Design Issues for the VLSI Implementation of Universal Approximator Fuzzy Systems

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Abstract: - Several VLSI realizations of fuzzy systems have been proposed in the literature in the recent years. They employ analog or digital circuitry, offering more or less programmability, implementing difference inference methods, with different types of membership functions as well as different antecedents' connectives. This paper centers this wide design space by fixing several parameters that allow efficient VLSI implementations of programmable fuzzy systems featuring first, second and third order accurate approximation. Hardware requirements are discussed and compared from the point of view of approximation capability or precision, thus attempting to a formalization that has never been applied before to the field of fuzzy hardware.

Key-Words: - Fuzzy systems, universal approximators, B-splines, VLSI design. *CSCC'99* Proc.pp..6471-6476

1. Introduction

A relevant feature of fuzzy systems is that they are universal approximators and, hence, potentially suitable for any application. The problem is how to design them [1]. On one side, the numbers of inputs, outputs, and corresponding linguistic labels covering them have to be selected. On the other side, the types of membership functions, antecedents' connective and fuzzy implication as well as the methods of rule aggregation and defuzzification have to be chosen.

VLSI implementations of fuzzy systems offer the advantages of high inference speed with low area and power consumption, but they lack of flexibility, that is, the previously commented design variables are fixed in a fuzzy chip (for instance the maximum number of inputs and outputs). Since flexibility is increased by programmability of several parameters, it is important to select efficiently these parameters to achieve a good trade-off between hardware simplicity and approximation capability.

Fuzzy implication and the methods of rule aggregation and defuzzification are fixed when selecting the Singleton (or zero-order Takagi-Sugeno's) Method of inference, which is the most suitable for hardware implementation. On the other side, the number of labels per input and output depends on the architecture. Architectures based on a grid partition of the input spaces are advantageous for several reasons. Considering knowledge representation, a grid partition has semantic meaning while considering approximation theory, the problem is simplified to local piecewise interpolation. Besides, from a hardware point of view, circuitry is considerably reduced since a rule-active driven architecture can be employed. Having selected singleton fuzzy systems with a grid architecture, we will resort to approximation theory to choose the programmable parameters efficiently.

This paper is organized as follows. Section 2 summarizes three types of singleton fuzzy systems that are first, second, and third order universal approximators as well as suitable for hardware implementation. VLSI realizations of these systems following two approaches are described in Section 3. Finally, Section 4 compares these approaches showing their range of applicability and giving general conclusions.

2. Fuzzy systems in the context of approximation theory

A singleton fuzzy system with u inputs, $\bar{x}=(x_1, ..., x_u)$ and one output (y) establishes in general a nonlinear relation between the input $(I_1, ..., I_u)$ and output (R) universes of discourse. Formally, this means that:

$$y = y(\bar{x}): I_1 \times I_2 \times \ldots \times I_u \subset R^u \rightarrow R \tag{1}$$

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Considering, as we have selected, a fuzzy system with a grid architecture, each input universe of discourse, I_i, is partitioned into L_i linguistic labels with a maximum overlapping degree of α. Hence, $L_i+1-\alpha$ intervals can be distinguished, as can be seen in Fig. 1. They are separated by the points $\{a_{i1}, a_{i2}, ..., a_{iNi}\}$, where N_i is $L_i-\alpha$. Given an input vector $\bar{x}_o$, the fuzzy system identifies the *u* intervals, that is, the particular grid cell, GC_p, to which the input belongs. As a second step, the system provides the corresponding output $y(\bar{x}_o)$ by evaluating the α^u active rules:

$$y(\bar{x}_o) = \left. \frac{\sum_{k=1}^{\alpha^u} h_{pk}(\bar{x}) \cdot c_{pk}}{\sum_{k=1}^{\alpha^u} h_{pk}(\bar{x})} \right|_{\bar{x}=\bar{x}_o} = y_p(\bar{x})\Big|_{\bar{x}=\bar{x}_o} \qquad (2)$$

where h_{pk} is the activation degree of one of the α^u active rules and c_{pk} is its corresponding singleton value.

The parameters that define this function $y_p(\bar{x})$ are a few of the global parameters that define the fuzzy system, $y(\bar{x})$. In this sense, a fuzzy system is viewed as a local piecewise interpolator that provides the piece y_p for each GC_p grid cell. The interpolation provided can be piecewise constant, piecewise linear, piecewise quadratic, and so on, depending on whether y_p is constant, linear or quadratic in $\bar{x}$.

