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## **A system to mathematically analyze negative income tax and guaranteed minimum income proposals.**

Ralph Richard Bravoco  
*University of Massachusetts Amherst*

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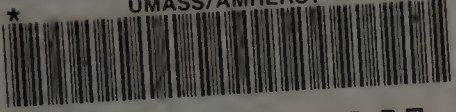
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A SYSTEM TO MATHEMATICALLY ANALYZE  
NEGATIVE INCOME TAX AND GUARANTEED  
MINIMUM INCOME PROPOSALS

A DISSERTATION PRESENTED

By

RALPH RICHARD BRAVOCO

Submitted to the Graduate School of the  
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NEGATIVE INCOME TAX AND GUARANTEED  
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By

RALPH RICHARD BRAVOCO

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The generous support of the Raytheon Company through their Ph.D. fellowship program enabled me to complete the Ph.D. program in two years. Without their assistance, the completion of this study would certainly have been made more difficult. However, though this dissertation was made under the Raytheon Fellowship, the conclusions, opinions, and other statements in this dissertation are solely those of the author.

To my wife, Vivian, I dedicate this dissertation. Without her encouragement and loving support, especially in typing the original draft, this dissertation would never have been completed.

## ABSTRACT

A System to Mathematically Analyze Negative Income Tax  
and Guaranteed Minimum Income Proposals

(August, 1971)

Ralph R. Bravoco, A.B., Northeastern University  
M.A., Northeastern University  
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Directed by: Dr. Van Court Hare, Jr.

The major purpose of this dissertation is the mathematical analysis of the transfer-by-taxation concept inherent in--and common to--contemporary proposals for a "guaranteed minimum income," or "negative tax." To facilitate that purpose the analytical mathematical model developed in this dissertation is incorporated into a computerized, time-sharing management information and control system. The combined system supplies the following to welfare policy makers:

- a) The population and income picture for each geographic area-family size division of the United States' population along with population and income figures for each family size or each geographic area. Of course, population and income figures are also given for the whole country.

- b) The ability to divide each geographic area-family size division of the total population into a to-be-subsidized segment and a non-subsidized segment as a function of a geographic area-family sized determined Breakeven Level of Incomes, and to determine the population and income figures for both segments.
- c) The additional income given to each geographic area-family size's subsidized segment due to the inclusion of a geographic area-family size determined minimum income, and the effect these additions have on total income for each geographic area-family size division, each geographic area, each family size, and for the whole country.
- d) The decrease in subsidy income that occurs in each geographic area-family size's subsidized population due to the levying of a geographic area-family size dependent negative tax to income earned while receiving a subsidy. This income effect is also determined for each geographic area, each family size, and for the whole country.
- e) The cost of any subsidy for each geographic



area-family size division, each geographic area, each family size, and for the whole country.

- f) The ability to apply varying marginal taxes for each geographic area-family size's non-subsidized segment's income so as to show what new taxes are needed if one wishes to finance the subsidy by additional taxation. The tax liability for each geographic area, family size, and whole country is also given.
- g) Varying subsidy responsibility is determined for corporations by assigning geographic area dependent corporate marginal taxes. The corporation subsidy-paying role is also determined for the whole country.
- h) The analysis is so done that even individual income bracket analysis may be done for a-f above.

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C H A P T E R I

INTRODUCTION

## THE URGENCY OF WELFARE REFORM

Criticisms of welfare are as old as welfare--perhaps older. In general, criticism takes the form of assertions that welfare is not doing enough or that it is doing too much. Such criticisms are directed toward the values implicit in welfare programs. On rare occasions--unfortunately much too rare--another kind of criticism emerges. This criticism runs to the effect that, whatever the welfare system is trying to do, it is not doing it very well.

In recent years, criticism of this second sort has been mounting. It is argued that the present system is neither economic nor equitable because programs are badly coordinated, many people are denied access to assistance on arbitrary grounds, levels of assistance vary widely from one area to another, attempts are made to maintain distinctions among individuals which are expensive to administer and lack much in obvious fairness, the administration of the system entails a costly and oppressive surveillance of beneficiaries, and assistance is given in ways that tend to assure continued dependency.

These criticisms are not from casual sources, but from the leading politicians, economists, and social action groups of this country. From the politicians' point of view the following is a statement made by the former

Secretary of Health, Education and Welfare, the Honorable Robert H. Finch before the Senate's Finance Committee hearing on President Nixon's Family Assistance Plan.<sup>1</sup>

As President Nixon said in his address to the Nation on August 8, 1969:

"Whether measured by the anguish of the poor themselves, or by the drastically mounting burden on the taxpayer, the present welfare system has to be judged a colossal failure.

What began on a small scale in the depression 1930's has become a huge monster in the prosperous 1960's. And the tragedy is not only that it is bringing States and cities to the brink of financial disaster, but also that it is failing to meet the elementary needs of the poor."

The President's assessment is borne out by the well-known fact that in this decade alone, total costs for the four federally aided welfare programs (Aid to Families with Dependent Children, Aid to the Aged, Aid to the Blind and Aid to the Disabled) have more than doubled, to a level estimated in 1970 at about \$6.6 billion. In the Aid for Families with Dependent Children program (AFDC), costs have more than tripled since 1960 (to almost \$4 billion annually at the present time) and the number of recipients has more than doubled (to some 7.1 million persons in November 1969). Since the President first proposed the family assistance plan, in August of last year, another million people have been added to the AFDC rolls. Even more disturbing is the fact that the proportion of children on AFDC is growing. In the 15 years since 1955, the proportion of children in the Nation receiving assistance has doubled--from 30 children per 1,000 to 60 per 1,000 at present.

Prospects for the future show no likelihood for relief from the present upward spiral. If present trends continue, AFDC costs will almost double again by fiscal year 1975, and caseloads will increase by 50 to 60 percent. Yet, despite these crushing costs, benefits remain below adequate levels in most States.

The crisis in our welfare system is, of course, not really surprising, because, in many ways, the



present AFDC program is built to fail. It contains structural defects which help to cause its own destruction.

....as all of you know, it is characterized by unjustifiable discrepancies between States. It is, as you know, not one welfare system but more than 50 different systems with no national standards for benefit levels. AFDC payments now vary from an average of \$46 per month for a family of four in Mississippi to \$265 for such a family in New Jersey. These gross disparities are aggravated by complicated State-by-State variations in criteria for eligibility and methods of administration. Each State has its own need standards, assets tests, incapacity tests, requirements for school attendance and age of children, and income exclusion of the program has varied widely in terms of equity and responsiveness to recipients' needs from State to State and locality to locality. For example, the proportion of children receiving AFDC varies from 2.2 percent in Indiana to 10.7 percent in New York. This variation simply cannot be blamed on differences in the incidence of poverty.

From the social agencies the following is an excerpt from the statement made by the Health and Welfare Council of the National Capital Area before the same Senate Finance Committee. This particular statement has been chosen since it seems to best present the opinion of the many social agencies of this country.<sup>2</sup>

Welfare reform is not a subject about which only the needy or only certain public officials are concerned. The private agencies represented by our council, and the very many volunteers who support these agencies, are also vitally concerned. We recognize the present welfare system and its inadequacies as a basic fact of life for our community and for a great many of the people served by our agencies. We recognize that voluntary agencies can make a full contribution to the solution of the welfare problems that face us only if there are adequate public programs on which to build.

We are concerned, of course, with the cost of developing an adequate public welfare system. But we

observe every day the cost of an inadequate system, not only in the constantly rising economic costs of half way help for families unable to break the welfare cycle, but also, painfully, in thousands of wasted lives. If the nation must tax itself more to truly reform the welfare system, or if it must back substantially other kinds of expenditures, then it must. We cannot afford a society in which so many exist without the means to support themselves in health and decency and without the prospect of any basic improvement in their lot.

These above statements of criticism of our present welfare system and a call to develop new programs is not something recent. A philosopher once described the problem of achieving welfare reforms as follows:<sup>3</sup>

"Like a diet prescribed by doctors which neither restores the patient nor allows him to succumb, so these doles that you are now distributing neither suffice to ensure your safety nor allow you to renounce them and try something else".

That was said by Demosthenes, over 2,200 years ago. Apparently, in all this time, man has been unable to get this job done right. It is now time to take advantage of growing unrest with our present welfare system and to give a willing President the tools to evaluate welfare replacement schemes. This dissertation develops, as will be pointed out in its chapters, that tool.

THE CASE FOR A NEGATIVE INCOME TAX OR GUARANTEED MINIMUM INCOME DEVICE.

It is because of the above mentioned, poor functioning of our present welfare system that today we find many alternatives set before us. Included in these alterna-

tives are additions to the present cumbersome system, child subsidy programs, but probably most realistic and by a wide margin most mentioned and accepted are income transfer programs such as negative income tax proposals (NIT) and its close equal, guaranteed minimum income (GMI) proposals.

"The persistence in the United States of millions of people who are poor is a basic reason for the interest in what is often called negative income taxation. In a society which is supposedly the most affluent in the world, the presence of 30 to 35 million people described as poor, depending on one's definition of poverty, scarcely accords with the American dream. In the minds of some, it may raise nagging doubts about both the political structure and the economic system that generates such results. Strong supporters of the American system find it less embarrassing to diagnose poverty as a consequence of some deficiency in poor people, such as laziness, and enjoin them to reform their behavior. But such a diagnosis implies that people are poor out of choice, that they happen to have a strong preference for leisure. Such a diagnosis scarcely fits the evidence.

The reasons why millions of people have low incomes can be enumerated almost indefinitely. People's mental and physical characteristics differ in innumerable ways. Individual talents and inclinations for obtaining gain are subject, as well, to a wide dispersion. The main problem in explaining the income distribution observed in a complex society is not how to explain low incomes. Rather, it is how to explain why the distribution does not have normal statistical properties and in particular why the tail of the low end, including the range of negative incomes, does not have properties similar to those of the upper tail.

Welfare programs in the United States, insofar as they have any rationale, have been based on the theory that the "causes" of low income can be enumerated. Specific measures related to those causes can supposedly provide adequate assistance. Thus there are the categorical aid programs, unemployment insurance, and old age and survivors' insurance. Since the causes are not exhaustive, some



cases of distress are inevitably left uncovered by public programs.

A welfare system aimed at alleviating poverty should adopt the premise that income dispersion, including negative income and small positive income, is normal and will not disappear next year or the year after. Since human productivity varies widely, as do other abilities, there are always some groups whose productivity will be low judged by some standard appropriate for "normal" people. If people of low productivity are to be employed in the absence of special subsidies, employers, to have an incentive to hire them, must be permitted to pay low wage rates. People's incomes from work will leave some of them and their dependents in poverty. To the extent they are denied the choice of working, by the establishment of minimum wages, their incomes will be even lower.

The income distribution as it naturally arises does not guarantee affluence for everyone in an affluent society. Government policy must be consistent with the fact that there are some able-bodied males who are not capable of earning as much as \$3,000 a year or even \$2,000 a year. Present welfare programs, geared as they are to various presumed causes of personal financial distress, presuppose that people not subject to special difficulties, such as ill health, old age, unemployment, etc., can earn adequate incomes and can bring up children who will develop into effective members of society. The presupposition is scarcely consistent with the facts. Mollie Orshansky found, for example, that 22.3 million people out of 27.9 million defined as poor were in families headed by a male. In addition: 'Of the 15 million children counted as poor in March 5.7 million were in the family of a worker who had a regular job in 1963 and was not out of work any time during the year.'<sup>4</sup> There are poor people who are unemployed or unemployable, but there are others who work and do so regularly. A wage structure that eliminates unemployment and underemployment of people with modest skills cannot be expected to end poverty levels of income, although such a pricing arrangement would result in a vast improvement for many presently disadvantaged groups. There is nothing in economic theory nor in the inherent characteristics of human beings that precludes equilibrium wage rates of one dollar an hour or less for some types of labor services.

If, then, the premise is adopted that the dispersion of income, including low income, is a normal feature of economic affairs, what social measures are appropriate to obtain a socially acceptable level of income for everyone? Negative income tax devices are techniques to solve this problem."<sup>5</sup>

This argument given by Professor Rolph of the University of California at Berkeley is but one calling for a NIT or GMI solution to our welfare problems. Other persuasive arguments have been given by such distinguished economists as Milton Friedman,<sup>6,7</sup> Christopher Green,<sup>5,12</sup> Robert Theobald,<sup>8,9</sup> Robert J. Lampman,<sup>10,11,12</sup> George Hildebrand,<sup>13</sup> Lady Rhys-Williams,<sup>14</sup> Robert R. Schultz,<sup>15</sup> Edward Schwartz<sup>16</sup> and many others. Some of which are detailed in Chapter II. President Nixon at the writing of this paper has already proposed a guaranteed minimum income proposal, and it has passed the House of Representatives. The Senate is investigating it more fully in the Finance Committee.

#### POSSIBLE RESOLUTIONS TO THE NIT OR GMI CONTROVERSIES

With all this interest being generated in NIT or GMI proposals, it becomes obvious that differences of technique and philosophy will occur in each individual's proposals even though general consensus is met as to the need of a NIT or GMI substitute for the present welfare system.

To the author there seems to be three distinct methods by which the present NIT or GMI controversies may be



resolved and thereby the welfare system made more proficient. They are:

- a) The development of an all-inclusive NIT or GMI proposal acceptable to all that corrects the welfare problem;
- b) The development of an alternate system that will eliminate NIT or GMI as feasible solutions; or
- c) The development of some sort of analytical tool that can test and compare present proposals and thereby select the most feasible one to be instituted.

After only a quick scrutiny of these three alternatives it becomes obvious that the latter proposal (c) is a must. Without it one would not know if (a) already existed or if investigations into (b) are needed.

There are many other reasons why an analysis such as the one mentioned above is needed at the present time. First, from the broad viewpoint of national economic goals, there are a number of arguments which state that contemporary economic policy should place greater emphasis on solving problems such as the inequitable distribution of income. Secondly, the individual proposals for various GMI plans have been based upon the various authors' particular sociopolitical points of view and therefore these plans have emphasized these aspects of

GMI and have not analyzed the GMI concept itself. Thirdly, the acceptance of any one of these proposals without some analytical comparison with the other could, if the proposal failed in use, simply place us in the position we are now or worse. Fourthly, the cost of implementing any negative tax proposal is such that a wrong selection as to program, judging from the cost of our present system, would be infinitely larger than the cost of a testing system. Finally, with the way antagonism has grown with the present program, it does not seem feasible to believe that the American populace would put up with another welfare failure.

With a GMI proposal already submitted by the President of the United States; its approval by the House of Representatives; and the Senate considering it, the author cannot but feel that not enough analysis of the issue has been done. How can we know that a proposal is the right one without comparing it to others? How do we know it will be adaptable to new tax legislation or new income levels in the country? How do we know it will pay for itself? From studying many plans the answer to all these questions is that we do not know. It is the author's opinion that any proposal should not be put into effect until analyzed by a system like the one developed in this paper or one similar to it.

AN OUTLINE OF THE DISSERTATION

The present welfare system as it exists today is to be considered unfeasible. To replace this unfeasible system many NIT or GMI proposals have been developed. Even though each one of these proposals differ somewhat in philosophy and technique and, thereby, no one has been universally accepted, some way must be developed to investigate and compare each and select a realistic alternative. To this objective this dissertation has been written.

In fact, the major purpose of this dissertation is the mathematical analysis of the transfer-by-taxation concept inherent in--and common to--contemporary proposals for a "guaranteed minimum income", or "negative tax". The following is what the model supplies decision makers:

- a) The population and income picture for each geographic area-family size division of the United States' population along with population and income figures for each family size or each geographic area. Of course, population and income figures are also given for the whole country.
- b) The ability to divide each geographic area-family size division of the total population into a to-be-subsidized segment and a non-subsidized segment as a function of a geographic area-family

sized determined Breakeven Level of Incomes, and to determine the population and income figures for both segments.

- c) The additional income given to each geographic area-family size's subsidized segment due to the inclusion of a geographic area-family size determined minimum income, and the effect these additions have on total income for each geographic area-family size division, each geographic area, each family size, and for the whole country.
- d) The decrease in subsidy income that occurs in each geographic area-family size's subsidized population due to the levying of a geographic area-family size dependent negative tax to income earned while receiving a subsidy. This income effect is also determined for each geographic area, each family size, and for the whole country.
- e) The cost of any subsidy for each geographic area-family size division, each geographic area, each family size, and for the whole country.
- f) The ability to apply varying marginal taxes for each geographic area-family size's non-subsidized segment's income so as to show what new taxes are needed if one wishes to finance the subsidy by



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g) Varying subsidy responsibility is determined for corporations by assigning geographic area dependent corporate marginal taxes. The corporation subsidy-paying role is also determined for the whole country.

h) The analysis is so done that even individual income bracket analysis may be done for a-f above.

So that the reader may more fully understand how this dissertation supplies the above results and what else is discussed in this dissertation the remainder of this chapter briefly describes the dissertation flow first in general, then in more detail for each chapter.

The first order of business described in the dissertation is a complete literature review divided into three areas:

- a) A historical background of NIT and GMI proposals;
- b) Present state of the art of NIT and GMI proposals;
- c) Present state of the art to analyze NIT and GMI proposals.

This literature search is presented in Chapter II.

Once a reader is familiar with NIT and GMI proposal terminology, a mathematical model encompassing both con-

cepts is developed and presented in Chapter III. In this development is the means of comparing all proposals and a means to select an optimal choice.

Since the mathematical model must represent the present income picture of the United States, income data segmented into family sizes and geographic areas must be analyzed. For the data to be useful for the mathematical model it must be described by parametric equations in a form recognizable as input to the mathematical model. Chapter IV of the proposed dissertation will explain this technique.

Once income data and the mathematical model are made compatible, it becomes necessary to present them in a form usable by decision makers. The form selected for this dissertation is a computerized, time-sharing or batch processing management information and control system depending upon system configuration. By using this technique the decision maker is allowed to change input quantities as the real world varies and immediately see their effect on NIT and GMI proposals. This management information and control system is defined in Chapter V.

Even though all input requirements and output reports are described fully in their respective chapters, a topical set of sample proposals is selected from the literature and analyzed by the management information and

control system. All steps are detailed and all input and output explained. These sample cases are presented in Chapter VI.

As in the case of most works, assumptions are made in the proposed dissertation. Therefore, the final chapter (VII) of this dissertation, considers the effect changes in major assumptions have on the system. Also considered in this final chapter is a review of the proposed system and possible extensions to the system.

Throughout the proposed dissertation whenever a topic is presented that is controversial, a brief outline of the controversy is given and references included. The most prevalent areas in which controversy is seen other than in the NIT and GMI proposals is in the areas of Fiscal Policy, Taxation, and Income Distribution.

## Chapter II Literature Searches

### A. Historical Background of NIT and GMI Proposals

The negative income tax plans advocated by Friedman, Lampman, Tobin, et al, designed to increase the efficiency and reduce the cost of current welfare programs; and the guaranteed minimum income plans advocated by Theobald, Schwartz, et al, designed to guarantee at least a minimum subsistence level of income to the poor are discussed to, in fact, form the definitions of the two varying proposal

types.

B. Present State of the Art for NIT and GMI Proposals

Those Income Transfer Programs such as President Nixon's Family Assistance Program being considered by the Ninety-First Congress; that program presented by the President's Commission on Income Maintenance Programs started during President Johnson's Administration and continuing into President Nixon's; and variations in each of these programs as to certain basic parameters by experts called to testify against and for the above proposals are detailed in this section.

C. Present State of the Art to Analyze NIT and GMI Proposals

This literature search shows that the analytical model presented in this dissertation is by far the most detailed and complete tool to analyze and compare NIT and GMI proposals available today. Areas of lacking in the present analytical literature is pointed out to further define the uniqueness of the analysis of this dissertation.

Chapter III The Mathematical Model

Since in this chapter both NIT and GMI proposals must be definable by one mathematical model, a discussion of their similarities and differences, mathematically, is made. This chapter proves that the two propos-



al types vary only in socio-political arguments and, therefore, can be singularly modeled. The differences in socio-political arguments are discussed in Chapter II, Section A.

In fact, by allowing the consideration of various Breakeven Levels of Income, various guaranteed minimum incomes, various negative taxes, various marginal personal taxes, various corporate taxes, and insuring a realistic representation of national income distribution segmented into geographic areas and family sizes; this mathematical model encompasses all NIT and GMI proposals, plus much more. The model, therefore, can compare any proposal available today or to be developed in the future.

#### Chapter IV Income Data Base Development

National income data is presented in a form that allows various income definitions. This form allows possible inclusion of salaries, interest on savings, dividends, rents, or a combination of these sources. For each geographic area-family size division of the United States population, a histogram of income frequencies is produced. From each income histogram two parametric functions are generated one for the to-be-subsidized segment, and one for the non-subsidized segment, by means of polynomial curve fitting techniques.

In order to generate the parametric equations from the income histograms, a time-sharing computer program is developed. This input storage by family size and geographic area on an assortment of computer devices is discussed. By storing the data in such a way, a decision maker can select any subset of family sizes and geographic areas; then, by inputting any Breakeven Level of Income for each histogram, compute the to-be-subsidized parametric equation segments and the not-to-be-subsidized parametric equation segments. These parametric equation segments along with the Breakeven Level of Income are stored in another set of data files used as input for the management information and control system.

A complete printout of the frequency input and their respective parametric equation fits are outputted in a report form. Also on this report form is a measure of the goodness of each fit along with a composite goodness of fit. This measure is calculated by a standard error about the means of the fitted data.

#### Chapter V The Management Information and Control System

To allow a decision maker who wishes to compare or evaluate any NIT or GMI proposal the most flexibility in selecting income distribution subsets of the Economy; negative, positive, and marginal tax functions; varied

guaranteed minimum income levels; varied Breakeven Levels of Income; and all-inclusive, immediately available output forms: a management information and control system computerized using timesharing, on-line techniques is part of the proposed dissertation. The management information and control system is designed so as to be self-instructing for the decision maker. The management information and control system is programmed in a universal language, FORTRAN IV, so that it is usable at almost any computer installation. To insure universality, the input design development of the parametric income distribution equation frequencies is multiple; first in the form of disk files for the fastest accessing, second in the form of drum files, thirdly in the form of magnetic tape records, and fourthly in the form of punched cards to magnetic core storage, and finally on paper tape readable by timesharing terminals. For those installations where timesharing is not available, the system changes required for batch processing use is discussed.

Incorporated, as the main element of the management information and control system, is the mathematical model described in Chapter III. To be solvable by computer, the mathematical integrals of the model are processed through a numerical polynomial integration subroutine.

By just changing the input parameters, the decision

maker can immediately see their effect by the changes in the output figures. By noting the changes, he can compare all NIT and GMI proposals given any state of the real world.

### Chapter VI System Testing

In this chapter actual NIT and GMI proposals are processed through the management information and control system. Their system input breakdown is explained along with the resulting output reports. This chapter also suggests possible comparison techniques. By following the test cases of this chapter, a decision maker can facilitate his using and understanding of the management information and control system. The test cases to be presented in this chapter are selected from those of Chapter II, Section B.

### Chapter VII Conclusions

This chapter discusses the validity of major assumptions made in the dissertation. It also explains how possible changes in these assumptions would alter the system. In areas where added work may be continued, this chapter suggests possible courses of action.



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C H A P T E R   I I

L I T E R A T U R E   S E A R C H

## HISTORICAL BACKGROUND OF NIT AND GMI PROPOSALS

For almost two hundred years the economic goal pursued most actively by technologically-advanced nations has been that of economic growth. While this goal may have been a logical one in the past times characterized by scarcities of material goods, a number of contemporary economists have questioned the wisdom of continuing this emphasis on growth. Emphasis on growth is thought by some to be an obsolescent<sup>1</sup> policy leading to adverse effects and entailing unreckoned costs.<sup>2</sup> It is argued that more emphasis today should be placed upon other goals. Optional allocation of resources between the public and private sectors has become an important need of nations suffering from air, water, and noise pollution--as well as the destruction of natural beauty--brought about by the current emphasis on economic growth. Further, the need for attention to the problem of inequities in the distribution of income in the United States has been dramatically revealed in the past several years. Again, economic growth has made the problem severe, first by creating a high standard of living, and then by endangering labor's share of income through the development of automated and cybernated techniques of production.

In "The Other America", Harrington<sup>3</sup> has described the conditions of poverty existing in the United States



today, and this description has been credited with the start of Federal government attention to the problem. Theobald<sup>4</sup> has argued that the problem of poverty will become even worse in the future as automation progressively reduces the demand for labor, and therefore, labor's share of the national product. He feels that the problem of income distribution will be a most serious national economic problem in the future, and has proposed the adoption of a guaranteed minimum income (plus other social services) as the solution to this problem.

The concept of government guarantees of a minimum standard of living is not a new one,<sup>5</sup> and in recent years several proposals have been made to incorporate such a plan into the income tax structure. Philosophically, these plans fall into two general categories: the "negative income tax" plans which are designed to increase the efficiency and reduce the cost of current welfare programs; and the "guaranteed minimum income" plans which are designed to guarantee at least a minimum subsistence level of income to the poor. The "negative income tax" has been advocated by Friedman,<sup>6</sup> Lampman,<sup>7</sup> and Tobin,<sup>8</sup> while the major spokesmen for the "guaranteed minimum income" have been Theobald,<sup>9</sup> and Schwartz.<sup>10</sup>

The negative income tax plan was proposed by Friedman because of his desire to reduce the size of government and to diminish its effect on the market place. In

his plan, if the total of a family's income-tax exemptions and standard deduction is more than its income, then the government would pay the family a percentage of the difference as a subsidy, that percentage being the negative tax rate. For a family of four the total amount of allowable exemptions and standard deduction is \$3,000, and Friedman proposed a maximum negative tax rate of 50%. His negative income tax plan is therefore as shown in Figure 2-1. Friedman proposed this negative tax plan as a replacement for existing welfare programs. Lampman, however,

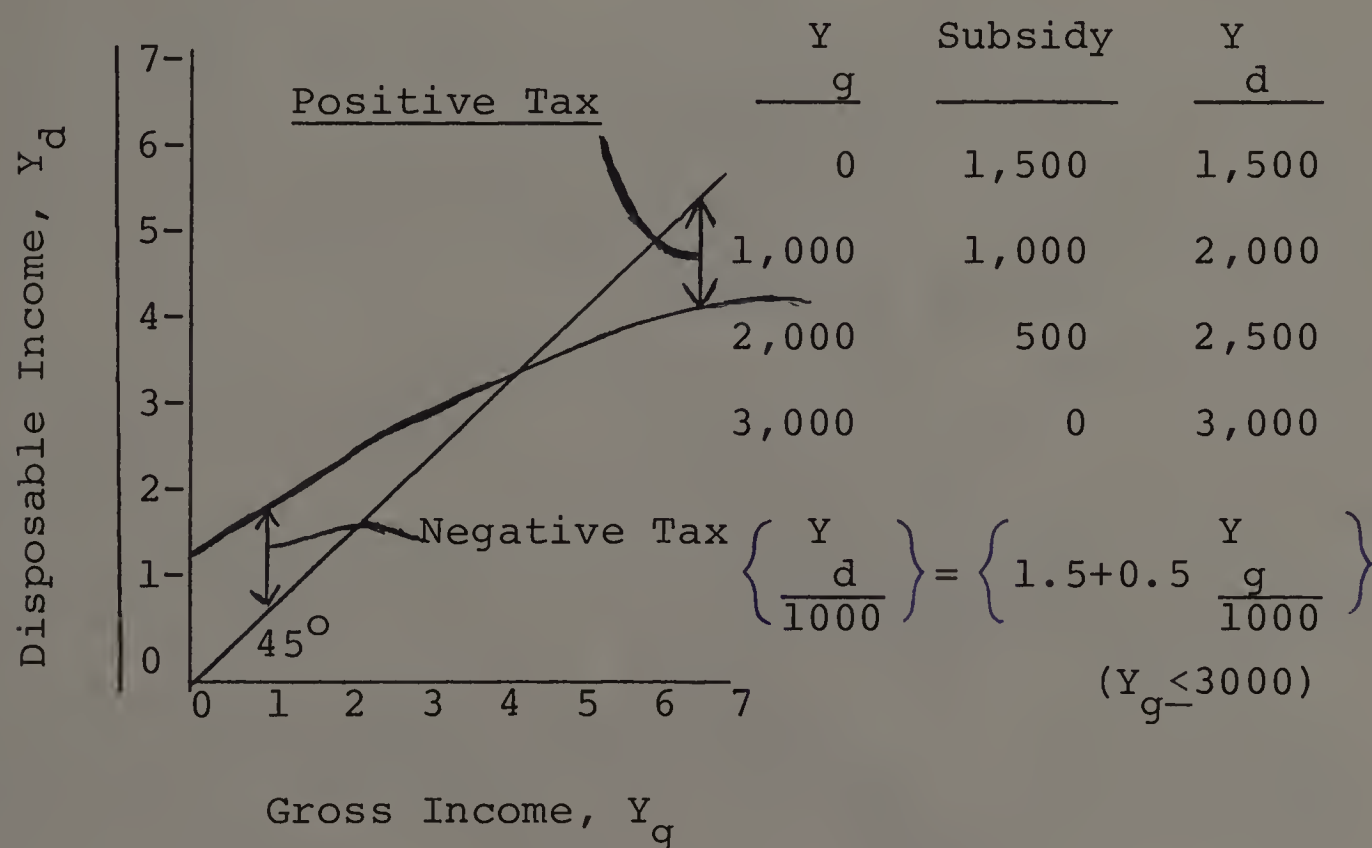


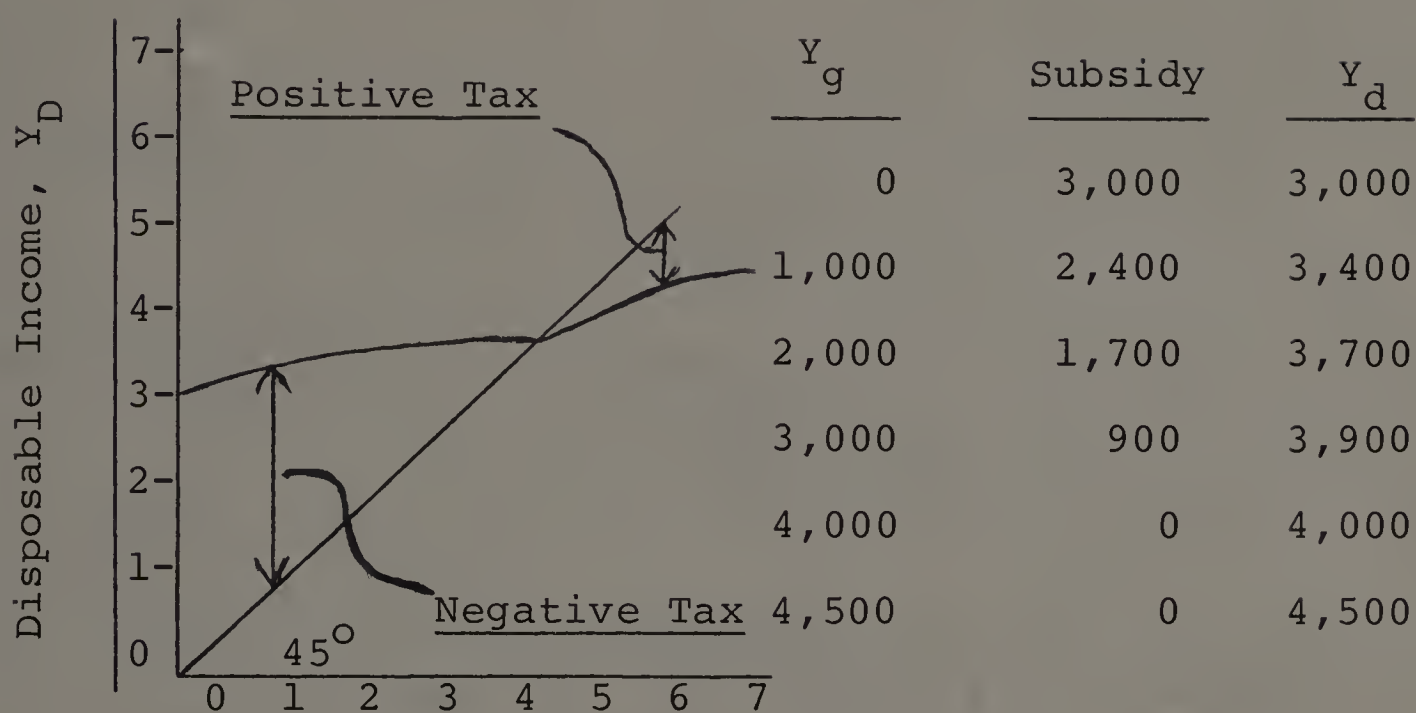
Figure 2-1: Friedman's Negative Income Tax Plan

believes it should both supplement and complement these programs. He points out that the existing tax structure discriminates against the poor, and that neither cuts in the (positive) tax rate nor increases in the dollar val-

ue of exemptions and standard deductions will do anything for them. Lampman's plan is similar to Friedman's except that he uses the poverty income gap as the basis for his negative income tax rate. In this manner he attempts to obtain greater equity in the treatment of the poor.

The "negative income tax" plans do not provide direct transfer payments sufficient to raise the family income to a minimum chosen level. (In Figure 2-1, this level is \$3,000, i.e., the maximum earned wage at which no positive tax is paid.) The "guaranteed minimum income" plans do raise the income of the poor to a predetermined minimum level, regardless of earnings.<sup>11</sup> Theobald sets the minimum level of income at that minimum point required for subsistence. If earned income is below this level then the government provides a subsidy sufficient to raise disposable income to the subsistence level. To provide a work incentive, Theobald proposed that the subsidy be augmented by a percentage of the earned income. Theobald's plan was stated as a preliminary one, and the plan proposed by Schwartz has become more of a standard model for GMI schemes. In an attempt to make Friedman's plan a more equitable one, Schwartz modified it by proposing a progressive negative tax, starting with a predetermined level of minimum disposable income. He called his plan a "Family Security Pro-

gram" and this plan is shown in Figure 2-2:



$$\text{Gross Income, } Y_g \quad \frac{Y_D}{1000} = \left\{ 3 + .045 \left( \frac{Y_G}{1000} \right) - .05 \left( \frac{Y_G}{1000} \right)^2 \right\}$$

( $Y_G < 4000$ )

Figure 2-2: Schwartz's Guaranteed Minimum Income Plan

The contemporary transfer-by-taxation plans described above have each been conceived from specific political and socio-economic points of view. Friedman, in the conservative tradition, wishes to reduce the size of government (e.g. replace all welfare agencies with a simple negative income tax scheme), and to diminish its effect upon the market place. Theobald is concerned that the advanced state of technology has caused a shortage of jobs, and that advances in automation and cybernation will drastically reduce the demand for labor in the future. To off-



set this he proposes a guaranteed minimum income as at least a first step in ensuring a continuing equity in the distribution of national income. Schwartz is concerned with individual dignity as well as with increased efficiency in the administration of welfare programs. Despite these divergent points of origin however, each of these transfer-by-taxation plans have marked similarities. As Green<sup>12</sup> points out, they share three common features:

- (1) A guaranteed minimum level of income, varying with family size and composition.
- (2) A tax rate applied against a tax base.
- (3) A breakeven level of individual income where the tax liability equals the guaranteed allowance.

The details of each of these features differ with each author, and these differences are significant in the determination of national welfare policies. The basic concept however, is consistent regardless of the name given to it. This paper develops a mathematical system to analyze this concept, free of any particular bias, and presents a solution which can be used by all proponents of transfer-by-taxation plans.

#### PRESENT STATE OF THE ART OF NIT AND GMI PROPOSALS

It is most fortunate that at the writing of this dis-

sertation that the Country is involved in welfare reform. In 1966, the Journal of Industrial Relations gathered together some of the leading names in income transfer research and put together in its February, 1967 issue a collection of papers by these authors entitled, "A Symposium: Negative Income Tax Proposals." In 1968 the Ninetieth Congress published its Hearings Before the Subcommittee on Fiscal Policy of the Joint Economic Committee entitled, "Income Maintenance Program." During these proceedings many of the plans similar in type to those presented above in the History of NIT and GMI Proposals were submitted and questioned. As a result of these hearings, President Johnson established a President's Commission on Income Maintenance Programs. This Commission continued its efforts under President Nixon and made its majority report along with minority recommendations in November, 1969.

As a result of the findings of the President's Commission on Income Maintenance Programs, the President along with the Department of Health, Education and Welfare, whose expertise was gained by conducting an income maintenance experiment in New Jersey submitted a Family Assistance Program to the Ninety-First Congress of the United States. This proposal has, at this writing, been passed by the House of Representatives in 1970 and has

been held up in the Senate's Subcommittee on Fiscal Policy since then.

What is to follow in this section of the dissertation is outlines of the above plans along with some pro's and con's from experts in this area. Also included are plans built upon the above but call for changes to some of the parameters.

One important fact must be pointed out here. Each one of the plans to be described here is justified by their proponents through methods I deem inconclusive. None use national income data and its projections in the years the program would be in effect. None include variations due to geographic area. It is because of these unsubstantiated proposals that the analytical tool developed in this dissertation is needed. The next section of this Chapter further points out the lack of analysis done to substantiate multi-billion dollar programs and points out how this dissertation is unique.

As the first proposal to be discussed, preference is given to that one which the author believes most likely to become law. That is President Nixon's recommended Program. The following is a summary.<sup>13</sup> The actual bill may be found in the Ninety-First Congress Subcommittee on Fiscal Policy Hearings.<sup>14</sup>

H.R. 16311. Messrs. Mills & Byrnes (Wisc.); 3/5/70,  
Ways and Means.

Family Assistance Act - Establishes a new Family Assistance Plan providing for payment of family assistance benefits by the Secretary of Health, Education, and Welfare and supplementary payments by the State.

Authorizes benefits to families with children payable at the rate of \$500 per year for each of the first two members of a family plus \$300 for each additional member.

Asserts that the family assistance benefit would be reduced by non-excluded income, so that families with more non-excludable income than these benefits would not be eligible for benefits. Provides that each family whose resources are less than \$1,500 shall be eligible.

Asserts that countable income would include both earned income (remuneration for employment and net earnings from self-employment) and unearned income.

Provides that in determining income, the following would be excluded (subject, in some cases to limitations by the Secretary): (1) all income of a student; (2) inconsequential or infrequent or irregular income; (3) income needed to offset necessary child care costs while in training or working; (4) earned income of the family at the rate of \$720 per year plus 1/2 the remainder; (5) food stamps and other public assistance or private charity; (6) special training incentives and allowances; (7) the tuition portion of scholarships and fellowships; and (8) home produced and consumed produce.

Provides that eligibility for and amount of benefits would be determined quarterly on the basis of estimates of income for the quarter, made in the light of the preceding period's income modified in the light of changes in circumstances and conditions.

Provides that eligible adult family members would be required to register with a public employment officer for manpower services and training or employment unless they belong to specified excepted groups. Provides assistance for: (1) ill, incapacitated, or aged persons; (2) the caretaker relative (usually the mother) of a child under 6; (3) the mother or other female caretaker of the child if an adult male (usually the father) who would have to register is there; (4) the caretaker for an ill household member; and (5) a child who is under 16.

Asserts that where the individual is disabled, referral for rehabilitation services would be made, provisions are also made for child care services to the extent the Secretary finds necessary in case of participation in



manpower services, training, or employment.

Provides that family assistance benefits would be denied with respect to any member of a family who refuses without good cause to register or to participate in suitable manpower services, training, or employment.

Provides that the Secretary would transfer to the Department of Labor funds which would otherwise be paid to families participating in employer-compensated on-the-job training if they were not participating. Asserts that these funds would be available to pay the training costs involved.

Provides that the individual States would have to agree to supplement the family assistance benefits under a new Part E of Title IV of the Social Security Act whenever the family assistance benefit level is below the previously existing Aid to Families with Dependent Children (AFDC) payment level.

Asserts that this supplementation is a condition which the State must meet in order to continue to receive Federal payments with respect to maternal and child health and crippled children's services (title V) and with respect to their State's plans for aid to the aged, blind, and disabled (title XVI), medical assistance (title XIX), and services to needy, families with children (part A of title IV).

Provides that the States would thus be required to supplement in the case of individuals eligible under the old AFDC-UF provisions; they would not have to supplement in case of the working poor.

Provides that some of the State plan requirements now applicable in the case of Aid and Services to Needy Families with Children would be made applicable to the agreement as follows: (1) Stateswideness; (2) Administration by a single State agency; (3) Fair hearing to dissatisfied claimants; (4) Methods of administration needed for proper and efficient operation, including personnel standards, training and effective use of subprofessional staff; (5) Reporting to Secretary as required; (6) Confidentiality of information relating to applicants; and (7) Opportunity to apply for the prompt furnishing of supplementary payments.

Authorizes the Secretary to pay to any State which has in effect an agreement, for each fiscal year, an amount equal to 30 percent of the total amount expended during such year pursuant to its agreement to the specified families with certain limitations based on a set poverty level promulgated by this Act. Provides, also, that 50 percent of its Administrative costs shall be paid by the Federal Government.

Provides for manpower services, training, and employment, and child care and related services for individuals eligible for the new Family Assistance Plan benefits or State supplementary payments to help them secure or retain employment or advancement in employment. Requires this to be done in a manner which will restore families with dependent children to self-supporting, independent, and useful roles in the community.

Requires the Secretary of Labor to develop an employability plan for each individual required to register or receiving supplementary payments pursuant to this Act. Asserts that the plan would describe the manpower services, training, and employment to be provided and needed to enable the individual to become self-supporting or attain advancement in employment.

Authorizes appropriations to the Secretary of Health, Education and Welfare for grants and contracts for up to 90 percent of the cost of projects for child care and related services for persons registered under the Family Assistance Plan and in manpower training or employment. Provides that the grants would go to any public or non-profit private agency or organization, and the contracts could be with any public or private agency or organization. Asserts that the cost of these services could include alteration, remodeling, and renovation of facilities, but no provision is made for wholly new construction. Provides that the Secretary of Health, Education, and Welfare could allow the non-Federal share of the cost to be provided in the form of services or facilities.

Asserts that the provision on inclusion of reasonable standards for determining eligibility and amount of aid would be replaced by one requiring a minimum benefit of \$110 per month, less any other income, and by another requiring that the standard of need not be lower than the standard applied under the State plan approved under the existing title XVI or (in case the State had not had such a plan) the appropriate one of the standards of need applied under the plans approved under titles I, X, and XIV.

Provides for the training and effective use of social service personnel, provision of technical assistance to State agencies and local subdivisions furnishing assistance or services, and provision for the development, through research or demonstrations, of new or improved methods of furnishing assistance or services. Provides for the use of a simplified statement for establishing eligibility and for adequate and effective methods of verification thereof. Requires evaluation of the State plan at least annually, with reports thereof being sub-

mitted to the Secretary together with any necessary modifications of the State plan, for establishment of advisory committees, including recipients as members; and for observing priorities and performance standards set by the Secretary in the administration of the State plan and in providing services thereunder.

The above plan when analyzed for a family of four (2 adults, 2 children) becomes very similar to that of Friedman's in that it establishes a minimum income of \$1,600 and a Breakeven Level of Income of \$2,710. Its negative tax for a family of four is,

$$0.5(I - \$720) \quad (2-1)$$

$$\text{where } \$720 \leq I \leq \$1,500$$

Therefore, the net income of a to-be-subsidized family of four is,

$$Y = \$1,600 + I - .5(I - \$720) \quad (2-2)$$

$$\text{where } I \leq \$1,500$$

The President's Plan has been acclaimed by many politicians and experts as being a step in the right direction. However, others believe it does not give enough to allow a family to survive. These arguments stem from the fact that the Social Security Administration had developed a poverty index based on the Department of Agriculture's measure of the cost of a temporary low-budget, nutritious diet for households of various sizes. In the family of four case being analyzed, this figure in 1968 was \$3,553 per year.<sup>15</sup> That means the Nixon figure of between \$1,600 and \$2,710 depending upon whether the



family can earn income is between \$1,953 to \$823 below subsistence. Toward this very question Senator Harris of Oklahoma said,

"First, the inadequate \$1,600 level of payments in H.R. 16311 should be increased. Over a 3-year period we should phase in payments which will provide an income at least to the poverty threshold level."<sup>16</sup>

This statement by Senator Harris was but one of many questioning the level of payments. In fact, some of the later proposals of this section were developed to offset these inadequacies.

Another point of the Nixon Proposal that drew comment was the fact that there was no mention in the bill of different subsidy payments for different geographic areas. The plan is based solely on family size.

With respect to this question of geographic area as a means to define subsidy payments, the following testimony occurred while Senator Miller of Iowa was questioning the Honorable Elliot L. Richardson, Secretary of Health, Education, and Welfare.

"Now, Mr. Secretary, generally speaking it appears that your position is that the number of people deriving benefits from welfare would increase from 12 million to 24 million, which would result in 12 percent of the national population deriving welfare benefits.

However, the picture in some States would be much worse. According to one of the tables it appears that about one out of every four citizens in the State of Kentucky would derive welfare benefits, and I have heard it estimated that in some counties the proportion might be as many as three out of four citizens deriving benefits from this program. I suggest to you that this poses a prob-



blem to a number of taxpayers.

I suggest further to you that one possible reason for this apparent aggravation could be that the national standards proposed by the bill need to be refined, taking into account the differences in the cost of living by areas.

I would suppose the cost of living in some counties of Kentucky, for example, would be considerably less than the cost of living in counties in other parts of the country.

