1-1-1992

Efficiency and frontier analysis with extension to strategic planning.

Catherine S. Lerme
University of Massachusetts Amherst

EFFICIENCY AND FRONTIER ANALYSIS

WITH EXTENSION TO

STRATEGIC PLANNING

A Dissertation Presented

by

CATHERINE S. LERME

Submitted to the Graduate School of the
University of Massachusetts in partial fulfillment
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

September 1992

School of Management

EFFICIENCY AND FRONTIER ANALYSIS WITH EXTENSION TO STRATEGIC PLANNING

A Dissertation Presented

by

CATHERINE S. LERME

Approved as to style and content by:

Agha Iqbal Ali

Agha Iqbal Ali, Chair

Stephen P. Coelen

Stephen P. Coelen, Member

Alan G. Robinson

Alan G. Robinson, Member

Lawrence Seiford

Lawrence Seiford, Member

C. Moore

Craig Moore, Department Head
General Business and Finance

This dissertation is dedicated to the memory of

Fran.

ACKNOWLEDGMENTS

I wish to thank the members of my committee, Iqbal Ali, Steve Coelen, Alan Robinson, and Larry Seiford for their encouragement and patience throughout this lengthy process. In particular Professor Ali earned my everlasting gratitude for his steady confidence in my ability to complete this work.

My deepest thanks go to my friends, Carol Donahue and her parents, Linda Downs-Bembury, Shirley Shmerling, and last but not least, Andrew Bendheim for forging the strongest, warmest, and most appreciated support system I could have hoped for.

I also wish to thank Steve Gengaro for his assistance in preparing the many tables of this Dissertation.

ABSTRACT

EFFICIENCY AND FRONTIER ANALYSIS

WITH EXTENSION TO

STRATEGIC PLANNING

SEPTEMBER 1992

CATHERINE S. LERME, B.S., LYCÉE MICHEL MONTAIGNE

M.S., ECOLE NATIONALE SUPÉRIEURE DE CHIMIE DE PARIS

M.B.A., UNIVERSITY OF MASSACHUSETTS

Ph.D., UNIVERSITY OF MASSACHUSETTS

directed by: Professor Agha Iqbal Ali

Whatever the economic entity, firm, industry, or nation, intensified worldwide competition has increased the need for effective competitive strategies and renders more pressing the need for methods to analyze swelling volumes of information prior to making any decision. A **successful** strategy is the equivalent of an **efficient** production plan, allowing a player to operate on the **frontier** of its feasible achievements. In practice however, such frontiers are not known and have to be estimated empirically.

Locating an empirical frontier is at the core of Data Envelopment Analysis (DEA), a mathematical programming technique developed by Charnes, Cooper et al. in 1978 to evaluate the **relative** performance of decision-making units (DMUs). Several models have since emerged, all aiming at the identification of which of **n** DMUs, each characterized by s outputs and **m**, determine an envelopment surface. **DEA therefore represents a methodological opportunity for the strategy field.**

The viability of DEA rests on its ability to foster sound economic decisions and the economic principles embedded in DEA performance evaluations must be clearly enunciated. The overall purpose of this research is hence twofold:

1) the integration of DEA with production theory via the concepts of efficiency.

2) the formalization of DEA as a tool for strategic planning.

This dissertation develops a new measure of efficiency that is shown to be superior to existing measures in terms of the number of properties it satisfies and also with respect to the economic interpretation it affords. A unifying perspective of DEA models is offered by means of a taxonomy which affords systematic connections between the various models and production theory, hence providing a consistent interpretation of all models and their limitations. A new model, called the Frontier model, is developed which strengthens the bridge between DEA and economics and addresses the measurement of economic efficiency. All developments are supported by numerical illustrations. Finally a new model, the Comparative Advantage model, is developed that adapts the methodology of DEA to identify a DMU's competitors and derive information regarding the DMU's comparative strengths and weaknesses to assist the unit in formulating its strategy. An application to regional economics using Census of Manufactures data is presented.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	v
ABSTRACT	vi
LIST OF TABLES	xi
LIST OF FIGURES	xvi
Chapter	
1 INTRODUCTION	1
1.1 Introduction	1
1.2 Integration of DEA with Production Theory	3
1.3 Extension to Strategic Planning	9
1.4 Organization of the Study	10
2 EFFICIENCY: CONCEPTS AND MEASURES	13
2.1 Introduction	13
2.2 Definition of the Production Correspondence	14
2.3 Technical Efficiency	17
2.4 Allocative Efficiency	20
2.5 Scale Efficiency	21
2.6 Measuring Technical Efficiency in the Input Space	23
2.6.1 Introduction	23
2.6.2 Desirable Properties of an Input-Oriented Measure of Technical Efficiency	26
2.6.3 Existing Input-Oriented Measures of Technical Efficiency	28
2.6.3.1 The Debreu-Farrell Measure	28
2.6.3.2 The Färe-Lovell Measure	29
2.6.3.3 The Zieschang Measure	32
2.6.4 An Admissible Input-Oriented Efficiency Measure	36
2.7 Measuring Technical Efficiency in the Output Space	43
2.7.1 Introduction	43
2.7.2 Desirable Properties of an Output-Oriented Measure of Technical Efficiency	43
2.7.3 An Admissible Output-Oriented Measure of Technical Efficiency	44

2.8 Measuring Global Technical Efficiency 46
2.9 Measuring Allocative Efficiency 49
2.10 Summary and Conclusions 51

3 MEASURING INEFFICIENCY IN PRACTICE
DATA ENVELOPMENT ANALYSIS 64

3.1 Introduction and DEA Terminology 64
3.2 Forms of Data Envelopment 66
3.3 Principles of Evaluation 70

3.3.1 Introduction 70
3.3.2 Orientation of the Evaluation 72
3.3.3 Aggregation across Non-Commensurable Measures 73

3.3.3.1 Introduction 73
3.3.3.2 Explicit Pricing 74
3.3.3.3 Constrained Implicit Pricing 79

3.4 Interpretation of Existing Models 80

3.4.1 Introduction 80
3.4.2 The CCR Model 81
3.4.3 The BCC Model 87
3.4.4 The ADD Model 91

3.5 From Envelopment to Frontier Analysis 93

3.5.1 Introduction 93
3.5.2 The Frontier Model 94
3.5.3 Computation of the Efficiency Score 108

3.6 Computational Illustration 109
3.7 Conclusions 115

4 EXTENSION TO STRATEGIC PLANNING 135

4.1 Introduction 135
4.2 Comparative Advantage and Strategies 136
4.3 Maximization of Comparative Advantage 144
4.4 Computational Illustration 151
4.5 Conclusion 153

5 DETERMINATION OF COMPARATIVE ADVANTAGE FOR THE
ECONOMY OF STATES IN THE U.S.A. 163

5.1 Introduction 163

5.2 Methodology 166
5.3 Proposed Analyses 171

5.3.1 Overall Economic Performance 172
5.3.2 Efficiency of an Industry Across States 176

5.4 Interpretation of Results 179

5.4.1 Overall Economic Performance of States 180
5.4.2 Efficiency of an Industry Across States 190

5.5 Conclusions 194

6 CONCLUSIONS 233

APPENDICES

A. MONOTONICITY OF DEBREU-FARRELL MEASURE OF EFFICIENCY 240
B. STRICT MONOTONICITY OF FÄRE-LOVELL MEASURE OF EFFICIENCY 241
C. THE INVARIANT ADDITIVE DEA MODEL 242
D. THE FRONTIER MODEL 244
E. THE COMPARATIVE ADVANTAGE MODEL 247
F. STANDARD INDUSTRIAL CLASSIFICATION MANUFACTURING DIVISION 250

REFERENCES 252

LIST OF TABLES

Table Page

3.1 Data Set 120

3.2 Evaluation of Inefficient Units VRS Envelopment/ Output Orientation/ Standard Pricing 121

3.3 Evaluation of Inefficient Units VRS Envelopment/ Output Orientation/ Invariant Pricing 122

3.4 Evaluation of Inefficient Units VRS Envelopment/ Output Orientation/ Normed Pricing Normed Prices(P_{x1} = 0.1652, P_{x2} = 0.2708, P_{y1} = 0.7220, P_{y2} = 0.7220) 123

3.5 Evaluation of Inefficient Units VRS Envelopment/ Input Orientation/ Standard Pricing 124

3.6 Evaluation of Inefficient Units VRS Envelopment/ Input Orientation/ Invariant Pricing 125

3.7 Evaluation of Inefficient Units VRS Envelopment/ Input Orientation/ Normed Pricing Normed Prices(P_{x1} = 0.1652, P_{x2} = 0.2708, P_{y1} = 0.7220, P_{y2} = 0.7220) 126

3.8 Evaluation of Inefficient Units VRS Envelopment/ Global Orientation/ Standard Pricing 127

3.9 Evaluation of Inefficient Units VRS Envelopment/ Global Orientation/ Invariant Pricing 128

3.10 Evaluation of Inefficient Units VRS Envelopment/ Global Orientation/ Normed Pricing Normed Prices(P_{x1} = 0.1652, P_{x2} = 0.2708, P_{y1} = 0.7220, P_{y2} = 0.7220) 129

3.11 Effect of Pricing Mechanism 130

3.12 Effect of Excess Inputs or Output Slacks on the Efficiency Score 130

3.13 Effect of the Orientation on the Efficiency Scores 130

3.14 Price Ratio Ranges Across Orientations 131

3.15 Selected Price Ratio Ranges 131

3.16 VRS Envelopment/ Frontier Model: Efficiency Prices 132

3.17 VRS Envelopment/ Frontier Model: Total Waste and Substitutions . . . 133

3.18 VRS Envelopment/ Frontier Model: Efficiency Price Ratios and Efficiency Scores . . . 134

4.1 Data Set . . . 160

4.2 Strategic Tradeoffs and Differences from Competitors . . . 161

4.3 Comparative Advantage Analysis . . . 162

5.1 States 1982: Original Data Set . . . 195

5.2 States 1987: Original Data Set . . . 196

5.3 SIC 367-1982 Data Electronic Components and Accessories . . . 197

5.4 States 1982: Transformed Data Set . . . 198

5.5 States 1987: Transformed Data Set . . . 199

5.6 States Study: Variables Definitions and Units of Measurement . . . 200

5.7 Industry Study: Variables Definitions and Units of Measurement . . . 201

5.8 Efficient States . . . 202

5.9 States 1982: Total Waste VRS Envelopment/ Global Orientation/ Invariant Pricing . . . 203

5.10 States 1987: Total Waste VRS Envelopment/ Global Orientation/ Invariant Pricing . . . 204

5.11 States 1982: Input Price Ratios of Efficient Units VRS Envelopment/ Global Orientation/ Invariant Pricing . . . 205

5.12 States 1987: Input Price Ratios of Efficient Units VRS Envelopment/ Global Orientation/ Invariant Pricing . . . 206

5.13 States 1982: Output Price Ratios of Efficient Units VRS Envelopment/ Global Orientation/ Invariant Pricing . . . 207

5.14 States 1987: Output Price Ratios of Efficient Units VRS Envelopment/ Global Orientation/ Invariant Pricing . . . 208

5.15 States Study: Limit-Price-Ratio Ranges . . . 209

5.16 States 1982: Total Waste and Substitutions VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation . . . 210

5.17 States 1987: Total Waste and Substitutions VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation . . . 211

5.18 States 1982: Efficiency Prices VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 212

5.19 States 1987: Efficiency Prices VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 213

5.20 States 1982: Output Price Ratios VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 214

5.21 States 1987: Output Price Ratios VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 215

5.22 States 1982: Input Price Ratios and Efficiency Scores VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 216

5.23 States 1987: Input Price Ratios and Efficiency Scores VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 217

5.24 States Study: Limit-Value-Ratio Ranges 218

5.25 States 1982: Total Waste and Substitutions VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation . . . 219

5.26 States 1987: Total Waste and Substitutions VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation . . . 220

5.27 States 1982: Efficiency Prices VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 221

5.28 States 1987: Efficiency Prices VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 222

5.29 States 1982: Output Price Ratios VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 223

5.30 States 1987: Output Price Ratios VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 224

5.31 States 1982: Input Price Ratios and Efficiency Scores VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 225

5.32 States 1987: Input Price Ratios and Efficiency Scores VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation 226

5.33 SIC367 Total Waste 1982 Data VRS Envelopment/ Global Orientation/ Invariant Pricing. 227

5.34 Industry Study: Limit-Value-Ratio Ranges 228

5.35 SIC367 Total Waste and Substitution/ Frontier Model Limit-Value-Ratios derived from Limit Value Ratios of DEA-Efficient Units 229

5.36 SIC367 Price Ratios VRS Envelopment/ Frontier Model Limit-Value-Ratios derived from Limit Value Ratios of DEA-Efficient Units 230

5.37 SIC367: Comparative Advantage 231

5.38 SIC367 1982 Data: Comparative Advantage 232

LIST OF FIGURES

Figure		Page
2.1	Input Set L(y)	54
2.2	Output Set P(x)	54
2.3	Weak and Strong Disposability of Inputs	55
2.4	Input Isoquants $Iso(y_1)$, $Iso(y_2)$, $y_2 \geq y_1$	56
2.5	Internal and External Inefficiencies	57
2.6	Example Input Set L(y)	58
2.7	Cone of Admissible Efficiency Prices in Input Space	59
2.8	Measuring Inefficiency	60
2.9	Cone of Admissible Efficiency Prices in Output Space	61
2.10	Components of Global Efficiency	62
2.11	Technical and Allocative Efficiency	63
3.1	Input set L(y) FDH Methodology	117
3.2	Inefficiency Evaluation with Input Substitution	118
3.3	Role of Prices in Inefficiency Measurement	119
4.1	Optimal Input Tradeoffs	155
4.2	Strategic Tradeoffs	156
4.3	Strategic Tradeoffs of a Border Unit	157
4.4	Comparative Advantage in Input Space	158
4.5	Border Unit Substitution	159

CHAPTER 1

INTRODUCTION

> "Like management science, the strategy field lies at the crossroads of a number of more established fields of study. Due to its stage of development, position, and focus, many theoretical and methodological opportunities exist which have yet to be tapped."
>
> [Day et al., 1990, p iii]

1.1 Introduction

Whatever the economic entity, whether it be firm, industry, region, or nation, intensified worldwide competition has increased the need for effective competitive strategies and renders more pressing the need for methods to compile, analyze and synthesize swelling volumes of information prior to making any decision. Borrowing from the concepts of production theory, a **strategy** is the equivalent of a production plan. Both can be seen as a set of decisions which define a planned course of action. Preliminary to the formulation of a competitive strategy are first, the identification of multiple, often conflicting, objectives, and second, an objective assessment of the current relative positioning of all the competing players. A **successful** strategy is the equivalent of an **efficient** production plan, allowing a player to operate on the **frontier** of its feasible achievements. Clearly, if all players can be characterized by measures of achievements of predefined objectives which define their respective production plans, then the successful strategy of a competing player enables the player to get ahead or, at a minimum, to persist at least as well as its competitors. In practice however, such frontiers of feasible achievements are not known. They can only be approximated by the observed best, non-dominated achievements in a Pareto-Koopmans sense if we exclude any a priori ranking of the predefined objectives.

The concept of locating an empirical frontier is at the core of Data Envelopment Analysis (DEA), a mathematical programming technique developed by Charnes, Cooper et al. in 1978 to evaluate the **relative** performance of decision-making units (DMUs). Several models, based on different sets of assumptions, have since emerged, all aiming at the identification of which of **n** DMUs determine an envelopment surface when each DMU is characterized by **s** "output" measures and **m** "input" measures. The identified envelopment surface represents an empirical frontier and the envelope-defining units identify efficient performances. **DEA therefore represents a methodological opportunity for the strategy field**. It offers a convenient and fast way to process empirical data thereby providing useful guidance in decision-making and strategy formulation, and allowing timely responses in fast changing environments. However, prior to the formalization of DEA as a tool for strategic planning, an assessment of the state of the art regarding the mathematics of DEA and its ties to production theory are warranted. Indeed the viability of DEA rests on its ability to foster sound economic decisions and it is crucial that the economic principles embedded in DEA performance evaluations be clearly enunciated. The overall purpose of this research is hence twofold:

1) the integration of Data Envelopment Analysis with production theory.

2) the formalization of Data Envelopment Analysis as a tool for strategic planning.

The remainder of this introductory chapter expands on the motivation and purpose of the research presented in the next four chapters.

1.2 Integration of DEA with Production Theory

Most textbooks for micro and managerial economics introduce production theory by defining a production function as the extremal relationship between the various levels of inputs (typically referred to as factors of production) used and the **maximum** level of output that can be attained. In the more general situation where multiple inputs combine into multiple outputs (typically referred to as joint products), it is customary to refer to the relationship between inputs and outputs as a production correspondence. This relationship is characteristic of the available technology which represents all the known ways of combining a predefined set of inputs to produce a predefined set of outputs. Each such combination is referred to as a technique of production or production process. A new technology corresponds to a modification of the input set and/or a modification of the output set. We assume in what follows that technology remains constant.

Assuming a production unit is characterized by an input vector $\mathbf{x}$, and an output vector, $\mathbf{y}$, the input possibility set of $\mathbf{y}$ and the output possibility set of $\mathbf{x}$ are respectively defined as the set of input vectors that may be substituted for one another to obtain $\mathbf{y}$ ($\mathbf{x}$ belongs to that set), and the set of output vectors attainable from $\mathbf{x}$ ($\mathbf{y}$ belongs to that set). Focusing for now on the engineering aspect of the technology, productive efficiency, also referred to as technical efficiency, is exclusive of value consideration for the inputs and outputs, and is synonymous with Pareto-Koopmans efficiency. It means that no output can be increased without simultaneously raising at least one input and/or decreasing at least another output, or that no input can be decreased without simultaneously decreasing at least one output and/or increasing at least another input. It follows that technically efficient production units define/map on the frontier of the production possibility sets. That is, a production unit (**x**,**y**) is technically efficient and defines the frontier of the production possibility set if it is **both** input efficient (i.e. **x** belongs to the frontier of the input possibility set of **y**) and output efficient (i.e. **y** belongs to the frontier of the output possibility set of **x**).

Assuming that the production function is known and well-defined by a single continuous function with continuous first and second order partial derivatives, production theory is then concerned with setting guidelines regarding the choice and levels of input utilization or the quantity of outputs to generate so that the producer may optimize some predefined objective. Profit maximization and cost minimization are the two most widely studied optimizing behaviors for the producer, and are generally studied within a specific institutional framework, typically a perfectly competitive environment. This particular choice of institutional framework offers the advantage that prices for the inputs and outputs can be considered fixed and given throughout the analyses. Within the context of prices for the various inputs and outputs productive efficiency is referred to as **allocative efficiency**. Allocative efficiency is tied to a valuation of the various inputs and outputs. It characterizes production units which exhibit an optimal behavior (cost minimization or profit maximization). It should be apparent that technical efficiency is a prerequisite to allocative efficiency.

It is important to emphasize that the guidelines set by production theory rest on the assumption that the production function is known. For instance, according to these guidelines, a producer wishing to minimize the firm's costs by controlling the consumption of its inputs needs to evaluate, from "his/her" production function, the marginal product of the various inputs. His only recourse is to rely on **estimated** production functions.

The empirical approach to production analysis is traditionally equated with the estimation of production functions and their associated cost functions. The primary technique employed is regression analysis whose limitations have often been publicized in the DEA literature and are summarized next.

The main argument against regression analysis is that it requires the prespecification of a functional form which typically relates a single dependent variable (i.e. a single output) to multiple independent variables (i.e. inputs). Multiple forms, for instance a Cobb-Douglas, translog, or constant elasticity of substitution function, are usually tested and the one that best fits the data is selected. However, the implicit and explicit assumptions underlying the function and their ensuing implications regarding relations among the estimated parameters are not simultaneously compatible with the empirical data [Drhymes, 1990]. The technique also assumes that all empirical data points are representative of technically efficient operations, and the deviations from the estimated average relation are attributed to statistical discrepancies. This assumption is not acceptable if one's purpose is precisely to distinguish betwen efficient and inefficient behaviors. To this end DEA offers an alternative approach to production frontier determination which offers greater flexibility than regression analysis in terms of less required assumptions.

The relation of DEA to economics is anchored in production theory particularly via the works of Shephard [1953, 1970] and Farrell [1957] concerning productive efficiency. Shephard, in the DEA literature, is traditionally credited with the introduction of a theoretical function whose purpose is to gauge the parsimony of an input vector in producing a corresponding output vector. Farrell's noted contribution, though limited to the single output case, is his attempt at measuring efficiency directly from observational data.

Production possibility sets (i.e. input possibility sets and output possibility sets) and hence their frontiers are seldom known a priori, underscoring the relevance of Farrell's efforts to determine them empirically. Instead of specifying a functional form for the production correspondence, thereby assuming existence and uniqueness of that function, the empirical approach of DEA is to assume properties for the production correspondence and estimate, from the available observations, the possibility sets that are consistent with these properties. Existence of a production function is then equivalent to consistency of the properties and uniqueness is equivalent to completeness of the set of properties.

Assuming for now a consistent and complete set of properties has been defined, Data Envelopment Analysis allows the determination of an empirical frontier enveloping a set of data points. The empirical frontier is then used to evaluate the relative efficiency of production units or decision-making units (DMUs). There are a variety of mathematical models which effect data envelopment. These models for DEA differ in their estimation of the empirical frontier (they assume different properties for the production correspondence) and/or their evaluation of inefficient DMUs. However, using any DEA model, the behavior, or performance, of all DMUs operating on the identified frontier is deemed satisfactory and earns those DMUs a status of efficiency. All DMUs operating beneath the frontier are labelled inefficient. If one is only interested in determining whether a production unit is technically efficient or not, then it suffices to check that no input can be reduced and still allow the production of the given output vector <u>and</u> that, given the available resources, more of any output could not have been obtained. However, if one is interested in determining the degree of technical inefficiency of a production unit, as summarized by a scalar index, then one is confronted with the problem of measurement of inefficiency, or amount of waste. Such a concern is highly relevant to provide, for instance, a criterion on which to base resource allocation, or to optimize the expected benefits of limited auditing resources by focusing on the identified inefficient production units, or simply to gauge the relative handicap of production units.

The first task is hence to define a measure of efficiency that captures its economic underpinnings. The measure should allow a meaningful ranking of DMUs that reflect their relative achievement with respect to behaviors prescribed by production theory. Essentially the measurement of inefficiency, which equates with the measurement of waste, involves aggregating quantities measured in different units, namely excess inputs and slack outputs. Defining a coherent method of aggregation across inputs and outputs is therefore central to the measurement of technical inefficiency.

An inefficient DMU lies in the interior of a production possibility set and the sources and extent of inefficiencies for such a DMU are identified by comparing the DMU's performance to that of an efficient DMU. Measuring the unit's degree of (in)efficiency hence necessitates the selection of a reference point on the frontier. Obviously infinitely many such reference points are feasible candidates. However, reasonable criteria can be introduced that will reduce the set of selectable referent points. Mathematically, these criteria result in a projection mechanism that, for each inefficient DMU, identifies a referent point against which the DMU's inefficiency is to be gauged. The DMU's shortcomings, which are the roots of its inefficiency, are revealed in the form of excesses in input consumption and deficiencies in output production. These revealed shortcomings can then constitute the core of any remedial plan of actions. It follows that each possible projection on the envelope can be seen as an optimal response to distinct managerial objectives. This projection mechanism effects the evaluation of the relative performance of the DMUs and is the embodiment of economic principles. These principles may focus primarily on controlling input consumption or primarily on output production, or on avoiding waste without distinction between inputs and outputs, they may be defined within a short-term horizon or within a long-term horizon.

The second task is hence the identification of the specific and inherent economic principles underlying the evaluations effected by existing models for DEA. It is essential to enunciate these principles since different principles, coupled with various sets of properties regarding the production function, define different projections, leading to the selection of different referent points and also to different efficiency scores. The knowledge and acceptance of the underlying principles and assumptions regarding the production correspondence are necessary to convince the DMUs to implement the recommended changes contained in the identified referent unit. These principles and assumptions then become the criteria for selecting a projection procedure and its associated efficiency assessments. Such knowledge also contributes to the unification and a better understanding of the theory of Data Envelopment Analysis, hence preventing its misuse, and fostering its reach to new areas of applications. Last but not least, it adds flexibility to the methodology of DEA and fosters the development of new models that can be tailored to reflect specific concerns of distinct economic environments. In particular concerns about allocative efficiency can be addressed by acknowledging the role of prices in performance evaluations and ensuring that realistic economic tradeoffs across inputs and across outputs are properly reflected. This concern is relevant to bring efficiency evaluations closer to economic efficiency ratings which should warrant sounder recommendations to inefficient operating units. Satisfying this need will strengthen the bridge between DEA and economics.

1.3 Extension to Strategic Planning

The methodology of Data Envelopment Analysis is by no means restricted to production processes. If the notion of input is generalized to encompass any measure such that the lower its level the better, and the notion of output is similarly generalized to encompass measures such that the higher their level the better, then DEA extends naturally to strategic planning applications. The measures correspond to objectives and their level can either reflect past or projected achievements or competitors'achievements. In this case however, no direct causal relationship can be assumed to exist between inputs and outputs, that is the inputs are not instrumental in "producing" outputs. By analogy with a production situation, a successful strategy allows the decision-making unit to position itself on the empirically determined frontier of feasible achievements. Such units therefore demonstrate, by being identified as efficient, that they enjoy some comparative advantage, relative to all other units, from which their efficiency derives. However, since it is unlikely that the frontier of feasible achievements remain static, these efficient units must engage in constant analysis and active planning if they are to sustain their efficiency. It is therefore important for an efficient unit to identify who its current competitors are that may threaten its efficiency. It is equally important to identify the sources of the unit's comparative advantage and the extent of its comparative advantage, that is whether it represents a comfortable cushion against competitors attacks or a precarious condition warranting undivided managerial attention. Finally the assessment of its current competitive environment should allow the efficient unit to define its future strategy in terms of choice of input mix, choice of output mix, and level of operations.

1.4 Organization of the Study

Chapter 2 takes an indepth look at concepts of efficiency and is concerned with measurement of efficiency. In particular the issue of existence of a measure of efficiency satisfying desirable properties is addressed by, first, reviewing existing measures and their relative merits with regard to these desirable properties, and, second, by developing a new measure that is shown to be superior to these existing measures in terms of the number of properties it satisfies and also with respect to the economic interpretation it affords.

Chapter 3 deals with the methodological aspects of efficiency measurement and focuses on Data Envelopement Analysis as a viable methodology to assess the relative efficiency of decision-making units. In particular Chapter 3 offers a unifying perspective of DEA models. It develops a taxonomy which affords systematic connections between the various models and production theory, hence providing a consistent interpretation of all models along with their limitations within the context of production theory. Finally, a new model, called the Frontier model, is developed which strengthens the bridge between DEA and economics by correctly determining and measuring economic efficiency. All theoretical developments are supported by numerical illustrations.

Chapter 4 builds on the results of Chapter 3 and formalizes the application of Data Envelopment Analysis to strategic planning. A new mathematical model, called the Comparative Advantage model, is developed that adapts the methodology of data envelopment to identify a DMU's most direct competitors and, hence, derive information regarding the DMU's comparative strengths and weaknesses. The assessment of its current competitive environment represents critical information to assist the unit in formulating its strategy.

An application to regional economics is presented in Chapter 5. It first evaluates the efficiency of the economy of states in the U.S. It then identifies, for a particular industry, which states are the leaders and what their comparative advantages are. Specific recommendations are formulated to improve the competitive position of those states found to be inefficient.

Chapter 6 summarizes all results and suggests directions to further the research.

CHAPTER 2

EFFICIENCY: CONCEPTS AND MEASURES

2.1 Introduction

The purpose of this chapter is to review and distinguish between the various types of efficiency: namely technical or productive efficiency, allocative efficiency, and scale efficiency. All discussions are with reference to a production correspondence described in Section 2. Technical efficiency is addressed in Section 3 within a Pareto-Koopmans framework. Allocative efficiency is introduced in Section 4 and scale efficiency is introduced in Section 5. In Section 6 desirable properties of an efficiency measure are identified. The section focuses first on input-oriented technical efficiency with inputs as the control variables. Existing measures are presented and their relative merits with regard to these desirable properties are discussed. A new measure of efficiency is introduced and shown to be superior to existing measures in terms of the number of properties it satisfies and also with respect to the economic interpretation it affords. The results are extended to output-oriented technical efficiency in Section 7, then to global technical efficiency in Section 8. Section 9 introduces a measure of allocative efficiency that is consistent with the proposed measure of technical efficiency. A summary and concluding remarks are presented in Section 10.

2.2 Definition of the Production Correspondence

We consider the situation of a production correspondence where a vector of outputs $\mathbf{y} \in \mathbb{R}^s_+$ is produced **jointly** from a vector of essential[1] inputs $\mathbf{x} \in \mathbb{R}^m_+$. All inputs are considered variable. Even those that, in economic terms, are thought of as fixed, can be thought of as variable by assuming that they are leased for infinitely divisible amounts, implying that input values equate input usage. Using standard notations we let $P(\mathbf{x})$ denote the output set of $\mathbf{x}$, i.e. the set of output vectors attainable from $\mathbf{x}$, and $L(\mathbf{y})$ denote the input set of $\mathbf{y}$, i.e. the set of input vectors $\mathbf{x}$ that may be substituted for one another to obtain $\mathbf{y}$. Typically [Russell, 1985] the input correspondence is assumed to satisfy the following set of properties[2]:

(IC1) $L(0^s) = \mathbb{R}^m_+$.

(IC2) If $\mathbf{y} \geq 0^s$, then $0^m \notin L(\mathbf{y})$.

(IC3) If $\mathbf{x} \in L(\mathbf{y})$, then $\delta\mathbf{x} \in L(\mathbf{y})$, $\forall\ \delta \in [1,\infty)$, $\forall\ \mathbf{y} \in \mathbb{R}^s_+$.

(IC4) $L(\mathbf{y})$ is closed, $\forall\ \mathbf{y} \in \mathbb{R}^s_+$.

The first property (IC1) states that zero output can always result from any input. (IC2), on the other hand, states that something cannot be obtained from nothing. (IC3) is a statement of weak disposability of the inputs, i.e. if an input vector is known to produce the output vector $\mathbf{y}$, then any input vector which is homothetically larger is assumed to produce at least $\mathbf{y}$, and, hence, to belong to the input set of $\mathbf{y}$. (IC4) states

[1] Input i is essential iff $\mathbf{y} > 0 \Rightarrow x_i > 0$

[2] $\mathbf{u}, \mathbf{v} \in \mathbb{R}^t$,
$\mathbf{u} > \mathbf{v}$ iff $u_i > v_i$ $i = 1,\ldots,t$
$\mathbf{u} \geq \mathbf{v}$ iff $u_i \geq v_i$ $i = 1,\ldots,t$ and $\mathbf{u} \neq \mathbf{v}$
$\mathbf{u} \geqq \mathbf{v}$ iff $u_i \geq v_i$ $i = 1,\ldots,t$

that the boundary of the input set is part of the input set and, in particular, that finite minimum amounts of inputs are necessary to produce **y**. Figure 2.1 on page 54 at the end of the chapter illustrates the situation of a two-input space where the shaded area corresponds to the input set L(**y**).

The output correspondence is the inverse correspondence of the input correspondence and, hence, can be assumed to satisfy a similar set of properties:

(OC1) $P(0^m) = 0^s$.

(OC2) $P(\mathbf{x})$ is bounded and closed, $\forall\ \mathbf{x} \in \mathbb{R}^m_+$.

(OC3) If $\mathbf{y} \in P(\mathbf{x})$ then $\theta\mathbf{y} \in P(\mathbf{x})$ for $\theta \in [0,1]$.

(OC4) $P(\lambda\mathbf{x}) \supseteq P(\mathbf{x})$ for $\lambda \geq 1$.

The first property states that only the null output can result from zero inputs. (OC2) states that only finite amounts of outputs can result from the combination of finite amounts of inputs. (OC3) states weak disposability of outputs and (OC4) states that the output set of any input vector is contained in the output set of any input vector that is homothetically larger. This property implies weak disposability of inputs. Figure 2.2 on page 54 illustrates the situation of a two-output space where the shaded area corresponds to the output set P(**x**).

These properties are very general and serve to describe a very broad class of technology [Russell, 1985]. Two additional properties which are often encountered in production studies deal respectively with the convexity of the production possibility sets and the homotheticity of the production correspondence. Their statement, labelled respectively "Convexity" and "Ray Unboundedness" is given below.

(Convexity) If $(\mathbf{x}_j, \mathbf{y}_j)$ $j = 1,\ldots,n$ are such that $\mathbf{x}_j \in L(\mathbf{y}_j)$ and $\mathbf{y}_j \in P(\mathbf{x}_j)$, and (α_j) $j = 1,\ldots,n$ are positive scalars such that $\Sigma_j \alpha_j = 1$, then $\Sigma_j \alpha_j\mathbf{x}_j \in L(\Sigma_j \alpha_j\mathbf{y}_j)$ and $\Sigma_j \alpha_j\mathbf{y}_j \in P(\Sigma_j \alpha_j\mathbf{x}_j)$.

(Ray Unboundedness) If $\mathbf{x} \in L(\mathbf{y})$ and $\mathbf{y} \in P(\mathbf{x})$ then $k\mathbf{x} \in L(k\mathbf{y})$ and $k\mathbf{y} \in P(k\mathbf{x})$ for any $k > 0$.

If we consider that economic data on consumption of inputs and production of outputs are in fact the representation of average **rates** of consumption and production over time, then it seems reasonable to consider that any convex combinations of observed production plans described by $(\mathbf{x}, \mathbf{y})$ is also an achievable production plan and therefore rightfully belongs to the production possibility set. Each observation is representative of a production facility dedicated to a technique so that its operations are characterized by steady state rates of production given its scale of operations. A convex combination of observations can then be interpreted as a time-sharing leasing agreement where the production facilities entering the combination are successively leased for subperiods of time over the period of measurement. This convexity property for the production possibility sets is hereafter assumed to hold. The second additional property deals with the global homotheticity (homogeneity of degree 1) of the production correspondence by assuming that any magnification or contraction of an observed production plan $(\mathbf{x}, \mathbf{y})$ is also a feasible production plan. This latter property is more restrictive regarding the class of technologies to which it applies to and will not be assumed to hold unless otherwise stated.

The input isoquant Iso(y) and output isoquant Iso(x) are defined as:

$$\mathrm{Iso}(\mathbf{y}) = \{\mathbf{x} \in \mathbb{R}^{m}{}_{+}\colon \mathbf{x} \in L(\mathbf{y});\ \theta\mathbf{x} \notin L(\mathbf{y}),\ \forall\ \theta \in [0,1)\}$$

$$\mathrm{Iso}(\mathbf{x}) = \{\mathbf{y} \in \mathbb{R}^{s}{}_{+}\colon \mathbf{y} \in P(\mathbf{x});\ \phi\mathbf{y} \notin P(\mathbf{x}),\ \forall\ \phi \in [1,\infty)\}$$

The input isoquant Iso(y) and output isoquant Iso(x) respectively represent the lower boundary of the input set L(y) and the upper boundary of the output set P(x) as illustrated on page 54. Having defined L(y), P(x), Iso(y), and Iso(x) we are now ready to introduce what an efficient output subset of P(x) is and, similarly, what an efficient input subset of L(y) is. Each of these subsets is a direct analytical interpretation of the Pareto-Koopmans concept of technical efficiency which is presented in the next section.

2.3 Technical Efficiency

Technical efficiency, also referred to as productive efficiency, is exclusive of value consideration for the inputs and outputs, and is synonymous with Pareto-Koopmans efficiency. A production unit characterized by (x, y) is deemed efficient in the Pareto-Koopmans sense if and only if producing more of any of the outputs necessitates more of at least one input <u>and</u> a reduction of the consumption of any input would necessarily lead to the reduction of at least one output. Analytically, the efficient subsets are defined as:

$$\mathrm{Eff}(\mathbf{y}) = \{\mathbf{x} \in \mathbb{R}^{m}{}_{+}\colon \mathbf{x} \in L(\mathbf{y});\ \mathbf{x}' \leq \mathbf{x} \Rightarrow \mathbf{x}' \notin L(\mathbf{y})\}$$

$$\mathrm{Eff}(\mathbf{x}) = \{\mathbf{y} \in \mathbb{R}^{s}{}_{+}\colon \mathbf{y} \in P(\mathbf{x});\ \mathbf{y}' \geq \mathbf{y} \Rightarrow \mathbf{y}' \notin P(\mathbf{x})\}$$

The set Eff($\mathbf{y}$) is the set of minimal input vectors that can be substituted for one another to obtain $\mathbf{y}$, and Eff($\mathbf{x}$) is the set of maximal output vectors attainable from $\mathbf{x}$. The relationship between the input set L($\mathbf{y}$) and its isoquant and efficient subset is given by:

$$\mathrm{Eff}(\mathbf{y}) \subseteq \mathrm{Iso}(\mathbf{y}) \subseteq L(\mathbf{y})$$

Similarly we have the relationship:

$$\mathrm{Eff}(\mathbf{x}) \subseteq \mathrm{Iso}(\mathbf{x}) \subseteq P(\mathbf{x})$$

A production unit ($\mathbf{x}$, $\mathbf{y}$) is Pareto-Koopmans efficient if and only if $\mathbf{y} \in \mathrm{Eff}(\mathbf{x})$ and $\mathbf{x} \in \mathrm{Eff}(\mathbf{y})$.

Clearly technical efficiency is geared toward the avoidance of waste. The notion of waste is related to the concept of strong disposability in economics which states that excess levels in resources can be disposed of independently of one another and that, similarly, slacks in products can be eliminated independently of one another. This concept is different from the concept of weak disposability which only considers radial or equiproportionate changes in inputs and outputs. These concepts are illustrated in Figure 2.3 on page 55. If the set of piecewise linear segments ABCD is known to belong to L($\mathbf{y}$), then the assumption of weak disposability of inputs implies that all input vectors in the shaded area delimited by $A^{\propto}ABCDD^{\propto}$ also belong to L($\mathbf{y}$). The assumption of strong disposability of inputs implies that all input vectors in the shaded area delimited by $A^{\|}ABCDD^{\|}$ also belong to L($\mathbf{y}$).

It is not always true that $\mathbf{x} \in \mathrm{Eff}(\mathbf{y}) \Leftrightarrow \mathbf{y} \in \mathrm{Eff}(\mathbf{x})$. This fact is illustrated in Figure 2.4 on page 56. In this example $\mathbf{x}_5 \in \mathrm{Eff}(\mathbf{y}_1)$ and $\mathbf{x}_5 \in \mathrm{Eff}(\mathbf{y}_2)$ with $\mathbf{y}_2 \geq \mathbf{y}_1$, it follows: $\mathbf{y}_1 \notin \mathrm{Eff}(\mathbf{x}_5)$ since $\mathbf{y}_2 \geq \mathbf{y}_1$ and $\mathbf{y}_2 \in P(\mathbf{x}_5)$. The relation is true under particular conditions. Färe [1983] showed that if inputs and outputs are strongly disposable then $\mathbf{x} \in \text{Eff}(\mathbf{y}) \Rightarrow \mathbf{y} \in \text{Eff}(\mathbf{x})$ if and only if the input efficiency sets are strictly monotonic with respect to inclusion, i.e. ordered output vectors lead to disjoint efficiency sets. Mathematically this condition translates into:

$$\mathbf{y}_2 \geq \mathbf{y}_1 \geq \mathbf{0} \Rightarrow \text{Eff}(\mathbf{y}_2) \cap \text{Eff}(\mathbf{y}_1) = \varnothing$$

Similarly the converse relationship, ($\mathbf{y} \in \text{Eff}(\mathbf{x}) \Rightarrow \mathbf{x} \in \text{Eff}(\mathbf{y})$), holds if and only if ordered input vectors lead to disjoint efficiency sets: ($\mathbf{x}_2 \geq \mathbf{x}_1 \geq \mathbf{0} \Rightarrow \text{Eff}(\mathbf{x}_2) \cap \text{Eff}(\mathbf{x}_1) = \varnothing$). We will hereafter assume that inputs and outputs are strongly disposable[3] but we do <u>not</u> assume that ordered input or output vectors lead to disjoint efficiency sets.

Following the Pareto-Koopmans definition of efficiency, the technical efficiency of a production unit $(\mathbf{y},\mathbf{x})$, can be decomposed into two components:

(i) the technical Input efficiency which is concerned with the membership of $\mathbf{x}$ in $\text{Eff}(\mathbf{y})$.

(ii) the technical Output efficiency which is concerned with the membership of $\mathbf{y}$ in $\text{Eff}(\mathbf{x})$.

A production unit is technically efficient if it is both input efficient and output efficient. If one is only interested in determining whether a production unit is technically efficient or not, then it suffices to check that no input can be reduced and still allow the production of the given output vector <u>and</u> that, given the available resources, more of any output could not have been obtained. However, if one is interested in determining the degree of technical inefficiency of a production unit, as summarized by a scalar index,

---

[3]One could argue that strong disposability of inputs is implied by (IC1).

then one is confronted with the problem of measurement of inefficiency, or amount of waste. Such a concern is highly relevant in selecting, for instance, a criterion on which to base resource allocation or in optimizing the expected benefits of limited auditing resources by targeting the most inefficient production units. Essentially the measurement of inefficiency involves aggregating quantities measured in different units, namely excess inputs and slack outputs. Defining a coherent method of aggregation across inputs and outputs is therefore central to the measurement of technical inefficiency. Market prices naturally come to mind as a convenient common denominator for all inputs and outputs which leads us to the concept of allocative efficiency presented in the next section.

2.4 Allocative Efficiency

Allocative efficiency is tied to a valuation of the various inputs and outputs. It simultaneously requires that the output mix be produced at minimal cost <u>and</u> that, given the available input mix, a most valuable output bundle be produced. It follows that technical efficiency is prerequisite to allocative efficiency and that the measurement of allocative efficiency rests on the availability of prices. Some developments e.g.[Banker and Maindiratta, 1988] consider the existence of a unique set of input and output prices thereby assuming a situation of a unique perfectly competitive market. However, if the operating units are spatially separated, we know that transportation and transaction costs allow for distinct local prices for the input and outputs. It follows that multiple input and output mixes may be found allocatively efficient. Furthermore if market prices are allowed to fluctuate to reflect changing supply and demand conditions, then it becomes reasonable to consider **ranges** of prices. Allocative efficiency is then contingent upon the existence of a set of prices within such acceptable ranges.

If the inputs and outputs are not traded, i.e. no market pricing occurs, then the concept of allocative efficiency can be extended to substitute relative societal values of the outputs and resource opportunity costs of the inputs for, respectively, the output and input prices. In this case, as well, spatially separated operations may reflect distinct environments and, hence, distinct relative valuations. The estimation of these distincts valuations, and, eventually, of acceptable **ranges** for these relative valuations presents an added difficulty for measuring allocative efficiency.

2.5 Scale Efficiency

The concepts of scale efficiency are intermediate between technical and allocative efficiencies. Indeed production theory offers two of definitions in terms of input quantities only or in terms of costs necessitating the knowledge of market prices.

When dealing exclusively with physical units, the scale of operations may be measured by either the volume of outputs or by the level of consumption of inputs, **assuming the transformation process is technically efficient**. An obvious difficulty arises when multiple outputs are produced and multiple inputs are consumed: How do we compare/define the scale of different mixes? Within this context efficiency is related to productivity. Banker [1984] introduced the concept of **most productive scale size** (mpss). It is independent of value consideration for the inputs and outputs. Banker's definition states that a production possibility ($\mathbf{x}_s$, $\mathbf{y}_s$) represents a mpss if and only if all production possibilities $(\alpha \mathbf{x}_s, \beta \mathbf{y}_s)$ are such that $\alpha/\beta \geq 1$. A production unit operating at a mpss is then deemed scale efficient. Hence scale efficiency, defined independently of market prices, is evaluated with respect to a given input and output mix.

The alternate concept of scale efficiency, standard in economics, takes into account the relative prices of inputs but assumes that a single output is produced. It states that the scale of production, unambiguously defined by the volume of output, is efficient if, for that volume of production, the per unit cost of production is minimum. A generalization of this cost based concept of scale efficiency to the case of multiple outputs has not been dealt with in the literature. A possible generalization is to associate scale efficiency with maximum return, that is maximum ratio of revenues to costs, for the given input and output mixes. If relative prices of inputs and outputs are fixed then a production possibility $(\mathbf{x}_s, \mathbf{y}_s)$ can be deemed scale-efficient if and only if all production possibilities $(\alpha \mathbf{x}_s, \beta \mathbf{y}_s)$ are such that $\alpha/\beta \geq 1$. Indeed if μ represents the output price vector and ν represents the input price vector, then we must have:

$$\mu \mathbf{y}_s \,/\, \nu \mathbf{x}_s \geq \beta \mu \mathbf{y}_s \,/\, \alpha \nu \mathbf{x}_s.$$

We immediately note that, so defined, the cost version of the concept of scale efficiency is equivalent to the physical units version since both versions identify identical sets of scale efficient production possibilities. However, if relative prices are allowed to vary, as would be the case for quantity discounts, then identification of scale-efficiency becomes a non-linear problem.

It should be apparent that technical inefficiency implies scale and allocative inefficiencies. It follows that the measurement of technical inefficiency is essential to any efficiency evaluation. Such measurement is developed in the next 3 sections.

2.6 Measuring Technical Efficiency in the Input Space

2.6.1 Introduction

We assume a given technology, i.e. production correspondence, of the type introduced in Section 2. This technology is operationalized through various techniques characterized by minimal proportions of the various essential inputs over some range of outputs. Technical inefficiency[4] of a production unit can be thought as consisting of two components:

1) Internal inefficiency. Internal inefficiency is proportional to the excess usage of inputs **given** the production unit's "own" technique, i.e. keeping constant the proportions of inputs characteristic of the operations of the unit. This component of inefficiency can be associated with quality where the excess usage of inputs corresponds to the amount of rework that has to be performed by the producing unit.

2) External inefficiency. External inefficiency reflects the degree of expertise or "know-how" embodied by the production unit's own technique. If the producing unit is operating with a state-of-the-art

[4]technical inefficiency (resp. efficiency) will be used instead of "input-oriented technical inefficiency (resp. efficiency) of producing **y**" whenever **y** is considered fixed.

technique, i.e. a technique with Pareto-optimal productivity or average products characteristics, then its external efficiency score ought to be maximal. It follows that external inefficiency is related to the deviations of the unit's input ratios from the minimal input proportions characteristics of a "closest" (i.e. most similar) efficient technique.

These concepts are illustrated in Figure 2.5 on page 57. Both unit A and unit B are technically inefficient. However, unit A is only internally inefficient and can gain efficiency by eliminating rework in the amount GA. Unit B on the other hand is both internally and externally inefficient. To gain internal efficiency it ought to eliminate rework in the amount HB. To eliminate external inefficiency unit B ought to adopt a new technique, namely E's technique described by E's input consumption ratios.

The measurement of technical efficiency (and, by complementarity, of technical efficiency) should, ideally, incorporate both components. There is no a priori unique way to combine these two components into a global measure of technical efficiency: it can be done either additively or multiplicatively. Neither is there any a priori unique way to compute a defined global measure and each of its components. Two possible procedures can be:

(GIE) Compute the global measure, then extract the internal efficiency component, then deduce the external efficiency component.

(IEG) Compute the internal efficiency component first, then the external component, then deduce the global technical efficiency score.

However, it should be apparent that the global efficiency score of an operating unit, once a measure has been defined, is dependent upon the selection of an efficient unit to be used as referent. The extent to which the procedure affects the selection of the referent unit has to be investigated. This is crucial since different procedures, leading to the selection of different referent points, may also lead to different efficiency assessments. The eventual dependence of the scores on the procedure employed may require the identification of the specific and inherent "principles" guiding that procedure. These inherent principles then become the criteria for selecting a procedure and its associated efficiency assessments.

It follows that the measurement of technical efficiency requires prior definition of:

i) A metric within the input space. This metric will define the method of aggregation across input measures.

ii) The expression for a global measure. Once defined this measure becomes the yardstick of efficiency for all operating units, independently of the procedure employed to calculate it.

It also requires the development of:

iii) A computational approach that is tractable and consistent with the definitions.

The remainder of this chapter will deal with i) and ii). Chapter 3 will focus on iii). The validity of a proposed expression for an efficiency measure may be tested by checking whether the measure satisfies some properties deemed desirable. Such properties and their ensuing implications for the definition of an efficiency measure are reviewed next.