In the context of mathematical approximation theory, popular piecewise interpolation methods usually employed to approximate a given function, $f(\bar{x})$, from which only several values, $f(\bar{x}_k)$, are known are Lagrange and Spline interpolation [2]. In particular, Lagrange interpolation of degree 1 is a B-spline interpolation of degree 1. Singleton fuzzy systems have been studied as B-spline interpolators [3]-[4]. This is summarized in the following.

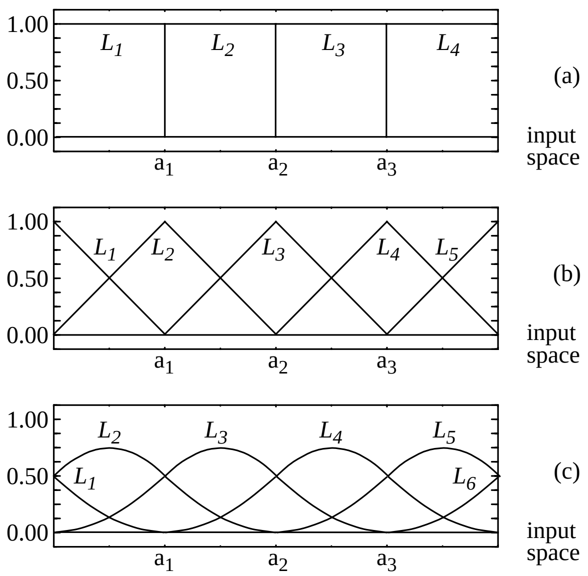

Fig. 1: Different coverings of the input space with overlapping degrees of (a) 1, (b) 2, and (c) 3.

2.1. Normalized B-splines as membership functions

Given an universe of discourse I_i partitioned by the points $\{a_1, a_2, ..., a_{Ni}\}$, and a positive entire k, a normalized B-spline of degree k on I_i is defined as:

$$N_p^{\,k}(x_i) = (a_{p+k+1} - a_p) \cdot$$

$$\cdot \sum_{m=p}^{p+k+1} \left[\prod_{n=p(n \neq m)}^{p+k+1} \frac{1}{a_n - a_m} \right] \cdot (x_i \Theta a_m)^k \qquad (3)$$

where p = -k+1, -k+2, ..., N_i -1; and $\{a_{-k+1}, a_{-k+2}, ..., a_0, a_{Ni+1}, ..., a_{Ni+k}\}$ are additional arbitrarily defined points. Operator Θ is a bounded difference operator, so that $(x_i \Theta a_m)$ is $x_i - a_m$ if $x_i > a_m$ and zero otherwise.

These B-spline functions can be employed as membership functions because they are local, positive, continuous, and monotonous [3]-[4]. They are poly-nomials of degree k within the interval $[a_p, a_{p+k+1}]$ and zero outside. Since we are interested in hardware implementation, we will focus on k=0, 1, and 2. Coverings of universes of discourse with this type of membership functions are shown, respectively, in Fig. 1a, b and c. The overlapping degree is $\alpha = k+1$.

Given an input x_i and once its interval $[a_p, a_{p+1}]$ is identified, the α membership degrees μ_j^{xi} (j=1, ..., α) inside the interval are the following:

(a) If k=0: $$\mu_1^{x_i} = 1 \qquad (4)$$

(b) If k=1: $$\mu_1^{x_i} = \frac{x_i - a_p}{a_{p+1} - a_p} \quad \text{and}$$

$$\mu_2^{x_i} = 1 - \mu_1^{x_i} \qquad (5)$$

(c)If k=2: $$\mu_1^{x_i} = \frac{(x_i - a_p)^2}{(a_{p+1} - a_p) \cdot (a_{p+2} - a_p)} ,$$

$$\mu_2^{x_i} = \frac{(x_i - a_{p+1})^2}{(a_{p+1} - a_p) \cdot (a_{p+1} - a_{p-1})} \quad \text{and}$$

$$\mu_3^{x_i} = 1 - \mu_1^{x_i} - \mu_2^{x_i} \quad (6)$$

Approximation capabilities of singleton fuzzy systems that employ these membership functions are summarized in the following.