Isn't it possible to refine this so we can avoid what happens to be a very serious number of people who are receiving benefits under this program in certain areas?"

".... it seems to me incredible that the bill that is now before us, with the amendments that have been sent over from HEW, will apparently make no differentiation between the cost of living in New York City or Chicago and the cost of living down here in some little town in Virginia or Kentucky."

"I am all for the cost-of-living differential, and I'm just wondering why we could not go a step further and within a region, say, Illinois, outside of Chicago, have a differential between those who live on a farm and those who do not."<sup>17</sup>

Secretary Richardson confirmed that no geographic area parameter was to be found in the bill and that individual State supplementation would probably handle the problem.

This question of having State supplementation instead of a single Federal program goes against the whole idea of welfare reform as previously pointed out in Chapter I by President Nixon and the former Secretary of Health, Education, and Welfare, the Honorable Robert H. Finch. To this question of State supplementation the National Association of Counties believes that it would simply cause State, County, and Municipal Governments to

be overburdened as they are now and insure the program's failure.

STATEMENT ON BEHALF OF THE NATIONAL ASSOCIATION OF COUNTIES

Howard Rourke Director, Department of Social Services Ventura County, Calif.	David Daniel Director, Department of Public Aid Cook County, Ill.
James Glover Director, Department of Social Services Nash County, N. C.	Bernard F. Hillenbrand Executive Director National Association of Counties

SUMMARY OF PRINCIPAL POINTS-FAMILY ASSISTANCE PLAN,  
NATIONAL ASSOCIATION OF COUNTIES

1. NACO is very concerned about the devastating effects of rapidly escalating costs of welfare at the state and county government levels. The increase in welfare costs have been jumping 20 to 30 percent annually during the past few years and last year almost doubled in some counties. NACO believes the issue of welfare reform can no longer be discussed purely in terms of assistance to the needy--equally important is the viability of local government to continue to function.

2. NACO strongly supports legislation which would provide for eventual full federal assumption of welfare costs. As a minimum interim step, we urge that all future increases in costs be assumed by the Federal Government.

3. NACO proposes long-range welfare reform involving two separate national programs:

A national program focused upon work and wage security for all who can be considered to be in the labor market.

A national program to assure basic necessities of life for those who are unable to work, or are needed at home to care for minor children or the aged, disabled and blind, ill and injured adults.

4. NACO supports the Family Assistance Program (H.R. 16311) as a step in the right direction. The legislation would provide a start in establishing uniform national standards of eligibility, would establish a minimum federal floor for aid payments, and would provide some fiscal relief to states and counties.

5. NACO, however, recommends several amendments to H.R. 16311:

Elimination of the two-tier system of benefit payments, the basic family assistance benefit and the state supplemental benefit;

Elimination of the food stamp program in favor of a more adequate cash payment;

Establishment of a single adult category of assistance replacing OAA, ATD, AB, and GA.

Retention of the federally-supported unemployed father program until FAP can absorb all supplementation;

Establishment of an absolute ceiling on total gross family income in determining eligibility.<sup>18</sup>

The following are outlines of programs recently suggested. They will simply be presented here, and some will be further exemplified in Chapter VI, System Testing. First is a plan introduced by Senator Harris of Oklahoma;

#### SUMMARY OF PROVISIONS IN NATIONAL BASIC INCOME BENEFITS ACT

Sec. 2001. Sets forth the purpose of the Act which is to establish a national program of basic income benefits entirely financed from Federal funds, uniformly administered throughout the nation by either Secretary or delegates, designed to assure that every individual and family will enjoy the level of living justified by American productivity, and will encourage persons to enter the labor market.

Sec. 2002. Provides that the determination of what constitutes a minimum living requirement shall be made by the Secretary of Health, Education, and Welfare. The determination shall be consistent with the incomes of non-farm families determined by the Social Security Administration for the latest year for which data is available as an index of poverty subject to such variables as age, composition of families, difference in cost of living in different regions, counties, etc.

Sec. 2003 (a). Provides authority for Secretary to determine what resources can be disregarded for purpose of determining qualification and level of need. Such items as the home, household goods and personal effects of an individual or family as well as other items which the Secretary may determine



warrants exclusion.

In addition, the earned income of any individual or any member of a family group during any month shall be disregarded to the extent of the first \$75., plus one-half of the next \$150., plus one-fourth of the remainder.

Sec. 2003 (b). Provides for conditions upon which a refusal to accept a job will not disqualify individuals to basic income benefits. Refusal to accept a job if it is vacant due to a labor dispute, if the wages are not in keeping with prevailing wages in area for similar job or if below minimum hourly rate, etc. will not be grounded for denial.

Refusal to participate in work-training program if the program would not prepare the individual for a suitable job which will be available when training is complete will not disqualify one to benefits.

Further, it is provided that: one under the age of 16 or over 65; one physically or mentally unable to work; a child attending school; a woman having in her care a preschool child or a child attending school; and that one required in the home because of the illness or incapacity of another, will not lose benefits by reason of refusal to participate in work-training program.

Sec. 2004. Provides that the amount of assistance to which an individual or family group is entitled to shall be equal to the minimum living requirement of such individual or group less the amount of other income and resources available to the individual or group.

Sec. 2005. Provides for responsibility of the Secretary, or his delegate to refer applicant for basic income benefits to other agencies, rehabilitative, etc., if he would benefit from services of the agency.

Sec. 2006. Provides for the filing of an applicant for benefits. The Secretary shall prescribe what information is to be contained in the application. Furthermore, the section requires the Secretary to act promptly on an application.

Sec. 2007. Provides that the Secretary or his delegate shall make the determination of eligibility for benefits based on information in the application and if the initial determination is incorrect the Secretary shall take appropriate action to assure that no more than the correct amount is paid.

Sec. 2008. Provides for review procedure of determination made by the Secretary or his delegate.



Judicial review of the Secretary's determination is provided for along with the right to legal representation.

Sec. 2009. Provides for the applicability of legal procedures otherwise provided for in Social Security Act to this Title.

Sec. 2010. Provides for authorization of appropriations. Such amounts as may be necessary beginning with the fiscal year which ends June 30, 1971, are authorized to be appropriated.

Sec. 2011. Provides for the publication of the of the regulations of the Secretary in the Federal Register in accordance with the provisions of subchapter 11 of title 5, United States Code.

Sec. 2012. Provides for authority for the Secretary to administer the program authorized by the Act. Authority is given to the Secretary to utilize the personnel, facilities, and services of another Federal Agency with the consent of the head of the Federal Agency concerned or to enter into agreements with States to administer the program.

Sec. 2013. Provides that the Secretary by regulation shall provide safeguards which restrict the use or disclosure of information concerning applicants or recipients.

Sec. 2014. Provides for a three year transition to the benefit levels prescribed in Section 2002, Section 2003, and Section 2004.

For the fiscal year 1971, the basic income benefits shall be the greater of 70% of the minimum living requirements or the benefits paid under the states plans as of January 1, 1970, based on the present poverty level this shall be \$2520. For fiscal year 1972 the greater of 85% of the then determined minimum living requirement or the states benefits.

For the years 1971 and 1972, the Secretary can enter into an agreement with the appropriate state agency to administer the plan.

The states will be reimbursed for the cost of administering the program including the payment of the benefits specified in the act in excess of the following percentages of expenditures which would have been made by such state under Titles, I, X, XIV, XVI, and part A of Title IV had this section not been in effect:

For the fiscal year 1971 - 80% of such expenditures.

For the fiscal year 1972 - 50% of such expenditures.

Should a state refuse to enter into agreement with the Secretary, then the Secretary can administer the program through employees of HEW and money expended, which would not have otherwise been expended had the state entered into an agreement, may be withheld

from amounts payable to such state under other Federal programs.

Sec. 2015. Provides for certain conforming amendments.<sup>19</sup>

The next plan outlined is the majority report of the President's Commission on Income Maintenance Programs.

This Commission's main recommendation is for the development of a universal income supplement program to be administered by the Federal Government, making payment to all members of the population with income needs.

It is clear to this Commission that such a program is needed in the United States to assist persons excluded from existing programs and to supplant other programs. It is time to design public policy to deal with the two basic facts of American poverty: the poor lack money, and most of them cannot increase their incomes themselves. These conditions can be remedied only when the government provides some minimum income to all in need. If we wish to eliminate poverty we must meet the basic income needs of the poor.

The only type of program which we believe can deal with the problem is a direct Federal cash transfer program offering payments to all, in proportion to their need. Such a program can be structured to provide increased cash incomes to all of the poor, and to maintain financial incentives to work. The basic payment should vary by family size, and the payments should be reduced by only 50 cents on the dollar as other income increases. Thus, positive incentives exist for work, and the further development of private savings and insurance, and social insurance systems is not discouraged. By making payments to all in need--regardless of demographic characteristics--incentives to modify family structure in order to become eligible for programs are reduced.

We recommend that such a program be enacted promptly at a level that provides an income of \$2,400 per year for a family of four with no other income. Benefits should be scaled to pay \$750 per adult and \$450 per child to families with no other incomes.<sup>20</sup>

This program in principle closely relates to the

Friedman Proposal. It has the same 50% negative tax, but its guaranteed income is \$2,400 instead of Friedman's \$1,500. Also its Breakeven Level of Income is \$4,800 instead of Friedman's \$3,000.

The next program outlined is a minority report of the President's Commission on Income Maintenance Programs.<sup>21</sup> This program was presented by Clifford L. Alexander, Jr. of Arnold and Porter, and was concurred in by David Sullivan, General President, Service Employees International Union, AFL-CIO, and A. Philip Randolph, President, A. Philip Randolph Institute.

This is to express where I differ from majority recommendation of the President's Commission on Income Maintenance Programs.

The Commission has recommended as a base income, as a part of a universal income supplement program, that a family of four receive \$2,400 annually. I recommend a base income of \$3,600 for a family of four (2 adults, 2 children) [The Figure of \$3,600 is derived from the following computation: \$1,125 per adult annually (as contrasted with \$750 under the majority Commission report) and \$675 per child (as opposed to \$450 per child annually in the Commission's majority submission). Therefore, the amount of income a family of one adult with three children would receive annually is \$3,150. A family of two adults and three children would receive annually \$4,275, etc.]

I believe there are two fundamental goals of the President's Commission on Income Maintenance Programs. One is the elimination of poverty for all Americans as soon as feasible. The other is the elimination in toto of a dehumanizing welfare system which turns millions of our citizens into skeptical beggars blamed for inadequacies not their making.

Setting the base income at \$3,600 for a family of four opposed to \$2,400 will substantially serve to



accomplish our twin goals. The government computation of the poverty level at present for a family of four is \$3,553. Certainly, we should not set our sights at a level below the Government's poverty line. The Majority's \$2,400 minimum level is by its own statement inadequate to meet individual needs. To strive for less than an adequate level is not to recognize the task we were given. On January 2, 1968, when this commission was established, the following statement was made by President Johnson:

"The welfare system in American is outmoded and in need of a major change...Look into all its aspects of existing welfare and related programs and make just and equitable recommendations for constructive improvements whenever needed and indicated. We must examine any and every plan, however unconventional, which could promise a constructive advance in meeting the income needs of all the American People."

Setting a bottom annual line of \$3,600 could effectively eliminate welfare programs throughout the United States. [The States with the highest welfare programs now pay a family of four a little over \$3,500 annually.] The \$2,400 minimum recommendation of the full commission would require the continuance of supplemental welfare programs in approximately 20 states. This would necessitate the expense of bureaucracies to run these remaining welfare programs. This could necessitate potentially complicated formulas for Federal assistance of State welfare programs. This would necessitate a complicated phasing-out of remaining programs if the base income were gradually elevated.

The next seven plans were summarized by Christopher Green.<sup>22</sup>

Plans A-1 and A-2 - Both of these are examples of a negative rate plan. Each would provide a guaranteed minimum income equal to 50 percent of a family's poverty line, which is the breakeven level of income. The allowance tax schedule, however, would take different forms for each. For plan A-1, a 50 percent flat rate would fill 50 percent of the family's poverty gap. For plan A-2, a regressive schedule would levy a 75 percent tax rate on the first third of poverty line income, and marginal rates of 50 and 25 percent, respectively, on the remaining thirds.

Plans B-1 and B-2 - These also are examples of a



negative rates plan. However, they would use the value of present tax exemptions and minimum standard deductions (EX-MSD) as breakeven lines, and they would guarantee a minimum income equal to 50 percent of the value of EX-MSD. In both plans, the negative tax rate would be a flat 50 percent applied to unused exemptions and deductions. The difference in these two plans lies in their treatment of the aged. In plan B-1, each member of the tax unit would be allowed a single exemption and minimum standard deduction. Plan B-2, as in the case of the present tax system, would double the allowance for EX-MSD for the aged--that is, for the filer and spouse who were 65 years old or more.

Plan C - This set-aside plan is a special form of a negative rates plan which, as a variant of plans A-1 and A-2, merits separate attention. A zero percent rate would be applied on the first half of income up to the family's poverty line and a 50 percent tax rate on the second half.

Plans D-1 and D-2 - Two examples of social dividend plans would guarantee a minimum income designed to "assure" that the family would not live in poverty. The guarantee under both would be equal to the poverty line, but the tax schedules would take different forms. Plan D-1 would levy a flat 50 percent allowance tax rate; this would require a 15 percent finance tax rate. Plan D-2 would depend on a single social dividend tax rate of  $33\frac{1}{3}$  percent, with no distinction between the allowance and finance tax schedules.

The plans discussed above in this section are the most current to be found and, in form, represent possible future plans. The mathematical model developed in Chapter III of this dissertation can analyze each of them.

#### PRESENT STATE OF THE ART TO ANALYZE NIT AND GMI PROPOSALS

In the above sections are presented the types of in-

come transfer programs available today. Just what information Congress requires before they will accept one is shown in the following paragraphs.

Congress's first anxiety in accepting these programs is Cost. The following statements point out where the concern is. The first statement is that of Senator Long of Louisiana, the Chairman of the Committee on Finance.<sup>23</sup>

"Members of the Committee on Finance will be most interested in the Department's estimate of the cost of the bill and the assumptions on which the cost estimates are based. The committee has already taken a preliminary look at the Department's cost estimates and has found what can only be considered unrealistic and contradictory assumptions. Cost is going to be a significant factor in the Committee's consideration of H.R. 16311; and for that reason, we need hard, reliable data. Up to now, the Department has been unable to supply it."

With respect to costing H.R. 16311, Senator Williams of Delaware expressed the following sentiment;<sup>24</sup>

"We want them accurate; we don't want the ballpark figures. That is the point. And we don't want it like estimates on programs presented here before, where the idea was to get something through the Congress and worry about the costs later.

I want the kind of figures that you wouldn't object at all if somebody got the idea to include in the bill a ceiling that if and when it went over these projected costs the law would become null and void and have to become re-enacted. I want the kind of figures you are willing to accept."

When it was suggested that cost not be placed in such an important position in the question of accepting an income transfer program, Senator Bennett of Utah had the following to say:<sup>25</sup>

"But we on this committee have the responsibility of providing income to balance the Federal budget. That is our primary responsibility. So, we have to deal with direct money costs, and we cannot say, 'Well, it is fine; we can have a budget deficit of \$25 billion next year because we have made this as a social investment.'"

With all the obvious concern over cost, the President's Proposal was tested by a small experiment in a confined region. This testing was done by the Office of Economic Opportunity and was called the New Jersey Experiment. This experiment was best summarized by the Honorable James D. Hodgson, Secretary of Labor.<sup>26</sup>

#### THE NEW JERSEY EXPERIMENT

The income maintenance experiment in New Jersey is an OEO funded test program operated by the Institute for Research on Poverty. The study involves 700 families over a three year period. The families report monthly on the amount of earned income they receive after which a subsidy is granted which brings their resources up to a minimum level deemed adequate to meet family needs.

It is the author's opinion that when one is concerned with a program that will cost upwards of 9 billion dollars per year and affect some 24 millions of people; a test of 700 families in one community cannot possibly be a sufficient costing procedure. An initial out-lay of funds, miniscule in proportion to funding an unsuccessful poverty program, can be made to collect national income data properly for varying family sizes within selected geographic



areas to more fully test the system. Since a great deal of this data collection is already done by the Census Bureau and Internal Revenue Service, the task is not formidable.

Once the above national income data is available the mathematical model and management information and control system of this dissertation may be used and if used would present valuable cost data so far unavailable. In fact, the only real investigation of various welfare program costs that are found in the literature are those by Gail R. Wilensky,<sup>27</sup> "An Income Transfer Computational Model", and Nelson McClung,<sup>28</sup> "Estimates of Income Transfer Effects." Both of these models, however, lack geographic area analysis, corporate responsibility, varying marginal positive taxes to pay for the subsidy, of individual income bracket analysis. Also neither is in the form of a management information and control system to allow on-line processing.

Other areas of concern to Congress are the level at which the minimum income should be set, the Breakeven Level of Income required for subsistence, the means to pay for the subsidy, and the changes that might occur in national income distribution. All these concerns can realistically be tested through the mathematical model of this dissertation. Therefore, this dissertation is an



innovation that if incorporated by Congress can be useful in selecting an optimal welfare program to aid the poor of this country.

To further point out the lack of an analytical system the following literature search was done. In the analytical areas such quantitative literature as The Journal of Management Science; the United State's, Canadian, and Indian Journals of Operations Research; Econometrica, and the Journal of Applied Mathematics have been researched. In the economic area books and journals in which outlines of various NIT and GMI proposals are to be found were investigated. Governmental hearings on income maintenance programs were read. Proceedings of various symposiums on NIT and GMI proposals were studied. Proposals and test results of individual proposals submitted to the Department of Health, Education, and Welfare were investigated. However, not once was a method found, never mind a system which could realistically compare NIT or GMI proposals. No solution at all was seen in the quantitative journals and in the economic literature most verbage was in justifying NIT or GMI proposals, not in finding comparisons amongst alternatives. The only analytical work found to examine NIT or GMI proposals other than the two mentioned above is that performed by each NIT or GMI proposer and then this

analytical work was just pertinent to the proposer's particular system.

Each proposal had its own guaranteed level of income, its own breakeven level, its own tax function, and ignored income distribution in the country. None of the proposers tried altering any of the above parameters and seeing their effect on his system. None of the investigated proposers considered the role of corporate taxes to pay for the system. And, finally, none of the investigated proposers considered the dynamics of inflation or recession on their system.

With this lack in the state of the art, the analytical system presented in this paper is a first step toward coordination. It may, in some areas, be incomplete, such as in the behavioral areas, but it is flexible enough to be updated, with little effort, once the initial vagueness found today is dissipated.

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11. This is the crux of the philosophical difference between the negative income tax and the guaranteed minimum income. The most emotionally-charged criticism of the GMI concept is that it "destroys the incentive to work." Friedman's negative income tax model has this incentive built in: "An extra dollar

earned always means more money available for expenditure." (op. cit.) Schwartz points out however, that this incentive is, "...obtainable only at the expense of sacrificing the assurance that all families will receive the income they need." (op. cit.) That is, if the Friedman plan is to be considered as a welfare plan, there is question as to the extent of its effectiveness.

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C H A P T E R   I I I  
T H E   M A T H E M A T I C A L   M O D E L

DEFINITION OF VARIABLES USED IN CHAPTER III

<u>VARIABLE</u>	<u>DEFINITION</u>
BLI	Breakeven Levels of Income or that income up to which a subsidy is to be given.
CS	Cost of the Subsidy
F	Family size indicator
G	Geographic area indicator
$I_m$	Guaranteed Minimum Income
K	Number of family sizes
L	Number of geographic areas
LIAB] <sub>B</sub> <sup>A</sup>	The tax liability between incomes A and B
MT <sub>C</sub>	Marginal tax on Corporations
MT <sub>n</sub>	Marginal tax on the non-subsidized population
S	Subsidy
TCS	Total Cost of the Subsidy
TS	Total Subsidy
T <sub>S</sub>	Negative tax on the subsidy income
Y <sub>A</sub>	Income added to the population due to a minimum income being added
Y <sub>AS</sub>	Income of subsidized population before the minimum income is added + Y <sub>A</sub>
Y <sub>AST</sub>	Y <sub>AS</sub> minus the negative tax on income earned
Y <sub>n</sub>	Income of the non-subsidized population
Y <sub>S</sub>	Income of the subsidized population

<u>VARIABLE</u>	<u>DEFINITION</u>
$Y_T$	Total income earned in the whole country
$Y_{TA}$	Total added income to the country due to adding a minimum income.
$Y_{TAS}$	Total $Y_{AS}$ for the country
$Y_{TAST}$	Total $Y_{AST}$ for the country
$Y_{Tn}$	Total $Y_n$ for the country
$Y_{TS}$	Total $Y_S$ for the country
$\emptyset$	Number of units earning a given yearly income for the country.
$\emptyset_n$	Number of units earning a given yearly income for the non-subsidized segments of the population.
$\emptyset_S$	Number of units earning a given yearly income for the subsidized segments of the population.
$\Phi$	Population variable
$\Phi_n$	Population of the non-subsidized segments of the population.
$\Phi_S$	Population of the subsidized segments of the population
$\Phi_T$	Total population of the country



THE MATHEMATICAL MODEL

In Chapter II, Section A the socio-political differences between negative income tax and guaranteed minimum income proposals were discussed. It will be shown in this chapter that no matter how different the two plan types are in arguments they are similar mathematically and can be represented by one mathematical model.

Each plan, for many various reasons, includes a term for some stated minimum income to be given the subsidized population. Each plan also advocates some maximum level of income up to which a subsidy is to be given. Also, each plan includes some tax function to be levied against the subsidized population for all income earned above the stated minimum income up to the maximum subsidized income.

It will be shown in the development how the above concepts, no matter how variant in argument between proposals, are mathematically the same. (Note to the reader: the analysis to be presented in this chapter will be performed in three phases. Each phase will build upon the last adding one more level of sophistication until, finally, in the last phase the model to be used in the management information and control system of Chapter V is presented. One may jump to phase three without any loss of continuity if "use" more than development is one's prime concern.)

## PHASE 1 - Model Development in Simplified Terms

As in all analyses that wish to be complete in development, the analysis to be performed here will start with the most simplified case and from there extend the concepts until in the end, one reaches a representation of the real world.

To be valid all proposals, must consider some actual distribution of income in the United States. In this simplified case, the distribution of income to be used will be a single model encompassing all earners, without classification, in the economy. Figure 3-1 represents this income distribution. Along the x-axis is income and along the y-axis is the frequency at which each income is earned in the United States. The data found in Figure 3-1 is found in the Census Bureau's pamphlet, "Consumer Income".<sup>1</sup> The data points were joined free-hand to show a continuous distribution. In Chapter IV exact data fitting techniques will be presented and in Chapter VI computerized plots are shown.

Once an income distribution is found, the first step of this simplified case is to add a graphical representation of a maximum subsidized income. To keep this analysis simple, it is assumed that this maximum subsidized income is the same for all earners in the country. Symbolically, this term is labeled "BLI" for "Breakeven

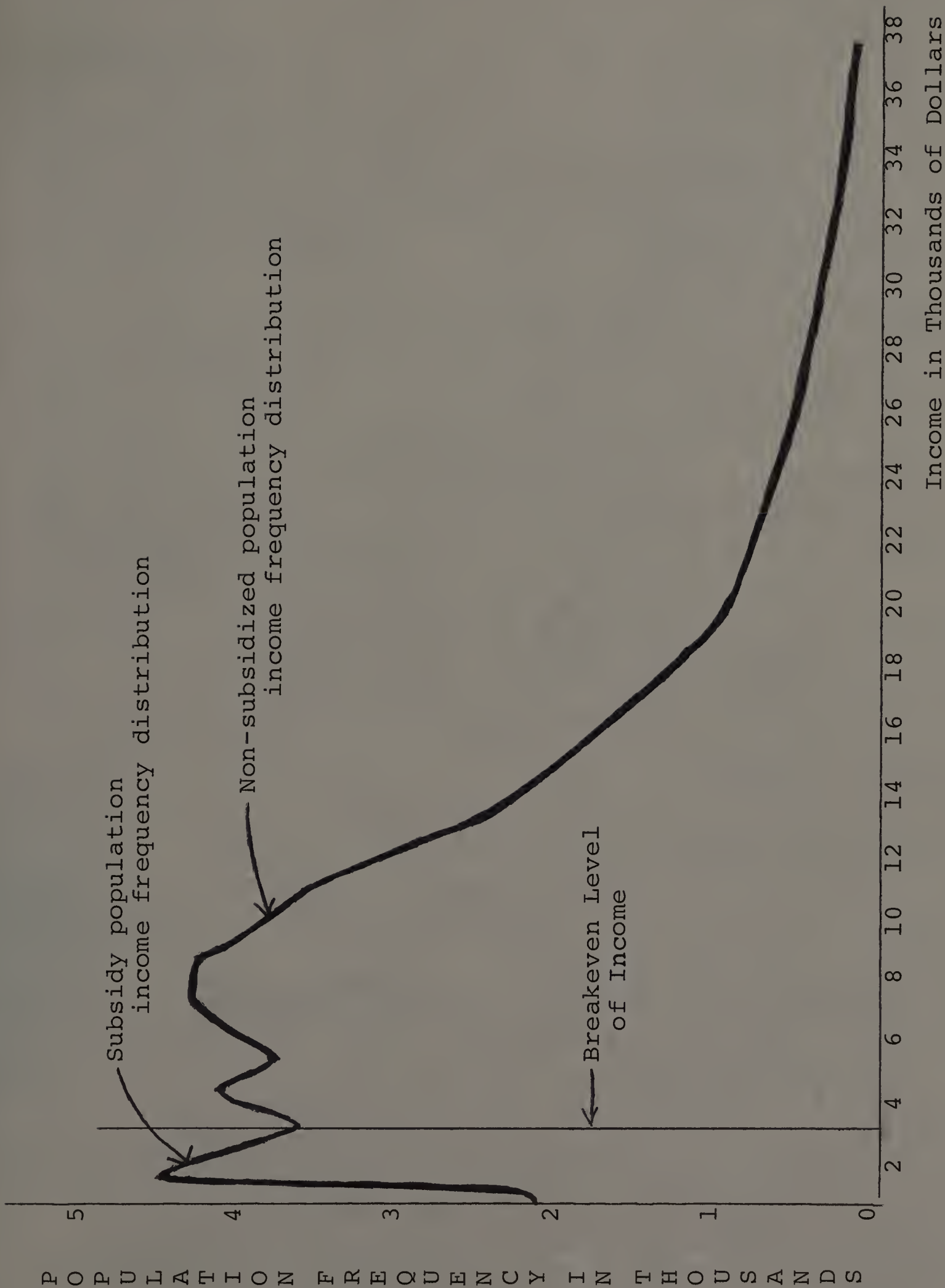


FIGURE 3-1

UNITED STATES INCOME FREQUENCY DATA WITH SEGMENTATION BY A BREAK-EVEN LEVEL OF INCOME (BLI)

Level of Income", the classifier used in most guaranteed minimum income proposals. Figure 3-1 also shows this addition to the all inclusive income frequency distribution.

As pointed out above, each negative income tax and guaranteed minimum income proposal set some level of income below which each member of the population must not earn. Symbolically, this minimum income is designated " $I_m$ ." Again to keep the model simple this term is made a constant for the whole subsidized population or that income given to each member of the population in Figure 3-1 earning less than the Breakeven Level of Income (BLI). Figure 3-2 represents the immediate effect this minimum income has on the income frequency distribution used for the United States.

Looking at Figure 3-2 one sees that the addition of a guaranteed minimum income results in a partial translation of axes from a zero x-axis reference to a x-axis reference at the minimum income ( $I_m$ ). However, once the new frequency distribution reaches the Breakeven Level of Income (BLI), the minimum income has introduced an additive effect. This additive effect creates an inequity socially (i.e., a subsidized member of the population earns more than a member who must pay for the subsidy especially at incomes approaching the Breakeven Level of



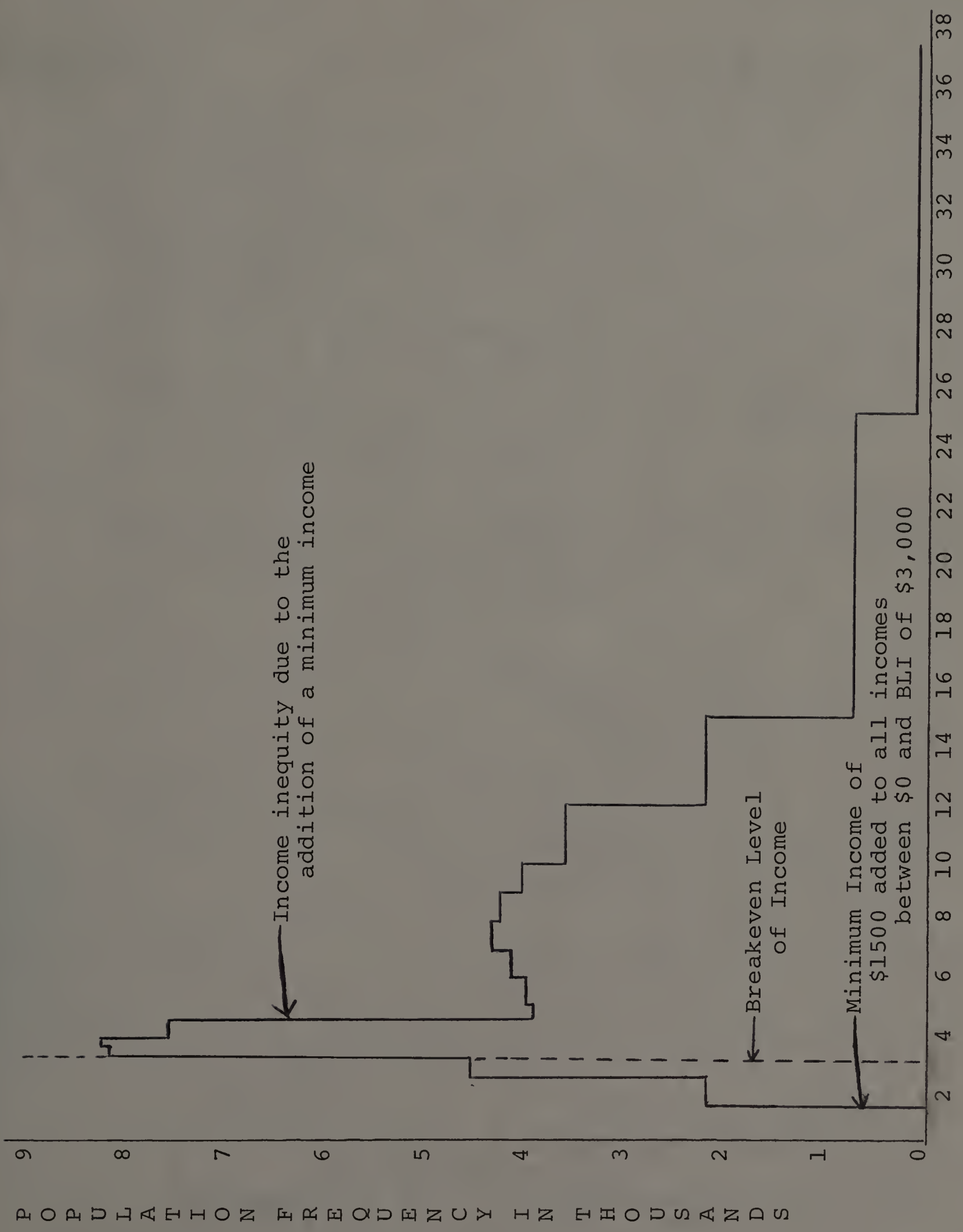


FIGURE 3-2

UNITED STATES INCOME FREQUENCY DATA SEGMENTED WITH THE ADDITION OF A MINIMUM INCOME

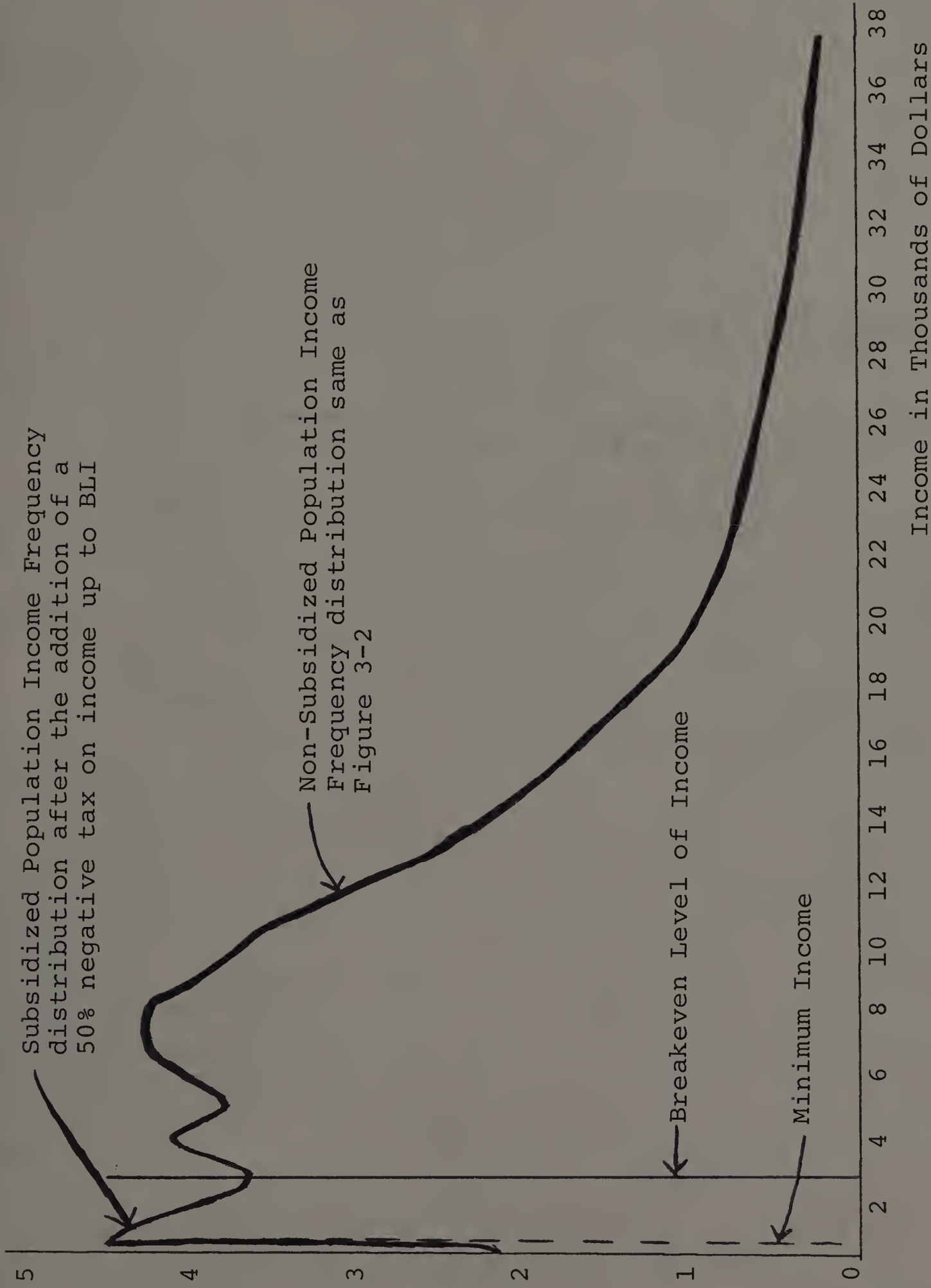
Income). This mathematical and social inequality is a basic reason why each proposal type has some tax it places on the subsidized population. This tax, mathematically, must be such that it negates the additive effect but does not tax income below the minimum income. Figure 3-3 shows the graphical result on the subsidized population.

The shape of Figure 3-3 between the minimum income and the Breakeven Level of Income will vary with respect to the kind of tax function applied. The management information and control system of Chapter V more fully describes different tax structures as does the system testing of Chapter VI.

The graphical representation of concepts inherent to each negative tax or guaranteed minimum income proposal help in giving an understanding of these concepts. However, to allow computer analysis of all concepts more than just graphs are required. What is needed is some mathematical model that accurately represents all concepts and that lends itself to computerization. Such a mathematical model is developed below.

As mentioned above each proposal to be realistic must base its findings on an accurate picture of income in the United States. Figure 3-1 shows a plot of a simplified frequency distribution of income. Mathematically,

POPULATION FREQUENCY IN THOUSANDS



Subsidized Population Income Frequency distribution after the addition of a 50% negative tax on income up to BLI

Non-Subsidized Population Income Frequency distribution same as Figure 3-2

Breakeven Level of Income

Minimum Income

Income in Thousands of Dollars

FIGURE 3-3

UNITES STATES INCOME FREQUENCY DATA SEGMENTED WITH AN ADDITION OF A MINIMUM INCOME AND A NEGATIVE TAX ON THE SUBSIDIZED POPULATION

this is the same as saying that there exists a total population of  $\Phi$  economic units. Each unit receives an "earned" yearly income, (i.e., the term for income must be dynamic enough to include all possible Internal Revenue Services' definitions of income such as salaries, interest on savings, dividends, rents, etc.). The number of units,  $\emptyset$ , which earns a given income depends upon the magnitude of the income, or mathematically,

$$\emptyset = \emptyset(I) \quad (3-1)$$

Once one has a mathematical representation of the frequency distribution as in equation 3-1, one has modeled Figure 3-1. From this frequency distribution it is now mathematically possible to get an equation of total population. Since the frequency distribution is a set of data points giving for each income the frequency of the population earning this income, one can simply sum over these data pairs and compute total population, or

$$\Phi = \sum_{L=1}^K \emptyset_L(I) \quad (3-2)$$

where K is the number of data points.

However, if one does not want the analysis' validity restricted to only the data points but wishes to be able to investigate any income breakdown between points, integral calculus can be introduced to supply another tool, or



$$\Phi = \int_0^{\infty} \emptyset(I) \cdot dI \quad (3-3)$$

where  $\emptyset(I)$  is a function determined by curve fitting techniques to be described in Chapter IV of this dissertation.

To be most realistic the remainder of this development will be in terms of integral calculus.

To this point the analysis has mathematically modeled the computation of total population given any income frequency distribution. However, Figure 3-1 shows that the total population is divided into two segments once one considers a Breakeven Level of Income. To represent this segmentation mathematically, consider first that portion of the population that is to receive a subsidy. This segment is modeled by curve fitting those data points between the incomes of zero and the Breakeven Level of Income. Given this fitted function or

$$\emptyset_S = \emptyset_S(I) \quad (3-4)$$

the total subsidized population becomes,

$$\Phi_S = \int_0^{\text{BLI}} \emptyset_S(I) \cdot dI \quad (3-5)$$

To model the non-subsidized portion of the population it is necessary to curve fit all data points for income above the Breakeven Level of Income and compute

its fitted income frequency distribution,

$$\phi_n = \phi_n(I) \quad (3-6)$$

Then the total non-subsidized population becomes,

$$\Phi_n = \int_{BLI}^{\infty} \phi_n(I) \cdot dI \quad (3-7)$$

Therefore, total population becomes,

$$\Phi = \Phi_S + \Phi_n \quad (3-8)$$

Or,

$$\Phi = \int_0^{BLI} \phi_S(I) \cdot dI + \int_{BLI}^{\infty} \phi_n(I) \cdot dI \quad (3-8')$$

Using integral calculus the total yearly income is calculated by the first moment of the total population distribution, or

$$Y_T = \int_0^{\infty} I \cdot \phi(I) \cdot dI \quad (3-9)$$

total income for the subsidized population becomes,

$$Y_S = \int_0^{BLI} I \cdot \phi_S(I) \cdot dI \quad (3-10)$$

and the total income for the non-subsidized population is,

$$Y_n = \int_{BLI}^{\infty} I \cdot \phi_n(I) \cdot dI \quad (3-11)$$

The total income for the country is then restated as

$$\begin{aligned} Y_T &= Y_S + Y_n \\ &= \int_0^{BLI} I \cdot \phi_S(I) \cdot dI + \int_{BLI}^{\infty} I \cdot \phi_n(I) \cdot dI \end{aligned} \quad (3-12)$$

Given the income picture for the simplified representation of the country, the model now considers the changes that occur when a subsidy or the guaranteed min-

imum income of Figure 3-2 is introduced. The immediate change is an addition to yearly income for the subsidized portion of the population. Since each  $\Phi_S$  member of the total population receives the subsidy, the added income is modeled by

$$Y_A = \int_0^{\Phi_S} I_m \cdot d\Phi_S \quad (3-13)$$

then,

$$Y_{AS} = Y_S + Y_A \quad (3-14)$$

and,

$$Y_T = Y_{AS} + Y_n \quad (3-15)$$

or

$$\begin{aligned} Y_T &= \int_0^{BLI} I \cdot \Phi_S(I) \cdot dI + \int_0^{\Phi_S} \cdot d\Phi_S + \int_{BLI}^{\infty} I \cdot \Phi_n(I) dI \\ &= \int_0^{BLI} I \cdot \Phi_S(I) \cdot dI + \int_{BLI}^{\infty} I \cdot \Phi_n(I) \cdot dI + I_m \cdot \Phi_S \end{aligned} \quad (3-16)$$

To model what many authors refer to as a negative tax (i.e., one that subtracts from a subsidy) so as to negate the subsidy proportionally as one earns more income the tax must not affect the minimum income, or mathematically

$$I - I \cdot T_S(I) \geq I_m \quad (3-17)$$

This tax must also assure that

$$I \cdot T_S(I) + I_m \leq BLI \quad (3-18)$$

That is the after tax income plus the subsidy must be less than or equal to the Breakeven Level of Income.

When this negative tax is placed on income

earned by the subsidized segment of the population, the income earned by that segment is altered as follows:

$$\begin{aligned}
 Y_{AST} &= \int_0^{BLI} I \cdot \phi_S(I) \cdot dI + \int_0^{\phi} I_m \cdot d\phi_S - \int_0^{BLI} I \cdot T_S(I) \cdot \phi_S(I) \cdot dI \\
 &= \int_0^{BLI} I \cdot \phi_S(I) \cdot [1 - T_S(I)] \cdot dI + I_m \cdot \phi_S \quad (3-19)
 \end{aligned}$$

One has now imposed on the economy a guaranteed minimum income, a negative tax, and some Breakeven Level of Income. Given these new additions it is now necessary to develop a term that will relate the subsidy cost to the country. From the above analysis it is readily seen that the subsidy is,

$$S = I_m \cdot \phi_S - \int_0^{BLI} I \cdot T_S(I) \cdot \phi_S(I) \cdot dI \quad (3-20)$$

Once one has determined the cost of the subsidy, one must consider the question as to how to pay for it. Here, the most critical assumption of the development must be made. One must assume that our present National Budget situation is in equilibrium before the cost of the subsidy is considered, or

$$\begin{aligned}
 &\text{Present Government Spending} \\
 &= \text{Present Government Income} \\
 &+ \text{Non-Subsidy Deficit} \quad (3-21)
 \end{aligned}$$

With this assumption one can isolate the cost of the subsidy and pay for it by adding a marginal tax to



the present progressive tax rate applied to the non-subsidized portion of the population and consider a marginal tax to the present corporate tax structure. Both of these concepts must be dynamic enough to vary from no subsidy responsibility to full subsidy responsibility. When both forms of marginal taxes are used to pay for the subsidy, one has the following equilibrium situation,

$$\begin{aligned} \text{Cost of Subsidy} &= \text{Marginal Non-Subsidy Tax} \\ &+ \text{Marginal Corporate Tax} \end{aligned} \quad (3-22)$$

or

$$CS = \int_{BLI}^{\infty} MT_n(I) \cdot I \cdot \phi_n(I) \cdot dI + MT_c \quad (3-22')$$

The use of integral calculus with respect to a marginal non-subsidy tax allows one also to determine the tax burden applied to any portion of the non-subsidized population,

$$T_{LIAB} \Big|_A^B = \int_A^B MT_n(I) \cdot I \cdot \phi_n(I) \cdot dI \quad (3-23)$$

where A and B are the lower and upper income bounds of any segment of the non-subsidized population

Equation 3-23 is very important because it can immediately test the marginal tax burden of any portion of the non-

subsidized population. By looking at a sequence of income segments it can give insight into the fairness of any particular marginal tax and, in fact, give a possible direction toward developing a more equitable tax structure. The management information and control system of Chapter V will show more fully how one may use this equation to its fullest.

The development of the first phase of the analysis is now complete. In the next two phases all of the above developed equations will be the basis from which more sophisticated models of our economy will be constructed.

#### PHASE 2 - Model Development to Include Family Size

The simplified assumptions used in the first phase of the mathematical development were in summary;

a) A single income frequency distribution including, without classification, all "earners" in the United States.

b) A single-valued Breakeven Level of Income for all "earners" in the United States.

c) A single-valued minimum income applied to each member of the to-be-subsidized population.

d) A negative tax applied to each member of the subsidized population as a function of income only.

e) Marginal taxes applied to both the subsidy pay-

ing population's present progressive tax and to corporate earnings without classifications.

No matter how simple and unrealistic the above assumptions may seem, one is able to derive from them the mathematics to answer the following questions;

a) Given any income frequency distribution of the United States; what is the total "earning population", and what is the total income of this "earning population"?

b) Adding a Breakeven Level of Income to segment the "earning population" into a to-be-subsidized population and the subsidy-paying population; what is the resulting population of each segment, and what total income does each segment attain?

c) Setting a minimum income to be given each member of the subsidized population; what is the resulting income now found in the subsidized segment of the population, and what is the subsidy's immediate effect on total income?

d) Introducing a negative tax on the subsidized population to eliminate the inequities resulting from the addition of a subsidy; what is the realized income of the subsidized population, and what is the total subsidy given?

e) Creating a marginal tax on both the earnings of the subsidy-paying population and on corporate earnings;

what is the added total cost to both to support the subsidy, and what is the cost of the subsidy to any income group within the subsidy-paying population?

