [13], have shown that the non Gaussian behavior of the noise leads to remarkable effects. For  $q = 1$ , the process  $\eta$  coincides with the OU one (with a correlation time equal to  $\tau$ ), while for  $q \neq 1$  it is a non Gaussian process. As shown in [10], for  $q < 1$  the stationary probability distribution has a bounded support, with a cut-off at  $|\eta| = \omega \equiv [(1-q)\tau/(2D)]^{-\frac{1}{2}}$ , with a form given by

$$P_q(\eta) = \frac{1}{Z_q} \left[ 1 - \left( \frac{\eta}{\omega} \right)^2 \right]^{\frac{1}{1-q}}, \quad (3)$$

for  $|\eta| < \omega$  and zero for  $|\eta| > \omega$  ( $Z_q$  is a normalization constant). Within the range  $1 < q < 3$ , the probability distribution is given by

$$P_q(\eta) = \frac{1}{Z_q} \left[ 1 + \frac{\tau(q-1)\eta^2}{2D} \right]^{\frac{1}{1-q}} \quad (4)$$

for  $-\infty < \eta < \infty$ , and decays as a power law (slower than a Gaussian distribution). Finally, for  $q > 3$ , this distribution can not be normalized.

Hence, we see that keeping  $D$  constant, the width or dispersion of the distribution increases with  $q$ . This means that, the higher the  $q$ , the stronger the “kicks” that the particle will receive. Figure 1 depicts the typical form of this distribution for  $q$  smaller, equal and larger than 1.

In [10] it was also shown that the second moment of the distribution (which can be interpreted as the “intensity” of the non Gaussian noise) is given by

$$D_{ng} \equiv \langle \eta^2 \rangle = \frac{2D}{\tau(5-3q)}, \quad (5)$$

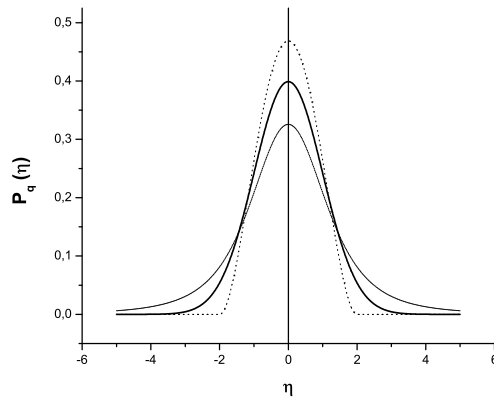


FIG. 1: PDF vs  $\eta$  for different values of  $q$ , and for  $D/\tau = 1$ . From top to bottom, the dotted line is for  $q = 0.5$ , the thick continuous line is for  $q = 1$  (Gaussian case), and the thin continuous line is for  $q = 1.5$ .

and diverges for  $q \geq 5/3$ . For the correlation time  $\tau_{ng}$  of the process  $\eta(t)$ , we were not able to find an analytical expression. However, it is known [10] that for  $q \rightarrow 5/3$  it also diverges as  $\sim (5 - 3q)^{-1}$ . In our analysis, we will only consider values of  $q < 5/3$ , in order to keep finite values of  $D_{ng}$  and  $\tau_{ng}$ .

To conclude this Section, we briefly sketch some results about the correlation time  $\tau_{ng}$ . As commented above, both the second moment of  $P_q(\eta)$  and the correlation time of the process  $\eta$ , diverge for  $q \rightarrow \frac{5}{3}$ . For the second moment,  $D_{ng}$ , we have the exact result shown in Eq. (5), while for the correlation time,  $\tau_{ng}$ , we have no analytical expression. However, in [10] we have observed, numerically, the behavior of the correlation function

$$C(t) = \frac{\langle \eta(t+t')\eta(t') \rangle}{\langle \eta(t')\eta(t') \rangle}, \quad (6)$$

in the stationary regime  $t' \rightarrow \infty$ . We have found that the exponential decay of the correlations in the OU process is still valid for  $q < 1$  where we can write  $C_q(t) \simeq \exp(-t/\tau_{ng})$ . This exponential behavior fails for  $q > 1$  where, on the other side,  $C_q(t)$  can be approximated by a “q-exponential” [16] as

$$C_q(t) \simeq \left[ 1 + (q-1) \frac{t}{\tau_{ng}} \right]^{\frac{1}{1-q}}. \quad (7)$$

The characteristic correlation time  $\tau_{ng}$  defined as

$$\tau_{ng} = \int_0^\infty dt C_q(t), \quad (8)$$

was shown to diverge for  $q \rightarrow 5/3$  as  $\tau_{ng} \approx 2/(5 - 3q)$ .