2.2. Fuzzy systems that are first order accurate approximators

In the context of mathematical approximation theory, given a function $f(\bar{x})$: $I \subset R^u \rightarrow R$, the fuzzy system $y(\bar{x})$ is said to be a k-th order accurate approximator for $f(\bar{x})$ if:

$$\| f(\bar{x}) - y(\bar{x}) \|_\infty = sup_{\bar{x} \in I} | f(\bar{x}) - y(\bar{x}) | < M_f \cdot h^k \quad (7)$$

where M_f is a constant that depends on the function f and $h = \max_i \{ \max_{ji} (a_{i,ji+1} - a_{i,ji}) \}$, with i = 1, ..., *u*; and j_i= 1, ..., N_i [4].

Fuzzy systems whose membership functions are rectangles or B-splines of degree 0 (Fig. 1a) are piecewise constant interpolators, that is, the output is constant for each grid cell:

$$y_p(\bar{x}) = \frac{\sum_{k=1}^{1} h_{pk}(\bar{x}) c_{pk}}{\sum_{k=1}^{1} h_{pk}(\bar{x})} = \sum_{k=1}^{1} \bar{h}_{pk}(\bar{x}) c_{pk} = c_p \quad (8)$$

Let us evaluate the error that this kind of fuzzy systems provides when approximating a differentiable function $f(\bar{x})$. Within each grid, GC_p, $f(\bar{x})$ can be expressed by its Taylor expansion around the point $\bar{x}_o$ where $f(\bar{x}_o) = c_p$, so that the error in that grid cell is given by:

$$\| f(\bar{x}) - y(\bar{x}) \|_\infty = \left\| \sum_{i=1}^{u} \left. \frac{\partial f}{\partial x_i} \right|_{\xi} \cdot (x_i - x_{io}) \right\|_\infty \le$$

$$\sum_{i=1}^{u} \left\| \frac{\partial f}{\partial x_i} \right\|_\infty \cdot h \le M_f \cdot h \quad (9)$$

where $|\xi_i - x_{io}| \le |x_i - x_{io}| \qquad \forall i = 1, \ldots, u$

Hence, they are first order accurate approximators. A feature of their output is discontinuity at the boundaries of the grid cells.

Singleton fuzzy systems are generally first order accurate approximators because:

$$\| f(\bar{x}) - y(\bar{x}) \|_\infty = \left\| f(\bar{x}) - \sum_k \bar{h}_{pk} \cdot c_{pk} \right\|_\infty =$$

$$\left\| \sum_k \bar{h}_{pk} \cdot f(\bar{x}) - \sum_k \bar{h}_{pk} \cdot c_{pk} \right\|_\infty \le$$

$$\sum_k \bar{h}_{pk} \cdot max_k \| f(\bar{x}) - c_{pk} \|_\infty \le \sum_{i=1}^{u} \left\| \frac{\partial f}{\partial x_i} \right\|_\infty \cdot h \quad (10)$$

For instance, fuzzy systems whose membership functions are B-splines of degree 1 and which connect them with the minimum operator provide first order accurate approximation [4]. In this case, output is continuous at the boundaries of the grid cells.

2.3. Fuzzy systems that are second order accurate approximators

Singleton fuzzy systems that employ normalized B-splines of degree 1 as membership functions and the product or the general meet operator [5] as connective, $\wedge$, among antecedents are piecewise multilinear interpolators. Considering, for simplicity, the case of two dimensions, the output is given by:

$$y_p(x_1, x_2) = c_{11} + \mu_1^{x_1}(c_{21} - c_{11}) +$$

$$\mu_1^{x_2}(c_{12} - c_{11}) + \mu_1^{x_1} \wedge \mu_1^{x_2}(c_{22} + c_{11} - c_{21} - c_{12}) \quad (11)$$

where c_{ij} are the four, α^2, singleton values (for instance, $c_{12}=y_p(a_{1,p}, a_{2,p+1})$) and μ_1^{xi} has the expression in Equation (5).

When the connective is the product, equation above can be expressed in function of x_1 and x_2 as follows:

$$y_p(x_1, x_2) = a x_1 x_2 + b x_2 + c x_1 + d \quad (12)$$

where a, b, c, and d are constants.