The answer to the above questions which are computable even in this simplified model, given real income data, are results not always accurately available to decision makers. Therefore, the mathematical model of Phase 1 is an important innovation in itself, but is, also, so structured that it can be improved to more fully model our economy as will be shown here in Phase 2 and again in Phase 3.

In reviewing the major assumptions of the model developed in Phase 1, it is found that all earners in the population were collected into one income frequency distribution. However, this single classification of earners is not in accord with most advocates of negative tax and minimum income proposals. The authors of these proposals believe that the number of people in a family largely affects the amount of subsidy they should receive, and the Breakeven Level of Income they need to survive. This dissertation will not make a judgement as to the validity of these beliefs, but must, to test all proposals, include in its modeling the ability to handle differences due to family sizes.



The first concept that is changed as a result of the consideration of family size in modeling proposals is how to distinguish family size in developing an income picture of the country. The answer is to collect the income data as a function of family size. Then, for each family size generate a graph similar to Figure 3-1. Mathematically, this is the same as saying there exists for each family size a total population of  $\Phi(F)$  economic units. Each unit in  $\Phi(F)$  receives a yearly income. The number of units,  $\emptyset(F)$ , which earns a given yearly income depends upon the magnitude of the income, or

$$\emptyset(F) = \emptyset(I, F) \quad (3-24)$$

It is now possible to curve fit each of these income frequency distributions and get the earning population of each family size segment, or

$$\Phi(F) = \int_0^{\infty} \emptyset(I, F) \cdot dI \quad (3-25)$$

and the total earning population of the economy becomes simply the summing over family sizes of the population of equation (3-27), or

$$\Phi_T = \sum_{F=1}^K \int_0^{\infty} \emptyset(I, F) \cdot dI \quad (3-26)$$

where  $K$  is the number of family sizes to be considered.

Given the income frequency distribution and earning population size for each family size along with the total earning population, it is now necessary to consider a way of introducing the concept Breakeven Level of Income or that income at which one stops subsidizing earners in the population. In Phase 1 this was a single-valued concept for the whole population. Now that family size is a classifier, the analysis must consider the possibility that one might wish to investigate different Breakeven Levels of Income for each family size. This is done by assigning a term for the Breakeven Level of Income to each income frequency distribution for different family sizes. Mathematically, this is the same as saying that the Breakeven Level of Income is a function of family size or,

$$BLI = BLI (F) \quad (3-28)$$

This assigning of a Breakeven Level of Income to each family size's income frequency distribution, also, divides each family size's income frequency distribution into two segments (i.e., that segment to receive a subsidy and that segment that pays for the subsidy). This is modeled by a curve fitting each segment of each family's income frequency distribution. For the subsidized segments, one has,

$$\phi_S(F) = \phi_S(I, F) \quad (3-29)$$

For the subsidy paying segments one has,

$$\phi_n(F) = \phi_n(I, F) \quad (3-30)$$

From these concepts compute the total earning population of each segment for each family size by

$$\phi_S(F) = \int_0^{\text{BLI}(F)} \phi_S(I, F) \cdot dI \quad (3-31)$$

and,

$$\phi_n(F) = \int_{\text{BLI}(F)}^{\infty} \phi_n(I, F) \cdot dI \quad (3-32)$$

Combining equations (3-31) and (3-32) the total earning population for each family size is,

$$\phi(F) = \phi_S(F) + \phi_n(F) \quad (3-33)$$

or

$$\phi(F) = \int_0^{\text{BLI}(F)} \phi_S(I, F) \cdot dI + \int_{\text{BLI}(F)}^{\infty} \phi_n(I, F) \cdot dI \quad (3-33')$$

From equations (3-33) and (3-33') and summing over family size the total population is

$$\begin{aligned} \phi_T &= \sum_{F=1}^K \phi(F) = \sum_{F=1}^K [\phi_S(F) + \phi_n(F)] \\ &= \sum_{F=1}^K \left[ \int_0^{\text{BLI}(F)} \phi_S(I, F) \cdot dI + \int_{\text{BLI}(F)}^{\infty} \phi_n(I, F) \cdot dI \right] \end{aligned} \quad (3-34)$$

Given the population and income frequency distribution equations, the yearly income equations become the

the first moments of the population equations. For the subsidized segments the yearly income for each family is,

$$Y_S(F) = \int_0^{BLI(F)} I \cdot \phi_S(I, F) \cdot dI \quad (3-35)$$

and for the subsidy paying segments the yearly income for each family size is,

$$Y_n(F) = \int_{BLI(F)}^{\infty} I \cdot \phi_n(I, F) \cdot dI \quad (3-36)$$

Therefore, the total income of the subsidized and subsidy paying population for all families are,

$$Y_{TS} = \sum_{F=1}^K Y_S(F) = \sum_{F=1}^K \int_0^{BLI(F)} I \cdot \phi_S(I, F) \cdot dI \quad (3-37)$$

and,

$$Y_{Tn} = \sum_{F=1}^K Y_n(F) = \sum_{F=1}^K \int_{BLI(F)}^{\infty} I \cdot \phi_n(I, F) \cdot dI \quad (3-38)$$

From equations (3-37) and (3-38) the total yearly income of the public sector of the economy is,

$$\begin{aligned} Y_T &= Y_{TS} + Y_{Tn} = \sum_{F=1}^K Y_S(F) + \sum_{F=1}^K Y_n(F) \\ &= \sum_{F=1}^K [Y_S(F) + Y_n(F)] \\ &= \sum_{F=1}^K \left[ \int_0^{BLI(F)} I \cdot \phi_S(I, F) \cdot dI + \int_{BLI(F)}^{\infty} I \cdot \phi_n(I, F) \cdot dI \right] \end{aligned} \quad (3-39)$$



It is now necessary to introduce the guaranteed minimum income concept into the analysis. In Phase 1, the minimum income was a constant given to all members of the to-be-subsidized population without classification. This constant minimum income models many of the proposals of Chapter II. However, there are some proposals that state that different family sizes should have different minimum incomes. Usually these differences are stated in terms of child subsidies. These varying views are modeled by assigning a guaranteed minimum income variable to each family size income frequency distribution. This results a graph similar to Figure 3-2 for each family size. By allowing the decision makers to set the value for each family size, the variable can include child subsidies as a simple addition. Therefore, the guaranteed minimum income becomes a function of family size or in the case of some authors the number of children. Mathematically this is,

$$I_m = I_m (F) \quad (3-40)$$

Since each member of a particular family size who is to-be-subsidized receives the minimum income one has,

$$Y_A (F) = \int_0^{\Phi_S (F)} I_m (F) \cdot d[\Phi_S (F)] \quad (3-41)$$

or,

$$Y_A (F) = I_m (F) \cdot \Phi_S (F) \quad (3-41')$$

Therefore, for each family size the total income for the subsidized population is,

$$\begin{aligned}
 Y_{AS}(F) &= Y_S(F) + Y_A(F) \\
 &= \int_0^{BLI(F)} I \cdot \phi_S(I, F) \cdot dI + I_m(F) \cdot \Phi_S(F)
 \end{aligned}
 \tag{3-42}$$

From equations (3-41) and (3-42) the total subsidy income, to this point, given the subsidized population by summing over family sizes is,

$$Y_{TA} = \sum_{F=1}^K Y_A(F) = \sum_{F=1}^K I_m(F) \cdot \Phi_S(F)
 \tag{3-43}$$

Then, the total income in the subsidized segment of the population is,

$$\begin{aligned}
 Y_{TAS} &= Y_{TA} + Y_{TS} \\
 &= \sum_{F=1}^K [I_m(F) \cdot \Phi_S(F) + \int_0^{BLI(F)} I \cdot \phi_S(I, F) \cdot dI]
 \end{aligned}
 \tag{3-44}$$

and the total income for the population is,

$$Y_T = Y_{TAS} + Y_{Tn}
 \tag{3-45}$$

Because of the addition of a guaranteed minimum income to each family size segment of the earning population, a question of equity arises in the opinion of many experts in the area. These experts point out that the

members of the subsidy-paying population at incomes near the Breakeven Level of Income for each family size may receive less income due to an across the board minimum income than those members at the high end of the subsidized segments who receive the minimum income. It is partly because of these questions of equity that negative income taxes are introduced into most proposals. These negative taxes are such that no member of the subsidized population will earn more than the Breakeven Level of Income as a result of the addition of a guaranteed minimum income, and also that no member of the subsidized population should earn less than the set guaranteed minimum income for each family size to which he belongs, or

$$I - I \cdot T_S(I, F) \geq I_m(F) \quad (3-46)$$

and,

$$I \cdot T_S(I, F) + I_m(F) \leq BLI(F) \quad (3-47)$$

From the analysis above it can be seen that the model allows a different negative tax ( $T_S$ ) to be placed on the income earned by the subsidized population within each family size. However, a single negative tax is also allowed.

Due to the addition of negative taxes to the analysis, the income picture of each family size segment is changed. The change in the subsidized segments is,

$$\begin{aligned}
Y_{AST}(F) &= \int_0^{BLI(F)} I \cdot \phi_S(I, F) \cdot dI \\
&+ \int_0^{\Phi_S(F)} I_m(F) \cdot d[\phi_S(F)] \\
&- \int_0^{BLI} I \cdot T_S(I, F) \cdot \phi_S(I, F) \cdot dI
\end{aligned} \tag{3-48}$$

or,

$$\begin{aligned}
Y_{AST}(F) &= \int_0^{BLI(F)} I \cdot \phi_S(I, F) [1 - T_S(I, F)] \cdot dI \\
&+ I_m(F) \cdot \Phi_S(F)
\end{aligned} \tag{3-48'}$$

and again the total subsidy is,

$$Y_{TAST} = \sum_{F=1}^K Y_{AST}(F) \tag{3-49}$$

and the new total income of the population is

$$Y_T = Y_{TAST} + Y_N \tag{3-50}$$

The economy is now divided into income frequency distributions by family sizes. To each of these income frequency distributions is imposed a guaranteed minimum income, a negative tax, and a Breakeven Level of Income. Given these new additions to each family size portion of the population the cost of the subsidy the country must absorb for each family size is,

$$S(F) = I_m(F) \cdot \Phi_S(F) - \int_0^{BLI(F)} I \cdot T_S(I, F) \cdot \phi_S(I, F) \cdot dI \tag{3-51}$$



and the total subsidy for the country is,

$$TS = \sum_{F=1}^K S(F) \quad (3-52)$$

Once the cost of the subsidy is determined, the question as to how to pay for it arises. Here as in Phase 1 the most critical assumption of the analysis is made. That the cost of the subsidy is an added cost to the economy and will be paid for by new taxation. Even if the subsidy cost is to be paid out of present tax dollars, how the cost is distributed must be determined. This cost distribution is accomplished by considering the taxes to pay for the subsidy to be in the form of a marginal tax on the progressive tax rate of earners in the non-subsidy segments and on corporate income. Therefore,

$$\begin{aligned} \text{Cost of Subsidy} &= \text{Marginal Non-Subsidy Tax} \\ &+ \text{Marginal Corporate Tax} \end{aligned} \quad (3-53)$$

or,

$$\begin{aligned} CS(F) &= \int_{BLI(F)}^{\infty} MT_n(I,F) \cdot I \cdot \phi_n(I,F) \cdot dI \\ &+ \frac{1}{K} MT_C \end{aligned} \quad (3-53')$$

where  $K$  is the number of family sizes

and the total cost is paid by,

$$TCS = \sum_{F=1}^K CS(F) \quad (3-54)$$

Equation (3-53) and (3-53') are very important. From these equations much subsection analysis to determine the tax burden placed on subsidy-paying earners in each family size segment between any income levels is possible using,

$$MT_{[LIAB]_A^B}(F) = \int_A^B MT_n(I,F) \cdot I \cdot \emptyset_n(I,F) \cdot dI \quad (3-55)$$

where A and B are the lower and upper income bounds of the segment to be analyzed

Using equation (3-55) and testing various sequences of segments of the population, one may be able to gain insights into the tax burdens. From these insights possible patterns may be recognized from which one might develop improved tax structures.

The development of the second phase of the analysis is now complete. In the next and last phase of the development the above analysis will become the basis from which the most realistic model of our economy will be constructed.

PHASE 3 - Model Development to Include Family size and Geographic Area.

The development of this third and final phase of the mathematical analysis of this dissertation is built upon the foundations set forth in Phases 1 and 2. So that this final phase may be understandable to those readers who have omitted the previous development of this chapter, a brief summary of those phases is given below. For those readers who have followed all the development the following review is helpful in pulling together the assumption changes between phases and thus, allows the reader to more readily see the additions to be found in this phase.

The Phase 1 summary is:

a) A single income frequency distribution including, without classification, all earners in the United States.

b) A single-valued Breakeven Level of Income for all earners in the United States.

c) A single-valued minimum income applied to each member of the to-be-subsidized population.

d) A negative tax applied to each member of the subsidized population as a function of income only.

e) Marginal taxes applied to both the subsidy-paying population's present progressive tax and to corporate earnings, without classification, to pay for the subsidy.

No matter how simple and unrealistic the above

assumptions may seem, the mathematics developed in Phase 1, based upon these assumptions, allow one to arrive at the following results:

a) A figure for the total earning population in the country and an accurate estimate of the income it earned.

b) The ability to divide the total population into a subsidized population segment and a subsidy-paying segment. Allowing the size of each segment to be determined by some arbitrary breakpoint called the Break-even Level of Income. The determination of the population size and income of each segment.

c) The income added to the subsidized segment of the population after the inclusion of a minimum income, and the effect this income addition has on total income.

d) The decrease in subsidy income due to the addition of a negative tax on earned income of the subsidized population and this decreases effect on total income.

e) The resulting total subsidy cost and the inclusion of a marginal tax on both the non-subsidized segment of the population and corporations to pay for it.

f) The subsidy responsibility of any income segment of the subsidy-paying segment of the population.

In Phase 2 of the development the above assumptions of Phase 1 were added to and the results intensified.



The added assumptions of Phase 2 are:

a) A set of income frequency distributions collected for each family size classification found in census records.

b) A separate Breakeven Level of Income assigned to each family size income frequency distribution.

c) A separate guaranteed minimum income assigned to each family size income frequency distribution.

d) A separate negative tax structure applied to the income earned in the subsidized segment of each family size's population.

e) A separate Marginal tax applied to the income earned in the non-subsidy segment of each family size's population to help pay for the subsidy given.

f) A Marginal corporate tax applied to income earned by all corporations to help pay for the subsidy.

From these added assumptions of Phase 2 it is possible to get the following results along with the results of Phase 1.

a) A population and income picture for each family size portion of the United States' population along with total population and income.

b) The ability to divide each family size's population into a subsidized segment and a subsidy-paying segment. Allowing the size of each segment within each

family size to vary according to the assigned Breakeven Level of Income for that family size and the calculation of population size and income for each segment.

c) The addition of income to each family size's subsidized segment due to the inclusion of a minimum income, and the effect these additions have on total income.

d) The decrease in subsidy income that occurs in each family size's subsidized population due to family size varying negative taxes applied to the income of these subsidized segments, and how these decreases affect total income.

e) The subsidy cost of each family size and the total subsidy cost of the country.

f) The ability to alter the subsidy-paying marginal taxes applied to each family size's non-subsidized segments to determine the proper tax structures to pay for the subsidy (i.e. larger family sizes may receive a high subsidy but the subsidy-paying segment of that family size will not bear the total brunt to pay for the subsidy. It can be spread out over smaller family sizes or one may penalize families who have more children than a stated limit, etc.).

g) The subsidy responsibility assigned to corporations.

h) Once marginal taxes are set for the subsidy-pay-

ing segments of each family size's population, the subsidy responsibility of any income segment in any family size.

Given the above summaries of the previous two phases, the major addition of this final phase of the analysis can be made.

When one looks at the income distribution within the United States and one wishes to develop programs to subsidize the poor within the income distribution, one first asks, is that part of the population who are to receive a subsidy such that one constant subsidy given to all is sufficient. As shown in phase 2 many authors do not believe so. They believe that the subsidy given should be a function of a family size. Their reasons to their coming to this conclusion is not the question here, but what is important in this dissertation is that their is a question. Therefore, there must be a means of considering that question. Phase two developed the means.

As there are authors who believe a single subsidy is not the answer but must be a function of family size, there are authors who even go beyond this means of classifying the poor. The authors now referred to are those who realize that the cost of living one sees in this country varies as one travels between geographic areas.

To some authors the geographic areas are large such as the Northeast, the Southeast, the Southwest, the Central States, etc. Then there are other authors who believe that differences occur amongst States of the Union. Still others believe even State classification is too large and they speak about county or city classification. Again this dissertation is not going to become involved in selecting proper classification, but does supply the tools to allow many types of analyses. Therefore, the addition to the previous developments of Phase 1 and 2 that is present here in Phase 3 is the presentation of geographic area.

The first concept that is changed as a result of the consideration of geographic area in modeling proposals is how to distinguish geographic area in developing an income picture of the country. The answer is to collect income data as a function of geographic area then further subsection these geographic area segments into family size income distributions. Then for each geographic area and family size a graph similar to Figure 3-1 can be generated. Mathematically, this is the same as saying there exists for each geographic area-family size segment a total earning population of  $\Phi(G,F)$  economic units. Each unit in  $\Phi(G,F)$  receives a yearly income. The number of  $\Phi(G,F)$ , which earns a given yearly income depends upon



the magnitude of the income, or

$$\emptyset(G,F) = \emptyset(I,G,F) \quad (3-56)$$

It is now possible to curve fit each of these frequency distributions (see Chapter IV of this dissertation) and obtain the earning population of each geographic area-family size segment, or

$$\Phi(G,F) = \int_0^{\infty} \emptyset(I,G,F) \cdot dI \quad (3-57)$$

and the total earning population of a particular geographic area is obtained by summing equation (3-57) over the number of family sizes, or

$$\Phi(G) = \sum_{F=1}^K \int_0^{\infty} \emptyset(I,G,F) \cdot dI \quad (3-58)$$

where K is the number of family sizes

also the total earning population of a particular family size is obtained by summing equation (3-57) over the number of geographic areas, or

$$\Phi(F) = \sum_{G=1}^L \int_0^{\infty} \emptyset(I,G,F) \cdot dI \quad (3-59)$$

where L is the number of geographic areas

The total earning population of the economy is the summation of equation (3-57) over both the number of family sizes and geographic areas, or

$$\Phi_T = \sum_{G=1}^L \sum_{F=1}^K \int_0^{\infty} \emptyset(I,G,F) \cdot dI \quad (3-60)$$

Given the income frequency distribution and earning population size for each geographic area-family size segment, and the total earning population for each geographic area, each family size, and for the whole country, it is

now necessary to consider a way of introducing the concept Breakeven Level of Income or that income at which one stops subsidizing earners in the population. Since one might wish to assign different Breakeven Levels of Income for each geographic area-family size segment of the earning population, the analysis must include a variable that can represent this division. This variable is assigned to each geographic area-family size graph similar to Figure 3-1. Mathematically, this is the same as saying that the Breakeven Level of Income is a function of family size and geographic area, or

$$BLI = BLI (G,F) \quad (3-61)$$

This addition of a Breakeven Level of Income to each geographic area-family size income frequency distribution divides each geographic area-family size income frequency distribution into two segments (subsidy and non-subsidy segments). This is modeled, mathematically, by curve fitting each segment of each geographic area-family size income frequency distribution. For the subsidized segments, one has

$$\phi_S (G,F) = \phi_S (I,G,F) \quad (3-62)$$

and the subsidy paying segment is

$$\phi_n (G,F) = \phi_n (I,G,F) \quad (3-63)$$

From these concepts the total earning population of each segment for each geographic area-family size are,

$$\Phi_S (G,F) = \int_0^{BLI(G,F)} \phi_S (I,G,F) \cdot dI \quad (3-64)$$

and

$$\Phi_n(G,F) = \int_{BLI(G,F)}^{\infty} \phi_n(I,G,F) \cdot dI \quad (3-65)$$

Combining equations (3-64) and (3-65) the total earning population of each geographic area-family size segment also is

$$\begin{aligned} \Phi(G,F) &= \Phi_S(G,F) + \Phi_n(G,F) \\ &= \int_0^{BLI(G,F)} \phi_S(I,G,F) \cdot dI \\ &\quad + \int_{BLI(G,F)}^{\infty} \phi_n(I,G,F) \cdot dI \end{aligned} \quad (3-66)$$

From equation (3-64) the total earning population to be subsidized for each family size and for each geographic area are computed. For each family size one has,

$$\begin{aligned} \Phi_S(F) &= \sum_{G=1}^L \Phi_S(G,F) \\ &= \sum_{G=1}^L \int_0^{BLI(G,F)} \phi_S(I,G,F) \cdot dI \end{aligned} \quad (3-67)$$

For each geographic area one has,

$$\begin{aligned} \Phi_S(G) &= \sum_{F=1}^K \Phi_S(G,F) \\ &= \sum_{F=1}^K \int_0^{BLI(G,F)} \phi_S(I,G,F) \cdot dI \end{aligned} \quad (3-68)$$

Also using equation (3-64) the total subsidized earning population is,

$$\begin{aligned} \Phi_S &= \sum_{G=1}^L \sum_{F=1}^K \Phi_S(G,F) \\ &= \sum_{G=1}^L \sum_{F=1}^K \left[ \int_0^{BLI(G,F)} \phi_S(I,G,F) \cdot dI \right] \end{aligned} \quad (3-69)$$

From equation (3-65) the total earning population which pays for the subsidy for each family size and for each geographic area are computed. For each family size one has,

$$\begin{aligned}\phi_n(F) &= \sum_{G=1}^L \phi_n(G,F) \\ &= \sum_{G=1}^L \int_{BLI(G,F)}^{\infty} \phi_n(I,G,F) \cdot dI\end{aligned}\quad (3-70)$$

For each geographic area one has,

$$\begin{aligned}\phi_n(G) &= \sum_{F=1}^K \phi_n(G,F) \\ &= \sum_{F=1}^K \int_{BLI(G,F)}^{\infty} \phi_n(I,G,F) \cdot dI\end{aligned}\quad (3-71)$$

Also using equation (3-65) the total earning population who pays for the subsidy is,

$$\begin{aligned}\phi_n &= \sum_{G=1}^L \sum_{F=1}^K \phi_n(G,F) \\ &= \sum_{G=1}^L \sum_{F=1}^K \left[ \int_{BLI(G,F)}^{\infty} \phi_n(I,G,F) \cdot dI \right]\end{aligned}\quad (3-72)$$

Combining equations (3-69) and (3-72) the total earning population of the country is,

$$\phi_T = \phi_S + \phi_n$$



$$\begin{aligned}
&= \sum_{G=1}^L \sum_{F=1}^K [\phi_S(G,F) + \phi_n(G,F)] \\
&= \sum_{G=1}^L \sum_{F=1}^K \left[ \int_0^{\text{BLI}(G,F)} \phi_S(I,G,F) \cdot dI \right. \\
&\quad \left. + \int_{\text{BLI}(G,F)}^{\infty} \phi_n(I,G,F) \cdot dI \right] \tag{3-73}
\end{aligned}$$

Given the various population and income frequency distribution equations, yearly income equations are developed which are the first moments of the population equations. For the subsidized segments the yearly income for each geographic area-family size division is,

$$Y_S(G,F) = \int_0^{\text{BLI}(G,F)} I \cdot \phi_S(I,G,F) \cdot dI \tag{3-74}$$

The subsidized segment's yearly income for each family size is,

$$\begin{aligned}
Y_S(F) &= \sum_{G=1}^L Y_S(G,F) \\
&= \sum_{G=1}^L \int_0^{\text{BLI}(G,F)} I \cdot \phi_S(I,G,F) \cdot dI \tag{3-75}
\end{aligned}$$

The subsidized segment yearly income for each geographic area is,

$$\begin{aligned}
Y_S(G) &= \sum_{F=1}^K Y_S(G,F) \\
&= \sum_{F=1}^K \int_0^{\text{BLI}(G,F)} I \cdot \phi_S(I,G,F) \cdot dI \tag{3-76}
\end{aligned}$$

The total earned income of the subsidy population is,

$$\begin{aligned}
 Y_{TS} &= \sum_{G=1}^L \sum_{F=1}^K Y_S(G,F) \\
 &= \sum_{G=1}^L \sum_{F=1}^K \int_0^{BLI(G,F)} I \cdot \phi_S(I,G,F) \cdot dI \quad (3-77)
 \end{aligned}$$

For the non-subsidized segments the yearly income for each geographic area-family size division is,

$$Y_n(G,F) = \int_{BLI(G,F)}^{\infty} I \cdot \phi_n(I,G,F) \cdot dI \quad (3-78)$$

The non-subsidy segment yearly income for each family size is,

$$\begin{aligned}
 Y_n(F) &= \sum_{G=1}^L Y_n(G,F) \\
 &= \sum_{G=1}^L \int_{BLI(G,F)}^{\infty} I \cdot \phi_n(I,G,F) \cdot dI \quad (3-79)
 \end{aligned}$$

The non-subsidy segment yearly income for each geographic area is,

$$\begin{aligned}
 Y_n(G) &= \sum_{F=1}^K Y_n(G,F) \\
 &= \sum_{F=1}^K \int_{BLI(G,F)}^{\infty} I \cdot \phi_n(I,G,F) \cdot dI \quad (3-80)
 \end{aligned}$$

The total earned income of the non-subsidy population is,

$$\begin{aligned}
Y_{Tn} &= \sum_{G=1}^L \sum_{F=1}^K Y_n(G,F) \\
&= \sum_{G=1}^L \sum_{F=1}^K \int_{BLI(G,F)}^{\infty} I \cdot \phi_n(I,G,F) \cdot dI \quad (3-81)
\end{aligned}$$

From equations (3-77) and (3-81) the total yearly income of the earning population in the United States is,

$$\begin{aligned}
Y_T &= Y_{TS} + Y_{Tn} \\
&= \sum_{G=1}^L \sum_{F=1}^K [Y_S(G,F) + Y_n(G,F)] \\
&= \sum_{G=1}^L \sum_{F=1}^K \int_0^{BLI(G,F)} I \cdot \phi_S(I,G,F) \cdot dI \\
&\quad + \int_{BLI(G,F)}^{\infty} I \cdot \phi_n(I,G,F) \cdot dI \quad (3-82)
\end{aligned}$$

Let us now introduce into the analysis the guaranteed minimum income concept. In Phase 1 the minimum income was a constant given to all members of the to-be-subsidized population without classification. In Phase 2 a constant guaranteed minimum income was assigned to the to-be-subsidized segment of each family size division. These guaranteed minimum incomes were such that they could include child subsidies. In this final phase of the model development a guaranteed minimum income is

assigned to each geographic area-family size division of the earning population. These guaranteed minimum incomes can also include child subsidies with the addition of further subsidies to overly depressed geographic areas.

Since the guaranteed minimum incomes to be assigned in this phase of the development are a function of family size and geographic area, mathematically, one has

$$I_m = I_m (G, F) \quad (3-83)$$

Now, since each member of the to-be-subsidized population of each geographic area-family size division of the earning population is to receive an assigned minimum income, the addition to income is,

$$\begin{aligned} Y_A (G, F) &= \int_0^{\Phi_S (G, F)} I_m (G, F) \cdot d[\Phi_S (G, F)] \\ &= I_m (G, F) \cdot \Phi_S (G, F) \end{aligned} \quad (3-84)$$

The addition to total income for each family size and for each geographic area then becomes

$$\begin{aligned} Y_A (F) &= \sum_{G=1}^L Y_A (G, F) \\ &= \sum_{G=1}^L I_m (G, F) \cdot \Phi_S (G, F) \end{aligned} \quad (3-85)$$

and



$$\begin{aligned}
Y_A(G) &= \sum_{F=1}^K Y_A(G,F) \\
&= \sum_{F=1}^K I_m(G,F) \cdot \Phi_S(G,F)
\end{aligned} \tag{3-86}$$

It is also evident from equation (3-84) that the total income added to the economy by introducing a subsidy in terms of a guaranteed minimum income is,

$$\begin{aligned}
Y_{TA} &= \sum_{G=1}^L \sum_{F=1}^K Y_A(G,F) \\
&= \sum_{G=1}^L \sum_{F=1}^K [I_m(G,F) \cdot \Phi_S(G,F)]
\end{aligned} \tag{3-87}$$

Due to the addition of a guaranteed minimum income all the income equations above are changed. For each subsidy segment of each geographic area-family size division of total earning population the new equation becomes

$$\begin{aligned}
Y_{AS}(G,F) &= Y_S(G,F) + Y_A(G,F) \\
&= \int_0^{BLI(G,F)} I \cdot \phi_S(I,G,F) \cdot dI \\
&\quad + I_m(G,F) \cdot \Phi_S(G,F)
\end{aligned} \tag{3-88}$$

The revised subsidized segment's yearly income for each family size becomes,

$$Y_{AS}(F) = \sum_{G=1}^L Y_{AS}(G,F) \quad (3-89)$$

and the revised subsidy segment's yearly income for each geographic area becomes,

$$Y_{AS}(G) = \sum_{F=1}^K Y_{AS}(G,F) \quad (3-90)$$

The revised total earned income of the subsidy population becomes,

$$Y_{TAS} = \sum_{G=1}^L \sum_{F=1}^K Y_{AS}(G,F) \quad (3-91)$$

Finally, the revised total yearly income of the earning population in the United States becomes,

$$Y_{TA} = Y_{TAS} + Y_{Tn} \quad (3-92)$$

or,

$$\begin{aligned} Y_{TA} = & \sum_{G=1}^L \sum_{F=1}^K \left[ \int_0^{BLI(G,F)} I \cdot \phi_S(I,G,F) \cdot dI \right. \\ & + \int_{BLI(G,F)}^{\infty} I \cdot \phi_n(I,G,F) \cdot dI \\ & \left. + I_m(G,F) \cdot \phi_S(G,F) \right] \quad (3-92') \end{aligned}$$

Because of the addition of a guaranteed minimum income to each geographic area-family size segment of the earning population, a question of equity arises in the opinion of many experts. These experts point out that

the members of the subsidy-paying population in each segment at incomes near the Breakeven Level of Income may receive less income due to an across the board minimum income than those members at the high end of the subsidized segment who receive the minimum income. It is partly because of these questions of equity that negative income taxes are introduced into most proposals. These negative taxes are such that no member of the subsidized population will earn more than the Breakeven Level of Income as a result of the addition of a guaranteed minimum income, and, also, that no member of the subsidized population should earn less than the set guaranteed minimum income for each geographic area-family size division to which he belongs, or

$$I - I \cdot T_S(I, G, F) \geq I_m(G, F) \quad (3-93)$$

and,

$$I \cdot T_S(I, G, F) + I_m(G, F) \leq BLI(G, F) \quad (3-94)$$

From the analysis above it can be seen that the model allows a different negative tax ( $T_S$ ) to be placed on the income earned by the subsidized population within each geographic area-family size division of the earning population. This concept of varying negative taxes per geographic area-family size division is not the case in a majority of the proposals of Chapter II. However, since

the analysis of this dissertation wishes to anticipate any argument, the analysis will continue to institute variations even though they increase the complexity of the analysis.

Again, due to the addition of negative taxes to the analysis, the income equations above are altered. For each geographic area-family size division the new income equation for the subsidized population is,

$$\begin{aligned}
 Y_{AST}(G,F) &= \int_0^{BLI(G,F)} I \cdot \phi_S(I,G,F) \cdot dI \\
 &+ \int_0^{\phi_S(G,F)} I_m(G,F) \cdot d[\phi_S(G,F)] \\
 &- \int_0^{BLI(G,F)} I \cdot T_S(I,G,F) \cdot \phi_S(I,G,F) \cdot dI \\
 &= \int_0^{BLI(G,F)} I \cdot \phi_S(I,G,F) [1 - T_S(I,G,F)] \cdot dI \\
 &+ I_m(G,F) \cdot \phi_S(G,F) \tag{3-95}
 \end{aligned}$$

The yearly income for each family size and geographic area subsidy segment is,

$$Y_{AST}(F) = \sum_{G=1}^L Y_{AST}(G,F) \tag{3-96}$$

and,

$$Y_{AST}(G) = \sum_{F=1}^K Y_{AST}(G,F) \tag{3-97}$$



The total income for the subsidy portion of the economy becomes,

$$Y_{TAST} = \sum_{G=1}^L \sum_{F=1}^K Y_{AST}(G,F) \quad (3-98)$$

Finally the revised total yearly income for the earning population of the United States becomes,

$$Y_T = Y_{TAST} + Y_n \quad (3-99)$$

The economy is now divided into income frequency distributions by geographic area-family size divisions. To each of these income frequency distributions is assigned a guaranteed minimum income, a negative tax, and a Breakeven Level of Income. Given these new additions to each geographic area-family size division of the population, equations that will evaluate the cost of the subsidy the country must absorb for each geographic area-family size division are

$$\begin{aligned} S(G,F) = & I_m(G,F) \cdot \Phi_S(G,F) \\ & - \int_0^{BLI(G,F)} I \cdot T_S(I,G,F) \cdot \theta_S(I,G,F) \cdot dI \end{aligned} \quad (3-100)$$

The total subsidy given to each family size is,

$$S(F) = \sum_{G=1}^L S(G,F) \quad (3-101)$$

And the total subsidy given to each geographic area is,

$$S(G) = \sum_{F=1}^K S(G,F) \quad (3-102)$$

The total subsidy for the country is,

$$TS = \sum_{G=1}^L \sum_{F=1}^K S(G,F) \quad (3-103)$$

Given the costs of the subsidy for various divisions of the economy, the question as to how to pay for it arises. Here as in Phase 1 and 2 of the development the most critical assumption of the analysis is made. It is assumed that the cost of the subsidy is an added cost to the economy and will be paid for by new taxation. Even if the subsidy cost is to be paid out of present tax dollars, the cost distribution must be determined. This is done by considering the taxes to pay for the subsidy to be in the form of a marginal tax on the progressive tax rates of earners in the non-subsidy segments and on corporate earnings.

The form the above marginal taxes take is not the question solved in this dissertation. However, mathematics are developed to model the marginal taxes. To be most complete, the marginal taxes on non-subsidy earnings are made a function of geographic area and family size, and the marginal tax on corporate earnings is made a function of geographic area. By use of the Management

Information and Control System of Chapter V, the decision maker is allowed many options including the setting of one marginal tax for the whole country.

The equation for the cost of the subsidy is

$$\begin{aligned} \text{COST OF SUBSIDY} &= \text{MARGINAL NON-SUBSIDY TAX} \\ &+ \text{MARGINAL CORPORATE TAX} \end{aligned}$$

or,

$$\begin{aligned} CS(G, F) &= \int_{BLI(F)}^{\infty} MT_n(I, G, F) \cdot I \cdot \phi_n(I, G, F) \cdot dI \\ &+ \frac{1}{K} MT_C(G) \end{aligned} \quad (3-104)$$

where K is the number of family sizes

The cost of the subsidy for all family sizes is,

$$CS(F) = \sum_{G=1}^L CS(G, F) \quad (3-105)$$

and the subsidy cost of all geographic areas is,

$$CS(G) = \sum_{F=1}^K CS(G, F) \quad (3-106)$$

The total subsidy cost is paid for by,

$$TCS = \sum_{G=1}^L \sum_{F=1}^K CS(G, F) \quad (3-107)$$

From the above equations of subsidy cost the analysis can be further segmented to determine the subsidy responsibility of individual income segments of each di-

vision. From these income segments information valuable to decision makers who wish to determine the marginal tax structures is available.

For the geographic area-family size divisions one has,

$$MT_{LIAB]_A^B}(G,F) = \int_A^B MT_n(I,G,F) \cdot I \cdot \phi_n(I,G,F) \cdot dI \quad (3-108)$$

where A and B are the lower and upper income bounds of the segment to be analyzed.

For family sizes it is,

$$MT_{LIAB]_A^B}(F) = \sum_{G=1}^L MT_{LIAB]_A^B}(G,F) \quad (3-109)$$

and for geographic areas it is,

$$MT_{LIAB]_A^B}(G) = \sum_{F=1}^K MT_{LIAB]_A^B}(G,F) \quad (3-110)$$

The tax liability for a particular income segment for the whole country is,

$$MT_{LIAB]_A^B} = \sum_{G=1}^L \sum_{F=1}^K MT_{LIAB]_A^B}(G,F) \quad (3-111)$$

The addition of marginal taxes on the public and private sectors of the economy have in effect equalized the addition of the subsidy. Therefore, the income equations of the country are simply those before a subsidy was added. What did change is the distribution of



income, and that new distribution has been shown above.

Phase 3 of this Chapter's development is the model used in The Management Information and Control System of Chapter V. It is in Chapter V that many of the variables such as marginal taxes, minimum income, negative taxes, Breakeven Levels of Incomes, and income frequency distribution are varied to fit all negative tax and guaranteed minimum income proposals available today. In Chapter VI, The Management Information and Control System of Chapter V is tested on actual proposals. This testing further defines how flexible the model developed in this Chapter can be.

## REFERENCES

1. United States Department of Commerce, Current Population Reports, Bureau of the Census, Consumer Income, Series P-60, No. 75, "Income in 1969 of Families and Persons in the United States," December 14, 1970, Table 39, p. 83.

C H A P T E R I V

I N C O M E D A T A B A S E D E V E L O P M E N T

### DATA COLLECTION AND STORAGE

In Chapter III of this dissertation the capability to investigate any negative income tax proposal or guaranteed minimum income proposal is developed. However, to do such investigations without the most up-to-date national income figures would be useless. Therefore, in this chapter of the dissertation, techniques to collect national income data; to store this data in digital computers; to curve fit this data by computer techniques so as to generate the income frequency distributions required by the mathematical model; and to convert the income frequency distributions into some parametric form useable by the management information and control system of Chapter V. is developed.

Of course, the first step in any data utilization scheme is the actual collection of data. Fortunately, a collection of national income statistics for many various categories is performed for the decision maker by the United States Census Bureau.<sup>1</sup> It is simply the decision maker's task to request from the Census Bureau the information he needs. For the mathematical model of Phase 3 of Chapter III of this dissertation the national income data the decision maker needs must be collected by family size within the geographic area segments the decision maker wishes to investigate.



Table 4-1 will be useful to the decision maker in his proper collection of data. For each geographic area and each family size fill in the proper income and income frequency data. Since a subsidy is usually given to just the low end of the income, it is suggested that the decision maker collect his data at smaller intervals there and then increase the intervals as the income is increased. Collecting the data at \$250 intervals from \$0 to \$10,000, at \$500 intervals \$10,000 to \$20,000; \$1,000 intervals from \$20,000 to \$50,000, and \$5,000 increments from \$50,000 to \$100,000 is one suggestion. This would give, for each geographic area-family size division of the national income picture, 100 data points. That income class over \$100,000 could be summed and averaged creating one more data point at some income figure with the combined frequency. However, the decision maker has complete freedom as to his collection desires, and the number of data points per geographic area-family may size up to 500 points.

Once the data is collected it must be stored in the digital computer. It is in this storage procedure that one becomes limited by the computer equipment one has available. Some equipment types are described in the following paragraphs along with their advantages and disadvantages. Appendix A describes the actual input sheets

TABLE 4-1  
NATIONAL INCOME DATA COLLECTION FOR  
GEOGRAPHIC AREA =  
FAMILY SIZE =

YEARLY INCOME IN DOLLARS	FREQUENCY AT WHICH THE INCOME IS EARNED

to utilize the computer subroutines to read the data onto the selected devices available at the University of Massachusetts Computer Center. Hints to use other devices are also found there.

#### NATIONAL INCOME DATA STORAGE ON MAGNETIC TAPE

Magnetic tape storage equipment is the most common auxiliary device by which data can be collected and used by digital computers. Not only are magnetic tapes transferable from one manufacturer's equipment to another's, they are easily handled and saved. Even if the tape structure of one manufacturer differs from another, most manufacturer's facilities have canned programs to convert other system's tapes to tapes usable by their system.

Before one chooses magnetic tape as his storage device some of the disadvantages of tapes must be enumerated. When one stores data on tape one stores it by logical records. Where each logical record is the smallest division of one's data one will need to use. In the analysis of this dissertation the smallest logical collection of data is the income frequency data collected by geographic area-family size divisions. Therefore, for each geographic area-family size division a logical record must be made containing each income and the frequency at which that income is earned. The tape would

then be structured as in Figure 4-1.

Assuming that there are  $K$  number of family sizes and  $L$  number of geographic areas there then would be  $K \cdot L$  logical records on the data tape. If the decision maker wishes to use all the data then tape processing time as compared to processing times of other storage techniques to be described in this chapter, would be comparable. However, if the decision maker wishes to select only a few data divisions, the tape would still have to be read in total up to the last record required. For example, if the decision maker just wished to investigate geographic area ( $L$ ) and family size ( $K$ ), data access processing time would be equal to the case where he investigated all cases. If then after examining this single ( $K \cdot L$ ) case the results make the decision maker wish to see another case, the tape would have to be rewound and reread in total up to the new case desired. This continued rewinding and reading of a magnetic tape becomes very costly with respect to computer time. Therefore, it is suggested that tape be used only if no other means is available; as the transfer agent from one system to another; or when total processing is to be done.



TAPE LABEL

UNIT XXX, NATIONAL INCOME FREQUENCY DATA

LOGICAL RECORD 1

Geographic Area (1), Family Size (1), number of  
data pairs, data

LOGICAL RECORD 2

Geographic Area (1), Family Size (2), number of  
data pairs, data

⋮

LOGICAL RECORD K

Geographic Area (1), Family Size (K), number of  
data pairs, data

LOGICAL RECORD K+1

Geographic Area (2), Family Size (1), number of  
data pairs, data

⋮

LOGICAL RECORD L·K

Geographic Area (L), Family Size (K), number of  
data pairs, data

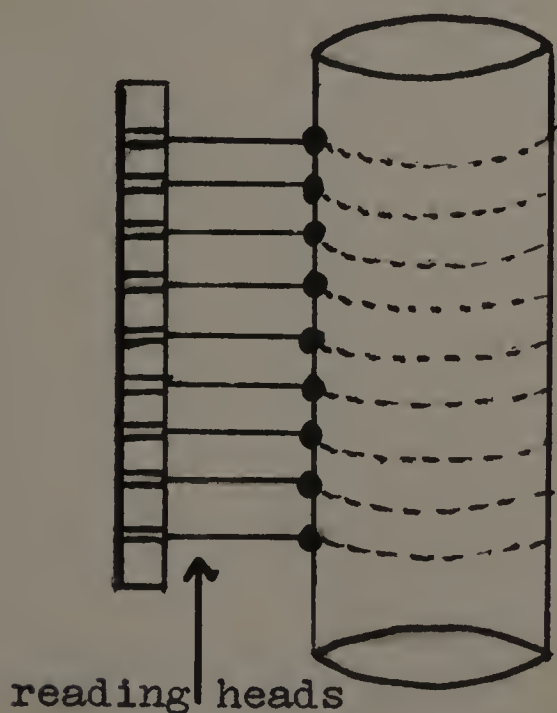
FIGURE 4-1

A TYPICAL NATIONAL INCOME FREQUENCY

DATA TAPE STRUCTURE

NATIONAL INCOME DATA STORAGE ON MAGNETIC DRUM

Many computer installations in the United States today are equipped with an auxiliary storage unit called a magnetic drum. Like magnetic tape it can be used to store logical records. However, where a magnetic tape must store data in series as represented in Figure 4-1 and the data must be accessed sequentially; a magnetic drum allows parallel storage with random access to any logical record being as quick as the first logical record. This random processing comes about due to the physical make-up of the magnetic drum. Figure 4-2 shows a typical drum system. From Figure 4-2 one can see the storage design of a Magnetic Drum. The drum is divided into many tracks of data



where ----- represents a single track of the drum

FIGURE 4-2

MAGNETIC DRUM DATA STORAGE EQUIPMENT

storage. External to the drum is a reading head. This reading head is so designed that by computer control it can be positioned at any track of storage allowing random processing. Most drum systems have more than one reading head thus allowing even quicker track selection. Another feature of the drum system is that each track is divided into smaller segments called arcs (or sectors) and these arcs are individually accessed by the reading head through computer control. Therefore, for the geographic area-family size logical records, this system must be able to access, one need simply to place each one in its own arc and create pointers to each. Then, when one requests a particular division, one can compute the pointer and the drum reading head will be positioned to read the information within microseconds. Since the drum is continually rotating one can then call for any other division without rewinding or reading through unnecessary records as with a magnetic tape system.

The above described magnetic drum seems perfect as the storage device to be used for this system. However, there are some disadvantages to using a magnetic drum. Besides its not being part of many computer systems, its physical make-up causes a magnetic drum not to be easily transferred from one system to another. Added to this is the fact that most magnetic drums are an integral part

of most computer operating systems and cannot be used for permanent storage. Because of the two above arguments, the use of a magnetic drum must be accompanied by the use of a magnetic tape. The magnetic tape is used as the permanent storage device. Before a user can do any processing the magnetic tape must be initially transcribed onto the magnetic drum and, of course, at the conclusion of processing the data must be retranscribed from magnetic drum to magnetic tape. This transcribing and retranscribing requires at least one tape reading and tape writing. In the case one owns the magnetic tape and wishes to use only that tape, a rewinding is also necessary. From the above restrictions it is suggested that a magnetic drum should not be used unless multiple random accessing is to be done and the following described magnetic disks are not a part of the user's system.