In Fig. 2 we show, after simulation results for the correlation function  $C_q$ , the dependence of  $\tau_{ng}$  on  $q$ , as well as the comparison with different approximations as follows

- $\tau_{ng} \simeq \frac{2\tau}{(5-3q)}$ ,
- $\tau_{ng} \simeq \frac{2\tau(2-q)}{(5-3q)}$ ,
- $\tau_{ng} \simeq \frac{[1+4(q-1)^2]\tau}{(5-3q)}$ ,

where the third, ad hoc, approximation corresponds to the best fit to the numerical data within the range  $0.8 < q < 5/3$ . This fitting result will be the one we will exploit after discussing the most direct approach.

### III. CASE I: CONSTANT $D$ AND $\tau$ .

As our first approach we will analyze the results for the current and efficiency as function of  $q$  for constant values of  $D$  and  $\tau$ . These are the parameters that could be adequately controlled, for example, in a designed technological

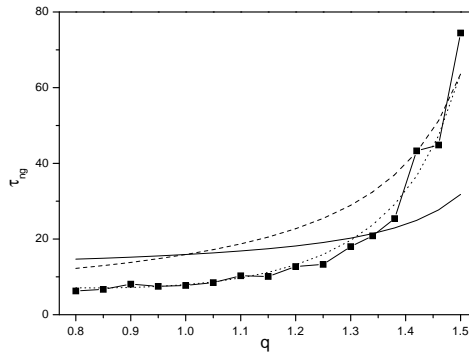


FIG. 2: Comparison of simulation and approximate results for  $\tau_{ng}$  vs  $q$ . Here  $D = 1$  and  $\tau = 15.9$ . Here  $\blacksquare$  correspond to the numerical data (the line joining the points is only to guide the eye), the broken line is for the first approximation, the continuous line for the second, and the dotted line for the third fit.

device. The analysis of the current and efficiency behavior for constant  $D_{ng}$  and  $\tau_{ng}$ , that will be discussed in the next Section, could be considered an even more relevant item –as  $D_{ng}$  and  $\tau_{ng}$  are parameters of the non Gaussian noise when we think such noise as the “primary source” acting on the Brownian particle.

In [4], essentially the same system described by Eq. (1) was analyzed for the case in which the time correlated forcing belongs to a general family of stochastic processes (non Gaussian in general) called “kangaroo processes”. Also in that work, and for the same kind of processes, a parameter called the “flatness” of the noise was defined as the ratio of the fourth moment to the square of the second moment of the stationary distribution. The dependence of the current on the flatness for the limit of small correlation time was discussed. It is interesting to note that, as defined in [4], the flatness of the stationary distribution of the process indicated by Eq. (2) is infinite for  $q > 7/5 = 1.4$ , since the fourth moment of the distribution diverges. As we will see, this corresponds to the parameter region where we observe a decay in the efficiency of the Brownian motor. However, it has to be noted that the process here considered is not a “kangaroo process”.

### A. Overdamped System

Firstly, we analyze the overdamped regime setting  $m = 0$  and  $\gamma = 1$ . For the ratchet potential in this case we will consider the same form as in [3] (with period  $L = 2\pi$ )

$$V(x) = V_1(x) = - \int^x dx' \left( \frac{\exp[\alpha \cos(x')]}{J_0(i\alpha)} - 1 \right), \quad (9)$$

where  $J_0(i\alpha)$  is the Bessel function, and  $\alpha = 16$ . The form of  $V_1(x)$  is shown in Fig. 3.a. The integrand in Eq.(9) is the ratchet force ( $-V'$ ) appearing in Eq.(1).