These systems have been proved to be second order accurate approximators [4]-[5]:

$$\| f(\bar{x}) - y(\bar{x}) \|_\infty \le M \cdot \sum_{i,j} \left\| \frac{\partial^2 f}{\partial x_i \partial x_j} \right\|_\infty \cdot h^2 \quad (13)$$

where M is a constant. If the connective is the product operator, the fuzzy system is equivalent to a piecewise multilinear Lagrange interpolator of degree 1 in x_i [4]. In both cases, the output is continuous but the first derivative is not continuous, in general, at the boundaries of the grid cells.

2.4. Fuzzy systems that are third order accurate approximators

Singleton fuzzy systems that employ normalized

B-splines of degree 2 are piecewise multiquadratic interpolators. Considering, for simplicity, the case of two dimensions, the output is given by:

$$y_p(x_1, x_2) = c_{33} + \mu_1^{x_1}(c_{13} - c_{33}) + \mu_2^{x_1}(c_{23} - c_{33}) + \mu_1^{x_2}(c_{31} - c_{33}) + \mu_2^{x_2}(c_{32} - c_{33}) + \mu_1^{x_1} \cdot \mu_1^{x_2}(c_{11} - c_{13} - c_{31} + c_{33}) + \mu_1^{x_1} \cdot \mu_2^{x_2}(c_{12} - c_{13} - c_{32} + c_{33}) + \mu_2^{x_1} \cdot \mu_1^{x_2}(c_{21} - c_{23} - c_{31} + c_{33}) + \mu_2^{x_1} \cdot \mu_2^{x_2}(c_{22} - c_{23} - c_{32} + c_{33}) \quad (14)$$

where c_{ij} are the nine, α^2, singleton values required and μ_j^{xi} takes the expressions in Equation (6).

The equation above can be expressed in function of x_1 and x_2 as follows:

$$y_p(x_1, x_2) = ax_1^2x_2^2 + bx_1x_2^2 + cx_2^2 + dx_1^2x_2 + ex_1x_2 + fx_2 + gx_1^2 + hx_1 + i \quad (15)$$

where *a*, ..., *i*, are constants.

Singleton fuzzy systems that employ normalized B-splines of degree k as membership functions and the product as connective among antecedents are (k+1)-th order accurate approximators [2], [4]. For the case k=2, the error is bounded as follows:

$$\|f(\bar{x}) - y(\bar{x})\|_\infty \le M \cdot \sum_{i,\,j,\,k} \left\| \frac{\partial^3 f}{\partial x_i \partial x_j \partial x_k} \right\|_\infty \cdot h^3 \quad (16)$$

where M is a constant. For k=2, the output and its first derivative are continuous at the boundaries of the grid cells.

3. Hardware realization of universal approximator fuzzy systems

There are basically two VLSI strategies to implement the universal approximator fuzzy systems described in the previous Section. One of them is to design a fuzzy chip that implements directly the Equations (8), (12) and (15). Programmable parameters of this approach are the points a_{ij} that allow identifying the active grid cell and the constants (for instance a, b, c, and d in Equation (12)) that define the output in that grid cell. These chips will be called memory-based fuzzy chips although they will also need scalers, adders, multipliers and squarers.

The other strategy is to design a fuzzy chip that implements the different stages of the singleton fuzzy inference mechanism, that is, they implement Equations like (12) and (15)). These stages are: calculation of the membership degrees, $\mu_j(x_i)$, (by membership function circuits, MFC's) and the activation degrees of the rules, $h_{pk}(\bar{x})$, (by connective circuits), scaling with the singleton values, c_{kl}, and addition. These fuzzy chips will be called MFC-based fuzzy chips because their main difference with the previous approach is that membership degrees are calculated explicitly. They will follow the massively parallel architecture described in [6]. The programmable parameters are again the points a_{ij} (which separate the L_i+1-α intervals) and the singleton values c_{kl}.