#### NATIONAL INCOME DATA STORAGE ON MAGNETIC DISKS

In the last few years of computer development, an auxiliary data storage device called a magnetic disk is more and more taking the place of magnetic tapes as the prime external storage device. The magnetic disk, like the magnetic drum, allows random data accessing. Therefore, one has almost instantaneous access to any logical



record. A magnetic disk's physical make-up differs from that of a magnetic drum, but its logical make-up is basically the same. Figure 4-3 shows the physical make-up of a magnetic disk. As one can see, the magnetic disk is divided into surfaces. Each surface is similar to a phonograph record except the grooves are replaced by magnetic storage and are called tracks. Unlike a phonograph record, the tracks are independent concentric circles and do not continue into the next track. Each surface is also divided into sectors and assigned an individual reading head. This reading head can be positioned to any track and sector of a surface. Therefore, the logical income data records required for one's analysis are the same for magnetic disk as for magnetic drum. However, unlike magnetic drums, magnetic disks come in units that may be transferred from system to system. These transferrable units are called "disk packs", and for the data base this dissertation requires is more than ample to store it.

A disadvantage of magnetic disks is that they are much more expensive to own than magnetic tape. Therefore, one may wish to use a system supplied magnetic disk as temporary storage while processing and, as described in the magnetic drum case, use magnetic tape as permanent storage.

TOP VIEW  
OF A DISK SURFACE

SIDE VIEW

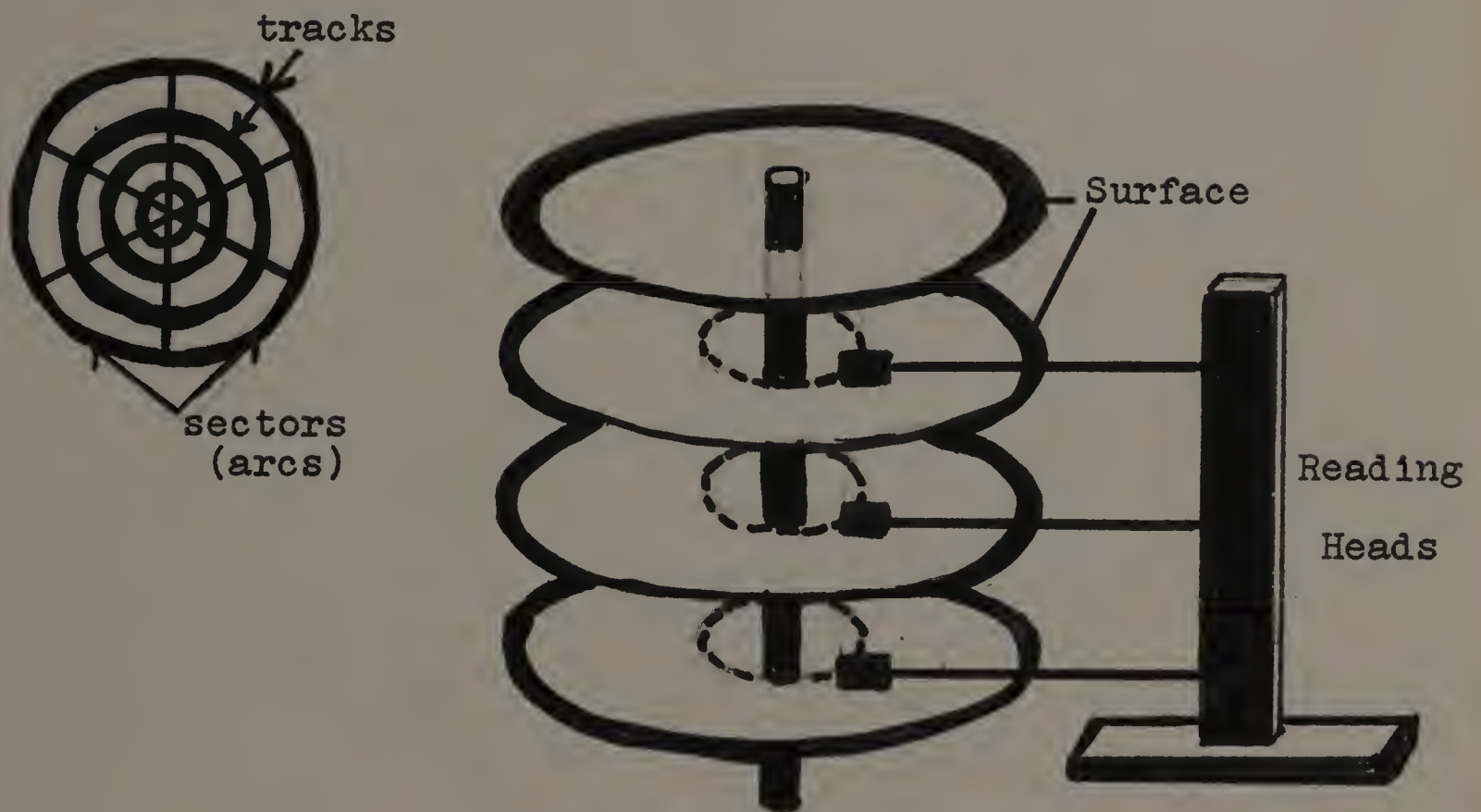


FIGURE 4-3

MAGNETIC DISK DATA STORAGE EQUIPMENT

It is the author's opinion that magnetic disk is the optimal auxiliary storage device that can be used by the system developed in this dissertation. However, two other auxiliary systems have been shown that may be used

if magnetic disk is not feasible for you, the user.

#### OTHER POSSIBLE NATIONAL INCOME STORAGE TECHNIQUES

The three types of auxiliary storage devices mentioned above are most appropriate when the data base is large. However, when the data base is small, data cards read directly into magnetic core storage with pointers created to locate each logical record is by far the most efficient system mentioned. Data card readers and magnetic core are a must for almost every computer system available today, and core accessing is both random and faster than any auxiliary storage device. Therefore, before one selects any storage technique one should determine how much data one has as compared to the available core storage. Where the available core storage is the core storage of the user's system minus program size. If the data fits in core, select this as one's storage.

Appendix A shows how to set up one's data base on cards to be read directly into core storage. It also supplies a subprogram and its flow diagram that can access core storage.

Before the question of data storage is complete, one more case where storage problems arise must be mentioned. That is the case of on-line processing using timesharing terminals. Since it is desired that all the computer

techniques developed in this dissertation be useable, both in batch processing and in on-line modes, Appendix A, contains programs to access data to be used by time-sharing terminals. These programs are two in kind. The first allows direct inputting of data via a timesharing terminal. This is done either by direct key board entry or by the terminal's auxiliary input device, paper tape. Where the paper tape may be created directly at the time-sharing terminal or from cards through a mini-computer. The second technique of developing a data base for time-sharing is to create disk files using input cards and a batch mode. These data disk files are so structured that timesharing software can access them.

#### DATA RESTRUCTURING

Now that national income data collection techniques have been illustrated and the many alternatives as to its storage on digital computers have been shown, how the data is to be processed to be applicable to the mathematical model developed in Chapter III must be defined.

The mathematical model of Chapter III, Phase 3 requires that national income data be represented as an income frequency distribution. That is, the actual data collected by incomes and the frequency at which that income is earned for each geographic area-family size divi-



sion of the population must be curve fitted to form a continuous parametric equation. Not only is this parametric equation required for the total distribution of incomes for each geographic area-family size division, it must also be segmentable into subsidy and non-subsidy portions of that population division as a function of some user defined Breakeven Level of Income.

To perform these curve fitting operations, computer programs have been developed for both batch and timesharing modes of use and are found in Appendix A along with their user's manuals. The input and output and the curve fitting model is the same whether batch or timesharing is used only the programming input/output instructions differ. Therefore, I shall confine the discussion of this chapter to the common elements and leave the varying means of handling the elements to Appendix A.

The inputs to the curve fitting routine are represented by Table 4-2 and the outputs are represented by Table 4-3.

Looking at Table 4-2, the first inputs  $[I, D(I, G, F)]$  are just the data points of our national income data base. BLI  $(G, F)$  is a parameter the decision maker must choose from his vast experience. Each of the input programs of Appendix A allow the decision maker to set these individually for each geographic area-family size or for

TABLE 4-2

## CURVE FITTING PROGRAM INPUTS

- I,  $D(I,G,F)$  - Income and Income frequency for family size (F) and geographic area (G).
- $BLI(G,F)$  - The Breakeven Level of Income at which one divides the geographic area-family size division of the population into a subsidized portion and a non-subsidized portion.
- $M(G,F)$  - = 1, curve fit whole geographic area-family size population frequency distribution.
- = 2, segment geographic area-family size population division into to-be-subsidized and non-subsidized segments and curve fit each individually.
- = 3, do both the 1 and 2 options of above.

larger divisions such as one  $BLI$  for each family size constant over all geographic areas; or vice versa, one for each geographic area constant over all family sizes within that geographic area; or one for the whole population regardless of family size and geographic area.

TABLE 4-3

## CURVE FITTING PROGRAM OUTPUTS

- $\emptyset (I,G,F)$  - The curve fitted income frequency distribution for family size (F) and geographic area (G) population.
- $n_1$  - The degree of the curve fitted polynomial for  $\emptyset (I,F,G)$ .
- $e_1$  - The standard error produced by curve fitting  $\emptyset (I,G,F)$ .
- $\emptyset_S(I,G,F)$  - The curve fitted income frequency distribution for the subsidy portion of family size (F) and geographic area (G) population division.
- $n_2$  - The degree of the curve fitted polynomial for  $\emptyset_S(I,F,G)$ .
- $e_2$  - The standard error produced by curve fitting  $\emptyset_S(I,G,F)$ .
- $\emptyset_n(I,G,F)$  - The curve fitted income frequency distribution for the non-subsidized portion of family size (F) and geographic area (G) population division.
- $n_3$  - The degree of the curve fitted polynomial  $\emptyset_n(I,G,F)$ .
- $e_3$  - The standard error produced by curve fitting  $\emptyset_n(I,G,F)$ .

The output of the curve fitting program are parametric polynomial equations of the form

$$a_1 + a_2x + a_3x^2 + a_4x^3 + \dots + a_nx^{n-1} \quad (4-1)$$

for the total geographic area-family size population's income frequency distribution and/or for the subsidized and non-subsidized portion of that population as a function of the inputted BLI. The choice of polynomial fits instead of, say, some other distribution such as logarithmic, gamma, or Raleigh sometimes found associated with population income frequency distributions is necessitated by the segmentation of total population into two, input controlled subsegments. These subsegments such as shown in Figure 3-1 are amenable to polynomial curve fitting with accuracies completely satisfactory for this dissertation's needs. The added accuracy possibly attainable by other curve fitting techniques such as exponential, logarithmic, or trigonometric is not such that it warrants the added expense of computation and their added complexity to the mathematical model.

The remaining output of the curve fitting routine are each curve fit's degree. That is the  $n-1$  of equation (4-1) for each fit. Also inputted is a measure of the goodness of each fit. This is simply the standard error produced by using least squares methods to generate discrete



data fitting polynomials. These measures of goodness of fit are, therefore, combinable to find total errors, due to fitting, for larger portions of the population.

Chapter VI of this dissertation along with Appendix A present the mathematics for these error terms along with examples as applied to real data.

NEW DATA BASE FOR THE MANAGEMENT INFORMATION AND CONTROL SYSTEM

Once the national income frequency data has been processed through the above curve fitting technique it again becomes necessary to develop file structures for the storage of this data so that it may be transferred to and used by the management information and control system of Chapter V. The same arguments concerning serial or random accessing; disk, drum, magnetic tape, paper tape, and card to core storage; and permanent or temporary storage as pointed out above for the non-parametric storing of national income data is also pertinent to the parametric storing of national income frequency distributions. For this latter case one merely has a change in the format of the logical record. The new minimum required logical record is now the national income frequency distribution for each geographic area-family size in sets of three; the distribution for the total population; for the subsidized

segment of the population; and for the non-subsidized segment of the population. Figure 4-4 represents the new file structure. One immediately sees when one compares this logical record set up with that of Figure 4-1 that each logical record is now two data records in length. The first record of each logical record locates the geographic area-family size division plus the type of curve fits available for this division. The type of curve fits were defined by  $M(G,F)$  of Table 4-2. The second record of each logical record then contains the available curve fits along with their degree and error term. Since the degree of the polynomial is given, all that need be stored to describe the parametric equation are the "a" coefficients. Therefore, a typical record 2 looks like Figure 4-5.

Appendix A supplies the programming techniques required to develop these file structures on some equipment types. Also to be found in Appendix A are programs to output any segment of the data base in a report format.

This Chapter, therefore, develops the means of taking a large national income data base where for each geographic area-family size there could be up to 500 data points and through curve fitting techniques develop much smaller records. Not only are these records smaller, they also can contain more information in the form of

FIGURE 4-4

## PARAMETRIC INCOME FREQUENCY DISTRIBUTIONS DATA

## STRUCTURE PER GEOGRAPHIC AREA-FAMILY SIZE

## LOGICAL RECORD 1

$$\left\{ \begin{array}{l} \text{Record 1 Geographic area (1), Family size (1), } M(1,1), \\ \quad \text{BLI (1,1)} \\ \text{Record 2 } n_1, e_1, \emptyset(I,1,1), n_2, e_2, \emptyset_S(I,1,1), n_3, e_3, \\ \quad \emptyset_n(I,1,1) \end{array} \right\}$$

## LOGICAL RECORD 2

$$\left\{ \begin{array}{l} \text{Record 1 Geographic area (1), Family size (2), } M(1,2), \\ \quad \text{BLI(1,2)} \\ \text{Record 2 } n_1, e_1, \emptyset(I,1,2), n_2, e_2, \emptyset_S(I,1,2), n_3, e_3, \\ \quad \emptyset_n(I,1,2) \end{array} \right\}$$

$$\vdots$$

## LOGICAL RECORD K

$$\left\{ \begin{array}{l} \text{Record 1 Geographic area (1), Family size (K), } M(1,K), \\ \quad \text{BLI(1,K)} \\ \text{Record 2 } n_1, e_1, \emptyset(I,1,K), n_2, e_2, \emptyset_S(I,1,K), n_3, e_3, \\ \quad \emptyset_n(I,1,K) \end{array} \right\}$$

## LOGICAL RECORD K+1

$$\left\{ \begin{array}{l} \text{Record 1 Geographic area (2), Family size (1), } M(2,1), \\ \quad \text{BLI(2,1)} \\ \text{Record 2 } n_1, e_1, \emptyset(I,2,1), n_2, e_2, \emptyset_S(I,2,1), n_3, e_3, \\ \quad \emptyset_n(I,2,1) \end{array} \right\}$$

$\vdots$   
 LOGICAL RECORD L·K  
 Record 1 Geographic area (L), Family size (K),  $M(L,K)$ ,  
 $BLI(L,K)$   
 Record 2  $n_1, e_1, \emptyset(I,L,K), n_2, e_2, \emptyset_S(I,L,K), n_3, e_3,$   
 $\emptyset_n(I,L,K)$

FIGURE 4-5

## RECORD 2 STRUCTURE

$$[n_1, e_1, a_1, a_2, a_3, \dots, a_{(n_1+1)}; n_2, e_2, a_1, a_2, \dots, a_{(n_2+1)}; \\
 n_3, e_3, a_1, a_2, \dots, a_{(n_3+1)}]$$

subsidy and non-subsidy segments. The next chapter of this dissertation utilizes these smaller records to allow the decision maker to use the mathematical model of Chapter III so as to compare negative income tax and guaranteed minimum income proposals.



## REFERENCES

1. United States Department of Commerce, Current Population Reports, Bureau of the Census, Consumer Income, Series P-60, No. 66, December 23, 1969, "Income in 1968 of Families and Persons in the United States"; No. 70, July 16, 1970, "Average Family Income up 9 Percent in 1969"; No. 72, August 14, 1970, "Household Income in 1969 and Selected Social and Economic Characteristics of Households"; No. 75, December 14, 1970, "Income in 1969 of Families and Persons in the United States"; No. 76, December 16, 1970, "24 Million Americans Poverty in the United States: 1969".

C H A P T E R V  
THE MANAGEMENT INFORMATION AND  
CONTROL SYSTEM

The Management Information and Control System to be outlined in this chapter is structured in two parts. The first part allows the decision maker to create national income frequency distributions for the whole population and also allows the user to create subsidy and non-subsidy segments as a function of arbitrary Breakeven Levels of Income.

The second part takes the outputs of the first parts and processes them through the mathematical model of Chapter III allowing the user to alter such parameters as the minimum income; negative, marginal non-subsidy, and marginal corporate tax rates, and to do individual income bracket analyses.

#### PART 1 - NATIONAL INCOME DATA CONSIDERATIONS

Chapter IV of this dissertation describes the programming techniques, the data file structure, and the input and output parameters required to convert national income frequency data sets to parametric polynomial equations approximating these data sets. However, in most cases the decision maker need not worry about these technical considerations, but he must be able to understand the flexibility of the system and, thereby, how to best use the system. This further understanding is the object of this section.

The curve fitting routine found in Appendix A, Section A3 and referred to in Chapter IV performs a least-squares analysis on the data and by computing a standard error (simply the standard deviation for a sample set) for each polynomial degree automatically selects the optimal polynomial fit where the degrees of this fit range from 1 to 15.

Since selecting optimal curve fits for one's data is a matter of convenience, the program structure of the management information and control system's first part to be described here is simply the development of techniques which allows the decision maker to communicate his desires as to analysis to the curve fitting techniques. The actual computer program that acts as this communicator is found in Appendix B. What follows is how it allows communication; what options it allows the decision maker; and what output reports it supplies.

The information and control system to be described here allows the user communication by the fact that it is designed to be used on time-sharing on-line equipment. This design allows the decision maker to investigate various geographic area-family size segments of the population and by changing parameters immediately see their effects on the population through output immediately printed for him. It then allows him to further change these inputs



until he is satisfied with his results. By continuously investigating various geographic area-family size populations the decision maker, in fact, generates his own particular data base he wishes to process through the mathematical model of Chapter III. He has complete control and need not waste time in analyzing unneeded situations as in the case with non-on-line systems where many instructions are pre-set.

How does one use this individualized communication system? Simply by using the following instructions:  
(Note: The following instructions are pertinent to the University of Massachusetts Computer Center's time-sharing system using CD -3600 and CD -3800 computers. However, altering the instructions for another computer system is not a major problem and most likely could be accomplished by the user's computer center's system analysts).

The first step is to get onto the time-sharing system. This is done by calling the University of Massachusetts's computer center at 617-545-1600. Once contact is made (one hears a ring followed by a loud continuous signal) place the telephone receiver into the data coupler situated at one's teletype terminal. Then turn the terminal LINE-OFF-LOCAL switch to LINE. The system will then call for an "USER NO.?". Type on the terminal keyboard R357. The system will then call for a "CODE NO.?". Type

on the terminal keyboard GEOR. You are now ready to use the information and control system. To do so type on the keyboard "FETCH MISCFT". The system will reply "OK" and the date the program was originally filed. To begin the analysis simply type on the keyboard "RUN".

The information and control system now develops a set of user pertinent instructions through which the user can now generate his own data base. The instructions and implications of these instructions are explained below.

The first instruction the decision maker is called on to respond to is:

```
TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA
FILE TYPE THE FILE NAME, THE GEOGRAPHIC AREA
INDEX, AND THE FAMILY SIZE INDEX
?
```

If the user followed the file generation naming instructions of Chapter IV, he would respond to this request by typing for example

```
G01F01 1 1
```

With this response the monitoring program automatically selects from prestored disk files the national income frequency data set for the geographic area designated 01 by the user and the family size make-up designated 01

by the user. Therefore, the user is allowed analysis on any geographic area-family size for which a data base is available.

The next instruction for user response is:

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE  
?

Of course this information could have been supplied with the data file itself. However, to allow the user as much flexibility as possible it was not. There may be the case that the user is not concerned with data above a particular income. Especially in the case of the family size classifications of individuals and large families where income frequency above \$40,000 per year is scarce. This data limiting results in tighter fits for these family size groups.

Once the data set is designated and its size determined the user is then asked to select the types of curve fits he wants by answering the following instruction:

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1  
TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...  
TYPE 2, TO CURVE FIT BOTH ABOVE...TYPE 3  
?

This instruction is always to be answered "3" if the user wishes to save the data to be used in Part 2 of the management information and control system. However, an option is allowed so that the user may first analyze his data base as to the applicability of polynomial fits to meet his accuracy requirements. By so investigating his files he can change his data base before storing any data if required accuracy is not met, without having to change two data files i.e., the original and the parametric.

Once the user has selected the curve fits he wishes to investigate, the monitoring program automatically generates the optimal fit within polynomials of degrees 1 through 15 and prints the following output reports.

When options 1 or 3 is selected, the first output report generated is:

THE FOLLOWING BEST FITS THE TOTAL POPULATION  
FOR GEOGRAPHIC AREA 01, FAMILY SIZE 01

Polynomial Degree = n	STD. Error = $S_{\bar{x}}$
Term Degree	Coefficients
0	$a_0$
1	$a_1$
⋮	⋮
n	$a_n$



## BACK SOLUTIONS FOR BEST FIT-DEGREE = n

<u>Weight</u>	<u>X Value</u>	<u>Y Value</u>	<u>Est. Y</u>	<u>Residual</u>
$w_1$	$x_1$	$y_1$	$Y_1$	$e_1$
$w_2$	$x_2$	$y_2$	$Y_2$	$e_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$w_n$	$x_n$	$y_n$	$Y_n$	$e_n$

When option 2 or 3 is selected, the decision maker is saying that he wishes to divide his geographic area-family size segment of the population into a to-be-subsidized portion and a non-subsidized portion. This division into subsidy and non-subsidy portion is user controlled. To portion his population he enters when called for by the following instruction,

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS  
GEOGRAPHIC AREA-FAMILY SIZE

?

a Breakeven Level of Income. The Breakeven Level of Income not only divides the population segment into portions but is also stored along with the parametric data to be used in the second phase of the management information and control system.

Once options 2 or 3 is selected the following output report is generated:

THE FOLLOWING BEST FITS THE SUBSIDY PORTION  
OF GEOGRAPHIC AREA 01, FAMILY SIZE 01

plus the best fit polynomial and back solution information for this fit as above and,

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION  
OF GEOGRAPHIC AREA 01, FAMILY SIZE 01

plus the best fit polynomial and back solution information for this fit as above.

The back solutions to each best fit simply allows the decision maker to not completely rely on the standard error to see how good his fit is, but he can actually see his inputted data pairs and the resulting estimated frequency data ( $Y_c$ ) for each income produced by the curve fitting. For his convenience the discrepancies between the inputted frequency data and calculated data are shown in the form of residual differences.

A weight for each data pair is also designated. However, this may be ignored by the decision maker and is automatically set to 1. It exists because the curve fitting routine is so structured that it may be used for many other systems that may require scaling.

The decision maker now has for this particular geographic area-family size segment of the total population output reports for all the options he requested. However,

to further allow him to investigate his parametric data the monitoring program includes various user controlled plot options.

The graphs generated through user control by the plot package included in this dissertation are printed directly on-line at the user's teletype terminal. Since the plots are on the terminal, certain limitations are imposed. First the program cannot handle more than 100 data pairs due to paper size limitations. Second the x-axis runs vertically on the paper and the y-axis horizontally. Therefore, the y frequency data variables are scaled depending upon their minimum and maximum value and the representation of the x-values will be skewed unless the x-value are equally spaced.

From the above restriction do not plot the original national income frequency data unless there are less than 100 data pairs, and the x-values of the data pairs are equally spaced. However, all the curve fits generated may be plotted.

To initiate plotting the monitoring program prints the following instructions:

```
DO YOU WISH TO PLOT THE ORIGINAL DATA..TYPE Y
FOR YES, N FOR NO
?
```

Then

```
DO YOU WISH TO PLOT THE CURVE FIT FOR THE
WHOLE GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...
TYPE Y FOR YES, N FOR NO
?
```

If just option 1 is selected, no data is stored for part 2 of the information and control system. The monitoring program upon completion of the plot options recycles itself and requests a new geographic area-family size segment be inputted. If the user does not wish to continue he simply need type after the "?" ENDDATA 0 0.

If options 2 or 3 are selected, the monitoring program asks for instructions as to further plotting for the subsidy and non-subsidy portions of this geographic area-family size segment of the population. The instruction requests are:

```
DO YOU WISH TO PLOT THE SUBSIDY PORTION...
TYPE Y FOR YES, N FOR NO
?
```

and

```
DO YOU WISH TO PLOT THE NON- SUBSIDY PORTION...
TYPE Y FOR YES, N FOR NO
?
```



Upon processing the above plot requests the monitoring program recycles itself if the curve fitting option was 2. If the curve fitting option is 3, the monitoring program asks for parametric data filing instructions by:

```
IF THE ABOVE CURVE FITS ARE TO BE SAVED, NAME  
THE FILE, IF NOT TYPE NO  
?
```

As was suggested in Appendix A, to name one's file by the notation GxxFxx, it is suggested here that one save the parametric data file by GxxFxxP. Where the P stands for parametric. By using this notation one can easily access the original data base along with its parametric equivalent without lengthy searching through a file name dictionary.

A "NO" option is allowed to the storing of parametric data files even for this option 3 of the curve fitting. This is done to allow the decision maker, who even after using this all-inclusive option, finds that the curve fits do not meet his accuracy requirements, and he wishes to alter the original data base to try curve fitting at a latter time.

Upon completion of this storing request of option 3, the monitoring program recycles itself to start a new geo-

graphic area-family size case. The program will continue to recycle, allowing the decision maker to perform all his immediate investigations and to build his parametric data base. When the decision maker wishes to quit he need simply type ENDDATA 0 0 to the initial request of a cycle.

The first phase of the management information and control system is now completely explained. If used properly it generates for each geographic area-family size a parametric income frequency distribution and depending upon user input as to a Breakeven Level of Income generates subsidy and non-subsidy parametric portions of this population segment. These three parametric representations are then stored if requested in data disk files to be used by the second part of the management information and control system to be described in the next section of this Chapter. Chapter VI of this dissertation points out more clearly the use of this first part of the management information and control system by applying it to real-world negative income tax and guaranteed minimum income proposals.

PART 2 - THE MANAGEMENT INFORMATION AND CONTROL SYSTEM

TO UTILIZE THE MATHEMATICAL MODEL OF CHAPTER III

Through part 1 of the management information and control system found in this chapter techniques are described to develop a national income frequency distribution and its subsidy and non-subsidy portion for geographic area-family size segments of the total population. These income frequency distributions are in the form of parametric equations. Therefore, the model generates the  $\emptyset(I,G,F)$ ,  $\emptyset_S(I,G,F)$ , and  $\emptyset_n(I,G,F)$  of Chapter III, equations (3-56), (3-62), and (3-63) respectively. It also creates from input, to perform the subsidy and non-subsidy portioning in part 1, the  $BLI(G,F)$  of Chapter III, equation (3-61). Hence, the first step of Part 2 of the management information and control system is to request from the user, one by one, the geographic area-family size parametric data files to be analyzed here from those he created in part 1 of the system, once the user is on the time-sharing terminal.

To utilize part 2 of the management information and control system the user follows the sign on instructions found in part 1 of this chapter. However, where he is told to "FETCH MISCFT" there, he, to access part 2, changes the request to "FETCH MISMTH".

Once the user types "RUN", the monitoring program of part 2 of the management information and control system begins requesting user input and as a result of this input generates user pertinent output reports.

The input requests and output reports of the system are presented below along with comments that make more clear their particular meanings. Chapter VI applies the system to NIT and GMI proposals being discussed in Congress today and shows how the system developed in this dissertation can answer many of the Congressional questions until now, unanswered.

The first instruction the user sees is

```

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA
FILE TYPE THE NAME FILE
?
```

The system is, here, calling for a national income frequency data file in parametric form containing a geographic area index, a family size index, a Breakeven Level of Income, and polynomial representations of  $\emptyset(I,G,F)$ ,  $\emptyset_S(I,G,F)$ , and  $\emptyset_n(I,G,F)$ . These files are created in part 1 of the management information and control system and if the user followed the instructions of section 1 of this chapter are denoted by "GxxFxxP". It is this "GxxFxxP" he types here. It is also here that the user can type



"ENDATA" once he wishes to stop his analysis.

Now that the data file is inputted the system requests

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR  
THE NON-SUBSIDY POPULATION  
?

The value inputted here by the user in decimal form is the maximum income value for which he has realistic data. This input corresponds closely with the call for number of data points in part 1 of the information and control system. It is used as the upper limit for integrating  $\emptyset(I, G, F)$  and  $\emptyset_n(I, G, F)$ .

Once the upper limit is inputted the system computes the geographic area-family size's population and income figures using equations (3-66) and (3-77), its subsidy portion's population and income figures using equations (3-64) and (3-74), and its non-subsidy portion's population and income figures using equations (3-68) and (3-78). The results are then printed in the following format:

POPULATION FIGURES FOR GEOGRAPHIC AREA II FAMILY  
SIZE II IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION	=
SUBSIDY POPULATION	=
NON-SUBSIDY POPULATION	=

INCOME FIGURES FOR GEOGRAPHIC AREA II FAMILY  
SIZE II IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME	=
SUBSIDY INCOME	=
NON-SUBSIDY INCOME	=

The system now requests minimum income information.  
In the first case processed the user receives the message

INPUT A MINIMUM INCOME FOR THIS SEGMENT  
?

This figure is any constant in dollars and in decimal form.  
From this input the system then computes the gross subsidy  
given the subsidy portion using equation (3-84) and prints  
the following result

THE SUBSIDY ADDED TO GEOGRAPHIC AREA II FAMILY  
SIZE II FOR MINIMUM INCOME X IS Y MILLION  
DOLLARS

The system then asks

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...  
TYPE Y FOR YES, N FOR NO  
?

This instruction allows the decision maker to obtain subsidy figures for many different minimum incomes that he may wish to investigate. For each Y answer he gives, the system requests a new minimum income and prints the resulting subsidy.

When the system is in its second or above case an added request occurs before the above sequence for minimum incomes occurs. That request is:

```
DO YOU WISH TO USE THE MINIMUM INCOME OF THE LAST
CASE... TYPE Y FOR YES, N FOR NO
?
```

This instruction allows the decision maker to hold the minimum income constant for a series of cases (e.g. for the same family size over many geographic areas).

Once a minimum income is added to the analysis the system then requests a negative tax be inputted in polynomial form to perform equation (3-95). When it is the first case, the instruction begins:

```
INPUT A NEGATIVE TAX IN POLYNOMIAL FORM
?
```

If the decision maker wishes the negative tax to be a constant, he responds "0 C" where C is the decimal value of the constant. For all other polynomials he responds

" $N a_1 a_2 \dots a_{n+1}$ " where  $N$  is the degree of the polynomial and  $a_i, i=1, \dots, n+1$  are the coefficients. Zero coefficients must be inputted in the proper sequence.

The system then requests

INPUT THE LOWEST LIMIT AT WHICH THE NEGATIVE  
TAX IS APPLIED  
?

The answer to this question is the decimal lower income limit in the subsidy portion at which the negative tax takes effect. In many proposals such as Friedman's, it is zero (0.). However, others allow income to be earned up to a certain point before negative taxation takes place. This input sets the lower bound for integration where the upper bound is the Breakeven Level of Income. The system then computes and prints the results of the negative tax by

THE NEGATIVE TAX ON THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA II FAMILY SIZE II IS X  
MILLION DOLLARS

The decision maker then is asked

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y,  
IF NOT TYPE N  
?



It is through this instruction the user may test many different negative tax structures. Each time he answers yes, he is requested to input a new negative tax and lower limit, and the system prints the results.

If this is not the first case the system responds:

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF  
NOT TYPE NO  
?

A yes response to this question generates the complete set of instructions for negative taxes above. A no response eliminates the first two instructions and uses the negative tax and lower limit of the last case.

Once a minimum income and negative tax is agreed upon the system then computes the net subsidy, equation (3-100), and prints the result by

THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS X MILLION DOLLARS

The system now develops instructions to pay for the subsidy by marginal taxation of the non-subsidy income and of corporations. If this is the first case, the system responds

INPUT THE MARGINAL TAX IN POLYNOMIAL FORM

?

The instructions for this question are those for the polynomial negative tax. The system then computes using equation (3-104) the resulting revenue and prints

THE ADDED REVENUE DUE TO A MARGINAL TAX IS  
X MILLION DOLLARS

The system then responds

IS A NEW MARGINAL TAX TO BE INPUTED...TYPE Y  
FOR YES, N FOR NO

?

As in the case of minimum incomes and negative taxes, the decision maker is again given the opportunity to try different marginal taxes to pay for the created subsidy. Each yes response generates the above instructions.

When it is not the first case the system responds

DO YOU WISH TO USE THE MARGINAL TAX OF THE  
LAST CASE....TYPE Y FOR YES, N FOR NO

?

A no response triggers all the above instructions for marginal taxes. A yes response uses the marginal tax of

the last case to compute the additional revenues.

Once the above marginal tax computations on the non-subsidy portion are performed, the system responds

IS TAXATION OF CORPORATE EARNINGS TO BE USED  
TO PAY FOR THE SUBSIDY...TYPE Y FOR YES, N  
FOR NO  
?

If a yes response is made, the system then requests

INPUT THE CORPORATE EARNINGS FOR THIS  
GEOGRAPHIC AREA  
?

The response to this request is a decimal constant representing corporate earnings in billion of dollars.

The system then responds

IS THE CORPORATE MARGINAL TAX TO CHANGE...  
TYPE Y FOR YES, N FOR NO  
?

If a yes reply is made, the system requests

INPUT THE CORPORATE MARGINAL TAX IN  
POLYNOMIAL FORM  
?

The instructions for this response are those for inputted negative taxes and marginal taxes. The system then computes using equation (3-104) the result of corporate taxation and prints

THE ADDED REVENUE DUE TO A MARGINAL TAX ON  
CORPORATIONS IS X MILLION DOLLARS

If the response to the marginal tax change question is no, the marginal tax of the last case is used. If the response to the initial corporate tax question was no, corporate marginal taxation is neglected.

Once marginal taxation is completed, the system then computes a deficit or surplus and prints either

MARGINAL TAXATION RESULTED IN A SURPLUS OF  
X MILLIONS

or

MARGINAL TAXATION RESULTED IN A DEFICIT OF  
X MILLIONS

All the major pertinent calculations of the segment are now completed. However, the decision maker might wish to see how these calculations effect particular income brackets within the segment. For this purpose the system responds:



DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME  
BRACKETS AS TO SUBSIDY RECEIVED OR SUBSIDY  
COST LEVIED...TYPE Y FOR YES, N FOR NO  
?

A yes response to this instruction triggers the following cycle of instructions. First, for the subsidy received, a sequence of the following instructions occurs until user ended

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS  
TO COMPUTE SUBSIDY RECEIVED, IF FINISHED IN-  
VESTIGATING SUBSIDY RECEIVED AREA TYPE -1. -1.  
?

The response to this instruction is the actual income range for which the user wishes to find out how much subsidy was received. The interval may be as low as one dollar. The calculations are made using equations (3-64) with the limits of integration changed and equation (3-66). The result is printed as

THE SUBSIDY RECEIVED BETWEEN X AND Y IS Z MILLION

Once the above cycle is ended by the user, the following instruction is printed to begin a new cycle

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS  
TO COMPUTE SUBSIDY PAID, IF FINISHED INVESTI-  
GATING SUBSIDY PAID AREA TYPE -1. -1.

?

The response to this instruction is the actual income range for which the user wishes to find out how much subsidy was paid. The interval as above may be as low as one dollar. The calculations are made using equation (3-108). The result is printed as

THE SUBSIDY PAID BETWEEN X AND Y IS Z MILLION

For the first case, once the above cycle is stopped, the system responds

ARE CUMULATIVE FIGURES TO BE KEPT...TYPE

Y FOR YES, N FOR NO

?

If a yes response is given the following cumulative figures are generated for all cases until the system is stopped. Total population of all segments and their subsidy and non-subsidy portions using equations (3-73), (3-72), and (3-69) respectively. The population figures for each family size and geographic area. The subsidy and non-subsidy population figures for each family size

using equations (3-67) and (3-70) and for each geographic area using equations (3-68) and (3-71). The total income of all segments and their subsidy and non-subsidy portions using equations (3-82), (3-81), and (3-77). The income figures for each family size and geographic area. The subsidy and non-subsidy income figures for each family size using equations (3-75) and (3-79) and for each geographic area using equations (3-76) and (3-80). Subsidy figures for all segments and each family size and geographic area using equation (3-87), (3-85), and (3-86). Negative tax figures for all segments and each family size and geographic area using equations (3-98), (3-96), and (3-97). Net subsidy figures for all segments and each family size and geographic area using equations (3-103), (3-101), and (3-102). Combine public and corporate marginal tax figures for all segments and for each family size and geographic area using equations (3-107), (3-105), and (3-106). And, finally, deficit or surplus figures for all segments and for each family size and geographic area.

If cumulative figures are kept, the system responds at the end of every case except the first the following instructions

DO YOU WISH TO PRINT THE CUMULATIVE FIGURES...

TYPE Y FOR YES, N FOR NO

?

If the response is yes, the system prints all the above cumulative figures in report form (see Chapter VI for sample output) preceded by the following line to denote the number of segments included in the accumulation.

THE CUMULATIVE FIGURES BELOW ARE FOR II  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENTS

Cumulative figures may be printed after every case or for particular, user pertinent, divisions such as one family size over many geographic areas.

The description of part 2 of the management information and control system is now complete. Therefore, in this chapter the management information and control system of this dissertation is completely described. By using the two parts, one may investigate any guaranteed minimum income or negative tax proposal. One may select any Breakeven Level of Income in part 1. Then in part 2, examine varying minimum incomes, negative taxes, marginal taxes, and income brackets for any geographic area-family size segment of the population. To further point out the system's flexibility, Chapter VI utilizes it to investigate pertinent proposals.



C H A P T E R VI  
SYSTEM TESTING

This chapter tests the Family Assistance Program submitted to Congress by President Nixon for a family size of four. It then alters some of the parameters as recommended by members of the Committee on Finance of the United States Senate to point out that the system developed in this dissertation is capable of supplying answers to cost questions not presently available. Chapter II, in the section devoted to the Present State of the Art of NIT and QMI Proposals, gives the background material necessary for this Chapter.

The first step of this test case is the collection of national income frequency data. The data used here is that of Appendix A, Tables A1-6, A1-13, A1-20, and A1-27 and are explained there and in Chapter IV. These tables are stored in disk files denoted by G01F04, G02F04, G03F04, and G04F04 respectively,

These disk files were then processed through part 1 of the management information and control system with a Breakeven Level of Income of \$2,750. The FAP figure for a family of four is \$2,710 made up of a \$1,600 minimum income, a \$720 tax exemption on initial earnings, and 50% of the earnings between \$720 and \$1,500. The discrepancy of \$40 between the test case and the actual case is made to add one more data point to the to-be-subsidized portions data base. In the event that a better data source

allowing more data points, as recommended in Chapter IV, becomes available the above concession can be eliminated.

Tables 6-1 through 6-4 present the results of part 1 of the management information and control system and at the same time exemplifies for the decision maker the user instructions of Chapter V, Section 1. Examining these results, especially the standard errors of the curve fits, show that even for a small sample size that the portioning of geographic area-family size segments into subsidy and non-subsidy portions adds greatly to one's accuracy. In no income level case did the errors in fitting exceed 15% and in all segments averaged less than 8%.

The parametric results of part 1 of the management information and control system are stored in G01F04P, G02F04P, G03F04P, and G04F04P for data bases G01F04, G02F04, G03F04, and G04F04 respectively. These parametric data bases are then used as input to part 2 of the management information and control system.

To determine the subsidy paid the FAP minimum income of \$1,600 is used. To determine the net subsidy the FAP negative tax of 50% between \$720 and the Break-even Level of Income is used. To pay for the subsidy a marginal tax of 0.4% is placed on the gross incomes of the non-subsidy section between the Breakeven Level of

TABLE 6-1

PARAMETRIC DATA GENERATION FOR FAMILY SIZE 4

GEOGRAPHIC AREA 1, BREAKEVEN LEVEL OF INCOME \$2,750

RUN MISCFT  
16K

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME,  
THE GEOGRAPHIC AREA INDEX, AND THE FAMILY SIZE INDEX  
?G01FO4 1 4

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE  
?19

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1  
TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...TYPE 2  
TO CURVE FIT BOTH ABOVE ...TYPE 3  
?3

THE FOLLOWING BEST FITS THE TOTAL POPULATION FOR  
GEOGRAPHIC AREA 1 FAMILY SIZE 4

POLYNOMIAL DEGREE = 6                      STD. ERROR = 8.4389

TERM	DEGREE	COEFFICIENTS
	0	.13363075E+02
	1	.13959270E-01
	2	-.67786090E-05
	3	.27188916E-08
	4	-.32662374E-12
	5	.15185604E-16
	6	-.24394717E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y



TABLE 6-1

(cont.)

BACK SOLUTIONS FOR BEST FIT - DEGREE = 6

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	17.92	13.36	4.55
1.00	500.00	17.92	18.97	-1.05
1.00	1250.00	15.44	24.78	-9.34
1.00	1750.00	20.44	28.78	-8.34
1.00	2250.00	43.00	33.90	9.10
1.00	2750.00	39.09	40.64	-1.54
1.00	3250.00	65.61	49.24	16.36
1.00	3750.00	58.40	59.76	-1.36
1.00	4500.00	76.45	78.73	-2.28
1.00	5500.00	106.90	108.23	-1.34
1.00	6500.00	129.94	139.13	-9.19
1.00	7500.00	159.18	167.28	-8.10
1.00	8500.00	198.74	188.79	9.95
1.00	9500.00	207.09	200.66	6.44
1.00	11000.00	195.72	196.94	-1.22
1.00	13500.00	135.37	139.63	-4.27
1.00	16500.00	53.43	51.49	1.93
1.00	20000.00	53.43	53.75	-.33
1.00	23000.00	53.43	53.40	.02

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS GEOGRAPHIC AREA-FAMILY SIZE  
?2750.

THE FOLLOWING BEST FITS THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA 1 FAMILY SIZE 4

POLYNOMIAL DEGREE = 5                      STD. ERROR = .0000

TERM	DEGREE	COEFFICIENTS
0		.17917616E+02
1		-.28480647E-01
2		.11452734E-03
3		-.15009695E-06
4		.76320622E-10
5		-.12779745E-13

TABLE 6-1

(cont.)

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 5

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	17.92	17.92	.00
1.00	500.00	17.92	17.92	-.00
1.00	1250.00	15.44	15.44	.00
1.00	1750.00	20.44	20.44	-.00
1.00	2250.00	43.00	43.00	.00
1.00	2750.00	39.09	39.09	-.00

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION OF  
GEOGRAPHIC AREA 1 FAMILY SIZE 4

POLYNOMIAL DEGREE = 8                      STD. ERROR = 8.1743

TERM	DEGREE	COEFFICIENTS
	0	-.20938563E+03
	1	.21922678E+00
	2	-.74194685E-04
	3	.12498575E-07
	4	-.87469831E-12
	5	.33831062E-17
	6	.26008269E-20
	7	-.12190670E-24
	8	.17297237E-29

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

TABLE 6-1

(cont.)

BACK SOLUTIONS FOR BEST FIT - DEGREE = 8

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	2750.00	39.09	43.81	-4.72
1.00	3250.00	65.61	54.73	10.87
1.00	3750.00	58.40	64.02	-5.62
1.00	4500.00	76.45	78.52	-2.07
1.00	5500.00	106.90	102.93	3.97
1.00	6500.00	129.94	133.05	-3.11
1.00	7500.00	159.18	164.37	-5.18
1.00	8500.00	198.74	190.52	8.22
1.00	9500.00	207.09	205.75	1.34
1.00	11000.00	195.72	201.24	-5.52
1.00	13500.00	135.37	133.08	2.29
1.00	16500.00	53.43	53.96	-.53
1.00	20000.00	53.43	53.36	.07
1.00	23000.00	53.43	53.44	-.01

DO YOU WISH TO PLOT...TYPE Y FOR YES, N FOR NO  
?Y

DO YOU WISH TO PLOT THE ORIGINAL DATA...TYPE Y FOR YES, N FOR NO  
?Y

15.437      47.380      79.322      111.265      143.208      175.151

+-----+-----+-----+-----+-----+-----+

0	I *								
500.0	I *								
1250.	I*								
1750.	I *								
2250.	I	*							
2750.	I	*							
3250.	I		*						
3750.	I	*							
4500.	I		*						
5500.	I			*					
6500.	I				*				
7500.	I					*			
8500.	I						*		*
9500.	I								
.1100E+05I									*
.1350E+05I						*			
.1650E+05I		*							
.2000E+05I		*							
.2300E+05I		*							

TABLE 6-1

(cont.)