We are interested on analyzing the dependence of the mean current  $J = \langle \frac{dx}{dt} \rangle / L$  and the efficiency  $\varepsilon$  on the different parameters, in particular, their dependence on the parameter  $q$ . The efficiency is defined as the ratio of the work (per unit time) done by the particle “against” the load force  $F$

$$\lim_{T_f \rightarrow \infty} \frac{1}{T_f} \int_{x=x(0)}^{x=x(T_f)} F dx(t),$$

to the mean power injected to the system through the external forcing  $\eta$

$$\lim_{T_f \rightarrow \infty} \frac{1}{T_f} \int_{x=x(0)}^{x=x(T_f)} \eta(t) dx(t).$$

For the numerator we get  $F \langle \frac{dx}{dt} \rangle = FJL$ , while for the denominator we obtain

$$\lim_{T_f \rightarrow \infty} \frac{1}{T_f} \int_0^{T_f} \eta(t) \frac{dx}{dt} dt = \frac{1}{\gamma} (\langle -V'\eta \rangle + \langle \eta^2 \rangle).$$

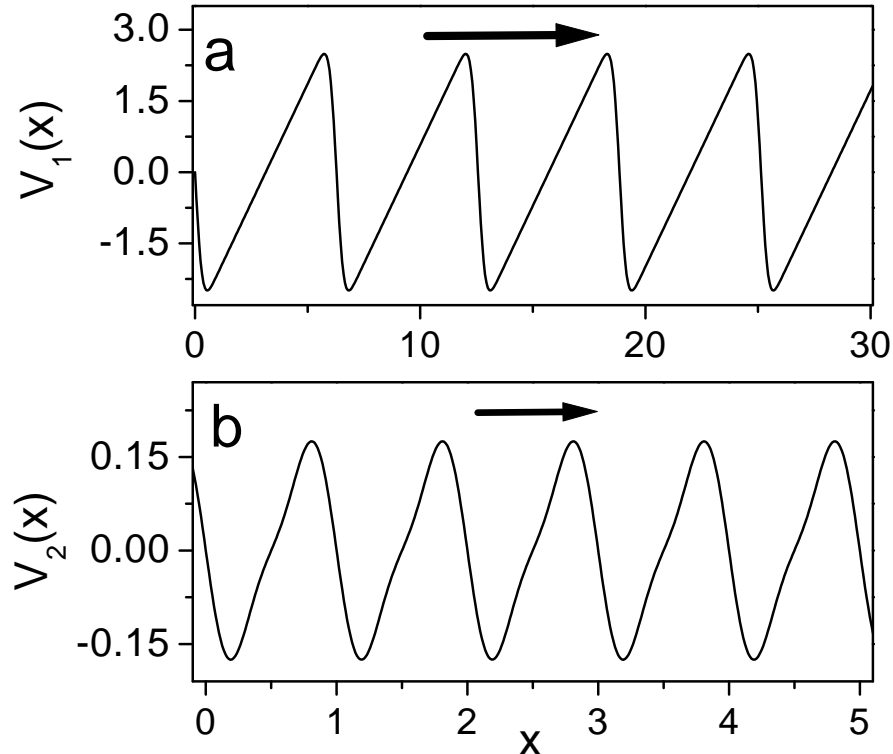


FIG. 3: The ratchet potentials considered in this work:  $V_1(x)$  (a) and  $V_2(x)$  (b). The arrows indicate the direction of the current in normal situations (that is, when there is no current “inversion”).

Simulations show that the time average of  $V'(x(t))\eta(t)$  is negligible in the latter equality (it is always several orders of magnitude lower than  $\langle\eta^2\rangle$ ) and we may approximate the denominator as  $\langle\eta^2\rangle/\gamma = 2D[\gamma\tau(5 - 3q)]^{-1}$ . Interesting and complete discussions on the thermodynamics and energetics of ratchet systems can be found in [20].