The objective in any case is to obtain general-purpose fuzzy chips that can be adjusted to a particular application by a digitally programming interface and whose inputs and output are analog signals. In particular, we will consider current-mode signals so that addition is reduced to wire connection and digitally programmed scaling is implemented by digitally programmed current mirrors or D/A converters [7]. When implementing second-order accurate approximators general meet operator or multiplier can be chosen as antecedents' connective. As a matter of fact, since the meet operator (in the two-dimensional case) is a piecewise linear approximation of the product operator with an error of ±6.25% (3 bits), we will focus on employing analog multipliers. Hardware is simplified if all the intervals L_i+1-α have the same width, h, specially in the MFC-based approach because the denominators in the expressions of the membership degrees (Equations (5)-(6)) are constant so that divider circuits are not required. We will first consider this type of partition. Taking into account that the output signal is always given with a resolution of q bits, the circuitry required is given in the following.

3.1. Memory-based approach

First order accurate approximators:

Look-up tables correspond to the memory-based approach to implement first-order accurate approximators. The input spaces are partitioned into 2^p intervals of the same width. The output is constant for everyone of the 2^{pu} grid cells. The hardware required is:

- A memory that stores 2^{pu} words of q bits.
- u A/D converters of p bits to address the memory.
- 1 D/A converter of q bits to obtain an analog output.

Second order accurate approximators:

Let us consider the implementation of a singleton fuzzy system whose membership functions are normalized B-splines of degree 1 and whose antecedents are connected by the product operator. The input spaces are partitioned into (L_i-1) intervals where L_i is the number of linguistic labels covering the i-th input space. Let us suppose, for simplicity, that $L=L_i \ \forall i$. Within each grid cell, the output is a u-linear function (like Equation (12)) that is defined by 2^u parameters. Considering that all the (L-1) intervals are of the same width, the hardware required is:

- A memory of $[2(L-1)]^u$ words of q bits that can be partitioned into 2^u memories for parallel processing.
- u A/D converters of $\log_2(L-1)$ bits that address the memory.
- 2^{u-1}-1 analog multipliers.
- 2^u D/A converters of q bits to provide the scaling with the 2^u parameters of each grid cell.

Considering for instance the case of two dimensions, this circuitry is required to obtain the output in Equation (12) by grouping the signals as follows:

$$y_p(x_1, x_2) = x_2(ax_1 + b) + (cx_1 + d) \qquad (17)$$

Third order accurate approximators:

Let us consider the implementation of a singleton fuzzy system whose membership functions are normalized B-splines of degree 2 and whose antecedents are connected by the product operator. The input spaces are partitioned into (L-2) intervals of the same width. Within each grid cell, the output is a u-quadratic function (like Equation (15)) that is defined by 3^u parameters. The hardware required is:

- A memory of $[3(L-2)]^u$ words of q bits that can be partitioned into 3^u memories.
- u A/D converters of $\log_2(L-2)$ bits that address the memory.
- $(2^{u-1}$-1)2= 2^u-2 analogue multipliers.
- 3^u D/A converters of q bits to provide the scaling with the 3^u parameters of each grid cell.
- u analog squaring circuits.

Considering for instance the case of two dimensions, this circuitry is required to obtain the output in Equation (15) by grouping the signals as follows:

$$y_p(x_1, x_2) = x_2^2(ax_1^2 + bx_1 + c) +$$

$$x_2(dx_1^2 + ex_1 + f) + gx_1^2 + hx_1 + i \qquad (18)$$

Several current mirrors are required, like in the former case of second order accurate approximators, to replicate signals such as x_1^2 or x_1.

3.2. MFC-based approach

Given L labels per input space distributed uniformly (defining intervals of the same width), the number of singletons which should be stored are L^u for any approximator. The global memory that stores these parameters should also be partitioned into α^u parts to allow massively parallel processing [6]. This approach requires less parameters to store, because each singleton parameter is employed within α^u grid cells. As a drawback, multiplexing circuitry have to be used to identify which operator blocks (MFC's and scalers) are associated with each singleton [6].

First order accurate approximators:

Let us consider the implementation of a singleton fuzzy system whose membership functions are normalized B-splines of degree 1 and whose antecedents are connected by a minimum operator. The hardware required is:

- A global memory that stores L^u words of q bits partitioned into 2^u memories.
- u A/D converters of $\log_2(L-1)$ bits that address the memory.
- 2^u minimum circuits.
- 2^u D/A converters of q bits to provide the scaling with the singleton values.
- u MFC's that provide the membership degrees, by calculating $\mu_1(x_i)$ (Equation (5)).
- Multiplexors (and current mirrors).