DO YOU WISH TO PLOT THE CURVE FIT FOR THE WHOLE  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...TYPE Y FOR YES, N FOR NO  
?Y

	13.363	44.736	76.110	107.483	138.856	170.229
0	I*					
800.0	I *					
1600.	I *	*				
2400.	I *	*				
3200.	I *	*				
4000.	I *	*	*			
4800.	I *	*	*	*		
5600.	I *	*	*	*	*	
6400.	I *	*	*	*	*	*
7200.	I *	*	*	*	*	*
8000.	I *	*	*	*	*	*
8800.	I *	*	*	*	*	*
9600.	I *	*	*	*	*	*
.1040E+05I	I *	*	*	*	*	*
.1120E+05I	I *	*	*	*	*	*
.1200E+05I	I *	*	*	*	*	*
.1280E+05I	I *	*	*	*	*	*
.1360E+05I	I *	*	*	*	*	*
.1440E+05I	I *	*	*	*	*	*
.1520E+05I	I *	*	*	*	*	*
.1600E+05I	I *	*	*	*	*	*
.1680E+05I	I *	*	*	*	*	*
.1760E+05I	I *	*	*	*	*	*
.1840E+05I	I *	*	*	*	*	*
.1920E+05I	I *	*	*	*	*	*



TABLE 6-1

(cont.)

DO YOU WISH TO PLOT THE SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	15.149	21.037	26.925	32.813	38.702	44.590
	+-----+-----+-----+-----+-----+-----+					
0	I		*			
110.0	I	*				
220.0	I	*				
330.0	I	*				
440.0	I		*			
550.0	I		*			
660.0	I		*			
770.0	I		*			
880.0	I		*			
990.0	I		*			
1100.	I	*				
1210.	I	*				
1320.	I	*				
1430.	I	*				
1540.	I	*				
1650.	I		*			
1760.	I		*			
1870.	I		*			
1980.	I		*			
2090.	I		*			
2200.	I		*			
2310.	I		*			
2420.	I		*			
2530.	I		*			
2640.	I		*			
2750.	I		*			
2860.	I		*			



TABLE 6-1

(cont.)

DO YOU WISH TO PLOT THE NON-SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	42.604	70.139	97.675	125.210	152.745	180.281
2750.	I*					
3640.	I	*				
4530.	I		*			
5420.	I			*		
6310.	I				*	
7200.	I					*
8090.	I					
8980.	I					*
9870.	I					
.1076E+05I						
.1165E+05I						*
.1254E+05I					*	
.1343E+05I				*		
.1432E+05I			*			
.1521E+05I		*				
.1610E+05I	*					
.1699E+05I	*					
.1788E+05I	*					
.1877E+05I	*					
.1966E+05I	*					
.2055E+05I	*					
.2144E+05I	*					
.2233E+05I*						

TABLE 6-2

PARAMETRIC DATA GENERATION FOR FAMILY SIZE 4

GEOGRAPHIC AREA 2, BREAKEVEN LEVEL OF INCOME \$2,750

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME,  
THE GEOGRAPHIC AREA INDEX, AND THE FAMILY SIZE INDEX

?G02F04 2 4

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE

?19

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1

TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...TYPE 2

TO CURVE FIT BOTH ABOVE ...TYPE 3

?3

THE FOLLOWING BEST FITS THE TOTAL POPULATION FOR  
GEOGRAPHIC AREA 2 FAMILY SIZE 4

POLYNOMIAL DEGREE = 7

STD. ERROR =

10.0345

TERM	DEGREE	COEFFICIENTS
	0	.17082318E+02
	1	.10884492E-01
	2	.11919092E-05
	3	-.36635130E-09
	4	.17043641E-12
	5	-.22990554E-16
	6	.11402373E-20
	7	-.19061169E-25

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?N

TABLE 6-2

(cont.)

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS GEOGRAPHIC AREA-FAMILY SIZE  
?2750.

THE FOLLOWING BEST FITS THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA 2 FAMILY SIZE 4

POLYNOMIAL DEGREE = 1 STD. ERROR = 8.8801

TERM	DEGREE	COEFFICIENTS
0		.16460207E+02
1		.12275237E-01

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 1

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	20.79	16.46	4.33
1.00	500.00	20.79	22.60	-1.81
1.00	1250.00	25.59	31.80	-6.21
1.00	1750.00	32.34	37.94	-5.60
1.00	2250.00	58.22	44.08	14.14
1.00	2750.00	45.36	50.22	-4.85

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION OF  
GEOGRAPHIC AREA 2 FAMILY SIZE 4

TABLE 6-2

(cont.)

POLYNOMIAL DEGREE = 6

STD. ERROR =

9.8921

TERM	DEGREE	COEFFICIENTS
0		-.13588677E+03
1		.16082299E+00
2		-.54545421E-04
3		.99097354E-08
4		-.86069407E-12
5		.34181214E-16
6		-.50236595E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 6

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	2750.00	45.36	55.90	-10.54
1.00	3250.00	83.38	66.61	16.77
1.00	3750.00	77.01	76.48	.52
1.00	4500.00	88.71	92.26	-3.55
1.00	5500.00	110.52	117.91	-7.39
1.00	6500.00	150.79	148.70	2.09
1.00	7500.00	175.00	181.21	-6.22
1.00	8500.00	221.52	210.31	11.21
1.00	9500.00	233.82	230.75	3.07
1.00	11000.00	230.31	236.55	-6.24
1.00	13500.00	172.57	173.90	-1.33
1.00	16500.00	56.78	54.41	2.36
1.00	20000.00	56.78	57.70	-.93
1.00	23000.00	56.78	56.60	.17

DO YOU WISH TO PLOT...TYPE Y FOR YES, N FOR NO  
?Y

TABLE 6-2

(cont.)

DO YOU WISH TO PLOT THE ORIGINAL DATA...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE CURVE FIT FOR THE WHOLE  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	16.460	22.311	28.163	34.014	39.865	45.716
	+-----+-----+-----+-----+-----+-----+					
0	I*					
110.0	I *					
220.0	I	*				
330.0	I	*				
440.0	I	*				
550.0	I		*			
660.0	I		*			
770.0	I		*			
880.0	I		*			
990.0	I			*		
1100.0	I			*		
1210.0	I			*		
1320.0	I			*		
1430.0	I			*		
1540.0	I			*		
1650.0	A				*	
1760.0	I				*	
1870.0	I				*	
1980.0	I				*	
2090.0	I				*	
2200.0	I				*	
2310.0	I				*	
2420.0	I				*	
2530.0	I				*	
2640.0	I				*	
2750.0	I				*	
2860.0	I				*	*



TABLE 6-2

(cont.)

DO YOU WISH TO PLOT THE NON-SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

-117.050      -57.891            1.267            60.426            119.584            178.743  
+-----+-----+-----+-----+-----+-----+

2750.	I			*						
3640.	I				*					
4530.	A					*				
5420.	I						*			
6310.	I							*		
7200.	I								*	
8090.	I									*
8980.	I									*
9870.	I									*
.1076E+05I										*
.1165E+05I										*
.1254E+05I										*
.1343E+05I									*	
.1432E+05I								*		
.1521E+05I							*			
.1610E+05I					*					
.1699E+05I				*						
.1788E+05I			*							
.1877E+05I			*							
.1966E+05I				*						
.2055E+05I					*					
.2144E+05I						*				
.2233E+05I							*			
.2322E+05I			*							
.2411E+05I*										

IF THE ABOVE CURVE FITS ARE TO BE SAVED, NAME THE FILE, IF NOT TYPE NO  
?G02F04P

TABLE 6-3

## PARAMETRIC DATA GENERATION FOR FAMILY SIZE 4

GEOGRAPHIC AREA 3, BREAKEVEN LEVEL OF INCOME \$2,750

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME,  
 THE GEOGRAPHIC AREA INDEX, AND THE FAMILY SIZE INDEX  
 ?G03F04 3 4

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE  
 ?19

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1  
 TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...TYPE 2  
 TO CURVE FIT BOTH ABOVE ...TYPE 3  
 ?3

THE FOLLOWING BEST FITS THE TOTAL POPULATION FOR  
 GEOGRAPHIC AREA 3 FAMILY SIZE 4

POLYNOMIAL DEGREE = 6                      STD. ERROR =                      16.5347

TERM	DEGREE	COEFFICIENTS
	0	.41896947E+02
	1	.14733304E-01
	2	.19381178E-06
	3	.97555049E-09
	4	-.17397973E-12
	5	.94336141E-17
	6	-.16440631E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
 ?N

TABLE 6-3

(cont.)

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS GEOGRAPHIC AREA-FAMILY SIZE  
?2750.

THE FOLLOWING BEST FITS THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA 3 FAMILY SIZE 4

POLYNOMIAL DEGREE = 3 STD. ERROR = 11.7192

TERM	DEGREE	COEFFICIENTS
0		.45494785E+02
1		-.37200551E-01
2		.61136938E-04
3		-.15979459E-07

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 3

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	43.77	45.49	-1.72
1.00	500.00	43.77	40.18	3.59
1.00	1250.00	64.65	63.31	1.34
1.00	1750.00	71.05	81.99	-10.94
1.00	2250.00	100.48	89.28	11.20
1.00	2750.00	69.76	73.22	-3.46

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION OF  
GEOGRAPHIC AREA 3 FAMILY SIZE 4

TABLE 6-3

(cont.)

POLYNOMIAL DEGREE = 5                      STD. ERROR =                      19.2610

TERM	DEGREE	COEFFICIENTS
0		.13863853E+03
1		-.79381096E-01
2		.30984906E-04
3		-.35968274E-08
4		.16301685E-12
5		-.25655979E-17

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 5

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	2750.00	69.76	88.78	-19.02
1.00	3250.00	131.40	101.71	29.69
1.00	3750.00	115.03	117.34	-2.31
1.00	4500.00	133.98	143.22	-9.24
1.00	5500.00	183.31	177.17	6.14
1.00	6500.00	210.53	205.22	5.31
1.00	7500.00	192.21	223.68	-31.48
1.00	8500.00	243.31	230.78	12.53
1.00	9500.00	242.55	226.34	16.21
1.00	11000.00	191.48	200.78	-9.30
1.00	13500.00	133.65	128.64	5.01
1.00	16500.00	46.29	52.23	-5.94
1.00	20000.00	46.29	43.14	3.15
1.00	23000.00	46.29	47.02	-.73

DO YOU WISH TO PLOT...TYPE Y FOR YES, N FOR NO  
?Y



TABLE 6-3

(cont.)

DO YOU WISH TO PLOT THE ORIGINAL DATA...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE CURVE FIT FOR THE WHOLE  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	39.302	47.653	56.004	64.355	72.705	81.056
0	I	*				
110.0	I	*				
220.0	I	*				
330.0	I	*				
440.0	I	*				
550.0	I	*				
660.0	I	*				
770.0	I	*				
880.0	I	*				
990.0	I	*				
1100.0	I	*				
1210.0	I	*				
1320.0	I	*				
1430.0	I	*				
1540.0	I	*				
1650.0	I	*				
1760.0	I	*				
1870.0	I	*				
1980.0	I	*				
2090.0	I	*				
2200.0	I	*				
2310.0	I	*				
2420.0	I	*				
2530.0	I	*				
2640.0	I	*				
2750.0	I	*				
2860.0	I	*				



TABLE 6-3

(cont.)

DO YOU WISH TO PLOT THE NON-SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

8.788      45.664      82.540      119.416      156.292      193.168

+-----+-----+-----+-----+-----+-----+

2750.	I		*						
3640.	I			*					
4530.	I				*				
5420.	I					*			
6310.	I						*		
7200.	I							*	
8090.	I								*
8980.	I								*
9870.	I								*
.1076E+05I								*	
.1165E+05I							*		
.1254E+05I						*			
.1343E+05I					*				
.1432E+05I			*						
.1521E+05I			*						
.1610E+05I		*							
.1699E+05I		*							
.1788E+05I	*								
.1877E+05I	*								
.1966E+05I	*								
.2055E+05I		*							
.2144E+05I		*							
.2233E+05I		*							
.2322E+05I	*								
.2411E+05I*									

IF THE ABOVE CURVE FITS ARE TO BE SAVED, NAME THE FILE, IF NOT TYPE NO  
?G03F04P

TABLE 6-4

PARAMETRIC DATA GENERATION FOR FAMILY SIZE 4

GEOGRAPHIC AREA 4, BREAKEVEN LEVEL OF INCOME \$2,750

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME,  
 THE GEOGRAPHIC AREA INDEX, AND THE FAMILY SIZE INDEX  
 ?GO4FO4 4 4

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE  
 ?19

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1  
 TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...TYPE 2  
 TO CURVE FIT BOTH ABOVE ...TYPE 3  
 ?3

THE FOLLOWING BEST FITS THE TOTAL POPULATION FOR  
 GEOGRAPHIC AREA 4 FAMILY SIZE 4

POLYNOMIAL DEGREE = 6                      STD. ERROR =                      8.0989

TERM	DEGREE	COEFFICIENTS
	0	.13046273E+02
	1	.66722098E-02
	2	-.25105900E-05
	3	.13295214E-08
	4	-.17059632E-12
	5	.81203560E-17
	6	-.13174219E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
 ?N

TABLE 6-4

(cont.)

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS GEOGRAPHIC AREA-FAMILY SIZE  
?2750.

THE FOLLOWING BEST FITS THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA 4 FAMILY SIZE 4

POLYNOMIAL DEGREE = 1 STD. ERROR = 7.0897

TERM	DEGREE	COEFFICIENTS
0		.12968481E+02
1		.51203711E-02

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 1

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	15.81	12.97	2.85
1.00	500.00	15.81	15.53	.28
1.00	1250.00	12.46	19.37	-6.91
1.00	1750.00	18.37	21.93	-3.56
1.00	2250.00	35.42	24.49	10.93
1.00	2750.00	23.46	27.05	-3.59

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION OF  
GEOGRAPHIC AREA 4 FAMILY SIZE 4

TABLE 6-4

(cont.)

POLYNOMIAL DEGREE = 6                      STD. ERROR = 9.3115

TERM	DEGREE	COEFFICIENTS
0		-.44970704E+02
1		.53601830E-01
2		-.16329312E-04
3		.32496852E-08
4		-.30586591E-12
5		.12782761E-16
6		-.19408489E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 6

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	2750.00	23.46	30.96	-7.50
1.00	3250.00	52.94	38.59	14.34
1.00	3750.00	43.11	46.23	-3.12
1.00	4500.00	53.98	58.25	-4.27
1.00	5500.00	76.85	75.62	1.23
1.00	6500.00	90.19	93.66	-3.47
1.00	7500.00	105.00	110.50	-5.50
1.00	8500.00	138.48	123.90	14.57
1.00	9500.00	130.41	131.86	-1.45
1.00	11000.00	123.59	130.79	-7.21
1.00	13500.00	98.27	95.68	2.59
1.00	16500.00	37.51	37.60	-.08
1.00	20000.00	37.51	37.68	-.17
1.00	23000.00	37.51	37.47	.04

DO YOU WISH TO PLOT...TYPE Y FOR YES, N FOR NO  
?Y



TABLE 6-4

(cont.)

DO YOU WISH TO PLOT THE ORIGINAL DATA...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE CURVE FIT FOR THE WHOLE  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	12.968	15.409	17.850	20.291	22.731	25.172
	+-----+-----+-----+-----+-----+-----+					
0	I*					
110.0	I *					
220.0	I	*				
330.0	I	*				
440.0	I	*				
550.0	I		*			
660.0	I		*			
770.0	I		*			
880.0	I		*			
990.0	I		*			
1100.0	I		*			
1210.0	A		*			
1320.0	I			*		
1430.0	I			*		
1540.0	I			*		
1650.0	I			*		
1760.0	I			*		
1870.0	I			*		
1980.0	I			*		
2090.0	I			*		
2200.0	I			*		
2310.0	I			*		
2420.0	I			*		
2530.0	I			*		
2640.0	I			*		
2750.0	I			*		
2860.0	I			*		





Income and \$23,000. The reason for this low upper limit is again the lack of data in the data base. A problem easily remedied by appropriating funds to collect a more extensive data base. What is a most interesting outcome is that such a small marginal tax, which does not include corporate earnings, more than paid for the net subsidy.

To further investigate the results and actual dollar figures of part 2 of the management information and control system for a family of four for the whole country, look at the cumulative figures of Table 6-5. Table 6-5 also gives pertinent results broken down by family size, geographic area, and geographic area-family size divisions not even considered in the FAP plan.

Up to this point the FAP proposal is investigated exactly as it is written. However, there are many critics of this proposal who wish to see how FAP reacts with respect to cost when its minimum income is changed or more realistically when its Breakeven Level of Income and minimum income is changed.

The above questions of the FAP plan are not only apropos, but also most evidently point out the uniqueness and wide scope of the system developed in this dissertation since they can be analyzed by this system.

To test the system as to changing the minimum income, the geographic area-family size represented by

## TABLE 6-5

## FAP TEST FOR FAMILY SIZE 4

USER NØ.?

ØØDE?\*

TERMINAL 022 PORT 013

TIME: 15:08, DATE: 06/26/71. ØFF AT 23:00.

FETCH MISMTH

ØK. DATE FILED: 06/26/71.

USE 16K

ØK

RUN

16K

TØ SELECT A GEØGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?GØ1FØ4P

INPUT AN UPPER INCOME LIMIT IN DØLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

\*PØPOPULATION FIGURES FOR GEØGRAPHIC AREA 1 FAMILY SIZE 4  
IN MILLIØNS OF PEOPLE ARE

SEGMENT PØPOPULATION =	2.229638
SUBSIDY PØPOPULATION =	.068912
NØN-SUBSIDY PØPOPULATION =	2.160726

\*\*INCOME FIGURES FOR GEØGRAPHIC AREA 1 FAMILY SIZE 4  
IN BILLIØNS OF DØLLARS ARE

SEGMENT INCOME =	24.475206
SUBSIDY INCOME =	.116575
NØN-SUBSIDY INCOME =	24.358631

## TABLE 6-5

(cont.)

INPUT A MINIMUM INCOME FOR THIS SEGMENT  
?1600.

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 1 FAMILY SIZE 4  
FOR MINIMUM INCOME 1600. IS 110.259233 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
?Y

INPUT A NEGATIVE TAX IN POLYNOMIAL FORM  
?0 .5

INPUT THE LOWEST LIMIT AT WHICH THE NEGATIVE TAX IS APPLIED  
?720.

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 1  
FAMILY SIZE 4 IS 55.999757 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
?Y

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS 54.259475 MILLION DOLLARS

INPUT THE MARGINAL TAX IN POLYNOMIAL FORM  
?0 .004

\*\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
IS 97.434525 MILLION DOLLARS

IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO  
?N



TABLE 6-5

(cont.)

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
 TYPE Y FOR YES, N FOR NO  
 ?N

\*\*MARGINAL TAXATION RESULTED IN A SURPLUS OF 43.175050 MILLIONS

DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
 SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO  
 ?Y

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS TO COMPUTE SUBSIDY-  
 RECEIVED, IF FINISHED INVESTIGATING SUBSIDY RECEIVED AREA  
 TYPE -1. -1.  
 ?500. 1000.

\*\*THE SUBSIDY RECEIVED BETWEEN 500. AND 1000. IS 14.710149 MILLION

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS TO COMPUTE SUBSIDY  
 RECEIVED, IF FINISHED INVESTIGATING SUBSIDY RECEIVED AREA  
 TYPE -1. -1.  
 ?2000. 2500.

\*\*THE SUBSIDY RECEIVED BETWEEN 2000. AND 2500. IS 33.758293 MILLION

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS TO COMPUTE SUBSIDY  
 RECEIVED, IF FINISHED INVESTIGATING SUBSIDY RECEIVED AREA  
 TYPE -1. -1.  
 ?-1. -1.

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS TO COMPUTE SUBSIDY  
 PAID IF FINISHED INVESTIGATING SUBSIDY PAID AREA  
 TYPE -1. -1.  
 ?3000. 4000.

\*\*THE SUBSIDY PAID BETWEEN 3000. AND 4000. IS .837108 MILLION

TABLE 6-5

(cont.)

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS TO COMPUTE SUBSIDY  
PAID IF FINISHED INVESTIGATING SUBSIDY PAID AREA  
TYPE -1. -1.  
?11000. 12000.

\*\*THE SUBSIDY PAID BETWEEN 11000. AND 12000. IS 8.821527 MILLION

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS TO COMPUTE SUBSIDY  
PAID IF FINISHED INVESTIGATING SUBSIDY PAID AREA  
TYPE -1. -1.  
?20000. 21000.

\*\*THE SUBSIDY PAID BETWEEN 20000. AND 21000. IS 4.273173 MILLION

INPUT THE INCOME RANGES LOWER AND UPPER LIMITS TO COMPUTE SUBSIDY  
PAID IF FINISHED INVESTIGATING SUBSIDY PAID AREA  
TYPE -1. -1.  
?-1. -1.

ARE CUMULATIVE FIGURES TO BE KEPT... TYPE Y FOR YES, N FOR NO  
?Y

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?G02F04P

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

\*\*POPULATION FIGURES FOR GEOGRAPHIC AREA 2 FAMILY SIZE 4  
IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION =	2.644188
SUBSIDY POPULATION =	.091681
NON-SUBSIDY POPULATION =	2.552507

TABLE 6-5

(cont.)

\*\*INCOME FIGURES FOR GEOGRAPHIC AREA 2 FAMILY SIZE 4  
IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME =	29.521440
SUBSIDY INCOME =	.147336
NON-SUBSIDY INCOME =	29.374104

DO YOU WISH TO USE THE MINIMUM INCOME OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 2 FAMILY SIZE 4  
FOR MINIMUM INCOME 1600. IS 146.690093 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
?N

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 2  
FAMILY SIZE 4 IS 70.770979 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
?Y

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS 75.919114 MILLION DOLLARS

DO YOU WISH TO USE THE MARGINAL TAX OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

(\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
IS 117.496415 MILLION DOLLARS



TABLE 6-5

(cont.)

IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO  
?N

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
TYPE Y FOR YES, N FOR NO  
?N

\*\*MARGINAL TAXATION RESULTED IN A SURPLUS OF 41.577301 MILLIONS

DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PRINT THE CUMULATIVE FIGURES...TYPE Y FOR YES, N FOR NO  
?N

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?G03F04P

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

\*\*POPULATION FIGURES FOR GEOGRAPHIC AREA 3 FAMILY SIZE 4  
IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION =	2.698658
SUBSIDY POPULATION =	.179793
NON-SUBSIDY POPULATION =	2.518865

\*\*INCOME FIGURES FOR GEOGRAPHIC AREA 3 FAMILY SIZE 4  
IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME =	26.467219
SUBSIDY INCOME =	.285631
NON-SUBSIDY INCOME =	26.181589



## TABLE 6-5

(cont.)

DO YOU WISH TO USE THE MINIMUM INCOME OF THE LAST CASE  
 ...TYPE Y FOR YES, N FOR NO  
 ?Y

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 3 FAMILY SIZE 4  
 FOR MINIMUM INCOME 1600. IS 287.669176 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
 ?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
 ?N

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 3  
 FAMILY SIZE 4 IS 137.488862 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
 ?Y

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
 IS 150.180314 MILLION DOLLARS

DO YOU WISH TO USE THE MARGINAL TAX OF THE LAST CASE  
 ...TYPE Y FOR YES, N FOR NO  
 ?Y

\*\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
 IS 104.726354 MILLION DOLLARS

IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO  
 ?N

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
 TYPE Y FOR YES, N FOR NO  
 ?N

TABLE 6-5

(cont.)

\*\*MARGINAL TAXATION RESULTED IN A DEFICIT OF -45.453960 MILLIONS

DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PRINT THE CUMULATIVE FIGURES...TYPE Y FOR YES, N FOR NO  
?N

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?G04F04P

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

\*\*POPULATION FIGURES FOR GEOGRAPHIC AREA 4 FAMILY SIZE 4  
IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION =	1.566769
SUBSIDY POPULATION =	.055025
NON-SUBSIDY POPULATION =	1.511745

\*\*INCOME FIGURES FOR GEOGRAPHIC AREA 4 FAMILY SIZE 4  
IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME =	17.532665
SUBSIDY INCOME =	.084533
NON-SUBSIDY INCOME =	17.448132

DO YOU WISH TO USE THE MINIMUM INCOME OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 4 FAMILY SIZE 4  
FOR MINIMUM INCOME 1600. IS 88.039564 MILLION DOLLARS

TABLE 6-5

(cont.)

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
?N

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 4  
FAMILY SIZE 4 IS 40.267245 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
?Y

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS 47.772319 MILLION DOLLARS

DO YOU WISH TO USE THE MARGINAL TAX OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
IS 69.792527 MILLION DOLLARS

IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO  
?N

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
TYPE Y FOR YES, N FOR NO  
?N

\*\*MARGINAL TAXATION RESULTED IN A SURPLUS OF 22.020208 MILLIONS

DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO  
?N



TABLE 6-5

(cont.)

DO YOU WISH TO PRINT THE CUMULATIVE FIGURES...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE CUMULATIVE FIGURES BELOW ARE FOR 4 GEOGRAPHIC AREA-FAMILY  
SIZE SEGMENTS

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR ALL SEGMENTS ARE

TOTAL =	9.139253
SUBSIDY =	.395411
NON-SUBSIDY =	8.743842

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR FAMILY SIZE 4 ARE

TOTAL =	9.139253
SUBSIDY =	.395411
NON-SUBSIDY =	8.743842

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR  
GEOGRAPHIC AREA 1 ARE

TOTAL =	2.229638
SUBSIDY =	.068912
NON-SUBSIDY =	2.160726

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR  
GEOGRAPHIC AREA 2 ARE

TOTAL =	2.644188
SUBSIDY =	.091681
NON-SUBSIDY =	2.552507

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR  
GEOGRAPHIC AREA 3 ARE

TOTAL =	2.698658
SUBSIDY =	.179793
NON-SUBSIDY =	2.518865



TABLE 6-5

(cont.)

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR  
GEOGRAPHIC AREA 4 ARE

TOTAL =	1.566769
SUBSIDY =	.055025
NON-SUBSIDY =	1.511745

\*\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR ALL SEGMENTS ARE

TOTAL =	97.996530
SUBSIDY =	.634075
NON-SUBSIDY =	97.362456

\*\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR FAMILY SIZE 4 ARE

TOTAL =	97.996530
SUBSIDY =	.634075
NON-SUBSIDY =	97.362456

\*(THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR GEOGRAPHIC AREA 1 ARE

TOTAL =	24.475206
SUBSIDY =	.116575
NON-SUBSIDY =	24.358631

\*\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR GEOGRAPHIC AREA 2 ARE

TOTAL =	29.521440
SUBSIDY =	.147336
NON-SUBSIDY =	29.374104

\*\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR GEOGRAPHIC AREA 3 ARE

TOTAL =	26.467219
SUBSIDY =	.285631
NON-SUBSIDY =	26.181589

TABLE 6-5

(cont.)

\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR GEOGRAPHIC AREA 4 ARE

TOTAL =	17.532665
SUBSIDY =	.084533
NON-SUBSIDY =	17.448132

\*\*THE TOTAL SUBSIDIES IN BILLIONS OF DOLLARS ARE

TOTAL =	.632658
FAMILY SIZE 4 =	.632658
GEOGRAPHIC AREA 1 =	.110259
GEOGRAPHIC AREA 2 =	.146690
GEOGRAPHIC AREA 3 =	.287669
GEOGRAPHIC AREA 4 =	.088040

\*\*THE TOTAL SUBSIDY IN BILLIONS OF DOLLARS PAID BY NEGATIVE TAXES ARE

TOTAL =	.304527
FAMILY SIZE 4 =	.304527
GEOGRAPHIC AREA 1 =	.056000
GEOGRAPHIC AREA 2 =	.070771
GEOGRAPHIC AREA 3 =	.137489
GEOGRAPHIC AREA 4 =	.040267

\*\*THE NET SUBSIDY FIGURES IN BILLIONS OF DOLLARS ARE

TOTAL =	.328131
FAMILY SIZE 4 =	.328131
GEOGRAPHIC AREA 1 =	.054259
GEOGRAPHIC AREA 2 =	.075919
GEOGRAPHIC AREA 3 =	.150180
GEOGRAPHIC AREA 4 =	.047772

TABLE 6-5

(cont.)

\*\*THE MARGINAL TAX FIGURES IN BILLIONS OF DOLLARS ARE

	TOTAL =	.389450
FAMILY SIZE	4 =	.389450
GEOGRAPHIC AREA	1 =	.097435
GEOGRAPHIC AREA	2 =	.117496
GEOGRAPHIC AREA	3 =	.104726
GEOGRAPHIC AREA	4 =	.069793

\*\*THE DEFICIT OR SURPLUS FIGURES IN BILLIONS OF DOLLARS ARE

	TOTAL =	.061319
FAMILY SIZE	4 =	.061319
GEOGRAPHIC AREA	1 =	.043175
GEOGRAPHIC AREA	2 =	.041577
GEOGRAPHIC AREA	3 =	-.045454
GEOGRAPHIC AREA	4 =	.022020

G01F04 is selected and rerun through part 2 of the management information and control system with minimum incomes of \$1,700 and \$1,800. Table 6-6 shows the results of this case and when compared with Table 6-5 shows that a \$100 increase in the minimum income results in an increase in net subsidy of 7 million. However, the program has the capability to also alter the negative tax or marginal taxes to pay for the subsidy to eliminate the added income due to an increase in the minimum income.

One further test is run to test the system for a change in both the Breakeven Level of Income and the guaranteed minimum income. Part 1 of the management information and control system is rerun for data files G01F04, G02F04, G03F04, and G04F04. The change is that a Breakeven Level of Income of \$3,750, the Social Security Administration's recommended level, is used. The parametric results are stored in G01F04X, G02F04X, G03F04X, and G04F04X. Tables 6-7 through 6-10 contain the actual data.

These new parametric data files are then used as input to part 2 of the management information and control system and are processed with a minimum income of \$2,600. The same FAP negative tax is used. However, a marginal tax of 1% is levied on the non-subsidy population to offset the higher subsidy. The results of this case is found



TABLE 6-6

## FAP TEST WITH MINIMUM INCOME CHANGE

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?G01F04P

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

\*\*POPULATION FIGURES FOR GEOGRAPHIC AREA 1 FAMILY SIZE 4  
IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION =	2.229638
SUBSIDY POPULATION =	.068912
NON-SUBSIDY POPULATION =	2.160726

\*\*INCOME FIGURES FOR GEOGRAPHIC AREA 1 FAMILY SIZE 4  
IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME =	24.475206
SUBSIDY INCOME =	.116575
NON-SUBSIDY INCOME =	24.358631

INPUT A MINIMUM INCOME FOR THIS SEGMENT  
?1700.

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 1 FAMILY SIZE 4  
FOR MINIMUM INCOME 1700. IS 117.150435 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?Y

TABLE 6-6

(cont.)

INPUT A MINIMUM INCOME FOR THIS SEGMENT  
?1800.

(\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 1 FAMILY SIZE 4  
FOR MINIMUM INCOME 1800. IS \*124.041637 MILLION DOLLARS-

\*

\*

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?Y

INPUT A MINIMUM INCOME FOR THIS SEGMENT  
?1700.

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 1 FAMILY SIZE 4  
FOR MINIMUM INCOME 1700. IS 117.150435 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
?Y

INPUT A NEGATIVE TAX IN POLYNOMIAL FORM  
?0 .5

INPUT THE LOWEST LIMIT AT WHICH THE NEGATIVE TAX IS APPLIED  
?820.

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 1  
FAMILY SIZE 4 IS 55.279106 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
?Y

## TABLE 6-6

(cont.)

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS 61.871329 MILLION DOLLARS

INPUT THE MARGINAL TAX IN POLYNOMIAL FORM  
?0 .004

\*\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
IS 97.434525 MILLION DOLLARS

IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO  
?N

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
TYPE Y FOR YES, N FOR NO  
?N

\*\*MARGINAL TAXATION RESULTED IN A SURPLUS OF 35.563197 MILLIONS

DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO  
?N

ARE CUMULATIVE FIGURES TO BE KEPT... TYPE Y FOR YES, N FOR NO  
?N



TABLE 6-7

PARAMETRIC DATA GENERATION FOR FAMILY SIZE 4

GEOGRAPHIC AREA 4, BREAKEVEN LEVEL OF INCOME \$3,750

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME,  
THE GEOGRAPHIC AREA INDEX, AND THE FAMILY SIZE INDEX

?G04F04 4 4

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE

?19

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1

TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...TYPE 2

TO CURVE FIT BOTH ABOVE ...TYPE 3

?3

THE FOLLOWING BEST FITS THE TOTAL POPULATION FOR  
GEOGRAPHIC AREA 4 FAMILY SIZE 4

POLYNOMIAL DEGREE = 6

STD. ERROR =

8.0989

TERM	DEGREE	COEFFICIENTS
	0	.13046273E+02
	1	.66722098E-02
	2	-.25105900E-05
	3	.13295214E-08
	4	-.17059632E-12
	5	.81203560E-17
	6	-.13174219E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO

?N

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS GEOGRAPHIC AREA-FAMILY SIZE  
?3750.



TABLE 6-7

(cont.)

THE FOLLOWING BEST FITS THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA 4 FAMILY SIZE 4

POLYNOMIAL DEGREE = 2                      STD. ERROR =                      8.9932

TERM	DEGREE	COEFFICIENTS
0		.14436851E+02
1		-.74797560E-03
2		.26938413E-05

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 2

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	15.81	14.44	1.38
1.00	500.00	15.81	14.74	1.08
1.00	1250.00	12.46	17.71	-5.25
1.00	1750.00	18.37	21.38	-3.01
1.00	2250.00	35.42	26.39	9.03
1.00	2750.00	23.46	32.75	-9.29
1.00	3250.00	52.94	40.46	12.48
1.00	3750.00	43.11	49.51	-6.41

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION OF  
GEOGRAPHIC AREA 4 FAMILY SIZE 4

POLYNOMIAL DEGREE = 5                      STD. ERROR =                      7.5887

TABLE 6-7

(cont.)

TERM	DEGREE	COEFFICIENTS
0		.27245032E+03
1		-.16468699E+00
2		.40062450E-04
3		-.38470876E-08
4		.15847219E-12
5		-.23583772E-17

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 5

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	3750.00	43.11	44.97	-1.86
1.00	4500.00	53.98	52.69	1.29
1.00	5500.00	76.85	71.64	5.20
1.00	6500.00	90.19	93.64	-3.45
1.00	7500.00	105.00	113.27	-8.28
1.00	8500.00	138.48	127.12	11.35
1.00	9500.00	130.41	133.44	-3.03
1.00	11000.00	123.59	128.35	-4.76
1.00	13500.00	98.27	91.44	6.83
1.00	16500.00	37.51	42.24	-4.73
1.00	20000.00	37.51	35.73	1.78
1.00	23000.00	37.51	37.86	-.35

DO YOU WISH TO PLOT...TYPE Y FOR YES, N FOR NO  
?Y

DO YOU WISH TO PLOT THE ORIGINAL DATA...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE CURVE FIT FOR THE WHOLE  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...TYPE Y FOR YES, N FOR NO  
?N

TABLE 6-7

(cont.)

DO YOU WISH TO PLOT THE SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	14.385	20.737	27.088	33.439	39.790	46.142
0	I*					
150.0	I*					
300.0	I*					
450.0	I*					
600.0	I *					
750.0	I *					
900.0	I *					
1050.	A	*				
1200.	A	"				
1350.	A	*				
1500.	I	*				
1650.	A	*				
1800.	I	*				
1950.	I	*				
2100.	A	*				
2250.	I	*				
2400.	I	*				
2550.	I	*				
2700.	I	*				
2850.	I	*				
3000.	I	*				
3150.	I	*				
3300.	I	*				
3450.	I	*				
3600.	I	*				
3750.	I	*				
3900.	I	*				

TABLE 6-7

(cont.)

DO YOU WISH TO PLOT THE NON-SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	6.017	27.306	48.595	69.884	91.173	112.461
3750 .	I		*			
4600 .	I			*		
5450 .	I				*	
6300 .	I					*
7150 .	I					
8000 .	I					*
8850 .	I					
9700 .	I					*
.1055E+05I						*
.1140E+05I						
.1225E+05I						*
.1310E+05I					*	
.1395E+05I				*		
.1480E+05I					*	
.1565E+05I			*			
.1650E+05I						*
.1735E+05I		*				
.1820E+05I		*				
.1905E+05I		*				
.1990E+05I			*			
.2075E+05I				*		
.2160E+05I			*			
.2245E+05I			*			
.2330E+05I		*				
.2415E+05I*						

IF THE ABOVE CURVE FITS ARE TO BE SAVED, NAME THE FILE, IF NOT TYPE NO  
?G04F04X



TABLE 6-8

PARAMETRIC DATA GENERATION FOR FAMILY SIZE 4  
 GEOGRAPHIC AREA 1, BREAKEVEN LEVEL OF INCOME \$3,750

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME,  
 THE GEOGRAPHIC AREA INDEX, AND THE FAMILY SIZE INDEX  
 ?GO1FO4 1 4

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE  
 ?19

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1  
 TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...TYPE 2  
 TO CURVE FIT BOTH ABOVE ...TYPE 3  
 ?3

THE FOLLOWING BEST FITS THE TOTAL POPULATION FOR  
 GEOGRAPHIC AREA 1 FAMILY SIZE 4

POLYNOMIAL DEGREE = 6                      STD. ERROR =                      8.4389

TERM	DEGREE	COEFFICIENTS
	0	.13363075E+02
	1	.13959270E-01
	2	-.67786090E-05
	3	.27188916E-08
	4	-.32662374E-12
	5	.15185604E-16
	6	-.24394717E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
 ?N

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS GEOGRAPHIC AREA-FAMILY SIZE  
 ?3750.

TABLE 6-8

(cont.)

THE FOLLOWING BEST FITS THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA 1 FAMILY SIZE 4

POLYNOMIAL DEGREE = 3                      STD. ERROR = 8.0433

TERM	DEGREE	COEFFICIENTS
0		.20356583E+02
1		-.24501346E-01
2		.20989233E-04
3		-.30740385E-08

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 3

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	17.92	20.36	-2.44
1.00	500.00	17.92	12.97	4.95
1.00	1250.00	15.44	16.52	-1.08
1.00	1750.00	20.44	25.28	-4.84
1.00	2250.00	43.00	36.47	6.53
1.00	2750.00	39.09	47.78	-8.69
1.00	3250.00	65.61	56.90	8.71
1.00	3750.00	58.40	61.53	-3.13

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION OF  
GEOGRAPHIC AREA 1 FAMILY SIZE 4

POLYNOMIAL DEGREE = 6                      STD. ERROR = 6.7541

TABLE 6-8

(cont.)

TERM	DEGREE	COEFFICIENTS
0		.28288837E+03
1		-.15704828E+00
2		.33787618E-04
3		-.19868368E-08
4		-.40611559E-13
5		.64451241E-17
6		-.13804424E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 6

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	3750.00	58.40	60.69	-2.29
1.00	4500.00	76.45	73.41	3.04
1.00	5500.00	106.90	102.09	4.81
1.00	6500.00	129.94	135.84	-5.90
1.00	7500.00	159.18	167.26	-8.08
1.00	8500.00	198.74	190.88	7.86
1.00	9500.00	207.09	203.25	3.84
1.00	11000.00	195.72	198.03	-2.31
1.00	13500.00	135.37	137.62	-2.26
1.00	16500.00	53.43	51.66	1.76
1.00	20000.00	53.43	53.99	-.56
1.00	23000.00	53.43	53.33	.10

DO YOU WISH TO PLOT...TYPE Y FOR YES, N FOR NO  
?Y

DO YOU WISH TO PLOT THE ORIGINAL DATA...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE CURVE FIT FOR THE WHOLE  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...TYPE Y FOR YES, N FOR NO  
?N

TABLE 6-8

(cont.)

DO YOU WISH TO PLOT THE SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	12.490	20.692	28.893	37.094	45.296	53.497
0	I	*				
150.0	I	*				
300.0	I	*				
450.0	I	*				
600.0	I	*				
750.0	I	*				
900.0	I	*				
1050.	I	*				
1200.	I	*				
1350.	I	*				
1500.	I	*				
1650.	I	*				
1800.	I	*				
1950.	I	*				
2100.	I	*				
2250.	I	*				
2400.	I	*				
2550.	I	*				
2700.	I	*				
2850.	I	*				
3000.	I	*				
3150.	I	*				
3300.	I	*				
3450.	I	*				
3600.	I	*				
3750.	I	*				
3900.	I	*				



TABLE 6-8

(cont.)

DO YOU WISH TO PLOT THE NON-SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

-43.836      -2.495      38.847      80.188      121.529      162.870

+-----+-----+-----+-----+-----+-----

3750.	I		*						
4600.	I			*					
5450.	I				*				
6300.	I					*			
7150.	I						*		
8000.	I							*	
8850.	I								*
9700.	I								*
.1055E+05I									*
.1140E+05I									*
.1225E+05I								*	
.1310E+05I							*		
.1395E+05I						*			
.1480E+05I					*				
.1565E+05I				*					
.1650E+05I			*						
.1735E+05I		*							
.1820E+05I		*							
.1905E+05I		*							
.1990E+05I			*						
.2075E+05I				*					
.2160E+05I				*	*				
.2245E+05I				*					
.2330E+05I		*							
.2415E+05I*									

IF THE ABOVE CURVE FITS ARE TO BE SAVED, NAME THE FILE, IF NOT TYPE NO  
?G01F04X

TABLE 6-9

PARAMETRIC DATA GENERATION FOR FAMILY SIZE 4

GEOGRAPHIC AREA 2, BREAKEVEN LEVEL OF INCOME \$3,750

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME,  
THE GEOGRAPHIC AREA INDEX, AND THE FAMILY SIZE INDEX

?G02F04 2 4

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE

?19

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1

TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...TYPE 2

TO CURVE FIT BOTH ABOVE ...TYPE 3

?3

THE FOLLOWING BEST FITS THE TOTAL POPULATION FOR  
GEOGRAPHIC AREA 2 FAMILY SIZE 4

POLYNOMIAL DEGREE = 7

STD. ERROR =

10.0345

TERM	DEGREE	COEFFICIENTS
	0	.17082318E+02
	1	.10884492E-01
	2	.11919092E-05
	3	-.36635130E-09
	4	.17043641E-12
	5	-.22990554E-16
	6	.11402373E-20
	7	-.19061169E-25

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?N

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS GEOGRAPHIC AREA-FAMILY SIZE  
?3750.

TABLE 6-9

(cont.)

THE FOLLOWING BEST FITS THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA 2 FAMILY SIZE 4

POLYNOMIAL DEGREE = 2                      STD. ERROR =                      10.6046

TERM DEGREE	COEFFICIENTS
0	.18367468E+02
1	.47784979E-02
2	.33824132E-05

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 2

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	20.79	18.37	2.42
1.00	500.00	20.79	21.60	-.81
1.00	1250.00	25.59	29.63	-4.04
1.00	1750.00	32.34	37.09	-4.75
1.00	2250.00	58.22	46.24	11.98
1.00	2750.00	45.36	57.09	-11.72
1.00	3250.00	83.38	69.62	13.76
1.00	3750.00	77.01	83.85	-6.84

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION OF  
GEOGRAPHIC AREA 2 FAMILY SIZE 4

POLYNOMIAL DEGREE = 6                      STD. ERROR =                      6.0843

TABLE 6-9

(cont.)

TERM	DEGREE	COEFFICIENTS
0		.21165236E+03
1		-.68938109E-01
2		.27730399E-05
3		.29089425E-08
4		-.41398169E-12
5		.19915131E-16
6		-.32263049E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 6

WEAGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	3750.00	77.01	77.54	-.53
1.00	4500.00	88.71	86.97	1.74
1.00	5500.00	110.52	112.83	-2.32
1.00	6500.00	150.79	147.34	3.44
1.00	7500.00	175.00	183.12	-8.12
1.00	8500.00	221.52	213.44	8.08
1.00	9500.00	233.82	232.99	.84
1.00	11000.00	230.31	235.36	-5.05
1.00	13500.00	172.57	170.02	2.55
1.00	16500.00	56.78	57.52	-.75
1.00	20000.00	56.78	56.64	.13
1.00	23000.00	56.78	56.79	-.01

DO YOU WISH TO PLOT...TYPE Y FOR YES, N FOR NO  
?Y

DO YOU WISH TO PLOT THE ORIGINAL DATA...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE CURVE FIT FOR THE WHOLE  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...TYPE Y FOR YES, N FOR NO  
?N



TABLE 6-9

(cont.)

DO YOU WISH TO PLOT THE SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	18.367	30.048	41.728	53.409	65.089	76.770
	+-----+-----+-----+-----+-----+-----+					
0	I*					
150.0	I *					
300.0	I *					
450.0	I *					
600.0	I *					
750.0	I *					
900.0	I *					
1050.0	I *					
1200.0	I *					
1350.0	I *					
1500.0	I *					
1650.0	I *					
1800.0	I *					
1950.0	I *					
2100.0	I *					
2250.0	I *					
2400.0	I *					
2550.0	I *					
2700.0	I *					
2850.0	I *					
3000.0	A *					
3150.0	I *					
3300.0	I *					
3450.0	I *					
3600.0	I *					
3750.0	I *					
3900.0	I *					

TABLE 6-9

(cont.)