2.6.2 Desirable Properties of an Input-Oriented Measure of Technical Efficiency

Assuming for now that there exists a measure of technical efficiency, E(x,y), defined over L(y), and such that $E(x,y) \leq 1$ as customary, Färe and Lovell [1978] identified four desirable properties for E(x,y) which, as Russell [1985] showed, can be condensed to only three given below:

(FL1) $E(x,y) = 1 \Leftrightarrow x \in Eff(y)$

(FL2) $E(\theta x,y) = 1/\theta \; E(x,y) \; \forall \; \theta\colon \theta x \in L(y),\; x \in L(y)$

(FL3) If $x \in L(y)$, $x' \in L(y)$, and $x \geq x'$, then $E(x',y) > E(x,y)$

An interpretation/justification of the above requirements for E(x,y) is offered next:

(FL1) is a direct interpretation of Koopmans' definition of input-oriented technical efficiency. Note that in practice Eff(y) may not be known a priori requiring the specification of a procedure to determine it.

(FL2) requires that the efficiency measure be homogeneous of degree -1. This condition is compatible with the measurement of the internal efficiency component introduced earlier. It suggests that the global technical efficiency measure ought to be **multiplicative** in its components so that the (-1)-homogeneity be preserved independently of the level of the external efficiency component.

(FL3) states the intuitive notion that the more waste the less the efficiency score. However, since there is no complete ordering[5] available for $\mathbb{R}^m$, the condition is restricted to the clear-cut cases where two input vectors can meaningfully be compared (i.e. by a consistent extension of the order relation on $\mathbb{R}$) and requires strict monotonicity of the efficiency score.

Another desirable property, often overlooked but of practical significance in empirical studies, is that the efficiency measure ought to remain invariant with respect to changes in the scales of measurement of the various input and output measures. For ease of exposition this property will hereafter be referred to as (UI4) for fourth property dealing with units invariance. Russell [1990] refers to this property as the "Commensurability" property.

(UI4) $E(Dx,y) = E(x,y)\ \forall\ x \in \mathbb{R}^m_+,\ \forall\ D$: (mxm) strictly positive diagonal.

Russell [1985] provides necessary and sufficient conditions for a measure to satisfy the first three conditions. He further states that the existence of "a measure satisfying these conditions for the broad class of technologies considered in (his) paper remains an open question". The only, but apparently crucial, distinctions between the class of technologies we are considering and that of Russell [1985] stems from our assumptions that all inputs are essential and that the production possibility sets are convex. With these additional assumptions we prove the existence of an efficiency

---

[5] A complete ordering of $\mathbb{R}^m$ would require that for any two vectors $\mathbf{u}$, $\mathbf{v}$ of $\mathbb{R}^m$ we have either $\mathbf{u} \geq \mathbf{v}$ or $\mathbf{v} \geq \mathbf{u}$.

measure satisfying all desirable properties by producing a closed form expression for it and verifying its properties. Before we proceed with the proof we review some of the existing, and commonly used, efficiency measures, the properties they satisfy, and we examine the reasons of their invalidity. The discussion will be supported throughout by an example[6] of an input set depicted in Figure 2.6 on page 58 at the end of the chapter.

2.6.3 Existing Input-Oriented Measures of Technical Efficiency

2.6.3.1 The Debreu-Farrell Measure. The most widely used measure is the one traditionally referred to as the Debreu-Farrell measure given by:

$$E_{DF}(x,y) = \text{Min}\ \{\theta : \theta x \in L(y)\ \} = \theta_{DF}$$

This measure satisfies only (FL2), the (-1)-homogeneity property [Russell, 1985], and (UI4), the units-invariance property. The main identified weakness of this measure is that the reference input vector ($\theta_{DF}x$) is only required to belong to Iso(y) while the complete elimination of waste demands that the reference point belong to Eff(y), i.e. $E_{DF}(x,y)$ fails (FL1). This fact is illustrated by point R in Figure 2.6 on page 58:

$$E_{DF}(R,y) \ = \ \frac{\overline{OG}}{\overline{OR}}, \quad G \in \text{Iso}(y), \quad G \notin \text{Eff}(y)$$

The Debreu-Farrell measure focuses exclusively on the internal component of technical efficiency and fails to address its external component: In the case of point R the external efficiency component would acknowledge the presence of slack in input x_1 in the amount

---

[6] This example is adapted from Russell (1985) by requiring that x_1 be an essential input.

FG. Concerning (FL3) it is easily shown that $E_{DF}(\mathbf{x},\mathbf{y})$ is not **strictly** monotonic in inputs. Consider points G and F in Figure 2.6 where G > F and $E_{DF}(\mathbf{G},\mathbf{y}) = E_{DF}(\mathbf{F},\mathbf{y})$. However, it is of interest to note that $E_{DF}(\mathbf{x},\mathbf{y})$ is nevertheless monotonically decreasing in inputs. The proof is provided in Appendix A on page 243.

2.6.3.2 <u>The Färe-Lovell Measure</u>. Färe and Lovell [1978] introduce an alternative measure that they label the Russell measure given by[7]:

$$E_{FL}(\mathbf{x},\mathbf{y}) = \text{Min} \; \{\frac{\sum_{i=1}^{m}\theta_i}{m} : \theta \odot \mathbf{x} \in L(\mathbf{y}),\; \theta_i \in (0,1],\; i = 1,\ldots,m\}$$

which, they claim, exhibits all desired properties. Let us denote $\boldsymbol{\theta}_{FL}$ an m-vector of optimal values θ_i, i = 1,...,m. The ith component of this vector, $(\boldsymbol{\theta}_{FL})_i$, represents the minimum proportion of input i consumed by the production unit that is necessary to produce y given the minimum necessary consumption of other inputs. The main advantage of this measure over the Debreu-Farrell measure is that $\boldsymbol{\theta}_{FL}\odot\mathbf{x}$ is guaranteed to belong to Eff(y), i.e. (FL1) is satisfied, hence guaranteeing complete elimination of input waste. Given the identified reference point $\mathbf{x}^*$ of Eff(y) we can write:

$$\mathbf{x}^* = \boldsymbol{\theta}_{FL}\odot\mathbf{x} = \mathbf{x} - \mathbf{e}^*.$$

Hence we have:

$$(\theta_{FL})_i = \frac{x_i^*}{x_i} = 1 - \frac{e_i^*}{x_i} \qquad i = 1,\ldots,m$$

It follows that alternate expressions for $E_{FL}(\mathbf{x},\mathbf{y})$ are:

---

[7] $(\theta\odot\mathbf{x})_i = \theta_i . x_i$

$$E_{FL}(\mathbf{x},\mathbf{y}) = \frac{1}{m}\sum_{i=1}^{m}\frac{x_i^*}{x_i} = \mathrm{Min}_{x^e \in Eff(y)}\left\{\frac{1}{m}\sum_{i=1}^{m}\frac{x_i^e}{x_i} : x_i^e \leq x_i \; i=1,\ldots,m\right\}$$

and

$$E_{FL}(\mathbf{x},\mathbf{y}) = 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{e_i^*}{x_i}$$

We can further note that $(\theta_{FL})_i$ is unitless, invariant to changes in the units of measurement of input i. Let us denote: $\theta'' = \mathrm{Max}\{ (\theta_{FL})_i, i = 1,\ldots,m \}$. The quantity $(1 - \theta'')$ represents the maximum equiproportionate reduction of all inputs consumed by the production unit that would still allow the production of $\mathbf{y}$.

$$\mathbf{x}^* = \theta_{FL} \odot \mathbf{x} = \theta''\mathbf{x} - \mathbf{e}'' \quad \text{with:}$$

$$(\theta_{FL})_i = \theta'' - \frac{e_i''}{x_i} \qquad i = 1,\ldots,m$$

It follows:

$$E_{FL}(\mathbf{x},\mathbf{y}) = \theta'' - \frac{1}{m}\sum_{i=1}^{m}\frac{e_i''}{x_i} = \theta''\left(1 - \frac{1}{m}\sum_{i=1}^{m}\frac{e_i''}{\theta'' x_i}\right)$$

It appears that θ'' could tentatively be associated with the internal efficiency component and $(1 - 1/m\, \Sigma^m_{i=1}\, e''_i/\theta'' x_i)$ with the external efficiency component. Note that the procedure implied in the definition of $E_{FL}(\mathbf{x},\mathbf{y})$ is a type GIE (Global, Internal, External) procedure described earlier where θ'' is derived from the computed global measure. It follows that, generally, we have:

$$\theta_{DF} \neq \theta''$$

For instance in the process of evaluating the technical efficiency of Q in Figure 2.6 on page 58 we get:

$$\theta_{FL} \odot Q = E\,[8] \text{ and } \theta_{DF} Q = F$$

with:

$$\theta'' = \text{Max}\,\{\frac{\overline{OW}}{\overline{OA}}, \frac{\overline{OU}}{\overline{OV}}\} = \frac{\overline{OU}}{\overline{OV}} > \frac{1}{2}$$

$$\theta_{DF} = \frac{\overline{OF}}{\overline{OQ}} = \frac{1}{2}$$

Moreover, Russell [1985] shows that the Färe-Lovell measure fails (FL2), the homogeneity condition. This fact is also illustrated by Q in Figure 2.6 on page 58. We have:

$$Q = 2F,\ \theta_{FL} \odot Q = E \text{ and } \theta_{FL} \odot F = F.$$

$$\text{Hence, } E_{FL}(2\mathbf{F},\mathbf{y}) = E_{FL}(\mathbf{Q},\mathbf{y}) = \frac{1}{2}(\frac{\overline{OW}}{\overline{OA}} + \frac{\overline{OU}}{\overline{OV}}) < \frac{1}{2}$$

$$\text{and } E_{FL}(\mathbf{F},\mathbf{y}) = 1.$$

It follows $E_{FL}(2\mathbf{F},\mathbf{y}) < 1/2\ E_{FL}(\mathbf{F},\mathbf{y})$.

Russell [1985] also showed that $E_{FL}(\mathbf{x},\mathbf{y})$ failed (FL3), the strict monotonicity condition but satisfied the weak monotonicity condition:

$$\mathbf{x} \leq \mathbf{x}' \Rightarrow E_{FL}(\mathbf{x},\mathbf{y}) \geq E_{FL}(\mathbf{x}',\mathbf{y})$$

However, his proof rests on the existence of at least one non-essential input. With the assumption of essentiality of all inputs $E_{FL}(\mathbf{x},\mathbf{y})$ is strictly monotonic in $\mathbf{x}$:

---

[8]E rather than F is the referent point for Q since:

$$\frac{1}{2}(\frac{\overline{OW}}{\overline{OA}} + \frac{\overline{OU}}{\overline{WE}}) < \frac{1}{2}(\frac{\overline{OX}}{\overline{OA}} + \frac{\overline{OT}}{\overline{XF}}) = \frac{1}{2}$$

$$x \leq x' \Rightarrow E_{FL}(x,y) > E_{FL}(x',y)$$

The proof of strict monotonicity is provided in Appendix B. Since all θ_i $i = 1,\ldots,m$ are unitless we can conclude that (UI4) is also satisfied.

It appears that, from a mathematical point of view, the only shortcoming of the Färe-Lovell measure, is its failure of the (-1)-homogeneity condition. The Färe-Lovell measure corresponds to a worst-case evaluation of the efficiency of the production unit. Its driving principle is the extraction of a maximal proportion of slack inputs without regard for the original proportions of the inputs. This priciple is reasonable in the case of a long-run evaluation where all inputs would become extremely scarce. Selecting a most frugal technique would hence be paramount. In the short-run, however, the original input proportions are likely to reflect local supply and demand equilibrium conditions which influence, if not dictate, the choice of the technique for operations. This point is illustrated by Q in Figure 2.6 on page 58. The Färe-Lovell technique compares Q to E while a comparison to F would appear more appropriate in the short-run since F and Q share the same technique. The Färe-Lovell technique does not systematically identify the "closest" (i.e. "most similar") efficient technique to Q.

2.6.3.3 The Zieschang Measure. In an attempt to correct for the flaws of both preceding measures, Zieschang [1984] offered a combination measure given below:

$$E_Z(x,y) = E_{DF}(x,y) . E_{FL}(E_{DF}(x,y)x,y)$$

This measure, following the above development is:

$$\theta_{DF}\left(1 - \frac{1}{m}\sum_{i=1}^{m} \frac{e_i^*}{\theta_{DF} x_i} \right)$$

where:

$$\theta_{DF} = E_{DF}(\mathbf{x},\mathbf{y}) = \text{Min}\ \{\theta : \theta\mathbf{x} \in L(\mathbf{y})\ \}$$

$$1 - \frac{1}{m}\sum_{i=1}^{m} \frac{e_i^*}{\theta_{DF} x_i} = \text{Min}_{x^*}\ \{\frac{1}{m}\sum_{i=1}^{m} \frac{x_i^*}{\theta_{DF} x_i}\ , x^* \in \text{Eff}(\mathbf{y}),\ x^* = \theta_{DF}\, x - e\ \}$$

This measure obviously requires that the Debreu-Farrell component, identifiable as the internal efficency component, be computed first, hence requiring an IEG (Internal, External, Global) procedure. This measure represents an improvement over the preceding two measures. It can be shown to satisfy (FL1), (FL2), and (UI4). However, concerning (FL3), the strict monotonicity condition, Zieschang notes, $E_Z(\mathbf{x},\mathbf{y})$ "does not always decrease in the nonslack inputs". The stated reason is that the rate of decrease of $E_{DF}(\mathbf{x},\mathbf{y})$ may be less than the rate of increase of $E_{FL}(\mathbf{x},\mathbf{y})$ as $\mathbf{x}$ increases in the non-slack inputs. A clarification of this statement is obtained by considering R and S in Figure 2.6 on page 58. In the case of these two input vectors, x_1 is identified as the slack input and x_2 as the non-slack input. Moving from S to R, x_2, the non-slack input, increases from B to C. As a result $E_{DF}(\mathbf{x},\mathbf{y})$ decreases:

$$E_{DF}(\mathbf{S},\mathbf{y}) = \frac{\overline{OH}}{\overline{OS}} > E_{DF}(\mathbf{R},\mathbf{y}) = \frac{\overline{OG}}{\overline{OR}}$$

and $E_{FL}(\mathbf{x},\mathbf{y})$ increases: $E_{DF}(\mathbf{S},\mathbf{y}).S = H$, $E_{DF}(\mathbf{R},\mathbf{y}).R = G$ and

$$E_{FL}(\mathbf{H},y) = \frac{1}{2}(\frac{\overline{OX}}{\overline{OZ}} + \frac{\overline{OT}}{\overline{OT}}) < E_{FL}(\mathbf{G},y) = \frac{1}{2}(\frac{\overline{OX}}{\overline{OY}} + \frac{\overline{OT}}{\overline{OT}})$$

However, it is important to note that the strict monotonicity condition is satisfied in a 2-input space. Indeed if we consider points S and R in Figure 2.6, an input consumption pattern to the right of R would rate lower than R on the Färe-Lovell component and no better than R on the Debreu-Farrell component. Hence if the strict monotonicity condition is violated then it will be by a point such as R with respect to S. From the preceding developments we have:

$$E_Z(\mathbf{S},y) = \frac{\overline{OH}}{\overline{OS}}\frac{1}{2}(\frac{\overline{OX}}{\overline{OZ}} + \frac{\overline{OT}}{\overline{OT}}) \text{ and } E_Z(\mathbf{R},y) = \frac{\overline{OG}}{\overline{OR}}\frac{1}{2}(\frac{\overline{OX}}{\overline{OY}} + \frac{\overline{OT}}{\overline{OT}})$$

We note $\frac{\overline{OZ}}{\overline{OH}} = \frac{\overline{OA}}{\overline{OS}}$ and, similarly, $\frac{\overline{OY}}{\overline{OG}} = \frac{\overline{OA}}{\overline{OR}}$.

Hence, $E_{DF}(\mathbf{S},y) = \frac{\overline{OH}}{\overline{OS}} = \frac{\overline{OZ}}{\overline{OA}}$ and $E_{DF}(\mathbf{R},y) = \frac{\overline{OG}}{\overline{OR}} = \frac{\overline{OY}}{\overline{OA}}$.

It follows:

$$E_Z(\mathbf{S},y) = \frac{1}{2}\frac{\overline{OZ}}{\overline{OA}}(\frac{\overline{OX}}{\overline{OZ}} + \frac{\overline{OT}}{\overline{OT}}) \text{ and } E_Z(\mathbf{R},y) = \frac{1}{2}\frac{\overline{OY}}{\overline{OA}}(\frac{\overline{OX}}{\overline{OY}} + \frac{\overline{OT}}{\overline{OT}}).$$

That is:

$$E_Z(\mathbf{S},y) = \frac{1}{2}(\frac{\overline{OX}}{\overline{OA}} + \frac{\overline{OZ}}{\overline{OA}}) \text{ and } E_Z(\mathbf{R},y) = \frac{1}{2}(\frac{\overline{OX}}{\overline{OA}} + \frac{\overline{OY}}{\overline{OA}}).$$

Violation of the strict monotonicity condition would require $E_Z(\mathbf{S},y) < E_Z(\mathbf{R},y)$ that is,

$$\overline{OZ} < \overline{OY}$$

which is impossible in a 2-input space.

Zieschang offers a modification to the above measure in order to ensure strict monotonicity for the global measure. The proposed change decreases the weight of the external efficiency component in the evaluation of the global efficiency score. The choice of weight is somewhat arbitrary and, more importantly, it may alter arbitrarily the efficiency ranking of the input vectors. The altered measure is given by:

$$E_Z'(x,y) = \theta^* \frac{n(1 - \frac{1}{m}\sum_{i=1}^{m} \frac{e_i}{\theta^* x_i}) + N}{n + N} = \theta^* (1 - \frac{n}{n+N} \frac{1}{m} \sum_{i=1}^{m} \frac{e_i}{\theta^* x_i})$$

For sufficiently large N we can write, employing $\epsilon = n/(n+N)m$:

$$E_Z'(x,y) = \theta^* - \epsilon \sum_{i=1}^{m} \frac{e_i}{x_i}$$

From the above expression, we can see that the Zieschang measure, by preemptively extracting slacks in accordance with the unit's own technique, ends up identifying an efficient technique that is generally "closer" to that of the unit than the one identified by the Färe-Lovell technique.

The next section offers an economic approach to the measurement of efficiency. This approach is worth considering for it leads to an expression for the measure of efficiency that satisfies all desirable properties.

2.6.4 An Admissible Input-Oriented Efficiency Measure

We start by defining a metric within the input space. The simplest way is to assign a **value** or **efficiency unit price** to each input and then, as customary in economics, define the efficiency score as **the ratio of the minimal total cost of producing y to the total cost associated with x**. The minimal cost is the "efficiency cost" associated with the input vector $\mathbf{x}^*$ in Eff(y) against which **x** ought to be evaluated.

$$E(\mathbf{x},\mathbf{y}) = \frac{\mathbf{p}\mathbf{x}^*}{\mathbf{p}\mathbf{x}}$$

This definition ensures that (FL1) is satisfied. However, **p**, from economic theory, is expected to reflect the implied trade-offs, or implied rates of substitution among inputs, and since all efficient input vectors $\mathbf{x}^*$ generally reflect different rates of substitution, it is impossible to find any single **p** that will be consistent with all $\mathbf{x}^*$. Each technically efficient input vector $\mathbf{x}^*$ implies a set of possible price vectors that would identify $\mathbf{x}^*$ as a (the) minimum cost alternative to producing **y**. This fact is illustrated in Figure 2.7 on page 59 at the end of the chapter. The price vector $\mathbf{p}_1 > 0$ represents a normal to the hyperplane defined by $\mathbf{x}^*_3$ and $\mathbf{x}^*_4$, $\mathbf{p}_2$ similarly represents a normal to the hyperplane defined by $\mathbf{x}^*_2$ and $\mathbf{x}^*_3$. Any price vector $\mathbf{p}_x$ such that:

$$\mathbf{p}_x = k_1\mathbf{p}_1 + k_2\mathbf{p}_2 \quad k_1, k_2 \geq 0, \; k_1 + k_2 \neq 0$$

ensures that $\mathbf{x}^*_3$ represents a minimum cost alternative to producing **y**.

It follows that the evaluation of **x** requires both the identification of $\mathbf{x}^* \in$ Eff(y) and the identification of $\mathbf{p}_{x^*}$ such that:

$$\mathbf{p}_{\mathbf{x}*}.\mathbf{x}^{*} \leq \mathbf{p}_{\mathbf{x}*}.\mathbf{x}^{e} \quad \forall\ \mathbf{x}^{e} \in \text{Eff}(\mathbf{y})$$

The identification of $\mathbf{x}^*$ in Eff(**y**) obviously assumes that Eff(**y**) can be determined.

From the knowledge of Eff(**y**) we can derive the range of efficiency prices compatible with Eff(**y**). Referring again to Figure 2.7 on page 59, any price vector **p** such that:

$$\mathbf{p} = k_1\mathbf{p}^l + k_2\mathbf{p}^L \quad k_1, k_2 \geq 0,\ k_1 + k_2 \neq 0$$

will identify either a whole facet of Eff(**y**) (e.g. $(x_1^* x_2^*)$, $(x_2^* x_3^*)$,..), or a unique face of Eff(**y**) (e.g. x_2^*, x_3^*,..) as minimium cost options to producing **y**. The price vectors $\mathbf{p}^l$ and $\mathbf{p}^L$ are determined as normals to the <u>extreme</u> (i.e. edge) facets of Eff(**y**). There are infinitely many (homothetic of one another) such normals to any facet but they all summarize to a unique set of price ratios. We further observe that given the cone of efficiency prices defined by $\mathbf{p}^l$ and $\mathbf{p}^L$, there exists no efficiency price vector that will uniquely determine $\mathbf{x}_1^*$ or $\mathbf{x}_4^*$ as the minimum cost option of producing **y**. Considering **p** such that:

$$\mathbf{p} = k_1\mathbf{p}^l - k_2\mathbf{p}^L\ , k_1, k_2 \geq 0,\ k_2 \neq 0$$

as a feasible price vector would allow $\mathbf{x}_1^*$ to become the unique minimum cost alternative to producing **y**. However, when the technology is not known nor described in closed form, input sets have to be derived from observations and hence offer no empirical ground on which to bound k_2. Without any theoretical nor empirical means to limit k_2 we would, by continuity, have to eventually consider $\mathbf{p} = (p_1,p_2)$ such that p_2, the efficiency price of input 2, is null. This extreme situation is not acceptable since the efficiency price of any input should equal the value of its marginal product which cannot be null in the case of an efficient input vector. Hence, for lack of further empirical or theoretical knowledge which would allow the extension of Eff(y) through $\mathbf{x}_1^*$ and $\mathbf{x}_4^*$, we choose to limit the cone of admissible efficiency price vectors to the minimum cone that will properly identify Eff(y), i.e. the cone defined by $\mathbf{p}^l$ and $\mathbf{p}^L$ in the case of Figure 2.7.

Within the cone of weak disposability the proposed efficiency measure coincides with the Debreu-Farrell measure. Moreover, within the cone of weak disposability, the efficiency prices of the unit being evaluated coincide with the efficiency prices of the referent unit in Eff(y). However, the recognition that the relevant efficiency prices are those defined by the set of efficient units allows an appropriate definition of the efficiency score, namely the ratio of minimal to actual cost at efficiency prices, to be extended to units mapping within the areas of strong disposability. Within these areas the efficiency score evaluates to strictly less than the Debreu-Farrell measure, and, more importantly, the efficiency score so computed satisfies **all** four desirable properties for an efficiency score.

Definition 2.1: The input efficiency measure for **x**, $E_I(\mathbf{x},\mathbf{y})$ is expressed as:

$$E_I(\mathbf{x},\mathbf{y}) = \frac{\mathbf{p}_{x*}\mathbf{x}^*}{\mathbf{p}_{x*}\mathbf{x}}$$

where:

x^* belongs to the efficient set of $L(y)$ and is such that it is representative of a production technique that is most similar to that of x in terms of input consumption ratios:

$$x^* = \theta^* x - e \quad \text{with } \theta^* = \mathrm{Min}_\theta \{\theta: \theta x \in L(y)\}$$

and p_{x^*} is a vector of efficiency prices such that at these prices x^* represents a minimum cost alternative of producing y.

<u>Theorem 2.1</u>: Given the class of technologies described by (IC1) - (IC4) and the assumption of strong disposability of essential inputs, the input efficiency measure $E_I(x,y)$ satisfies all four properties (FL1) - (FL3), (UI4) stated below:

(FL1) $E_I(x,y) = 1 \Leftrightarrow x \in Eff(y)$

(FL2) $E_I(\theta x,y) = 1/\theta\, E_I(x,y)\ \forall\ \theta: \theta x \in L(y),\ x \in L(y)$

(FL3) If $x \in L(y)$, $x' \in L(y)$, and $x \geq x'$, then $E_I(x',y) > E_I(x,y)$

(UI4) $E_I(Dx,y) = E_I(x,y)\ \forall\ x \in \mathbb{R}^m_+,\ \forall$ D: (mxm) strictly positive diagonal.

<u>Proof</u>: The proof is supported by Figure 2.8, on page 60 at the end of the chapter, which illustrates the case of a 2-dimensional input space. The argument can easily be adapted to an m-input space.

(FL1) is trivially proven. If $x \in Eff(y)$ then $x^* = x$ and $E_I(x,y) = 1$. Conversely if $E_I(x,y) = 1$ then $p_{x^*}.(x^* - x) = 0$ with $x^* - x \leq 0$ and $p_{x^*} > 0$. It follows $x^* - x = 0$ hence $x = x^*$ and $x \in Eff(y)$.

To prove (FL2), let us first consider $\mathbf{x}$ and $\mathbf{x}'$ in $L(y)$ such that $\mathbf{x} = \theta'\mathbf{x}'$, $\mathbf{x}^* = \theta^*\mathbf{x}$, $\mathbf{x}^* \in Eff(y)$, and $E_I(x,y) = \theta^*$. This situation is illustrated by I_o, J, and P_o in Figure 2.8.

$$E_I(I_o,y) = \theta^* = \frac{\overline{OP_o}}{\overline{OI_o}} \text{ and } I_o = \theta'J = \frac{\overline{OI_o}}{\overline{OJ}}.J$$

$\mathbf{x}^* = \theta^*\,\mathbf{x} = \theta^*\theta'\mathbf{x}'$ with $\theta^* = \text{Min}\{\,\theta{:}\theta\mathbf{x} \in L(y)\}$. It follows $(\theta^*\theta') = \text{Min}\{\,\theta{:}\theta\mathbf{x}' \in L(y)\}$. Hence, $E_I(x',y) = \theta^*\theta' = \theta' E_I(x,y)$, i.e. $1/\theta'\ E_I(x',y) = E_I(\theta'x',y)$.

The second situation corresponds to the case where $\mathbf{x}$ does not belong to the cone of weak disposability. We can then write $\mathbf{x}^* = \theta^*\mathbf{x} - \mathbf{e}$ or $\mathbf{x} = 1/\theta^*\,(\mathbf{x}^* + \mathbf{e})$ with $\mathbf{x}^* \in Eff(y)$ and $\theta^* = \text{Min}\{\,\theta{:}\theta\mathbf{x} \in L(y)\}$. Let $\mathbf{p}_{x*}$ represent a vector of efficiency prices such that at these prices $\mathbf{x}^*$ represents a minimum cost alternative of producing y, the efficiency score of $\mathbf{x}$ is then expressed by:

$$E_I(x,y) = \frac{\mathbf{p}_{x*}.\mathbf{x}^*}{\mathbf{p}_{x*}.\mathbf{x}} = \frac{\mathbf{p}_{x*}.\mathbf{x}^*}{\frac{1}{\theta}(\mathbf{p}_{x*}.\mathbf{x}^* + \mathbf{p}_{x*}.\mathbf{e})} = \frac{\theta}{(1 + \frac{\mathbf{p}_{x*}.\mathbf{e}}{\mathbf{p}_{x*}.\mathbf{x}^*})}$$

If we now consider $\mathbf{x}'$ in $L(y)$ such that $\mathbf{x} = \theta'\mathbf{x}'$, then we have $\mathbf{x}^* = \theta^*\theta'\mathbf{x}' - \mathbf{e}$ with $\theta^* = \text{Min}\{\,\theta{:}\theta\theta'\mathbf{x}' \in L(y)\}$, implying $(\theta^*\theta') = \text{Min}\{\,\theta{:}\theta\mathbf{x}' \in L(y)\}$. Hence:

$$E_I(x',y) = \frac{\mathbf{p}_{x*}.\mathbf{x}^*}{\mathbf{p}_{x*}.\mathbf{x}'} = \frac{\mathbf{p}_{x*}.\mathbf{x}^*}{\frac{1}{\theta\theta'}(\mathbf{p}_{x*}.\mathbf{x}^* + \mathbf{p}_{x*}.\mathbf{e})} = \frac{\theta\theta'}{(1 + \frac{\mathbf{p}_{x*}.\mathbf{e}}{\mathbf{p}_{x*}.\mathbf{x}^*})} = \theta' E_I(x,y)$$

i.e. $1/\theta'\ E_I(x',y) = E_I(\theta'x',y)$ in this situation as well.

To prove (FL3), we study the variations of $E_I(\mathbf{x},\mathbf{y})$ as one input is increased. Starting at I_o in Figure 2.8 we progressively increase input 1. The input consumption is represented by a point moving from I_o to I_1, I_2, I_3, and so on.

Between I_o and I_2 radial reduction of input consumption projects I onto the same facet E_2E_3 of the efficient set where the same price vector, $\mathbf{p}_{x*}$ is valid. We then have:

$$E_I(x_1,y) = \frac{p_{x*}x^*}{p_{x*}x_1} = \frac{\overline{OP_1}}{\overline{OI_1}} \text{ and } E_I(x_o,y) = \frac{\overline{OP_o}}{\overline{OI_o}} = \frac{\overline{OH_1}}{\overline{OI_1}} > \frac{\overline{OP_1}}{\overline{OI_1}}$$

Increasing input 1 further and beyond I_2 leads to a new relevant price vector since proportional input reduction projects the inefficient unit to a new facet of Eff(y): E_3E_4. With these prices the inefficiency of I_3 evaluates to :

$$E_I(x_3,y) = \frac{\overline{OP_3}}{\overline{OI_3}} < \frac{\overline{OH_3}}{\overline{OI_3}} = E_I(x_o,y)$$

The convexity of the isoquant ensures that the ratio of the cost of the projected efficient point to the cost of the inefficient point is strictly decreasing as the consumption of one input increases.

Finally as the input consumption increases further, the area of strong disposability is reached (see I_4 in Figure 2.8). Proportional input reduction is not sufficient to attain efficiency:

$$x^* = \theta x - e \text{ or } x = 1/\theta\,(x^* + e)$$

As the input consumption increases further θ remains constant and **e** absorbs the extra consumption. However, provided a unique efficiency price vector associated with x^* has been identified (a normal to the extreme facet) then we have:

$$E_I(x,y) = \frac{p_{x*}x^*}{p_{x*}x} = \frac{p_{x*}x^*}{\frac{1}{\theta}(p_{x*}x^* + p_{x*}e)} = \frac{\theta}{(1 + \frac{p_{x*}e}{p_{x*}x^*})}$$

which shows that the efficiency measure is, in this case as well, **strictly** decreasing in input: as **e** increases strictly $E_I(x,y)$ decreases strictly.

Regarding (UI_4), as long as the procedure ensures that prices reflect an inverse relationship to units of measurement $E_I(x,y)$ will be units-invariant.

Q.E.D.

The pricing mechanism for slack quantities is critical. The measures, $E_{DF}(x,y)$, $E_{FL}(x,y)$, $E_Z(x,y)$, either ignored or undervalued excess inputs leading to an overestimation of the efficiency score. This shortcoming is the primary reason why these three measures do not satisfy either the homogeneity or the strict monotonicity condition for an efficiency score.

The Debreu-Farrell measure can be identified as the internal efficiency component and the residual reduction of the input vector as the external component. Hence the input-oriented technical efficiency measure is equivalent to the Debreu-Farrell measure **only** over the cone of weak disposability. In **all** circumstances it is measured by $E_I(x,y) = p_{x*}x^* / p_{x*}x$ where x^* is the referent unit in Eff(y) against which the inefficiency of

$\mathbf{x}$ is gauged, and where $\mathbf{p}_{x^*}$ is a vector of efficiency prices which allows $\mathbf{x}^*$ to be identified as a minimum cost alternative to producing $\mathbf{y}$.

2.7 Measuring Technical Efficiency in the Output Space

2.7.1 Introduction

By analogy with the input-oriented measure of technical efficiency, the output-oriented measure ought to incorporate two components:

1) Internal efficiency. Internal efficiency is proportional to the shortage of outputs **given** the production unit's "own" mix, i.e. keeping constant the proportions of outputs characteristic of the operations of the production unit. This component of efficiency can be associated with productivity.

2) External efficiency. External efficiency reflects the scheduling expertise of the production unit in that it allows the unit to produce an efficient mix. External efficiency is inversely proportional to the deviations of the unit's output ratios from the output ratios characteristic of the "closest" efficient scheduling/mix.

2.7.2 Desirable Properties of an Output-Oriented Measure of Technical Efficiency

Desirable properties for the output-oriented efficiency measure, $E_O(\mathbf{x},\mathbf{y})$, can similarly be derived from the corresponding properties for the input-oriented measure:

(OP1) $\quad E_O(\mathbf{x},\mathbf{y}) = 1 \Leftrightarrow \mathbf{y} \in \mathrm{Eff}(\mathbf{x})$

(OP2) $\quad E_O(\mathbf{x},\phi\mathbf{y}) = \phi\, E_O(\mathbf{x},\mathbf{y}) \;\forall\; \phi\text{: } \phi\mathbf{y} \in P(\mathbf{x}),\ \mathbf{y} \in P(\mathbf{x})$

(OP3) If $y \in P(x)$, $y' \in P(x)$, and $y' \geq y$, then $E_O(x,y') > E_O(x,y)$

(OP4) $E_O(x,Dy) = E_O(x,y)\ \forall\ y \in \mathbb{R}^s_+$, $\forall$ D: (sxs) strictly positive diagonal

(OP1) is a direct interpretation of Koopmans'definition of output-oriented technical efficiency. In this case as well Eff(x) may not be known a priori requiring the specification of a procedure to identify it.

(OP2) requires the efficiency measure to be homogeneous of degree 1. This condition is compatible with the measurement of the internal efficiency component introduced above. It also suggests that the output-oriented technical efficiency measure ought to be **multiplicative** in its components so that (1)-homogeneity be preserved independently of the level of the external efficiency component.

(OP3) requires strict monotonicity of the efficiency score and, finally, (OP4) requires that the efficiency score be invariant with respect to changes in the units of measurement of outputs.

2.7.3 An Admissible Output-Oriented Measure of Technical Efficiency

An output-oriented measure that is worth considering is one easily adapted from the admissible input-oriented measure introduced earlier. We define a metric within the output space. The simplest way is to assign a value or efficiency price to each output and then define the efficiency score as **the ratio of the total revenue associated with y to the maximal total revenue of producing y**. The maximal revenue is the revenue associated to the output vector y^* in Eff(x) against which y is being evaluated.

Definition 2.2: The output efficiency measure for $\mathbf{y}$, $E_O(\mathbf{x},\mathbf{y})$ is expressed as:

$$E_O(\mathbf{x},\mathbf{y}) = \frac{\mathbf{p}\mathbf{y}}{\mathbf{p}\mathbf{y}^*}$$

where:

$\mathbf{y}^*$ belongs to the efficient set of $P(\mathbf{x})$ and is such that it is representative of a production technique that is most similar to that of $\mathbf{y}$ in terms of output mix:

$$\mathbf{y}^* = \phi^*\mathbf{y} + \mathbf{s} \quad \text{with } \phi^* = \text{Max}_\phi \, \{\phi\text{: } \phi\mathbf{y} \in P(\mathbf{x})\}$$

and $\mathbf{p}$ is a vector of efficiency prices such that, at these prices, $\mathbf{y}^*$ represents a maximum revenue alternative of consuming $\mathbf{x}$.

This definition ensures that (OP1) is satisfied. However, $\mathbf{p}$, from economic theory, is expected to reflect the implied tradeoffs, or implied rates of transformation among outputs, and since most efficient output vectors $\mathbf{y}^*$ reflect different rates of transformation, it is impossible to find any $\mathbf{p}$ that will be consistent with all $\mathbf{y}^*$. Each technically efficient output vector $\mathbf{y}^*$ implies a set of possible price vectors that would identify $\mathbf{y}^*$ as a (the) maximum revenue alternative to consuming $\mathbf{x}$. This fact is illustrated in Figure 2.9 on page 61 at the end of the chapter. The price vector $\mathbf{p}_1 > 0$ represents a normal to the hyperplane defined by $\mathbf{y}^*_2$ and $\mathbf{y}^*_3$, $\mathbf{p}_2$ similarly represents a normal to the hyperplane defined by $\mathbf{y}^*_1$ and $\mathbf{y}^*_2$. Any price vector $\mathbf{p}_y$ such that:

$$\mathbf{p}_{\mathbf{y}*} = k_1\mathbf{p}_1 + k_2\mathbf{p}_2 \quad k_1,\, k_2 \geq 0,\; k_1 + k_2 \neq 0$$

ensures that $\mathbf{y}^*_2$ represents a maximum revenue alternative to consuming $\mathbf{x}$. It follows that the evaluation of $\mathbf{y}$ requires both the identification of $\mathbf{y}^* \in \text{Eff}(\mathbf{x})$ and the identification of $\mathbf{p}_{\mathbf{y}*}$ such that:

$$p_y . y^* \geq p_y . y^e \quad \forall\, y^e \in \text{Eff}(x)$$

For the same reasons as in the case of input-oriented technical efficiency we choose to limit the cone of admissible efficiency price vectors to the minimum cone that will properly identify Eff(x), i.e. the cone defined by $\mathbf{p}^l$ and $\mathbf{p}^L$ in the case of Figure 2.9. With efficiency prices restricted to that cone and a development that parallels the case of the input-oriented measure, it is easily shown that the proposed output-oriented measure of technical efficiency satisfies all four properties (OP1), (OP2), (OP3), and (OP4).

2.8 Measuring Global Technical Efficiency

The input-oriented efficiency measure focuses exclusively on input waste, and, similarly, the output-oriented efficiency measure focuses exclusively on output waste. A complete measure of efficiency requires that both subspaces, inputs and outputs, be considered so that **all** waste be accounted for. A logical approach in defining a global measure is to combine the definitions of the input-oriented measure and of the output-oriented measure. The evaluation of (x,y) requires the identification of (x*,y*) that is simultaneously input and output efficient, and of price vectors $(\mathbf{p}_{x^*}, \mathbf{p}_{y^*})$ such that, at these prices, **x*** is a least cost alternative to producing **y*** $\geq$ y, and **y*** is a maximum revenue alternative to consuming **x*** $\leq$ x. We then have:

$$E(x,y) = E_I(x,y).E_O(x,y) = \frac{p_x . x^*}{p_x . x} . \frac{p_y . y}{p_y . y^*}$$

It is important to note that **x*** belongs to the efficiency set of L(**y***), not simply L(**y**), and that **y*** belongs to the efficiency set of P(**x***), not simply P(**x**). The resulting properties of this global efficiency measure derive from the properties of its component efficiency measures with the added stipulation that some of these properties are **contingent upon the procedure** employed to identify (**x***,**y***). These properties, summarized by statements Efficiency 1 - Efficiency 5, are given below.

(Efficiency 1) $E(\mathbf{x},\mathbf{y}) = 1 \Leftrightarrow \mathbf{x} \in Eff(\mathbf{y})$ and $\mathbf{y} \in Eff(\mathbf{x})$.

(Efficiency 2) If waste is identified in proportion of current input consumption first and if $\mathbf{x} \in L(\mathbf{y})$, $\mathbf{x}' \in L(\mathbf{y})$, and $\mathbf{x} \geq \mathbf{x}'$, then $E_I(\mathbf{x}',\mathbf{y}) > E_I(\mathbf{x},\mathbf{y})$ but it is not always true that $E(\mathbf{x}',\mathbf{y}) > E(\mathbf{x},\mathbf{y})$.

(Efficiency 3) If waste is identified in proportion of current output mix first and if $\mathbf{y} \in P(\mathbf{x})$, $\mathbf{y}' \in P(\mathbf{x})$, and $\mathbf{y}' \geq \mathbf{y}$, then $E_O(\mathbf{x},\mathbf{y}') > E_O(\mathbf{x},\mathbf{y})$ but it is not always true that $E(\mathbf{x},\mathbf{y}') > E(\mathbf{x},\mathbf{y})$.

(Efficiency 4) $E(\theta\mathbf{x},\mathbf{y}) = 1/\theta\ E(\mathbf{x},\mathbf{y})\ \forall\ \theta$: $\theta\mathbf{x} \in L(\mathbf{y})$, $\mathbf{x} \in L(\mathbf{y})$ if waste is identified in proportion of current input consumption first.

(Efficiency 5) $E(\mathbf{x},\phi\mathbf{y}) = \phi\ E(\mathbf{x},\mathbf{y})\ \forall\ \phi$: $\phi\mathbf{y} \in P(\mathbf{x})$, $\mathbf{y} \in P(\mathbf{x})$ if waste is identified in proportion of current output mix first.

Property Efficiency 1 states that the efficiency score E(**x**,**y**) reveals (**x**,**y**) to be efficient if and only if it is input and output efficient. It suffices to note that both the input component and output component of the efficiency measure are bounded above by 1, and that efficiency prices are constrained to be strictly positive. It follows that (**x**,**y**) is identical to the efficient unit against which its efficiency is gauged. Efficiency 2 states that even when the strict monotonicity property of the input-oriented efficiency measure holds, the global measure may not depict this property. The fact that strict monotonicity does not extend to the global measure stems from the fact that the output-oriented efficiency measure may reverse the order of the input-oriented score. A situation where such a reversal could occur is depicted in Figure 2.10 on page 62 at the end of the chapter. In the input space the two input bundles, I_1 and I_2 are such that $I_1 < I_2$ and, accordingly $E_I(I_1,\mathbf{y}) = OX_1/OI_1 > E_I(I_2,\mathbf{y}) = OX_2/OI_2$. However, in the output space, **y** belongs to Eff(X_2) but **y** does not belong to Eff(X_1). It follows $E_O(X_1,\mathbf{y}) < 1$ and $E_O(X_2,\mathbf{y}) = 1$, so that globally we may have $E(I_1,\mathbf{y}) = E_I(I_1,\mathbf{y}).E_O(X_1,\mathbf{y}) < E(I_2,\mathbf{y}) = E_I(I_2,\mathbf{y}).E_O(X_2,\mathbf{y})$. Property Efficiency 3 parallels property Efficiency 2. Property Efficiency 4 states that E(**x**,**y**) statisfies the homogeneity property regarding input consumptions provided the input-oriented component is evaluated first. Since the input component of efficiency, which satisfies the homogeneity condition, is assessed first it leads to the same efficient input vector, $\mathbf{x}^*$, for both (**x**,**y**) and (θ**x**,**y**). The output component of efficiency, which is assessed second, will therefore evaluate to the same value for both (**x**,**y**) and (θ**x**,**y**), ensuring that the global measure satisfies the homogeneity condition. Property Efficiency 5 parallels property Efficiency 4.

Given the scale of operations defined by **x** and **y** the best return or yield observable is exhibited by the efficient unit (**x***, **y***) evaluated at efficiency prices given by (p_{x^*}, p_{y^*}). The performance of a production unit characterized by (x,y) is then appropriately gauged against a best observed performance. The sources of inefficiency can then be attributed to the pattern of input consumption whenever $x \neq x^*$ and/or to the output mix whenever $y \neq y^*$. The recommendations to an inefficient unit are fully contained in the identified efficient unit (x^*,y^*).

2.9 Measuring Allocative Efficiency

The distinction between technical and allocative efficiency is well understood and the literature abounds in explanatory illustrations [e.g. Banker and Maindiratta, 1988]. As mentioned earlier, allocative efficiency requires technical efficiency. Moreover allocative efficiency rests on the existence/availability of prices or valuations for the various inputs and outputs that ensure a maximum yield or return for the unit. Namely (x,y) is allocatively efficient if and only if there exits (p_x, p_y) representative of market conditions or societal values such that:

$$\frac{p_y y}{p_x x} \geq \frac{p_y y_t}{p_x x_t} \text{ for all } (x_t, y_t) \text{ technically efficient.}$$

It follows that if the set of technical efficiency price vectors is contained in the set of acceptable market price vectors then technical efficiency implies allocative efficiency. However, it is conceivable that some technical efficiency price vector may be representative of relative values which are **not** compatible with the ranges of market or societal relative values. This situation is illustrated in Figure 2.11 on page 63 at the end of the chapter by unit E_1. If the maximum value of input 1 relative to input 2 is limited to $\mathbf{p}_1$ which, in this case represent a normal to the facet E_2E_3 of the technical efficiency set, then E_1, despite its technical efficiency, is evaluated as allocatively inefficient. All techniques spanning the facet E_2E_3 are cheaper/more economical ways of producing **y**. The technique represented by E_1 can be construed as obsolete. The "closest" allocatively efficient technique is that of E_2.

The evaluation of (**x**,**y**) requires the identification of a "closest" (in terms of input and output ratios) allocatively efficient technique, (**x***,**y***), and of acceptable price vectors $(\mathbf{p}_{x*},\mathbf{p}_{y*})$ such that, at these prices, **x*** is a least cost alternative to generating revenue $\mathbf{p}_{y*}\mathbf{y}^* \geq \mathbf{p}_{y*}\mathbf{y}$, and **y*** is a maximum revenue alternative to spending $\mathbf{p}_{x*}\mathbf{x}^* \leq \mathbf{p}_{x*}\mathbf{x}$. Following the format of the technical efficiency measure, allocative efficiency of (**x**,**y**) can then be measured by E_A(**x**,**y**) given by the following expression:

$$E_A(\mathbf{x},\mathbf{y}) = \frac{\dfrac{\mathbf{p}_{y*}\mathbf{y}}{\mathbf{p}_{x*}\mathbf{x}}}{\dfrac{\mathbf{p}_{y*}\mathbf{y}^*}{\mathbf{p}_{x*}\mathbf{x}^*}}$$

Note that for (**x**,**y**) technically but not allocatively efficient, the referent allocatively efficient unit, (**x***,**y***), does not satisfy $\mathbf{x}^* \leq \mathbf{x}$ and $\mathbf{y}^* \geq \mathbf{y}$. To attain allocative efficiency (**x**,**y**) is required to implement some degree of substitution across inputs or across outputs.

Measuring allocative efficiency hence requires the ability, for the procedure, to handle some control over the pricing mechanism so that identified efficiency prices be compatible with/restricted to the set of market or societal values.

2.10 Summary and Conclusions

Assuming a class of technologies described by (IC1) - (IC4), (Convexity) and strong disposability of essential inputs, the evaluation of the technical efficiency of an economic unit described by an input consumption vector, **x**, characteristic of the unit's technique and scale of production, and by an output production vector, **y**, characteristic of the unit's output mix and productivity, is effected by searching for an efficient unit described by (**x***, **y***) such that:

i) $\mathbf{x}^* \leq \mathbf{x}$, $\mathbf{y}^* \geq \mathbf{y}$.

ii) **x*** is representative of a technique that is closest to that of **x** in terms of input ratios.

iii) **y*** is representative of a product mix that is closest of that of **y** in terms of output ratios.

iv) There exist efficiency prices $\mathbf{p}_x > 0$ such that $\mathbf{p}_x\mathbf{x}^* \leq \mathbf{p}_x\mathbf{x}$ for all **x** in L(**y***).

v) There exist efficiency prices $\mathbf{p}_y > 0$ such that $\mathbf{p}_y\mathbf{y}^* \geq \mathbf{p}_y\mathbf{y}$ for all **y** in P(**x***).

The technical efficiency of the production unit (**x**,**y**) is then the product of its input technical efficiency and its output technical efficiency:

$$E(\mathbf{x},\mathbf{y}) = E_I(\mathbf{x},\mathbf{y}).E_O(\mathbf{x},\mathbf{y}) = \frac{\mathbf{p}_x . \mathbf{x}^*}{\mathbf{p}_x . \mathbf{x}} . \frac{\mathbf{p}_y . \mathbf{y}}{\mathbf{p}_y . \mathbf{y}^*} = \frac{\dfrac{\mathbf{p}_y . \mathbf{y}}{\mathbf{p}_x . \mathbf{x}}}{\dfrac{\mathbf{p}_y . \mathbf{y}^*}{\mathbf{p}_x . \mathbf{x}^*}}$$

The technical efficiency measure satisfies the following set of properties:

(1) $E(\mathbf{x},\mathbf{y}) = 1 \Leftrightarrow \mathbf{x} \in \mathrm{Eff}(\mathbf{y})$ and $\mathbf{y} \in \mathrm{Eff}(\mathbf{x})$.