Second order accurate approximators:

The hardware required to implement a singleton fuzzy system with B-spline membership functions of degree 1 connected by the product is very similar to the previous one. Grouping signals conveniently, it is the following:

- A global memory that stores L^u words of q bits partitioned into 2^u memories.
- u A/D converters of $\log_2(L-1)$ bits that address the memory.
- 2^{u-1}-1 analogue multipliers.
- 2^u D/A converters of q bits to provide the scaling with the singleton values.
- u MFC's that provide the membership degrees, by calculating μ_1^{xi}. They are very simple linear circuits since the denominator in Equation (5) is a constant.
- Multiplexors (and current mirrors).

Third order accurate approximators:

The hardware required to implement a singleton fuzzy system with B-spline membership functions of degree 2 connected by the product is the following:

- A global memory that stores L^u words of q bits partitioned into 3^u memories.
- u A/D converters of $\log_2(L-2)$ bits that address the memory.
- $(2^{u-1}-1)2= 2^u-2$ analog multipliers (by grouping the signals conveniently).
- 3^u D/A converters of q bits to provide the scaling with the singleton values.
- 2u MFC's (squaring circuits) that provide the membership degrees, by calculating μ_1^{xi} and μ_2^{xi} in Equation (6) (denominators are again constant) .
- Multiplexors (and current mirrors).

4. Discussion and conclusions

Given a range of applications, that is, a group of functions to approximate, the VLSI designer has to evaluate which implementation approach is more efficient. Considering that derivatives of the functions to approximate can be estimated, the designer can apply Equations (10), (13) and (16) to obtain the value of h for a given precision and for each type of approximator. Once h, and hence L, are known the results of previous Section can be used to choose an efficient realization. General results are the following.

Comparing the memory-based approaches, look-up tables are not usually efficient since they offer a low order accurate approximation with a very high memory consumption. This is illustrated in Fig. 2, where the memory size (in bytes) is shown versus the number of labels, given p=q=7 bits, u=2 (Fig. 2a) and u=3 (Fig. 2b).

The number and complexity of the operators increase in any approach as the order of the approximation is bigger. This is justified if less intervals are required for a given precision in the approximation so that the memory size decreases. MFC-based approach save a lot of memory by slightly increasing the number of operators (see Fig. 2). Hence, it is usually more efficient in terms of area and power although more difficult to design.

It can be concluded from Equations (10), (13), and (16) that unequally spaced points a_{ij} (distributed so as to minimize the values of the derivatives of the function to approximate) may achieve a given precision with less intervals. Implementing this flexibility with a memory-based approach does not complicate very much the circuitry (it requires to store the points a_{ij} in an additional memory and to slightly complicate the circuitry to obtain the digital code of the grid cell). On the contrary, the MFC-based approach also requires more complex MFC's that include divider circuits (which are usually very costly). This complexity is again justified if the number of labels decreases sufficiently or if the design objective is to realize implementations with on-chip tuning [7].

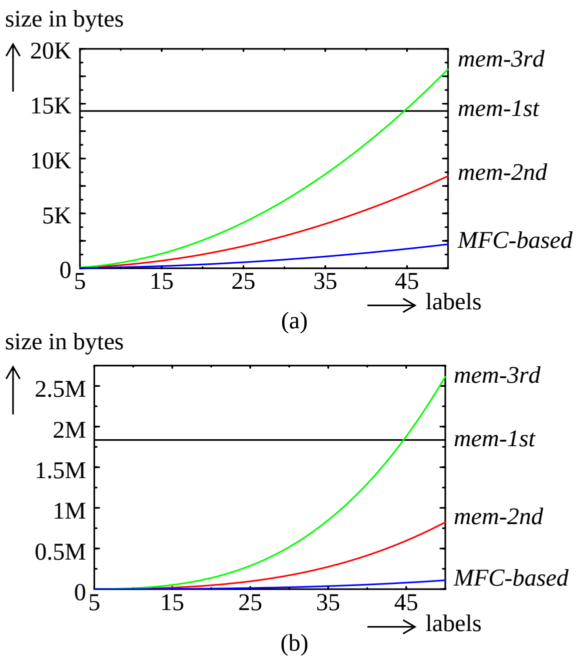

Fig. 2: Memory requirements for memory- and MFC-based approaches for p = q = 7 bits and (a) u = 2, (b) u = 3.

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