In the overdamped regime we are able to give an approximate analytical solution for the problem, which is expected to be valid in the large correlation time regime ( $\frac{\tau}{D} \gg 1$ ): we perform the adiabatic approximation of solving the Fokker-Planck equation associated to Eq. (1) assuming a constant value of  $\eta$  [21], analogous to the one used in [22]. This leads us to obtain an  $\eta$ -dependent value of the current  $J(\eta)$  that is then averaged over  $\eta$  using the distribution  $P_q(\eta)$  [10] with the desired value of  $q$

$$J = \int d\eta J(\eta) P_q(\eta).$$

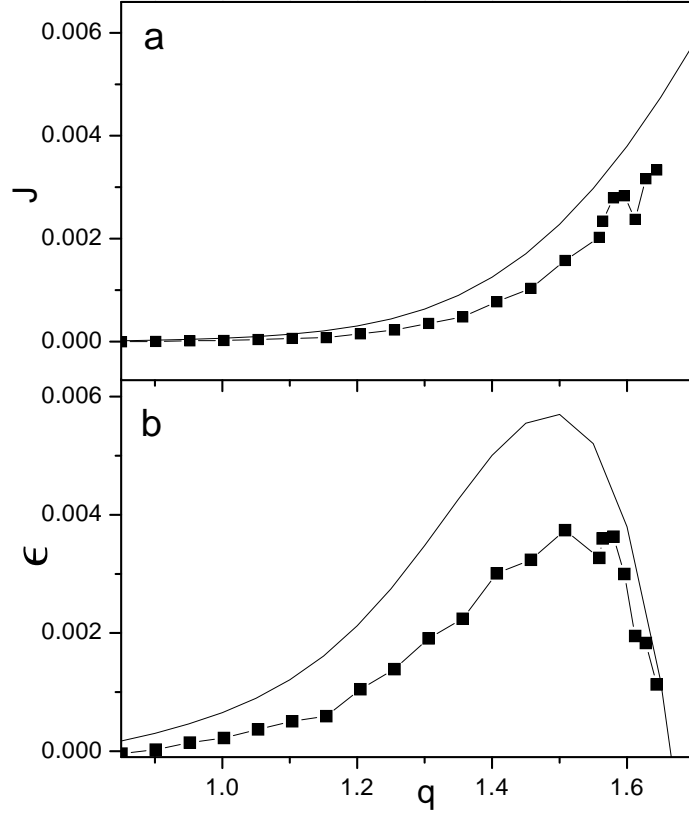


FIG. 4: Current (a) and efficiency (b) as functions of  $q$ . The solid line corresponds to the adiabatic approximation, the line with squares shows results from simulations. All calculations are for  $m = 0, \gamma = 1, kT = 0.5, F = 0.1, D = 1$  and  $\tau = 100/(2\pi)$ .

In Fig. 4, we show typical analytical results for the current and the efficiency as functions of  $q$  together with results coming from numerical simulations (for the complete system given by Eqs.(1) and (2)). Calculations have been done in a parameter region similar to that studied in [3] but considering (apart from the difference provided by the non Gaussian noise) a non-zero load force that leads to a non-vanishing efficiency. As can be seen, although there is not a quantitative agreement between theory and simulations, the adiabatic approximation predicts qualitatively very well the behavior of  $J$  and  $\epsilon$  as  $q$  is varied. As shown in the figure, the current grows monotonously with  $q$  (at least for  $q < 5/3$ ) while there is an optimal value of  $q$  ( $> 1$ ) which gives the maximum efficiency. This fact could be interpreted as follows: when  $q$  is increased, the width of the  $P_q(\eta)$  distribution grows and high values of the non Gaussian noise become more frequent, leading to an improvement of the current. Although the mean value of  $J$  increases monotonously with  $q$ , the efficiency decays when  $q$  approaches  $5/3$  since the denominator in the definition of  $\epsilon$ , which is essentially the dispersion of the noise distribution, diverges.

In Fig. 5 we show results from simulations for  $J$  and  $\epsilon$  as functions of  $q$  for different values of  $D$ , the intensity of the white noise in Eq. (2). The results correspond to  $T = 0$ , hence, the only noise present in the system is the non Gaussian one. On the curve corresponding to the results for  $J$  (Fig. 5.a), we indicate with error bars the dispersion of the results.