DO YOU WISH TO PLOT THE NON-SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

-88.815      -34.301      20.212      74.726      129.239      183.753  
+-----+-----+-----+-----+-----+-----+

3750.	I			*					
4600.	I				*				
5450.	I					*			
6300.	I						*		
7150.	I							*	
8000.	I								*
8850.	I								*
9700.	I								*
.1055E+05I									*
.1140E+05I									*
.1225E+05I									*
.1310E+05I								*	
.1395E+05I							*		
.1480E+05I						*			
.1565E+05I					*				
.1650E+05I				*					
.1735E+05I			*						
.1820E+05I		*							
.1905E+05I		*							
.1990E+05I			*						
.2075E+05I				*					
.2160E+05I					*				
.2245E+05I				*					
.2330E+05I		*							
.2415E+05I*									

IF THE ABOVE CURVE FITS ARE TO BE SAVED, NAME THE FILE, IF NOT TYPE NO  
?G02F04X

TABLE 6-10

PARAMETRIC DATA GENERATION FOR FAMILY SIZE 4  
 GEOGRAPHIC AREA 3, BREAKEVEN LEVEL OF INCOME \$3,750

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME,  
 THE GEOGRAPHIC AREA INDEX, AND THE FAMILY SIZE INDEX  
 ?G03F04 3 4

HOW MANY DATA PAIRS ARE THERE IN THIS DATA FILE  
 ?19

TO CURVE FIT THE WHOLE DATA FILE...TYPE 1  
 TO CURVE FIT THE SUBSIDY AND NON-SUBSIDY SEGMENTS...TYPE 2  
 TO CURVE FIT BOTH ABOVE ...TYPE 3  
 ?3

THE FOLLOWING BEST FITS THE TOTAL POPULATION FOR  
 GEOGRAPHIC AREA 3 FAMILY SIZE 4

POLYNOMIAL DEGREE = 6                      STD. ERROR =                      16.5347

TERM	DEGREE	COEFFICIENTS
	0	.41896947E+02
	1	.14733304E-01
	2	.19381178E-06
	3	.97555049E-09
	4	-.17397973E-12
	5	.94336141E-17
	6	-.16440631E-21

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
 ?N

INPUT A BREAKEVEN LEVEL OF INCOME FOR THIS GEOGRAPHIC AREA-FAMILY SIZE  
 ?3750.

TABLE 6-10

(cont.)

THE FOLLOWING BEST FITS THE SUBSIDY PORTION OF  
GEOGRAPHIC AREA 3 FAMILY SIZE 4

POLYNOMIAL DEGREE = 1                      STD. ERROR =                      16.2870

TERM DEGREE	COEFFICIENTS
0	.37767067E+02
1	.21791500E-01

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 1

WEIGHT	X VALUE	Y VALUE	EST. Y	RESIDUAL
1.00	0	43.77	37.77	6.00
1.00	500.00	43.77	48.66	-4.89
1.00	1250.00	64.65	65.01	-.36
1.00	1750.00	71.05	75.90	-4.85
1.00	2250.00	100.48	86.80	13.68
1.00	2750.00	69.76	97.69	-27.94
1.00	3250.00	131.40	108.59	22.81
1.00	3750.00	115.03	119.49	-4.45

THE FOLLOWING BEST FITS THE NON-SUBSIDY PORTION OF  
GEOGRAPHIC AREA 3 FAMILY SIZE 4

POLYNOMIAL DEGREE = 5                      STD. ERROR =                      16.5800



TABLE 6-10

(cont.)

TERM	DEGREE	COEFFICIENTS
0		.21019700E+03
1		-.12218386E+00
2		.39631677E-04
3		-.43687339E-08
4		.19435909E-12
5		-.30380354E-17

DO YOU WISH TO SEE THE BACK SOLUTIONS...TYPE Y FOR YES, N FOR NO  
?Y

BACK SOLUTIONS FOR BEST FIT - DEGREE = 5

WEIGHT	X VALUE	Y VALUE	EST. Y	RESADUAL
1.00	3750.00	115.03	115.13	-.10
1.00	4500.00	133.98	138.90	-4.93
1.00	5500.00	183.31	172.76	10.56
1.00	6500.00	210.53	202.37	8.16
1.00	7500.00	192.21	222.91	-30.71
1.00	8500.00	243.31	231.84	11.46
1.00	9500.00	242.55	228.56	13.99
1.00	11000.00	191.48	203.16	-11.68
1.00	13500.00	133.65	128.25	5.40
1.00	16500.00	46.29	49.44	-3.15
1.00	20000.00	46.29	45.06	1.23
1.00	23000.00	46.29	46.54	-.25

DO YOU WISH TO PLOT...TYPE Y FOR YES, N FOR NO  
?Y

DO YOU WISH TO PLOT THE ORIGINAL DATA...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PLOT THE CURVE FIT FOR THE WHOLE  
GEOGRAPHIC AREA-FAMILY SIZE SEGMENT...TYPE Y FOR YES, N FOR NO  
?N

TABLE 6-10

(cont.)

DO YOU WISH TO PLOT THE SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	37.767	51.932	66.096	80.260	94.425	108.589
0	I*					
150.0	I *					
300.0	I	*				
450.0	I	*				
600.0	I	*				
750.0	I	*	*			
900.0	I	*	*			
1050.	I	*	*			
1200.	I	*	*			
1350.	I	*	*			
1500.	I	*	*	*		
1650.	I	*	*	*		
1800.	I	*	*	*	*	
1950.	I	*	*	*	*	
2100.	I	*	*	*	*	*
2250.	I	*	*	*	*	*
2400.	I	*	*	*	*	*
2550.	I	*	*	*	*	*
2700.	I	*	*	*	*	*
2850.	I	*	*	*	*	*
3000.	I	*	*	*	*	*
3150.	I	*	*	*	*	*
3300.	I	*	*	*	*	*
3450.	I	*	*	*	*	*
3600.	I	*	*	*	*	*
3750.	I	*	*	*	*	*
3900.	I	*	*	*	*	*

TABLE 6-10

(cont.)

DO YOU WISH TO PLOT THE NON-SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO  
?Y

	-4.556	34.880	74.317	113.753	153.189	192.625
	+-----+-----+-----+-----+-----+-----+					
3750.	I			*		
4600.	I				*	
5450.	I					*
6300.	I					*
7150.	I					*
8000.	I					*
8850.	I					*
9700.	I					*
.1055E+05I						*
.1140E+05I						*
.1225E+05I					*	
.1310E+05I					*	
.1395E+05I				*		
.1480E+05I			*			
.1565E+05I		*				
.1650E+05I		*				
.1735E+05I	*					
.1820E+05I	*					
.1905E+05I	*					
.1990E+05I	*					
.2075E+05I	*					
.2160E+05I	*	*				
.2245E+05I	*	*				
.2330E+05I	*					
.2415E+05I*						

IF THE ABOVE CURVE FITS ARE TO BE SAVED, NAME THE FILE, IF NOT TYPE NO  
?G03F04X

in Table 6-11.

Comparing Tables 6-6 with 6-7 allows the decision maker to compare the cost effects of the two different proposals. Comparing the different geographic area-family size segments with each case also gives information as to how the subsidy is distributed throughout the country.



TABLE 6-11

FAP TEST WITH BREAKEVEN LEVEL OF INCOME  
AND MINIMUM INCOME CHANGE

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?G01F04X

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

\*\*POPULATION FIGURES FOR GEOGRAPHIC AREA 1 FAMILY SIZE 4  
IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION =	2.265949
SUBSIDY POPULATION =	.121038
NON-SUBSIDY POPULATION =	2.144911

\*\*INCOME FIGURES FOR GEOGRAPHIC AREA 1 FAMILY SIZE 4  
IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME =	25.425486
SUBSIDY INCOME =	.294193
NON-SUBSIDY INCOME =	25.131293

INPUT A MINIMUM INCOME FOR THIS SEGMENT  
?2600.

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 1 FAMILY SIZE 4  
FOR MINIMUM INCOME 2600. IS 314.697966 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
?Y

TABLE 6-11

(cont.)

INPUT A NEGATIVE TAX IN POLYNOMIAL FORM  
?0 .5

INPUT THE LOWEST LIMIT AT WHICH THE NEGATIVE TAX IS APPLIED  
?720.

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 1  
FAMILY SIZE 4 IS 145.336804 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
?Y

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS 169.361162 MILLION DOLLARS

INPUT THE MARGINAL TAX IN POLYNOMIAL FORM  
?0 .01

\*\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
IS 251.312928 MILLION DOLLARS

IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO  
?N

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
TYPE Y FOR YES, N FOR NO  
?N

\*\*MARGINAL TAXATION RESULTED IN A SURPLUS OF 81.951766 MILLIONS-

\*  
\*  
DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO  
?N

TABLE 6-11

(cont.)

ARE CUMULATIVE FIGURES TO BE KEPT... TYPE Y FOR YES, N FOR NO  
Y

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?G02F04X

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

\*\*POPULATION FIGURES FOR GEOGRAPHIC AREA 2 FAMILY SIZE 4  
IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION =	2.628540
SUBSIDY POPULATION =	.161933
NON-SUBSIDY POPULATION =	2.466607

\*\*INCOME FIGURES FOR GEOGRAPHIC AREA 2 FAMILY SIZE 4  
IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME =	29.262775
SUBSIDY INCOME =	.380365
NON-SUBSIDY INCOME =	28.882411

DO YOU WISH TO USE THE MINIMUM INCOME OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 2 FAMILY SIZE 4  
FOR MINIMUM INCOME 2600. IS 421.026581 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
?N



## TABLE 6-11

(cont.)

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 2  
FAMILY SIZE 4 IS 187.391016 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
?Y

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS 233.635564 MILLION DOLLARS

DO YOU WISH TO USE THE MARGINAL TAX OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
IS 288.824106 MILLION DOLLARS

IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO  
?N

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
TYPE Y FOR YES, N FOR NO  
?N

\*\*MARGINAL TAXATION RESULTED IN A SURPLUS OF 55.188542 MILLIONS

DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PRINT THE CUMULATIVE FIGURES...TYPE Y FOR YES, N FOR NO  
?N

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?G03F04X



TABLE 6-11

(cont.)

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

\*\*POPULATION FIGURES FOR GEOGRAPHIC AREA 3 FAMILY SIZE 4  
IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION =	2.705660
SUBSIDY POPULATION =	.294848
NON-SUBSIDY POPULATION =	2.410812

\*\*INCOME FIGURES FOR GEOGRAPHIC AREA 3 FAMILY SIZE 4  
IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME =	26.531623
SUBSIDY INCOME =	.648603
NON-SUBSIDY INCOME =	25.883019

DO YOU WISH TO USE THE MINIMUM INCOME OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 3 FAMILY SIZE 4  
FOR MINIMUM INCOME 2600. IS 766.604764 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
?N

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 3  
FAMILY SIZE 4 IS 318.051483 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
?Y

## TABLE 6-11

(cont.)

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS 448.553280 MILLION DOLLARS

DO YOU WISH TO USE THE MARGINAL TAX OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
IS 258.83091 MILLION DOLLARS-

←  
\*  
IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO  
?N

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
TYPE Y FOR YES, N FOR NO  
?N

\*\*MARGINAL TAXATION RESULTED IN A DEFICIT OF -189.723090 MILLIONS

DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO  
?N

DO YOU WISH TO PRINT THE CUMULATIVE FIGURES...TYPE Y FOR YES, N FOR NO  
?N

TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE DATA FILE TYPE THE FILE NAME  
?G04F04X

INPUT AN UPPER INCOME LIMIT IN DOLLARS FOR THE NON-SUBSIDY PORTION  
?23000.

TABLE 6-11

(cont.)

\*\*POPULATION FIGURES FOR GEOGRAPHIC AREA 4 FAMILY SIZE 4  
IN MILLIONS OF PEOPLE ARE

SEGMENT POPULATION =	1.548306
SUBSIDY POPULATION =	.096232
NON-SUBSIDY POPULATION =	1.452075

\*\*INCOME FIGURES FOR GEOGRAPHIC AREA 4 FAMILY SIZE 4  
IN BILLIONS OF DOLLARS ARE

SEGMENT INCOME =	17.217710
SUBSIDY INCOME =	.221541
NON-SUBSIDY INCOME =	16.996169

DO YOU WISH TO USE THE MINIMUM INCOME OF THE LAST CASE  
...TYPE Y FOR YES, N FOR NO  
?Y

\*\*THE SUBSIDY ADDED TO GEOGRAPHIC AREA 4 FAMILY SIZE 4  
FOR MINIMUM INCOME 2600. IS 250.202336 MILLION DOLLARS

DO YOU WISH TO TRY ANOTHER MINIMUM INCOME...TYPE Y FOR YES, N FOR NO  
?N

IF A NEW NEGATIVE TAX IS NEEDED TYPE Y, IF NOT TYPE N  
?N

\*\*THE NEGATIVE TAX ON THE SUBSIDY PORTION OF GEOGRAPHIC AREA 4  
FAMILY SIZE 4 IS 108.855278 MILLION DOLLARS

IF THE NEGATIVE TAX IS SATISFACTORY TYPE Y, IF NOT TYPE N  
?Y

\*\*THE SUBSIDY AFTER THE NEGATIVE TAX IS ADDED  
IS 141.347058 MILLION DOLLARS



## TABLE 6-11

(cont.)

DO YOU WISH TO USE THE MARGINAL TAX OF THE LAST CASE  
 ...TYPE Y FOR YES, N FOR NO

?Y

\*\*THE ADDED REVENUE DUE TO A MARGINAL TAX  
 IS 169.961692 MILLION DOLLARS

IS A NEW MARGINAL TAX TO BE INPUTED ...TYPE Y FOR YES, N FOR NO

?N

IS TAXATION OF CORPORATE EARNINGS TO BE USED TO PAY FOR THE SUBSIDY  
 TYPE Y FOR YES, N FOR NO

?N

\*\*MARGINAL TAXATION RESULTED IN A SURPLUS OF 28.614634 MILLIONS

DO YOU WISH TO INVESTIGATE INDIVIDUAL INCOME BRACKETS AS TO  
 SUBSIDY RECEIVED OR SUBSIDY COST LEVIED...TYPE Y FOR YES, N FOR NO

?N

DO YOU WISH TO PRINT THE CUMULATIVE FIGURES...TYPE Y FOR YES, N FOR NO

?Y

\*\*THE CUMULATIVE FIGURES BELOW ARE FOR 4 GEOGRAPHIC AREA-FAMILY  
 SIZE SEGMENTS

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR ALL SEGMENTS ARE

TOTAL =	9.148455
SUBSIDY =	.674051
NON-SUBSIDY =	8.474405



TABLE 6-11

(cont.)

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR FAMILY SIZE 4 ARE

TOTAL =	9.148455
SUBSIDY =	.674051
NON-SUBSIDY =	8.474405

\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR GEOGRAPHIC AREA 1 ARE

TOTAL =	2.265949
SUBSIDY =	.121038
NON-SUBSIDY =	2.144911

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR GEOGRAPHIC AREA 2 ARE

TOTAL =	2.628540
SUBSIDY =	.161933
NON-SUBSIDY =	2.466607

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR GEOGRAPHIC AREA 3 ARE

TOTAL =	2.705660
SUBSIDY =	.294848
NON-SUBSIDY =	2.410812

\*\*THE POPULATION FIGURES IN MILLIONS OF PEOPLE FOR GEOGRAPHIC AREA 4 ARE

TOTAL =	1.548306
SUBSIDY =	.096232
NON-SUBSIDY =	1.452075

\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR ALL SEGMENTS ARE

TOTAL =	98.437593
SUBSIDY =	1.544701
NON-SUBSIDY =	96.892892

TABLE 6-11

(cont.)

\*\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR FAMILY SIZE 4 ARE

TOTAL =	98.437593
SUBSIDY =	1.544701
NON-SUBSIDY =	96.892892

\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR GEOGRAPHIC AREA 1 ARE

TOTAL =	25.425486
SUBSIDY =	.294193
NON-SUBSIDY =	25.131293

\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR GEOGRAPHIC AREA 2 ARE

TOTAL =	29.262775
SUBSIDY =	.380365
NON-SUBSIDY =	28.882411

\*\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR GEOGRAPHIC AREA 3 ARE

TOTAL =	26.531623
SUBSIDY =	.648603
NON-SUBSIDY =	25.883019

\*\*THE INCOME FIGURES IN BILLIONS OF DOLLARS FOR GEOGRAPHIC AREA 4 ARE

TOTAL =	17.217710
SUBSIDY =	.221541
NON-SUBSIDY =	16.996169

\*\*THE TOTAL SUBSIDIES IN BILLIONS OF DOLLARS ARE

TOTAL =	1.752532
FAMILY SIZE 4 =	1.752532
GEOGRAPHIC AREA 1 =	.314698
GEOGRAPHIC AREA 2 =	.421027
GEOGRAPHIC AREA 3 =	.766605
GEOGRAPHIC AREA 4 =	.250202

TABLE 6-11

(cont.)

\*\*THE TOTAL SUBSIDY IN BILLIONS OF DOLLARS PAID BY NEGATIVE TAXES ARE

TOTAL	=	.759635
FAMILY SIZE	4 =	.759635
GEOGRAPHIC AREA	1 =	.145337
GEOGRAPHIC AREA	2 =	.187391
GEOGRAPHIC AREA	3 =	.318051
GEOGRAPHIC AREA	4 =	.108855

\*\*THE NET SUBSIDY FIGURES IN BILLIONS OF DOLLARS ARE

TOTAL	=	.992897
FAMILY SIZE	4 =	.992897
GEOGRAPHIC AREA	1 =	.169361
GEOGRAPHIC AREA	2 =	.233636
GEOGRAPHIC AREA	3 =	.448553
GEOGRAPHIC AREA	4 =	.141347

\*\*THE MARGINAL TAX FIGURES IN BILLIONS OF DOLLARS ARE

TOTAL	=	.968929
FAMILY SIZE	4 =	.968929
GEOGRAPHIC AREA	1 =	.251313
GEOGRAPHIC AREA	2 =	.288824
GEOGRAPHIC AREA	3 =	.258830
GEOGRAPHIC AREA	4 =	.169962

\*\*THE DEFICIT OR SURPLUS FIGURES IN BILLIONS OF DOLLARS ARE

TOTAL	=	-.023968
FAMILY SIZE	4 =	-.023968
GEOGRAPHIC AREA	1 =	.081952
GEOGRAPHIC AREA	2 =	.055189
GEOGRAPHIC AREA	3 =	-.189723
GEOGRAPHIC AREA	4 =	.028615

C H A P T E R VII

CONCLUSIONS



DISSERTATION SUMMARY

As stated in Chapter I the purpose of this dissertation is the development of a means to investigate income transfer programs which are being considered to replace the present welfare system of the United States. Chapter II defined the types of theoretical transfer programs such as negative income tax and guaranteed minimum income proposals being considered as replacements. Also in Chapter II the present income maintenance plan of President Nixon, along with income maintenance plans presented by Senator Harris of Oklahoma, the President's Commission on Income Maintenance Programs, the minority plan as a result of the President's Commission on Income Maintenance Programs, and various other plans as summarized by Christopher Green are found. With respect to President Nixon's Plan various alternative courses of action are mentioned by the Senate's Finance Committee members, the National Association of Counties, numerous U.S. Congressmen, Governors, Mayors, and interested citizens.

As a result of these debates as to determining a plan to replace our present welfare system, a literature search was undertaken to find an analytical technique that might possibly aid in selecting an optimal plan and at the same time answer the types of cost questions

being asked by decision makers responsible for this selection. Such an analytical system that could answer all cost questions and could also analyze and compare all income transfer plans was not found. It was then decided that this all inclusive system would be developed.

The all inclusive analytical system development is to be found in Chapter III. What follows is a brief summary of this development, its testing parameters, and its conclusions.

The mathematical model to test and compare income transfer programs was developed in three phases. Each phase adds one more level of sophistication. The first phase considered the United States as a whole. The second phase divided the population into family size segments. Finally, the third phase further divided the population into geographic area-family size segments. By so dividing the population the model allows analysis of any income transfer program mentioned in the literature. Since the phase three model includes the analysis of the other two phases, it will be the one summarized here.

The phase three model looks at the total population of the United States as being made up of many geographic area-family size segments. Therefore, each segment can

be analyzed separately, and by including summation techniques the model also gives results for individual family sizes, individual geographic areas, and for the whole United States.

Every income transfer program presented today includes a guaranteed minimum income, a Breakeven Level of Income, and a negative tax applied to the income earned by the subsidy receiving members of the population. Therefore, the mathematical model of phase three further divides the population of each geographic area-family size segment into subsidy and non-subsidy portions. With this portioning the decision maker can apply different minimum incomes, Breakeven Levels of Income, and negative taxes to each geographic area-family size's subsidy portion. He is also allowed to make each of these variables pertinent to any larger divisions he wishes to analyze. For example, he may hold the negative tax rate constant for the country and vary the minimum income as a function of family size regardless of geographic area and, in effect, model President Nixon's Plan.

Since the population can be so finely divided as to a subsidy and non-subsidy portion within geographic area-family size segments of the total United States population, a means to pay for the subsidy presents itself. This means is marginal taxes applied to income earned in

the non-subsidy portions, and marginal taxes applied to corporate earnings. Also available from the model is income bracket analysis as to the subsidy received or the subsidy paid pertinent to each geographic area-family size segment of the population.

In total the mathematical model of Chapter III supplies the decision maker the following information:

a) A population and income picture for each geographic area-family size segment of the United States along with population and income figures for each geographic area, each family size, and for the United States as a whole.

b) The ability to divide each geographic area-family size segment into subsidy and non-subsidy portions. This partitioning is a function of some Breakeven Level of Income assignable to each geographic area-family size segment or any larger divisions the decision maker requires.

c) Once the partitioning into subsidy and non-subsidy portions is performed, the population and income figures of each is computed for every geographic area-family size segment. These figures are also accumulated for each geographic area, each family size, and for the United States as a whole.



d) A guaranteed minimum income can be given to each geographic area-family size's subsidy portion or to larger divisions of the population. The resulting additions to income is computed for each geographic area-family size segment, each geographic area, each family size, and for the United States as a whole. This guaranteed minimum income can also be such that it penalizes the having of more children as some programs suggest.

e) A negative tax can be levied on the income earned in the subsidy segment for any division of the population down to geographic area-family size segments. The resulting negative tax liability and net subsidy (guaranteed minimum income additions minus negative tax liability) are computed for each geographic area-family size segment, each geographic area, each family size, and for the United States as a whole.

f) To pay for the subsidy a marginal tax can be applied to the income earned in the non-subsidy portion of each geographic area-family size segment or any larger division. The resulting revenue from this marginal taxation is computed for each geographic area-family size segment, each geographic area, each family size, and for the United States as a whole.

g) To pay for the subsidy a marginal tax can also be applied to corporate earnings within each geographic

area. The resulting revenue is computed for each geographic area and for the United States as a whole.

h) The resulting combined marginal tax revenues is then computed for each geographic area-family size segment (this is done by allocating the geographic corporate revenues equally among the family sizes of the geographic area), each geographic area, each family size, and for the United States as a whole.

i) The resulting deficit or surplus (marginal tax revenues minus net subsidy) is computed for each geographic area-family size segment, each geographic area, each family size, and for the United States as a whole.

j) The subsidy responsibility or subsidy received by any income level within any geographic area-family size segment can be computed. This analysis allows the decision maker to examine possible inequities within plans. By looking at the income levels just before the Breakeven Level of Income and just after he can determine an equalizing negative tax structure analytically rather than guessing at one.

The results and variability of the analysis presented above more than satisfies those needed to test any income transfer program. By being so variable it might even persuade plan developers to include smaller breakdowns of the population in their proposals to more

realistically model the diversity of income needs within this country.

Once the above mathematical model was developed it was equally important to develop a means to use the model that allowed its variability to be brought forth. To that means in Chapter IV and Chapter V techniques to create a population data base on the geographic area-family size level and an on-line, timesharing management information and control system were developed.

In particular, Chapter IV developed the techniques to collect geographic area-family size population and income frequency data. Also found in Chapter IV are the computer program designs to store this data on various computer storage equipment. The storage equipment considered are punched data cards, drum files, disk files, magnetic tapes, and punched paper tapes. The actual programs and their user instructions are found in Appendix A.

Chapter V developed the management information and control system in two parts. The first part allows the decision maker to create national income frequency distributions in parametric equation form from the data base created in Chapter IV. It is through this part of the management information and control system that subsidy and non-subsidy portioning takes place. The technique

used is to input a Breakeven Level of Income and perform separate curve fits for each portion. The reason for using separate curve fits instead of just one for the whole geographic area-family size segment is simply a question of accuracy. The two fits do much better than one when the data base is small. However, if a large enough data base can be collected the one fit might be sufficient. That is why the parametric data base created by this part of the management information and control system includes all three curve fits, the whole geographic area-family size segment and its subsidy and non-subsidy portions. Part two of the management information and control system is the means to use the mathematical model of Chapter III.

As a result of the dual partitioning of Chapter V it, in fact, becomes the user's manual for the complete management information and control system. For those decision makers who already understand the concepts inherent to income transfer programs Chapter V, along with Appendix B which lists the computer programs for Chapter V, is all he needs to investigate any possible welfare replacement program.

Chapter VI has been included not only to prove that the system accomplishes its goal of analyzing welfare replacement proposals, but also to give the decision



maker a means to sharpen his understanding of the model. To be most up to date Chapter VI used as its test case President Nixon's Plan which is presently being considered by Congress.

#### AUTHOR'S STATEMENT

The results of Chapter VI confirms that the mathematical model developed in this dissertation supplies cost and benefit information currently needed by Congress to select a replacement for the Country's poor operating-present welfare program. It proves that the system is easy to operate and at the same time inexpensive to run.

By inexpensive, it is meant that the total computer time used to generate the test cases of this dissertation was less than one minute of Central Processor Time (CPU time), the most expensive computer cost. Using this cost to completely evaluate the FAP proposal before Congress concludes in a CPU time of much less than 30 minutes or less than \$500. Therefore, in considering if this system should be employed cost figures are most favorable.

Given that cost figures are favorable and that the system supplies needed information, I hope that this dissertation will be evaluated by Government and pertinent Decision Makers and then if found worthy, be used

to assist this Nation in improving the lot of its poor.

This dissertation does not recommend or develop a welfare replacement. However, it has been written so that: by its use such a replacement may be easier found with a greater confidence.

#### MODEL ASSUMPTIONS EXPLAINED

In the course of developing the mathematical model of this dissertation, certain major assumptions are made which must be justified. The most important assumption, of course, is that the system works. That is proven throughout the dissertation. However, once the model is proved operable, it must next be proven feasible. That can only be done by looking at its underlying foundation.

The reader first notices that the system is based upon gross income figures. That is negative taxation and marginal taxation are applied to gross national income figures. With respect to negative taxation being applied to gross income the system is justified by the fact that most NIT and GMI proposals utilize this method. With respect to the marginal taxation being applied to gross income one defense is that it is a matter of convenience since this data is more readily available and justified through Internal Revenue Sources, and another is that it is preceded by our present Social Security

deductions.

Another major assumption is that marginal taxation is the means to pay for the subsidy and that both the non-subsidy segment and corporations are considered. These assumption are, however, strictly suggestions and in no way defeat the purpose of the analysis to compute gross and net subsidy costs. The figures Congress needs. They do, however, give one an idea of a possible source of revenue and as shown in the sample cases found in this dissertation, result in figures of amazingly small proportions for the important benefits garnished.

The model, in its mathematics, assumes polynomials be used to present the income frequency distributions of the subsidy and non-subsidy segments, the negative taxes to be applied against the subsidy portion, and the marginal non-subsidy and corporate taxes to pay for the subsidy. In the first case, that of modeling income distribution, the small errors of the test case for a small sample size, more than verify polynomial applicability. In the second case, that of negative taxes, polynomials more than adequately represent possible negative taxes found in the literature. And, in the final case, that of marginal taxation it is used as a matter of convenience since polynomial integration and multiplication cause no additive errors but are performed in closed

form.

When the model considered paying for the subsidy by marginal taxation, it assumed a balanced government fiscal situation. This is feasible since it wishes only to consider the added cost of a GMI or NIT program to the government and is not interested in solving deficit problems arising out of other government areas.

The last assumption to be justified is that of selecting geographic areas as part of the model when only few plans presently available for testing consider it. The justification to this assumption is answered in Chapter II of this dissertation when Senator Harris could not believe that it was not used in the FAP program. It is, therefore, the author's opinion that a realistic NIT or GMI proposal must include geographic area differentiations and that this dissertation's system must be prepared to analyze those programs that will be developed to be equitable.

#### FURTHER ANALYSIS SUGGESTIONS

It is immediately obvious to the trained decision maker that the national income frequency data base used in this dissertation lacks data points. As a result of this lack of data points testing of NIT and GMI proposals suffers. It is, therefore, recommended that funds



be appropriated to collect national income data by family size and geographic area for income intervals suggested in Chapter IV. Once a proper data base is collected a study should be made in total of the present FAP proposal along with tests of other major proposals available. From these tests comparison tests can then be performed to, thereby, select an optimal proposal with respect to cost and benefits.

Once the above study is performed one more important step should be taken before a final plan is put into effect. That is, a behavioral study using the data of the management information and control system utilizing an accurate data base to determine work incentives of each program. With proper cost and benefit data a behavioral field study might be successful in determining an accurate work incentive index for each tested proposal. Thus, optimizing the cross relationship between cost-benefit and work incentives will allow an improved means to select the NIT or GMI proposal to be used in this country.

APPENDIX A  
DATA BASE DEVELOPMENT

Al-INCOME DATA GENERATION

In searching through the most up to date Consumer Income Data Publications of the Department of Commerce, national income data collected by family sizes within geographic areas was not to be found. However, income and population figures for the country as a whole and these total figures broken down into income and population figures for four geographic areas were available. Table Al-1<sup>1</sup> details this information. Total income and population figures of the country broken down into family sizes (2 through 7 +) was also available. Table Al-2<sup>2</sup> represents these figures.

Using the two above tables and assuming:

- a) That the population distribution within each income range of Table Al-1 and Table Al-2 are uniform distributions and
- b) The distribution of the total population per income range within each geographic area for each family size follows the distribution per income range as presented for the whole country in Table Al-1.

The following algorithm to generate geographic area-family size income frequency distributions from Tables Al-1 and Al-2 is developed. Tables Al-3 through Al-30 represent these generated distributions.

TABLE A1-1

## FAMILIES AND UNRELATED INDIVIDUALS BY TOTAL MONEY INCOME IN 1969

TOTAL MONEY INCOME	FAMILIES					UNRELATED INDIVIDUALS				
	TOTAL	NORTH- EAST	NORTH- CENTRAL	SOUTH	WEST	TOTAL	NORTH- EAST	NORTH- CENTRAL	SOUTH	WEST
NUMBER... THOUSANDS	51 237	12 373	14 358	15 770	8 736	14 452	3 609	4 014	3 922	2 908
PERCENT	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Under \$1,000	1.6	1.2	1.2	2.3	1.5	13.7	11.4	12.6	19.8	9.8
\$1,000 To \$1,499	1.3	0.7	1.0	2.3	0.8	12.8	11.0	14.4	14.9	9.9
\$1,500 To \$1,999	1.8	1.1	1.5	3.0	1.4	10.6	9.6	12.2	9.7	10.9
\$2,000 To \$2,499	2.4	1.8	2.1	3.3	2.1	7.9	7.8	7.8	6.5	10.1
\$2,500 To \$2,999	2.2	2.0	2.0	2.8	1.7	5.8	5.6	5.4	6.0	6.4
\$3,000 To \$3,499	2.6	2.1	2.3	3.3	2.4	5.1	4.9	5.4	5.5	4.4
\$3,500 To \$3,999	2.7	2.2	2.5	3.4	2.3	5.1	5.3	5.2	5.2	4.5
\$4,000 To \$4,999	5.4	4.8	4.8	6.6	4.8	7.7	8.5	7.3	8.0	6.9
\$5,000 To \$5,999	5.9	5.5	4.9	7.4	5.6	6.7	8.2	6.3	5.9	6.6
\$6,000 To \$6,999	6.4	5.9	5.9	7.5	5.8	6.1	7.0	6.0	5.0	6.5
\$7,000 To \$7,999	7.3	7.6	7.2	7.2	7.1	5.0	5.6	5.1	4.4	5.0
\$8,000 To \$8,999	7.4	7.6	7.3	7.3	7.5	3.4	3.7	3.6	2.5	4.1
\$9,000 To \$9,999	7.0	7.4	7.2	6.8	6.6	2.1	2.0	2.0	1.4	3.3
\$10,000 To \$11,999	13.0	14.2	14.4	10.9	12.7	3.1	4.1	2.6	2.2	3.7
\$12,000 To \$14,999	13.7	14.2	15.6	11.0	14.6	2.5	2.8	1.9	1.4	4.4
\$15,000 To \$24,999	15.6	17.8	16.3	12.1	17.7	1.8	1.9	1.9	1.1	2.8
\$25,000 To \$49,999	3.2	3.4	3.3	2.2	4.8	0.5	0.7	0.2	0.6	0.6
\$50,000 and over	0.4	0.5	0.5	0.4	0.5	0.1	(Z)	-	0.1	(Z)

- Equals 0

Z Equals  $\leq 0.01$



## TABLE A1-1

(cont.)

GEOGRAPHIC REGIONS. The four major regions of the United States, for which data are represented in this Table, represent groups of States, as follows:

NORTHEAST: Connecticut, Maine, Massachusetts, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, Vermont.

NORTH CENTRAL: Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, Ohio, South Dakota, Wisconsin.

SOUTH: Alabama, Arkansas, Delaware, District of Columbia, Florida, Georgia, Kentucky, Louisiana, Mississippi, Maryland, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, West Virginia.

WEST: Arizona, Colorado, California, Idaho, Montana, Nevada, New Mexico, Oregon, Utah, Washington, Wyoming, Alaska and Hawaii.

TABLE A1-2

## SIZE OF FAMILY--FAMILIES BY TOTAL MONEY INCOME IN 1969

TOTAL MONEY INCOME	TOTAL FAMILIES	FAMILIES HAVING SPECIFIED NUMBER OF PERSONS					
		2	3	4	5	6	7 OR MORE
UNITED STATES							
NUMBER... THOUSANDS.....	51 237	17 654	10 688	9 893	6 426	3 467	3 109
PERCENT.....	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Under \$1,000.....	1.6	1.5	1.0	0.8	0.8	1.1	0.8
\$1,000 To \$1,499.....	1.3	0.9	0.6	0.6	0.6	0.6	0.9
\$1,500 To \$1,999.....	1.8	1.3	0.7	0.8	0.8	0.8	0.7
\$2,000 To \$2,499.....	2.4	2.1	1.2	0.8	0.7	0.7	1.2
\$2,500 To \$2,999.....	2.2	2.0	0.9	0.9	1.0	1.0	1.5
\$3,000 To \$3,499.....	2.6	2.2	1.7	1.3	1.7	1.7	2.3
\$3,500 To \$3,999.....	2.7	2.1	1.5	1.5	1.8	1.8	2.6
\$4,000 To \$4,999.....	5.4	5.2	3.6	3.4	4.2	4.2	5.4
\$5,000 To \$5,999.....	5.9	5.6	4.8	4.7	5.0	5.0	6.1
\$6,000 To \$6,999.....	6.4	6.8	5.9	5.7	5.5	5.5	6.8
\$7,000 To \$7,999.....	7.3	8.4	6.4	7.4	7.2	7.2	7.1
\$8,000 To \$8,999.....	7.4	7.4	8.1	8.2	7.0	7.0	6.9
\$9,000 To \$9,999.....	7.0	7.3	8.2	7.3	7.9	7.9	6.4
\$10,000 To \$11,999.....	13.0	14.2	15.0	14.6	13.6	13.6	13.2
\$12,000 To \$14,999.....	13.7	14.0	16.4	16.6	16.3	16.3	15.4
\$15,000 To \$24,999.....	15.6	15.6	19.6	20.8	20.6	20.6	17.0
\$25,000 To \$49,999.....	3.2	2.9	3.9	4.1	4.2	4.2	5.0
\$50,000 and over.....	0.4	0.3	0.4	0.6	0.9	0.9	0.6

Geographic Area-Family Size Income Frequency Data Algorithm

Let:

- $F(I,J)$  = Number of families per income range per family size.
- $G(I,K)$  = Number of families per income range per geographic area.
- $I$  = Income range index,  $I=1,2,\dots,NI$
- $IN(I,K)$  = Number of individuals per income range per geographic area.
- $J$  = Family size index,  $J=1,2,\dots,NF$
- $K$  = Geographic area index,  $K=1,2,\dots,NG$
- $MP(I)$  = Mid-point of the income ranges
- $NF$  = Number of family sizes
- $NG$  = Number of geographic areas
- $NI$  = Number of income ranges
- $PC(I)$  = Percent of total families earning within a particular income range.
- $PF(I,J)$  = Percent of total families per income range per family size.
- $PG(I,K)$  = Percent of total families per income range per geographic area.
- $PIN(I,K)$  = Percent of Individuals per income range per geographic area.
- $RDV(I)$  = Dollar value width of each income range

$RG(I,K)$  = Ratio of the total family population at each income range to be found in each geographic area.

$TC(I)$  = Number of families per income range for the country

$TF(J)$  = Total number of families per family size

$TG(K)$  = Total number of families per geographic area

$TGF(I,J,K)$  = Number of families per income range per geographic area per family size.

$TIN(K)$  = Total number of individuals per geographic area

$TP$  = Total number of families for the country

Then, for each income range of Table A1-1 determine the number of families in the total population earning, within a given range, or

$$TC(I) = PC(I) \cdot TP \quad (A1-1)$$

Again, for each income range of Table A1-1 determine the number of families in each geographic area earning within that income range, or

$$G(I,K) = PG(I,K) \cdot TG(K) \quad (A1-2)$$

From equations (A1-1) and (A1-2) compute the ratio of the total family population at each income range to be found in each geographic area by,



$$RG(I,K) = \frac{G(I,K)}{TC(I)} \quad (A1-3)$$

Given this geographic ratio breakdown of total family population for each income range, the geographic area breakdown for each family size at each income range can be computed. To do so, first calculate the family distribution of the country at each income range for each family size. This is done by determining, using Table A1-2, the percent of families at each income range for each family size and multiplying this by the total number of families for each family size, or

$$F(I,J) = TF(J) \cdot PF(I,J) \quad (A1-4)$$

where  $J=2, \dots, NF$

Given equation (A1-3) and (A1-4) the desired result is,

$$FG(I,J,K) = F(I,J) \cdot RG(I,K) \quad (A1-5)$$

where  $J=2, \dots, NF$

It is immediately obvious from the above calculations that one now has national income data frequency sets for each geographic area-family size division of the population except for the divisions encompassing individuals. To get these divisions one must again turn to Table A1-2 and for each income range for each geographic area determine the total number of individuals

earning at that range, or

$$IN(I,K) = PIN(I,K) \cdot TIN(K) \quad (A1-6)$$

To keep the data stored in just one array one need only perform the following transformation:

$$FG(I,1,K) = IN(I,K) \quad (A1-7)$$

Given the total individual and family population at each income range for each family size within each geographic area, income frequency distributions can be generated by selecting the mid-points of each income range as the x-coordinates and the above  $FG(I,J,K)$  income frequency data divided by the dollar-value of the income range as the y-coordinates, (this follows the uniform assumption), or

$$X(I,J,K) = MP(I)$$

and

$$Y(I,J,K) = \frac{FG(I,J,K)}{RDV(I)} \quad (A1-8)$$

From the above algorithm a time-sharing computer program has been written in FORTRAN IV and is found in Table A1-31. This program contains comment cards which are especially helpful in setting up one's data. The data of Table A1-1 and A1-2 are used as the sample case, and Tables A1-3 through A1-30 are the resulting national income frequency distributions.