(2) $E(Dx,D'y) = E(x,y)$ $\forall$ $x \in \mathbb{R}^m_+$, $\forall$ D: (mxm) strictly positive diagonal, $\forall$ $y \in \mathbb{R}^s_+$, $\forall$ D': (sxs) strictly positive diagonal.

If waste is preemptively identified across inputs, that is we can write:

$x^* = \theta^* x - e$ with $\theta^* = \mathrm{Min}_\theta \{\theta: \theta x \in L(y)\}$, then

(3) $E_I(\theta x,y) = 1/\theta\ E_I(x,y)$, $E_O(\theta x,y) = E_O(x,y)$, $E(\theta x,y) = 1/\theta\ E(x,y)$ $\forall$ θ: $\theta x \in L(y)$, $x \in L(y)$.

(4) If $x \in L(y)$, $x' \in L(y)$, and $x \geq x'$, then $E_I(x',y) > E_I(x,y)$ but it is not always true that $E_O(x',y) > E_O(x,y)$ and $E(x',y) > E(x,y)$.

If waste is preemptively identified across outputs, that is we can write:

$y^* = \phi^* y + s$ with $\phi^* = \mathrm{Max}_\phi \{\phi: \phi y \in P(x)\}$, then

(5) $E_O(x,\phi y) = \phi\ E_O(x,y)$, $E_I(x,\phi y) = E_I(x,y)$, $E(x,\phi y) = \phi\ E(x,y)$ $\forall$ ϕ: $\phi y \in P(x)$, $y \in P(x)$.

(6) If $y \in P(x)$, $y' \in P(x)$, and $y' \geq y$, then $E_O(x,y') > E_O(x,y)$ but it is not always true that $E_I(x,y') > E_I(x,y)$ and $E(x,y') > E(x,y)$.

Allocative efficiency requires technical efficiency. The evaluation of (x,y) requires the identification of a "closest" (in terms of input and output ratios) allocatively efficient technique, (x^*,y^*), and of acceptable price vectors (p_{x^*},p_{y^*}) such that, at these prices, x^* is a least cost alternative to generating revenue $p_{y^*}y^* \geq p_{y^*}y$, and y^* is a maximum revenue alternative to spending $p_{x^*}x^* \leq p_{x^*}x$. Following the format of the technical efficiency measure, allocative efficiency of (x,y) can then be measured by $E_A(x,y)$ given by the following expression:

$$E_A(x,y) = \frac{\dfrac{p_y \cdot y}{p_x \cdot x}}{\dfrac{p_y \cdot y^*}{p_x \cdot x^*}}$$

If (x,y) is technically but not allocatively efficient, the referent allocatively efficient unit, (x^*,y^*), does not satisfy $x^* \leq x$ and $y^* \geq y$. Some degree of substitution across inputs or across outputs is then required to attain allocative efficiency.

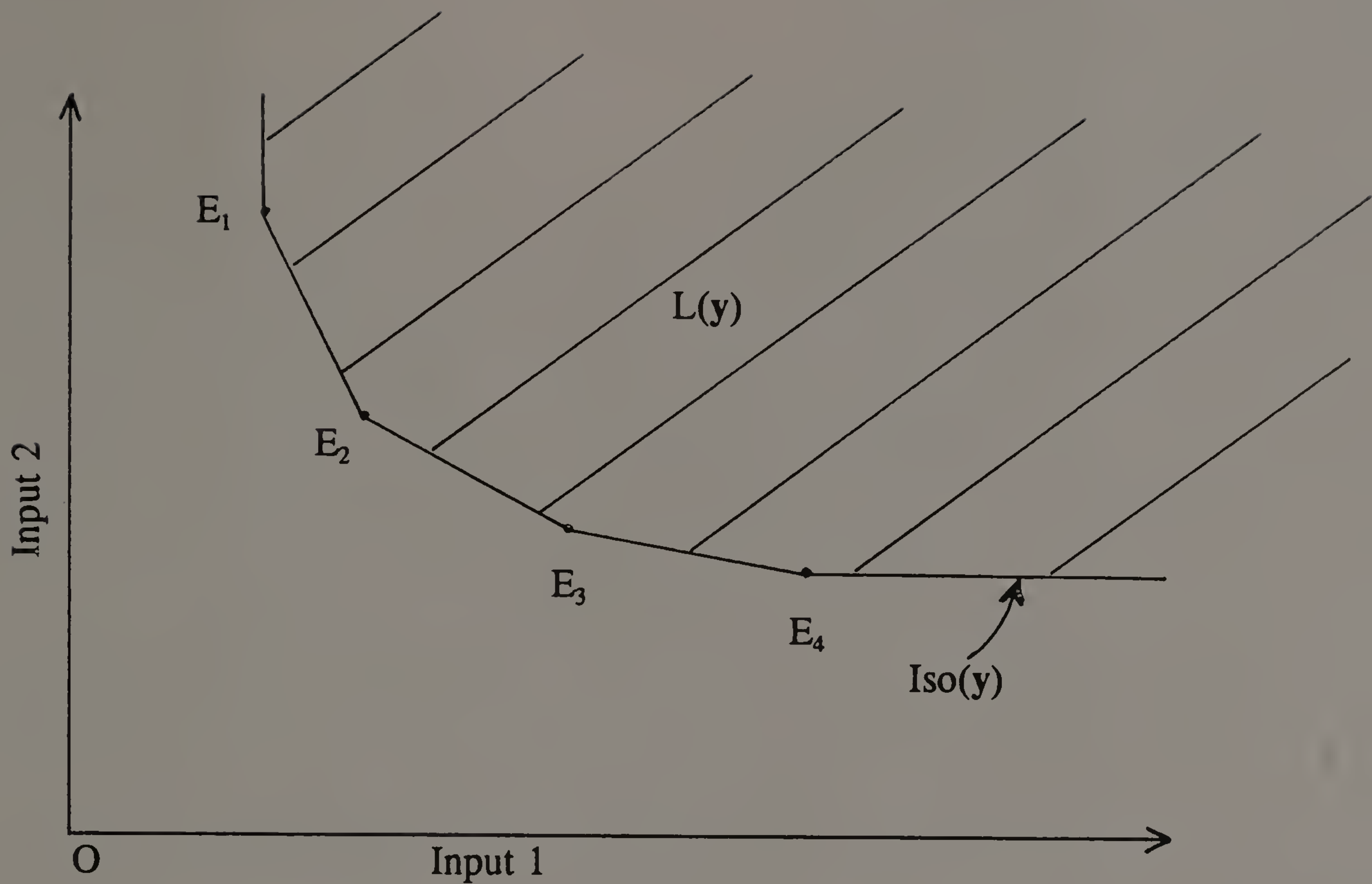

Figure 2.1 Input Set L(**y**)

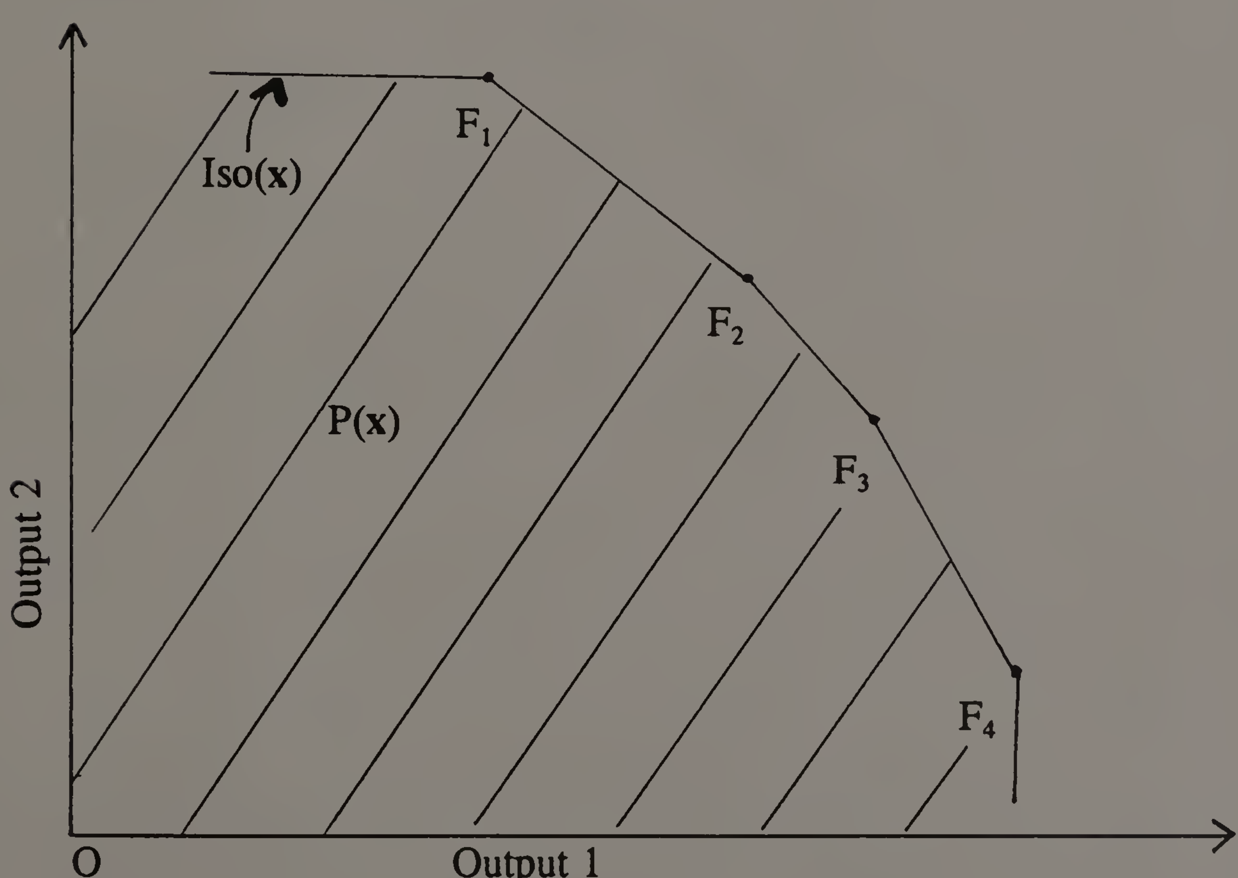

Figure 2.2 Output Set P(**x**)

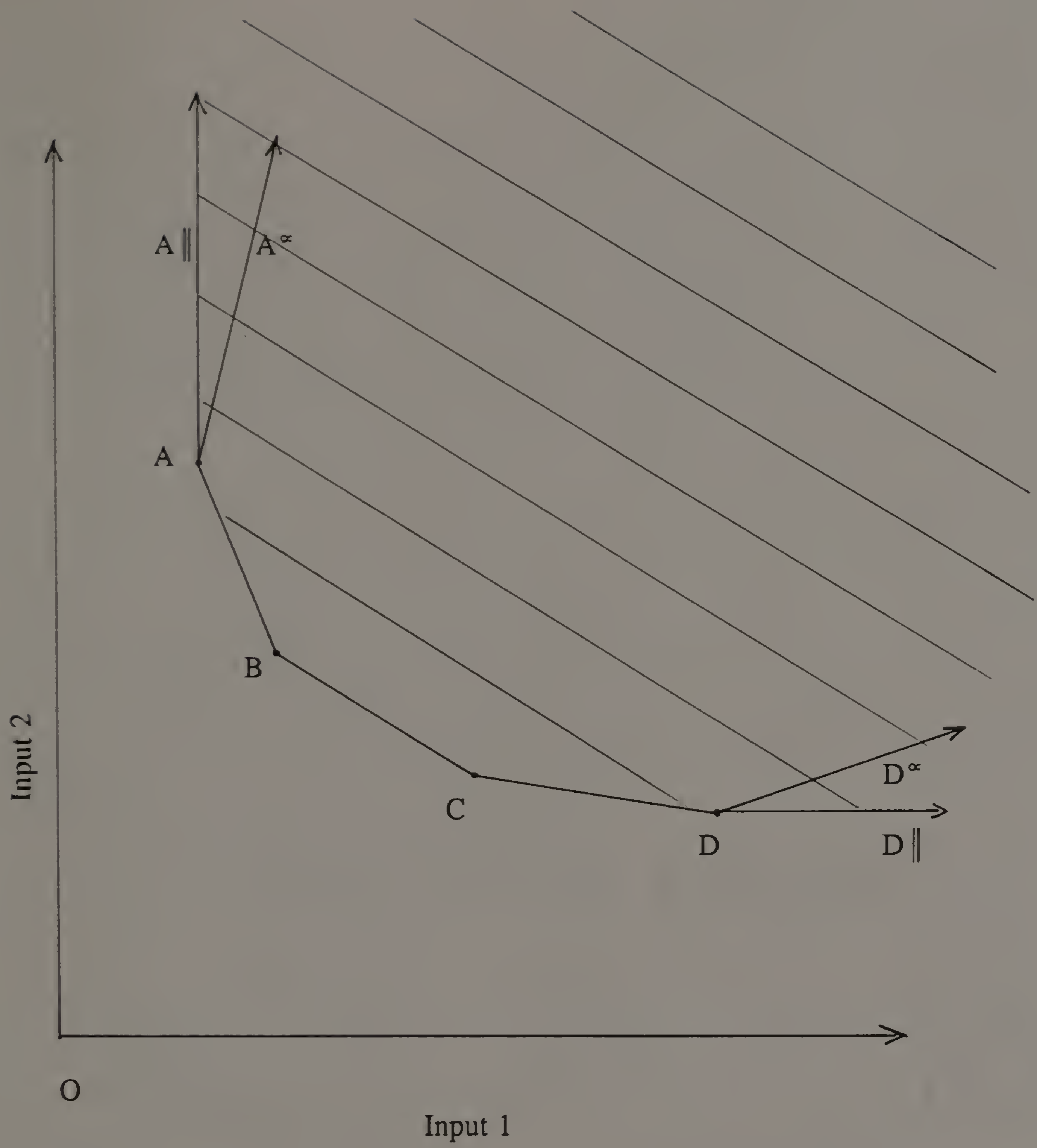

Figure 2.3 Weak and Strong Disposability of Inputs

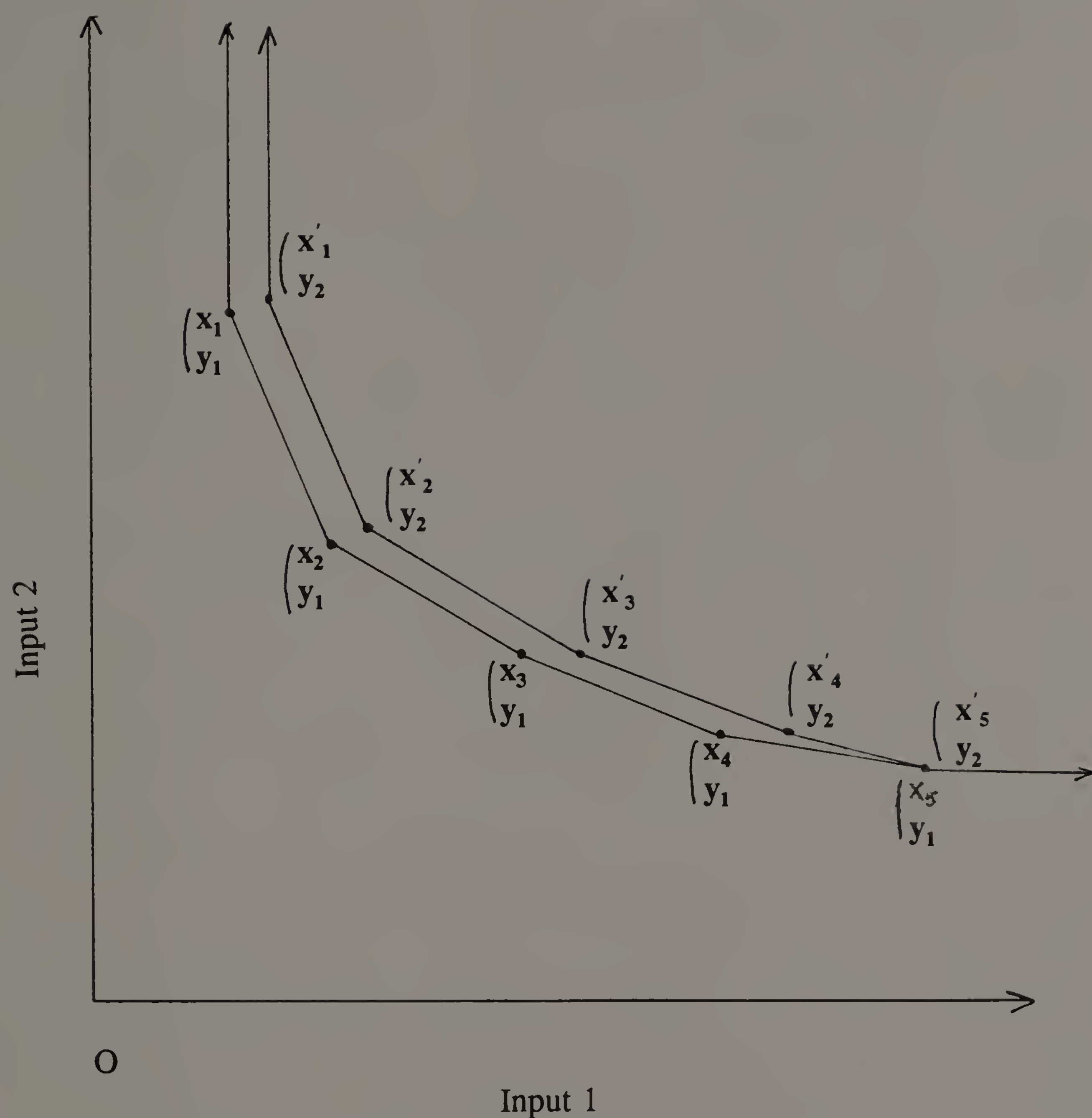

Figure 2.4 Input Isoquants Iso(y_1), Iso(y_2), $y_2 \geq y_1$

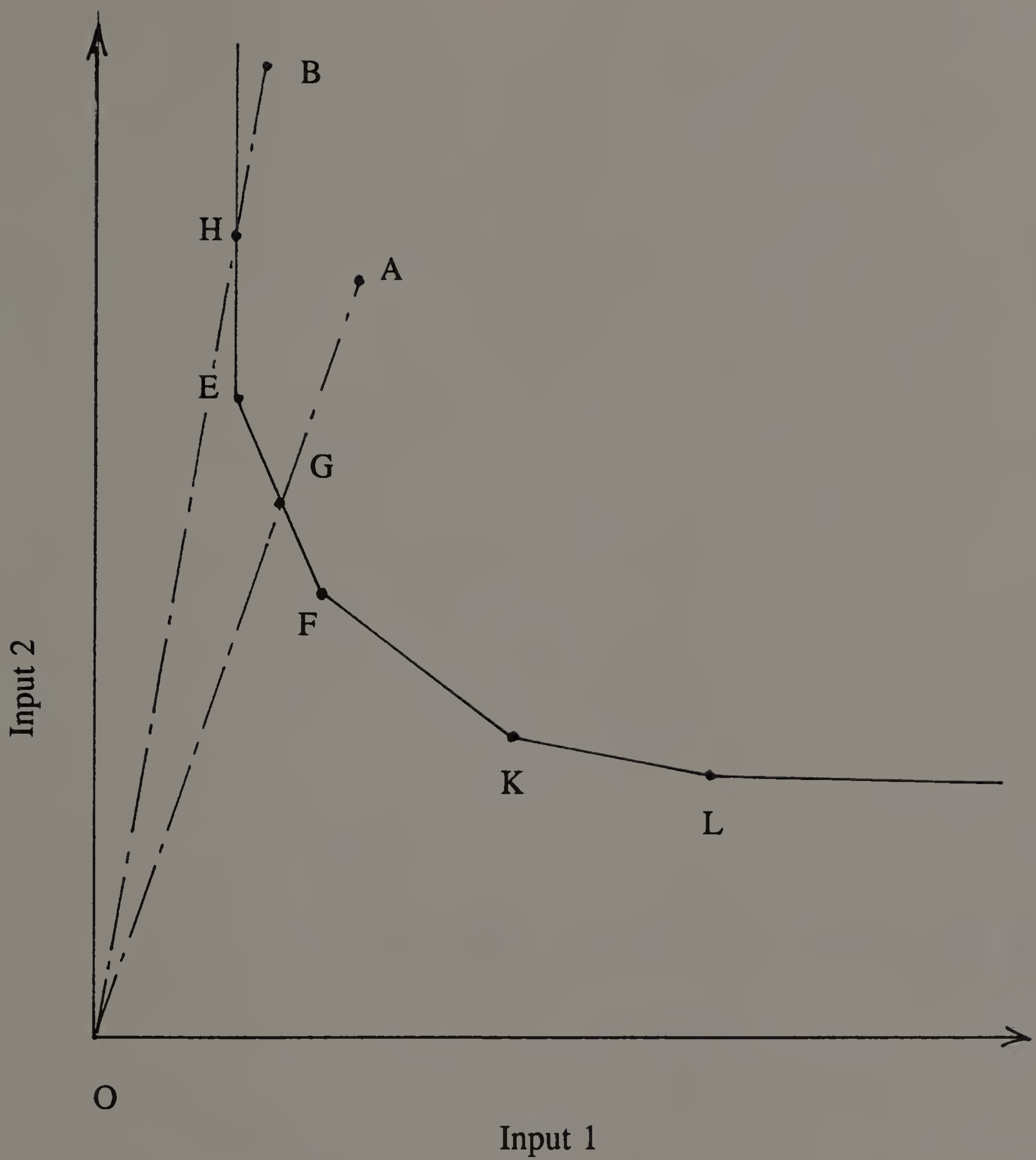

Figure 2.5 Internal and External Inefficiencies

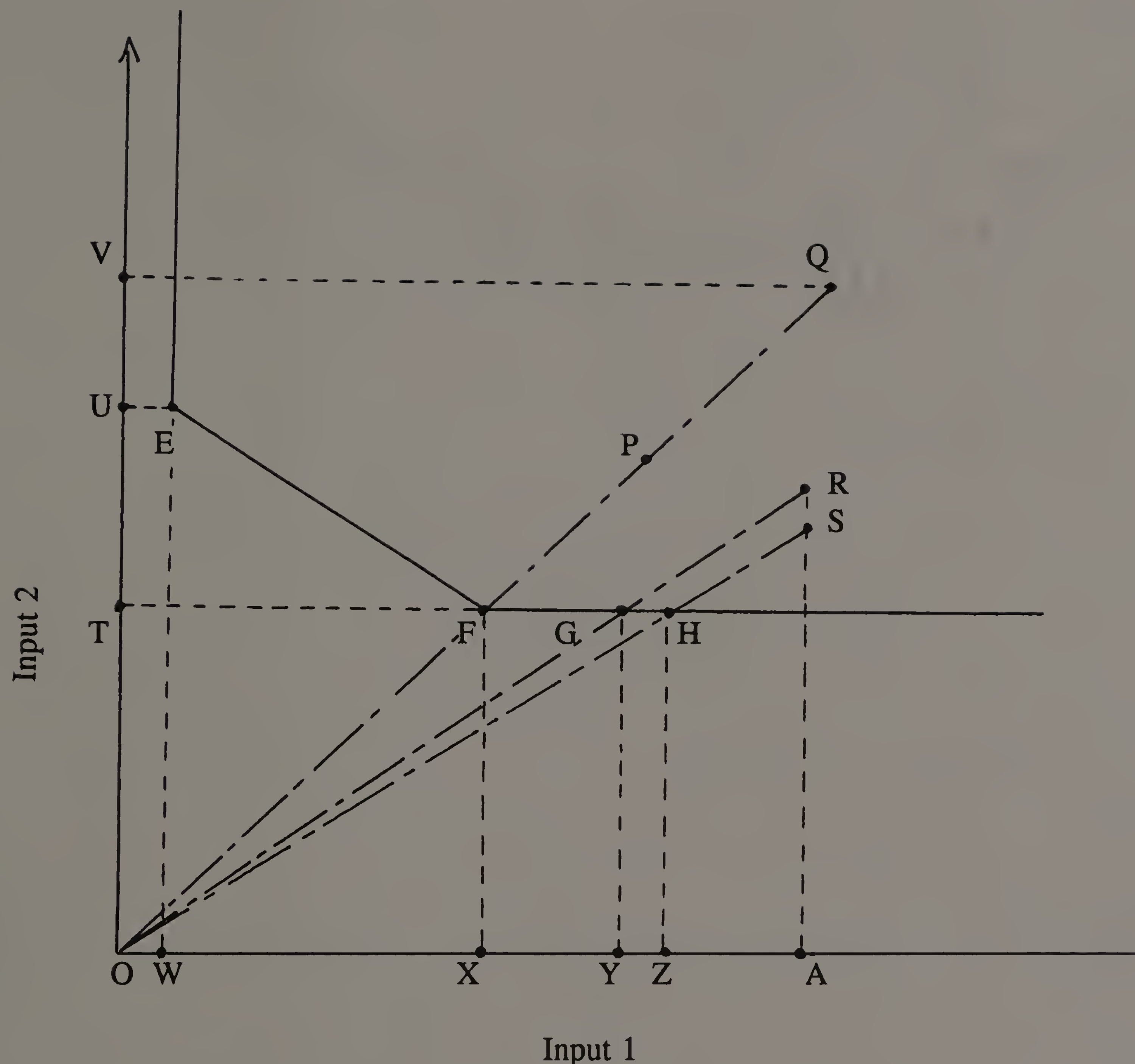

Figure 2.6 Example Input Set L(y)

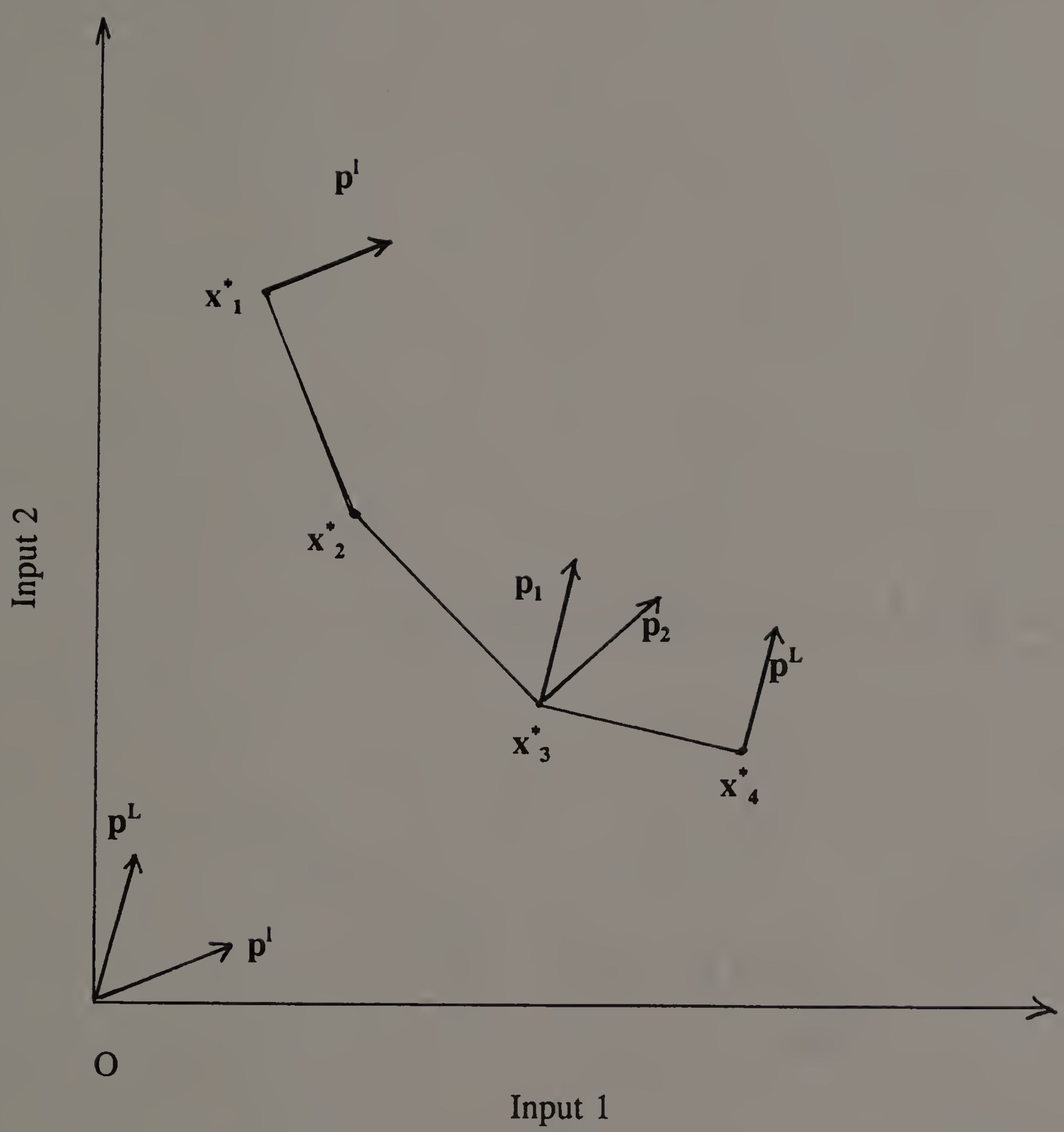

Figure 2.7 Cone of Admissible Efficiency Prices in Input Space

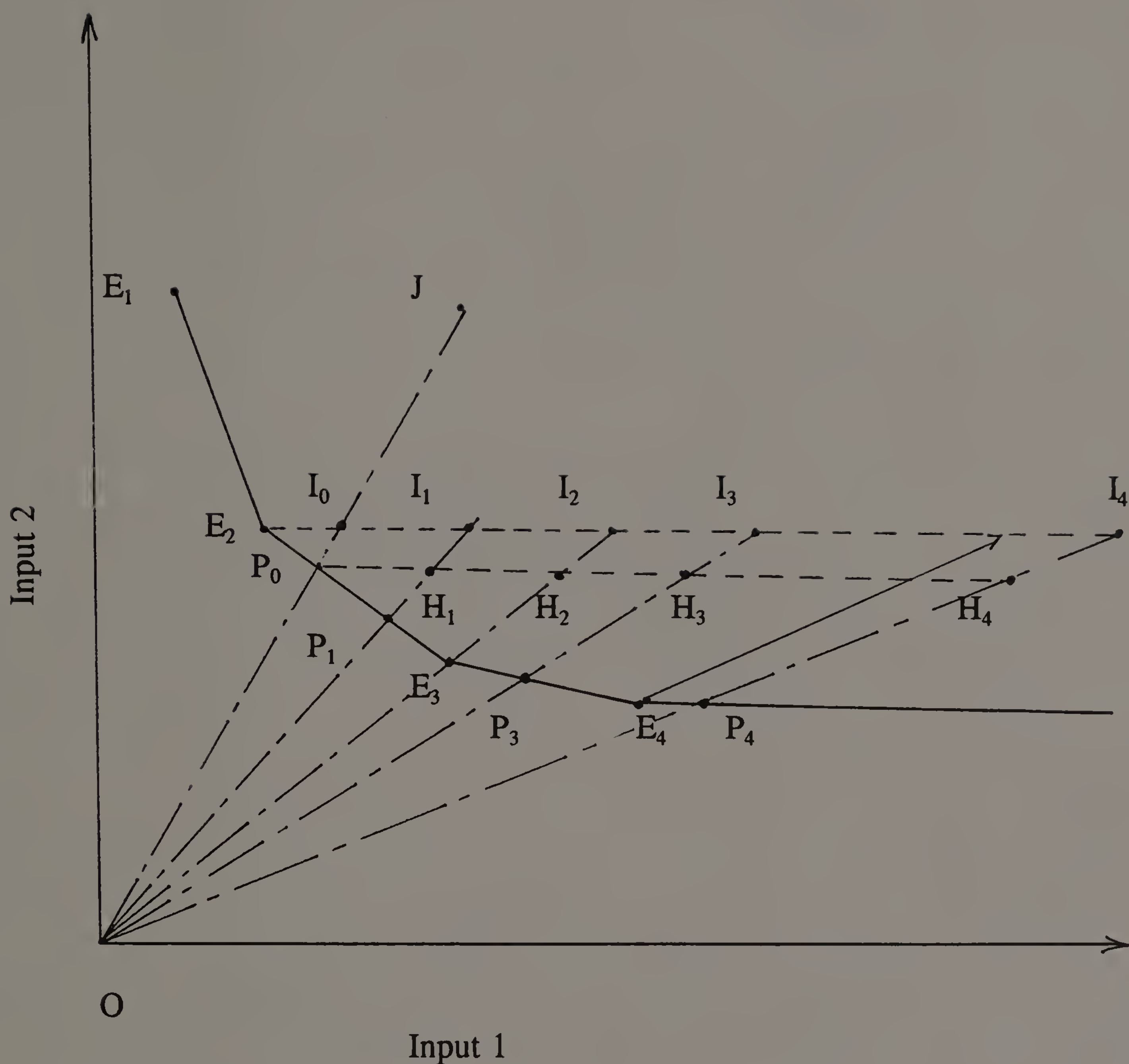

Figure 2.8 Measuring Inefficiency

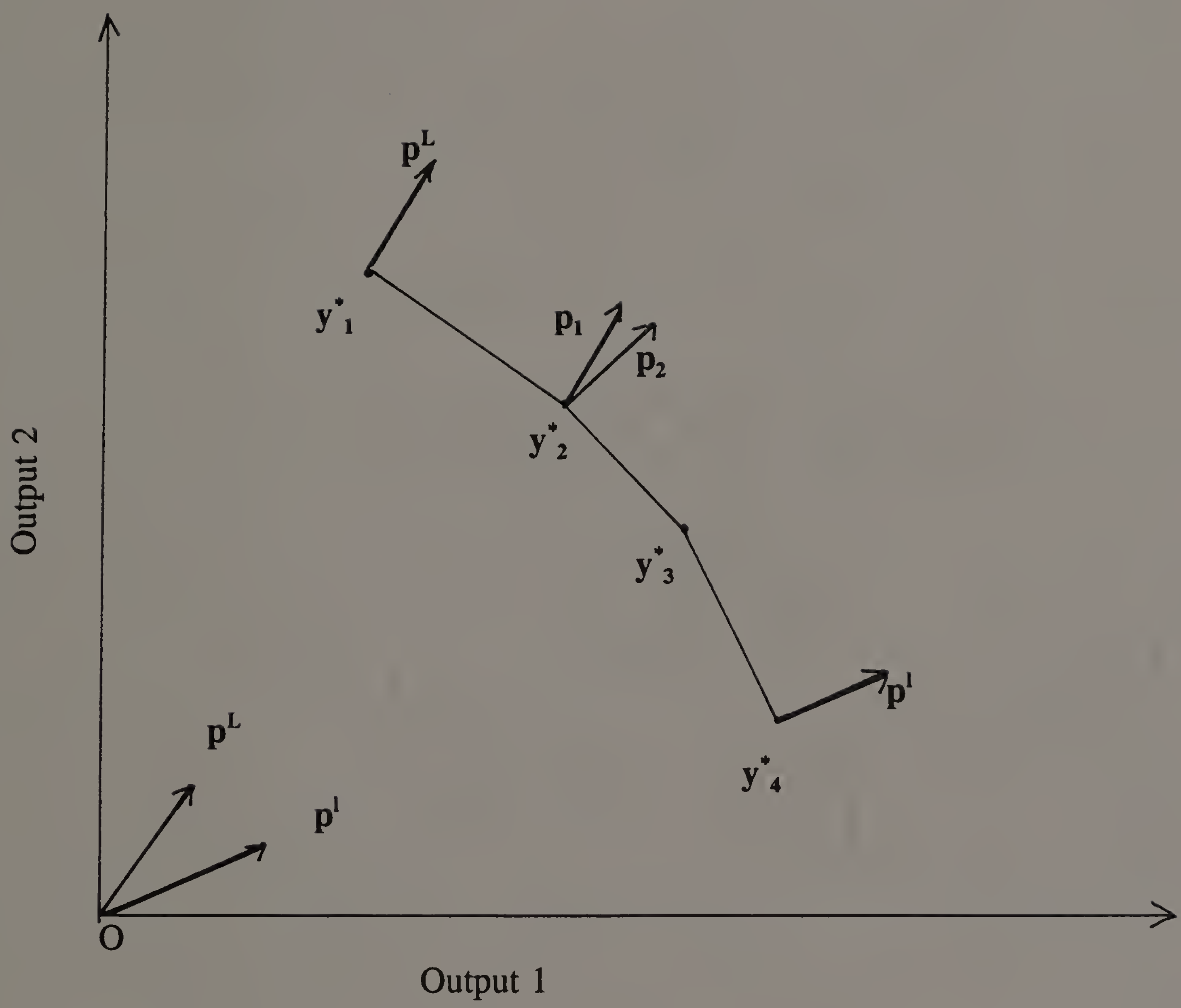

Figure 2.9 Cone of Admissible Efficiency Prices in Output Space

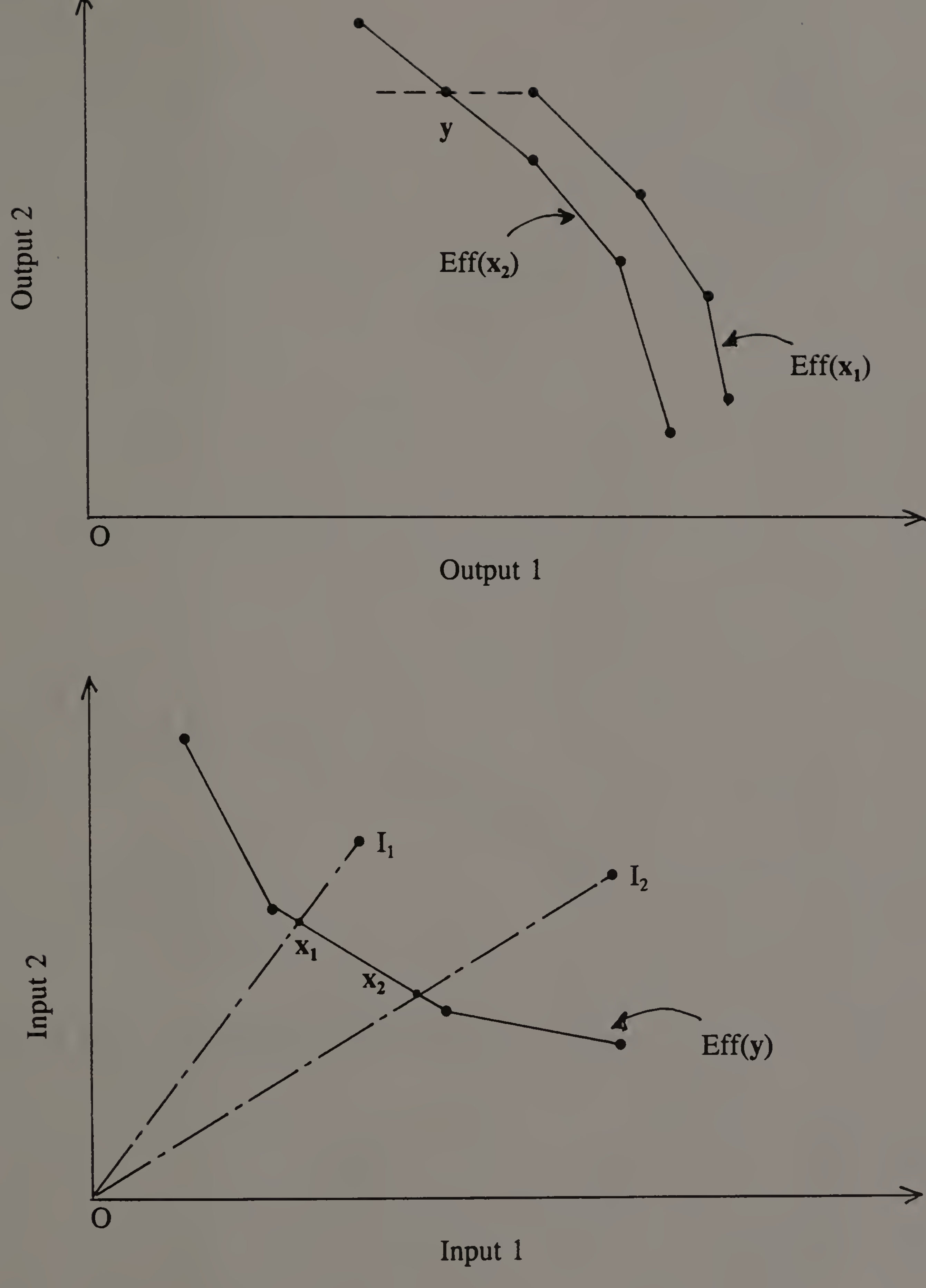

Figure 2.10 Components of Global Efficiency

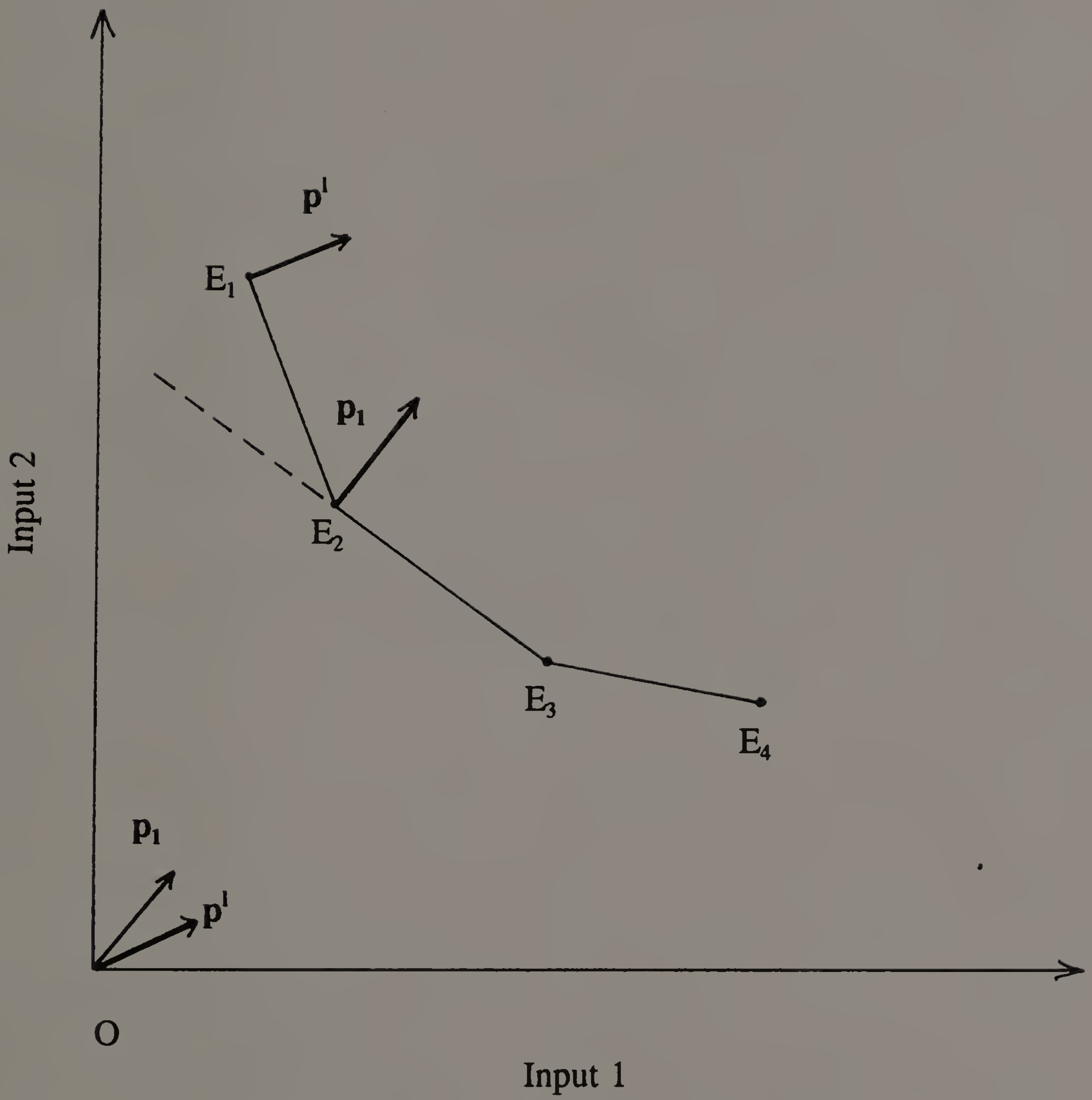

Figure 2.11 Technical and Allocative Efficiency

CHAPTER 3

MEASURING INEFFICIENCY IN PRACTICE

DATA ENVELOPMENT ANALYSIS

3.1 Introduction and DEA Terminology

The preceding chapter developed a measure to gauge the efficiency (or degree of inefficiency) of a production unit characterized by input consumption levels and output production levels. The measure is expressed with respect to these levels and with respect to the consumption and production levels of a reference unit operating on the frontier of feasible achievements. The actual computation of the efficiency score therefore rests on, first, the ability to locate the frontier of feasible achievements and, second, on the definition of a mechanism to select the referent point on the identified frontier. This chapter addresses these two concerns.

The concept of locating an empirical frontier is at the core of Data Envelopment Analysis (DEA), a mathematical programming technique developed by Charnes, Cooper, and Rhodes in 1978 to evaluate the relative performance of decision-making units (DMUs). A variety of data envelopment analysis models has appeared in the literature as have numerous studies employing the technique [Banker, Charnes, Cooper, Swarts, Thomas, 1989], [Seiford 1990]. Each of the various models for data envelopment analysis (DEA) seeks to determine which of n decision making units (DMUs) determine an **envelopment surface** when considering **m** inputs and **s** outputs. The statement of these models carries at the outset assumptions regarding the form of the envelopment surface. Units that lie on (determine) the surface are deemed **efficient** in DEA terminology. The assumed form of the envelopment surface bears a direct relationship to the set of efficient units. Units that do not lie on the surface are termed **inefficient** and the analysis provides measures of their relative efficiency. The forms of the envelopment surface that are most commonly used are presented in section 2.

For decision making unit l, $x_{il} > 0$ denotes the i^{th} input value and $y_{rl} > 0$ denotes the r^{th} output value and $\mathbf{x}_l$ and $\mathbf{y}_l$ denote, respectively, the vectors of input and output values. A solution to a DEA model for $(\mathbf{x}_l, \mathbf{y}_l)$ identifies a point $(\mathbf{x}^*,\mathbf{y}^*)$ on the envelopment surface in accordance with **the evaluation principles** implicit in the model. The evaluation principles are criteria that guide the determination and measurement of efficiency. Within the mathematical programs they define the manner in which projected points on the envelopment surface are obtained for DMUs that are inefficient. A variety of evaluation principles are introduced in section 3.

Three of the DEA models that are most often associated with the DEA methodology are CCR, BCC and ADD models. The form of the envelopment surface for the BCC and the ADD models is exactly the same and different from the one underlying the CCR model. This implies that the DMUs determined to be efficient are exactly the same in both the BCC and ADD models. However, these two models differ in their implicit evaluation principles. Both the CCR and the BCC models obey the same evaluation principles. Each of these models is reviewed in section 4.

A more recent class of models commonly referred to as cone-ratio models [Charnes et al.,1990] and assurance-region models [Thompson et al., 1986] are introduced in Section 5. These models are intended to bring efficiency evaluations closer to economic efficiency ratings. However, to this date, no general formulation and economic interpretation of such models has appeared in the literature. Section 5 fulfills this need. It develops a new model in the context of production theory that strengthens the bridge between DEA and economics. A general formulation is offered that underscores the flexibility of this class of models in terms of evaluation principles, and their properties and advantages in terms of efficiency measurement.

The respective behavior of these models is constrasted in section 6 by means of a computational illustration based on a synthetic data set figuring four input isoquants in a 2 x 2 input-output space. Their performance with respect to the derived efficiency scores is given particular emphasis.

Section 7 summarizes all findings and concludes on the viability of Data Envelopment Analysis as a methodology to assess the relative efficiency of decision-making units.