It is possible to observe that a huge growth on the fluctuations of  $J$  occurs for the same values of  $q$  for which the efficiency decays. This result induces to think of a connection between the enhancement of the fluctuations on  $J$  and the growth of the width of  $P_q(\eta)$  occurring for  $q \rightarrow 5/3$  (which, as stated above, is the origin of the efficiency

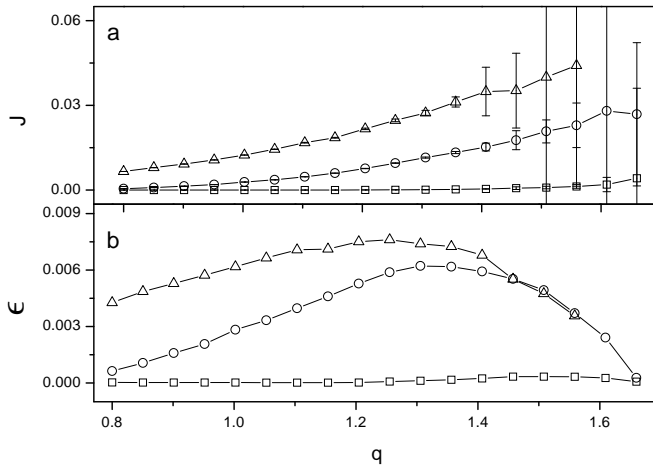


FIG. 5: Current (a) and efficiency (b) as functions of  $q$ . Results from simulations at  $T = 0$  for  $D = 1$  (squares),  $D = 10$  (circles), and  $D = 20$  (triangles). All calculations are for  $m = 0$ ,  $\gamma = 1$ ,  $F = 0.1$  and  $\tau = 100/(2\pi)$ .

decay). A simple, however useful, interpretation of this connection can be given: for  $q \rightarrow 5/3$ , in spite of having a large (positive) mean value of the current, for a given realization of the process, the transport of the particle towards the desired direction is far from being assured (due to the fluctuations on  $J$ ). Hence, we could say that the transport mechanism ceases to be efficient.

It is worth recalling here that  $q = 1$  corresponds to the Gaussian OU noise [9]. Hence, these results for constant  $D$  and  $\tau$  seem to show that the transport mechanism becomes more efficient when the stochastic forcing has a non Gaussian distribution with  $q > 1$ . Let us now discuss the case with inertia and after that we will return to this aspect through an alternative approach.

## B. Inertial System

Now we turn to study the  $m \neq 0$  case, that is, the situations in which the inertia effects are relevant. Hence, we consider the complete form of Eq. (1). In Fig. 6 we show the dependence of the current  $J$  on the mass  $m$  for different values of  $q$  at constant  $D$  and  $\tau$ . The results are from simulations for zero temperature and without load force. It can be seen that, as  $m$  is increased from 0, the inertial effects initially contribute to increase the current, until an optimal value of  $m$  is reached. As expected, for high values of  $m$ , the motion of the particle becomes difficult and, for  $m \rightarrow \infty$ , the current vanishes.

An interesting effect appears for  $q = 1.3$ . For a well defined interval of the value of the mass (ranging approximately from  $m = 100$  to  $m = 5000$ ), a negative current is observed. This can be explained as a consequence of the high value of the mass, that makes inertial effects much more important than those of the ratchet potential. The same velocity's change of sign for high inertia has been observed in [24] for an OU noise ( $q = 1$ ) for different values of the parameters. However, in the region of parameters here considered, the effect occurs only if we consider a non Gaussian noise with a value of  $q > 1$ . In [24], the authors give an explanation of this phenomenon in terms of the so called “running states” (i.e. transitions between wells that are separated by several periods) which is also valid here as was observed in results from simulations for single realizations of the process.

These results suggest the possibility that the mass separation capability of a ratchet system may be in general enhanced by the inclusion of non Gaussian noises, since we have found mass separation in a region of parameters where it is not observed for OU noise. But the separation of masses found here, occurs for particles with a ratio of masses of the order of 10 or more (say 800/80 in Fig. 6), while it has been shown in [23, 24] that, with a load force and considering simply OU noise, particles of much more similar masses can be separated by ratchets.