TABLE A1-3  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 1  
FAMILY SIZE = 1

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	411
1250	793
1750	692
2250	563
2750	404
3250	353
3750	382
4500	306
5500	295
6500	252
7500	202
8500	133
9500	72
11000	73
13500	33
20000	6
37500	1
75000	0

TABLE A1-4  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 1  
FAMILY SIZE = 2

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	76
1250	110
1750	182
2250	281
2750	302
3250	275
3750	298
4500	280
5500	294
6500	267
7500	315
8500	302
9500	270
11000	242
13500	148
20000	49
37500	4
75000	0



TABLE A1-5  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 1  
FAMILY SIZE = 3

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	29
1250	25
1750	41
2250	81
2750	93
3250	91
3750	88
4500	119
5500	134
6500	161
7500	225
8500	196
9500	199
11000	200
13500	124
20000	45
37500	3
75000	0

TABLE A1-6  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 1  
FAMILY SIZE = 4

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	17
1250	15
1750	20
2250	43
2750	39
3250	65
3750	58
4500	76
5500	106
6500	129
7500	159
8500	198
9500	207
11000	195
13500	135
20000	53
37500	3
75000	0

TABLE A1-7  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 1  
FAMILY SIZE = 5

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	9
1250	10
1750	15
2250	18
2750	25
3250	32
3750	38
4500	47
5500	68
6500	81
7500	120
8500	131
9500	120
11000	124
13500	89
20000	37
37500	2
75000	0

TABLE A1-8  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 1  
FAMILY SIZE = 6

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	6
1250	5
1750	8
2250	8
2750	15
3250	22
3750	24
4500	31
5500	39
6500	42
7500	62
8500	60
9500	69
11000	62
13500	47
20000	19
37500	1
75000	0



TABLE A1-9  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 1  
FAMILY SIZE = 7

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	4
1250	7
1750	6
2250	13
2750	20
3250	27
3750	31
4500	36
5500	42
6500	47
7500	55
8500	53
9500	50
11000	54
13500	39
20000	14
37500	1
75000	0

TABLE A1-10  
 NATIONAL INCOME DATA COLLECTED FOR  
 GEOGRAPHIC AREA = 2  
 FAMILY SIZE = 1

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	805
1250	1156
1750	979
2250	626
2750	433
3250	433
3750	417
4500	292
5500	252
6500	240
7500	204
8500	144
9500	80
11000	52
13500	25
20000	7
37500	6
75000	0

TABLE A1-11  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 2  
FAMILY SIZE = 2

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	89
1250	182
1750	288
2250	380
2750	350
3250	350
3750	393
4500	325
5500	304
6500	310
7500	346
8500	336
9500	305
11000	284
13500	189
20000	52
37500	4
75000	0

TABLE A1-12  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 2  
FAMILY SIZE = 3

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	33
1250	41
1750	64
2250	110
2750	108
3250	116
3750	116
4500	138
5500	139
6500	187
7500	248
8500	218
9500	224
11000	235
13500	159
20000	48
37500	3
75000	0



TABLE A1-13  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 2  
FAMILY SIZE = 4

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	20
1250	25
1750	32
2250	58
2750	45
3250	83
3750	77
4500	88
5500	110
6500	150
7500	174
8500	221
9500	233
11000	230
13500	172
20000	56
37500	4
75000	0

TABLE A1-14  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 2  
FAMILY SIZE = 5

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	10
1250	16
1750	24
2250	25
2750	29
3250	41
3750	50
4500	54
5500	70
6500	95
7500	132
8500	146
9500	135
11000	146
13500	114
20000	39
37500	3
75000	0

TABLE A1-15  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 2  
FAMILY SIZE = 6

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	8
1250	8
1750	12
2250	11
2750	17
3250	29
3750	32
4500	36
5500	40
6500	49
7500	68
8500	67
9500	78
11000	73
13500	60
20000	20
37500	1
75000	0

TABLE A1-16  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 2  
FAMILY SIZE = 7

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	5
1250	12
1750	10
2250	18
2750	23
3250	35
3750	41
4500	41
5500	44
6500	54
7500	61
8500	59
9500	57
11000	63
13500	50
20000	15
37500	1
75000	0



TABLE A1-17  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 3  
FAMILY SIZE = 1

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	776
1250	1168
1750	760
2250	509
2750	470
3250	431
3750	407
4500	313
5500	231
6500	196
7500	172
8500	98
9500	54
11000	43
13500	18
20000	4
37500	0
75000	0

TABLE A1-18  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 3  
FAMILY SIZE = 2

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	187
1250	461
1750	633
2250	657
2750	539
3250	551
3750	588
4500	491
5500	504
6500	432
7500	380
8500	369
9500	316
11000	236
13500	146
20000	42
37500	3
75000	0

TABLE A1-19  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 3  
FAMILY SIZE = 3

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	70
1250	104
1750	142
2250	189
2750	167
3250	183
3750	173
4500	209
5500	231
6500	262
7500	272
8500	240
9500	233
11000	195
13500	123
20000	39
37500	2
75000	0

TABLE A1-20  
 NATIONAL INCOME DATA COLLECTED FOR  
 GEOGRAPHIC AREA = 3  
 FAMILY SIZE = 4

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	22
1250	42
1750	53
2250	43
2750	45
3250	65
3750	75
4500	82
5500	117
6500	132
7500	145
8500	160
9500	141
11000	121
13500	88
20000	32
37500	2
75000	0



TABLE A1-21  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 3  
FAMILY SIZE = 5

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	43
1250	64
1750	71
2250	100
2750	69
3250	131
3750	115
4500	133
5500	183
6500	210
7500	192
8500	243
9500	242
11000	191
13500	133
20000	46
37500	3
75000	0

TABLE A1-22  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 3  
FAMILY SIZE = 6

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	16
1250	22
1750	28
2250	20
2750	27
3250	46
3750	48
4500	54
5500	66
6500	68
7500	75
8500	73
9500	81
11000	60
13500	46
20000	17
37500	1
75000	0

TABLE A1-23  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 3  
FAMILY SIZE = 7

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	11
1250	30
1750	22
2250	31
2750	36
3250	55
3750	62
4500	63
5500	73
6500	76
7500	67
8500	65
9500	59
11000	52
13500	39
20000	12
37500	1
75000	0

TABLE A1-24  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 4  
FAMILY SIZE = 1

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	284
1250	575
1750	633
2250	587
2750	372
3250	255
3750	261
4500	200
5500	191
6500	189
7500	145
8500	119
9500	95
11000	53
13500	42
20000	8
37500	0
75000	0



TABLE A1-25  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 4  
FAMILY SIZE = 2

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	67
1250	88
1750	163
2250	231
2750	181
3250	222
3750	220
4500	197
5500	211
6500	185
7500	207
8500	210
9500	170
11000	152
13500	107
20000	34
37500	4
75000	0

TABLE A1-26  
 NATIONAL INCOME DATA COLLECTED FOR  
 GEOGRAPHIC AREA = 4  
 FAMILY SIZE = 3

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	25
1250	20
1750	36
2250	66
2750	56
3250	74
3750	65
4500	84
5500	96
6500	112
7500	148
8500	136
9500	125
11000	126
13500	90
20000	32
37500	3
75000	0

TABLE A1-27  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 4  
FAMILY SIZE = 4

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	15
1250	12
1750	18
2250	35
2750	23
3250	52
3750	43
4500	53
5500	76
6500	90
7500	104
8500	138
9500	130
11000	123
13500	98
20000	37
37500	3
75000	0

TABLE A1-28  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 4  
FAMILY SIZE = 5

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	8
1250	8
1750	13
2250	15
2750	15
3250	26
3750	28
4500	33
5500	49
6500	56
7500	79
8500	91
9500	75
11000	78
13500	64
20000	26
37500	2
75000	0



TABLE A1-29  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 4  
FAMILY SIZE = 6

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	6
1250	4
1750	7
2250	7
2750	9
3250	18
3750	18
4500	22
5500	28
6500	29
7500	41
8500	41
9500	44
11000	39
13500	34
20000	13
37500	1
75000	0

TABLE A1-30  
NATIONAL INCOME DATA COLLECTED FOR  
GEOGRAPHIC AREA = 4  
FAMILY SIZE = 7

INCOME VALUE AT MID-RANGE	POPULATION FREQUENCY AT MID-RANGE
500	3
1250	5
1750	5
2250	11
2750	12
3250	22
3750	23
4500	25
5500	30
6500	32
7500	36
8500	37
9500	31
11000	34
13500	28
20000	10
37500	1
75000	0

TABLE A1-31

PROGRAM TO GENERATE GEOGRAPHIC AREA-FAMILY SIZE  
NATIONAL INCOME FREQUENCY DATA SETS

```

005 PROGRAM DATASM
010*
015*PROGRAM TO COMPUTE INCOME FREQUENCY DATA SETS
020*FOR GEOGRAPHIC AREA - FAMILY SIZE DIVISIONS OF THE
030*POPULATION.
035*
040 DIMENSION TG(5),G(18,5),TF(7),F(18,7),FG(18,7,4),MF(18)
045 DIMENSION RDV(18)
050*
055*READ TOTAL POPULATION FIGURES FOR THE COUNTRY, FOR THE
060*NORTHEAST, NORTH CENTRAL, SOUTH, AND WEST GEOGRAPHIC AREAS
065*
070 READ, (TG(N),N=1,5)
075*
080*READ INCOME PERCENTAGES AT EACH INCOME RANGE FOR
085*THE TOTAL POPULATION AND FOR EACH GEOGRAPHIC AREA
090*
095 READ, ((G(I,J),J=1,5),I=1,18)
100*
105*COMPUTE NO. OF PEOPLE/INCOME RANGE FOR THE COUNTRY.
110*COMPUTE NO. OF PEOPLE/INCOME RANGE/GEOGRAPHIC AREA.
115*COMPUTE THE PERCENTAGE OF PEOPLE/INCOME RANGE/GEOGRAPHIC
120*AREA.
125*
130 DO 1 J=1,5
135 DO 1 I=1,18
140 G(I,J) = G(I,J) * (TG(J)/100.)
142 IF(J.EQ.1)GO TO 1
145 G(I,J) = G(I,J) / G(I,1)
147+1 CONTINUE
150*
155*READ TOTAL POPULATION/FAMILY SIZES(2-7+).
160*
165 READ,(TF(J),J=2,7)
170*
175*READ INCOME PERCENTAGES AT EACH INCOME RANGE
180*FOR EACH FAMILY SIZE.
185*
190 READ,((F(I,J),J=2,7),I=1,18)
195*
200*COMPUTE NO. OF PEOPLE AT EACH INCOME RANGE

```

TABLE A1-31

(cont.)

```

205*FOR EACH FAMILY SIZE. THEN DIVIDE THIS
210*POPULATION INTO GEOGRAPHIC AREAS BY
215*PERCENTAGES COMPUTED ABOVE.
220*
225 D0 2 J=2,7
230 D0 2 I=1,18
235 F(I,J) = F(I,J) * (TF(J)/100.)
240 D0 2 K=1,4
245+2 FG(I,J,K) = F(I,J) * G(I,K+1)
250*
255*FOR INDIVIDUALS, READ TOTAL POPULATION
260*OF EACH GEOGRAPHIC AREA.
265*
270 READ, (TG(K),K=1,4)
275*
280*READ INCOME PERCENTAGES AT EACH INCOME RANGE
285*FOR INDIVIDUALS PER GEOGRAPHIC AREA.
290*
295 READ, ((G(I,J),J=1,4),I=1,18)
300*
305*COMPUTE NO. OF INDIVIDUALS/INCOME RANGE/GEOGRAPHIC AREA.
310*
315 D0 3 K=1,4
320 D0 3 I=1,18
325+3 FG(I,1,K) = (G(I,K)/100.) * TG(K)
330*
335*READ MID-POINTS OF THE INCOME RANGE AND
340*THE DOLLAR VALUE OF THE RANGE.
345*
350 READ, (MP(I),RDV(I),I=1,18)
355*
360*COMPUTE THE POPULATION FREQUENCY
365*AT EACH RANGES MID-POINT.
370*
375 D0 4 K=1,4
380 D0 4 J=1,7
385 D0 4 I=1,18
390+4 FG(I,J,K) = FG(I,J,K) / RDV(I)
395*
400*PRINT THE GEOGRAPHIC AREA-FAMILY SIZE
405*INCOME FREQUENCY DATA TABLES.
410*
415 L=3
420 D0 6 K=1,4
425 D0 6 J=1,7

```



TABLE A1-31

(cont.)

```

430 PRINT 100,L
435 PRINT 101
440 PRINT 102,K
445 PRINT 103,J
450 PRINT 104
455 PRINT 105
460 PAUSE
465*
470* THE PAUSES ARE TO ALLOW PAPER TAPE OUTPUT OF
475* GEOGRAPHIC AREA-FAMILY SIZE FREQUENCY DATA. AFTER
480* PAUSE IS EXECUTED, ACTIVATE THE TAPE PUNCH. THEN
485* PRESS CARRIAGE RETURN. AT THE SECOND PAUSE SHUT
490* OFF TAPE PUNCH AND ADVANCE THE TERMINAL PAPER.
495*
500 DO 5 I=1,18
505 IFG = FG(I,J,K)
510+5 PRINT 106,MP(I),IFG
515 PAUSE
520 L=L+1
525+6 CONTINUE
530+100 FORMAT(30X*TABLE A1-*,I2,/)
535+101 FORMAT(18X*NATIONAL INCOME DATA COLLECTED FOR*,/)
540+102 FORMAT(26X*GEOGRAPHIC AREA = *,I2,/)
545+103 FORMAT(30X*FAMILY SIZE = *,I1,/)
550+104 FORMAT(14X*INCOME VALUE AT*,10X,*POPULATION*,
555+104A * FREQUENCY*)
560+105 FORMAT(17X*MID-RANGE*,17X,*AT MID-RANGE*,/)
565+106 FORMAT(18X,I6,23X,I6,/)
570 END
575 FNDPR0G
580 51237000.,12373000.,14358000.,15770000.,8736000.
585 1.6,1.2,1.2,2.3,1.5
590 1.3,.7,1.,2.3,.8
595 1.8,1.1,1.5,3.,1.4
600 2.4,1.8,2.1,3.3,2.1
605 2.2,2.,2.,2.8,1.7
610 2.6,2.1,2.3,3.3,2.4
615 2.7,2.2,2.5,3.4,2.3
620 5.4,4.8,4.8,6.6,4.8
625 5.9,5.5,4.9,7.4,5.6
630 6.4,5.9,5.9,7.5,5.8
635 7.3,7.6,7.2,7.2,7.1
640 7.4,7.6,7.3,7.3,7.5
645 7.,7.4,7.2,6.8,6.6
650 13.,14.2,14.4,10.9,12.7
655 13.7,14.2,15.6,11.,14.6

```

TABLE A1-31

(cont.)

660	15.6, 17.8, 16.3, 12.1, 17.7
665	3.2, 3.4, 3.3, 2.2, 4.8
670	.4, .5, .5, .4, .5
675	17654000., 10688000., 9893000., 6462000., 3467000.
680	3109000.
685	2.4, 1.5, 1., .8, 1.1, .8
690	2.4, .9, .6, .6, .6, .9
695	3.5, 1.3, .7, .8, .8, .7
700	4.4, 2.1, 1.2, .8, .7, 1.2
705	3.9, 2., .9, .9, 1., 1.5
710	4., 2.2, 1.7, 1.3, 1.7, 2.3
715	4.3, 2.1, 1.5, 1.5, 1.8, 2.6
720	7.4, 5.2, 3.6, 3.4, 4.2, 5.4
725	7.4, 5.6, 4.8, 4.7, 5., 6.1
730	6.8, 6.8, 5.9, 5.7, 5.5, 6.8
735	7.1, 8.4, 6.4, 7.4, 7.2, 7.1
740	6.9, 7.4, 8.1, 8.2, 7., 6.9
745	6., 7.3, 8.2, 7.3, 7.9, 6.4
750	10.4, 14.2, 15., 14.6, 13.6, 13.2
755	10.1, 14., 16.4, 16.6, 16.3, 15.4
760	10.2, 15.6, 19.6, 20.8, 20.6, 17.
765	2.3, 2.9, 3.9, 4.1, 4.2, 5.
770	.4, .3, .4, .6, .9, .6
775	3609000., 4014000., 3922000., 2908000.
780	11.4, 12.6, 19.8, 9.8
785	11., 14.4, 14.9, 9.9
790	9.6, 12.2, 9.7, 10.9
795	7.8, 7.8, 6.5, 10.1
800	5.6, 5.4, 6., 6.4
805	4.9, 5.4, 5.5, 4.4
810	5.3, 5.2, 5.2, 4.5
815	8.5, 7.3, 8., 6.9
820	8.2, 6.3, 5.9, 6.6
825	7.0, 6.0, 5.0, 6.5
830	5.6, 5.1, 4.4, 5.
835	3.7, 3.6, 2.5, 4.1
840	2., 2., 1.4, 3.3
845	4.1, 2.6, 2.2, 3.7
850	2.8, 1.9, 1.4, 4.4
855	1.9, 1.9, 1.1, 2.8
860	.7, .2, .6, .6
865	.001, .001, .1, .001
870	500, 1000., 1250, 500., 1750, 500., 2250, 500., 2750, 500.
875	3250, 500., 3750, 500., 4500, 1000., 5500, 1000.
880	6500, 1000., 7500, 1000., 8500, 1000., 9500, 1000.
890	11000, 2000., 13500, 3000., 20000, 10000.
895	37500, 25000., 75000, 50000.

A2-DISK STORAGE PROCEDURE FOR NATIONAL INCOME FREQUENCY  
DATA

There are two possible ways to store national income frequency data onto disk storage. The first technique is required when the program of Appendix A, Section A1, Table A1-31 is used to generate the data base. In this case one need simply replace the paper tape punching procedure between the PAUSE statements at sequence numbers 460 and 515 with the statements of Table A2-1.

TABLE A2-1

DISK FILE GENERATION PROCEDURE  
USING THE PROGRAM OF TABLE A1-31

```

460 PRINT 109,K,J
465*109 FORMAT(*SELECT A DISK FILE NAME FOR GEOGRAPHIC AREA = *,
470*109A I2,* FAMILY SIZE = *,I2)
475 INPUT(61,108) NAME
480*108 FORMAT(A6)
485 WRITE(5) (FLOAT(MP(I)),FG(I,J,K),I=1,18)
490 CALL CLOSE (5,NAME)
495
500
505
510
515

```

These new instructions place each geographic area-family size national income frequency data set into a disk location defined by the user. The user must simply type, when called for the message, "SELECT A DISK FILE FOR GEOGRAPHIC AREA = II FAMILY SIZE = II", some name that will specify the geographic area-family size connotation of the file. It is suggested that the form "GXXFXX" be used. Where "GXX" specifies a particular geographic area by the symbol "G" and an alphanumeric code "XX", for the geographic area so assigned as to be meaningful to the user. An example of this would be denoting the four geographic areas of Table A1-2 by G01, G02, G03, and G04 respectively. "FXX" would therefore denote the family sizes of Table A1-1 by F02 through F07 and F01 would denote the individuals of Table A1-2. Therefore, by using the above format one has up to 100 family sizes within each of 100 geographic areas if just numbers were used to denote particular geographic areas and family sizes. However, since "GXXFXX" is alphanumeric, combinations using the alphabet plus any other useable characters of the user's FORTRAN system are possible.

The second way to file national income frequency data is necessitated when accurate census data is available and no preprocessing is required to generate geographic area-family size national income frequency sets of the form as







TABLE A2-2  
 NATIONAL INCOME FREQUENCY DATA CARDS  
 TO DISK FILES PROGRAM

```

C SUPPLY BEFORE THIS CARD YOUR SYSTEM OPERATING CARDS
C
C READ THE FILE NAME CARD AND NO. OF DATA PAIRS IN THE FILE
C
1 DIMENSION XMP(500),FG(500)
2 READ 100, NAME,NPAIRS
100 FORMAT(A6,2X,I3)
    IF (NAME .EQ. 6HENDATA) STOP
C
C READ THE DATA PAIRS
C
    READ 101, (XMP(I),FG(I),I=1,NPAIRS)
C
C PLACE THE DATA SET INTO DISK FILE STORAGE
C
    WRITE(5) (XMP(I),FG(I),I=1,NPAIRS)
    CALL CLOSE (5,NAME)
    GO TO 2
101 FORMAT(4(I6,2X,I6,2X))
    END
  
```

The second method to file national income frequency data sets directly is by reading prepunched paper tapes through a time-sharing terminal. This may be done by the following instructions:

To generate the paper tape;

- a) Set the terminal LINE-OFF-LOCAL switch to LOCAL, position the paper tape in the punch unit (if necessary), and press the ON button

on the punch unit.

b) Generate a short (3-to 4-inch) leader on the tape by holding down the REPT and RUBOUT keys on the keyboard. (This leader aids in the placement of the tape in the paper tape reader.)

c) Punch the data exactly as formatted for the card program of Table A2-2 but remember to follow the following added steps for each card.

(1) For time-sharing each line of data must have a sequence number. Therefore, before punching the data punch in a sequence number starting at 100 increment by 5 for each line. Then punch a space and the data, i.e.

```
100 NAME, NPAIRS
```

```
105 IIIIIIIΔΔIIIIIIIΔΔIIIIIIIΔΔIIIIIII, etc.
```

(2) Each line of data must be ended on the tape by depressing the following sequence of keys: CARRIAGE RETURN, LINE FEED, RUBOUT, RUBOUT.

d) The last line of the paper tape should be a KEY, or KBOARD command to return control from paper tape to the terminal keyboard.

e) Once the tape contains all the desired information, generate a (3-to 4-inch) trailer by



holding the REPT and RUBOUT keys. (This action ensures that no information will be lost when the tape is torn off the punch unit.

- f) Tear the tape off the punch unit, identify it as to its content, and indicate its beginning-of-tape.

To generate the disk files use the program of Table A2-2 after sequence numbers have been added (See Table A2-3). Then read in the paper tape generated by the above instructions using the following instructions:

- a) With the paper tape reader set to OFF position the previously prepared tape in the reader by carefully placing the small central holes in the tape into the reader sprockets. The start of the leader points toward the user.
- b) Issue the TAPE or PTAPE system command and turn the reader switch to START.

TABLE A2-3

NATIONAL INCOME FREQUENCY DATA PUNCHED TAPE  
TO DISK PROGRAM

```
010*  
020* READ THE FILE NAME AND NO. OF DATA PAIRS IN THE FILE  
030*  
040 DIMENSION XMP(500),FG(500)  
050*2 READ 100, NAME NPAIRS  
060*100 FORMAT(A6,2X,I3)  
070 IF (NAME .EQ. 6HENDATA) STOP  
080*  
090* READ THE DATA PAIRS  
100*  
110 READ 101, (XMP(I),FG(I),I=1,NPAIRS)  
120*101 FORMAT(4(I6,2X,I6,2X))  
130*  
140* PLACE THE DATA SET INTO DISK FILE STORAGE  
145*  
150 WRITE(5) (XMP(I),FG(I),I=1,NPAIRS)  
160 CALL CLOSE (5,NAME)  
170 GO TO 2  
180 END
```

A3-CURVE FITTING THE NATIONAL INCOME FREQUENCY DATA

The curve fitting technique employed by this dissertation is that of Least-Squares using polynomial interpolation. A general purpose FORTRAN IV time-sharing subroutine to perform the curve fitting is found in Table A3-1 and by comments inserted in the program is self-explanatory. However, to more fully describe the technique and its accuracy in fitting data the following mathematics is

presented.

## FUNCTIONAL APPROXIMATION - LEAST-SQUARES TECHNIQUES<sup>4</sup>

Polynomial interpolation is a method of approximating the value of a function at a point by means of a polynomial passing through known functional values. A major virtue of this method of approximation is its ease of implementation. Another virtue is that it leads to an expression for the truncation error in the approximation which can often be estimated or bounded.

The reader may be familiar with the principle of least-squares as applied to continuous functions over an interval  $[a,b]$ . However, the analysis here is entirely concerned with the principles of least-squares as applied to functions known only at a discrete set of points. My reason for emphasizing discrete rather than continuous case is the composition of the data I am to fit.

Let me first make precise my heuristic definition of least-squares approximations. Let  $f(x)$  be a function and  $\{x_i\}$ ,  $i=1,\dots,n$  be a sequence of data points at which are observed values of  $f(x)$  which generally will be in error. Denote  $f(x_i)$ , the true value at  $x_i$ , and denote the observed value at  $x_i$  by  $\bar{f}_i$ . Then define  $E_i = f_i - \bar{f}_i$ . One more assumption must be made, that is, the errors at different

data points are uncorrelated (i.e., independent).

Let  $\{\phi_j(x)\}$ ,  $j=0,1,\dots$  be a (generally finite) sequence of functions defined for every  $x_i$ . Then the object is to approximate  $\bar{f}_i$  by a linear combination of the  $\{\phi_j(x)\}$ , or

$$\bar{f}_i \approx \sum_{j=0}^m a_j^{(m)} \phi_j(x_i) \quad , \quad i=1,\dots,n \quad (\text{A3-1})$$

with the  $a_j^{(m)}$  to be determined so that

$$\begin{aligned} H(a_0^{(m)}, \dots, a_m^{(m)}) &\equiv \sum_{i=1}^n w(x_i) \left[ \bar{f}_i - \sum_{j=0}^m a_j^{(m)} \phi_j(x_i) \right]^2 \\ &= \sum_{i=1}^n w(x_i) R_i^2 \end{aligned} \quad (\text{A3-2})$$

is minimized. The function  $w(x)$  is called the "weight function" and is assumed to be such that  $w(x_i) \geq 0, i=1,\dots,n$ . The quantity  $R_i$  is called the "residual" at  $x_i$ . The superscript  $m$  on  $a_j^{(m)}$  denotes the fact that the coefficients of  $\phi_j(x)$  depends on  $m$ .

Once one determines the  $a_j^{(m)}$  so as to satisfy equation (A3-2), one then has the approximation

$$y_m(x) = \sum_{j=0}^m a_j^{(m)} \phi_j(x) \quad (\text{A3-3})$$

which is called a least-squares approximation to  $f(x)$  over  $\{x_i\}$ .



To calculate the  $a_j^{(m)}$ 's take the partial derivative of  $H$  in equation (A3-2) with respect to  $a_k^{(m)}$  and set it equal to zero, thereby obtaining

$$\frac{\partial H}{\partial a_k^{(m)}} = -2 \sum_{i=1}^n w_i [\bar{f}_i - \sum_{j=0}^m a_j^{(m)} \phi_j(x_i)] \phi_k(x_i) = 0$$

$$\text{where } k=0, \dots, m; \quad w_i = w(x_i) \quad (\text{A3-4})$$

Equation (A3-4) is a system of  $m+1$  linear equations for the  $m+1$  unknowns  $a_j^{(m)}$ 's. This system is called the normal equations. If the determinant of the coefficients does not vanish, one can solve for the  $a_j^{(m)}$ 's and quite easily show that this solution is indeed a minimum by looking at the signs of the second order terms.

The result of the above discussion is the basic assumption that for some unknown value of  $m$ , say  $M$ , the true function  $f(x)$  can be expressed as a finite linear combination of the set of functions  $\{\phi_j(x)\}$ . That is, assume

$$f(x) = \sum_{j=0}^M a_j^{(M)} \phi_j(x) \quad (\text{A3-5})$$

Now, let's consider the case in which  $\phi_j(x)$  is a polynomial of degree  $j$ . In particular consider the case where  $\phi_j(x) = x^j$  and  $w(x) = 1$ . Then equation (A3-4) becomes after cancelling the  $-2$ ,

$$\sum_{i=1}^n [\bar{f}_i - \sum_{j=0}^m a_j^{(m)} x_i^j] x_i^k = 0; \quad k=0, \dots, m \quad (\text{A3-6})$$

Interchanging summations,

$$\sum_{j=0}^m a_j^{(m)} \left( \sum_{i=1}^n x_i^{j+k} \right) = \sum_{i=1}^n \bar{f}_i x_i^k; \quad k=0, \dots, m \quad (\text{A3-7})$$

Using the notion

$$g_{jk} = \sum_{i=1}^n x_i^{j+k} \quad P_k = \sum_{i=1}^n \bar{f}_i x_i^k \quad (\text{A3-8})$$

the normal equations may be written

$$\sum_{j=0}^m g_{jk} a_j^{(m)} = P_k; \quad k=0, \dots, m \quad (\text{A3-9})$$

Using matrix calculus, it can be proved that the least-squares problem and, thus the system (equation (A3-9)) has a unique solution.

At this point the least-squares problem for the case  $\phi_j(x) = x^j$ ,  $w(x)=1$  is solved. All that is left to do is to perform the perhaps tedious calculations required to solve the normal equations (A3-9).

A major question that must still be answered is how to choose the degree of the polynomial to obtain the best fit given a data set of  $n$  points. The basic hypothesis is that the true function  $f(x)$  is a polynomial of degree

$M < n$  or at least can be accurately represented by such a polynomial. A priori one does not know what  $M$  is; the problem is to find it. If one chooses value of  $m < M$ , then clearly it is impossible to get a good representation of the true function. On the other hand, choosing a value of  $m > M$  also defeats our purpose. It can be shown that choosing  $m = n - 1$  makes

$$\delta_m^2 = \sum_{i=1}^n w_i R_i^2 = \sum_{i=1}^n w_i \left[ \bar{f}_i - \sum_{j=0}^m a_j^{(m)} x_i^j \right]^2 \quad (\text{A3-10})$$

equal to zero. But in so doing, all smoothing properties of least-squares approximations is lost. In fact, any value of  $m > M$  sacrifices some smoothing.

Using powers of  $x$ , equation (A3-5) becomes,

$$f(x) = \sum_{j=0}^M a_j^{(M)} x^j \quad (\text{A3-11})$$

Therefore, if one knew  $M$  and calculated the least-squares approximation

$$y_{M+1}(x) = \sum_{j=0}^{M+1} a_j^{(M+1)} x^j \quad (\text{A3-12})$$

using the observed data  $\{\bar{f}_i\}$ , then statistically  $a_{M+1}^{(M+1)}$  should be zero. That is, if there were no errors in the data, it would be zero, but because of these errors, it will not be zero even if the assumption that  $f(x)$  has the form of equation (A3-12) is correct. One would like then

to test the statistical hypothesis that  $a_{M+1}^{(M+1)}=0$ . In order to be able to do this, one must make one further assumption that the errors  $E_i$  are normally distributed with zero mean and variance  $\sigma^2/w_i$ . This assumption is reasonable because more accurate measurements (i.e., those with small variances) will usually be more heavily weighted.

This statistical hypothesis to be tested is often called the null hypothesis. It can be tested using maximum-likelihood statistical methods [see Wilks, S.S., Mathematical Statistics, rev. ed., John Wiley & Sons, Inc., New York, 1962]. Here the result is only stated, that, if the null hypothesis is correct, then the expected value of

$$\sigma_m^2 = \delta_m^2 / (n-m-1) \quad (\text{A3-13})$$

will be independent of  $m$  for  $m=M, M+1, \dots, n-1$ . Thus, in practice, since one does not know  $M$ , one will wish to solve the normal equations (A3-9) for  $m=1, 2, \dots$ , compute  $\sigma_m^2$ , and continue as long as  $\sigma_m^2$  decreases significantly with increasing  $m$ . As soon as a value of  $m$  is reached after which no significant decrease occurs in  $\sigma_m^2$ , then this  $m$  is that of the null hypothesis, and one has the desired least-squares approximation.



Computationally, that means one must compute the solution of the normal equations for a sequence of values of  $m$ . The computer program of Table A3-1 does exactly this. It then compares the errors of each degree fit and selects the optimal choice automatically.

The only restrictions of the curve fitting program are that the number of data points cannot exceed 500 and the degrees of polynomials search is between 1 and 15. However, if the number of data points is less than 15 the degrees of polynomials searched is between 1 and number of points -2.

The calling sequence of the curve fitting subroutine is

```
CALL CUFIT (X,Y,NPAIRS,SMIN,MDEG,B,W)
```

where X = an array of the X-coordinates to be inputed

Y = an array of the Y-coordinates to be inputed

NPAIRS = number of X,Y coordinates to be inputed

SMIN = the error of the best curve fit to be returned as output

MDEG = the degree of the best curve fit to be returned as output

B = the array of coefficients for the best curve fit to be returned as output

W = the array of weights for the data set to

be inputed. Set these values to zero if no weighting is required.

Besides returning the above outputs the curve fitting program also prints the best curve fit's coefficients, degree, and measure of error. Also printed is a comparison between the initial data pairs to be curve fitted and the estimated values as a result of curve fitting.

TABLE A3-1

A FORTRAN IV GENERAL PURPOSE SUBROUTINE TO PERFORM  
POLYNOMIAL CURVE FITTING

```

2000 SUBROUTINE CUFIT (X,Y,NPAIRS,SMIN,MDEG,B,W)
2010 DIMENSION X(500),Y(500),A(16,16),B(16)
2020 DIMENSION SUMX(31),SUMY(15),W(500)
2030*
2040*  INITIAL CALCULATIONS
2050*
2060 ITEST = 0
2070 MAXDEG = NPAIRS-1
2080 IF (MAXDEG .GT. 15) MAXDEG = 15
2090 DO 1 I=1,MAXDEG
2100 SUMX(I) = 0.
2110*1 SUMY(I) = 0.
2120 DO 2 I=1,NPAIRS
2130 IF (W(I)) 3,4,3
2140*4 W(I) = 1.
2150*3 SUMX(1) = SUMX(1) + W(I)
2160 SUMX(2) = SUMX(2) + W(I)*X(I)
2170 SUMX(3) = SUMX(3) + W(I)*X(I)*X(I)
2180 SUMY(1) = SUMY(1) + W(I)*Y(I)
2190*2 SUMY(2) = SUMY(2) + W(I)*X(I)*Y(I)
2200*
2210*  BEGIN POLYNOMIAL ITERATION
2220*
2230 K = 0
2240 NORD = 1
2250*5 L = NORD + 1
2260 KK = L + 1
2270 DO 7 I=1,L
2280 DO 6 J=1,L
2290 IK = J-1+I
2300*6 A(I,J) = SUMX(IK)
2310*7 A(I,KK) = SUMY(I)
2320 DO 10 I=1,L
2330 A(KK,I) = -1.
2340 KKK = I+1
2350 DO 8 J=KKK,KK
2360*8 A(KK,J) = 0.
2370 C = 1./A(1,I)
2380 DO 9 II=2,KK
2390 DO 9 J=KKK,KK

```

TABLE A3-1

(cont.)

```

2400*9 A(II,J) = A(II,J)-A(1,J)*A(II,1)*C
2410 DO 10 II=1,L
2420 DO 10 J=KKK,KK
2430*10 A(II,J) = A(II+1,J)
2440 S2 = 0.
2450 DO 12 J=1,NPAIRS
2460 S1 = 0.
2470 S1 = S1 + A(1,KK)
2480 DO 11 I=1,NORD
2490*11 S1 = S1 + A(I+1,KK)*X(J)**I
2500*12 S2 = S2 + (S1-Y(J))*(S1-Y(J))
2510 B1 = NPAIRS-L
2520 S2 = (S2/B1)**.5
2540 IF (NORD .EQ. 1) GO TO 20
2550 IF (SMIN .LE. S2) GO TO 14
2560*20 MDEG = NORD
2570 SMIN = S2
2580 DO 13 I=1,MDEG+1
2590*13 B(I) = A(I,KK)
2595 IF (MDEG .LT. MAXDEG) GO TO 21
2600*14 IF (NORD .GE. MAXDEG) GO TO 16
2610 ITEST = ITEST + 1
2620 IF (ITEST .GE. 2) GO TO 16
2630*21 NORD = NORD + 1
2640 J = 2*NORD
2650 SUMX(J) = 0.
2660 SUMX(J+1) = 0.
2670 SUMY(NORD+1) = 0.
2680 DO 15 I=1,NPAIRS
2690 SUMX(J) = SUMX(J) + X(I)**(J-1)*W(I)
2700 SUMX(J+1) = SUMX(J+1) + X(I)**J*W(I)
2710*15 SUMY(NORD+1) = SUMY(NORD+1) + Y(I)*X(I)**NORD*W(I)
2720 GO TO 5
2730*
2740* PRINT BEST FIT INFORMATION
2750*
2760*16 PRINT 100,MDEG,SMIN
2770 PRINT 110
2780 DO 17 I=1,MDEG+1
2790 J=I-1
2800*17 PRINT 120,J,B(I)
2802 PRINT 160
2804 INPUT,N1
2806 IF (N1 .EQ. 1HN) GO TO 31

```



TABLE A3-1

(cont.)

```
2810 PRINT 130,MDEG
2820 PRINT 140
2830 DO 19 I=1,NPAIRS
2835 S1 = 0.
2840 S1 = B(1)
2850 DO 18 J=1,MDEG
2860*18 S1 = S1 + B(J+1)*X(I)**J
2870 S3 = Y(I) - S1
2880*19 PRINT 150,W(I),X(I),Y(I),S1,S3
2890*
2900*   FORMAT STATEMENTS
2910*
2920*100 FORMAT(///,*POLYNOMIAL DEGREE = *,I2,9X,
2930*100A *STD. ERROR = *,F12.4)
2940*110 FORMAT(//,1X,*TERM DEGREE*,5X,*COEFFICIENTS*)
2950*120 FORMAT(1X,I6,6X,E16.8)
2960*130 FORMAT(///,*BACK SOLUTIONS FOR BEST FIT - DEGREE =*
2970*130A ,I4,/)
2980*140 FORMAT(4X,*WEIGHT*,5X,*X VALUE*,5X,*Y VALUE*,6X,
2990*140A *EST. Y*,4X,*RESIDUAL*,/)
3000*150 FORMAT(5(F12.2))
3002*160 FORMAT(//,*DO YOU WISH TO SEE THE BACK SOLUTIONS*
3004*160A *...TYPE Y FOR YES, N FOR NO*)
3010*31 RETURN
3020 END
```

## REFERENCES

1. United States Department of Commerce, Current Population Reports, Bureau of the Census, Consumer Income, "Income in 1969 of Families and Persons in the United States," p. 60, No. 70, December 14, 1970, p. 78.
2. Consumer Income, op. cit., p. 42.
3. Consumer Income, op. cit., p. 83.
4. Ralston, Anthony, A First Course in Numerical Analysis, McGraw-Hill Book Company, 1965, pp. 228-235.

APPENDIX B

THE MANAGEMENT INFORMATION AND CONTROL SYSTEM'S  
TIME-SHARING COMPUTER PROGRAMS

B1-PROGRAMS FOR PART 1 OF THE MANAGEMENT INFORMATION AND CONTROL SYSTEM

The use of part 1 of the management information and control system of this dissertation is presented in Section 1 of Chapter IV. This section of Appendix B presents the program listings and in the case of subroutines the calling sequences and the variables transferred between programs.

The complete part 1 management information and control system program is broken down into four distinct units. They are the monitoring program, MISFIT; the curve fitting subroutine, CUFIT; the plotting subroutine, PLOT; and finally the program to select from a set of data points the minimum and maximum values, DMIMA.

The curve fitting subroutine, CUFIT is explained in Appendix A, Section 3 and in Chapter IV. The remaining segments are found here.

The monitoring program, MISIFT is used to allow communication between the computer and the user. It generates user instructions and upon receiving answers to these instructions generates the pertinent answers by properly using the associated subroutines. The user instructions and resulting outputs are presented in Chapter V and Chapter VI of this dissertation. The actual program is



found in Table B1-1.

The plotting subroutine, PLOT is found in Table B1-2. This routine is designed to be useable in other programming systems besides that of this dissertation. Its limitations are presented in Chapter V. What follows is its calling sequence

```
nn CALL PLOT (X,Y,N,SCALE,BIAS,IHOLD,ICHAR)
```

where

nn = the line number;

X = an array containing the values of the independent variable;

Y = an array of values of the dependent variable to be plotted (0 in the first CALL PLOT line);

N = the number of data points to be plotted (less than or equal to 100);

SCALE =  $60./(YMAX-YMIN)$  where YMAX is the maximum and YMIN the minimum Y-value in any of the functions to be plotted (used as below to scale values of Y);

BIAS =  $-YMIN$  where YMIN is the minimum Y-value in any of the functions to be plotted (used to scale values of Y in the formula  $(Y+BIAS)SCALE+.5=XPOS$  where XPOS is the column position of Y);

IHOLD = -1 in the first CALL PLOT line;  
1 in all succeeding CALL PLOT lines except the last; and  
0 in the last CALL PLOT LINE;

ICCHAR (the character to be printed)  
= 0 in the first CALL PLOT line;  
IH\*(where \* is any printable character) in all succeeding CALL PLOT lines.

- NOTES:
- 1) X-values which are out of range are printed at the closest printable value.
  - 2) On multiple plots, the character printed at points where the curves intersect is that of the last function to CALL PLOT with the value in question. No message is given when this occurs.
  - 3) All curves plotted must be tabulated at the same value of X, and X must be equally spaced.

(NOTE: There must be an initial CALL PLOT line followed by one CALL PLOT line for each Y-array)

The maximizing and minimizing subroutine, DMIMA, is found in Table B1-3. It simply determines the minimum and maximum elements of an array. The calling sequence is:

```
nn CALL DMIMA (Y,N,AMI,AMA)
```

where

nn = the line number

Y = an array of less than 500 words to be  
inputed

N = number of words in the array to be  
inputed

AMI = the minimum Y value selected and out-  
puted

AMA = the maximum Y value selected and out-  
puted

TABLE B1-1

THE MONITORING PROGRAM FOR PART 1 OF THE  
MANAGEMENT INFORMATION AND CONTROL SYSTEM

```

NAME MISCFT
OK
LIST 010,1580
010 PROGRAM MISCFT
020*
030* THIS PROGRAM ACCEPTS GEOGRAPHIC AREA-FAMILY SIZE
040* NATIONAL INCOME FREQUENCY DATA FROM DISK FILES AS
050* REQUESTED BY THE USER AND CURVE FITS THIS DATA FOR
060*     1) THE WHOLE GEOGRAPHIC AREA-FAMILY SIZE, OR
070*     2) A SUBSIDY AND NON-SUBSIDY PORTION OF (1)
080*     PORTIONED BY A USER INPUT - A BREAKEVEN
090*     LEVEL OF INCOME, OR
100*     3) BOTH (1) AND (2) ABOVE
110* AND STORES THE RESULTING CURVE FITS IN DISK FILES
120* NAMED BY THE USER.
130*
140* INPUT GEOGRAPHIC AREA-FAMILY SIZE DATA
150*
160 DIMENSION X(500),Y(500),W(500),AP(16),AS(16),AN(16)
170 DIMENSION XS(300),YS(300)
180*1 PRINT 101
190 MCONT = 0
200 INPUT,NAME,IG,IF
210 IF (NAME .EQ. 6HENDATA) STOP
220 PRINT 103
230 INPUT, NPAIRS
240 CALL OPEN (5,NAME,-1)
250 READ(5) (X(I),Y(I),I=1,NPAIRS)
260 CALL CLOSE (5,NAME)
270 PRINT 104
280 INPUT, M
290 GO TO (2,4,3),M
300*
310* PERFORM REQUESTED CURVE FITS
320*
330*2 MCONT = -1
340*3 PRINT 105,IG,IF
350 CALL CUFIT (X,Y,NPAIRS,STERRP,MDEGP,AP,W)
360 IF (MCONT) 11,4,4
370*4 PRINT 106

```

## TABLE B1-1

(cont.)

```
380 INPUT, BLI
390 NPS = 0
400 NPN = 0
410 DO 5 I=1, NPAIRS
420 IF (X(I) .GT. BLI) GO TO 6
4305 NPS = NPS+1
4406 NPN = (NPAIRS-NPS)+1
450 DO 7 I=1, NPS
460 XS(I) = X(I)
4707 YS(I) = Y(I)
480 PRINT 107, IG, IF
490 CALL CUFIT (XS, YS, NPS, STERRS, MDEGS, AS, W)
500 DO 8 I=NPS, NPAIRS
510 XS((I-NPS)+1) = X(I)
5208 YS((I-NPS)+1) = Y(I)
530 PRINT 108, IG, IF
540 CALL CUFIT (XS, YS, NPN, STERRN, MDEGN, AN, W)
550*.
560* PLOT INSTRUCTIONS
570*
57211 PRINT 114
574 INPUT, N1
576 IF (N1 .EQ. 1HN) GO TO 19
580 PRINT 110
590 INPUT, N1
600 IF (N1 .EQ. 1HN) GO TO 12
610 CALL DMIMA (Y, NPAIRS, AMI, AMA)
620 SCALE = 60./(AMA-AMI)
630 BIAS = -AMI
640 CALL PLOT (X, 0, NPAIRS, SCALE, BIAS, -1, 0)
650 CALL PLOT (X, Y, NPAIRS, SCALE, BIAS, 0, 1H*)
66012 PRINT 111
670 INPUT, N1
680 IF (N1 .EQ. 1HN) GO TO 14
683 XS(1) = 0.
686 YS(1) = AP(1)
690 DO 13 I=2, 25
700 XI = I-1
710 XS(I) = XI*1000.
720 YS(I) = AP(1)
730 DO 13 J=2, MDEGP+1
74013 YS(I) = YS(I) + AP(J)*XS(I)**(J-1)
750 CALL DMIMA (YS, 25, AMI, AMA)
760 SCALE = 60./(AMA-AMI)
```



## TABLE B1-1

(cont.)

```
770 BIAS = -AMI
780 CALL PLOT (XS,YS,25,SCALE,BIAS,-1,0)
790 CALL PLOT (XS,YS,25,SCALE,BIAS,0,1H*)
800*14 IF (MCONT) 1,15,15
810*15 PRINT 112
820 INPUT,N1
830 IF (N1 .EQ. 1HN) GO TO 17
840 XINC = IFIX(BLI/250.)*10
843 XS(1) = 0.
846 YS(1) = AS(1)
850 DO 16 I=2,27
860 XI = I-1
870 XS(I) = XI*XINC
880 YS(I) = AS(1)
890 DO 16 J=2,MDEGS+1
900*16 YS(I) = YS(I) + AS(J)*XS(I)**(J-1)
910 CALL DMIMA (YS,27,AMI,AMA)
920 SCALE = 60./(AMA-AMI)
930 BIAS = -AMI
940 CALL PLOT (XS,0,27,SCALE,BIAS,-1,0)
950 CALL PLOT (XS,YS,27,SCALE,BIAS,0,1H*)
960*17 PRINT 113
970 INPUT,N1
980 IF (N1 .EQ. 1HN) GO TO 19
990 XINC = IFIX((25000.-BLI)/250.)*10
1000 DO 18 I = 1,25
1010 XI = I-1
1020 XS(I) = XI*XINC + BLI
1030 YS(I) = AN(1)
1040 DO 18 J= 2,MDEGN+1
1050*18 YS(I) = YS(I) + AN(J)*XS(I)**(J-1)
1060 CALL DMIMA (YS,25,AMI,AMA)
1070 SCALE = 60./(AMA-AMI)
1080 BIAS = -AMI
1090 CALL PLOT (XS,0,25,SCALE,BIAS,-1,0)
1100 CALL PLOT (XS,YS,25,SCALE,BIAS,0,1H*)
1110*19 IF (M .EQ. 2) GO TO 1
1120*
1130* STORE THE DATA FITS IN PARAMETRIC FORM
1140*
1150*9 PRINT 109
1160 INPUT,N1
1170 IF (N1 .EQ. 2HNO) GO TO 1
1180 WRITE(5) (IG,IF,M,BLI)
```

## TABLE B1-1

(cont.)

```

1190*10 WRITE(5) (MDEGP, STERRP, (AP(I), I=1, MDEGP+1),
1200*10A MDEGS, STERRS, (AS(J), J=1, MDEGS+1),
1210*10B MDEGN, STERNR, (AN(K), K=1, MDEGN+1))
1220 CALL CLOSE (5, N1)
1230 GO TO 1
1240*
1250*     FORMAT STATEMENTS
1260*
1270*101 FORMAT(//, *TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE *
1280*101A *DATA FILE TYPE THE FILE NAME, /*THE GEOGRAPHIC *
1290*101B *AREA INDEX, AND THE FAMILY SIZE INDEX*)
1300*103 FORMAT(//, *HOW MANY DATA PAIRS ARE THERE IN *
1310*103A *THIS DATA FILE*)
1320*104 FORMAT(//, *TO CURVE FIT THE WHOLE DATA FILE*
1330*104A *...TYPE 1 /*TO CURVE FIT THE SUBSIDY AND *
1340*104B *NON-SUBSIDY SEGMENTS...TYPE 2*/
1350*104C *TO CURVE FIT BOTH ABOVE ...TYPE 3*)
1360*105 FORMAT(//, *THE FOLLOWING BEST FITS THE TOTAL *
1370*105A *POPULATION FOR /*GEOGRAPHIC AREA *, I2,
1380*105B * FAMILY SIZE *, I2)
1390*106 FORMAT(//, *INPUT A BREAKEVEN LEVEL OF INCOME *
1400*106A *FOR THIS GEOGRAPHIC AREA-FAMILY SIZE*)
1410*107 FORMAT(//, *THE FOLLOWING BEST FITS THE SUBSIDY *
1420*107A *PORTION OF /*GEOGRAPHIC AREA *, I2,
1430*107B * FAMILY SIZE *, I2)
1440*108 FORMAT(//, *THE FOLLOWING BEST FITS THE NON-*
1450*108A *SUBSIDY PORTION OF /*GEOGRAPHIC AREA *, I2,
1460*108B * FAMILY SIZE *, I2)
1470*109 FORMAT(//, *IF THE ABOVE CURVE FITS ARE TO BE *
1480*109A *SAVED, NAME THE FILE, IF NOT TYPE NO*)
1490*110 FORMAT(//, *DO YOU WISH TO PLOT THE ORIGINAL *
1500*110A *DATA...TYPE Y FOR YES, N FOR NO*)
1510*111 FORMAT(//, * DO YOU WISH TO PLOT THE CURVE FIT *
1520*111A *FOR THE WHOLE /* GEOGRAPHIC AREA-FAMILY SIZE *
1530*111B *SEGMENT...TYPE Y FOR YES, N FOR NO*)
1540*112 FORMAT(//, *DO YOU WISH TO PLOT THE SUBSIDY *
1550*112A *PORTION...TYPE Y FOR YES, N FOR NO*)
1560*113 FORMAT(//, *DO YOU WISH TO PLOT THE NON-*
1570*113A *SUBSIDY PORTION...TYPE Y FOR YES, N FOR NO*)
1572*114 FORMAT(//, *DO YOU WISH TO PLOT...TYPE Y FOR YES, *
1574*114A *N FOR NO*)
1580 END

```

TABLE B1-2

ON-LINE PLOT SUBROUTINE INCORPORATED IN PART 1  
OF THE MANAGEMENT INFORMATION AND CONTROL SYSTEM

```

3100†10 SUBROUTINE PLOT(X,Y,N,SCALE,BIAS,HOLD,ICHAR)
3110†20 DIMENSION MAX(100)
3120†30 DIMENSION X(100),Y(N),LINES(100,9)
3130†35 REAL MAX
3140†40 INTEGER HOLD
3150†41 IMODF(X,Y)=((X/Y)-FLOAT(IFIX(X/Y)))*Y
3170*IF HOLD = 0 PRINT PLOT
3180*IF HOLD = 1 HOLD IT
3190*IF HOLD = -1 CLEAR PAGE IMAGE
3200†50 MSK=77777777777777777777B
3210†51 IF (N.GT.100) GOTO 1040
3220†60 IF(HOLD.NE.-1) GO TO 110
3230†70 DO 100 I=1,N
3240†80 DO 90 J=1,9
3250†90 LINES(I,J) = 8H
3260†95 MAX(I)=0
3270†100 CONTINUE
3280†105 RETURN
3290†110 DO 500 K=1,N
3300†120 XPOS=(Y(K)+BIAS)*SCALE+.5
3310†130 IPOSR = 42-IMODF(XPOS,8.)*6
3320†140 IPOSR=XPOS/8.+1.
3330†145 IF (IPOS.GT.9) GOTO 1020
3340†146 IF (IPOS.LE.0) GOTO 1030
3350†200 MSKOUT=-IBYTE(6,MSK,IPOSR,IPOSR)
3360†210 . INTER = LINES(K,IPOS)*MSKOUT
3370†220 . LINES (K,IPOS)= IBYTE(6,ICHAR,52,IPOSR)+INTER
3380†230 IF (MAX(K).LT.XPOS)MAX(K)=XPOS
3390†231 IF (K.GT.100) STOP
3400†500 CONTINUE
3410†501 IF (HOLD.NE.0) RETURN
3420†502 FORMAT(*-. 0.000*,2X)
3430†504 PRINT 505
3440†505 FORMAT(1H-,7X)
3450†507 SC= - BIAS
3460†508 DO 515 M=1,6
3470†509 IF (SC.NE.0.) GOTO 511
3480†510 PRINT 502; GOTO 513
3490†511 PRINT 512, SC
3500†512 FORMAT (1H-F8.3,2X)
3510†513 SC=SC+10./SCALE

```



TABLE B1-2

(cont.)

```

3520*515 CONTINUE
3530*516 PRINT 517
3540*517 FORMAT(/,1H-,11X,5(*+-----*))
3550*518 PRINT 519
3560*519 FORMAT(1X,*+-----*)
3570*520 DO 1000 I= 1,N
3580*525 PRINT 526,X(I)
3590*526 FORMAT (1H-,G10.4*I*)
3600*530 IF (MAX(I).EQ.0.)GOTO 700
3610*540 MAXW= MAX(I)/8.+1.
3620*545 IS =1H
3630*550 IREM = IMODF(MAX(I),8.)+1
3640*555 IF (IREM.EQ.8) GOTO 600
3650*560 IS =1H-
3660*570 MAXW=MAXW-1
3670*600 PRINT 605,IS,(LINES(I,J),J=1,MAXW)
3680*605 FORMAT(A1,8A8)
3690*610 IF (IREM.EQ.8) GOTO 1000
3700*620 L= MAXW+1
3710*630 PRINT 635,(IBYTE(6,LINES(I,L),48-6*J,42),J=1,IREM)
3720*635 FORMAT (1X,8A1)
3730*640 GOTO 1000
3740*700 PRINT 705
3750*705 FORMAT (1H )
3760*1000 CONTINUE
3770*1005 IF (1.EQ.1) RETURN
3780*1006 STOP
3790*1007 N=0
3800*1020 XPOS =8*8+6;IPOS =9 ; IPOSR = 6 ; GOTO 200
3810*1030 IPOS =1 ; IPOSR= 42 ; GO TO 200
3820*1040 PRINT 1045,N ;STOP
3830*1045 FORMAT (* MAX NO. OF POINTS ACCEPTED =100, NO. GIVEN =*17)
3840*1050 END
4000 SUBROUTINE DMIMA (Y,NPOINT,AMI,AMA)
4010 DIMENSION Y(100)
4020 AMI = Y(1)
4030 AMA = Y(1)
4040 DO 1 I=1,NPOINT
4050 IF (Y(I) .GT. AMA) AMA = Y(I)
4060 IF (Y(I) .LT. AMI) AMI = Y(I)
4070*1 CONTINUE
4080 RETURN
4090 END
4100 ENDPROG

```

TABLE B1-3

FORTRAN IV SUBROUTINE TO CHOOSE THE MINIMUM VALUE AND  
MAXIMUM VALUE OF A SET OF DATA POINTS

```
4000 SUBROUTINE DMIMA (Y,NPOINT,AMI,AMA)
4010 DIMENSION Y(100)
4020 AMI = Y(1)
4030 AMA = Y(1)
4040 DO 1 I=1,NPOINT
4050 IF (Y(I) .GT. AMA) AMA = Y(I)
4060 IF (Y(I) .LT. AMI) AMI = Y(I)
4070+1 CONTINUE
4080 RETURN
4090 END
4100 ENDPROG
```



B2-PROGRAMS FOR PART 2 OF THE MANAGEMENT INFORMATION AND  
CONTROL SYSTEM

The use of part 2 of the management information and control system of this dissertation is presented in Section 2 of Chapter V. This section of Appendix B presents the program listings and in the case of subroutines the calling sequences and the variables transferred between programs.