3.2 Forms of Data Envelopment

Acceptable forms of envelopes for the data derive directly from accepted properties regarding the production possibility set. If the description of that set rests on the observation of empirical data points then the least costly, in terms of number of assumptions, is to declare efficient (i.e. envelope-defining) any point that is not dominated in at least one measure by any other point. Conversely a rating of inefficiency for a point DMU_o implies that there exists another observed point, DMU_l, that dominates DMU_o. That is, DMU_l exhibits lower consumption of all inputs and higher production of all outputs with strict comparisons in at least one input and/or at least one output. In the literature this approach to describe the production possibility set is known as the "free disposal hull" (FDH) methodology [Tulkens, 1990]. It leads to an envelope that wraps the data very closely and identifies a large number of efficient points relative to other envelopment forms. As an illustration let us consider the case where observations are limited to the input set L(y) represented in Figure 3.1 on page 117 at the end of the chapter. In this situation the points E, F, and U are identified as efficient and hence, as envelope-defining observations. Obviously this methodology allows for non-convex isoquants which is not in accordance with traditional assumptions of economic theory. This approach will not be given any further consideration.

Building upon the traditionally accepted assumptions of convex input sets and convex output sets characterizing substitutions across inputs and across outputs respectively, the consideration of, and assumptions regarding, the relationship between outputs and inputs for any technique are then sufficient to complete the description of the production possibility set and define acceptable forms of envelope for the data.

The particular feature of the production function that defines the form of the envelope is the characterization of returns-to-scale. As mentioned in Peterson [1990] a property of ray unboundedness for the production possibility set, that is, as defined in Chapter 2, the assumption that any multiple of an observed point belongs to the production possibility set as well, is equivalent to the assumption of non-decreasing returns to scale for the technique described by the input ratios of the observed point when proportionally larger input vectors are considered, and to the assumption of non-increasing returns to scale when proportionally smaller input vectors are considered. These assumptions, coupled with the empirical knowledge that the general behavior for a production technique is to successively exhibit increasing, constant, and decreasing returns to scale as inputs increase proportionately, lead to the deduction that the assumption of ray unboundedness for the production possibility set is equivalent to the assumption of constant returns to scale for the technology. It follows that the exclusion of the ray unboundedness property, from the set of properties concerning the production technology, acknowledges non-constant returns to scale for the technology, that is the possibility of any technique exhibiting variable returns to scale as inputs increase proportionately.

Finally, convexity of the production possibility set is justified by considering that economic data on consumption of inputs and production of outputs are in fact the representation of average **rates** of consumption and production over time. Each observation is representative of a production facility dedicated to a technique so that its operations are characterized by steady state rates of production given its scale of operations. A convex combination of observations can then be interpreted as a time-sharing leasing agreement where the production facilities entering the combination are successively leased for subperiods of time over the period of measurement.

In practice existing DEA models address envelopment surfaces of two types. In this exposition, these surfaces are referred to as **constant returns-to-scale** (CRS) and **variable returns-to-scale** (VRS) surfaces.

The CRS envelopment surface consists of hyperplanes in R^{m+s} which are particular facets of the **convex polyhedral cone** determined by the vectors $(\mathbf{x}_j,\mathbf{y}_j)$, j = 1,...n, and which are consistent with the postulate of ray unboundedness for the production possibility set. This type of envelope is the loosest to the extent that it identifies the smallest set of efficient units relative to other envelopment forms.

The VRS envelopment surface consists of hyperplanes which are particular facets of the **convex hull** of the points $(\mathbf{x}_j,\mathbf{y}_j)$, j = 1,...n in R^{m+s}. This type of envelopment is intermediate between the FDH type and the CRS type in terms of the size of the set of efficient units. In fact there is a relation of inclusion among the sets of efficient units for these three types of envelopes. We have:

$$E_{CRS} \subset E_{VRS} \subset E_{FDH}$$

where E denotes the set of efficient points.

For each of the above types of surfaces, to determine whether DMU_l is on the envelopment surface, independently of measuring the unit's degree of inefficiency if it is not, Charnes, Cooper and Thrall [1986] solve a simple linear program that seeks to identify a positive linear combination of observed points that dominates DMU_l in the Pareto-Koopmans sense. In the mathematical statements of the models, the s x n matrix of outputs for the n observed data points is denoted **Y** and the m x n matrix of inputs is denoted **X**. The vector $\boldsymbol{\lambda}$ identifies which units constitute the linear combination and the vectors **s** and **e** represent respectively the identified output slacks and input excesses. In the case of the constant returns-to-scale (CRS) envelopment the following model is solved:

$$
\begin{aligned}
&\text{Min } \lambda_1 \\
&Y\lambda - s = Y_1 \\
&-X\lambda - e = -X_1 \\
&\lambda \geq 0,\ e \geq 0,\ s \geq 0
\end{aligned}
$$

and, in the case of the VRS envelopment, the following model is solved:

$$
\begin{aligned}
&\text{Min } \lambda_1 \\
&Y\lambda - s = Y_1 \\
&-X\lambda - e = -X_1 \\
&1\lambda = 1 \\
&\lambda \geq 0,\ e \geq 0,\ s \geq 0
\end{aligned}
$$

The convexity relation explicitly stated in the VRS formulation is masked by the ray unboundedness postulate in the CRS formulation. In both cases DMU_1 is on the envelope if and only if **s** = 0, and **e** = 0 at optimality, for all optimal solutions.

3.3 Principles of Evaluation

3.3.1 Introduction

Once the form of the envelopment has been prespecified the above models unambiguously report on the efficiency status of any DMU, that is they provide an answer to the binary question of whether a DMU is efficient or not. When it is recognized that a DMU is **not** efficient the next question that arises is the assessment of the unit's inefficiency. The arguments of Chapter 2 contribute to the answer by demonstrating that DMU_1's efficiency can be measured by:

$$E(x_l, y_l) = \frac{p_{x*}x^*}{p_{x*}x_l} \cdot \frac{p_{y*}y_l}{p_{y*}y^*}$$

where ($\mathbf{x^*} = \mathbf{X}\lambda, \mathbf{y^*} = \mathbf{Y}\lambda$) is representative of an efficient DMU and ($\mathbf{p}_{x^*}$, $\mathbf{p}_{y^*}$) are admissible "efficiency prices", that is prices that identify $\mathbf{x^*}$ as a minimum cost alternative to producing $\mathbf{y^*}$ and $\mathbf{y^*}$ as a maximum revenue alternative to consuming $\mathbf{x^*}$. An inefficient DMU maps in the interior of the production possibility set and measuring its degree of efficiency hence necessitates the selection of a reference point on the envelope. Obviously infinitely many such reference points are feasible candidates. However, reasonable criteria can be introduced that will reduce the set of selectable referent points. These criteria summarize to a projection mechanism that, for each inefficient DMU, identifies a referent point against which the DMU's inefficiency is to be gauged. This projection mechanism is the embodiment of the evaluation principles or criteria. These principles may focus primarily on controlling input consumption or primarily on output production, or on avoiding waste without distinction between inputs and outputs, they may be defined within a short-term horizon or within a long-term horizon, finally they may define explicit tradeoffs across inputs and outputs respectively or they may allow for these tradeoffs to be defined endogeneously. The purpose of the next subsections is to explore, define, and operationalize evaluation principles, that is criteria that will afford the selection of ($\mathbf{x^*}, \mathbf{y^*}$) on the envelope and of ($\mathbf{p}_{x^*}$, $\mathbf{p}_{y^*}$) to aggregate input and output measures. The identification of ($\mathbf{x^*}, \mathbf{y^*}$) on the envelope to serve as reference to evaluate DMU$_l$'s efficiency immediately points to DMU$_l$'s weaknesses in the form of revealed excesses in input consumption and deficiencies in output production. These revealed weaknesses can then constitute the core of any remedial plan of actions. It follows that each possible projection on the envelope can be seen as an optimal response to distinct managerial objectives.

3.3.2 Orientation of the Evaluation

The orientation of the evaluation deals with the "quantity" side of managerial objectives as opposed to the "pricing" or tradeoff side. It states whether waste is to be identified first among inputs, or among outputs, or across all measures without any prioritization.

An **input-oriented** evaluation seeks $(\mathbf{x}^*,\mathbf{y}^*)$, against which the inefficiency of $(\mathbf{x}_l,\mathbf{y}_l)$ will be gauged, on the envelope so that $\mathbf{x}^*$ be representative of an efficient technique that is "closest" to the technique characterized by the input ratios of $\mathbf{x}_l$. Equivalently input-orientation seeks a projected point such that the proportional reduction in inputs is maximized. The primary concern of management implicit in this orientation is that the DMU being evaluated keep operating with its current technique, characterized by the actual input ratios, and gain efficiency by maintaining its current levels of outputs and decreasing its inputs.

An **output-oriented** evaluation seeks a projected point such that the proportional augmentation in outputs is maximized. In this situation, the primary objective is to reach efficiency by focusing on productivity gains while preserving the current output mix. For either orientation, satisfaction of the primary objective may not be sufficient to reach the envelopment surface or attain efficiency. Indeed it may happen that, for instance, after the smallest proportional input vector has been identified in the case of an input-oriented evaluation, that minimum input vector would allow production of a larger output vector than $\mathbf{y}_l$. In that case the primary objective alone failed to project DMU_l on the envelopement surface, and there might still be infinetely many options to reach that surface. When that happens, management has to decide on how to reach the envelope, that is identify and account for the remaining waste so that the full extent of the unit's inefficiency be gauged. The final choice will come from the stated secondary managerial objectives.

A **global orientation** for the evaluation does not prioritize the identification of waste over input or output measures. Instead it focuses on the "valuation" of the bundle of excess inputs and unrealized outputs. For input or ouput orientations the valuation of the remaining waste may become the focus of the secondary managerial objectives. This concern of "valuation" is addressed in the next subsection.

3.3.3 Aggregation across Non-Commensurable Measures

3.3.3.1 Introduction. The manner in which an inefficient unit is projected on the envelopement surface depends not only on the orientation of the evaluation but also on the aggregation method to account for any remaining waste. This method defines a metric within the input and output spaces and is entirely described by the efficiency price vectors, $(\mathbf{p}_x, \mathbf{p}_y)$. For instance if $(\mathbf{x}^*, \mathbf{y}^*)$ represents the projected point associated with the evaluation of $(\mathbf{x}_l, \mathbf{y}_l)$, given the chosen orientation and aggregation method characterized by $(\mathbf{p}_{x^*}, \mathbf{p}_{y^*})$, then the global waste revealed in DMU$_l$'s operations can be accounted by:

$$\mathbf{p}_{x^*}(\mathbf{x}_l - \mathbf{x}^*) + \mathbf{p}_{y^*}(\mathbf{y}^* - \mathbf{y}_l).$$

The vectors $(\mathbf{p}_{x^*}, \mathbf{p}_{y^*})$ are the vectors used in computing DMU$_l$'s efficiency and from the arguments presented in the preceding chapter, $\mathbf{p}_{x^*}$ ought to be such that $\mathbf{p}_{x^*}.\mathbf{x}^*$is minimum over L(y*) and $\mathbf{p}_{y^*}.\mathbf{y}^*$ is maximum over P(**x***). These vectors obviously play the role of prices representative of the relative valuation of inputs and outputs at the point (**x***, **y***). Depending on the knowledge and constraints of the environment, varying degrees of freedom may accompany the determination of $(\mathbf{p}_{x^*}, \mathbf{p}_{y^*})$ in practice. We distinguish between two main approaches which may respectively be associated with a short-term perspective and a long-term perspective regarding the efficiency evaluations. In the short-term prices tend to be sticky (in the economics sense). Some prices may even be considered fixed and exogenous. By constrast, in the long-run, relative prices as well as the price level tend to be granted more freedom. These perspectives are presented in the next subsections.

3.3.3.2 Explicit Pricing. This first approach reflects a short-term perspective where the evaluation is effected according to **preset** or fixed "prices" for the excess inputs and preset "opportunity costs" for unrealized outputs, that is within a framework of a fixed price level. The preset excess input prices can be interpreted as current equilibrium prices enforced in a centralized second market for inputs. Similarly the preset opportunity costs may represent going offer prices on a centralized market for the various outputs. A multiplicity of pricing schemes has found its way into the DEA literature. It is worth noting that the influence of pricing is secondary to that of the orientation to the extent that the pricing scheme is effective only if there is waste remaining after the proportional input reduction and/or proportional output augmentation. Whenever the unit is efficient, or successfully reaches the envelope after maximum proportional input reduction or maximum proportional output augmentation, pricing is endogenous but constrained to a minimum level defined by the explicit pricing scheme. This observation will be further clarified in the next section when we interpret the pricing schemes of the most commonly used DEA models. The remainder of this section presents the three most common pricing schemes.

Standard models are such that $(\mathbf{p}_x)_{il} = 1$, $i = 1,\ldots, m$, when $e_{il} > 0$, and $(\mathbf{p}_y)_{rl} = 1$, $r = 1,\ldots, s$ when $s_{rl} > 0$. They implicitly assume that the marginal worths of each unit of the non-zero output slacks and nonzero excess inputs are identical and equal to 1, independently of the DMU being evaluated and independently of its scale of operations. This pricing mechanism implies that a unit of slack of any measure, be it input or output, is equated to a unit of waste, and that an objective of the evaluation is simply to account for global waste. This pricing mechanism therefore mirrors the particular conversion system defined by the scale and units of measurement of the various inputs and outputs. For instance, if input 1 measures annual wages in millions of dollars, input 2 measures annual salaries in thousands of dollars and input 3 measures the number of hours worked in thousands of hours, then whenever there is slack in these inputs, one million dollars of annual wages is equivalent to one thousand dollars of annual salaries, and equivalent to one thousand hours of work. It follows that caution should be exercised in selecting units of measurement if a standard model is to be used without manipulation of the data. To fully "standardize" the model and prevent the aggregation of non-commensurable measures some preprocessing of data may be required. Two possibilities are discussed below:

(i) Standardize each input and output measure. The resulting data will be unitless and the values of output slacks and excess inputs will capture the deviations from efficient levels within and across measures. A minor drawback is that the transformed data will exhibit both negative and positive values which will require further manipulation before LP-based DEA models are used. A more serious drawback stems from the lack of justification/interpretation that can be granted to the averages of each measure. The average values, themselves, are aggregates of both efficient and inefficient observed levels and therefore can only be representative of an inefficient theoretical DMU.

(ii) Prorate each input measure by the minimum observed value and each output measure by the maximum observed value. The transformed data will be unitless and the slacks will capture the deviations from efficient levels within and, relatively, across measures. The drawbacks of preprocessing data by standardizing inputs and outputs are completely avoided by preprocessing data by prorating.

Invariant models are such that $(p_x)_{il} = 1/x_{il}$, $i = 1,\ldots, m$, when $e_{il} > 0$, and $(p_y)_{rl} = 1/y_{rl}$, $r = 1,\ldots, s$ when $s_{rl} > 0$. This pricing scheme assumes that the marginal values of non-zero output slack and excess input variables are not identical. Moreover these prices are DMU specific, reflecting the fact that different techniques demand different input and output mixes. Consequently, different relative values are assigned to the various inputs and outputs. These prices are consistent with an emphasis on proportional reduction of inputs and proportional augmentation of outputs for each DMU. Indeed, assuming that waste can be entirely eliminated through proportional input reductions, these prices are such that for all inputs i, j, we have:

$$\frac{e_i}{x_i} = \frac{e_j}{x_j} \qquad i, j = 1,\ldots,m \quad i \neq j$$

where $e_i = x_i - x^*_i$, $i = 1,\ldots,s$ represent the revealed excess consumption for each input. These prices are then expressing that the marginal value of the last unit of waste is the same across all inputs.

Finally **normed** models are such that prices are given by the reciprocal coordinates of the barycenter of all <u>efficient</u> units, namely:

$$(p_x)_{il} = \frac{1}{n_e \sum_{j \in E} x_{ij}} \qquad i = 1,\ldots,m, \text{ when } e_{il} > 0$$

$$(p_y)_{rl} = \frac{1}{n_e \sum_{j \in E} y_{rj}} \qquad r = 1,\ldots,s, \text{ when } s_{rl} > 0$$

where $E = \{j \mid DMU_j \text{ is efficient}\}$ and n_e is the cardinality of E. These prices possess the following characteristics: they are the same for all DMUs, they distinguish between inputs and outputs in terms of relative value. The relative weighting is established by retaining information on efficient units only (implying the prior identification of efficient units). Possible arguments against this choice of prices deal with their uniqueness across DMUs and their artificiality. A unique set of prices applicable to all inefficient DMUs ignores the eventual reality of spatially separated markets which allow for different supply and demand conditions and equilibria and, hence, for multiple, or ranges of, relative prices. The barycenter of efficient points from which the prices are derived corresponds to an artificial DMU that is definitely inefficient since it is interior to the polytope defined by the efficient units.

As mentioned above, normed models require knowledge of the envelopment surface. It is therefore necessary to first determine the set of efficient units. This is done by performing a DEA analysis with any one of the models for the particular form of the surface decided upon (CRS or VRS), since efficiency is independent of the chosen pricing scheme. Prices for the normed model are then computed from the barycenter of the identified efficient units.

Each of the above pricing schemes can lead to different projected points on the envelope. This is expected since each mechanism reflects a different valuation system for the identified waste. However, when using any one system, the projected point ought to remain the same regardless of the units of measurement of the data. In the case of alternate optima the set of projected points should remain the same. This remains true with the invariant and the normed models since their valuation systems are invariant to units of measurements. It is true for the standard models <u>only</u> when the data are unitless.

3.3.3.3 Constrained Implicit Pricing. This second approach reflects a long-term perspective where the price level is determined endogeneously in the course of the evaluation, and where the evaluation explicitly incorporates recommendations to change the technique of production whenever appropriate, i.e. whenever the DMU, using the terminology introduced in Chapter 2, is relatively inefficient. These recommendations are formulated by means of substitutions across inputs and across outputs, hence allowing for strict augmentation of some input consumptions and strict diminution of some output productions. It follows that this approach departs from a Pareto-Koopmans evaluation.

The only constraints on pricing stem from the specification of **ranges** of price ratios defining acceptable tradeoffs across inputs and across outputs respectively. Ranges rather than unique price ratios are considered to acknowledge, in this case as well, the reality of spatially separated markets with differing supply and demand conditions and equilibria. Substitutions, that is recommendations to change technique, will occur whenever the current pattern[1] of input consumption or output production cannot be justified by any acceptable price vector as a cost-minimizing and/or revenue-maximizing pattern. The specification of the ranges may derive from historical records, or may be imposed by a central authority, or may be arrived at by consensus across all DMUs prior to the evaluation.

[1] A pattern is defined by the relative consumptions of inputs and relative productions of outputs.

We will next review the most commonly used DEA models that operationalize combinations of these evaluation principles.

3.4 Interpretation of Existing Models

3.4.1 Introduction

Data Envelopment Analysis models are linear programs which effect relative evaluations of the operations of decision-making units. In fact performing data envelopment analysis requires the solution of a linear programming model for each decision-making unit. The primal models can be characterized as **projection models**. The associated dual models as **pricing models**. An optimal solution of a primal DEA model to evaluate a DMU, DMU_l, may include θ, the minimum proportion of $\mathbf{x}_l$ allowing the production of $\mathbf{y}_l$, ϕ, the maximum multiple of $\mathbf{y}_l$ attainable from $\mathbf{x}_l$, the m-vector $\mathbf{e}$ which may represent either the total excess input consumption for each input or the remaining excesses after the proportional input reduction has been effected, the s-vector $\mathbf{s}$ which may represent either the total output deficiency for each output or the remaining deficiency after the proportional output augmentation has been effected, and the n-vector $\boldsymbol{\lambda}$ which indicates which envelope-defining DMUs form the referent point to evaluate DMU_l. An optimal dual solution includes the m-vector $\boldsymbol{\nu}$ of input efficiency prices, the s-vector $\boldsymbol{\mu}$ of output efficiency prices, and, for the VRS envelopments, the variable ω. The set of values $\boldsymbol{\mu}$, $\boldsymbol{\nu}$,(ω), are the coefficients of hyperplanes that define the facets of the envelopment surface. The interpretation of these solutions derive from the principles of evaluation that are operationalized by the formulation. The next subsections provide illustrations and comments on the most commonly used DEA models.

3.4.2 The CCR Model

The CCR model assumes a CRS envelopment, it is input-oriented and incorporates a standard pricing mechanism where all units of waste remaining after a maximal proportional input reduction are priced at a same value, ϵ. In the process of evaluating DMU_l characterized by $(\mathbf{x}_l,\mathbf{y}_l)$, an efficient unit given by $(\sum^{n}_{j=1}\lambda_j\mathbf{x}_j,\ \sum^{n}_{j=1}\lambda_j\mathbf{y}_j)$ is identified and establishes that DMU_l could have produced its levels of output **and** additional amounts given by $(s_r)_{r\,=\,1,\dots,s}$ by consuming less than its levels of inputs. In fact DMU_l could have reduced its inputs by $(1-\theta)\%$ across the board **and** by additional amounts given by $(e_i)_{i\,=\,1,\dots,m}$.

The solution of the primal program (CCR Primal) given below indicates that DMU_l is efficient, i.e. determines the frontier, if and only if the efficient production plan solution of the program is that of DMU_l itself. Mathematically this necessary and sufficient condition translates into:

$$\theta = 1 \text{ and } \mathbf{s} = 0 \text{ and } \mathbf{e} = 0 \text{ at optimality.}$$

In the context of the dual mathematical program of (CCR Primal), called (CCR Dual) and given below, the evaluation of DMU_l corresponds to the determination of a set of virtual prices $((\mu_r)_{r\,=\,1,\dots,s},\ (\nu_i)_{i\,=\,1,\dots,m})$ such that the aggregated value of DMU_o's **inputs** be normalized to unity and the aggregated value of DMU_l's outputs be maximized without exceeding unity. The dual formulation is more amenable to a geometric characterization of the empirical frontier as a cone through the origin whose facets are defined by the intersection of hyperplanes of the form:

$$\sum_{r=1}^{s} \mu_r y_r - \sum_{i=1}^{m} \nu_i x_i = 0$$

The facets of the empirical frontier are delimited by adjacent/contiguous efficient production plans. The prices hence define a normal to a facet of the empirical frontier.

$$\mathrm{Min}_{\theta,\lambda,s,e} \quad \theta - \epsilon \left(\sum_{r=1}^{s} s_r + \sum_{i=1}^{m} e_i\right)$$

$$\text{st} \begin{cases} \sum_{j=1}^{n} y_{rj}\lambda_j - s_r = y_{rl} & r = 1,\ldots,s \\ \theta x_{il} - \sum_{j=1}^{n} x_{ij}\lambda_j - e_i = 0 & i = 1,\ldots,m \\ \lambda_j \geq 0 & j = 1,\ldots,n \\ s_r \geq 0 & r = 1,\ldots,s \\ e_i \geq 0 & i = 1,\ldots,m \end{cases} \qquad \text{(CCR Primal)}$$

$$\mathrm{Max}_{\mu_r,\nu_i} \quad z = \sum_{r=1}^{s} \mu_r y_{rl}$$

$$\text{st} \begin{cases} \sum_{i=1}^{m} \nu_i x_{il} = 1 & \\ \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} \leq 0 & j=1,\ldots,n \\ \mu_r \geq \epsilon & r=1,\ldots,s \\ \nu_i \geq \epsilon & i=1,\ldots,m \end{cases} \qquad \text{(CCR Dual)}$$

The role of ϵ deserves further comment. It represents a non-Archimedean quantity, that is a number whose magnitude is smaller than that of any data of the problem and is introduced to ensure that the proportional reduction of inputs is the

primary goal in the search for an efficient production plan. If the model CCR primal is solved with a prespecified value for ϵ, the chosen value bears a direct impact on the shape and size of the cone of efficiency prices. If we accept the solution of CCR Dual as admissible efficiency prices, that is accept (ν, μ) as $(\mathbf{p}_x, \mathbf{p}_y)$ then we are lead to derive:

$$E(x,y) = \frac{\nu x^*}{\nu x_l} \cdot \frac{\mu y_l}{\mu y^*} = z = \theta - \epsilon(\mathbf{1s} + \mathbf{1e}) = \theta - \epsilon$$

An often used "selling" point for this model is that it provides a best case evaluation for all DMUs since the efficiency score is equal to the dual objective function value which is maximized at optimality. However, we recognize the modified Zieschang measure of efficiency and are reminded of the caveats of that measure, namely that does not satisfy the strict monotonicity property. It follows that any ranking of DMUs derived from the selection of an arbitrary value for ϵ is a priori debatable. This reservation is confirmed by the open controversy in the literature on the use and role of ϵ [Boyd and Färe, 1984], [Charnes and Cooper, 1984].

When ϵ is considered as a modelling construct, a two-phase approach is taken to solve the program. The first phase corresponds to the minimization of θ, that is the extremal proportional reduction of inputs. The second phase proceeds to select the efficient production plan on the frontier against which DMU_l is evaluated. Hence from a computational point of view two sets of linear programs are solved for each DMU. They are respectively (Phase I Primal, Phase I Dual) and (Phase II Primal, Phase II Dual) given below.

$$\mathrm{Min}_{\theta,\lambda,s,e} \quad \theta$$

$$\text{st}\begin{cases} \sum_{j=1}^{n} y_{rj}\lambda_j - s_r = y_{rl} & r = 1,\ldots,s \\ \theta x_{il} - \sum_{j=1}^{n} x_{ij}\lambda_j - e_i = 0 & i = 1,\ldots,m \\ \lambda_j \geq 0 & j = 1,\ldots,n \\ s_r \geq 0 & r = 1,\ldots,s \\ e_i \geq 0 & i = 1,\ldots,m \end{cases}$$

(Phase I Primal)

$$\mathrm{Max}_{\mu_r,\nu_i} \quad z_l = \sum_{r=1}^{s} \mu_r y_{rl}$$

$$\text{st}\begin{cases} \sum_{i=1}^{m} \nu_i x_{il} = 1 & \\ \sum_{r=1}^{s} \mu_r\, y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} \leq 0 & j=1,\ldots,n \\ \mu_r \geq 0 & r=1,\ldots,s \\ \nu_i \geq 0 & i=1,\ldots,m \end{cases}$$

(Phase I Dual)

$$\mathrm{Min}_{\lambda,s,e} \quad -\left(\sum_{r=1}^{s} s_r + \sum_{i=1}^{m} e_i\right)$$

$$\text{st}\begin{cases} \sum_{j=1}^{n} y_{rj}\lambda_j - s_r = y_{rl} & r = 1,\ldots,s \\ -\sum_{j=1}^{n} x_{ij}\lambda_j - e_i = -\theta^{*} x_{il} & i = 1,\ldots,m \\ \lambda_j \geq 0 & j = 1,\ldots,n \\ s_r \geq 0 & r = 1,\ldots,s \\ e_i \geq 0 & i = 1,\ldots,m \end{cases}$$

(PhaseII Primal)

$$\mathrm{Max}_{\mu_r,\nu_i} \quad z_l = \sum_{r=1}^{s} \mu_r y_{rl} + \sum_{i=1}^{m} \nu_i \theta^* x_{il}$$

$$\text{st} \begin{cases} \sum_{r=1}^{s} \mu_r\, y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} \le 0 & j=1,\ldots,n \\ \mu_r \ge 1 & r=1,\ldots,s \\ \nu_i \ge 1 & i=1,\ldots,m \end{cases} \qquad \text{(Phase II Dual)}$$

The efficiency score is then expressed as:

$$E(x_l,y_l) = \frac{\nu x^*}{\nu x_l} \cdot \frac{\mu y_l}{\mu y^*} = \theta - \frac{1s + 1e}{\nu x_l}$$

We recognize the efficiency score ι (iota) introduced by Ali et al. [1992]. When the inefficient unit maps within the cone of weak disposability we recover the Debreu-Farrell measure θ since $\mathbf{s} = 0$ and $\mathbf{e} = 0$. When the unit maps within an area of strong disposability the efficiency score is further reduced from θ to account for any remaining slack output and excess input.

Other caveats of the model stem from the dependence of the evaluation on units of measurement of the data, and more importantly from the possible occurrence of prices implying relative values across inputs and across outputs that are beyond any reasonable justification. However, if CRS envelopment appears reasonable, if the data is presented in units that are conform to common usage then the model represents an excellent means to assess the implied relative values of inputs and outputs for the various operating units. Such knowledge is relevant to decide on the allocation of a limited supply of some input or the requisition of some output.

The output-oriented version of the model is given by:

$$\mathrm{Max}_{\phi,\lambda,s,e} \quad \phi + \epsilon\left(\sum_{r=1}^{s} s_r + \sum_{i=1}^{m} e_i\right)$$

$$\text{st}\left\{\begin{array}{rll} \phi y_{rl} - \sum_{j=1}^{n} y_{rj}\lambda_j + s_r = 0 & r = 1,\ldots,s \\ \sum_{j=1}^{n} x_{ij}\lambda_j + e_i = x_{il} & i = 1,\ldots,m \\ \lambda_j \geq 0 & j = 1,\ldots,n \\ s_r \geq 0 & r = 1,\ldots,s \\ e_i \geq 0 & i = 1,\ldots,m \end{array}\right. \qquad (\mathrm{CCR_o\ Primal})$$

The envelopement surface is to be reached by stretching the output vector proportionately first, and then according to the pricing of any remaining output slack and input excesses explicit in the objective function. The same comments as in the case of an input orientation apply to ϵ. If the program is considered a one-phase program and if we accept the solution of CCR Dual as admissible efficiency prices, that is accept (ν, μ) as ($\mathbf{p}_x$, $\mathbf{p}_y$) then we are lead to derive:

$$E(\mathbf{x}_l,\mathbf{y}_l) = \frac{1}{\phi + \epsilon(\mathbf{1s} + \mathbf{1e})} = \frac{1}{\phi + \epsilon}$$

If a two-phase procedure is used, corresponding to a preemptive proportional output augmentation, then at optimality of the second phase we have:

$$\mathbf{1s} + \mathbf{1e} = -\phi\mu\mathbf{y}_l + \nu\mathbf{x}_l \Rightarrow \mu\mathbf{y}_l = \frac{1}{\phi}[\nu\mathbf{x}_l - (\mathbf{1s} + \mathbf{1e})]$$

with still:

$$\mu\mathbf{y}* - \nu\mathbf{x}* = 0$$

It follows:

$$E(x_l,y_l) = \frac{\nu x^*}{\nu x_l} . \frac{\mu y_l}{\mu y^*} = \frac{1}{\phi}(1 - \frac{\mathbf{1s} + \mathbf{1e}}{\nu x_l})$$

We note that the expression of the efficiency score in the case of two-phase solution of the output oriented version of the CCR model is different from o (omicron) introduced by Ali et al. [1992] as a counterpart to ι.

3.4.3 The BCC Model

A first extension to the CCR model was offered by Banker, Charnes, and Cooper [1984]. It represents a generalization to the extent that their model rests on a reduced number of postulates for the production possibility set by eliminating the ray unboundedness postulate. The BCC model assumes a VRS envelopment, it is input-oriented and incorporates a standard pricing mechanism where all units of waste remaining after a maximal proportional input reduction are priced at a same value, ϵ. The same interpretation as for (CCR Primal) applies with the added stipulation that the efficient production plan sought in the evaluation, in accordance with the type of envelopment, be a **convex** combination of known feasible production plans instead of a positive linear one. The set of dual programs to determine the new envelope or empirical frontier are (BCC Primal) and (BCC Dual) given below.

The characterization of efficiency is still made via the necessary and sufficient condition that $\theta = 1$ and $\mathbf{s} = 0$ and $\mathbf{e} = 0$ at optimality.

$$
\text{Min}_{\theta,\lambda,s,e} \quad \theta - \epsilon \left(\sum_{r=1}^{s} s_r + \sum_{i=1}^{m} e_i\right)
$$

$$
\text{st}
\begin{cases}
\displaystyle\sum_{j=1}^{n} y_{rj}\lambda_j - s_r = y_{rl} & r = 1,\ldots,s \\
\displaystyle\theta x_{il} - \sum_{j=1}^{n} x_{ij}\lambda_j - e_i = 0 & i = 1,\ldots,m \\
\displaystyle\sum_{j=1}^{n} \lambda_j = 1 & \\
\lambda_j \geq 0 & j = 1,\ldots,n \\
s_r \geq 0 & r = 1,\ldots,s \\
e_i \geq 0 & i = 1,\ldots,m
\end{cases}
\qquad \text{(BCC Primal)}
$$

$$
\text{Max}_{\mu_r,\nu_i,\omega_l} \sum_{r=1}^{s} \mu_r y_{rl} - \omega_l
$$

$$
\text{st}
\begin{cases}
\displaystyle\sum_{i=1}^{m} \nu_i x_{il} = 1 & \\
\displaystyle\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} - \omega_l \leq 0 & j=1,\ldots,n \\
\mu_r \geq \epsilon & r=1,\ldots,s \\
\nu_i \geq \epsilon & i=1,\ldots,m \\
\omega_l \ \text{unrestricted} &
\end{cases}
\qquad \text{(BCC Dual)}
$$

The additional variable, ω_l, in (BCC Dual) has been associated with the characterization of returns-to-scale prevailing in the interior of the facet defined by the efficient production plan against which DMU_l is compared. Following Banker and Thrall's definition [1990], returns-to-scale at (x^*, y^*) on the envelope are said to be increasing if $[(1+\delta)x^*, (1+\delta)y^*]$ maps on the frontier for $\delta > 0$ but not for $\delta < 0$, δ small. They are said to be constant if $[(1+\delta)x^*, (1+\delta)y^*]$ belongs to the production possibility set for $\delta > 0$ and for $\delta < 0$, δ small, and they are said to be decreasing if $[(1+\delta)x^*, (1+\delta)y^*]$ for $\delta < 0$ but not for $\delta > 0$, δ small. This particular characterization of returns-to-scale is captured by ω_l. If there exists a solution to (BCC

Dual) such that $\omega_l < 0$ (respectively $= 0$, > 0) then increasing (respectively constant, decreasing) returns-to-scale prevail.

From an economic/accounting point of view ω_l can be attributed a profit related interpretation. Indeed it seems natural to assimilate $\mu\mathbf{y}$ to a revenue fucntion and $\nu\mathbf{x}$ to a cost fucntion. Regarding $\nu\mathbf{x}$, if variable costs are the only costs accounted for, then the difference $\mu\mathbf{y}^* - \nu\mathbf{x}^*$ is a gross profit function equal to the net profit plus the fixed costs of operations. If $\nu\mathbf{x}$ accounts for the total costs of operations then $\mu\mathbf{y}^* - \nu\mathbf{x}^*$ directly expresses net profit. In either situation we have:

$$\mu\mathbf{y}^* - \nu\mathbf{x}^* = -\omega_l$$

It follows that $-\omega_l$ represents either the **maximum** gross profit or net profit attainable by any DMU given input and output prices, ν and μ, and given the scale of operations of DMU_l.

Since the model was first introduced it has been customary to use as an efficiency score for DMU_l the following expression:

$$\frac{\mu\mathbf{y}_l + \omega_l}{\nu\mathbf{x}_l} = \theta - \epsilon$$

The motivation for such a practice may be traced to the fact that the score exhibits the following properties:

i) The score is maximized since it evaluates to the objective function value of the dual program. It hence helps "sell" the analysis by guaranteeing each DMU a best case evaluation.

ii) The score is strictly between 0 and 1.

iii) The score is equal to 1 if and only if DMU_l is efficient.

However, the same caveats as for the CCR model apply regarding the non-Archimedean ϵ. When the model is solved as a two-phase program, where the second phase identifies the referent point on the envelope and the associated efficiency price vectors, the above efficiency score, equivalent in this case as well to ι, may become negative. More importantly the above score does **not** have any economic interpretation/justification as $E(x,y) = \nu x^*/\nu x_l \,.\, \mu y_l/\mu y^*$ universally does.

Moreover it is worth noting that there is no simple relationship linking the efficiency score E(x,y) to the objective function of the BCC model. Indeed at optimality the following relationships obtain:

$$\begin{aligned} \nu x_l &= 1 \\ \mu y_l &= \theta - \epsilon(1s + 1e) - \omega_l \\ \mu y^* &= \nu x^* - \omega_l \end{aligned}$$

It follows:

$$E(x_l,y_l) = \frac{\nu x^*}{\nu x_l} . \frac{\mu y_l}{\mu y^*} = \frac{\theta - \omega_l - \epsilon(1s + 1e)}{1 - \dfrac{\omega_l}{\nu x^*}} = \frac{\theta - \omega_l - \epsilon}{1 - \dfrac{\omega_l}{\nu x^*}}$$

Again because of the non-Archimedean quantity the monotonicity properties of the efficiency measure are hindered. The acceptance of the conclusions of the evaluation on the part of the DMUs has to stem from an understanding and acceptance of the evaluation principles: form of the envelope, orientation and pricing mechanism. In the case of a two-phase approach different relationships obtain at optimality, namely:

$$\mu y_l = \theta \nu x_l - \omega_l - (1s + 1e)$$
$$\mu y^* = \nu x^* - \omega_l$$
$$\nu x^* = \theta \nu x_l - 1e$$

It follows:

$$E(x_l,y_l) = \frac{\nu x^*}{\frac{1}{\theta}(\nu x^* + 1e)} \frac{\nu x^* - 1s - \omega_l}{\nu x^* - \omega_l} = \frac{\theta}{1 + \frac{1e}{\nu x^*}} (1 - \frac{1s}{\nu x^* - \omega_l})$$

3.4.4 The ADD Model

A third model, referred to as the ADD model and developed by Charnes, Cooper, Golany, Seiford, and Stutz [1985], rests on the same set of postulates as the BCC model but distinguishes itself from the previous two models by **departing from the preemptive proportional inputs reduction.** The ADD model assumes a VRS envelopment, it follows a global orientation seeking to identify waste across all measures without prioritization between inputs and outputs. The projection to the envelope is entirely driven by the pricing mechanism reflected in the primal objective function. In the case of the original ADD model stated below, we recognize a standard pricing mechanism, namely $p_{xi} = 1$ if $e_i > 0$ and $p_{yr} = 1$ if $s_r > 0$, warranting the usual caution with regard to units of measurement of the data.

The necessary and sufficient conditions for efficiency are $s = 0$ **and** $e = 0$ at optimality.

$$\mathrm{Min}_{\lambda,s,e} \; - \left(\sum_{r=1}^{s} s_r + \sum_{i=1}^{m} e_i \right)$$

$$\text{st} \begin{cases} \sum_{j=1}^{m} y_{rj}\lambda_j - s_r = y_{rl} & r = 1,\ldots,s \\ -\sum_{j=1}^{n} x_{ij}\lambda_j - e_i = -x_{il} & i = 1,\ldots,m \\ \sum_{j=1}^{n} \lambda_j = 1 & \\ \lambda_j \geq 0 & j = 1,\ldots,n \\ s_r,\ e_i \geq 0 & r = 1,\ldots,s \; i = 1,\ldots,m \end{cases} \qquad \text{(ADD Primal)}$$

$$\mathrm{Max}_{\mu,\nu,\omega_l} \; \sum_{r=1}^{s} \mu_r y_{rl} - \sum_{i=1}^{m} \nu_i x_{il} + \omega_l$$

$$\text{st} \begin{cases} \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} + \omega_l \leq 0 & j = 1,\ldots,n \\ \mu_r \geq 1 & r = 1,\ldots,s \\ \nu_i \geq 1 & i = 1,\ldots,m \\ \omega_l \ \text{unrestricted} & \end{cases} \qquad \text{(ADD Dual)}$$

To the extent that the principles of evaluation of the ADD model are looser than those underlying the BCC model, that is the ADD model does not impose an input orientation, one could expect the ADD model to lead to worse efficiency evaluation since it has the freedom to reach the envelope where the revealed bundle of waste will be the largest in terms of total number of wasted units of the various measures. However, this is not necessarily the case since the efficiency score takes into account the efficiency prices of all measures and these generally evaluate to different relative values for different projected points. This emphasizes the fact that the **explicit pricing** mechanisms, be they standard, invariant, or normed, **apply exclusively** to the identified **excess input** consumptions and **slack output** productions. The pricing of all measures reflecting envelope levels is, on the other hand, completely endogenous. This complete freedom may itself lead to extremal/abnormal valuations. It may indeed happen that these efficiency prices bear no relations to the price level implied by "waste" prices, and more importantly, their relative values may reflect absurd rates of substitution across inputs and across outputs. This drawback, common to all explicit pricing mechanisms, is widely acknowledged in the literature (e.g. Roll, Cook, and Golany [1991], Wong and Beasley [1990]). In the next section a new class of DEA models is developed. These models are aimed at bringing more control over the pricing mechanism and circumvent the drawbacks mentioned above.

3.5 From Envelopment to Frontier Analysis

3.5.1 Introduction

The purpose of this section is to develop models that correctly determine efficiency evaluations and reflect realistic economic tradeoffs across inputs and outputs. This concern is relevant to bring efficiency evaluations closer to economic efficiency ratings which should warrant sounder recommendations to inefficient operating units. The developed models can be thought of as extensions of original DEA models in the vein of previous work by Charnes, Cooper, Huang, and Sun [1990], and by Thompson, Singleton, Thrall, and Smith [1986], who respectively introduced cone-ratio models and assurance regions to prevent unreasonable evaluations in terms of disproportionate efficiency prices. Recent work by Charnes, et al. [1991] presents elaborate theorems establishing the equivalence of both lines of research. However, to this date, no general formulation and economic interpretation of such model has appeared in the literature. Satisfying this need will strengthen the bridge between DEA and economics. In the next subsections we develop a new model in the context of production theory, justify its mathematical properties, and show how its solution values elegantly yield the efficiency score.

3.5.2 The Frontier Model

Any DEA model approximates the envelope of the production possibility set by a set of facets defined by empirical observations. Each identified facet implies ranges of values for rates of substitution across inputs and across outputs. Production theory refers to these rates as marginal rates of technical substitution (MRTS) and marginal rates of product transformation (MRPT) where, respectively, $MRTS_{ij}$, is the rate at which input i is substituted for input j while still producing the same levels of outputs and keeping other inputs constant, and $MRPT_{kl}$ represents the rate at which output k must be sacrificed to obtain more of output l while keeping the consumption of all inputs and the level of all other outputs constant.

$$MRTS_{i,j} = - \frac{\partial x_j}{\partial x_i} \bigg| \; x_g, \; g = 1,...m \text{ constant, } y \text{ constant}$$

$$MTPT_{k,l} = - \frac{\partial y_k}{\partial y_l} \bigg| \; y_m, \; m = 1,...,s \text{ constant, } x \text{ constant}$$

Assuming a technology characterized by variable returns to scale, each facet can be described analytically by an expression of the form:

with

$$\sum_{r=1}^{s} \mu_r y_r - \sum_{i=1}^{m} \nu_i x_i + \omega = 0$$

$$\underline{x_i} \leq x_i \leq \overline{x_i} \quad i = 1,\ldots,m$$
$$\underline{y_r} \leq y_r \leq \overline{y_r} \quad r = 1,\ldots,s$$

The ranges on the inputs and outputs serve to identify the scales (i.e. levels) of operations at which the techniques described by the facet are efficient. It follows that across a facet the rates of substitution are constant and are easily derived from the analytical expression of the facet.

$$MRTS_{ij} = \frac{\nu_i}{\nu_j} \quad i, j = 1,\ldots,m \quad i < j$$
$$MRPT_{kl} = \frac{\mu_k}{\mu_l} \quad k, l = 1,\ldots,s \quad k < l$$

If one is concerned exclusively with technical efficiency then engineering and best practice knowledge may be extensive enough to set bounds on the rates of substitution, that is constants are known:

$$\underline{r^{ij}},\ \overline{r_{ij}},\ \underline{r^{kl}},\ \overline{r_{kl}}$$

such that, given the existing technology, we must have:

$$\underline{r^{ij}} \leq MRTS_{ij} \leq \overline{r_{ij}}$$
$$\underline{r^{kl}} \leq MRPT_{kl} \leq \overline{r_{kl}}$$

If one is concerned with allocative efficiency and prices or societal valuations are known for inputs and outputs, then production theory stipulates that profit maximization in perfectly competitive input markets require that the MRTS for every pair of inputs, holding the levels of all outputs and all other inputs constant, must equal the ratio of their prices, and the MRPT for every pair of outputs, holding the levels of all inputs and all other outputs constant, must equal the ratio of their prices. To acknowledge, again, the possibility of spatially separated (perfectly competitive) markets or societal environments, profit or utility maximization, typical of efficient operations, requires that the MRTSs and MRPTs belong to preset ranges of admissible price ratios. It follows that in both cases of technical or allocative efficiency, realistic assessments may be obtained by restricting explicitly the ranges of efficiency price ratios, ν_i/ν_j, i,j = 1,...,m, i ≠ j, and μ_k/μ_l, k,l = 1, ...,s, k ≠ l. This will still allow an endogenous determination of prices, which completely avoids fixing a price level arbitrarily or imposing a priori rates of substitution across inputs and across outputs.

For any two inputs i, j, and any two outputs k, l, lower and upper bounds for their respective rates of substitution can be estimated/predefined that translate into the following constraints on input and output prices:

$$\underline{r}^{ij} \leq \frac{\nu_i}{\nu_j} \leq \overline{r}_{ij} \quad i,\ j = 1,\ldots,m \quad i < j$$
$$\underline{r}^{kl} \leq \frac{\mu_k}{\mu_l} \leq \overline{r}_{kl} \quad k,\ l = 1,\ldots,s \quad k < l$$

These conditions can easily be converted into linear constraints on the efficiency prices to be estimated:

$$-\nu_i + \underline{r}^{ij}\nu_j \leq 0 \qquad \nu_i - \overline{r}_{ij}\nu_j \leq 0 \quad i,\ j = 1,\ldots,m \quad i < j$$
$$-\mu_k + \underline{r}^{kl}\mu_l \leq 0 \qquad \mu_k - \overline{r}_{kl}\mu_l \leq 0 \quad k,\ l = 1,\ldots,s \quad k < l$$

Such constraints summarize to a global matrix format:

$$\nu \mathbf{R}_i \leq 0 \qquad \mu \mathbf{R}_o \leq 0$$

where R_i and R_o are respectively m x $2\binom{m}{2}$ and s x $2\binom{s}{2}$. These constraints are added to the dual model and translate into the inclusion of substitution variables, summarized by the $2\binom{m}{2}$ vector σ_i, and the $2\binom{s}{2}$ vector σ_o, in the primal formulation.

At the core of the evaluation is still the identification of waste in the form of excessive inputs consumption and unrealized outputs. The amount of waste is to be measured by θ which characterizes the extent of proportional input reduction from a unit's current levels, and by ϕ which characterizes the extent of proportional output augmentation from the unit's current levels. The emphasis on proportional changes allows a unit to be evaluated with respect to most similar efficient units in terms of production technique and output mix.