In order to compare results, we analyze the same system studied in [23, 24] but considering the non Gaussian forcing described by Eq.(2). Hence, we study the system in Eq.(1) with

$$V(x) = V_2(x) = -\frac{2}{\pi}[\sin(2\pi x) + 0.25 \sin(4\pi x)]$$

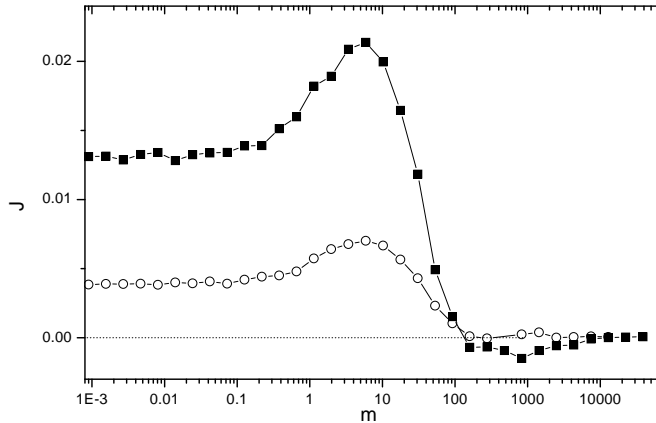


FIG. 6: Current as function of the mass for  $D = 1$  and  $\tau = 100/2\pi$  and for different values of  $q$ : circles  $q = 1$  (Gaussian noise case), squares  $q = 1.3$ .

as the ratchet potential, which is shown in Fig. 3.b. We focus on the region of parameters where, in [23] (for  $q = 1$ ), separation of masses was found. We fix  $\gamma = 2, T = 0.1, \tau = 0.75$ , and  $D = 0.1875$  and assume the values of the masses  $m = m_1 = 0.5$  and  $m = m_2 = 1.5$  as in [23]. Our main result is that the separation of masses is enhanced when a non-Gaussian noise with  $q > 1$  is considered. In Fig. 7.a we show  $J$  as function of  $q$  for  $m_1 = 0.5$  and  $m_2 = 1.5$ . It can be seen that there is an optimum value of  $q$  that maximizes the difference of currents. This value, which is close to  $q = 1.25$ , is indicated with a vertical double arrow. Another double arrow indicates the separation of masses occurring for  $q = 1$  (Gaussian OU forcing). We have observed that when the value of the load force is varied, the difference between the curves remains approximately constant but both are shifted together to positive or negative values (depending on the sign of the variation of the loading). By controlling this parameter it is possible to achieve, for example, the situation shown in Fig. 7.b, where, for the value of  $q$  at which the difference of currents is maximal, the heavy “species” remains static on average (has  $J = 0$ ), while the light one has  $J > 0$ . Also it is possible to get the situation shown in Fig. 7.c, at which, for the optimal  $q$ , the two species move in opposite directions with equal average velocity.

#### IV. CASE II: CONSTANT $D_{ng}$ AND $\tau_{ng}$ .

We consider here the second point of view. It corresponds to studying the behavior of current and efficiency as function of  $q$  when  $D_{ng}$  and  $\tau_{ng}$  are kept constant (instead of  $D$  and  $\tau$  as was done in the previous section). This approach is more consistent with the image of  $\eta$  as a primary (natural) source of noise, and it isolates the effects of the non-Gaussian character of the noise distributions by keeping the dispersion and correlation time at fixed values.

##### A. Overdamped Case

We consider Eq. (1) with  $m = 0$  and  $V(x)$  as in Fig. 3.a (see Section III.A, overdamped regime). In the simulations, for each value of  $q$ , we have adapted the values of  $D$  and  $\tau$  in order to obtain the desired values of  $D_{ng}$  and  $\tau_{ng}$ . This was done by inverting Eq. (5) together with the equation for the best fit for  $\tau_{ng}$  (the third one, presented at the end of Section II).

As  $q$  is varied for constant  $D_{ng}$ , the efficiency is essentially proportional to the current. Hence, we only present results for  $J$ . In Fig. 8 we show the results for the current as function of  $q$  for  $\tau_{ng} = 2\pi/100$  and different values of  $D_{ng}$ . An interesting result is found for low values of  $D_{ng}$ . For  $D_{ng} < 0.5$  it is observed that the current grows monotonously with  $q$  through most of the range studied (it decays for  $q$  very close to  $5/3$ ). This means that, for  $q > 1$  we found an enhancement of  $J$  with respect to the Gaussian noise situation ( $q = 1$ ). This enhancement has to be attributed essentially to the non-Gaussian character of the noise  $\eta(t)$ , as for every  $q$  on each curve we have considered the same values of dispersion and correlation time.