The complete part 2 management information and control system program is broken down into three distinct units. They are the monitoring program, MISMTH, the polynomial integration subroutine, POLINT, and the polynomial multiplication subroutine, POLMUL.

The monitoring program MISMTH is used to allow communication between the computer and the user. It generates user instructions as to guaranteed minimum income and negative tax testing procedures and upon receiving answers to these instructions utilizes the mathematical model found in Phase 3, Chapter III. The user instructions are presented in Chapter's V and VI of this dissertation. The actual program is found in Table B2-1. The polynomial integration subroutine, POLINT, is found in Table B2-2. The mathematics used is found in Section 3 of this Appendix. The calling sequence of this sub-

routine is

```
nn CALL POLINT (MDEG, SL, SU, C, ANS)
```

where nn = the line number;

MDEG = the degree of the polynomial to be integrated;

SL = the lower limit for closed form integration;

SU = the upper limit for closed form integration;

C = the coefficients of the polynomial to be integrated;

ANS = the solution to the integral.

The polynomial multiplication subroutine, POLMUL, is found in Table B2-3. The mathematics used is found in Section 4 of this Appendix. The calling sequence of this subroutine is

```
nn CALL POLMUL (MDEGA,A,MDEGB,B,MDEGC,C)
```

where nn = the line number;

MDEGA = the degree of the A polynomial to be multiplied by B;

A = the coefficients of the A polynomial;

MDEGB = the degree of the B polynomial to be multiplied by A;

B = the coefficients of the B polynomial;

MDEGC = the degree of the product,  $A \cdot B$ , polynomial;

C = the coefficients of the product polynomial.

TABLE B2-1

THE MONITORING PROGRAM FOR PART 2 OF THE  
MANAGEMENT INFORMATION AND CONTROL SYSTEM

## LIST MISMTH

0010 PROGRAM MISMTH .

0020\*

0030\* THIS PROGRAM COMPUTES FOR EACH GEOGRAPHIC AREA-  
0040\* FAMILY SIZE SEGMENT OF THE POPULATION OF THE UNITED  
0050\* STATES ITS POPULATION AND INCOME. THE POPULATION AND  
0060\* INCOME OF ITS SUBSIDY AND NON-SUBSIDY PORTIONS. THE  
0070\* INCOME ADDED TO THE SUBSIDY PORTION DUE TO THE  
0080\* INCORPORATION OF A MINIMUM INCOME. THE TAX LIABILITY  
0090\* ON THE NON-SUBSIDY PORTION TO PAY FOR THE SUBSIDY. THE  
0100\* TAX LIABILITY ON CORPORATIONS TO PAY FOR THE SUBSIDY.  
0110\* THE TAX LIABILITY OR SUBSIDY RECEIVED FOR PARTICULAR  
0120\* INCOME LEVELS WITHIN THE NON-SUBSIDY AND SUBSIDY  
0130\* PORTIONS RESPECTIVELY. IT WILL ALSO ACCUMULATE ALL  
0140\* THE ABOVE FOR THE WHOLE COUNTRY, FOR GEOGRAPHIC AREAS,  
0150\* AND FOR FAMILY SIZES UPON REQUEST. ALL OF THE ABOVE  
0160\* COMPUTATIONS ARE BASED UPON USER INPUTS TO REQUESTS  
0170\* MADE BY THIS SYSTEM. THE REQUESTS ARE SELF EXPLANATORY.

0180\*

0190 DIMENSION POPSF(99),POPSG(99),POPNF(99),POPNG(99),  
0200A YSF(99),YSG(99),YNF(99),YNG(99),YAF(99),YAG(99),  
0210B YFNT(99),YGNT(99),SF(99),SG(99),CSF(99),CSG(99),  
0220C DEFF(99),DEFG(99),INF(99),ING(99)

0225 DIMENSION CSC(31)

0230 DIMENSION AP(16),AN(16),AS(16),ANT(16),AMTN(16),  
0240A AMTC(16),YS(31),YN(31),YTAX(31),CS(31),CI(2)

0250\*

0260\* INPUT A GEOGRAPHIC AREA-FAMILY SIZE FILE AND INITIALIZE.

0270\*

0275 TM=1.0E6

0280 ISEG=0

0285 TB=1.0E9

0290 IF=0

0300 IG=0

0310 POPST=POPNT=POPCT=YTS=YTN=YT=YTA=YTNT=TS=TCS=TDEF=0.0

0320 DO 2 I=1,99

0330 POPSF(I)=POPSG(I)=POPNF(I)=POPNG(I)=YSF(I)=YSG(I)=0.0

0340 YNF(I)=YNG(I)=YAF(I)=YAG(I)=YFNT(I)=YGNT(I)=SF(I)=0.0

0350 SG(I)=CSF(I)=CSG(I)=DEFF(I)=DEFG(I)=0.0

0360\*2 CONTINUE

0370 CI(1)=0.0



TABLE B2-1

(cont.)

```

0380 CI(2)=1.0
0390:1 PRINT 100
0400 INPUT,NAME
0410 IF (NAME .EQ. 6HENDATA) STOP
0420 CALL OPEN (5,NAME,-1)
0430 READ(5) (IG,IF,M,BLI)
0440 READ(5) (MDEGP,STERRP,(AP(I),I=1,MDEGP+1),
0450A           MDEGS,STERRS,(AS(J),J=1,MDEGS+1),
0460B           MDEGN,STERRN,(AN(K),K=1,MDEGN+1))
0470 CALL CLOSE (5,NAME)
0500 ISEG=ISEG+1
0530*
0540* COMPUTE THE POPULATION FIGURES FOR EACH
0550* GEOGRAPHIC AREA-FAMILY SIZE SEGMENT ALONG
0560* WITH THE SUBSIDY AND NON-SUBSIDY PORTIONS.
0570*
0580* THE SUBSIDY PORTION POPULATION.
0590*
0600 CALL POLINT (MDEGS,0.,BLI,AS,POPGFS)
0610*
0620* THE NON-SUBSIDY PORTION POPULATION
0630*
0640 PRINT 101
0650 INPUT,ULN
0660 CALL POLINT (MDEGN,BLI,ULN,AN,POPGFN)
0670*
0680* THE GEOGRAPHIC AREA-FAMILY SIZE SEGMENT POPULATION.
0690*
0700 POPGF=POPGFS+POPGFN
0710*
0720* PRINT THE POPULATION DATA
0730*
0740 PRINT 102,IG,IF
0750 PRINT 103,POPGF/TM,POPGFS/TM,POPGFN/TM
0760*
0770* COMPUTE THE INCOME FIGURES FOR EACH GEOGRAPHIC
0780* AREA-FAMILY SIZE SEGMENT ALONG WITH ITS SUBSIDY
0790* AND NON-SUBSIDY PORTIONS.
0800*
0810* THE SUBSIDY PORTION INCOME.
0820*
0830 CALL POLMUL (MDEGS,AS,1,CI,MDEGYS,YS)
0840 CALL POLINT (MDEGYS,0.,BLI,YS,YGFS)
0850*
0860* THE NON-SUBSIDY PORTION INCOME.
0870*
0880 CALL POLMUL (MDEGN,AN,1,CI,MDEGYN,YN)

```

## TABLE B2-1

(cont.)

```

0890 CALL POLINT (MDEGYN,BLI,ULN,YN,YGFN)
0900*
0910* THE GEOGRAPHIC AREA-FAMILY SIZE SEGMENT INCOME.
0920*
0930 YGF=YGFS+YGFN
0940*
0950* PRINT THE INCOME DATA
0960*
0970 PRINT 105,IG,IF
0980 PRINT 106,YGF/TB,YGFS/TB,YGFN/TB
0990*
1000* COMPUTE THE ADDITION OF INCOME AS A RESULT OF THE
1010* INCLUSION OF AN INPUTED MINIMUM INCOME
1020*
1030 IF (ISEG .EQ. 1) GO TO 44
1032 PRINT 180
1034 INPUT,N1
1036 IF(N1 .EQ. 1HY) GO TO 45
1038 IF(N1 .EQ. 1HN) GO TO 44
1040+4 PRINT 109
1050 INPUT,N1
1060 IF(N1 .EQ. 1HN) GO TO 5
1070+44 PRINT 107
1080 INPUT,GMI
1090+45 YAGF=GMI*POPGFS
1100*
1110* PRINT THE ADDITION TO INCOME.
1120*
1130 PRINT 108,IG,IF,GMI,YAGF/TM
1140 GO TO 4
1150*
1160* INPUT A NEGATIVE TAX AN POLYNOMIAL FORM
1170*
1180+5 PRINT 150
1190 INPUT ,N1
1200 IF (N1 .EQ. 1HN) GO TO 55
1210+50 PRINT 110
1220 INPUT,MDEGNT,(ANT(I),I=1,MDEGNT+1)
1222 PRINT 170
1225 INPUT,TL
1230*
1240* COMPUTE THE INCOME SUBTRACTED FROM THE SUBSIDY
1250* PORTION BY THE ADDITION OF A NEGATIVE TAX
1260* TO INCOME EARNED.
1270*
1280+55 CALL POLMUL (MDEGNT,ANT,MDEGYS,YS,MDEGYT,YTAX)
1290 CALL POLINT (MDEGYT,TL,BLI,YTAX,YGFNT)

```

## TABLE B2-1

(cont.)

```
1300*
1310* PRINT THE RESULT OF THE NEGATIVE TAX.
1320*
1330 PRINT 111,IG,IF,YGFNT/TM
1340 PRINT 160
1350 INPUT,N1
1370 IF (N1 .EQ. 1HN) GO TO 50
1380*
1390* COMPUTE THAT PART OF THE SUBSIDY TO BE PAID FOR
1400* BY THE NON-SUBSIDY SEGMENT AND PRINT THE RESULTS.
1410*
1420 SGF=YAGF-YGFNT
1430 PRINT 112,SGF/TM
1440*
1450* INPUT A MARGINAL TAX ON THE NON-SUBSIDY EARNINGS
1460* AND ON CORPORATE EARNINGS TO PAY FOR THE SUBSIDY.
1470*
1471 IF(ISEG .EQ. 1) GO TO 67
1472 PRINT 200
1474 INPUT,N1
1476 IF(N1 .EQ. 1HY) GO TO 66
1478 IF(N1 .EQ. 1HN) GO TO 67
1480:6 PRINT 113
1490 INPUT,N1
1500 IF(N1 .EQ. 1HN) GO TO 7
1510:67 PRINT 114
1520 INPUT,MDGMTN,(AMTN(I),I=1,MDGMTN+1)
1530*
1540* COMPUTE THE ADDED REVENUE OF THE MARGINAL TAX
1550* FROM THE NON-SUBSIDY PORTION.
1560*
1570:66 CALL POLMUL (MDGMTN,AMTN,MDEGYN,YN,MDEGCS,CS)
1580 CALL POLINT (MDEGCS,BLI,ULN,CS,CSN)
1590*
1600* PRINT THE ADDED REVENUE OF THE MARGINAL TAX
1610*
1620 PRINT 115,CSN/TM
1630 CSGF = CSN
1640 GO TO 6
1650*
1660* COMPUTE THE ADDED REVENUE OF THE MARGINAL TAX
1670* ON CORPORATIONS IF REQUESTED.
1680*
1690:7 PRINT 116
1700 INPUT,N1
1710 IF (N1 .EQ. 1HN) GO TO 9
1715 CSC=0.
```



TABLE B2-1

(cont.)

```
1720 PRINT 117
1730 INPUT,CEGF
1740 PRINT 120
1750 INPUT,N1
1760 IF (N1 .EQ. 1HN) GO TO 8
1770 PRINT 118
1780 INPUT,MDGMTC,(AMTC(I),I=1,MDGMTC+1)
1783 DO 30 I=1,MDGMTC+1
1786+30 CSC(I)=0.0
1790*
1800* COMPUTE THE ADDED REVENUE OF THE MARGINAL TAX ON
1810* CORPORATE EARNINGS TO PAY FOR THE SUBSIDY.
1820*
1830+8 CALL POLMUL (MDGMTC,AMTC,0,CEGF,MDEGCS,CSC)
1840*
1850* PRINT THE ADDED REVENUE DUE TO A MARGINAL CORP. TAX.
1860*
1870 PRINT 119,CSC/TM
1880*
1890* COMPUTE THE TOTAL REVENUE FROM MARGINAL TAXES.
1900*
1910 DO 31 I=1,MDEGCS+1
1915+31 CSGF=CSGF+CSC(I)
1920*
1930* COMPUTE THE DEFICIT OR SURPLUS AS A
1940* RESULT OF MARGINAL TAXATION.
1950*
1960+9 DEFGF=CSGF-SGF
1970*
1980* PRINT THE DEFICIT OR SURPLUS.
1990*
2000 IF (DEFGF .LT. 0.) PRINT 121,DEFGF/TM
2010 IF (DEFGF .GE. 0.) PRINT 122,DEFGF/TM
2020*
2030* COMPUTE MARGINAL TAX AND SUBSIDY FIGURES
2040* FOR INDIVIDUAL INCOME BRACKETS UPON REQUEST.
2050*
2060 PRINT 123
2070 INPUT,N1
2080 IF (N1 .EQ. 1HN) GO TO 12
2090*
2100* ANPUT THE LIMITS FOR SUBSIDY RECEIVED.
2110*
2120+10 PRINT 124
2130 INPUT,SLL,SUL
2140 IF (SLL .LT. 0.) GO TO 11
2150*
```



## TABLE B2-1

(cont.)

```
2160* COMPUTE SUBSIDY RECEIVED BETWEEN SLL AND SUL.
2170*
2180 CALL POLINT (MDEGS,SLL,SUL,AS,ANS)
2190 SUBLU = GMI*ANS
2200*
2210* PRINT THE SUBSIDY RECEIVED BETWEEN SLL AND SUL.
2220*
2230 PRINT 125,SLL,SUL,SUBLU/TM
2240 GO TO 10
2250*
2260* INPUT THE LIMITS FOR SUBSIY PAID.
2270*
2280:11 PRINT 126
2290 INPUT,SLL,SUL
2300 IF (SLL .LT. 0.) GO TO 12
2310*
2320* COMPUTE SUBSIDY PAID BETWEEN SLL AND SUL.
2330*
2340 CALL POLINT (MDEGCS,SLL,SUL,CS,TLIAB)
2350*
2360* PRINT THE SUBSIDY PAID BETWEEN SLL AND SUL.
2370*
2380 PRINT 127,SLL,SUL,TLIAB/TM
2390 GO TO 11
2400*
2410* COMPUTE CUMULATIVE FIGURES FOR FINAL CASES ABOVE
2420* IF REQUESTED BY USER.
2430*
2440:12 IF (ISEG .GT. 1 ) GO TO 13
2450 PRINT 104
2460 INPUT,NCUM
2470:13 IF (NCUM .EQ. 1HN)GO TO 1
2480*
2490* POPULATION CUMULATIVE FIGURES.
2500*
2510 POPSF(IF)=POPSF(IF)+POPGFS
2520 POPSG(IG)=POPSG(IG)+POPGFS
2530 POPNF(IF)=POPNF(IF)+POPGFN
2540 POPNG(IG)=POPNG(IG)+POPGFN
2550 POPST=POPST+POPGFS
2560 POPNT=POPNT+POPGFN
2570 POPCT=POPCT+POPGF
2580*
2590* INCOME CUMULATIVE FIGURES.
2600*
2610 YSF(IF)=YSF(IF)+YGFS
2620 YSG(IG)=YSG(IG)+YGFS
```

## TABLE B2-1

(cont.)

2630  $YNF(IF) = YNF(IF) + YGFN$   
 2640  $YNG(IG) = YNG(IG) + YGFN$   
 2650  $YTS = YTS + YGFS$   
 2660  $YTN = YTN + YGFN$   
 2670  $YT = YT + YGF$   
 2680 \*  
 2690 \* ADDITION TO INCOME CUMULATIVE FIGURES.  
 2700 \*  
 2710  $YAF(IF) = YAF(IF) + YAGF$   
 2720  $YAG(IG) = YAG(IG) + YAGF$   
 2730  $YTA = YTA + YAGF$   
 2740 \*  
 2750 \* NEGATIVE TAX CUMULATIVE FIGURES.  
 2760 \*  
 2770  $YFNT(IF) = YFNT(IF) + YGFNT$   
 2780  $YGNT(IG) = YGNT(IG) + YGFNT$   
 2790  $YTNT = YTNT + YGFNT$   
 2800 \*  
 2810 \* NET SUBSIDY CUMULATIVE FIGURES.  
 2820 \*  
 2830  $SF(IF) = SF(IF) + SGF$   
 2840  $SG(IG) = SG(IG) + SGF$   
 2850  $TS = TS + SGF$   
 2860 \*  
 2870 \* MARGINAL TAX CUMULATIVE FIGURES.  
 2880 \*  
 2890  $CSF(IF) = CSF(IF) + CSGF$   
 2900  $CSG(IG) = CSG(IG) + CSGF$   
 2910  $TCS = TCS + CSGF$   
 2920 \*  
 2930 \* DEFICIT OR SURPLUS CUMULATIVE FIGURES  
 2940 \*  
 2950  $DEFF(IF) = DEFF(IF) + DEFGF$   
 2960  $DEFG(IG) = DEFG(IG) + DEFGF$   
 2970  $TDEF = TDEF + DEFGF$   
 2980 \*  
 2990 \* PRINT THE CUMULATIVE FIGURES UPON REQUEST.  
 3000 \*  
 3010 IF (ISEG .EQ. 1) GO TO 1  
 3020 PRINT 128  
 3030 INPUT, N1  
 3040 IF (N1 .EQ. 1HN) GO TO 1  
 3050 \*  
 3060 \* PRINT THE NUMBER OF GEOGRAPHIC AREA-FAMILY SIZE  
 3070 \* SEGMENTS FOR WHICH CUMULATIVE FIGURES ARE AVAILABLE.  
 3080 \*  
 3090 PRINT 129, ISEG

## TABLE B2-1

(cont.)

```
3100*
3110* PRINT THE POPULATION FIGURES.
3120*
3130 PRINT 130
3140 PRINT 134,POPCT/TM,POPST/TM,POPNT/TM
3150*
3160* PRINT THE FAMILY SIZE POPULATION FIGURES.
3170*
3180 DO 14 I=1,7
3185 IF (POPSF(I) .LE. 0.) GO TO 14
3190 POPF=POPSF(I)+POPNF(I)
3200 PRINT 133,I
3210 PRINT 134,POPF/TM,POPSF(I)/TM,POPNF(I)/TM
3215+14 CONTINUE
3220*
3230* PRINT THE GEOGRAPHIC AREA POPULATION FIGURES.
3240*
3250 DO 15 I=1,4
3255 IF(POPSG(I) .LE. 0.) GO TO 15
3260 POPG=POPSG(I)+POPNG(I)
3270 PRINT 135,I
3280 PRINT 134,POPG/TM,POPSG(I)/TM,POPNG(I)/TM
3285+15 CONTINUE
3290*
3300* PRINT THE INCOME FIGURES.
3310*
3320 PRINT 131
3330 PRINT 134,YT/TB,YTS/TB,YTN/TB
3340*
3350* PRINT THE FAMILY SIZE INCOME FIGURES.
3360*
3370 DO 16 I=1,7
3375 IF(YSF(I) .LE. 0.) GO TO 16
3380 YF=YSF(I)+YNF(I)
3390 PRINT 132,I
3400 PRINT 134,YF/TB,YSF(I)/TB,YNF(I)/TB
3405+16 CONTINUE
3410*
3420* PRINT THE GEOGRAPHIC AREA INCOME FIGURES.
3430*
3440 DO 17 I=1,4
3445 IF(YSG(I) .LE. 0.) GO TO 17
3450 YG=YSG(I)+YNG(I)
3460 PRINT 136,I
3470 PRINT 134, YG/TB,YSG(I)/TB,YNG(I)/TB
3475+17 CONTINUE
3480*
```



## TABLE B2-1

(cont.)

```
3490* PRINT THE CUMULATIVE ADDITIONS TO REVENUE.
3500*
3510 PRINT 137
3520 PRINT 138,YTA/TB
3530 DO 18 I=1,7
3535 IF(YAF(I) .LE. 0.) GO TO 18
3540 PRINT 139,I,YAF(I)/TB
3545:18 CONTINUE
3550 DO 19 I=1,4
3555 IF(YAG(I) .LE. 0.) GO TO 19
3560 PRINT 140,I,YAG(I)/TB
3565:19 CONTINUE
3570*
3580* PRINT THE CUMULATIVE NEGATIVE TAX FIGURES.
3590*
3600 PRINT 141
3610 PRINT 138,YTNT/TB
3620 DO 20 I=1,7
3625 IF(YFNT(I) .LE. 0.) GO TO 20
3630 PRINT 139,I,YFNT(I)/TB
3635:20 CONTINUE
3640 DO 21 I=1,4
3645 IF(YGNT(I) .LE. 0.) GO TO 21
3650 PRINT 140,I,YGNT(I)/TB
3655:21 CONTINUE
3660*
3670* PRINT THE CUMULATIVE NET SUBSIDY FIGURES.
3680*PRINT 142
3700 PRINT 138,TS/TB
3710 DO 22 I=1,7
3715 IF(SF(I) .LE. 0.) GO TO 22
3720 PRINT 139,I,SF(I)/TB
3725:22 CONTINUE
3730 DO 23 I=1,4
3735 IF(SG(I) .LE. 0.) GO TO 23
3740 PRINT 140,I,SG(I)/TB
3745:23 CONTINUE
3750*
3760* PRINT THE MARGINAL TAX CUMULATIVE FIGURES.
3770*
3780 PRINT 143
3790 PRINT 138,TCS/TB
3800 DO 24 I=1,7
3815 IF(CSF(I) .LE. 0.) GO TO 24
3820 PRINT 139,I,CSF(I)/TB
3825:24 CONTINUE
3830 DO 25 I=1,4
```



TABLE B2-1

(cont.)

```

3835 IF(CSG(I) .LE. 0.) GO TO 25
3840 PRINT 140,I,CSG(I)/TB
3845:25 CONTINUE
3850* PRINT THE DEFICIT OR SURPLUS CUMULATIVE FIGURES.
3860*
3870 PRINT 144
3880 PRINT 138,TDEF/TB
3890 DO 26 I=1,7
3895 IF(DEFF(I) .EQ. 0.) GO TO 26
3900 PRINT 139,I,DEFF(I)/TB
3905:26 CONTINUE
3910 DO 27 I=1,4
3915 IF(DEFG(I) .EQ. 0.) GO TO 27
3920 PRINT 140,I,DEFG(I)/TB
3925:27 CONTINUE
3930 GO TO 1
3940*
3950*          FORMAT STATEMENTS
3960*
3970:100 FORMAT(//,*TO SELECT A GEOGRAPHIC AREA-FAMILY SIZE
3980:100A DATA FILE TYPE THE FILE NAME*)
3990:101 FORMAT(//,*INPUT AN UPPER INCOME LIMIT IN DOLLARS
4000:101A FOR THE NON-SUBSIDY PORTION*)
4010:102 FORMAT(//,2H**,*POPULATION FIGURES FOR GEOGRAPHIC
4020:102A AREA *,I2,* FAMILY SIZE *,I2,/,3X,
4030:102B *IN MILLIONS OF PEOPLE ARE*)
4040:103 FORMAT(//,10X,*SEGMENT POPULATION = *,F12.6,/,10X,
4050:103A *SUBSIDY POPULATION = *,F12.6,/,6X,
4060:103B *NON-SUBSIDY POPULATION = *,F12.6)
4070:104 FORMAT(///,*ARE CUMULATIVE FIGURES TO BE KEPT...
4080:104A TYPE Y FOR YES, N FOR NO*)
4090:105 FORMAT(//,2H**,*INCOME FIGURES FOR GEOGRAPHIC AREA *
4100:105A ,I2,* FAMILY SIZE *,I2,/,3X,
4110:105B *IN BILLIONS OF DOLLARS ARE*)
4120:106 FORMAT(//,10X,*SEGMENT INCOME = *,F12.6,/,10X,
4130:106A *SUBSIDY INCOME = *,F12.6,/,6X,
4140:106B *NON-SUBSIDY INCOME = *,F12.6)
4150:107 FORMAT(///,*INPUT A MINIMUM INCOME FOR THIS
4160:107A SEGMENT*)
4170:108 FORMAT(//,2H**,*THE SUBSIDY ADDED TO GEOGRAPHIC
4180:108A AREA *,I2,* FAMILY SIZE *,I2,/,3X,*FOR MINIMUM
4190:108B INCOME *,F6.0,* IS *,F12.6,* MILLION DOLLARS*)
4200:109 FORMAT(//,*DO YOU WISH TO TRY ANOTHER MINIMUM
4210:109A INCOME...TYPE Y FOR YES, N FOR NO*)
4230:110 FORMAT(//,*INPUT A NEGATIVE TAX IN POLYNOMIAL FORM*)
4240:111 FORMAT(//,2H**,*THE NEGATIVE TAX ON THE SUBSIDY
4250:111A PORTION OF GEOGRAPHIC AREA *,I2,/,3X,*FAMILY SIZE *

```

TABLE B2-1

(cont.)

4260†111B ,I2,\* IS \*,F12.6,\* MILLION DOLLARS\*)  
 4270†112 FORMAT(//,2H\*\*,\*THE SUBSIDY AFTER THE NEGATIVE TAX  
 4280†112A IS ADDED\*,/,3X,\*IS \*,F12.6,\* MILLION DOLLARS\*)  
 4290†113 FORMAT(//,\*IS A NEW MARGINAL TAX TO BE INPUTED  
 4300†113A ...TYPE Y FOR YES, N FOR NO\*)  
 4310†114 FORMAT(//,\*INPUT THE MARGINAL TAX IN POLYNOMIAL FORM\*)  
 4320†115 FORMAT(//,2H\*\*,\*THE ADDED REVENUE DUE TO A MARGINAL  
 4330†115A TAX \*,/,3X,\*IS \*,F12.6,\* MILLION DOLLARS\*)  
 4350†116 FORMAT(//,\*IS TAXATION OF CORPORATE EARNINGS TO BE  
 4360†116A USED TO PAY FOR THE SUBSIDY\*,/,\*TYPE Y FOR YES,  
 4370†116B N FOR NO\*)  
 4380†117 FORMAT(//,\*INPUT THE CORPORATE EARNINGS FOR THIS  
 4390†117A GEOGRAPHIC AREA\*)  
 4400†118 FORMAT(//,\*INPUT THE CORPORATE MARGINAL TAX IN  
 4410†118A POLYNOMIAL FORM\*)  
 4420†119 FORMAT(//,2H\*\*,\*THE ADDED REVENUE DUE TO A MARGINAL  
 4430†119A TAX ON CORPORATIONS\*,/,3X,\*IS \*,F12.6,  
 4435†119B \* MILLION DOLLARS\*)  
 4440†120 FORMAT(//,\*IS THE CORPORATE MARGINAL TAX TO CHANGE  
 4450†120A ...TYPE Y FOR YES, N FOR NO\*)  
 4460†121 FORMAT(//,2H\*\*,\*MARGINAL TAXATION RESULTED IN A  
 4470†121A DEFICIT OF \*,F12.6,\* MILLIONS\*)  
 4480†122 FORMAT(//,2H\*\*,\*MARGINAL TAXATION RESULTED IN A  
 4490†122A SURPLUS OF \*,F12.6,\* MILLIONS\*)  
 4500†123 FORMAT(//,\*DO YOU WISH TO INVESTIGATE INDIVIDUAL  
 4510†123A INCOME BRACKETS AS TO\*,/\*SUBSIDY RECEIVED OR  
 4520†123B SUBSIDY COST LEVIED...TYPE Y FOR YES,N FOR NO\*)  
 4530†124 FORMAT(//,\*INPUT THE INCOME RANGES LOWER AND UPPER  
 4540†124A LIMITS TO COMPUTE SUBSIDY\*/\*RECEIVED, IF FINISHED  
 4550†124B INVESTIGATING SUBSIDY RECEIVED AREA\*/\*TYPE -1. -1.\*)  
 4560†125 FORMAT(//,2H\*\*,\*THE SUBSIDY RECEIVED BETWEEN \*,F6.0,  
 4570†125A \* AND \*,F6.0,\* IS \*,F12.6,\* MILLION\*)  
 4580†126 FORMAT(//,\*INPUT THE INCOME RANGES LOWER AND UPPER  
 4590†126A LIMITS TO COMPUTE SUBSIDY\*,/\*PAID IF FINISHED  
 4600†126B INVESTIGATING SUBSIDY PAID AREA\*/\*TYPE -1. -1.\*)  
 4610†127 FORMAT(//,2H\*\*,\*THE SUBSIDY PAID BETWEEN \*,F6.0,  
 4620†127A \* AND \*,F6.0,\* IS \*,F12.6,\* MILLION\*)  
 4630†128 FORMAT(//,\*DO YOU WISH TO PRINT THE CUMULATIVE  
 4635†128A FIGURES...TYPE Y FOR YES, N FOR NO\*)  
 4640†129 FORMAT(//,2H\*\*,\*THE CUMULATIVE FIGURES BELOW ARE FOR \*  
 4650†129A ,I2,\* GEOGRAPHIC AREA-FAMILY\*,/,3X,\*SIZE SEGMENTS\*)  
 4660†130 FORMAT(//,2H\*\*,\*THE POPULATION FIGURES IN MILLIONS  
 4670†130A OF PEOPLE FOR ALL SEGMENTS ARE\*)  
 4680†131 FORMAT(//,2H\*\*,\*THE INCOME FIGURES IN BILLIONS OF  
 4690†131A DOLLARS FOR ALL SEGMENTS ARE\*)  
 4700†132 FORMAT(//,2H\*\*,\*THE INCOME FIGURES IN BILLIONS OF  
 4710†132A DOLLARS FOR FAMILY SIZE \*,I2,\* ARE\*)  
 4720†133 FORMAT(//,2H\*\*,\*THE POPULATION FIGURES IN MILLIONS OF



TABLE B2-1

(cont.)

4730†133A PEOPLE FOR FAMILY SIZE \*,I2,\* ARE\*)  
 4740†134 FORMAT(/,12X,\*TOTAL = \*,F12.6,/,10X,\*SUBSIDY = \*,  
 4750†134A F12.6,/,6X,\*NON-SUBSIDY = \*,F12.6)  
 4760†135 FORMAT(/,2H\*\*,\*THE POPULATION FIGURES IN MILLIONS  
 4770†135A OF PEOPLE FOR\*,/,3X,\*GEOGRAPHIC AREA \*,I2,\* ARE\*)  
 4780†136 FORMAT(/,2H\*\*,\*THE INCOME FIGURES IN BILLIONS OF  
 4790†136A DOLLARS FOR GEOGRAPHIC AREA \*,I2,\* ARE\*)  
 4800†137 FORMAT(/,2H\*\*,\*THE TOTAL SUBSIDIES IN BILLIONS OF  
 4810†137A DOLLARS ARE\*)  
 4820†138 FORMAT(/,19X,\*TOTAL = \*F12.6)  
 4830†139 FORMAT(10X,\*FAMILY SIZE \*,I2,\* = \*,F12.6)  
 4840†140 FORMAT(6X,\*GEOGRAPHIC AREA \*,I2,\* = \*,F12.6)  
 4850†141 FORMAT(/,2H\*\*,\*THE TOTAL SUBSIDY IN BILLIONS OF  
 4860†141A DOLLARS PAID BY NEGATIVE TAXES ARE\*)  
 4870†142 FORMAT(/,2H\*\*,\*THE NET SUBSIDY FIGURES IN BILLIONS  
 4875†142A OF DOLLARS ARE\*)  
 4880†143 FORMAT(/,2H\*\*,\*THE MARGINAL TAX FIGURES IN  
 4885†143A BILLIONS OF DOLLARS ARE\*)  
 4890†144 FORMAT(/,2H\*\*,\*THE DEFICIT OR SURPLUS FIGURES IN  
 4895†144A BILLIONS OF DOLLARS ARE\*)  
 4900†150 FORMAT(/,\*IF A NEW NEGATIVE TAX IS NEEDED TYPE Y,  
 4910†150A IF NOT TYPE N\*)  
 4920†160 FORMAT(/,\*IF THE NEGATIVE TAX IS SATISFACTORY TYPE  
 4930†160A Y, IF NOT TYPE N\*)  
 4935†170 FORMAT(/,\*INPUT THE LOWEST LIMIT AT WHICH THE  
 4937†170A NEGATIVE TAX IS APPLIED\*)  
 4940†180 FORMAT(/,\*DO YOU WIS@ TO USE THE MINIMUM INCOME  
 4945†180A OF THE LAST CASE\*,/,\*...TYPE Y FOR YES,  
 4950†180B N FOR NO\*)  
 4955†200 FORMAT(/,\*DO YOU WISH TO USE THE MARGINAL TAX OF  
 4960†200A THE LAST CASE\*,/,\*...TYPE Y FOR YES, N FOR NO\*)  
 4970 END

TABLE B2-2

## SUBROUTINE TO INTEGRATE IN CLOSED FORM POLYNOMIALS

```
5000 SUBROUTINE POLINT (MDEG,SL,SU,C,ANS)
5020*
5040* THIS PROGRAM INTEGRATES POLYNOMIALS OF UP TO DEGREE 30
5060* BETWEEN AN INPUTED LOWER AND UPPER LIMIT.
5080*
5100* INITIALIZE
5120*
5130 DIMENSION C(31)
5140 TEMP=0.
5160 TEMP1=0.
5180*
5200* INTEGRATE
5220*
5240 DO 1 K = 1,MDEG+1
5250 XK = K
5260 TEMP = TEMP + (C(K)*SU**K*(1./XK))
5270 TEMP1 = TEMP1 + (C(K)*SL**K*(1./XK))
5280+1 CONTINUE
5300 ANS = TEMP - TEMP1
5320 RETURN
5340 END
```



TABLE B2-3

## SUBROUTINE TO MULTIPLY TWO POLYNOMIALS

```
5500 SUBROUTINE POLMUL (MDEGA,A,MDEGB,B,MDEGC,C)
5510*
5520* THIS PROGRAM COMPUTES THE PRODUCT OF TWO POLYNOMIALS
5530*
5540* INITIALIZE THE COEFFICIENTS TO ZERO AND DETERMINE
5550* THE DEGREE OF THE PRODUCT POLYNOMIAL.
5560*
5570 DIMENSION A(16),B(16),C(31)
5580 MDEGC = MDEGA + MDEGB
5590 DO 1 I=1,MDEGC+1
5600 C(I)=0.0
5610+1 CONTINUE
5620*
5630* COMPUTE THE C COEFFICIENTS.
5640*
5650 DO 2 I=1,MDEGA+1
5660 DO 2 J=1,MDEGB+1
5670 C(I+J-1)=C(I+J-1)+A(I)*B(J)
5677+2 CONTINUE
5680 RETURN
5690 END
5700 ENDPROG
```

B3-POLYNOMIAL INTEGRATION ALGORITHM

Closed form integration is represented by

$$y = \int_L^U f(x) dx \quad (\text{B3-1})$$

for this dissertation  $f(x)$  is a polynomial of the form

$$f(x) = a_1 + a_2x + a_3x^2 + \dots + a_{n+1}x^n \quad (\text{B3-2})$$

where  $a_i, i=1, \dots, n+1$  are constant coefficients and  $n$  is the degree of the polynomial.

Once  $f(x)$  is defined as a polynomial one has the following

$$y = \int_L^U (a_1 + a_2x + a_3x^2 + \dots + a_{n+1}x^n) dx \quad (\text{B3-3})$$

By the rules of Riemann integration in closed form the solution is

$$y = a_1x + \frac{a_2x^2}{2} + \frac{a_3x^3}{3} + \dots + \frac{a_{n+1}x^{n+1}}{n+1} \Bigg]_L^U \quad (\text{B3-4})$$

This is rewritten as

$$y = a_1U + \frac{a_2U^2}{2} + \frac{a_3U^3}{3} + \dots + \frac{a_{n+1}U^{n+1}}{n+1}$$

$$- (a_1 L + \frac{a_2 L^2}{2} + \frac{a_3 L^3}{3} + \dots + \frac{a_{n+1} L^{n+1}}{n+1})$$

(B3-5)

The generation of an algorithm to perform polynomial integration on a digital computer is simply the task of writing equation (B3-5) in summation notation

$$y = \sum_{i=1}^{n+1} \frac{a_i}{i} x^i \quad (\text{B3-6})$$

and then programming

$$y = \sum_{i=1}^{n+1} \frac{a_i}{i} U^i - \sum_{i=1}^{n+1} \frac{a_i}{i} L^i \quad (\text{B3-7})$$

B4-POLYNOMIAL MULTIPLICATION ALGORITHM

Given two polynomials of varying degree and constant coefficients, the multiplication of these polynomials is represented by

$$\begin{aligned}
 & a_1 + a_2x + a_3x^2 + \dots + a_{n+1}x^n \\
 \times & \underline{b_1 + b_2x + b_3x^2 + \dots + b_{m+1}x^m} \\
 = & c_1 + c_2x + c_3x^2 + \dots + c_{n+m+1}x^{n+m} \qquad (B4-1)
 \end{aligned}$$

where  $n$  and  $m$  are the degrees of the polynomials and  $a_i, i=1, \dots, n$  and  $b_j, j=1, \dots, m$  are the respective constant coefficients.

It is immediately obvious that the degree of the resulting polynomial is simply the summation of the degrees of the two polynomials to be multiplied. This follows from the laws of exponential multiplication.

The remaining problem is the determination of the  $c_k, k=1, \dots, m+n+1$  coefficients. The algorithm used in POLMUL, the polynomial multiplication subroutine of this dissertation is:

$$c_k = \sum a_i b_j \qquad (B4-2)$$

for all  $i, j$  such that  $k = i+j-1$ .



APPENDIX C  
THE DATA TABLES USED TO GENERATE THE  
GRAPHS OF CHAPTER III

TABLE C-1

PRIMARY FAMILIES AND INDIVIDUALS BY TOTAL MONEY INCOME  
IN 1969

TOTAL MONEY INCOME	ALL RACES			
	PRIMARY FAMILIES AND INDIVIDUALS			SECON- DARY INDI- VIDUALS
	TOTAL	PRIMARY FAMILIES	PRIMARY INDI- VIDUALS	
NUMBER....THOUSANDS...	62 875	51 110	11 765	2 687
PERCENT.....	100.0	100.0	100.0	100.0
Under \$1,000.....	3.4	1.5	11.6	23.0
\$1,000 To \$1,499.....	3.6	1.3	13.6	9.1
\$1,500 To \$1,999.....	3.5	1.8	10.8	9.6
\$2,000 To \$2,499.....	3.4	2.4	8.0	7.7
\$2,500 To \$2,999.....	2.9	2.2	5.7	6.3
\$3,000 To \$3,499.....	3.1	2.6	5.3	4.1
\$3,500 To \$3,999.....	3.2	2.7	5.2	4.4
\$4,000 To \$4,999.....	5.8	5.4	7.6	8.3
\$5,000 To \$5,999.....	6.0	5.9	6.5	7.5
\$6,000 To \$6,999.....	6.4	6.4	6.5	4.3
\$7,000 To \$7,999.....	6.8	7.3	4.9	5.5
\$8,000 To \$8,999.....	6.7	7.4	3.6	2.8
\$9,000 To \$9,999.....	6.1	7.0	2.3	1.4
\$10,000 To \$11,999....	11.2	13.0	3.3	2.3
\$12,000 To \$14,999....	11.6	13.7	2.6	2.2
\$15,000 To \$24,999....	13.1	15.7	2.0	1.1
\$25,000 To \$49,999....	2.8	3.3	0.5	0.3
\$50,000 and over.....	0.4	0.4	0.1	0

TABLE C-2

## THE INCOME FREQUENCY DISTRIBUTION OF TABLE A5-1

INCOME	PERCENT OF TOTAL	RANGE POPULATION	PER DOLLAR POPULATION WITHIN INCOME RANGES	MID RANGES
UNDER 1,000.....	3.4	2,137,750	2,138	500
1,000 To 1,499.....	3.6	2,263,500	4,527	1,250
1,500 To 1,999.....	3.5	2,200,625	4,401	1,750
2,000 To 2,499.....	3.4	2,137,750	4,276	2,250
2,500 To 2,999.....	2.9	1,823,375	3,647	2,750
3,000 To 3,499.....	3.1	1,949,125	3,898	3,250
3,500 To 3,999.....	3.2	2,012,000	4,024	3,750
4,000 To 4,999.....	5.8	3,646,750	3,647	4,500
5,000 To 5,999.....	6.0	3,772,500	3,773	5,500
6,000 To 6,999.....	6.4	4,024,000	4,024	6,500
7,000 To 7,999.....	6.8	4,275,500	4,276	7,500
8,000 To 8,999.....	6.7	4,212,625	4,213	8,500
9,000 To 9,999.....	6.1	3,835,375	3,835	9,500
10,000 To 11,999...	11.2	7,042,000	3,521	11,000
12,000 To 14,999...	11.6	7,293,500	2,431	13,500
15,000 To 24,999...	13.1	8,236,625	824	20,000
25,000 To 49,999...	2.8	1,760,500	71	37,500
50,000 and over....	0.4	251,500	10	75,000

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