The formulations of the Frontier model presented next assume a VRS envelopment since the referent unit is required to be a convex combination of frontier defining units; they effect a globally oriented evaluation by not giving priority to the input reduction (minimizing θ) or to the output augmentation (maximizing ϕ , i.e. minimizing $-\phi$), and they implement an implicit pricing mechanism constrained by R_i and R_o. The interpretation and illustration of the workings of the programs follow their statement.

$$\mathrm{Min}_{\theta,\phi,\lambda,\sigma_i,\sigma_o} \ \theta - \phi$$

$$\text{st}\begin{cases} -\phi \mathbf{y}_l + \mathbf{Y}\boldsymbol{\lambda} + \mathbf{R}_o\sigma_o = 0 \\ \theta \mathbf{x}_l - \mathbf{X}\boldsymbol{\lambda} + \mathbf{R}_i\,\sigma_i = 0 \\ \mathbf{1}.\boldsymbol{\lambda} = 1 \\ \phi \geq 1 \\ -\theta \geq -1 \\ \boldsymbol{\lambda} \geq 0 \\ \sigma_o,\ \sigma_i \geq 0 \end{cases} \qquad \text{(Frontier Primal)}$$

$$\mathrm{Max}_{\mu,\nu,\omega_l,R,C} \ \omega_l + R - C$$

$$\text{st}\begin{cases} \boldsymbol{\mu}\mathbf{Y} - \boldsymbol{\nu}\mathbf{X} + \omega_l \leq 0 \\ \boldsymbol{\nu}\mathbf{x}_l - C \leq 1 \\ -\boldsymbol{\mu}\mathbf{y}_l + R \leq -1 \\ \boldsymbol{\nu}\mathbf{R}_i \leq 0 \\ \boldsymbol{\mu}\mathbf{R}_o \leq 0 \\ R,\ C \geq 0 \\ \boldsymbol{\mu},\ \boldsymbol{\nu},\ \omega_l \ \text{unrestricted} \end{cases} \qquad \text{(Frontier Dual)}$$

In the primal formulation the role the substitution variables, σ_i and σ_o, play is critical to the evaluation process. After a proportional reduction of input consumption has been effected, they allow the conversion, i.e. substitution, of any remaining excess input in such a way that the reduced input vector is "brought" back into the cone spanned by the efficient units given the set of price ratios. Note that, given the price ratios, this cone becomes the cone of weak disposability. This situation is illustrated in Figure 3.2 on page 118 at the end of the chapter. Proportional input reductions for unit L (with respect to L's input ratios) in the amount E′L/OL is followed by substitutions to reach E on the frontier. Alternately, the evaluation may be interpreted as identifying an input reduction (with respect to L's input ratios) in the amount ML/OL. This reduction leads to point M which is inefficient with respect to its consumption of input 2. By substituting out its excess input 2 following the tradeoffs implied by **p**, unit M is transformed into unit N which can withstand further proportional (with respect to N's input ratios) reduction until point E on the frontier is reached. These new proportions command efficiency prices which are bound by the tradeoff constraints for all measures that required substitution. It follows that input inefficiency can entirely be gauged by the extent of proportional input reduction: ML/OL + EN/ON. A third interpretation considers that the substitution is effected first and brings L to L′ in the cone of weak disposability where subsequent proportional input reduction (with respect to L′'s input ratios) brings the unit to E. All these paths to reach the frontier evaluate to the same degree of input inefficiency since we have: $1 - \theta = E'L/OL = ML/OL + EN/ON = NL'/ON$. Similarly on the output side, the substitution variables will point to modifications in the output mix that will allow output deficiency to be entirely gauged by the extent of feasible proportional augmentation given the new mix.

In the dual program R - C represents the profit level attained by DMU_l being evaluated given the identified input and output prices ν, and μ. This statement will be proved later in the study of the mathematical properties of the model. For all units $-\omega_l$ represents the largest profit level attainable given the identified optimal input and output prices ν, and μ since we have $-\omega_l \geq \mu y - \nu x$ for all DMUs characterized by (x,y). The dual objective function can then be rewritten as:

$$\text{Max } R - C - (-\omega_l) = -\text{ Min } (-\omega_l) - (R - C)$$

The right-hand side expression emphasizes the purpose of the dual program, namely the identification of an optimal set of prices (ν, μ) such that the comparative/relative **disadvantage** of DMU_l is minimized. Indeed the profit function $\mu\mathbf{y}$ - $\nu\mathbf{x}$ allows the measurement of a unit's achievement once a set of prices (ν, μ) is determined. The comparative disadvantage of a unit, for set of prices (ν, μ), can then be gauged by the difference between the achievement of the unit being evaluated and the maximum observed achievement across all units operating at a scale close to that of the unit being evaluated. It follows that the dual program effects a best case evaluation of DMU_l. The identified optimal prices minimize the unit's comparative disadvantage. It is worth emphasizing that the maximum attained profit is not necessarily positive, i.e., ω_l is unrestricted in sign and that profit maximization is not a stated objective of the model.

The workings of the pricing mechanism are illustrated next. Restricting ourselves to the input space for ease and clarity of the exposition, we examine the evaluation process applied to three inefficient DMUs: I_1, I_2, and I_3 represented in Figure 3.3 on page 119 at the end of the chapter. I_1 is inefficient in input 2 but efficient in input 1. The pricing most favorable to I_1 is the one that will maximize the value of input 1 and minimize the cost of consuming input 2 given that the total input consumption costs 1 and that the relative value of input 1 with respect to input 2, p_1/p_2, is bounded upward by some predefined value. The identified set of prices to evaluate I_1 will be homothetic of $\mathbf{p}_A$. The relevant efficient facet against which I_1 will be compared is EF. All points along that facet lead to a same cost which is the smallest observable cost given $\mathbf{p}_A$. The comparative disadvantage of I_1 is measured by the difference in cost which evaluates to:

$$\theta_1 = \frac{\overline{OE}}{\overline{OP_1}} = \frac{\overline{OF}}{\overline{OP_2}}$$

From a cost point of view it follows that all points on the facet EF are suitable reference points to evaluate I_1. With respect to any other facet the difference in cost, hence, the comparative disadvantage, would be larger. From an input consumption point of view, however, E is the "best" reference point since it is the one that minimizes the change from I_1's operations characterized by I_1's input consumption ratio.

I_2 is inefficient with respect to both inputs. Comparative disadvantage will be minimized with prices homothetic of $\mathbf{p}_B$ identifying the facet FG as the reference facet to evaluate I_2's inefficiency. Along that facet J is identified as the optimum reference point since it is the only point sharing I_2's technique characterized by I_2's input consumption ratios. It follows that no input substitution is required in accordance with relative prices strictly within allowed limits.

I_3 is also inefficient with respect to both inputs but with the added particularity that an infinity of price vectors would lead to the same minimum comparative disadvantage (all vectors which are positive combinations of $\mathbf{p}_B$ and $\mathbf{p}_C$ with the added condition that the "cost" of I_3 is 1). However, it is important to note that the referent point is unique despite the multiplicity of prices.

The evaluation of any inefficient unit summarizes to one of the above three situations. It follows that in any case the recommendations to an inefficient unit are unambiguous and fully contained in the **unique** identified referent point.

The study of the mathematical properties of the Frontier model is effected by expressing and manipulating the complementary slackness conditions. These conditions allow the derivation of some important results regarding the behavior of the model. In particular it will be shown that at optimality the efficiency prices are strictly positive, hence all problems stemming from the stipulation of ϵ in earlier models are circumvented. All sources of inefficiency are identified and priced implicitly with the total cost of inefficiency summarized by θ and ϕ. This implies that there is no need for extraneous slack and excess variables in the formulation, i.e. $\mathbf{s} = 0$ and $\mathbf{e} = 0$, always, at optimality. The proof of these results requires a slight modification of the formulation of the frontier model to incorporate explicit slack and excess variables. These modified formulations (Frontier Primal-0 and Frontier Dual-0) are given below with their corresponding complementary slackness conditions.

$$\mathrm{Min}_{\theta,\phi,\lambda,\sigma_i,\sigma_o} \ \theta - \phi$$

$$st \left\{ \begin{array}{rcl} -\phi \mathbf{y}_l + \mathbf{Y}\lambda + \mathbf{R}_o \sigma_o - \mathbf{s} & = & 0 \\ \theta \mathbf{x}_l - \mathbf{X}\lambda + \mathbf{R}_i \sigma_i - \mathbf{e} & = & 0 \\ \mathbf{1}.\lambda & = & 1 \\ \phi & \geq & 1 \\ -\theta & \geq & -1 \\ \lambda & \geq & 0 \\ \sigma_o, \ \sigma_i, \mathbf{s}, \mathbf{e} & \geq & 0 \end{array} \right. \qquad \text{(Frontier Primal-0)}$$

$$\text{Max}_{\mu,\nu,\omega_1,R,C} \quad \omega_1 + R - C$$

$$st \begin{cases} \mu \mathbf{Y} - \nu \mathbf{X} + \omega_1 \leq 0 \\ \nu \mathbf{x}_1 - C \leq 1 \\ -\mu \mathbf{y}_1 + R \leq -1 \\ \nu \mathbf{R}_i \leq 0 \\ \mu \mathbf{R}_o \leq 0 \\ \mu, \nu, R, C \geq 0 \\ \omega_1 \text{ unrestricted} \end{cases} \qquad \text{(Frontier Dual-0)}$$

The complementary slackness conditions follow:

$$-\phi \mu \mathbf{y}_1 + \mu \mathbf{y}^* + \mu \mathbf{R}_o \sigma_o - \mu \mathbf{s} = 0 \quad (1)$$
$$\theta \nu \mathbf{x}_1 - \nu \mathbf{x}^* + \nu \mathbf{R}_i \sigma_i - \nu \mathbf{e} = 0 \quad (2)$$
$$R(\phi - 1) = 0 \quad (3)$$
$$C(1 - \theta) = 0 \quad (4)$$

$$\phi(\mu \mathbf{y}_1 - (1 + R)) = 0 \quad (5)$$
$$\theta(1 + C - \nu \mathbf{x}_1) = 0 \quad (6)$$
$$\lambda(\mu \mathbf{Y} - \nu \mathbf{X} + \omega_1) = 0 \quad (7)$$
$$\mu \mathbf{R}_o \sigma_o = 0 \quad (8)$$
$$\nu \mathbf{R}_i \sigma_i = 0 \quad (9)$$

Lemma 3.1: At optimality of the primal and dual Frontier programs, the output efficiency price vector, μ, is strictly positive.

Proof: Since ϕ is constrained from below by 1, condition (5) coupled with the non-negativity constraint on R ensures that the total output value of unit 1 ($\mu \mathbf{y}_1$) is bounded from below by 1, implying that at least one output efficiency price is strictly positive. The ratio constraints on the output efficiency prices then ensure that **all** output efficiency prices are strictly positive, for if at least one such price is negative then all are by cascading through the ratio constraints.

Q.E.D.

Lemma 3.2: At optimality of the primal and dual Frontier programs, the input efficiency price vector, ν, is strictly positive.

Proof: Similarly, since θ is strictly positive at optimality ($\theta = 0$ would mean that outputs can be generated out of nothing!), condition (6) coupled with the non-negativity constraint on C ensures that the total input value of unit l ($\nu \mathbf{x}_l$) is bounded from below by 1, implying that at least one input efficiency price is strictly positive. The ratio constraints on the input efficiency prices then ensure that **all** input efficiency prices are strictly positive, for if at least one such price is negative then all are by cascading through the ratio constraints.

Q.E.D.

Theorem 3.1: At optimality of the Frontier programs, the vectors of excess input consumption, **e**, and of output slacks, **s**, are null:

$$\mathbf{e} = 0 \text{ and } \mathbf{s} = 0 \text{ at optimality.}$$

Proof: Conditions (3) and (4) show that whenever unit l is output inefficient then its output bundle is valued at 1 (R = 0), and whenever the unit is input inefficient then its input bundle is valued at the lower bound value of 1 (C = 0). It follows that, at optimality, if the output bundle is valued at strictly more than 1 then the unit is output efficient, and similarly, if the input bundle is valued at strictly more than 1 then the unit is input efficient.

The proof rests on the equation:

$$-\phi + \theta = (1 - \phi)\mu \mathbf{y}_l + (\theta - 1)\nu \mathbf{x}_l - \mu \mathbf{s} - \nu \mathbf{e} \quad (10)$$

derived from the equality of the primal and dual objective function values at optimality:

$$\theta - \phi = \omega_l + R - C$$

by first substituting $-(\mu \mathbf{y}^* - \nu \mathbf{x}^*)$ for ω_l, as allowed by conditions (7), and $\mu \mathbf{y}_l - \nu \mathbf{x}_l$ for R - C according to conditions (5) and (6) after noting that θ and ϕ are strictly positive, leading to:

$$\theta - \phi = -(\mu \mathbf{y}^* - \nu \mathbf{x}^*) + \mu \mathbf{y}_l - \nu \mathbf{x}_l$$

The transition to (10) derives from conditions (1), (2), (8), and (9) by extracting from these equations equivalent expressions for $\mu \mathbf{y}^*$ and $\nu \mathbf{x}^*$.

Four situations can occur, namely: $\theta = \phi = 1$, $\theta = 1$ and $\phi > 1$, $\theta < 1$ and $\phi = 1$, and lastly, $\theta < 1$ and $\phi > 1$.

If $\theta = \phi = 1$, then we have: $-\mu \mathbf{s} - \nu \mathbf{e} = 0$. From Lemmas 3.1 and 3.2 $\mu > 0$ and $\nu > 0$, hence we must have $\mathbf{s} = 0$ and $\mathbf{e} = 0$.

If $\theta = 1$ and $\phi > 1$, then we have $\mu \mathbf{y}_l = 1$ and therefore (10) reduces to:

$$-\phi + 1 = (1 - \phi)1 + (1 - 1)\nu \mathbf{x}_l - \mu \mathbf{s} - \nu \mathbf{e}$$

implying $-\mu \mathbf{s} - \nu \mathbf{e} = 0$. Hence $\mathbf{s} = 0$ and $\mathbf{e} = 0$.

If $\theta < 1$ and $\phi = 1$, then we have $\nu \mathbf{x}_l = 1$ and therefore:

$$-1 + \theta = (1 - 1)\mu \mathbf{y}_l + (\theta - 1)1 - \mu \mathbf{s} - \nu \mathbf{e}$$

also implying $-\mu \mathbf{s} - \nu \mathbf{e} = 0$. Hence $\mathbf{s} = 0$ and $\mathbf{e} = 0$.

If $\theta < 1$ and $\phi > 1$, then we have $\mu y_l = 1$ and $\nu x_l = 1$ and therefore:

$$-\phi + \theta = (1 - \phi)1 + (\theta - 1)1 - \mu \mathbf{s} - \nu \mathbf{e}$$

implying as well $- \mu \mathbf{s} - \nu \mathbf{e} = 0$. Hence $\mathbf{s} = 0$ and $\mathbf{e} = 0$.

It follows that in all four situations we have $\mathbf{s} = 0$ and $\mathbf{e} = 0$.

Q.E.D.

<u>Corollary 3.1</u>:

$$\theta - \phi = (\theta - 1)\nu \mathbf{x}_l + (1 - \phi)\mu \mathbf{y}_l \quad \text{at optimality}$$

These results allow the objective of the global orientation to be interpreted as the maximization of the global waste value since at optimality we can write:

$$\mathbf{x}^* = \theta \mathbf{x}_l + \mathbf{R}_i \sigma_i$$
$$\mathbf{y}^* = \phi \mathbf{y}_l + \mathbf{R}_o \sigma_o$$

It follows:

$$\nu \mathbf{x}^* = \theta \nu \mathbf{x}_l + \nu \mathbf{R}_i \sigma_i = \theta \nu \mathbf{x}_l$$
$$\mu \mathbf{y}^* = \phi \mu \mathbf{y}_l + \mu \mathbf{R}_o \sigma_o = \phi \mu \mathbf{y}_l$$

The global waste is then valued at:

$$(1 - \theta)\nu \mathbf{x}_l + (\phi - 1)\mu \mathbf{y}_l = \phi - \theta$$

which is maximized since, by referring to the objective of the primal program we have:

$$\text{Max } \phi - \theta \equiv - \text{Min } \theta - \phi$$

The global orientation therefore corresponds to a conservative efficiency evaluation from the point of view of the primal formulation.

An input-orientation can easily be implemented by considering a two-phase solution of the model. The proportional input reduction is maximized, i.e. θ is minimized in the first phase. In the second phase the following pair of dual programs are solved:

$$\text{Min}_{\theta,\phi,\lambda,\sigma_i,\sigma_o}\ \theta - \phi$$

$$st \begin{cases} -\phi \mathbf{y}_l + \mathbf{Y}\lambda + \mathbf{R}_o\sigma_o = 0 \\ \theta \mathbf{x}_l - \mathbf{X}\lambda + \mathbf{R}_i\ \sigma_i = 0 \\ \mathbf{1}.\lambda = 1 \\ \phi \geq 1 \\ -\theta = -\theta^* \\ \lambda \geq 0 \\ \sigma_o,\ \sigma_i \geq 0 \end{cases} \qquad (\text{Frontier}_{\text{Input/II}}\ \text{Primal})$$

$$\text{Max}_{\mu,\nu,\omega_l,R,C}\ \ \omega_l + R - C\theta^*$$

$$st \begin{cases} \mu \mathbf{Y} - \nu \mathbf{X} + \omega_l \leq 0 \\ \nu \mathbf{x}_l - C \leq 1 \\ -\mu \mathbf{y}_l + R \leq -1 \\ \nu \mathbf{R}_i \leq 0 \\ \mu \mathbf{R}_o \leq 0 \\ R \geq 0 \\ \mu,\ \nu,\ C,\ \omega_l \text{ unrestricted} \end{cases} \qquad (\text{Frontier}_{\text{Input/II}}\ \text{Dual})$$

If the tradeoff constraints expressed by R_i and R_o are consistent with the VRS envelope identified using any of the BCC or ADD models, then all efficient units remain efficient and the pricing of inefficient units is consistent with the pricing of their associated efficient units. If the ranges of ratios are made more restrictive, then the efficient set may be reduced. In particular the efficient units which may have been rated efficient simply because of a minimum consumption of one input relative to other units may now be identified as inefficient. In practice, state of the art technology, on one hand, may allow the definition of "technical" tradeoffs leading to an evaluation of technical efficiency. Conditions of supply and demand on the other hand may define different ranges of acceptable tradeoffs for the various inputs and outputs. A second analysis, based on ranges which are intersections of technical and economic ranges, will provide an evaluation of economic/allocative efficiency.

3.5.3 Computation of the Efficiency Score

The efficiency score can easily be computed from the optimal solution values. Its expression emphasizes once more that the evaluation is not geared toward making a unit look its best, nor its worst, for minimizing $\theta - \phi$ does not necessarily imply that $E(x,y)$ is minimized as well.

At optimality, as mentioned earlier, we have:

$$x^* = \theta x_l + R_i \sigma_i$$
$$y^* = \phi y_l + R_o \sigma_o$$

with:

$$\nu x^* = \theta \nu x_l + \nu R_i \sigma_i = \theta \nu x_l$$
$$\mu y^* = \phi \mu y_l + \mu R_o \sigma_o = \phi \mu y_l$$

hence:

$$E(x,y) = \frac{\nu x^*}{\nu x_l} \cdot \frac{\mu y_l}{\mu y^*} = \frac{\theta}{\phi}$$

3.6 Computational Illustration

The exposition in this section is supported throughout by a numerical example. All data related to this example are summarized in Table 3.1 on page 120 at the end of the chapter and illustrate a production possibility set described by four isoquants in a 2-input x 2-output space. These isoquants are representative of a VRS type-envelopment since isoquants associated with proportional output vectors are not homothetic of one another. For instance unit E3, which belongs to isoquant (1,1), consumes inputs in the amounts (3,3.4) while unit H2, which belongs to isoquant (2,2), only consumes inputs in the amounts (6,3.6), less than double the amount consumed by E3. The assumption of a CRS-type envelopment would lead to a reduced set of efficient units and different isoquants, but the results developed below for the VRS envelopment would apply.

A first set of analyses focuses on explicit pricing mechanisms, standard pricing, invariant pricing, and normed pricing respectively. The set of efficient units remains the same and includes: E_1 through E_{10} belonging to isoquant (1,1), F_1 through F_6 belonging to isoquant (1,2), G_1 through G_6 belonging to isoquant (2,1), and H_1 through H_4 belonging to isoquant (2,2). The output to these analyses presents the revealed waste for each measure, the corresponding efficiency prices, the proportional output augmentation or input reduction, or aggregated waste, as well as the resulting input efficiency score, output efficiency score, and global efficiency score. This information is summarized for all inefficient units in Table 3.2 through Table 3.10 on pages 121 through 129 at the end of the chapter, with output-oriented analyses gathered in Table 3.2 through Table 3.4, input-oriented analyses gathered in Table 3.5 through Table 3.7, and analyses based on a global orientation gathered in Table 3.8 through Table 3.10.

<u>Lemma 3.3</u>: For a given orientation, the selection of an efficient unit on the envelope against which a unit's inefficiency will be gauged depends on the pricing mechanism.

<u>Proof</u>: The importance of the pricing mechanism is illustrated by I_8 which, in the case of an output-orientation, is projected to different points on the envelopment surface depending on which pricing mechanism is used. The obtained results are summarized in Table 3.11 on page 130 at the end of the chapter.

All projections correspond to a same proportional augmentation of outputs (ϕ = 2.8571) but the final recommendations to eliminate waste differ drastically and justifiably according to the relative pricing implied by the various pricing mechanisms.

<u>Lemma 3.4</u> For a same form of envelopment and a given pricing mechanism different orientations lead to different projected points.

<u>Proof</u> This fact is clearly illustrated by tables 3.2, 3.5, and 3.8, respectively on page 121, 124, and 127, which, for a same (standard) pricing mechanism, exhibit different amount of identified waste for all DMUs but one (I_1) across the three orientations.

Lemma 3.5 All input-oriented (respectively output-oriented) evaluations lead to the same value of the maximum proportional input reduction (respectively output augmentation) for any DMU independently of the explicit pricing mechanism and we have:

$$\theta = E_I(x,y) \text{ iff } \mathbf{e} = 0 \qquad \frac{1}{\phi} = E_o(x,y) \text{ iff } \mathbf{s} = 0$$

Proof The proportional input reduction (output augmentation) preempts the pricing of excess input s and slack outputs. It is therefore independent of the pricing mechanism. In input-oriented evaluations: $x^* = \theta x - e$. It follows:

$$E_I(x,y) = \frac{\nu x^*}{\nu x} = \frac{\theta \nu x - \nu e}{\nu x} = \theta - \frac{\nu e}{\nu x}$$

Hence

$$E_I(x,y) = \theta \Leftrightarrow \mathbf{e} = 0 \text{ and } \mathbf{e} > 0 \Rightarrow E_I(x,y) < \theta$$

Similarly for output-oriented evaluations we have:

$$y^* = \phi y + s$$

$$E_O(x,y) = \frac{\mu y}{\mu y^*} = \frac{1}{\phi + \dfrac{\mu s}{\mu y}}$$

$$E_O(x,y) = \frac{1}{\phi} \Leftrightarrow s = 0 \text{ and } s > 0 \Rightarrow E_O(x,y) < \frac{1}{\phi}$$

Lemma 3.6 Given an orientation and a same projected point, the efficiency score may differ across pricing mechanisms if and only if excess input consumption or output slacks are present after the proportional change consistent with the orientation has been effected.

Proof Let us consider the case of an input orientation and represent the pricing mechanism by $(\mathbf{p_x}, \mathbf{p_y})$. We have:

$$E(\mathbf{x},\mathbf{y}) = \frac{\theta - \epsilon(\mathbf{p_x e} + \mathbf{p_y s}) - \omega_1}{1 - \dfrac{\omega_1}{\nu \mathbf{x}^*}}$$

If $\mathbf{e} = 0$ and $\mathbf{s} = 0$ then $\mathbf{x}^* = \theta\mathbf{x}$, $\nu\mathbf{x}^* = \theta\nu\mathbf{x}_1 = \theta$. It follows $E(\mathbf{x},\mathbf{y}) = \theta$. The case where $\mathbf{e} \neq 0$ or $\mathbf{s} \neq 0$ is illustrated by I_5 in the case of an input orientation. The values of the identified waste and the corresponding efficiency prices using respectively the invariant and the normed pricing mechanisms, are summarized in Table 3.12 on page 130. We note that for both mechanisms the same relative input prices are revealed ($\nu_1/\nu_2 = 0.40$). However, the explicit component of the pricing mechanism comes into effect with the existence of output slack and is responsible for the difference in the efficiency score.

More important than the discrepancy in the computed efficiency score is the discrepancy in the categorization of the inefficient units based on their efficiency score, once again emphasizing the criticality of the selected pricing mechanism. For instance if we consider 9 categories with catogory I grouping DMUs which exhibit score of 0.95

and above, category II for scores between 0.85 and 0.95, category III for scores between 0.75 and 0.85, and so on, down to category IX grouping DMUs which exhibit scores strictly lower than 0.25, then, in the case of the invariant pricing mechanism, the obtained scores, given in Table 3.6 on page 125, allow the assignment of I_{15} to category I, of units I_1 and I_2 to category II, of I_{10} and I_{12} to category III, of I_{11} to category IV, of I_3, I_{13}, I_{14} in category V, and of I_4, I_5, I_6, I_7, I_8, and I_9 to category VIII. However, in the case of the normed pricing mechanism, the obtained scores, given in Table 3.7 on page 126, lead to a different categorization of I_1 which would drop to category III, and of I_5, I_6, I_8, and I_9 which would jump to category VII. If bonuses are to be awarded based on categories units 1 will lobby for the invariant pricing mechanism while units 5, 6, 8, and 9 will favor the normed pricing mechanism.

It is of interest to see how different orientations affect the efficiency scores, namely the input-oriented efficiency score, $E_I(x,y)$, the output-oriented efficiency score, $E_O(x,y)$, and the global efficiency score, $E(x,y)$.

<u>Lemma 3.7</u> Given a pricing mechanism, i) $E_I(x,y)$ is the smallest for the input orientation, ii) $E_O(x,y)$ is the smallest for the output orientation, but iii) $E(x,y)$ is **not** necessarily the smallest over the global orientation.

<u>Proof</u> i) and ii) are trivial from the formulation of the objective function of the primal program for respectively the input and output orientations.

iii) is evidenced by the efficiency scores of I_{10} when considering a normed pricing mechanism. These scores are summarized in Table 3.13 on page 130.

A second set of analyses focuses on constrained implicit pricing mechanisms. Three sets of price ranges were considered deriving respectively from the observed price ratios associated with the efficient units under the standard, invariant, and normed price mechanisms. For each orientation and price mechanism, the observed ranges across efficient units are summarized in Table 3.14 on page 131.

The selected ranges cover the observed ranges across orientations for each price mechanism and are summarized in Table 3.15 on page 131.

For each set of price ranges: S, N, and I, the partition of efficient and inefficient units remains the same. The recommendations to the inefficient units in terms of input reduction and substitutions, output augmentation and substitutions, the derived efficiency prices, price ratios, and efficiency scores are summarized in Tables 3.16 through 3.18 on pages 132 through 134 at the end of the chapter.

We observe that the recommendations in the case of range set S differ drastically from those in the case of sets N and I. The recommendations in the case of set N are the same as those for set I. The obvious explanation is that the allowed output price ratio of set S are very forgiving regarding the valuation of output waste, making input waste more valuable by comparison, hence directing/orienting the search for waste toward the inputs. The output price ratio ranges are much closer in the case of sets N and I and, accordingly, we observe more commonality in the identified sources of inefficiency, i.e., the waste, in Table 3.17. In fact in all but two cases (I_{11} and I_{14}) the projected points are the same and, as expected, both exceptions involve substitutions which are forced by the limit price ratios: Different limit ratios imply different extent of required substitutions.

The role of the limit price ratios is further emphasized by observing that, despite same projected points, the efficiency scores differ reflecting different tradeoffs for the input and output measures. The tighter the range of allowed price ratios the lower the efficiency score of units requiring substitution.

3.7 Conclusions

Once the form of the envelopment surface is specified, DEA models gauge the efficiency of DMUs by implementing evaluation procedures which differ in terms of embedded economic principles and implicit managerial objectives. In particular in the process of identifying excessive input consumption and deficient output production, DEA models may focus primarily on controlling input consumption or primarily on output production, or on avoiding waste without distinction between inputs and outputs. The accounting of the identified waste is effected via a pricing mechanism that offers various degrees of flexibility regarding the relative tradeoffs for the inputs and outputs the estimated prices imply.

The Frontier model offers numerous advantages over the earlier DEA models. In particular pricing is consistently effected across all units and completely avoids unrealistic implicit tradeoff values across inputs and across inputs. More importantly, the recommendations to inefficient units are fully contained in the **unique** identified referent point on the frontier. There is therefore no ambiguity left in the results of the evaluation of inefficent units once the objectives of the evaluation are clearly stated: find the **closest** technique and production schedule (respectively characterized by input consumption ratios and output mix ratios) such that comparative disadvantage (measured by the discrepancy between attained and maximum attainable profit) is minimized. In so doing relative prices are identified for inputs and outputs which are compatible with predefined acceptable ranges for such relative prices. These prices maximize the value of total identified waste, namely the sum of savings that can be obtained with regard to input consumption and of unrealized revenues stemming from inefficient output mix and deficient throughtput.

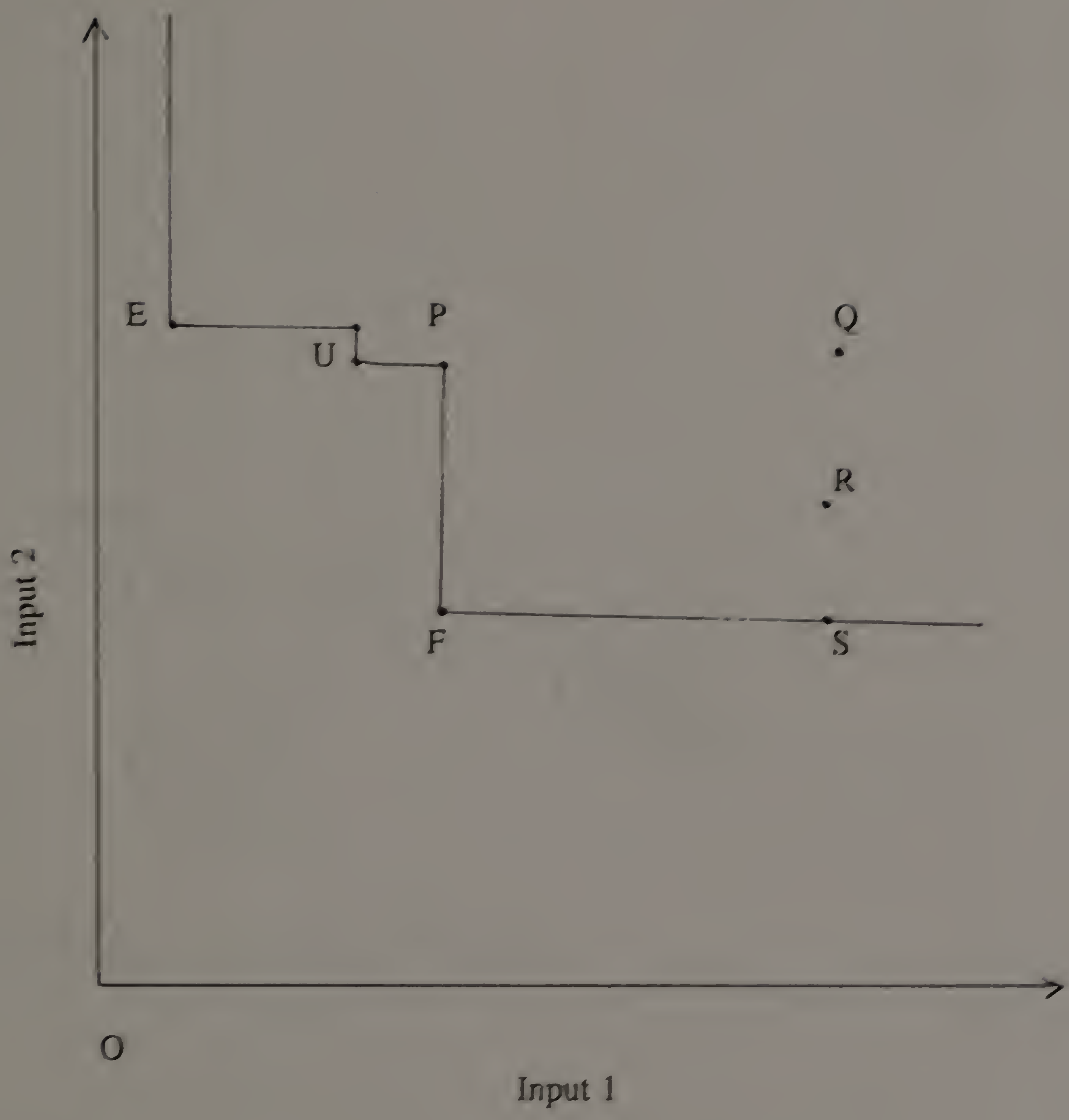

Figure 3.1 Input Set L(y) FDH Methodology

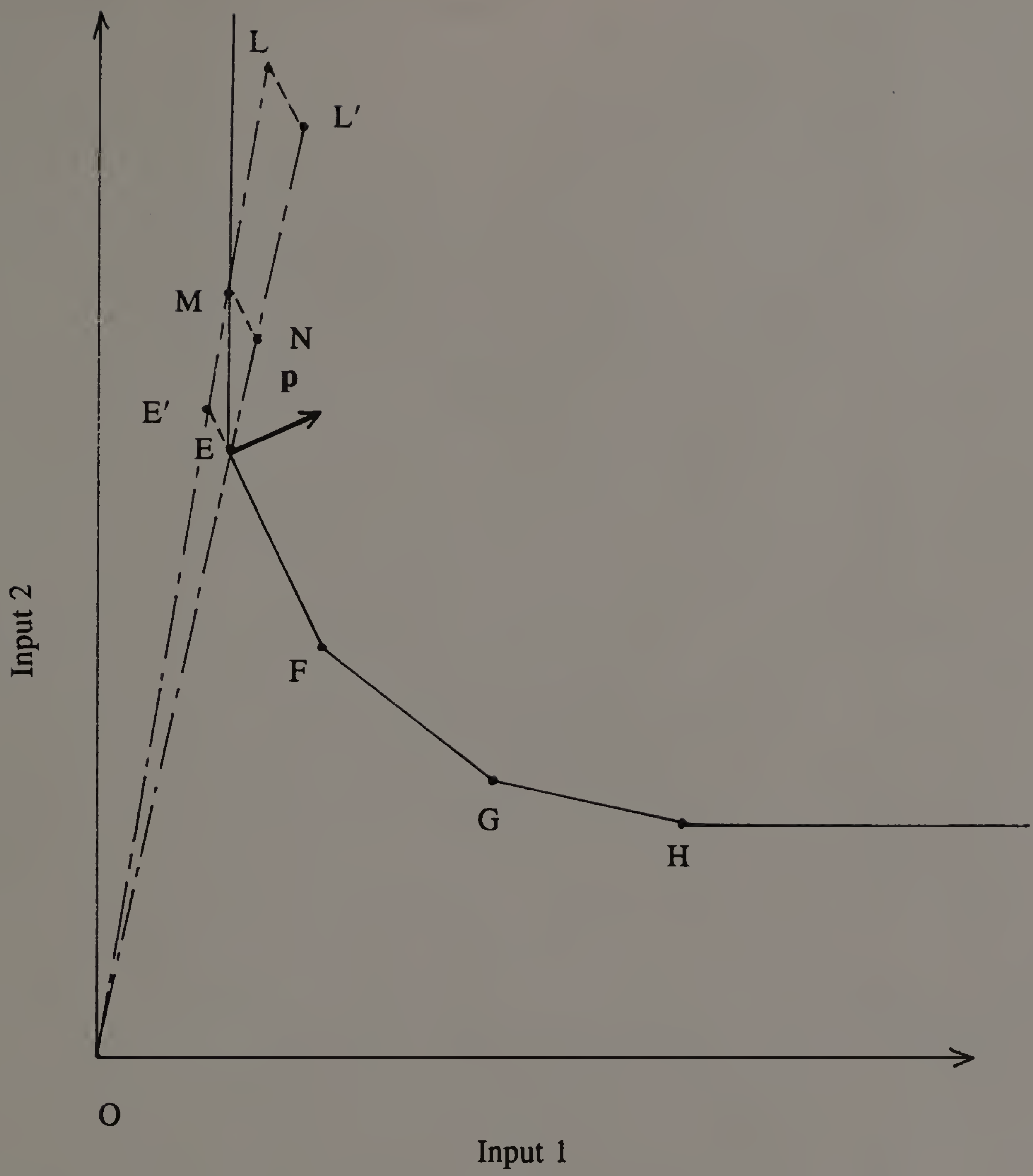

Figure 3.2 Inefficiency Evaluation with Input Substitution

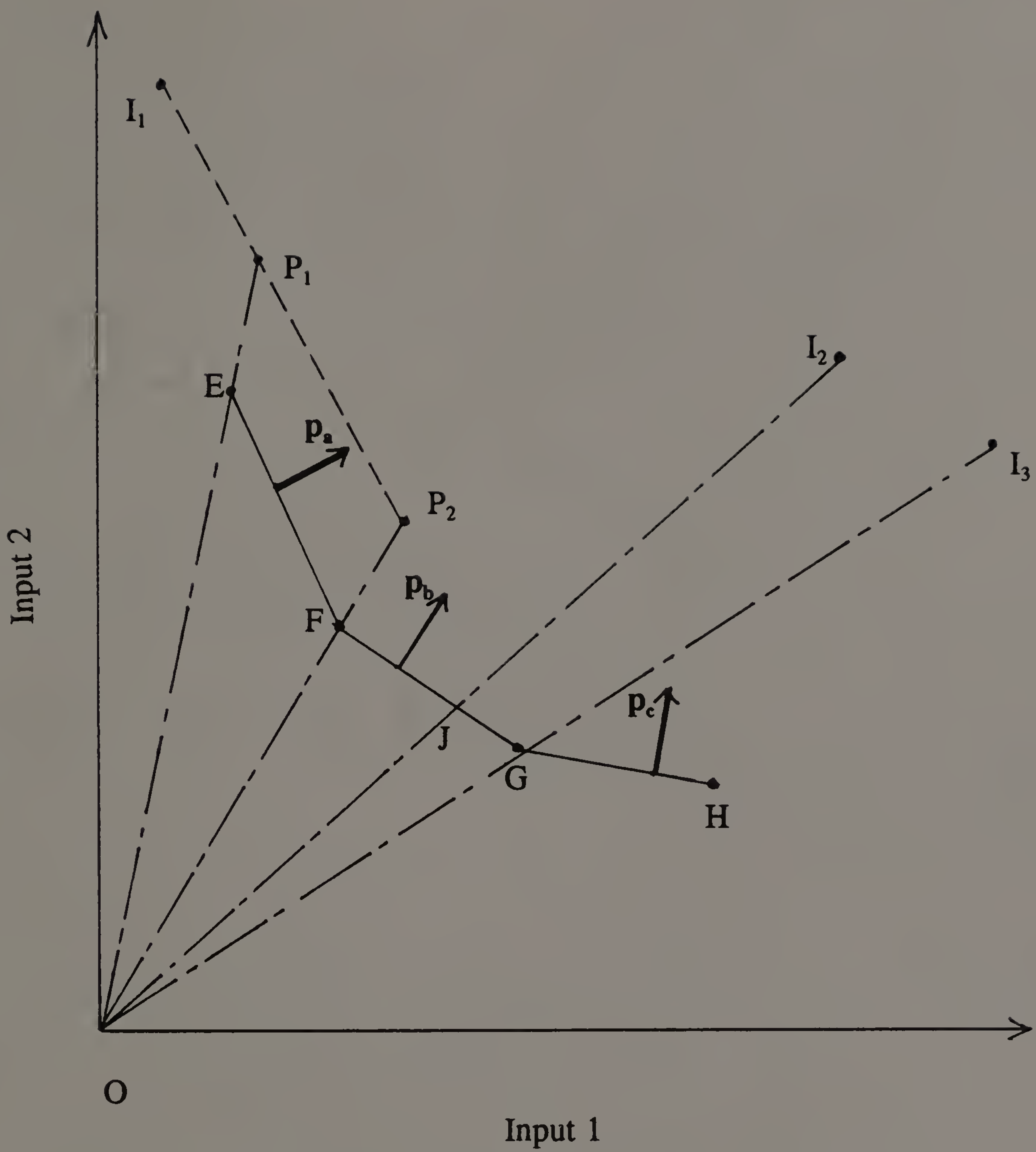

Figure 3.3 Role of Prices in Inefficiency Measurement

Table 3.1 Data Set

DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
E1	1.00	7.00	1.00	1.00
E2	2.00	4.60	1.00	1.00
E3	3.00	3.40	1.00	1.00
E4	5.00	2.40	1.00	1.00
E5	7.00	1.60	1.00	1.00
E6	10.00	1.20	1.00	1.00
E7	12.00	1.00	1.00	1.00
E8	1.74	5.22	1.00	1.00
E9	3.27	3.27	1.00	1.00
E10	5.21	2.32	1.00	1.00
F1	2.00	8.00	1.00	2.00
F2	3.00	6.00	1.00	2.00
F3	4.00	4.40	1.00	2.00
F4	6.00	2.80	1.00	2.00
F5	9.00	2.40	1.00	2.00
F6	13.00	2.20	1.00	2.00
G1	2.40	8.00	2.00	1.00
G2	2.80	5.40	2.00	1.00
G3	4.40	3.80	2.00	1.00
G4	7.00	2.80	2.00	1.00
G5	10.20	2.00	2.00	1.00
G6	14.00	1.80	2.00	1.00
H1	4.40	5.00	2.00	2.00
H2	6.00	3.60	2.00	2.00
H3	8.00	3.00	2.00	2.00
H4	11.00	2.80	2.00	2.00
I1	1.00	9.00	1.00	1.00
I2	2.00	6.00	1.00	1.00
I3	9.00	4.00	1.00	1.00
I4	9.00	4.00	.50	.50
I5	9.00	4.00	.40	.80
I6	9.00	4.00	.80	.40
I7	5.00	5.00	.50	.50
I8	5.00	5.00	.70	.40
I9	5.00	5.00	.40	.70
I10	6.00	4.00	1.20	1.40
I11	18.00	1.20	.90	1.00
I12	6.00	2.88	1.00	1.00
I13	8.00	3.84	1.00	1.00
I14	3.00	8.00	1.00	1.00
I15	4.70	3.13	1.20	1.40

Table 3.2 Evaluation of Inefficient Units
VRS Envelopment/ Output Orientation/ Standard Pricing

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	0.00	0.23	0.23
I3	3.46	0.00	1.00	1.00
I4	3.46	0.00	1.50	1.50
I5	4.50	0.00	0.60	1.20
I6	4.80	0.00	1.20	0.60
I7	0.60	0.00	1.50	1.50
I8	1.63	0.00	1.30	0.74
I9	0.60	0.00	1.60	1.30
I10	0.77	0.00	0.51	0.60
I11	5.92	0.00	0.13	0.15
I12	0.00	0.00	0.55	0.55
I13	2.27	0.00	1.00	1.00
I14	0.00	1.09	0.68	0.68
I15	0.00	0.00	0.03	0.04
	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	2.40	1.00	1.00	1.00
I2	2.40	1.00	2.72	3.40
I3	1.00	1.14	1.86	1.37
I4	1.00	1.14	1.86	1.37
I5	1.00	1.25	1.00	2.25
I6	1.00	1.00	1.80	1.20
I7	1.00	1.14	1.86	1.37
I8	1.00	1.00	1.80	1.20
I9	1.00	1.00	1.00	2.00
I10	1.00	1.14	1.09	2.14
I11	1.00	20.00	10.00	25.00
I12	1.00	2.50	2.00	2.00
I13	1.00	1.14	1.86	1.37
I14	6.50	1.00	12.60	10.00
I15	1.00	2.00	1.60	1.80
DMU	ϕ	E_i	E_o	E
I1	1.00	0.82	1.00	0.82
I2	1.23	1.00	0.81	0.81
I3	2.00	0.75	0.50	0.37
I4	4.00	0.75	0.25	0.19
I5	2.50	0.68	0.40	0.27
I6	2.50	0.63	0.40	0.25
I7	4.00	0.94	0.25	0.24
I8	2.86	0.84	0.35	0.29
I9	2.86	0.94	0.30	0.28
I10	1.43	0.93	0.70	0.65
I11	1.15	0.86	0.87	0.75
I12	1.55	1.00	0.65	0.65
I13	2.00	0.82	0.50	0.41
I14	1.68	0.96	0.59	0.57
I15	1.03	1.00	0.97	0.97

Table 3.3 Evaluation of Inefficient Units
VRS Envelopment/ Output Orientation/ Invariant Pricing

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	0.00	0.23	0.23
I3	3.00	0.40	1.00	1.00
I4	3.00	0.40	1.50	1.50
I5	3.00	0.40	1.60	1.20
I6	3.00	0.40	1.20	1.60
I7	0.60	0.00	1.50	1.50
I8	0.60	0.00	1.30	1.60
I9	0.60	0.00	1.60	1.30
I10	0.00	0.40	0.80	0.60
I11	5.92	0.00	0.13	0.15
I12	0.00	0.00	0.55	0.55
I13	2.00	0.24	1.00	1.00
I14	0.00	1.09	0.68	0.68
I15	0.00	0.00	0.03	0.03
	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	1.00	0.11	1.00	1.00
I2	0.88	0.37	1.00	1.25
I3	0.11	0.25	1.00	1.00
I4	0.11	0.25	2.00	2.00
I5	0.11	0.25	2.50	1.25
I6	0.11	0.25	1.25	2.50
I7	0.20	0.23	2.00	2.00
I8	0.20	0.23	1.43	2.50
I9	0.20	0.23	2.50	1.43
I10	0.17	0.25	0.83	0.71
I11	0.06	2.04	1.11	2.50
I12	0.50	1.25	1.00	1.00
I13	0.13	0.26	1.00	1.00
I14	0.81	0.13	1.58	1.25
I15	0.52	1.04	0.83	0.94
DMU	ϕ	E_I	E_o	E
I1	1.00	0.89	1.00	0.89
I2	1.23	1.00	0.81	0.81
I3	2.00	0.78	0.50	0.39
I4	4.00	0.78	0.25	0.20
I5	2.50	0.78	0.27	0.21
I6	2.50	0.78	0.27	0.21
I7	4.00	0.94	0.25	0.24
I8	2.86	0.94	0.25	0.24
I9	2.86	0.94	0.25	0.24
I10	1.43	0.95	0.65	0.61
I11	1.15	0.90	0.87	0.79
I12	1.55	1.00	0.65	0.65
I13	2.00	0.84	0.50	0.42
I14	1.68	0.96	0.59	0.57
I15	1.03	1.00	0.97	0.97

Table 3.4 Evaluation of Inefficient Units
VRS Envelopment/ Output Orientation/ Normed Pricing
Normed Prices (P_{x1} = 0.1652, P_{x2} = 0.2708, P_{y1} = 0.7220, P_{y2} = 0.7220)

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	0.00	0.23	0.23
I3	3.00	0.40	1.00	1.00
I4	3.00	0.40	1.50	1.50
I5	3.00	0.40	1.60	1.20
I6	3.00	0.40	1.20	1.60
I7	0.00	0.53	1.50	1.50
I8	0.00	0.53	1.30	1.60
I9	0.00	0.53	1.60	1.30
I10	0.00	0.40	0.80	0.60
I11	5.92	0.00	0.13	0.15
I12	0.00	0.00	0.55	0.55
I13	2.00	0.24	1.00	1.00
I14	0.00	1.09	0.68	0.68
I15	0.00	0.00	0.03	0.04
	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.65	0.27	0.72	0.72
I2	0.65	0.27	0.74	0.92
I3	0.17	0.27	0.72	0.72
I4	0.17	0.27	0.72	0.72
I5	0.17	0.27	0.72	0.72
I6	0.17	0.27	0.72	0.72
I7	0.24	0.27	0.72	0.72
I8	0.24	0.27	0.72	0.72
I9	0.24	0.27	0.72	0.72
I10	0.17	0.27	0.72	0.72
I11	0.17	3.30	1.65	4.13
I12	0.36	0.90	0.72	0.72
I13	0.17	0.27	0.72	0.72
I14	1.76	0.27	3.41	2.70
I15	0.45	0.90	0.72	0.81
DMU	ϕ	E_i	E_o	E
I1	1.00	0.82	1.00	0.82
I2	1.23	1.00	0.81	0.81
I3	2.00	0.77	0.50	0.38
I4	4.00	0.77	0.25	0.19
I5	2.50	0.77	0.30	0.23
I6	2.50	0.77	0.30	0.23
I7	4.00	0.94	0.25	0.24
I8	2.86	0.94	0.28	0.26
I9	2.86	0.94	0.28	0.26
I10	1.43	0.95	0.65	0.62
I11	1.15	0.86	0.87	0.75
I12	1.55	1.00	0.65	0.65
I13	2.00	0.83	0.50	0.42
I14	1.68	0.96	0.59	0.57
I15	1.03	1.00	0.97	0.97

Table 3.5 Evaluation of Inefficient Units
VRS Envelopment/ Input Orientation/ Standard Pricing