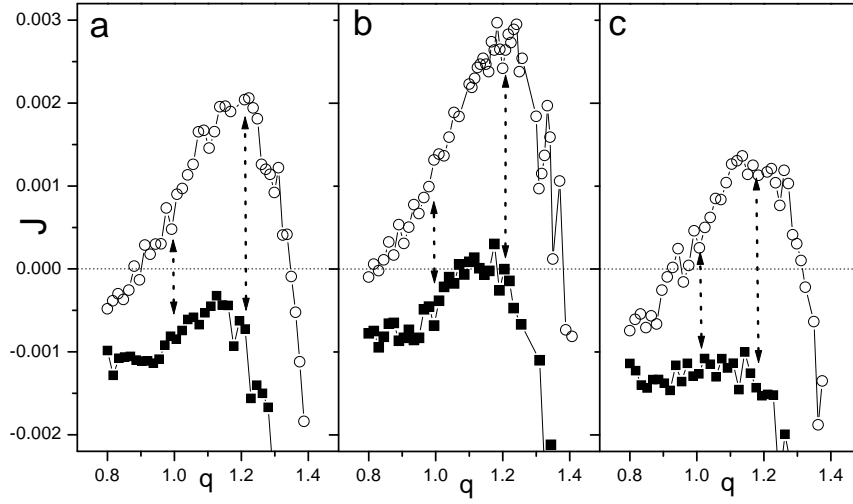


FIG. 7: Separation of masses: results from simulations for the current as a function of  $q$  for particles of masses  $m = 0.5$  (hollow circles) and  $m = 1.5$  (solid squares). Calculations for three different values of the load force:  $F = 0.025$  (a),  $F = 0.02$  (b) and  $F = 0.03$  (c).

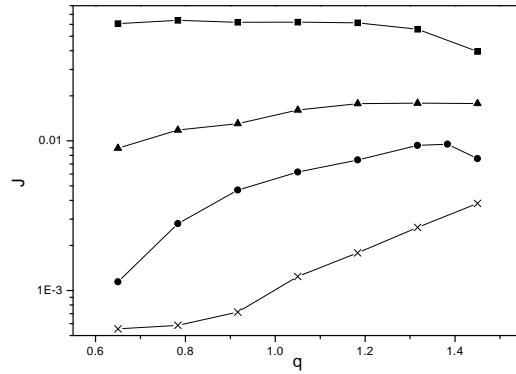


FIG. 8: Current as a function of  $q$  for fixed  $\tau_{ng} = 100/(2\pi)$  and different fixed values of  $D_{ng}$ . From top to bottom, the curves are for  $D_{ng} = 1, D_{ng} = 0.5, D_{ng} = 0.35$  and  $D_{ng} = 0.2$ . All calculations are for  $m = 0, \gamma = 1, T = 0.1$  and  $F = 0.1$ .

For higher values of  $D_{ng}$  the effect disappears: for ( $D_{ng} \sim 1$ ) the dependence of  $J$  on  $q$  becomes flat in most of the range analyzed. This means that the non-Gaussian character does not play a relevant role in this region of parameters, and the current and efficiency are essentially determined by the intensity and correlation time of the noise source  $\eta$ , independently of the detailed statistical characteristics of the process  $\eta(t)$ . For even higher values of  $D_{ng} (\sim 2)$  the optimum value of  $q$  that maximizes the current tends toward values of  $q < 1$  (results not shown). However in the range of  $q$  analyzed the corresponding  $J(q)$  curves are essentially flat and the differences with the  $q = 1$  case are not distinguishable at all. Hence, we can remark the enhancement effect occurring for  $q > 1$  at low values of  $D_{ng}$ .

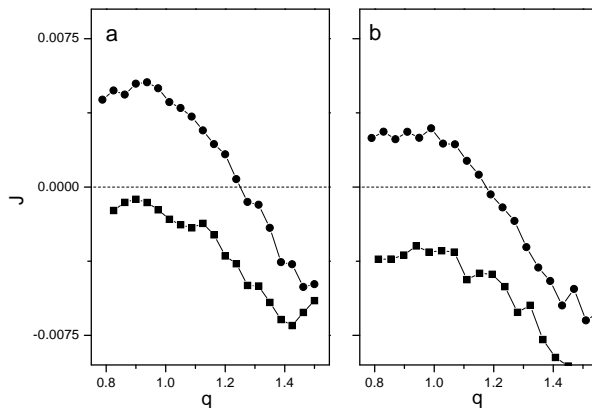


FIG. 9: Current as a function of  $q$  for fixed  $\tau_{ng} = 0.75$  and  $D_{ng} = 0.1875$ . Figure (a) is for  $F = 0.015$  and (b) is for  $F = 0.025$ . In both cases circles correspond to  $m = 0.5$  while squares correspond to  $m = 1.5$ .