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.26	0.78	0.00	0.00
I3	3.79	1.68	0.00	0.00
I4	3.79	1.68	0.50	0.50
I5	3.79	1.68	0.60	0.20
I6	3.79	1.68	0.20	0.60
I7	1.73	1.73	0.50	0.50
I8	1.73	1.73	0.30	0.60
I9	1.73	1.73	0.60	0.30
I10	1.35	0.90	0.00	0.00
I11	6.00	0.20	0.10	0.00
I12	1.00	0.48	0.00	0.00
I13	3.00	1.44	0.00	0.00
I14	1.15	3.05	0.00	0.00
I15	0.05	0.34	0.00	0.00
	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	2.40	1.00	1.00	1.00
I2	2.40	1.00	1.00	3.40
I3	1.00	2.50	1.00	1.00
I4	1.00	2.50	1.00	1.00
I5	1.00	2.50	1.00	1.00
I6	1.00	2.50	1.00	1.00
I7	1.00	2.00	1.00	1.00
I8	1.00	2.00	1.00	1.00
I9	1.00	2.00	1.00	1.00
I10	1.00	2.00	1.60	1.80
I11	1.00	10.00	1.00	1.00
I12	1.00	2.00	1.00	1.00
I13	1.00	2.00	1.00	1.00
I14	2.40	1.00	1.00	1.00
I15	1.00	2.00	1.60	1.80
DMU	θ	E_i	E_o	E
I1	1.00	0.82	1.00	0.82
I2	0.87	0.87	1.00	0.87
I3	0.58	0.58	1.00	0.58
I4	0.58	0.58	0.50	0.29
I5	0.58	0.58	0.60	0.35
I6	0.58	0.58	0.60	0.35
I7	0.65	0.65	0.50	0.33
I8	0.65	0.65	0.55	0.36
I9	0.65	0.65	0.55	0.36
I10	0.77	0.77	1.00	0.77
I11	0.83	0.83	0.95	0.70
I12	0.83	0.83	1.00	0.83
I13	0.63	0.63	1.00	0.63
I14	0.62	0.62	1.00	0.62
I15	0.99	0.99	1.00	0.99

Table 3.6 Evaluation of Inefficient Units
VRS Envelopment/ Input Orientation/ Invariant Pricing

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.26	0.78	0.00	0.00
I3	3.79	1.68	0.00	0.00
I4	3.79	1.68	0.50	0.50
I5	3.79	1.68	0.60	0.20
I6	3.79	1.68	0.20	0.60
I7	1.73	1.73	0.50	0.50
I8	1.73	1.73	0.30	0.60
I9	1.73	1.73	0.60	0.30
I10	1.35	0.90	0.00	0.00
I11	6.00	0.20	0.10	0.00
I12	1.00	0.48	0.00	0.00
I13	3.00	1.44	0.00	0.00
I14	1.15	3.05	0.00	0.00
I15	0.05	0.34	0.00	0.00
	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	1.00	0.11	1.00	1.00
I2	0.88	0.37	1.00	1.25
I3	0.50	1.25	1.00	1.00
I4	1.00	2.50	2.00	2.00
I5	0.94	2.34	2.50	1.25
I6	1.25	3.13	1.25	2.50
I7	1.18	2.35	2.00	2.00
I8	1.39	2.78	1.43	2.50
I9	1.16	2.31	2.50	1.43
I10	0.52	1.04	0.83	0.94
I11	0.06	1.25	1.11	1.00
I12	0.59	1.18	1.00	1.00
I13	0.59	1.18	1.00	1.00
I14	0.88	0.37	1.00	1.00
I15	0.52	1.04	0.83	0.94
DMU	θ	E_i	E_o	E
I1	1.00	0.89	1.00	0.89
I2	0.87	0.87	1.00	0.87
I3	0.58	0.58	1.00	0.58
I4	0.58	0.58	0.50	0.29
I5	0.58	0.58	0.53	0.31
I6	0.58	0.58	0.53	0.31
I7	0.65	0.65	0.50	0.33
I8	0.65	0.65	0.51	0.33
I9	0.65	0.65	0.51	0.33
I10	0.77	0.77	1.00	0.77
I11	0.83	0.77	0.95	0.73
I12	0.83	0.83	1.00	0.83
I13	0.63	0.63	1.00	0.63
I14	0.62	0.62	1.00	0.62
I15	0.99	0.99	1.00	0.99

Table 3.7 Evaluation of Inefficient Units
VRS Envelopment/ Input Orientation/ Normed Pricing
Prices = (P_{x1} = 0.1652, P_{x2} = 0.2708, P_{y1} = 0.7220, P_{y2} = 0.7220)

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.26	0.78	0.00	0.00
I3	3.79	1.68	0.00	0.00
I4	3.79	1.68	0.50	0.50
I5	3.79	1.68	0.60	0.20
I6	3.79	1.68	0.20	0.60
I7	1.73	1.73	0.50	0.50
I8	1.73	1.73	0.30	0.60
I9	1.73	1.73	0.60	0.30
I10	1.35	0.90	0.00	0.00
I11	6.00	0.20	0.10	0.00
I12	1.00	0.48	0.00	0.00
I13	3.00	1.44	0.00	0.00
I14	1.15	3.05	0.00	0.00
I15	0.05	0.03	0.00	0.00
	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.65	0.27	0.72	0.72
I2	0.65	0.27	0.72	0.92
I3	0.36	0.90	0.72	0.72
I4	0.36	0.90	0.72	0.72
I5	0.36	0.90	0.72	0.72
I6	0.36	0.90	0.72	0.72
I7	0.42	0.85	0.72	0.72
I8	0.42	0.85	0.72	0.72
I9	0.42	0.85	0.72	0.72
I10	0.45	0.90	0.72	0.82
I11	0.17	1.65	0.72	0.72
I12	0.42	0.85	0.72	0.72
I13	0.42	0.85	0.72	0.72
I14	0.65	0.27	0.72	0.72
I15	0.45	0.90	0.72	0.82
DMU	θ	E_i	E_o	E
I1	1.00	0.82	1.00	0.82
I2	0.87	0.87	1.00	0.87
I3	0.58	0.58	1.00	0.58
I4	0.58	0.58	0.50	0.29
I5	0.58	0.58	0.60	0.35
I6	0.58	0.58	0.60	0.35
I7	0.65	0.65	0.50	0.33
I8	0.65	0.65	0.55	0.36
I9	0.65	0.65	0.55	0.36
I10	0.77	0.77	1.00	0.77
I11	0.83	0.73	0.95	0.70
I12	0.83	0.83	1.00	0.83
I13	0.63	0.63	1.00	0.63
I14	0.62	0.62	1.00	0.62
I15	0.99	0.99	1.00	0.99

Table 3.8 Evaluation of Inefficient Units
VRS Envelopment/ Global Orientation/ Standard Pricing

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	1.40	0.00	0.00
I3	6.00	0.60	0.00	0.00
I4	6.00	0.60	0.50	0.50
I5	6.00	0.60	0.60	0.20
I6	6.00	0.60	0.20	0.60
I7	2.00	1.60	0.50	0.50
I8	2.00	1.60	0.30	0.60
I9	2.00	1.60	0.60	0.30
I10	2.47	0.00	0.13	0.00
I11	8.00	0.00	0.10	0.00
I12	1.96	0.00	0.00	0.00
I13	5.00	0.44	0.00	0.00
I14	0.00	4.60	0.00	0.00
I15	0.12	0.00	0.00	0.00
	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	2.40	1.00	1.00	1.00
I2	2.38	1.00	2.70	3.40
I3	1.00	1.00	1.00	1.00
I4	1.00	1.00	1.00	1.00
I5	1.00	1.00	1.00	1.00
I6	1.00	1.00	1.00	1.00
I7	1.00	1.00	1.00	1.00
I8	1.00	1.00	1.00	1.00
I9	1.00	1.00	1.00	1.00
I10	1.00	1.00	1.00	2.00
I11	1.00	7.50	1.00	1.00
I12	1.00	2.00	1.00	1.00
I13	1.00	1.00	1.00	1.00
I14	1.20	1.00	1.00	1.00
I15	1.00	2.00	1.60	1.80
DMU	$P_x e + P_y s$	E_i	E_o	E
I1	2.00	0.82	1.00	0.82
I2	1.40	0.87	1.00	0.87
I3	6.60	0.49	1.00	0.49
I4	7.60	0.49	0.50	0.25
I5	7.40	0.49	0.60	0.30
I6	7.40	0.49	0.60	0.30
I7	4.60	0.64	0.50	0.32
I8	4.50	0.64	0.55	0.35
I9	4.50	0.64	0.55	0.35
I10	2.60	0.75	0.97	0.73
I11	8.10	0.70	0.95	0.67
I12	1.96	0.83	1.00	0.83
I13	5.44	0.54	1.00	0.54
I14	4.60	0.60	1.00	0.60
I15	0.12	0.99	1.00	0.99

Table 3.9 Evaluation of Inefficient Units
VRS Envelopment/ Global Orientation/ Invariant Pricing

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	0.00	0.52	0.00
I3	3.00	0.40	1.00	1.00
I4	3.00	0.40	1.50	1.50
I5	3.00	0.40	1.60	1.20
I6	3.00	0.40	1.20	1.60
I7	0.60	0.00	1.50	1.50
I8	0.60	0.00	1.30	1.60
I9	0.60	0.00	1.60	1.30
I10	0.00	0.40	0.80	0.60
I11	5.50	0.00	0.35	0.00
I12	0.00	0.00	0.55	0.55
I13	2.00	0.24	1.00	1.00
I14	0.00	1.25	0.42	1.00
I15	0.00	0.00	0.08	0.00
	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	1.00	0.11	1.00	1.00
I2	0.88	0.37	1.00	1.25
I3	0.11	0.25	1.00	1.00
I4	0.11	0.25	2.00	2.00
I5	0.11	0.25	2.50	1.25
I6	0.11	0.25	1.25	2.50
I7	0.20	0.20	2.00	2.00
I8	0.20	0.23	1.43	2.50
I9	0.20	0.23	2.50	1.43
I10	0.22	0.25	0.83	0.71
I11	0.06	1.25	1.11	1.08
I12	0.50	1.25	1.00	1.00
I13	0.13	0.26	1.00	1.00
I14	0.57	0.13	1.00	1.00
I15	0.52	1.04	0.83	0.94
DMU	$P_x e + P_y s$	E_i	E_o	E
I1	0.22	0.89	1.00	0.89
I2	0.52	1.00	0.81	0.81
I3	2.43	0.78	0.50	0.39
I4	6.43	0.78	0.25	0.20
I5	5.93	0.78	0.27	0.21
I6	5.93	0.78	0.27	0.21
I7	6.12	0.94	0.25	0.24
I8	5.98	0.94	0.25	0.24
I9	5.98	0.94	0.25	0.24
I10	1.20	0.96	0.65	0.62
I11	0.69	0.88	0.84	0.74
I12	1.10	1.00	0.65	0.65
I13	2.31	0.84	0.50	0.42
I14	1.57	0.94	0.59	0.55
I15	0.06	1.00	0.97	0.97

Table 3.10 Evaluation of Inefficient Units
VRS Envelopment/ Global Orientation/ Normed Pricing
Prices = P_{x1} = 0.1652, P_{x2} = 0.2708, P_{y1} = 0.7220, P_{y2} = 0.7220)

	TOTAL WASTE			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	1.40	0.00	0.00
I3	3.00	0.40	1.00	1.00
I4	3.00	0.40	1.50	1.50
I5	3.00	0.40	1.60	1.20
I6	3.00	0.40	1.20	1.60
I7	0.00	0.53	1.50	1.50
I8	0.00	0.53	1.30	1.60
I9	0.00	0.53	1.60	1.30
I10	0.00	0.40	0.80	0.60
I11	8.00	0.00	0.10	0.00
I12	0.00	0.00	0.55	0.55
I13	2.00	0.24	1.00	1.00
I14	0.00	2.65	1.00	0.13
I15	0.00	0.00	0.08	0.00

	EFFICIENCY PRICES			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.65	0.27	0.72	0.72
I2	0.65	0.27	0.74	0.92
I3	0.17	0.27	0.72	0.72
I4	0.17	0.27	0.72	0.72
I5	0.17	0.27	0.72	0.72
I6	0.17	0.27	0.72	0.72
I7	0.24	0.27	0.72	0.72
I8	0.24	0.27	0.72	0.72
I9	0.24	0.27	0.72	0.72
I10	0.24	0.27	0.72	0.72
I11	0.17	1.24	0.72	0.72
I12	0.36	0.90	0.72	0.72
I13	0.17	0.27	0.72	0.72
I14	0.52	0.27	0.72	0.72
I15	0.45	0.90	0.72	0.81

DMU	$P_x e + P_y s$	E_i	E_o	E
I1	0.54	0.82	1.00	0.82
I2	0.38	0.87	1.00	0.87
I3	2.05	0.77	0.50	0.38
I4	2.77	0.77	0.25	0.19
I5	2.63	0.77	0.30	0.23
I6	2.63	0.77	0.30	0.23
I7	2.31	0.94	0.25	0.24
I8	2.24	0.94	0.28	0.26
I9	2.24	0.94	0.28	0.26
I10	1.12	0.96	0.65	0.62
I11	1.39	0.70	0.95	0.67
I12	0.79	1.00	0.65	0.65
I13	1.84	0.83	0.50	0.42
I14	1.53	0.81	0.64	0.52
I15	0.05	1.00	0.97	0.97

Table 3.11 Effect of Pricing Mechanism

VRS/Output-Oriented		Input 1	Input 2	Output 1	Output 2
I8	DATA	5.000	5.000	.700	.400
Standard Pricing	WASTE	1.629	0.000	1.300	.743
	PRICE	1.000	1.000	1.800	1.200
Invariant Pricing	WASTE	0.600	0.000	1.300	1.600
	PRICE	0.200	0.229	1.429	2.500
Normed Pricing	WASTE	0.000	0.525	1.300	1.600
	PRICE	0.237	0.237	0.722	0.722

Table 3.12 Effect of Excess Inputs or Output Slacks on the Efficiency Score

Input Orient.(I_5)	Input 1	Input 2	Output 1	Output 2	E
WASTE	3.789	1.684	0.600	0.200	
INVARIANT PRICES	0.937	2.344	2.500	1.250	0.309
NORMED PRICES	0.361	0.902	0.722	0.722	0.347

Table 3.13 Effect of the Orientation on the Efficiency Scores

Normed Pricing (I_{10})	E_I	E_O	E
Output-Oriented	0.948	0.650	0.616
Input-Oriented	0.774	1.000	0.774
Global Orientation	0.957	0.650	0.623

Table 3.14 Price Ratio Ranges Across Orientations

Input Price Ratio Range Output Price Ratio Range	ORIENTATION		
	INPUT	OUTPUT	GLOBAL
STANDARD	0.05 - 6.50 0.04 - 17.2	0.05 - 6.50 0.04 - 17.2	0.05 - 6.50 0.04 - 17.2
NORMED	0.05 - 6.50 0.17 - 3.94	0.05 - 6.50 0.17 - 3.94	0.05 - 6.50 0.17 - 3.94
INVARIANT	0.04 - 7.00 0.41 - 2.70	0.04 - 7.00 0.41 - 2.89	0.04 - 7.00 0.41 - 2.40

Table 3.15 Selected Price Ratio Ranges

Price Ratio Ranges	ν_1/ν_2	μ_1/μ_2
S	0.05 - 6.50	0.04 - 17.2
N	0.05 - 6.50	0.17 - 3.94
I	0.04 - 7.00	0.41 - 2.89

Table 3.16 VRS Envelopment/ Frontier Model: Efficiency Prices

S Range	INPUT PRICE RATIOS: [0.05 - 6.50] OUTPUT PRICE RATIOS: [0.04 - 17.20]			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.42	0.06	0.65	0.35
I2	0.39	0.16	0.44	0.56
I3	0.04	0.15	0.93	0.07
I4	0.04	0.15	0.12	1.88
I5	0.03	0.19	0.09	1.21
I6	0.05	0.15	1.21	0.07
I7	0.09	0.11	1.87	0.13
I8	0.10	0.10	1.36	0.12
I9	0.09	0.11	0.09	1.38
I10	0.09	0.11	0.09	0.64
I11	0.03	0.59	0.29	0.74
I12	0.25	0.63	0.50	0.50
I13	0.05	0.16	0.13	0.87
I14	0.29	0.04	0.56	0.44
I15	0.23	0.45	0.36	0.41
N Range	INPUT PRICE RATIOS: [0.05 - 6.50] OUTPUT PRICE RATIOS: [0.17 - 3.94]			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.42	0.06	0.65	0.35
I2	0.39	0.16	0.44	0.56
I3	0.04	0.15	0.80	0.20
I4	0.04	0.15	0.29	1.71
I5	0.04	0.15	0.20	1.15
I6	0.04	0.15	1.11	0.28
I7	0.09	0.11	1.60	0.40
I8	0.09	0.11	1.25	0.32
I9	0.09	0.11	0.22	1.30
I10	0.09	0.11	0.11	0.62
I11	0.03	0.59	0.29	0.74
I12	0.25	0.63	0.50	0.50
I13	0.05	0.16	0.15	0.85
I14	0.29	0.04	0.56	0.44
I15	0.23	0.45	0.36	0.41
I Range	INPUT PRICE RATIOS: [0.04 - 7.00] OUTPUT PRICE RATIOS: [0.41 - 2.89]			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.44	0.06	0.68	0.33
I2	0.39	0.16	0.44	0.56
I3	0.04	0.15	0.74	0.26
I4	0.04	0.15	1.49	0.51
I5	0.04	0.15	0.43	1.04
I6	0.04	0.15	1.07	0.37
I7	0.09	0.11	1.49	0.51
I8	0.09	0.11	1.19	0.41
I9	0.09	0.11	0.47	1.16
I10	0.09	0.11	0.22	0.53
I11	0.02	0.59	0.30	0.73
I12	0.25	0.63	0.50	0.50
I13	0.05	0.16	0.74	0.26
I14	0.28	0.04	0.56	0.44
I15	0.23	0.45	0.36	0.41

Table 3.17 VRS Envelopment/ Frontier Model: Total Waste and Substitutions

S Range	INPUT PRICE RATIOS: [0.05 - 6.50] OUTPUT PRICE RATIOS: [0.04 - 17.20]			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	0.00	0.23	0.23
I3	1.75	0.78	1.00	1.00
I4	1.75	0.78	1.50	1.50
I5	2.77	1.23	0.60	1.20
I6	2.25	1.00	1.20	0.85
I7	0.28	0.28	1.50	1.50
I8	0.81	0.81	1.30	0.74
I9	0.71	0.71	0.74	1.30
I10	0.43	0.29	0.51	0.60
I11	6.59	-0.33	0.29	0.32
I12	0.00	0.00	0.55	0.55
I13	1.07	0.52	1.00	1.00
I14	-0.32	2.08	0.73	0.73
I15	0.00	0.00	0.03	0.04
N Range	INPUT PRICE RATIOS: [0.05 - 6.50] OUTPUT PRICE RATIOS: [0.17 - 3.94]			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	0.00	0.23	0.23
I3	1.75	0.78	1.00	1.00
I4	1.75	0.78	1.50	1.50
I5	1.75	0.78	1.60	1.20
I6	1.75	0.78	1.20	1.60
I7	0.28	0.28	1.50	1.50
I8	0.28	0.28	1.30	1.60
I9	0.28	0.28	1.60	1.30
I10	0.26	0.17	0.80	0.60
I11	6.59	-0.33	0.29	0.32
I12	0.00	0.00	0.55	0.55
I13	1.08	0.52	1.00	1.00
I14	-0.32	2.08	0.73	0.73
I15	0.00	0.00	0.03	0.04
I Range	INPUT PRICE RATIOS: [0.04 - 7.00] OUTPUT PRICE RATIOS: [0.41 - 2.89]			
DMU	INPUT 1	INPUT 2	OUTPUT 1	OUTPUT 2
I1	0.00	2.00	0.00	0.00
I2	0.00	0.00	0.23	0.23
I3	1.75	0.78	1.00	1.00
I4	1.75	0.78	1.50	1.50
I5	1.75	0.78	1.60	1.20
I6	1.75	0.78	1.20	1.60
I7	0.28	0.28	1.50	1.50
I8	0.28	0.28	1.30	1.60
I9	0.28	0.28	1.60	1.30
I10	0.26	0.17	0.80	0.60
I11	6.03	-0.24	0.26	0.29
I12	0.00	0.00	0.55	0.55
I13	1.08	0.52	1.00	1.00
I14	-0.19	1.36	0.73	0.73
I15	0.00	0.00	0.32	0.04

Table 3.18 VRS Envelopment/Frontier Model
Efficiency Price Ratios & Efficiency Scores

S Range	INPUT PRICE RATIOS: [0.05 - 6.50] OUTPUT PRICE RATIOS: [0.04 - 17.20]				
DMU	ν_1/ν_2	μ_1/μ_2	θ	ϕ	E
I1	6.50	1.87	0.87	1.00	0.87
I2	2.40	0.80	1.00	1.23	0.81
I3	0.30	12.40	0.81	2.00	0.40
I4	0.30	0.06	0.81	4.00	0.20
I5	0.13	0.07	0.69	2.50	0.28
I6	0.32	17.20	0.75	2.52	0.30
I7	0.88	14.63	0.94	4.00	0.24
I8	1.00	11.33	0.84	2.86	0.29
I9	0.80	0.06	0.86	2.86	0.30
I10	0.80	0.14	0.93	1.43	0.65
I11	0.05	0.40	1.00	1.32	0.76
I12	0.40	1.00	1.00	1.55	0.65
I13	0.30	0.15	0.87	2.00	0.43
I14	6.50	1.26	1.00	1.73	0.58
I15	0.50	0.89	1.00	1.03	0.97
N Range	INPUT PRICE RATIOS: [0.05-6.50] OUTPUT PRICE RATIOS: [0.17-3.94]				
DMU	ν_1/ν_2	μ_1/μ_2	θ	ϕ	E
I1	6.50	1.87	0.87	1.00	0.87
I2	2.40	0.80	1.00	1.23	0.81
I3	0.30	3.94	0.81	2.00	0.40
I4	0.30	0.17	0.81	4.00	0.20
I5	0.30	0.17	0.81	2.70	0.30
I6	0.30	3.94	0.81	2.78	0.29
I7	0.88	3.94	0.94	4.00	0.24
I8	0.88	3.94	0.94	3.13	0.30
I9	0.88	0.17	0.94	3.05	0.31
I10	0.88	0.17	0.96	1.46	0.66
I11	0.05	0.40	1.00	1.32	0.76
I12	0.40	1.00	1.00	1.55	0.65
I13	0.30	0.17	0.87	2.00	0.43
I14	6.50	1.26	1.00	1.73	0.58
I15	0.50	0.89	1.00	1.03	0.97
I Range	INPUT PRICE RATIOS: [0.04-7.00] OUTPUT PRICE RATIOS: [0.41-2.89]				
DMU	ν_1/ν_2	μ_1/μ_2	θ	ϕ	E
I1	7.00	2.08	0.88	1.00	0.88
I2	2.40	0.80	1.00	1.23	0.81
I3	0.30	2.89	0.81	2.00	0.40
I4	0.30	2.89	0.81	4.00	0.20
I5	0.30	0.41	0.81	2.93	0.28
I6	0.30	2.89	0.81	2.87	0.28
I7	0.88	2.89	0.94	4.00	0.24
I8	0.88	2.89	0.94	3.21	0.29
I9	0.88	0.41	0.94	3.26	0.29
I10	0.88	0.41	0.96	1.49	0.64
I11	0.04	0.42	1.00	1.29	0.78
I12	0.40	1.00	1.00	1.55	0.65
I13	0.30	2.89	0.87	2.00	0.43
I14	7.00	1.25	1.00	1.73	0.58
I15	0.50	0.89	1.00	1.03	0.97

CHAPTER 4

EXTENSION TO STRATEGIC PLANNING

4.1 Introduction

The frontier model introduced in the preceding chapter determines whether an economic unit is efficient or not and, in the case that it is not, proceeds to identify sources and extent of inefficiencies. The inefficiencies are with respect to an empirically determined frontier of feasible achievements, and with respect to evaluation principles which embody managerial objectives and tradeoff information. However, in the case that the unit is efficient, no waste in terms of inefficient input consumption or unrealized production is uncovered, and only congratulations may be delivered to the unit with the implied message: "Keep doing whatever it is that you are doing". It seems reasonable to ask whether more constructive advice could be provided to help that unit in sustaining efficiency. The goal of sustaining efficiency is falsely innocuous for it requires constant analysis and active planning since it is unlikely that the frontier of feasible achievements remain static. The mere fact of operating on the frontier implies that the unit enjoys some comparative advantage, relative to all other units, from which its efficiency derives. Therefore it is important for an efficient unit to identify who its current competitors, that may threaten its efficiency, are. It is equally important to identify the sources of the unit's comparative advantage and the extent of its comparative advantage, that is whether it represents a comfortable cushion against competitors attacks or a precarious condition warranting undivided managerial attention. Finally the assessment of its current competitive environment should allow the efficient unit to define its future strategy in terms of choice of technique of production, choice of output mix, and level of operations.

The purpose of this chapter is to develop an analytical tool that will provide an efficient unit with an assessment of its current competitive environment and with critical information to assist the unit in formulating its strategy. Section 2 develops the notion of comparative advantage and its implication to strategy and introduces notation and definitions. A new mathematical program, referred to as the Comparative Advantage model, is developed in Section 3 and its use in assessing the competitive environment is discussed. Section 4 presents a computational illustration using a synthetic data set. A summary and conclusions are stated in Section 5.

4.2 Comparative Advantage and Strategies

Each decision-making unit is characterized, as before, by an m-vector **x** of input consumption levels, and by an s-vector **y** of produced outputs. Using, for instance, the Frontier model developed in the preceding chapter, the set of efficient units is identified, that is the set of units that define the empirical frontier of feasible achievements. This frontier is described by a set of facets whose analytical expressions, assuming a variable returns to scale technology, are given by hyperplanes of the form:

$$\sum_{r=1}^{s} \mu_r y_r - \sum_{i=1}^{m} \nu_i x_i + \omega = 0$$

For each such hyperplane, the Frontier model identifies efficiency price vectors, μ and ν, from which the marginal rates of technical substitution (MRTSs) and marginal rates of product transformation (MRPTs)[1], characteristic of the techniques described by the facet, can be derived. These prices define tradeoffs across inputs and across outputs that allow an efficient unit to maintain its efficient status as long as substitutions, effected according to these tradeoffs, keep the unit within the facet where these tradeoffs are valid. Alternatively a facet of the frontier may be assimilated to an isoprofit surface such that, at these efficiency prices, the efficient units on the facet are the only units to exhibit no comparative disadvantage relative to any other unit in terms of profit achievement. All efficient units on the facet achieve the maximum observed profit level given by $-\omega$. Because the methodology to estimate the frontier is based on a finite set of observations (x,y), some efficient units belong to several facets which raises the question of choice among the multiple admissible sets of tradeoffs for these efficient units (one set of tradeoffs per facet). This situation is illustrated in Figure 4.1 on page 155 at the end of the chapter where we restrict ourselves to a 2-dimensional isoquant. The set of observations is given by $\{E_1, E_2, E_3, E_4, F, G, H\}$. The frontier is defined by 3 facets delimited by E_1, E_2, E_3, and E_4. All efficient points which are interior to a facet, such as F, G, or H face a unique input tradeoff implied respectively by the efficiency price vectors $\mathbf{p}_a$, $\mathbf{p}_b$, and $\mathbf{p}_c$ which coincide with normals to the 3 identified facets. Efficient points E_2 and E_3, on the other hand, face multiple options. In the case of E_2, any input

---

1

$$MRTS_{ij} = -\frac{\partial x_i}{\partial x_j}\Big| x_g, \text{ y constant } i, j = 1,\ldots,m, \quad i \neq j$$

$$MRPT_{kl} = -\frac{\partial y_l}{\partial y_k}\Big| y_m, \text{ x constant } k, l = 1,\ldots,s, \quad k \neq l$$

tradeoff associated with a positive linear combination of $\mathbf{p}_a$ and $\mathbf{p}_b$ (any input tradeoff associated with a positive linear combination of $\mathbf{p}_b$ and $\mathbf{p}_c$ in the case of E_3) would allow E_2 to maintain its efficiency. The choice of any of these admissible tradeoffs may be so as to satisfy a particular managerial objective. The formulation of one such managerial objective derives from the primary concern for maintaining efficiency and is examined in the following developments.

Assuming for now that outputs are fixed and that decisions are restricted to the 2-input space pictured in Figure 4.1, a situation of competition among the decision-making units would require, as a condition for their survival as efficient units, that these units differentiate themselves from one another in terms of input mix and technique, and that they operate at minimum cost. The measurement of cost necessitates the identification of input prices which themselves imply particular tradeoffs across inputs. Moreover this cost function can serve to gauge the degree of differentiation of a unit by evaluating the cost incurred by all other observed efficient units. The minimum cost difference between the unit and all other efficient units becomes the distinguishing feature of the unit and gauges the unit's comparative advantage. It follows that the selection of particular tradeoffs across inputs for an efficient unit can be guided by the legitimate desire of the unit to maximize its comparative advantage. Referring to Figure 4.1, unit E_2 would select a price vector which is a **strictly** positive linear combination of $\mathbf{p}_a$ and $\mathbf{p}_b$ so that the minimum of the cost differences, between E_2 and F, and between E_2 and G (F and G are the observed efficient units tahat are closest to E_2), is maximized. At prices given by $\mathbf{p}_a$, E_2 would exhibit minimum cost but so would F and any unit mapping on the facet E_1E_2, and, hence, E_2's comparative advantage would be zero. Similarly, at prices given by $\mathbf{p}_b$, E_2 would still exhibit minimum cost but so would G and E_3. At any prices "between" $\mathbf{p}_a$ and $\mathbf{p}_b$ E_2 will be the unique observed efficient unit exhibiting minimum cost and therefore will be guaranteed a strictly positive comparative advantage. It is worth noting that the process of identifying optimal tradeoffs that will maximize a unit's comparative advantage also identifies the unit's closest competitors, i.e. the efficient units that limit the extent of the unit's comparative advantage. In the case of E_2 in Figure 4.1, the closest competitors can easily be identified as F and G.

The situation where inputs are fixed and decisions are restricted to an s-dimensional output space is parallel to that where outputs are fixed and decisions are restricted to the input space as illustrated in Figure 4.1. In this situation competition among the decision-making units would require, as a condition for their survival as efficient units, that these units differentiate themselves from one another in terms of output mix, and that they generate as much revenue as possible. The measurement of revenue necessitates the identification of output prices which themselves imply particular tradeoffs across outputs. This revenue function is analogous to the cost function in the input space and can serve to gauge the degree of differentiation of a unit by evaluating the revenue generated by all other observed efficient units. The minimum revenue difference between the unit and all other efficient units becomes the distinguishing feature of the unit, and, in the context of the output space, gauges the unit's comparative advantage. It follows that the selection of particular tradeoffs across outputs for an efficient unit can be guided by the legitimate desire of the unit to maximize its comparative advantage.

Finally the global and general situation, where decisions are to be made with respect to both inputs and outputs, can be dealt with logically by assimilating comparative advantage with the minimum difference between an efficient unit's profit and the profit of its closest competitors. As mentioned earlier, after assessing the current competitive environment, that is after having identified frontier-defining units, an efficient unit faces the task of defining its future strategy in terms of choice of technique of production, choice of output mix, and level of operations. The level of outputs is traditionally determined exogenously by demand conditions and it seems reasonable to assume that, over the medium term, a range of fluctuations in the output levels can be efficiently accommodated at the current scale of operations. It follows that if we assume that no unit is planning a change in the scale of their operations then a unit's strategy is described by its choice of technique of production, and by its choice of output mix. As before, the technique is entirely described by its input ratios. That technique is to be implemented efficiently and ought to be economically efficient as well (i.e. a minimum cost technique). Similarly on the output side, the output mix is to reflect technical and economic (i.e. maximum revenue) efficiency as well. It is fair to assume that given its current level and technique of operations, an efficient unit is concerned with adapting/repositioning itself to maximize its relative profit, that is maximize its comparative advantage measured by the difference between the unit's profit and the profit of its closest competitors. The analysis therefore hinges on the determination of prices for the various inputs and outputs to allow the computation of profit and of comparative advantage. Conditions may be imposed on these prices in accordance with the restrictions that may apply to the tradeoffs that the prices imply across inputs and across outputs (since the tradeoffs are given by the ratios of prices). For instance the MRTSs and MRPTs may be bound by specific values defined by state-of-the-art technology and/or societal valuations. It is also conceivable that a unit might want to impose further constraints on the tradeoffs that would be representative of the unit's preferences regarding choice of technique and output mix.

A unit's strategy can hence be further described by the tradeoffs across inputs and across outputs that the unit should abide by in defining its new positioning to guarantee a maximum comparative advantage over its closest competitors. The exact positioning will be defined by the extent of substitutions across inputs and across outputs, from the current proportions, that the unit elects to carry through following the identified optimal tradeoffs. These concepts are illustrated in Figure 4.2 on page 156 at the end of the chapter where we again restrict ourselves to the situation of a 2-input isoquant for ease and clarity of exposition.

The frontier is defined by the observed efficient units E_1, E_2, E_3, and E_4. Unit 2 may have identified s, for instance, as representative of its strategic tradeoff. Units E_1 and E_3 are identified as E_2's closest competitors and, at prices described by s, the difference in cost between E_1 and E_2, and between E_1 and E_3 are identical. Assuming unit E_2 has the opportunity to substitute input 1 for input 2 following the tradeoff implied by s, then unit E_2 ought to take advantage of the opportunity and change its input mix up to the point, E'_2, where unit E_3 becomes dominated and ceases to be efficient. Beyond that point the competitive environment is changed and a new assessment is warranted. Units E_1 and E_4 represent particular cases to the extent that these units are not entirely "surrounded" by observed efficient units as E_2 is by E_1 and E_3. This concept is defined next.

Definition 4.1: An efficient unit E_0 is said to be surrounded in the input space if at least one other efficient unit can be found, in any input direction, that consumes more of that input and less of some others than E_0, and generates at least the same revenue as E_0. Similarly an efficient unit P_0 is said to be surrounded in the output space if at least one other efficient unit can be found, in any output direction, that produces less of that output and more of some others than P_0, and incurs no more cost than P_0. In the global input and output space, a unit is surrounded if it surrounded both in the input space and in the output space.

Definition 4.2: A border unit is a unit which is not surrounded.

Border units can be pictured as sitting on/ defining the edge of the frontier. These border units, such as E_1 or E_4 in Figure 4.2, are likely to identify limit tradeoffs as strategic tradeoffs. Indeed these units appear as novices or opposites of specialists in the directions where they are not surrounded. For instance, at their scale of operations, they consume the most of one input or produce the least of one output. It should hence be the inclination of these units to attribute as low a relative value as possible to these inputs or outputs. This situation is illustrated in Figure 4.3 on page 157 at the end of the chapter. Assuming that limit strategic tradeoffs, s_l and s_L, are in effect, the border unit E_1 will identify s_l as its strategic tradeoff. This tradeoff is the only one among the admissible tradeoffs that minimizes the drawbacks of E_1's lack of expertise regarding the consumption of input 2. That is the cost advantage of E_1 over E_2 is maximized when input prices are homothetic to s_l.

The identification of "strategic" tradeoffs is at the core of the analysis. From its current mode of operations, that is technique of production described by input ratios and its current output mix, an efficient unit competes with all other identified efficient units to maintain a comparative advantage. The extent of comparative advantage can be measured by the additional profit that the unit was able to achieve in comparison with all other observed efficient units. A closer look at the origins of this extra profit will provide valuable information as to the sources of comparative advantage and, hence, help a unit discern its strengths and weaknesses. The extra profit may result from lower cost operations or higher revenues or both. In the situation where lower costs are present the unit's technique is a source of its comparative advantage. The differential in cost between the unit and its closest competitors has a quantity equivalent in terms of additional proportion of inputs the unit's competitors would consume if they were to act in total cooperation and attempt to replicate the unit's technique. This extra porportion gauges the unit's comparative advantage in the input space and offers a basis to compare the extent of comparative advantage, in the input space, across efficient units. This perspective is illustrated in Figure 4.4, on page 158 at the end of the chapter, in the case of a cost advantage. In evaluating E_2 we observe that all other efficient units in total cooperation can**not** replicate E_2's operations efficiently. Their best performance is given by the input consumption pattern depicted by R which corresponds to a minimum proportional augmentation:

$$\theta = \frac{\overline{OR}}{\overline{OE_2}} > 1$$

of input consumption from E_2's input consumption pattern. It follows that E_2's technique is a source of comparative advantage for E_2. Similarly, any revenue differential between the unit and its closest competitors has a quantity equivalent in terms of proportion of the output bundle the unit's competitors would fail to turn out if they were to act in total cooperation to replicate the unit's output mix.

The procedure developed to identify a unit's closest competitors, its strategic tradeoffs, sources and extent of comparative advantage is contained in a pair of dual mathematical programs, named the Comparative Advantage model, whose formulations are presented in the next section.

4.3 Maximization of Comparative Advantage

In the global inputs-outputs space a unit, DMU_l, seeks to identify strategic tradeoffs among inputs and among outputs that will enable it to maintain its efficient status. These tradeoffs derive from input and output prices, denoted respectively by the m-vector ν and the s-vector μ, which are to estimated as well. As in the case of the Frontier model, tradeoffs which imply disproportionate valuations of inputs or outputs, or which are not reflective of realistic supply and demand conditions, are not admissible. It follows that price ratios representative of tradeoffs among inputs and among outputs have to evaluate within preset lower and upper bounds. These conditions on price ratios can easily be converted into linear constraints on the efficiency prices to be estimated. Such constraints summarize to a global matrix format:

$$\nu \mathbf{R}_i \leq \mathbf{0} \quad \mu \mathbf{R}_o \leq \mathbf{0}$$

where $\mathbf{R}_i$ and $\mathbf{R}_o$ are respectively m x $2\binom{m}{2}$ and s x $2\binom{s}{2}$. Moreover these prices ought to allow DMU_l to maximize its comparative advantage measured by the difference between DMU_l's profit and the highest profit attained by all other efficient units.

All efficient units are described by their m-vector $\mathbf{x}$ of input consumption levels and their s-vector $\mathbf{y}$ of output levels. Assuming n efficient units have been identified (using the Frontier model for instance) and that unit l described by ($\mathbf{x}_l$, $\mathbf{y}_l$) seeks to evaluate its comparative advantage, all other efficient units will cooperate to the best of their abilities to at least replicate DMU_l's operations. Such a coalition is represented by a convex combination of the operations of the n-1 efficient units. The achievement of this coalition will be measured by θ, the extent of proportional input **augmentation** from DMU_l's levels it consumes, and by ϕ, the extent of proportional output **reduction** from DMU_l's levels it produces. The direction of the differences stems from the fact that DMU_l is an efficient unit and is therefore not dominated by any other efficient unit. The coalition can only map beneath or at the same level as DMU_l in the output space, and above or at the same level in the input space. Moreover, the emphasis on proportional differences from DMU_l's levels allows DMU_l to gauge its sources of comparative advantage in terms of production technique and output mix. DMU_l's comparative advantage can be attributed to its technique whenever $\theta > 1$ and/or to its output mix whenever $\phi < 1$ at optimality. Finally the search for a **minimum** θ and/or **maximum** ϕ across all possible coalitions ought to allow the identification of DMU_l's closest, i.e. most threatening, competitors in terms of production technique and output mix.

As in the case of efficiency determination, identifying comparative advantage requires the solution of a pair of dual programs for each efficient unit. The formulations of the Comparative Advantage model, given below, effect a global assessment since they do not prioritize between the input space and the ouput space regarding the minimization of θ or the maximization of ϕ. The coalition of closest competitors is represented by $(\mathbf{X}^{-}\boldsymbol{\lambda}, \mathbf{Y}^{-}\boldsymbol{\lambda})$ with $\mathbf{1}\boldsymbol{\lambda} = 1$, and where $\mathbf{X}^{-}$, and $\mathbf{Y}^{-}$ are m x (n-1) and s x (n-1) matrices respectively, representative of the operations of the n-1 other efficient units[2]. The programs implement an implicit pricing mechanism represented by $\mathbf{R}_i$ and $\mathbf{R}_o$. These constraints require the inclusion of substitution variables, summarized by the $2\binom{m}{2}$-vector σ_i, and the $2\binom{s}{2}$-vector σ_o in the primal formulation. The role of these substitution variables will be illustrated later.

The assessment of DMU_l's comparative advantage is carried out by solving the following pair of dual models, (C-A Primal), and (C-A Dual).

---

[2] Removing the DMU being assessed from the reference set is done by Adolphson, Cornia, and Walters, Lovell, Walters, and Wood, and also by Andersen and Petersen.

$$\mathrm{Min}_{\theta,\phi,\lambda,\sigma_i,\sigma_o} \ \theta - \phi$$

$$st \left\{ \begin{array}{rcl} -\phi \mathbf{y}_l + \mathbf{Y}^- \lambda + \mathbf{R}_o \sigma_o & = & 0 \\ \theta \mathbf{x}_l - \mathbf{X}^- \lambda + \mathbf{R}_i \sigma_i & = & 0 \\ \mathbf{1}.\lambda & = & 1 \\ -\phi & \geq & -1 \\ \theta & \geq & 1 \\ \lambda, \sigma_o, \sigma_i & \geq & 0 \end{array} \right. \qquad \text{(C-A Primal)}$$

$$\mathrm{Max}_{\mu,\nu,\omega_l,R,C} \ \omega_l - (R - C)$$

$$st \left\{ \begin{array}{rcl} \mu \mathbf{Y}^- - \nu \mathbf{X}^- + \omega_l & \leq & 0 \\ \nu \mathbf{x}_l + C & \leq & 1 \\ \mu \mathbf{y}_l + R & \geq & 1 \\ \nu \mathbf{R}_i & \leq & 0 \\ \mu \mathbf{R}_o & \leq & 0 \\ R, C, \mu, \nu & \geq & 0 \\ \omega_l \text{ unrestricted} & & \end{array} \right. \qquad \text{(C-A Dual)}$$

In the primal formulation, the constraints that bound θ below by 1 and ϕ above by 1, force the unit being assessed to "face" its competitors by looking upward in the input space and by looking downward in the output space. These constraints are matched with dual variables, C and R respectively, which are related to the cost and revenue levels of the unit being assessed. Their exact role as well as the behavior of the model can be further explained by expressing and manipulating the complementary slackness conditions whose statement follows.

$$\begin{array}{rll} R(1 - \phi) & = 0 & (1) \\ C(\theta - 1) & = 0 & (2) \\ \lambda(\mu \mathbf{Y}^- - \nu \mathbf{X}^- + \omega_l) & = 0 & (3) \\ \theta[(1 - C) - \nu \mathbf{x}_l)] & = 0 & (4) \\ \phi(\mu \mathbf{y}_l - (1 - R)] & = 0 & (5) \\ \nu \mathbf{R}_i \sigma_i & = 0 & (6) \\ \mu \mathbf{R}_o \sigma_o & = 0 & (7) \end{array}$$

Since θ and ϕ are strictly positive (θ is constrained from below by 1 and $\phi = 0$ would mean that no output was generated from non-zero inputs, which is clearly inefficient), conditions (4) and (5) indicate that the cost of unit l's operations evaluates to 1-C and that its revenue evaluates to 1-R. It follows that DMU_l's profit is given by:

$$\mu y_l - \nu x_l = (1 - R) - (1 - C) = -(R - C)$$

Condition (3), on the other hand, stipulates that any other efficient unit which is part of the coalition against DMU_l, that is any unit i such that $\lambda_i \neq 0$, allows the coalition, defined by $(x^*, y^*) = (X^- \lambda, Y^- \lambda)$, to reach a maximum profit level given by:

$$\mu y^* - \nu x^* = -\omega_l$$

It follows that the maximum comparative advantage for DMU_l is attained when the tradeoffs across inputs and across outputs implied by ν and μ are such that ω_l - (R - C) is maximized which is precisely the objective of the dual program.

Conditions (6) and (7) show that whenever the tradeoffs evaluate within the admissible ranges then the substitution variables are null. It follows that these substitution variables take on non-zero values whenever a border unit is assessed, hence serving to identify border units. The border components of the unit have to be substituted out to allow comparison with the other efficient units. This situation is illustrated in Figure 4.5 on page 159 at the end of the chapter. In the process of evaluating E_1 in this 2-input space, limit strategic tradeoffs implied by s_l are identified as those that maximize E_1's comparative advantage since they maximize the difference in cost between E_1 and its closest competitor, E_2. E_1's comparative advantage is measured by $\theta = OE'_2/OE_1 > 1$. However, E'_2 is not an observed efficient unit nor does it correspond to a convex combination of other efficient units. E'_2, therefore, cannot play the role of closest competitor to E_1. The closest competitor is E_2 which derives from E'_2 through substitution of input 2 (E_1's border component) for input 1 according to the tradeoff implied by s_l. At prices given by s_l, E'_2 and E_2 incur the same cost. Hence the comparative advantage of E_1 with respect to E'_2, and with respect to E_2, is the same.

Border units when assessed can also be recognized by directly examining the discrepancies between the unit's and the identified competitor's achievements. These discrepancies are given by ($\mathbf{x}_l$ - $\mathbf{X}^{\cdot}\boldsymbol{\lambda}$, $\mathbf{Y}^{\cdot}\boldsymbol{\lambda}$ - $\mathbf{y}_l$). It follows that whenever a component of $\mathbf{x}_l$ - $\mathbf{X}^{\cdot}\boldsymbol{\lambda}$ evaluates to a strictly positive value, then unit l is revealed to be a border unit in that component. Whenever the component evaluates to a negative number unit l consumes less of the corresponding input than the oalition of its closest competitors. Similarly, whenever a component of $\mathbf{Y}^{\cdot}\boldsymbol{\lambda}$ - $\mathbf{y}_l$ evaluates to a strictly positive value, unit l is revealed to be a border unit in that component, and whenever the discrepancy evaluates to a negative value, unit l is producing more of that output than the coalition of its closest competitors. It follows that negative discrepancies, in both the input space and output space, represent an advantage for the unit. The relative criticality of each input and of each output can be assessed by the ratios $\nu_i(x_{li} - (\mathbf{X}^{\cdot}\boldsymbol{\lambda})_i)/\nu\mathbf{x}_l$, regarding the inputs, and by the ratios $\mu_r((\mathbf{Y}^{\cdot}\boldsymbol{\lambda})_r - y_{lr})/\mu\mathbf{y}_l$, regarding the outputs. The lower the value (in algebraic sense) the better for unit l.

The behavior of the model regarding the identification of strategic prices from which the strategic tradeoffs are derived is examined next.

Lemma 4.1: At optimality of the primal and dual Comparative Advantage programs, the strategic input price vector, ν, is strictly positive if the unit being assessed has a strictly positive comparative advantage in inputs.

Proof: The result is ensured by complementary slackness conditions (2) and (4). Since a comparative advantage in inputs is indicated by $\theta > 1$, condition (2) requires $C = 0$ and therefore $\nu x_l = 1$ from condition (4). This strictly positive cost implies that at least one input efficiency price is strictly positive. Cascading through the ratio constraints on the input efficiency prices then ensure that all input efficiency prices are strictly positive.

Lemma 4.2: At optimality of the primal and dual Comparative Advantage programs, the strategic output price vector, μ, is strictly positive if the unit being assessed has a strictly positive comparative advantage in outputs.

Proof: The result is ensured by complementary slackness conditions (1) and (5). Since a comparative advantage in outputs is indicated by $\phi < 1$ and $\phi \leq 0$ is infeasible, condition (1) requires $R = 0$ and therefore $\mu y_l = 1$ from condition (5). This strictly positive revenue implies that at least one output efficiency price is strictly positive.

Cascading through the ratio constraints on the output efficiency prices then ensure that all output efficiency prices are strictly positive.

Whenever $\theta = 1$ at optimality the efficient unit being assessed lacks any comparative advantage in inputs. The unit's cost of operations is matched by that of its closest competitor. In particular it means that there are **no** strategic prices (hence no tradeoffs) that will create a strictly positive comparative advantage in inputs. That is we have:

$$\mathbf{x}_l = \mathbf{X}^{-}\boldsymbol{\lambda} \text{ hence } \nu\mathbf{x}_l = \nu\mathbf{X}^{-}\boldsymbol{\lambda} \text{ for all } \nu.$$

This indicates that the unit maps in the interior of facet of the frontier. The solution of the dual program could show $\nu = 0$ at optimality. In this situation, innovation, in terms of a new technique or of the same technique with more productive inputs, is the only recourse if an efficient unit is to gain a comparative advantage in the input space. A similar situation obtain in the output space whenever $\phi = 1$ at optimality. In that case the dual program could show $\mu = 0$ at optimality.

4.4 Computational Illustration

The Comparative Advantage model is applied to a data set composed of twenty six efficient units, representative of 4 isoquants in a 2-input-2-output space. The data set is given in Table 4.1 on page 160 at the end of the chapter. These units are found efficient when evaluated using the Frontier model and assuming a VRS envelopment, a global orientation and constrained implicit pricing with allowable price ratio constrained to [0.05 - 6.50] regarding ν_1/ν_2, and to [0.17 - 3.94] regarding μ_1/μ_2. The analysis of comparative advantage is effected under the same assumptions regarding the form of the envelopment, the orientation, and the pricing mechanism. The results are gathered in Tables 4.2 and 4.3, respectively on page 161 and 162 at the end of the chapter. Table 4.2 lists the evaluated strategic prices and the discrepencies with the closest competitor. Table 4.3 gives the input measure of comparative advantage θ, its output equivalent, ϕ, the strategic tradeoffs corresponding to the evaluated prices given in Table 4.2, and, finally, lists, for each unit, which observed efficient units attain the profit level that is closest to that of the unit at the evaluated stratagic prices.

All units but five (E8, E_9, E_{10}, $F_{6,\,G1}$) exhibit either a comparative advantage with respect to input consumption (indicated by $\theta > 1$) or a comparative advantage with respect to output mix (indicated by $\phi < 1$). Unit E_{10} appears to be in the most precarious situation with no comparative advantage and as many as 5 competitors.

Border units can be identified by looking at the discrepancies in inputs and outputs between the unit and its closest competitor, that is the unit which represents the coalition of all other efficient units. Such border units are characterized by strictly positive discrepancies in their border components. For instance E_1 in Table 4.2 shows a positive discrepancy in input 2, E_7 in input 1. From the data set given in Table 4.1 we can verify that E_1 consumes more of input 1 than any other efficient unit in its isoquant. Similarly E_7 consumes more of input 2 than any other efficient unit in its isoquant.

If we examine the particular case of H2 belonging to isoquant (2,2) and consuming 6 units of input 1 and 3.6 units of input 2, the analysis identifies G3 from isoquant (2,1) and H1 and H3 from isoquant (2,2) as H2's most threatening competitors. H2 has a comparative advantage with respect to technique and consumption of inputs since θ evaluates to $1.074 > 1$, but does not have any comparative advantage with respect to output mix and production since $\phi = 1$. H2 is not a border unit since it consumes strictly less inputs than the coalition of its closest competitors (0.442 units of input 1 and 0.265 units of input 2). The input strategic tradeoff ν_1/ν_2 is estimated at 0.556 with $\nu_1 = 0.08$ and $\nu_2 = 0.144$[3]. The overall cost advantage of H2 evaluates to 7.35% of H2's cost ($- 100 \times \theta[x_{H2} - (X\lambda)]/\nu x_{H2}$) with 3.54% attributable to the unit's consumption of input 1 ($- 100 \times \theta_1[x_{H2} - (X\lambda)]_1/\nu x_{H2}$) and 3.82% attributable to the consumption of input 2 ($- 100 \times \theta_2[x_{H2} - (X\lambda)]_2/\nu x_{H2}$). It follows that its pattern of consumption of input 2 represents H2's relative strength.

4.5 Conclusion

The Comparative Advantage model offers valuable information to efficient units to help them maintain their efficiency and eventually sharpen their competitive edge. It allows an efficient unit to identify who its current competitors are that may threaten its efficiency by means of similar low cost operations or high revenue output mixes. It also allows the extent of the unit's comparative advantage to be gauged providing the unit with a means to control the progress and success of its strategy -defined in terms of choice of technique of production (described by its input ratios) and its choice of output

---

[3] These input prices include some rounding error since νx_{H2} evaluates to 0.9984 and should evaluate to 1 whenever a unit is found to have a comparative advantagewith respect to inputs, i.e. such that $\theta > 1$.

mix. Finally, central to the formulation of a strategy are the identification of strategic prices which define tradeoffs across inputs and across outputs. The Comparative Advantage model by evaluating such strategic prices allows the derivation of these tradeoffs. An efficient unit can maintain its efficient status and protect its comaprative advantage as long as substitutions, effected according to these tradeoffs, keep the unit within the facet where these tradeoffs are valid.

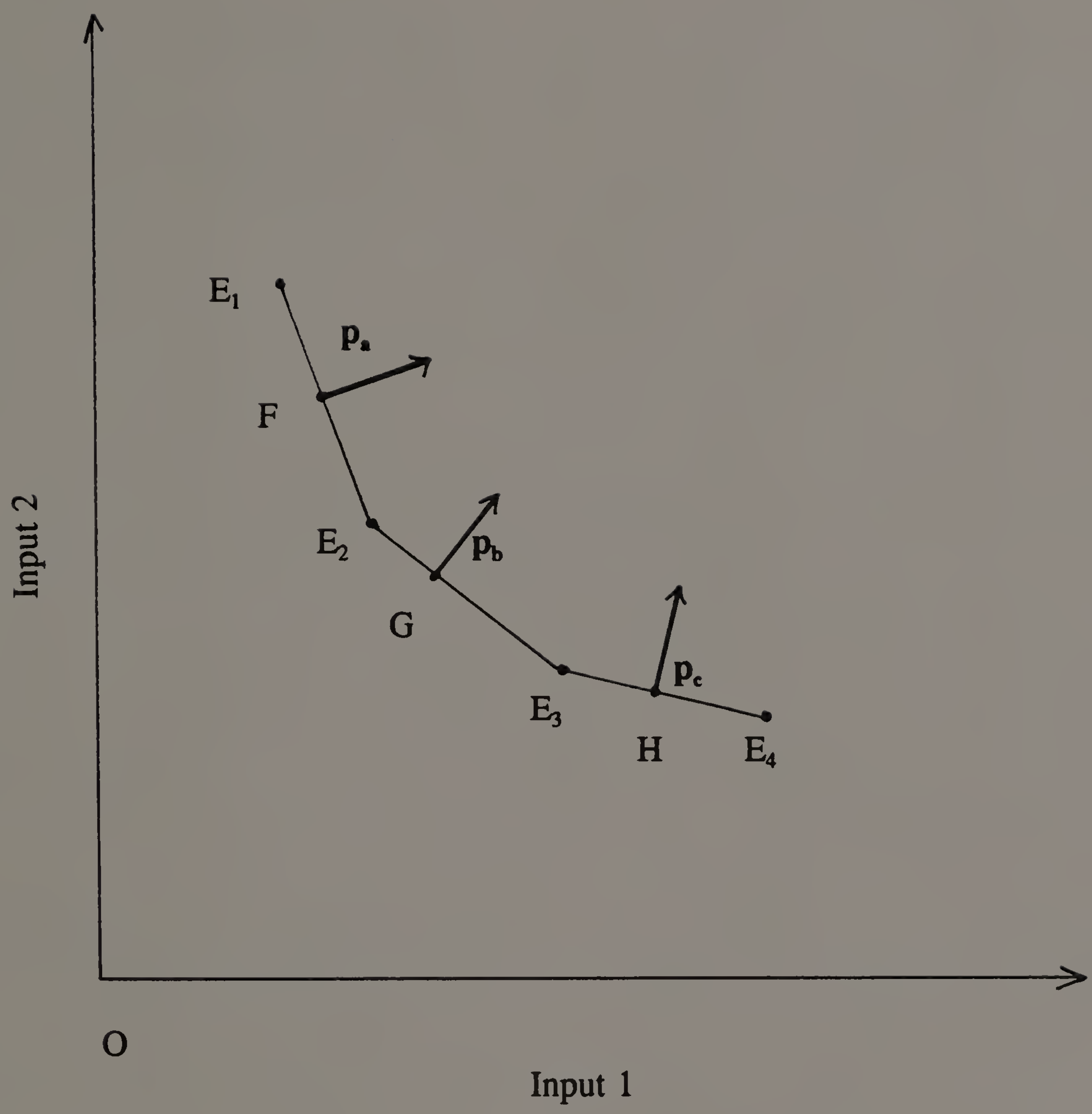

Figure 4.1 Optimal Input Tradeoffs

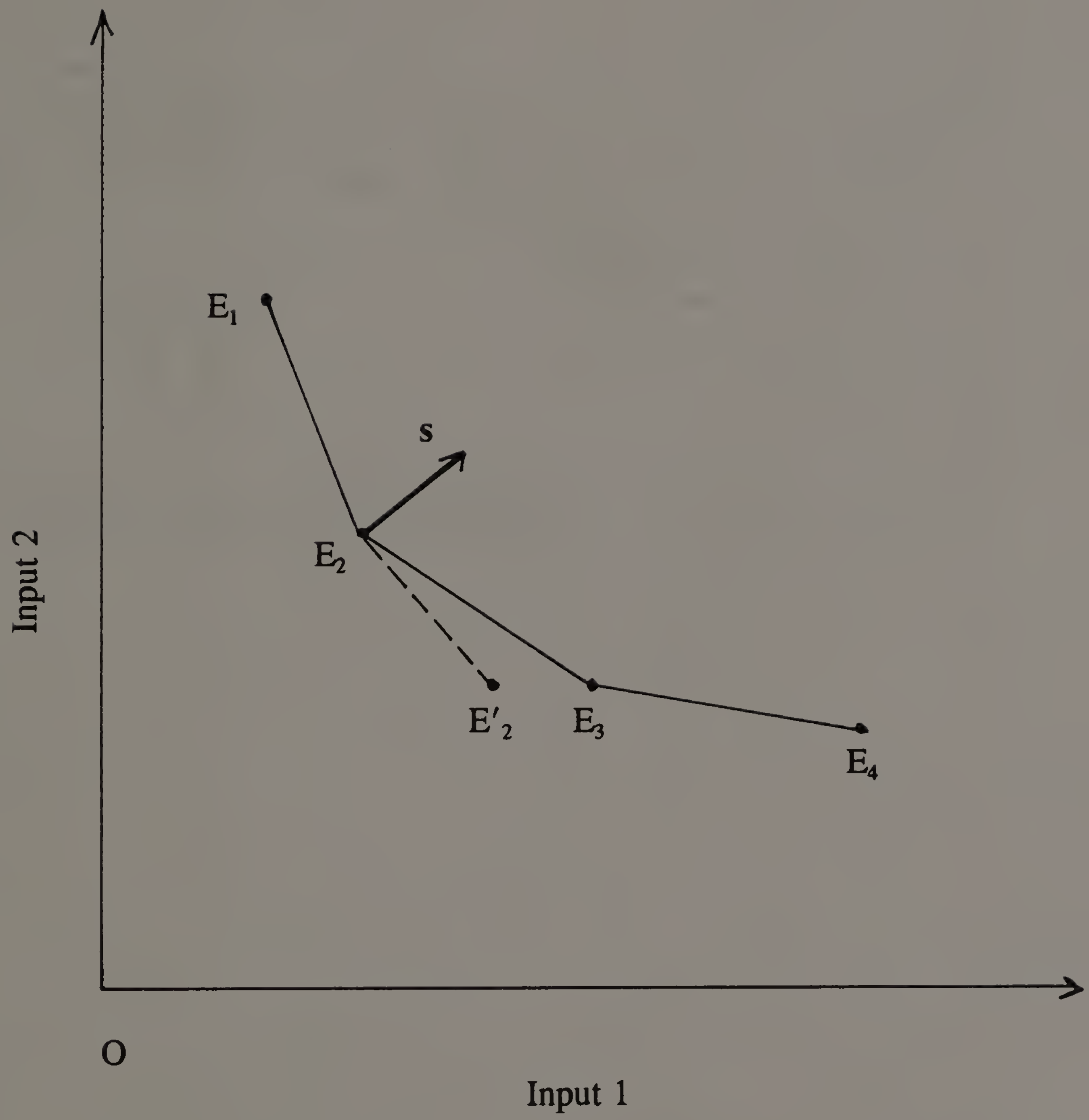

Figure 4.2 Strategic Tradeoffs

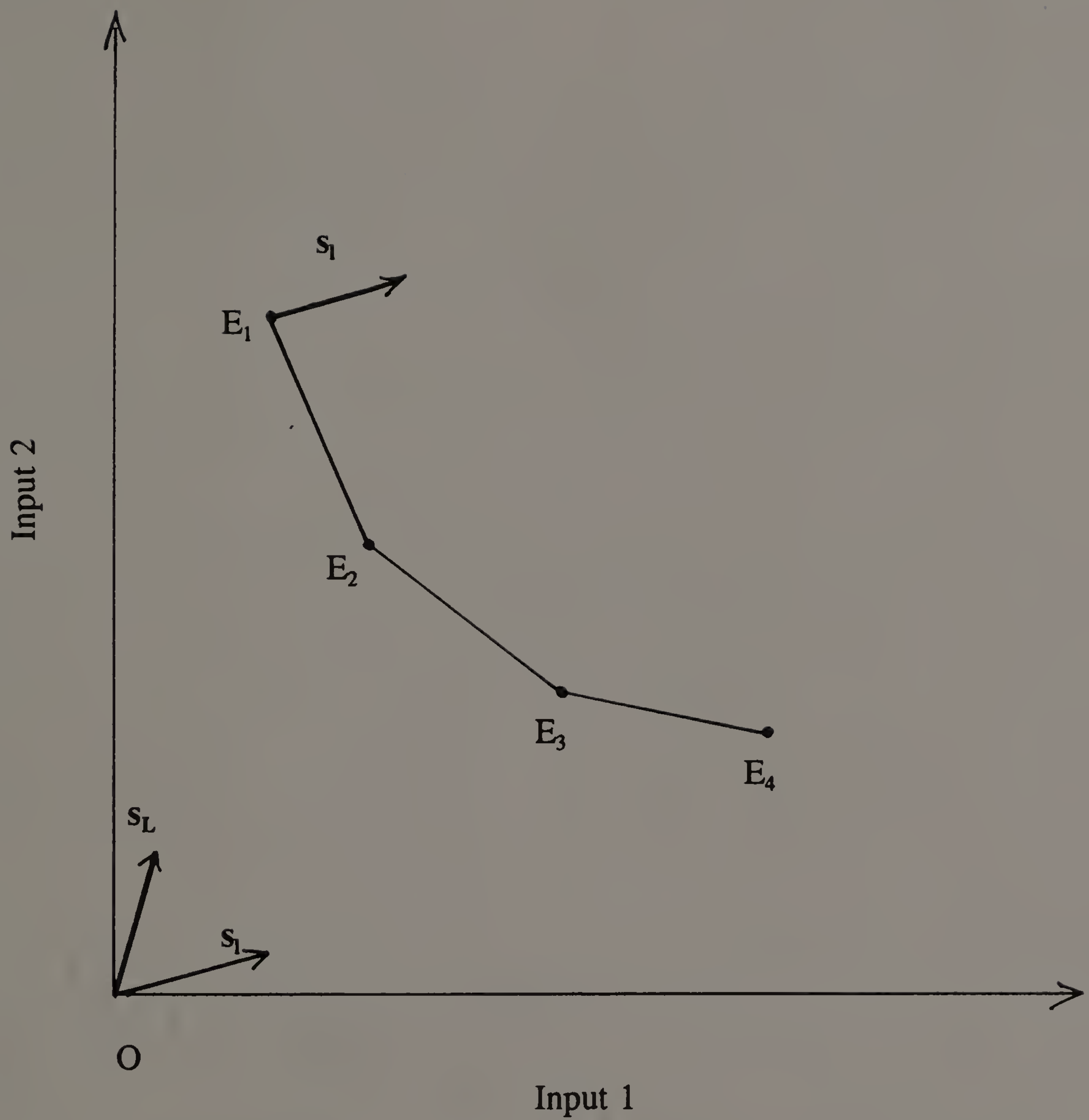

Figure 4.3 Strategic Tradeoffs of a Border Unit

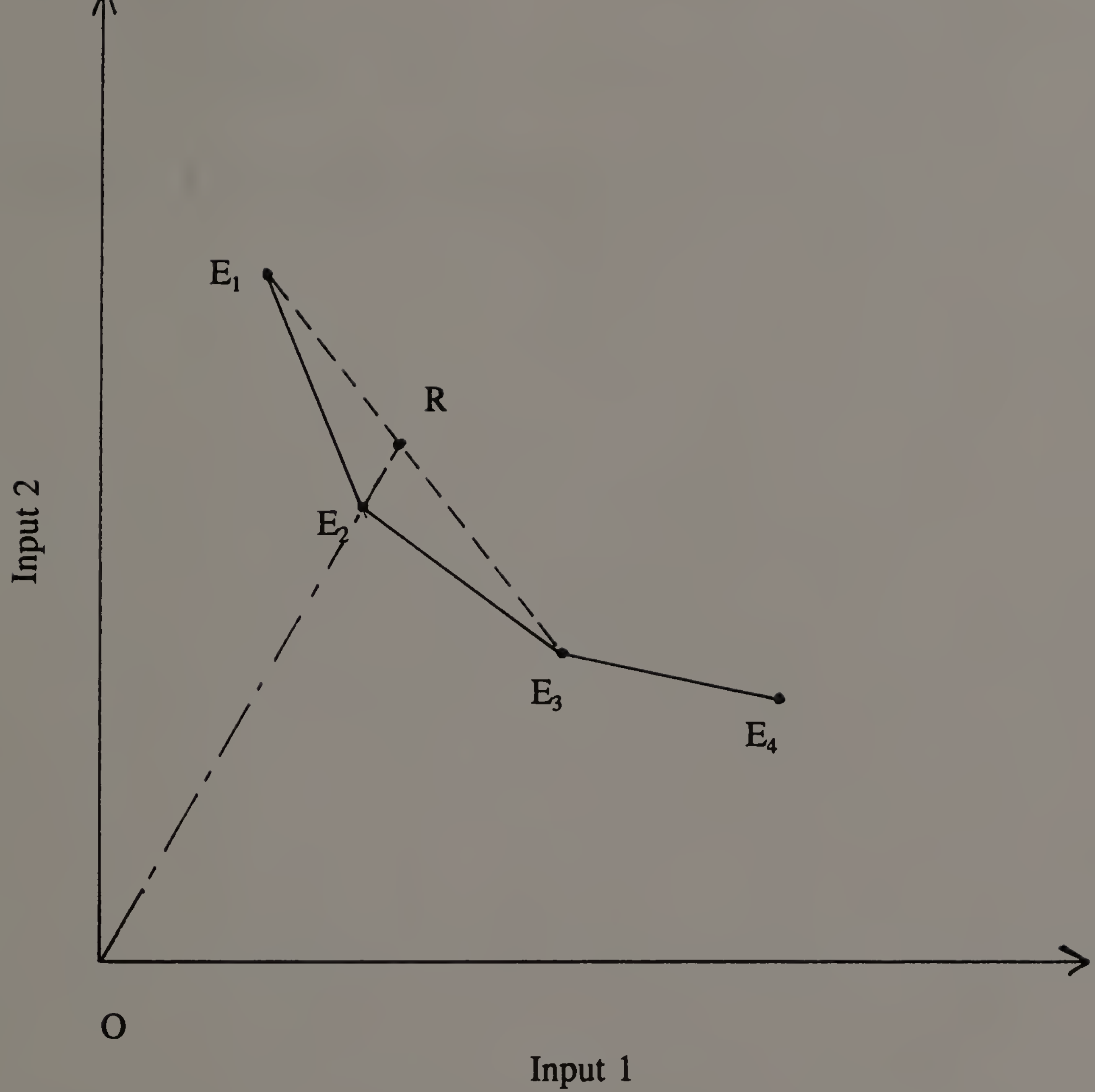

Figure 4.4 Comparative Advantage in Input Space

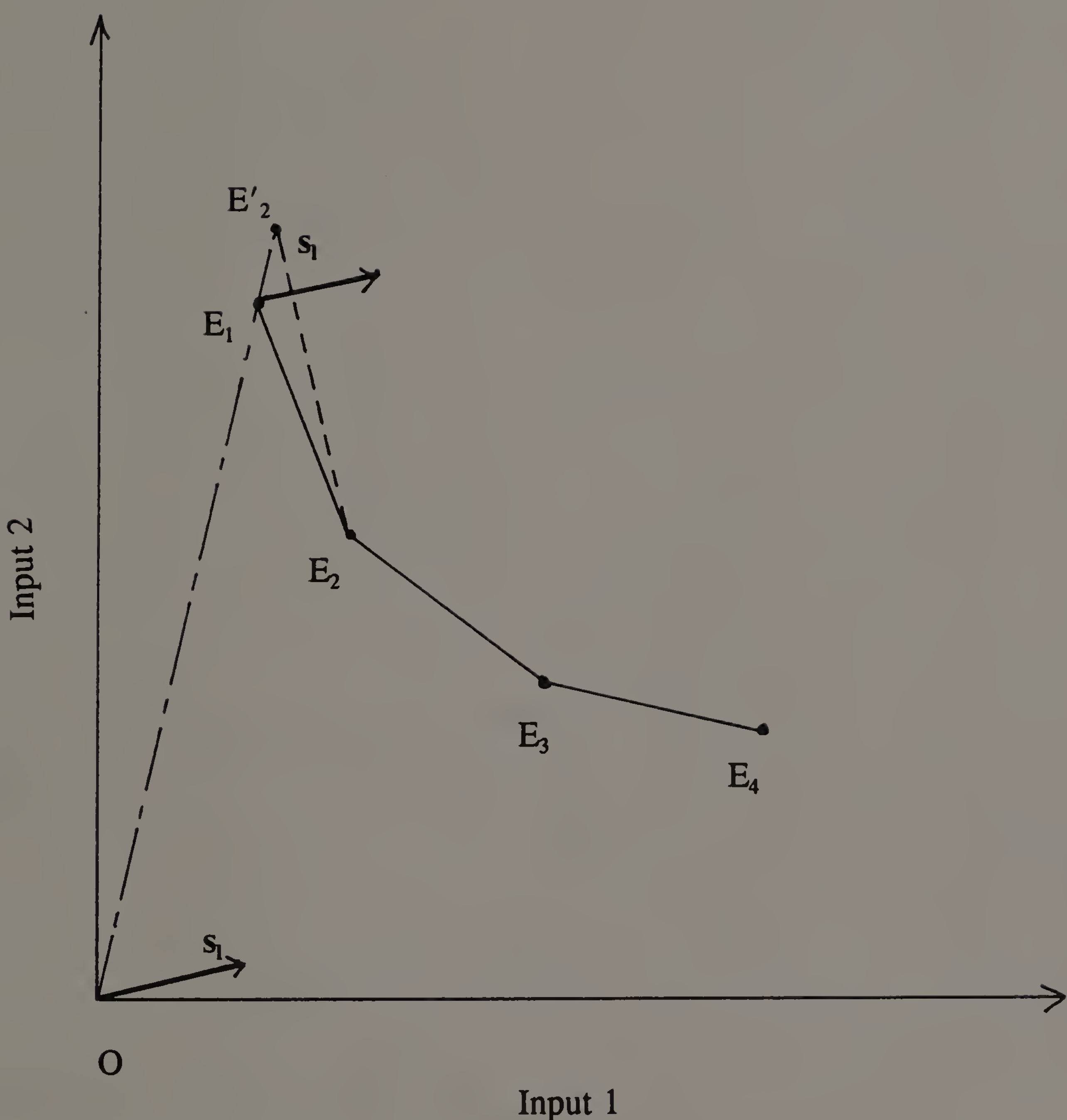

Figure 4.5 Border Unit Substitution

Table 4.1 Data Set

DMU	Input 1	Input 2	Output 1	Output 2
E1	1.00	7.00	1.00	1.00
E2	2.00	4.60	1.00	1.00
E3	3.00	3.40	1.00	1.00
E4	5.00	2.40	1.00	1.00
E5	7.00	1.60	1.00	1.00
E6	10.00	1.20	1.00	1.00
E7	12.00	1.00	1.00	1.00
E8	1.74	5.22	1.00	1.00
E9	3.27	3.27	1.00	1.00
E10	5.21	2.32	1.00	1.00
F1	2.00	8.00	1.00	2.00
F2	3.00	6.00	1.00	2.00
F3	4.00	4.40	1.00	2.00
F4	6.00	2.80	1.00	2.00
F5	9.00	2.40	1.00	2.00
F6	13.00	2.20	1.00	2.00
G1	2.40	8.00	2.00	1.00
G2	2.80	5.40	2.00	1.00
G3	4.40	3.80	2.00	1.00
G4	7.00	2.80	2.00	1.00
G5	10.20	2.00	2.00	1.00
G6	14.00	1.80	2.00	1.00
H1	4.40	5.00	2.00	2.00
H2	6.00	3.60	2.00	2.00
H3	8.00	3.00	2.00	2.00
H4	11.00	2.80	2.00	2.00

Table 4.2 Strategic Tradeoffs and Differences from Competitors

	PRICES				Discrepancies With Competitor			
DMU	Input 1	Input 2	Output 1	Output 2	Input 1	Input 2	Output 1	Output 2
E1	0.481	0.074	0.000	0.000	-0.741	1.778	0.000	0.000
E2	0.193	0.133	0.000	0.000	-0.066	-0.152	0.000	0.000
E3	0.161	0.152	0.000	0.000	-0.067	-0.076	0.000	0.000
E4	0.101	0.206	0.224	0.057	-0.019	-0.009	0.000	0.000
E5	0.072	0.310	0.000	0.000	-0.648	-0.148	0.000	0.000
E6	0.050	0.417	0.000	0.000	-0.167	-0.020	0.000	0.000
E7	0.031	0.625	0.000	0.000	2.000	-0.200	0.000	0.000
E8	0.255	0.106	0.000	0.000	0.000	0.000	0.000	0.000
E9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
E10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
F1	0.250	0.038	0.078	0.461	-0.250	1.625	0.000	-0.375
F2	0.158	0.088	0.051	0.298	-0.053	-0.105	0.000	0.000
F3	0.123	0.115	0.051	0.300	-0.246	-0.271	0.686	0.000
F4	0.068	0.212	0.052	0.305	-0.708	-0.330	0.000	-0.117
F5	0.027	0.315	0.069	0.405	-0.405	-0.108	0.000	0.000
F6	0.000	0.000	0.000	0.000	4.000	-0.200	0.000	0.000
G1	0.275	0.042	0.444	0.113	-0.400	2.600	0.000	0.000
G2	0.186	0.089	0.376	0.095	-0.437	-0.843	0.000	0.000
G3	0.103	0.144	0.268	0.068	-0.295	-0.255	-0.148	0.583
G4	0.061	0.204	0.277	0.070	-0.223	-0.089	-0.202	0.798
G5	0.041	0.289	0.347	0.088	-1.012	-0.198	0.000	0.000
G6	0.020	0.400	0.364	0.092	3.800	-0.200	0.000	0.000
H1	0.080	0.012	0.266	0.234	0.000	0.000	-0.222	-0.222
H2	0.080	0.144	0.327	0.173	-0.442	-0.265	0.000	0.000
H3	0.037	0.234	0.283	0.217	-0.523	-0.196	0.000	0.000
H4	0.014	0.278	0.250	0.250	2.889	-0.144	-0.028	-0.028

Table 4.3 Comparative Advantage Analysis

DMU	θ	ϕ	ν_1/ν_2	μ_1/μ_2	Competitors
E1	1.225	1.000	6.500	0.000	E8
E2	1.033	1.000	1.447	0.000	E3,E8
E3	1.022	1.000	1.053	0.000	E2,E9
E4	1.004	1.000	0.489	3.940	E9,E10,G3
E5	1.093	1.000	0.233	0.000	E6,E10
E6	1.017	1.000	0.120	0.000	E5,E7
E7	1.063	1.000	0.500	0.000	E6
E8	1.000	1.000	2.400	0.000	E1,E2
E9	1.000	1.000	0.000	0.000	E2,E3,E4
E10	1.000	1.000	0.000	0.000	E4,E5,E9,G3,H2
F1	1.000	0.827	6.500	0.170	E1,F2
F2	1.018	1.000	1.800	0.170	E2,F1,F3
F3	1.062	1.000	1.067	0.170	E3,F2,F4
F4	1.118	1.000	0.319	0.170	E5,F5,H2
F5	1.045	1.000	0.086	0.170	E7,F4,F6
F6	1.000	1.000	0.000	0.000	E7,F5,H4
G1	1.000	1.000	6.500	3.940	G2
G2	1.156	1.000	2.100	3.940	E2,G1,G3
G3	1.067	1.000	0.710	3.940	E3,G2,H2
G4	1.032	1.000	0.300	3.940	E5,H2,H3
G5	1.099	1.000	0.143	3.940	E4,G4,G6
G6	1.004	1.000	0.050	3.940	E7,G5
H1	1.000	0.889	6.500	1.137	F1,G1,G2,H2
H2	1.074	1.000	0.556	1.889	G3,H1,H3
H3	1.065	1.000	0.160	1.306	G5,H2,H4
H4	1.000	0.986	0.050	1.000	E7,G6,H3

CHAPTER 5

DETERMINATION OF COMPARATIVE ADVANTAGE FOR THE ECONOMY OF STATES IN THE U.S.A.

5.1 Introduction

Expansion of market boundaries of U.S. companies coupled with the elimination of trade barriers underscore the growing reality of global economic competition. The ever increasing volume and importance of international exchanges are evidenced by monthly reports of national merchandise deficit, the recent US-Canadian Trade Agreement and the formation of the European Economic Community. As tariffs, quotas, voluntary export restraints, and most other protectionist measures (often proven inefficient and counterproductive [Cline, 1989], [Lawrence, 1989]) are gradually repealed, it becomes imperative that American firms sharpen their competitive edge with respect to foreign market penetration.

Active participation by state governments in the revitalization of their industry base is necessary because achieving competitiveness is not merely the responsibility of a firm. The vitality of firms is inherently tied to that of the state which can offer many forms of assistance and incentives to foster economic activity. Despite recent attention given to export development by policy makers, no concentrated effort at the assessment of a state's performance regarding international trade, nor identification of its areas of comparative advantage vis-à-vis other states has been made. Before corrective or prescriptive policies can be devised it is necessary to (i) assess a state's overall economic performance compared to other states; and (ii) assess any industry's performance in a state with respect to the same industry in other states. Such assessment requires simultaneous consideration of multiple measures (criteria) such as volume shipments, value-added, profits, etc.

Measures such as percentage change in employment, and percentage change in output [Plaut and Pluta, 1983], [Wasylenko and McGuire, 1985], [Erickson, 1989] reflect a state's ability to maintain and foster conditions enabling economic growth and development. Their levels are determined by a complex set of interactions between laws, regulations, institutional structure, industrial organization, and resource endowments and allocations. A state's endowments of natural resources influence the composition of its industry mix. However, it is the composition of its workforce with respect to skills, education, and allocation across industries which ultimately determines a state's overall success and wealth.

Manufacturing activities are the most important of industrial activities in terms of contribution to GNP (Statistical Abstract of the U.S., 1989)[1]. Measures of consumption of labor, capital, material inputs and measures of contribution of value-added, products shipments and overall profits allow a perspective of a state's manufacturing base. It is these same measures which gauge the performance of a specific industry across states. A leading industry in a state, in terms of activity and share of labor demand, may not operate efficiently when compared to the same industry in other states.

---

[1] Manufacturing is one of 10 industrial activities as defined by the Standard Industrial Classification.

As such, a subsidiary industry may prove more competitive and a better candidate for growth and development.

Classical econometric techniques do not adequately address efficient frontier determination in the case of multiple dependent measures. In application, prespecified functional forms (Cobb-Douglas, translog, or constant elasticity of substitution (CES) function) are introduced to study particular economic effects such as scale or factor substitution [Fuss et. al., 1978]. However, the implicit and explicit assumptions underlying the function and their ensuing implications regarding relations among the estiminated parameters are not simultaneously compatible with the empirical data [Dryhmes, 1990]. Data envelopment analysis, on the other hand, offers flexibility in the sense of less maintained hypotheses to address efficient frontier determination.

This chapter presents two levels of analysis, using data envelopment analysis (DEA), to determine comparative advantage for the economy of the states of the US. Section 2 introduces the methodology used in the analysis. Mathematical programs comprising the methodology are given in Appendices C-E. Section 3 details the two levels of analysis: the first level assesses a state's overall economic performance compared to other states. Comparative-statics analysis across states provides the basis for comparative assessment of yearly achievement measures such as per capita income, output per capita, growth in total value-added and employment. A state can then critically evaluate its allocation of current stocks of capital and labor, the adequacy of labor supply, and its potential to generate value-added. The second level allows evaluation of an industry in a state with respect to the same industry in other states.

Such analysis identifies sources of inefficiency, such as inadequate levels of shipments, for the industry. Efficiency, here, refers to a simultaneous maximization of measures such as volume of shipments, value-added, and profits, and minimization of measures such as labor, wages, salaries, and material inputs. Section 4 reports the results of analysis using Census of Manufactures data for 1982 and 1987. Conclusions are drawn in Section 5.

5.2 Methodology

As mentioned earlier classical econometric techniques do not adequately address efficient frontier determination in the case of multiple dependent measures. Other techniques such as shift-share analysis only aim at estimating average behaviors. Data Envelopment analysis offers a different approach by aiming at the identification of extreme behaviors.

Since the seminal paper by Charnes, Cooper, and Rhodes [1978], a variety of data envelopment analysis models has appeared in the literature as have numerous studies employing the technique [Banker, Charnes, Cooper, Swarts, Thomas, 1989], [Seiford, 1990]. Each of the various models for data envelopment analysis (DEA) seeks to determine which of n decision-making units (DMUs) determine an envelopment surface when considering m inputs and s outputs. Each DMU $i = 1,\ldots,n$ is then characterized by the m-vector $\mathbf{x}_i$ denoting its input values, and by the s-vector $\mathbf{y}_i$ denoting its output values. Units that lie on (determine) the surface are deemed efficient in the DEA terminology. The statement of the models for DEA carries at the outset assumptions regarding the form of the envelopment surface. The assumed form of the envelopment bears a direct relationship to the set of efficient units. This study considers a VRS-type envelopment surface. This type assumes that the envelope consists of hyperplanes which are particular facets of the convex hull of the points (x_i, y_i), $i = 1, \ldots, n$. Units that do not lie on the surface are deemed inefficient and the analysis provides measures of their relative inefficiency. The inefficiencies stem from the identification of excessive input levels and deficient output levels. These excess inputs and slack outputs represent waste that bar the DMU from attaining efficiency, i.e. map on the envelopment surface.

A solution to a DEA model for a DMU identifies a point on the envelope in accordance with evaluation principles implicit in the model. Within the mathematical programs these pinciples define the manner in which projected points on the envelope are obtained for a DMU that is inefficient. This projection mechanism requires the specification of 2 components: the orientation and the pricing mechanism. The orientation refers to a prioritization for the search of waste across measures. An evaluation may be input-oriented if waste is to be identified among inputs first, or it may be output-oriented if waste is to be identified among outputs first, or it may follow a global orientation in which case the pricing mechanism directs the projection. The pricing mechanism identifies relative values (efficiency prices) for the various measures and allows the aggregation of non-commensurable inputs and outputs. The pricing mechanism may itself be defined in various ways. Following the taxonomy introduced in Chapter 3 we distinguish between two classes of pricing mechanisms: explicit pricing mechanism and constrained implicit pricing mechanisms. An explicit pricing mechanism attributes explicit values to the measures for which waste has been identified but it allows free determination of prices when no waste is present. Models known in the literature as CCR [Charnes, Cooper, Rhodes, 1978], BCC [Banker, Charnes, Cooper, 1979], ADD [Charnes, Cooper, Golany, Seiford, Stutz, 1981] follow such a pricing mechanism. A constrained implicit pricing mechanism recognizes that price ratios across measures represent tradeoffs across measures and that these tradeoffs are constrained to some acceptable ranges, either by market conditions or by societal valuations. Consequently this mechanism allows prices to be determined freely across measures subject to the condition that they evaluate to permissible tradeoffs. Models kown as the cone-ratio model [Charnes, Cooper, Huang, and Sun, 1990], the assurance region models [Thompson, Singleton, Thrall,and Smith, 1986], the Frontier model developed in Chapter 3 follow these pricing principles.

Common to most mathematical models for DEA is the assumption that a DMU's current input ratios and output ratios are characteristic of the economic environment and supply and demand conditions in which the DMU operates[2]. It follows that waste is to be eliminated proportionately across all input and across all output measures. Given an orientation and pricing mechanism, the evaluation of a DMU leads to the identification of a projected point on the envelope $(\mathbf{x}^*,\mathbf{y}^*)$, and of a set of price vectors $(\mathbf{p}_{x^*},\mathbf{p}_{y^*})$ such that, at these prices, the value of global waste is minimized. A mathematical statement of global waste valuation is given by:

[2] An exception is the additive model which follows a global orientation and an explicit pricing mechanism.

$$p_{x*}(x - x*) + p_{y*}(y* - y).$$

Further the efficient point (x*,y*) is such that no other efficient unit would exhibit a higher profit level. Profit is defined as the difference between the aggregated values of outputs, hereafter called revenues, and the aggregated values of inputs, hereafter referred to as costs. Mathematically there is no efficient unit (x',y') such that:

$$(p_{y*}y* - p_{x*}x*) - (p_{y*}y' - p_{x*}x') < 0.$$

Which leads to:

$$p_{x*}(x' - x*) + p_{y*}(y* - y') < 0.$$

This latter condition expresses that, at prices (p_{x*},p_{y*}), x* represents a minimum cost alternative to producing y*, and y* represents a maximum revenue alternative to consuming x*.

The efficiency, or degree of inefficiency of the DMU is then gauged by a global score E(x,y) which is the product of an input efficiency score E_I(x,y), and of an output efficiency score E_O(x,y) as defined in Chapter 2. Input efficiency is measured by the ratio of the minimal cost of producing y*, p_{x*}x*, to the cost associated with x. That is:

$$E_I(x,y) = \frac{p_{x*}\,x*}{p_{x*}\,x}$$

Output efficiency is measured by the ratio of revenue associated with y to the maximal revenue attainable from x*. That is:

$$E_O(x,y) = \frac{p_{y*}\,y}{p_{y*}\,y*}$$

When a constrained implicit pricing mechanism is used $E_I(\mathbf{x},\mathbf{y})$ evaluates to the global proportional input reduction, θ, that would render the DMU input-efficient, that is $E_I(\mathbf{x},\mathbf{y}) = \theta$. Similarly, $E_O(\mathbf{x},\mathbf{y})$ evaluates to the reciprocal of the global proportional output augmentation, ϕ, that would render the DMU output-efficient, that is $E_O(\mathbf{x},\mathbf{y}) = 1/\phi$. This study employs the Frontier model developed in Chapter 2 and summarized in Appendix D, which yields both θ, and ϕ.

The Frontier model is concerned mainly with gauging inefficiency and identifying sources of inefficiency. However, whenever a DMU is found efficient the evaluation summarizes to an identified price vector reflecting feasible tradeoffs across measures that are sufficient to ensure efficiency. It is fair to assume that given its current level and technique of operations, an efficient unit is concerned with adapting/repositioning itself so as to maximize its comparative advantage measured, in this situation, by its relative profit, that is the difference between its profit and the profit of its closest competitors among other efficient units. This task involves the identification of the unit's closest competitors and of a set of price vectors ($\mathbf{p}_x$, $\mathbf{p}_y$) reflecting admissible tradeoffs across inputs and across outputs. If a closest competitor is characterized by ($\mathbf{x}^*$, $\mathbf{y}^*$), then the quantity:

$$(\mathbf{p}_y \mathbf{y} - \mathbf{p}_x \mathbf{x}) - (\mathbf{p}_y \mathbf{y}^* - \mathbf{p}_x \mathbf{x}^*)$$

is maximized. That is,

$$\mathbf{p}_y(\mathbf{y} - \mathbf{y}^*) + \mathbf{p}_x(\mathbf{x}^* - \mathbf{x})$$

is maximized.

The tradeoffs implicit in the identified prices ($\mathbf{p}_x$, $\mathbf{p}_y$) can then guide the unit $(\mathbf{x},\mathbf{y})$ in its plans for expansion or contraction. This study employs the Comparative Advantage model developed in Chapter 4 and summarized in Appendix E which yields, for each efficient DMU, a list of identified closest competitors, the identified set of price vectors, values of discrepancies between the unit's and the competitor's levels for all measures. The model summarizes these discrepancies to a global score which is the product of an input score, θ, and of an output score, ϕ. The input score θ represents the minimum proportional augmentation from the DMU's input levels that would match the competitor's input levels. It follows that whenever $\theta > 1$, it can be inferred that the DMU's technique is a source of its comparative advantage. Similarly, the output score ϕ represents the minimum proportional reduction from the DMU's output levels that would match the competitors levels. Hence, whenever $\phi < 1$ it can be inferred that the DMU's current output mix is a source of its comparative advantage.

5.3 Proposed Analyses

The Bureau of Economic Analysis provides yearly estimates of population levels and income information for various states. The Bureau of Census conducts "censuses of manufactures" every five years. The proposed analyses use such available data for 1982 and 1987 on the manufacturing activity of all states and the District of Columbia. The relevant measures (definition of inputs and outputs) for each level of analysis vary and are discussed in the following two subsections.

5.3.1 Overall Economic Performance

To evaluate a state's overall economic performance, each state represents a decision making unit. The measures which focus on the share and strength of the manufacturing base of each state are classified as input or output measures for the purpose of this analysis. The output measures are:

i) **Per Capita Income** (Output 1). Per capita income is an obvious indicator of the wealth of a state. It is usually associated with high per capita disposable income, hence high levels of consumption which stimulate all economic activities.

ii) **Value Added per Employee** (Output 2). This output measures relates to the productivity of the global manufacturing base.

iii) **Ratio of Manufacturing Employment to Total Population** (Output 3). The extent to which the manufacturing sector contributes to sustaining the state's population can be measured by the proportion of the population it employs. The higher this proportion, the higher the chances of the state to gain significantly from increased export activity.

iv) **Average Level of New Capital Expenditures per Manufacturing Establishment** (Output 4). This ratio reflects the commitment to keep abreast of technological advancements and the degree of involvement in medium to long-term planning. It shows how actively the firms invest in their future instead of merely reacting to current economic conditions.

v) **Ratio of Total Number of Manufacturing Establishments to Total Population** (Output 5). This ratio depicts the average level of concentration across all manufacturing industries. A relatively high ratio can be indicative of smaller establishments which are traditionally seen as more flexible to adapt to fluctuating demand conditions, or it can be indicative of a very diversified and competitive manufacturing base counting a relatively large number of establishments. Conversely, a relatively small ratio can reflect a pattern of larger and/or fewer establishments benefiting from economies of scale and having successfully overcome entry barriers of the sunk capital cost type.

The input variables consist of two trend variables and two cost related measures:

i) **Percentage Loss in Manufacturing Employment** (Input 1).

ii) **Percentage Decline in Total Manufacturing Value-Added** (Input 2).

iii) **Hourly Wages** (Input 3).

iv) **Ratio of Cost of Material Inputs to the Total Dollar Value of Shipments** (Input 4).

The trend variables emphasize that while most states are concerned with managing growth, other states strive to prevent shrinkage of their manufacturing base. The percentages are calculated over a five-year period and reflect the expansion path of a state. Negative values for these variables indicate growth. The hourly wages are computed from a total wages paid to production workers up through the line-supervisor level.

The first step of the analysis is to identify which states define the envelope, i.e. exhibit a DEA-efficient behavior. This is accomplished by solving the invariant additive model which assumes a VRS envelope, a global orientation and an invariant pricing mechanism. The model is given in Appendix C. The global orientation rather than an input or output orientation is more appropriate for an expost evaluation of performance and whenever the input and output measures are not directly controllable variables. The invariant pricing mechanism is the one tailored to each unit by considering that the "inefficiency valuation" of each measure for a DMU is given by the reciprocal of the value of that measure for that DMU. It is easily implemented in the absence of further information regarding the tradeoff values for the various measures. The model has been solved to identify the efficient units in 1982 and 1987. A comparison across the years will show how dynamic the economy was over that time period and to which extent economic strength and vitality tend to be localized geographically or to shift. After the identification of envelope-defining units, the ranges of efficiency price ratios over these units can be derived. These price ratios represent evaluated tradeoffs across measures that are consistent with the efficient rating of the envelope-defining units.

The second step of the analysis is then concerned with identifying sources of inefficiency and measuring the degree of inefficiency of DMUs mapping beneath the envelope. This is accomplished for both 1982 and 1987 by solving the Frontier model with pricing constrained by the ranges of efficiency price ratios of the units found efficient in the first step. These ranges will hereafter be referred to as Limit-Price-Ratios. These Limit-Price-Ratios are likely to preserve the efficient rating of the units found efficient in the first step of the analysis. Comparing the results over both years

1982 and 1987 for selected units will show their relative progress or recess and illustrate whether any industrial policy recommendation can emerge from such analysis.

It is essential to stress that the evaluation process does not lend "absolute" results. The results depend on the accepted and stated ranges that constrain the relative values of efficiency prices estimated by the frontier model. This important point is illustrated by conducting the evaluation anew with a different set of constraints on price ratio ranges. Using the second set of price ratio ranges further emphasizes that the price ratio constraints can serve to "reward" some behaviors or measure the tendency of units to efficiently exhibit that behavior. The second set, considered for both years 1982 and 1987 is derived from the ranges of ratios of input consumption levels and ouput production levels respectively over the DEA-efficient units. That is the range for input price ratios ν_i/ν_j, $[(\nu_i/\nu_j)_{min}, (\nu_i/\nu_j)_{max}]$ for all inputs i, j, i $\neq$j is given by:

$$\left(\frac{\nu_i}{\nu_j}\right)_{min} = \text{Min}\left\{ \frac{x_{kj}}{x_{ki}} : \text{DMU}_k \text{ is DEA-efficient}\right\}$$
$$\left(\frac{\nu_i}{\nu_j}\right)_{max} = \text{Max}\left\{ \frac{x_{kj}}{x_{ki}} : \text{DMU}_k \text{ is DEA-efficient}\right\}$$

Similarly the range for output price ratios μ_t/μ_u, $[(\mu_t/\mu_u)_{min}, (\mu_t/\mu_u)_{max}]$, for all outputs, t, u, t $\neq$ u is given by:

$$\left(\frac{\mu_t}{\mu_u}\right)_{min} = \text{Min}\left\{ \frac{y_{kt}}{y_{ku}} : \text{DMU}_k \text{ is DEA-efficient}\right\}$$
$$\left(\frac{\mu_t}{\mu_u}\right)_{max} = \text{Max}\left\{ \frac{y_{kt}}{y_{ku}} : \text{DMU}_k \text{ is DEA-efficient}\right\}$$

These ranges will hereafter be referred to as Limit-Value-Ratios ranges. There is no theoretical justification from an economics point-of-view for the selection of these boundaries on the efficiency price ratios. However, from a practical point-of-view, they are easily computed and consistent with the intuitive notion that an efficient unit ought to consume more of the cheaper inputs and produce more of the more valuable outputs. The set of efficient units under these Limit-Value-Ratio constraints is expected to be different from the original efficient set. In this case as well, industrial policy recommendations to inefficient units will be sought and constrasted with those stemming from the evaluation under the Limit-Price-Ratio ranges.

5.3.2 Efficiency of an Industry Across States

This level of analysis aims at identifying the states in which an industry group is efficient. Note that this does not imply that the industry is preeminent in that state. An industry group corresponds to the 3-digit level classification in the Standard Industrial Classification (SIC)[3]. In this analysis a decision making unit represents an industry group in a particular state. The number of decision making units is determined by the number of states where the particular industry is active. This analysis, then, provides a comparative assessment of industry performance across states in terms of market share

[3]SIC distinguishes between 20 manufacturing major groups, each identified by a 2-digit code. The 2-digit major groups are themselves combinations of several industry groups each identified by a 3-digit code. These industry groups are the aggregate of several industries (identified by a 4-digit SIC code). In the manufacturing division, classification at the 3-digit level is based on process. The Standard Industrial Classification in the Manufacturing Division is given in Appendix F.

as indicated by total shipments and profitability, and in terms of operating efficiency as indicated by labor, and material input utilization.

The output variables are:

i) **Total Shipments** (Output 1). Shipments refer to the dollar valuation of the shipments (receivable net selling values f.o.b.) and therefore represents the aggregated revenues of an industry. This variable is traditionally employed as a proxy for market share.

ii) **Value-Added** (Output 2). Value-added is the difference between shipments and the cost of material inputs. It represents the amounts of additional labor and know-how embodied in the shipments. According to the Bureau of the Census, "Value-added is considered to be the best value measure available for comparing the relative economic importance of manufacturing among industries and geographic areas."

iii) **Profits** (Output 3). Profits is the share of value-added returning to the owners of capital after labor is full compensated (wages and salaries). Profit maximization is a classical assumption of production theory.