### B. Inertial systems

We consider now the case  $m \neq 0$ , and study the current as a function of  $q$  for fixed values of  $D_{ng}$  and  $\tau_{ng}$ . We fix  $\gamma = 1$  and consider again the potential  $V(x) = V_2(x)$  defined in Section III.B.

In Fig. 9 we show the results for  $J(q)$  at fixed  $D_{ng} (= 0.1875)$  and  $\tau_{ng} (= 0.75)$ , for different values of the masses and the external force  $F$ . It can be seen that, when considering  $q \neq 1$ , no remarkable enhancement of the mass separation capabilities of the system is found. Moreover, for  $q > 1$  the mass separation effect decreases. However, an interesting fact is that we find an inversion of current when considering large enough values of  $q$  (depending on the mass and  $F$ ). This inversion of current is due essentially to the variation of the non-Gaussian properties of the noise distribution, as  $D_{ng}$  and  $\tau_{ng}$  are kept fixed.

## V. CONCLUSIONS

We have systematically studied the effect of a colored non Gaussian noise source on the transport properties of a Brownian motor using two alternative points of view. In the first one, we analyze the results for the current, efficiency and mass separation as functions of  $q$ , for constant values of  $D$  and  $\tau$ , which are the parameters that could be adequately controlled, for example, in a designed technological device. The second point of view corresponds to studying those behaviors as functions of  $q$  when  $D_{ng}$  and  $\tau_{ng}$  are kept constant. This is more consistent with the image of  $\eta$  as a primary (natural) source of noise, and it isolates the effects of the non-Gaussian character of the noise distributions by keeping the dispersion and correlation time at fixed values.

Considering the first, direct, point of view, what we have found is that a departure from Gaussian behavior, in particular given by a value of  $q > 1$ , induces a remarkable increase of the current together with an enhancement of the motor efficiency. The latter shows, in addition, an optimum value for a given degree of non Gaussianity. When inertia is taken into account we have also found a considerable increment in the mass separation capability.

The second point of view is analogous to the one used in Ref. [11] to study stochastic resonance in an activator-inhibitor system, where it was shown that the signal-to-noise ratio shows an enhancement as a function of  $q$ . Here, by keeping the distribution's width  $D_{ng}$  and the correlation time  $\tau_{ng}$  constant, we have compared the results for "equivalent" Gaussian and non Gaussian noises. We have observed that non Gaussian noises with  $q > 1$  produce an enhancement of the current when compared to the Gaussian case. This effect is observed for relatively low values of the noise intensity  $D_{ng}$  and lead us to interpret that, at low values of  $D_{ng}$ , the increment of the probability of having arbitrary high values of the noise that occurs for  $q > 1$  (with respect to the Gaussian case) plays a significative role in the determination of the current. In contrast, for higher values of  $D_{ng}$ , the fluctuations dominate the dynamics in such a way that the Gaussian or non Gaussian character of the noise produces no relevant differences.

When studying inertial systems at constant  $D_{ng}$  and  $\tau_{ng}$  we have not observed relevant effects on the mass separation capability of the system, in the region of parameters considered: the effect of mass separation seems to be governed essentially by the noise intensity and the correlation time.

Another remarkable fact is the occurrence of an inversion of current as a consequence of varying the parameter  $q$  alone (keeping  $D_{ng}$  and  $\tau_{ng}$  fixed). This clearly shows the relevance that the details of the noise distribution may have in the determination of the transport properties in ratchet systems. Or, equivalently, how sensitive the ratchet systems could be to the detailed properties of the noises.

We think that these studies could be of interest for their possible relation to biologically motivated problems [1, 18, 19] as well as for the potential technological applications, for instance in “nanomechanics” [8, 9]. More specific studies on these areas will be the subject of further work.

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