The input measures stem from the assumption of cost minimization. They are:

i) **Cost of Material Inputs** (Input 1). These consist of direct charges to the production process and include energy and freight charges.

ii) **Wages** (Input 2). These are the total wages paid to production workers up through the line-supervisor level.

iii) **Salaries** (Input 3). This measure represents non-production wages and salaries.

iv) **Hours** (Input 4). This measure represents the total number of hours worked by production workers up through the line-supervisor level.

v) **Workers** (Input 5). This measure represents the total number of production workers up through line-supervisor level.

vi) **Overhead** (Input 6). This measure represents the total number of non-production workers.

Capital costs are not included in the model due to the difficulty in defining and measuring such a variable. Indeed beside the differences in productivity associated with various vintages of capital there is a general lack of data concerning the levels of utilization of the existing stocks of plant and equipment. We may also note that given the short-term irreversibility of capital investment, labor is left as the adjustable input.

The procedure of this second analysis follows that of the overall economic performance analysis. We start by identifying the states which operate on the envelope for an industry by solving the invariant-additive model for each state reporting activities in that industry in 1982. We then proceed to identify sources of inefficiency and measure the degree of inefficiency of units mapping beneath the envelope using the Frontier model with the Limit-Value-Ratio constraints. Finally the issue of recommendations to efficient units will be addressed by solving the Comparative Advantage model in the case of the Limit-Value-Ratio constraints.

5.4 Interpretation of Results

Both levels of analysis proposed in the previous section were performed[4] using the data tabulated in Tables 5.1, 5.2, and 5.3 respectively on pages 195, 196, and 197 at the end of the chapter. The comparative economic performance of the states was performed using 1982 and 1987 data given in Tables 5.1 and 5.2. The values of the variables tabulated are derived from data compiled by the Bureau of Census and the Bureau of Economic Analysis.

Regarding the first level of analysis Input 1 depicts the decrease in manufacturing employment (from 1977 to 1982 in Table 5.1 and from 1982 to 1987 in Table 5.2) in a state and is stated as a percentage. Input 2 represents similarly the percentage decrease in value added per manufacturing employee. Input 3 (hourly wages) is expressed in dollars per hour are Input 4 (cost ratio) is in tenths. Output 1 (income per capita) is reported in dollars, Output 2 (value-added per manufacturing sector employee) is in thousands of dollars per employee, Output 3 (ratio of total population employed in the manufacturing sector) is a percentage, Output 4 (new capital expenditures per manufacturing establishment) is reported in hundreds of thousands of dollars and are averages across all establishments, and Output 5 (number of establishments per capita) is per 100,000 people. In order to perform the analysis the data sets have been transformed so that all measures are expressed by positive numbers of similar magnitude. The transformed data sets are shown in Tables 5.4 and 5.5, respectively on page 198 and

[4] All data envelopment analysis were performed using IDEAS [Ali, 1991] on a COMPUA-DD 286-12 personal computer equipped with a math co-processor.

199. In particular Input 1 and Input 2 were translated by 100 in 1982 and by 50 in 1987, then scaled by 100 in 1982 and 10 in 1987. It follows that values less than 1 for Input 1 and Input 2 in 1982, and less than 5 in 1987, represent growth. Input 4 is scaled by 10 so that the cost ratio becomes unitless. Output 1 and Output 2 are converted into tens of thousands of dollars per capita and tens of thousands of dollars per manufacturing employee respectively. Definitions and units of measurement of all measures are summarized in Table 5.6 on page 200.

The results of the analyses are discussed in subsection 5.4.1.

The second level of analysis concerns the comparative assessment of an industry group performance across states. The analysis is illustrated by focusing on the Electronic Components and Accessories (SIC367) group, which, in, 1982, represented the growing area oh high technology. This analysis is performed using 1982 data. The data is given in Table 5.3 on page 197. Each of the six variables, profits, shipments, value-added, cost of material, wages, and salaries, is reported in millions of dollars. The number of hours is in millions, the number of production and non-production workers are in thousands. Definition of all measures are summarized in Table 5.7 on page 201.

The Electronic Components and Accessories (SIC367) is analyzed in subsection 5.4.2.

5.4.1 Overall Economic Performance of States

Table 5.8 on page 202 lists the states found efficient in 1982 and those found efficient in 1987 when considering a VRS envelopment, a global orientation and an invariant pricing mechanism for the evaluation. In 1982 25 of the 51 states were efficient and defining the envelope[5]. The 10 states which changed from a classification of efficient in 1982 to inefficient in 1987 are marked by †. The 9 states which exhibited the reverse phenomenon are marked with an asterisk.

The input price ratios resulting from the evaluation of these efficient units are reported in Table 5.11 on page 205 and in Table 5.12 on page 206 for 1982 and 1987 respectively. The associated output price ratios are reported in Table 5.13 on page 207 and in Table 5.14 on page 208. The observed ranges for these ratios are given in Table 5.15 page 209. The disparity of the ranges across measures and across years is quite wide and offers valuable information regarding the span of behaviors exhibited by the envelope-defining units.

For instance the minimum tradeoff between Output 1 (per capita income) and Output 3 (participation of population to manufacturing labor force) went from 0.155 to 1.438 between 1982 and 1987. This change can be interpreted as a sign that Output 3 is losing ground as a potential source of efficiency, that is as a source of strength relative to Output 1. The relative (with respect to other output measures) decline of Output 3's effectiveness as a source of efficiency is confirmed by the sliding of the ranges of ratios μ_3/μ_4 and μ_3/μ_5 toward lower values between 1982 and 1987 and the leap toward higher values of μ_2/μ_3. This phenomenon is easily justified by the strong average decline in manufacturing employment through this time period. On the input side the growing

[5] Tables 5.9 and 5.10 on page 203 and 204 report the revealed waste (excess inputs and output slacks) for each state for 1982 and 1987 respectively. States with zero waste are efficient.

importance of retaining the manufacturing employment base is illustrated by the relative prices of Input 1 (percentage decrease in manufacturing employment) which reach extremely high values in 1987.

Given these observed ranges of relative prices, the identification of sources of inefficiency and measurement of global inefficiency is effected for all states by solving the Frontier model. Tables 5.16 and 5.17 on pages 210 and 211 report the total identified waste and substitutions recommended to inefficient units in 1982 and 1987 respectively. Tables 5.18 and 5.19, on pages 212 and 213, display the efficiency prices resulting from the evaluation for both years, Tables 5.20 and 5.21, on pages 214 and 215, report the corresponding output price ratios and, finally Table 5.22 and 5.23, on pages 216 and 217, show the input price ratios and efficiency scores of all units for 1982 and 1987.

In Tables 5.16 and 5.17, which report the amount of identified waste and recommended substitutions, substitutions appear as negative numbers. For instance in Table 5.16 Nevada in 1982 would have been better off by sacrificing, i.e. substituting out 0.019 of Output 2 ($190 per employee of value-added) but boosting Output 3, 4, and 5 which exhibit slack values of 0.293, 1.858, and 0.033 respectively. This is confirmed on the input side where a higher level of Input 2 (percentage decrease in value-added per employee) would be preferable along with lower levels of Input 1, 3, and 4 as indicated by a substitution value of -0.007 for Input 2 and excess input values of 0.112, 0.064, and 0.198 for Input 1, 3, and 4 respectively. General inferences across all states about each measure can be made by examining total idenfied waste and recommended substitutions.

For example, the relatively large excesses for Input 1 (decline in manufacturing employment) across states in 1982 reflect large inter-state migrations of the manufacturing labor force over the preceding 5-year period. In 1987, the excesses for Input 2 (value-added/employee), which does not show a marked effect in 1982, indicate a new emphasis on productivity. By 1987, the distribution of the labor force had stabilized geographically and the discriminating factor for productivity had shifted to value-added.

As expected, since the Limit-Price-Ratio ranges were obtained from the additive invariant model, the sets of efficient units for 1982 and 1987 are very similar to those identified under the invariant pricing evaluation. In fact we observe that all states that were on the envelope under the invariant pricing evaluation, with the exception of Indiana in 1982, are still found efficient under the Frontier model constrained by the Limit-Price-Ratio ranges. However, we note that, for these efficient units, the identified price vectors report different tradeoffs from those identified under the invariant pricing evaluation. For instance Texas, which is efficient in 1982, exhibits maximum price ratios for μ_1/μ_2 and μ_1/μ_3 (82.67 and 199.87 respectively) under the invariant pricing mechanism (Table 5.13) but sees these ratios drop to a minimum and low value of 0.011 and 0.308 repectively when evaluated with the Frontier model (Table 5.20). This illustrates that multiple price vectors, representative of different tradeoffs are feasible for efficient units. Hence, caution must be exercised in interpreting these revealed relative prices. Returning to the case of Texas in 1982, 3 output price ratios were evaluated at a bound: lower bound for μ_1/μ_2, upper bound for μ_2/μ_3 and for μ_2/μ_5 suggesting that Output 2 was a relative strength of Texas. Output 2 (value-added per employee) for

Texas in 1982 represents the 5th highest value over all states confirming the inference. However, alternate price vectors could have been identified by the model that would not have made this strength so apparent by not showing any price ratio related to Output 2 at a bound. It follows that revealed for efficient units strengths should not be considered exhaustive.

To illustrate the inferences which may be drawn about a specific state from Tables 5.16 through 5.23 (pages 210-217), we focus on Texas, Michigan, Washington, and Pennsylvania.

In 1982 Texas was efficient. The pattern of output price ratios reveals μ_1/μ_2 to be minimum and μ_2/μ_3 and μ_2/μ_5 to be maximum in accordance with a high level of Output 2 relative to other states (the level of Output 2, value-added per employee, in Texas in 1982 is the fifth largest across all states). The pattern of input price ratios indicates a low efficiency price for Input 4 (cost ratio) and a high efficiency price for Input 3 (hourly wages) relative to other inputs. These prices reflect that Texas' manufacturing operations in 1982 were characterized by high value of the cost ratio variable and low hourly wages relative to other states. These observations are supported by the industrial profile of Texas with the predominance of the oil industry and electronics assembly industry, and also by the fact that Texas experienced a large influx of workers between 1977 and 1982. However, by 1987, Texas had lost its efficient status. The evaluation shows $\theta = 1$ in 1987, indicating input efficiency. However, the non zero excess input and substitutions suggest that a repositioning of inputs, given Texas' identified relative prices, would allow the state to boost its output levels. Texas' output inefficiency in 1987 is 0.887, indicating a 12% deficiency in outputs relative to efficient states. In particular paying higher hourly wages (-0.036 for Input 3 in Table 5.17) would afford a higher participation of the population to the manufacturing labor force and higher growth in value-added per employee. The data shows that between 1982 and 1987 Texas experienced a loss of manufacturing employment and growth in value-added per employee. However, the analysis reveals that this absolute growth is not at par with that of efficient states (Output 3, value-added per employee exhibits a slack value of 2.133 in 1987). Between 1982 and 1987 it appears that Texas suffered a drainage of skilled labor.

Michigan was inefficient in 1982 but gained efficiency by 1987. In 1982 global input inefficiency was measured by θ = 0.88 with the primary source of inefficiency stemming from Input 3 (hourly wages). In fact Michigan exhibited in 1982 the highest level of hourly wages across all states. The identified excess is 2.05 for that measure corresponding to a recommended reduction to 82% (the partial input inefficiency measured by θ_3 = (level - excess)/level = (11.47 -2.05)/11.47 = 0.82) of its original level.

On the output side global inefficiency is measured by $E_o = 1/\phi = 0.99$ (ϕ = 1.01) indicating an average deficiency in ouputs of 1%. Looking at individual outputs we note that the identified source of output deficiency is Output 1 (per capita income) with the partial output inefficiency measured by ϕ_1 = (level + slack)/level = (1.11 + 0.07)/1.11 = 1.063. On the other hand, Output 4 (new capital expenditures per establishment) shows no slack and therefore represents a source of relative strength for the state. This is supported by the output price ratios μ_1/μ_4 and μ_4/μ_5 which are evaluated at their minimum and maximum allowable values repectively, and by the evaluated prices μ_1 and μ_4 which are respectively low and high when compared to the evaluated output 1 and output 4 prices for other states.

By 1987, despite the fact that Michigan still had the highest level of hourly wages, the state became efficient ($\theta = 1$, and $\phi = 1$). From the particular (alternate prices are possible) price vector that is identified we observe that μ_2/μ_3 and μ_2/μ_4 are at their minimum allowable level while ν_1/ν_2, ν_1/ν_3, and ν_1/ν_4 are at their maximum level. These data indicate that Output 2 (value-added per employee) is a weakness of Michigan relative to Output 3 (labor force participation) and Output 4 (new capital expenditures per establishment), and that Input 1 (percentage decrease in employment) is a relative input strength, i.e. manufacturing employment was maintained at a satisfactory level. Looking at the evaluated prices themselves in Table 5.19, we further note that μ_2 for Michigan is low relative to other states while μ_3 and μ_4 are relatively high, and, on the input side, ν_1 for Michigan is among the lowest across states. This information is consistent with the interpretation that Michigan benefited from an influx of workers between 1982 and 1987, contributing to a low value of Input 1 (and high ν_1) and a high value of Output 3 (Labor force participation). However, the productivity of this workforce as measured by Output 2 (value-added per employee) appears to be trailing behind the level of new capital expenditures.

Washington state was inefficient both in 1982 and in 1987 but much less so in 1987. In 1982 the state's evaluation lead to θ = 0.77, ϕ = 1.08 for a global efficiency score of 0.72, second to worst. By 1987 the scores had become θ = 0.86, ϕ = 1 for a global efficiency score of 0.86 placing the state ahead of 12 others.

The sources of inefficiency in 1982 reside mainly with insufficient growth in value-added (Input 2), a low participation level (Output 3) and low density of manufacturing establishments (Output 5): the identified waste for individual measures are such that the partial inefficiency measure for Input 2 and Outputs 3 and 5 evaluate to θ_2 = 0.61, ϕ_3 = 1.15 and ϕ_5 = 1.14. However, by 1987, Washington state had gained output efficiency and at that time its level of Output 5 (density of manufacturing establishments) is found high relative to other states. Indeed the 1987 evaluation requires that some of that variable be substituted out (substitution of 0.068 of Output 5 in Table 5.17) to allow input inefficiency to be revealed. Input inefficiency is again concentrated, and more severely so than in 1982, on insufficient growth (i.e. too high percentage decrease) in value-added per manufacturing employee.

Pennsylvania was inefficient in 1982 and its efficiency deteriorated by 1987. In 1982 the state's evaluation lead to θ = 0.96, ϕ = 1.04 for a global score of 0.93 positioning the state ahead of 20 others. However, by 1987 those scores had become

0.98, 1.187 and 0.825 respectively, dropping the state to eigth before last. For both years looking at partial input and output efficiency scores we observe that Input 3 (hourly wages) which appeared as a relative weakness of Pennsylania in 1982 (θ_3 = 1 - 0.94/8.95 = 0.895 < θ) had become a relative strength by 1987 (θ_3 = 0.983). Input 2 (percentage decrease in value-added per employee) is revealed to be very low for Pennsylvania (θ_2 = 0.33). On the output side inefficiency spread evenly to all measures except Output 5 (density of manufacturing establishments) which is more severely affected. Pennsylvania appears in dire needs of revitalization of its manufacturing industry base.

The overall economic performance of states in 1982 and 1987 was reassessed under a new set of price ratio constraints given by the Limit-Value-Ratio ranges. These ranges are summarized in Table 5.24 on page 218 and are radically different (generally much narrower) from the Limit-Price-ratio ranges given in Table 5.15 on page 209.

The identified waste and substitutions are given in Tables 5.25 and 5.26 on pages 219 and 220, estimated prices are given in Tables 5.27 and 5.28 on pages 221 and 222, their corresponding ratios for output measures are given in Tables 5.29 and 5.30 on pages 223 and 224, and their corresponding ratios for input measures as well as their associated efficiency scores are given in Tables 5.31 and 5.32 on pages 225 and 226.

Of the 24 states found efficient in 1982 with the Limit-Price-ratio ranges only 8 remain efficient with the Limit-Value-Ratio ranges. These 8 states are: California, Connecticut, Delaware, Louisiana, Massachusetts, North Carolina, New York, and Ohio. Some of their strengths become apparent in Table 5.27, on page 221, reporting their identified efficiency prices. For instance, California exhibits very high prices for Input

1 and Input 2 (percentage decrease in employment and in value-added respectively) relative to its other prices and relative to Input 1 and Input 2 prices of other states. We can infer that, had California not benefited from a hige influx of workers between 1977 and 1982, the state might not have been found efficient. North Carolina and Ohio similarly price Input 2 very high while Louisiana's strengths can be traced to Output 2 (value-added per employee) and Output 4 (new capital expenditures).

Alaska and New Jersey which were found efficient with the Limit-Price-Ratio ranges are now "penalized" for "excessive" Output 1 (per capita income). New Jersey also shows an excessive level of Output 5 (number of establishments to total population). These observations have to be tempered with considerations of cost-of-living for Alaska and a large out-of-state commuting labor force in New Jersey. Hawaï on the other hand, owed its previous efficient rating to an extremely low level of Input 3 (hourly wages). This is confirmed by the data in Table 5.4, on page 198, which shows that Hawaï had the lowest level of hourly wages across states in 1982. Finally the results for Texas, which also lost its efficient rating, confirm the inferences drawn in the evaluation with the Limit-Price-Ratios. In particular, paying higher hourly wages would afford a higher participation of the population to the manufacturing labor force.

The evaluation of units inefficient under the Limit-Price-Ratio ranges reveals the same weaknesses under the Limit-Value-Ratio constraints but the extent of these weaknesses is generally exacerbated due to the tighter ranges of allowable tradeoffs. Moreover, measures that were identified earlier as sources of relative (with respect to other measures) strengths for a unit still appear as such by showing as candidates for substitution. This is illustrated by Michigan for which Output 4 (new capital expenditures) had been identified as a relative strength. The evaluation with the Limit-Value-Ratio ranges now suggests that Michigan could bear a lower level for that output (substitution of -0.18 for Output 4 for Michigan in Table 5.25 on page 219).

The evaluation of the states in 1987 with the corresponding Limit-Value-Ratio ranges given in Table 5.24, on page 218, lead to similar conclusions. Of the 24 states that were efficient in 1987 with the Limit-Price-Ratio ranges only 5 remain efficient with the Limit-Value-ratio ranges. These 5 states are: Arizona, Indiana, Louisiana, North Carolina, and New Hampshire.

The Limit-Value-ratio ranges, as mentioned earlier, are much tighter than the Limit-Price-Ratio ranges. They bar "specialist" states, that is states which exhibit extreme levels in a measure, highest level across units for an output or lowest level across units for an input, from reaping an efficient rating, by preventing unreasonable prices to be attributed to these specialty measures. The above examples illustrate that the specification of the allowable ranges for the tradeoffs across measures is critical to the evaluation process. These ranges may easily be set to filter out undesirable behaviors.

5.4.2 Efficiency of an Industry Across States

This analysis concerns the comparative assessment of the industry group SIC 367, Electronic Components and Accessories, across all states for the year 1982. Decision-making units are the industry group in the various states and will be identified by the state's name. In 1982, the industry was present in 42 states. Since data for 8 of these states is suppressed, analysis is performed across the remaining 34[6]. The envelope determined using a global orientation and an explicit pricing mechanism is defined by 12 of the 34 states[7]. These DEA-efficient states are: California, Georgia, Kansas, North Carolina, New Mexico, New York, Oklahoma, Oregon, Pennsylvania, Texas, Virginia, and Washington. To identify the sources and extent of inefficiencies of other states we apply the frontier model constrained by the Limit-Value-Ratio ranges derived from the set of DEA-efficient units. These ranges are given in Table 5.34 on page 228. Table 5.35, on page 229, reports the identified total waste and substitutions, and Table 5.36, on page 230, reports the evaluated output and input price ratios. Of the 12 states that were efficient, only 10 remain efficient with respect to the Limit-Value-Ratio constraints imposed with the Frontier model (Oklahoma and Washington are no longer efficient). Both states appear input-efficient but output inefficient. The output deficiency for Oklahoma is revealed by allowing the evaluation to compare Oklahoma with units which consume relatively more Input 1 (i.e. exhibit higher costs of material inputs) and more of Input 6 (number of non-production workers). It follows that these 2 measures represent Oklahoma's relative strengths. Similarly we can infer that Washington's

---

[6] Data is suppressed for Arkansas, Idaho, Kentucky, Mississippi, North Dakota, South Dakota, Vermont, and Wisconsin.

[7] Table 5.33, on page 227, report the revealed waste (input excesses and output slacks) for each state reporting activity in that industry. States with zero total waste are efficient in that industry group.

relative strengths in 1982 reside in Input 2 (low level of wages paid out), Inputs 3, 4 (low levels of production and non-production workers), and Output 2 (high level of value-added). However, these strengths failed to translate into sufficiently high levels of Output 1 (shipments) and Output 3 (profits) to rate Washington as efficient relative to all other states.

The results of the analysis show in fact that all states, except Missouri, are input-efficient ($\theta = 1$), and that inefficiency originated from deficient output levels ($\phi > 1$). We may conclude that by 1982 the technology of industry group 367 was mastered by all players and that the degrees of inefficiency were tied to the outcomes of battles for market share.

If we define large-scale producers in that industry group as the states posting at least 1 billion dollars in value-added, we count 7 such states in 1982 (California, Illinois, Massachusetts, Missouri, New York, Pennsylvania, and Texas), 3 of which are inefficient (Illinois, Massachusetts, and Missouri). The efficiency scores of Illinois and Massachusetts are evaluated at 0.821 and 0.814 respectively. The output price ratios of Illinois evaluate at lower bound for μ_1/μ_2 and upper bound for μ_2/μ_3, leading to the conclusion that Output 2 (value-added) represents Illinois' relative strength, and that Outputs 1 (shipments) and 3 (profits) are deficient. The output price ratios for Massachusetts evaluate at lower bound for μ_1/μ_3 and μ_2/μ_3, indicating that Output 3 (profits) represent Massachusetts' relative strength and that the state's output deficiencies stem essentially from Outputs 1 (shipments) and 2 (value-added). These inferences are confirmed by the partial output inefficiency scores[8]. For Illinois these scores evaluate to 1.422, 1.187, 1.276 for Outputs 1, 2, and 3 respectively. The same calculations for Massachusetts give 1.376, 1.232, and 1.189 respectively.

Keeping the same set of price ratio ranges, the Limit-Value-Ratio ranges, the Comparative Advantage model (given in Appendix E) is solved over the set of efficient units. This model produces an input score θ and an output score ϕ such that whenever $\theta > 1$, it can be inferred that the DMU's technique is a source of its comparative advantage. Similarly, whenever $\phi < 1$ it can be inferred that the DMU's current output mix is a source of its comparative advantage. These scores for the states efficient in industry group 367 are given in Table 5.37 on page 231. The model also identifies, for each efficient unit, which other units are its closest competitors. This information is also provided in Table 5.37. Table 5.38, on page 232, gives the identified discrepancies for all measures between a unit's and its closest competitor's levels[9], and the evaluated price ratios across all measures.

From Table 5.37 we observe that 3 states emerge with a comparative advantage in inputs: Georgia, Kansas, and New Mexico, which are all small-scale producers (with value-added less than $100 million). All other states have a comparative advantage in outputs. California stands alone, more than twice the scale of its closest competitor, New York. In Table 5.38, on page 232, negative discrepancies represent relative

---

[8] The partial inefficiency score relative to Output r is given by ϕ_r = (Output r level + Output r slack) / Output r level.

[9] If (x_l, y_l) denote the unit's input and output vectors , and (x^*, y^*) denote its closest competitor's input and output vectors, then the discrepancies are given by $(x_l - x^*)$ and $(y^* - y^l)$.

advantages relative to competitors and positive discrepancies represent relative disadvantages. Such information is valuable to help a state design assistance programs and target incentives that will promote growth and strength of the state's economy.

5.5 Conclusions

This study has illustrated the use of data envelopment analysis to evaluate a state's overall economic performance compared to other states, and the performance of a specific industry across states. The illustrative studies show that the technique can be used to gauge the effect of changes over time. Economic data series which are produced annually, while providing current information, are typically limited in scope (geographic and industry detail). Relevant data, unfortunately, is often delayed. For effective use of the proposed analyses in regional planning, the timeliness of the collection of such data is crucial. The Frontier model is geared toward the identification of sources of comparative disadvantage for the decision-making units. It offers flexibility through the specification of ranges for allowable tradeoffs across measures. The specification of these ranges is critical to the evaluation and allows filtering of performances. The Comparative Advantage model is geared toward the identification of sources of comparative advantage for decision-making units that exhibit frontier efficient behavior. Such information is valuable to help a state design assistance programs and target incentives that will promote growth and strength of the state's economy.

Table 5.1 States 1982: Original Data Set

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK82	2564.10	992.80	10.74	0.70	17370	60.90	2.87	1.30	1.00
AL82	3.50	-30.20	7.50	0.59	8836	36.55	8.36	2.80	1.40
AR82	79.70	8.40	6.92	0.61	8604	40.86	8.22	2.10	1.44
AZ82	127.60	36.40	8.62	0.52	10314	41.14	5.19	2.20	1.18
CA82	-83.00	-91.10	8.87	0.53	13238	47.08	8.09	1.80	1.92
CO82	78.20	4.30	9.04	0.55	12638	42.09	6.23	2.00	1.43
CT82	-19.70	-48.70	8.55	0.46	14459	38.58	13.56	1.60	2.14
DC82	1941.90	805.20	10.44	0.39	14695	55.60	2.66	1.10	0.82
DE82	402.20	240.80	9.21	0.70	11953	36.33	11.28	5.10	1.05
FL82	-25.00	-53.60	6.98	0.53	11315	39.86	4.34	1.40	1.31
GA82	-32.20	-56.20	6.90	0.61	9867	38.15	8.90	2.30	1.51
HI82	1344.90	650.80	7.27	0.68	11760	47.44	2.36	0.90	0.97
ID82	616.40	304.80	8.71	0.60	9284	43.62	4.87	1.00	1.44
IL82	-68.10	-82.40	9.50	0.57	12355	44.64	9.31	1.80	1.62
IN82	-41.70	-67.40	10.30	0.58	10293	44.00	10.67	2.80	1.45
IO82	60.20	-30.40	10.22	0.62	10868	56.76	7.33	1.90	1.24
KS82	99.90	-0.40	8.98	0.68	11811	49.45	7.08	1.90	1.34
KY82	38.30	-28.90	8.99	0.60	9278	47.93	6.67	2.80	0.95
LA82	68.80	-28.50	9.72	0.79	10234	58.19	4.61	6.20	0.94
MA82	-47.00	-67.60	7.84	0.46	12751	40.36	11.20	1.50	1.92
MD82	45.50	-17.10	9.43	0.53	12735	43.32	5.49	1.80	0.91
ME82	209.40	108.20	7.46	0.54	9598	36.64	9.69	2.50	1.77
MI82	-61.40	-78.50	11.47	0.60	11101	44.26	9.69	2.30	1.66
MN82	-2.60	-45.30	9.11	0.56	11548	43.91	8.47	1.80	1.64
MO82	-16.00	-54.20	8.80	0.58	10867	45.17	8.21	1.90	1.43
MS82	69.10	7.40	6.61	0.59	8006	38.79	7.86	3.70	1.22
MT82	1588.10	1078.10	9.81	0.81	10083	35.32	2.51	0.70	1.35
NC82	-57.30	-70.50	6.54	0.56	9284	35.68	13.27	2.50	1.68
ND82	2204.10	1189.10	7.84	0.74	10520	44.06	2.20	1.70	0.87
NE82	274.30	89.10	8.16	0.70	11054	48.79	5.73	1.30	1.21
NH82	217.50	109.90	7.28	0.47	11592	37.28	11.33	1.20	2.09
NJ82	-54.80	-73.40	8.55	0.55	13966	41.98	10.15	1.40	2.04
NM82	933.30	501.40	7.10	0.64	9509	42.35	2.41	1.00	0.89
NV82	1571.60	874.50	8.46	0.51	12498	42.28	2.33	0.90	0.97
NY82	-76.00	-86.60	8.23	0.48	12703	44.34	8.07	1.30	1.86
OH82	-69.10	-83.10	10.42	0.55	10932	45.05	10.23	2.50	1.57
OK82	73.20	3.20	8.80	0.65	11357	41.36	6.09	2.00	1.29
OR82	84.20	5.40	9.81	0.55	10581	43.07	6.93	1.10	2.12
PA82	-71.10	-81.20	8.95	0.55	11440	37.99	9.93	1.90	1.49
RI82	199.60	121.60	6.68	0.50	11168	33.33	11.94	0.60	3.00
SC82	-7.20	-31.20	6.85	0.56	8710	33.25	11.41	3.60	1.31
SD82	1291.80	664.00	8.03	0.63	9488	44.91	3.53	0.80	1.08
TN82	-26.10	-52.80	7.31	0.56	9208	38.61	9.89	3.20	1.37
TX82	-67.80	-84.20	8.60	0.69	11684	50.41	6.88	4.00	1.32
UT82	309.90	143.30	8.13	0.61	9041	41.53	5.34	1.70	1.26
VA82	-12.80	-51.30	7.75	0.53	11630	44.12	7.13	2.70	1.01
VT82	628.60	312.70	7.72	0.45	10110	43.52	8.98	2.50	2.12
WA82	17.00	-33.30	11.08	0.67	12018	43.22	6.81	1.60	1.59
WI82	-31.30	-62.70	9.65	0.56	11073	45.39	10.46	1.80	1.83
WV82	255.90	107.60	10.15	0.58	9006	42.27	4.88	2.40	0.85
WY82	3344.40	1961.80	8.73	0.81	12235	41.18	1.94	1.00	1.00

Table 5.2 States 1987: Original Data Set

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK87	18.5	-7.0	11.5	0.7	18461	76.59	2.06	1.60	0.81
AL87	-5.7	-35.2	8.9	0.5	12039	53.16	8.56	7.10	0.51
AR87	-7.7	-26.6	8.2	0.6	11421	51.38	8.61	2.60	1.42
AZ87	-18.7	-44.9	10.0	0.5	14322	60.67	5.42	2.00	1.22
CA87	-5.0	-29.6	10.6	0.5	17770	63.47	7.63	1.70	1.81
CO87	4.4	-32.2	11.2	0.5	15680	64.76	5.57	1.70	1.43
CT87	9.9	-26.7	10.9	0.4	21258	57.84	12.03	1.90	2.10
DC87	-1.8	-40.4	11.2	0.3	19543	93.59	2.74	0.90	0.79
DE87	0.1	-37.3	11.7	0.6	16305	58.01	10.46	4.00	1.04
FL87	-9.0	-35.2	8.5	0.5	15594	55.99	4.15	1.20	1.30
GA87	-11.4	-43.4	8.8	0.6	14387	59.76	9.11	2.70	1.47
HI87	6.8	-20.7	8.8	0.6	15569	63.85	2.04	1.00	0.94
ID87	-9.7	-32.0	9.6	0.6	11797	57.98	5.27	1.60	1.50
IL87	8.5	-25.1	10.9	0.5	16394	64.61	8.51	2.40	1.59
IN87	-2.7	-34.9	11.5	0.5	13987	65.73	10.88	3.90	1.56
IO87	3.3	-16.7	11.0	0.6	14028	70.37	7.30	2.30	1.26
KS87	-9.7	-34.6	10.7	0.6	15089	68.32	7.63	3.10	1.32
KY87	-2.1	-34.7	10.5	0.6	11996	71.86	6.77	4.70	0.99
LA87	26.0	-28.6	11.5	0.7	11506	102.64	3.60	3.70	0.86
MA87	9.2	-27.5	10.1	0.4	19131	60.74	10.06	2.00	1.88
MD87	1.30	-28.0	10.9	0.5	18217	60.82	5.10	2.10	0.94
ME87	8.6	-23.3	9.6	0.5	13996	51.88	8.56	2.50	1.83
MI87	-10.0	-35.6	13.3	0.6	15558	61.87	10.66	3.00	1.74
MN87	-6.4	-34.0	10.5	0.5	15789	62.22	8.81	2.50	1.68
MO87	-3.0	-29.3	10.4	0.6	14630	61.98	8.19	2.20	1.43
MS87	-8.3	-25.4	7.8	0.6	10301	47.71	8.38	1.90	1.26
MT87	0.5	-36.0	10.3	0.7	12304	55.50	2.48	0.80	1.53
NC87	-5.2	-39.8	8.1	0.5	13353	56.16	13.15	2.70	1.71
ND87	-3.9	-33.8	8.6	0.6	12825	63.99	2.30	0.80	0.93
NE87	1.4	-22.9	9.5	0.6	14100	64.17	5.63	1.70	1.17
NH87	-0.9	-48.8	10.1	0.4	18083	72.13	10.27	1.50	2.21
NJ87	10.1	-25.5	10.8	0.5	20277	62.04	8.92	1.60	1.88
NM87	-4.6	-18.7	9.1	0.6	11889	49.69	2.31	1.50	0.88
NV87	-13.9	-7.1	9.7	0.5	16359	39.17	2.36	1.10	0.97
NY87	11.7	-20.7	10.2	0.5	17943	62.42	7.12	1.40	1.66
OH87	0.5	-30.7	12.1	0.6	14575	65.33	10.14	2.80	1.62
OK87	26.8	-17.2	10.6	0.6	12607	63.33	4.77	1.50	1.15
OR87	-9.1	-31.0	10.3	0.6	13906	56.76	7.48	1.20	2.33
PA87	14.0	-22.4	10.3	0.5	15198	55.77	8.67	1.90	1.50
RI87	2.2	-22.3	8.4	0.5	15683	43.82	11.30	0.90	2.92
SC87	-0.5	-36.1	8.4	0.5	12078	51.78	10.78	3.50	1.33
SD87	-10.3	-25.5	8.0	0.6	12414	54.07	3.85	1.00	1.07
TN87	-4.8	-34.2	8.9	0.5	12977	55.85	9.99	2.80	1.41
TX87	16.0	-16.5	10.1	0.6	13840	70.07	5.44	2.20	1.21
UT87	-7.0	-28.9	9.3	0.5	11530	54.29	5.33	1.90	1.24
VA87	-8.5	-36.4	9.5	0.5	16539	63.49	7.23	2.50	1.04
VT87	-3.7	-19.9	9.5	0.5	14267	52.35	8.88	2.70	2.31
WA87	-6.0	-30.1	10.0	0.6	15634	58.14	6.83	1.60	1.68
WI87	-3.1	-28.3	11.0	0.6	14674	61.30	10.67	2.20	1.91
WV87	14.9	-25.3	11.4	0.5	11013	65.03	4.39	2.20	0.85
WY87	26.9	-16.1	11.1	0.8	12836	62.27	1.59	1.30	1.02

Table 5.3 SIC 367-1982 Data
Electronic Components and Accessories

DMU	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6	Output 1	Output 2	Output 3
AL367	158.1	41.6	26.8	7.8	3.7	1.0	125.3	340.8	193.7
AZ367	369.2	169.8	340.9	21.8	12.2	11.6	318.4	1189.1	829.1
CA367	3615.7	1290.2	1489.8	159.4	82.2	52.9	3469.6	9797.8	6249.6
CO367	81.0	31.8	33.5	5.9	3.0	1.4	29.6	174.8	94.9
CT367	249.1	113.7	79.9	18.0	8.9	3.3	226.6	671.0	420.2
FL367	226.3	135.1	111.8	21.5	10.8	5.2	239.2	734.6	486.1
GA367	44.2	11.3	6.4	1.7	0.9	0.3	28.6	91.1	46.3
IL367	601.8	240.0	157.1	34.5	18.5	6.8	457.9	1448.5	855.0
IN367	253.0	105.6	49.3	14.8	7.8	2.3	153.8	559.0	308.7
IO367	45.1	22.1	11.7	3.3	1.6	0.7	30.2	109.5	64.0
KS367	25.1	8.9	5.5	1.6	0.8	0.3	11.6	51.2	26.0
MA367	573.5	296.3	241.9	42.6	21.8	9.8	565.5	1659.4	1103.7
MD367	48.2	22.3	18.4	3.4	1.8	0.8	34.5	123.6	75.2
ME367	70.0	35.8	22.7	6.3	3.2	1.0	112.8	232.6	171.3
MI367	47.6	16.6	13.5	2.8	1.4	0.6	32.1	109.6	62.2
MN367	115.5	67.8	48.1	10.3	5.3	2.1	88.2	323.1	204.1
MO367	205.8	117.4	57.4	11.3	6.1	2.0	193.2	580.4	368.0
NC367	886.0	94.2	160.7	13.2	6.9	5.1	424.5	1417.1	679.4
NE367	28.5	15.3	23.1	3.5	1.7	1.3	34.0	100.7	72.4
NH367	100.5	56.9	29.9	10.0	5.4	1.4	77.0	263.3	163.8
NJ367	369.0	162.5	152.6	23.9	12.5	5.9	289.5	975.1	604.6
NM367	15.3	10.5	5.7	1.8	0.9	0.3	17.7	50.0	33.9
NY367	2679.7	428.0	807.8	57.4	28.7	28.4	1297.3	5201.1	2533.1
OH367	208.5	101.2	56.9	13.0	6.8	2.5	197.8	564.3	355.9
OK367	36.3	15.2	7.7	2.6	1.4	0.3	23.4	81.4	46.3
OR367	49.4	30.0	72.8	4.2	2.2	2.9	215.1	364.1	317.9
PA367	824.1	371.0	220.7	40.6	22.0	8.1	820.7	2245.8	1412.4
RI367	43.1	14.5	17.4	2.8	1.5	1.1	23.7	102.1	55.6
SC367	174.7	82.5	46.0	13.1	6.4	2.3	86.7	381.9	215.2
TN367	51.1	20.8	15.1	3.1	1.7	0.6	21.7	106.2	57.6
TX367	601.5	276.8	404.9	36.6	18.2	15.0	866.7	2140.0	1548.4
UT367	55.8	34.4	43.3	5.3	2.9	1.6	36.0	167.9	113.7
VA367	160.7	57.5	28.8	7.4	3.8	1.2	248.9	493.2	335.2
WA367	145.7	30.7	53.7	4.4	2.5	1.9	158.5	374.2	242.9

Table 5.4 States 1982: Transformed Data Set

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK82	26.64	10.93	10.74	7.00	1.74	6.01	2.87	1.30	1.00
AL82	1.04	0.70	7.50	5.90	0.88	3.66	8.36	2.80	1.40
AR82	1.80	1.08	6.92	6.10	0.86	4.09	8.22	2.10	1.44
AZ82	2.28	1.36	8.62	5.20	1.03	4.11	5.19	2.20	1.18
CA82	0.17	0.09	8.87	5.30	1.32	4.71	8.09	1.80	1.92
CO82	1.78	1.04	9.04	5.50	1.26	4.21	6.23	2.00	1.43
CT82	0.80	0.51	8.55	4.60	1.45	3.86	13.56	1.60	2.14
DC82	20.42	9.05	10.44	3.90	1.47	5.56	2.66	1.10	0.82
DE82	5.02	3.41	9.21	7.00	1.20	3.63	11.28	5.10	1.05
FL82	0.75	0.46	6.98	5.30	1.13	3.99	4.34	1.40	1.31
GA82	0.68	0.44	6.90	6.10	0.99	3.82	8.90	2.30	1.51
HI82	14.45	7.51	7.27	6.80	1.18	4.74	2.36	0.90	0.97
ID82	7.16	4.05	8.71	6.00	0.93	4.36	4.87	1.00	1.44
IL82	0.32	0.18	9.50	5.70	1.24	4.46	9.31	1.80	1.62
IN82	0.58	0.33	10.30	5.80	1.03	4.40	10.67	2.80	1.45
IO82	1.60	0.70	10.22	6.20	1.09	5.68	7.33	1.90	1.24
KS82	2.00	1.00	8.98	6.80	1.18	4.95	7.08	1.90	1.34
KY82	1.38	0.71	8.99	6.00	0.93	4.79	6.67	2.80	0.95
LA82	1.69	0.72	9.72	7.90	1.02	5.82	4.61	6.20	0.94
MA82	0.53	0.32	7.84	4.60	1.28	4.04	11.20	1.50	1.92
MD82	1.46	0.83	9.43	5.30	1.27	4.32	5.49	1.80	0.91
ME82	3.09	2.08	7.46	5.40	0.96	3.66	9.69	2.50	1.77
MI82	0.39	0.22	11.47	6.00	1.11	4.43	9.69	2.30	1.66
MN82	0.97	0.55	9.11	5.60	1.15	4.39	8.47	1.80	1.64
MO82	0.84	0.46	8.80	5.80	1.09	4.52	8.21	1.90	1.43
MS82	1.69	1.07	6.61	5.90	0.80	3.88	7.86	3.70	1.22
MT82	16.88	11.78	9.81	8.10	1.01	3.53	2.51	0.70	1.35
NC82	0.43	0.30	6.54	5.60	0.93	3.57	13.27	2.50	1.68
ND82	23.04	12.89	7.84	7.40	1.05	4.41	2.20	1.70	0.87
NE82	3.74	1.89	8.16	7.00	1.11	4.88	5.73	1.30	1.21
NH82	3.18	2.10	7.28	4.70	1.16	3.73	11.33	1.20	2.09
NJ82	0.45	0.27	8.55	5.50	1.40	4.20	10.15	1.40	2.04
NM82	10.33	6.01	7.10	6.40	0.95	4.24	2.41	1.00	0.89
NV82	16.72	9.75	8.46	5.10	1.25	4.23	2.33	0.90	0.97
NY82	0.24	0.13	8.23	4.80	1.27	4.43	8.07	1.30	1.86
OH82	0.31	0.17	10.42	5.50	1.09	4.51	10.23	2.50	1.57
OK82	1.73	1.03	8.80	6.50	1.14	4.14	6.09	2.00	1.29
OR82	1.84	1.05	9.81	5.50	1.06	4.31	6.93	1.10	2.12
PA82	0.29	0.19	8.95	5.50	1.14	3.80	9.93	1.90	1.49
RI82	3.00	2.22	6.68	5.00	1.12	3.33	11.94	0.60	3.00
SC82	0.93	0.69	6.85	5.60	0.87	3.33	11.41	3.60	1.31
SD82	13.92	7.64	8.03	6.30	0.95	4.49	3.53	0.80	1.08
TN82	0.74	0.47	7.31	5.60	0.92	3.86	9.89	3.20	1.37
TX82	0.32	0.16	8.60	6.90	1.17	5.04	6.88	4.00	1.32
UT82	4.10	2.43	8.13	6.10	0.90	4.15	5.34	1.70	1.26
VA82	0.87	0.49	7.75	5.30	1.16	4.41	7.13	2.70	1.01
VT82	7.29	4.13	7.72	4.50	1.01	4.35	8.98	2.50	2.12
WA82	1.17	0.67	11.08	6.70	1.20	4.32	6.81	1.60	1.59
WI82	0.69	0.37	9.65	5.60	1.11	4.54	10.46	1.80	1.83
WV82	3.56	2.08	10.15	5.80	0.90	4.23	4.88	2.40	0.85
WY82	34.44	20.62	8.73	8.10	1.22	4.12	1.94	1.00	1.00

Table 5.5 States 1987: Transformed Data Set

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK87	6.85	4.30	11.47	7.00	1.85	7.66	2.06	1.60	0.81
AL87	4.43	1.48	8.93	5.40	1.20	5.32	8.56	7.10	0.51
AR87	4.23	2.34	8.16	5.90	1.14	5.14	8.61	2.60	1.42
AZ87	3.13	0.51	10.03	4.60	1.43	6.07	5.42	2.00	1.22
CA87	4.50	2.04	10.58	4.80	1.78	6.35	7.63	1.70	1.81
CO87	5.44	1.78	11.20	4.90	1.57	6.48	5.57	1.70	1.43
CT87	5.99	2.33	10.93	4.10	2.13	5.78	12.03	1.90	2.10
DC87	4.82	0.96	11.18	2.70	1.95	9.16	2.74	0.90	0.79
DE87	5.01	1.27	11.67	6.40	1.63	5.80	10.46	4.00	1.04
FL87	4.10	1.48	8.50	5.10	1.56	5.60	4.15	1.20	1.30
GA87	3.86	0.66	8.76	5.60	1.44	5.98	9.11	2.70	1.47
HI87	5.68	2.93	8.78	5.90	1.56	6.39	2.04	1.00	0.94
ID87	4.03	1.80	9.64	5.70	1.18	5.80	5.27	1.60	1.50
IL87	5.85	2.49	10.91	5.20	1.64	6.46	8.51	2.40	1.59
IN87	4.73	1.51	11.49	5.30	1.40	6.57	10.88	3.90	1.56
IO87	5.33	3.33	10.96	5.90	1.40	7.04	7.30	2.30	1.26
KS87	4.03	1.54	10.65	5.90	1.51	6.83	7.63	3.10	1.32
KY87	4.79	1.53	10.53	5.70	1.20	7.19	6.77	4.70	0.99
LA87	7.60	2.14	11.49	6.80	1.15	10.26	3.60	3.70	0.86
MA87	5.92	2.25	10.12	4.40	1.91	6.07	10.06	2.00	1.88
MD87	5.13	2.20	10.87	5.00	1.82	6.08	5.10	2.10	0.94
ME87	5.86	2.67	9.64	5.10	1.40	5.19	8.56	2.50	1.83
MI87	4.00	1.44	13.33	5.90	1.56	6.19	10.66	3.00	1.74
MN87	4.36	1.60	10.52	5.20	1.58	6.22	8.81	2.50	1.68
MO87	4.70	2.07	10.39	5.70	1.46	6.20	8.19	2.20	1.43
MS87	4.17	2.46	7.77	5.70	1.03	4.77	8.38	1.90	1.26
MT87	5.05	1.40	10.27	6.80	1.23	5.55	2.48	0.80	1.53
NC87	4.48	1.02	8.11	5.10	1.34	5.62	13.15	2.70	1.71
ND87	4.61	1.62	8.85	6.30	1.28	6.40	2.30	0.80	0.93
NE87	5.14	2.71	9.46	6.30	1.41	6.42	5.63	1.70	1.17
NH87	4.91	0.12	10.07	3.60	1.81	7.21	10.27	1.50	2.21
NJ87	6.01	2.45	10.75	4.90	2.03	6.20	8.92	1.60	1.88
NM87	4.54	3.13	9.14	5.90	1.19	4.97	2.31	1.50	0.88
NV87	3.61	4.29	9.69	4.90	1.64	3.92	2.36	1.10	0.97
NY87	6.17	2.93	10.23	4.60	1.79	6.24	7.12	1.40	1.66
OH87	5.05	1.93	12.08	5.50	1.46	6.53	10.14	2.80	1.62
OK87	7.68	3.28	10.59	6.00	1.26	6.33	4.77	1.50	1.15
OR87	4.09	1.90	10.34	5.50	1.39	5.68	7.48	1.20	2.33
PA87	6.40	2.76	10.25	5.20	1.52	5.58	8.67	1.90	1.50
RI87	5.22	2.77	8.39	4.80	1.57	4.38	11.30	0.90	2.92
SC87	4.95	1.39	8.38	5.40	1.21	5.18	10.78	3.50	1.33
SD87	3.97	2.45	8.04	6.20	1.24	5.41	3.85	1.00	1.07
TN87	4.52	1.58	8.88	5.30	1.30	5.59	9.99	2.80	1.41
TX87	6.60	3.35	10.14	6.10	1.38	7.01	5.44	2.20	1.21
UT87	4.30	2.11	9.32	5.30	1.15	5.43	5.33	1.90	1.24
VA87	4.15	1.36	9.52	4.90	1.65	6.35	7.23	2.50	1.04
VT87	4.63	3.01	9.47	4.60	1.43	5.24	8.88	2.70	2.31
WA87	4.40	1.99	10.02	6.20	1.56	5.81	6.83	1.60	1.68
WI87	4.69	2.17	10.99	5.50	1.47	6.13	10.67	2.20	1.91
WV87	6.49	2.47	11.39	5.20	1.10	6.50	4.39	2.20	0.85
WY87	7.69	3.39	11.07	7.70	1.28	6.23	1.59	1.30	1.02

Table 5.6 States Study: Variables Definitions and Units of Measurement

		Units of Measurement	
	Definition	1982	1987
Input 1	Decrease in Manufacturing Employment	$^{o}/_{ooo} + 1$ < 1 : growth	$^{o}/_{ooo} + 5$ < 5 : growth
Input 2	Decrease in Value-Added per Manufacturing Employee	$^{o}/_{ooo} + 1$ < 1 : growth	$^{o}/_{ooo} + 5$ < 5 : growth
Input 3	Hourly Wages	$/hour	$/hour
Input 4	Cost Ratio: Materials/Shipments	unitless	unitless
Output 1	Income per Capita	$10,000	$10,000
Output 2	Value-Added per Manufacturing Employee	$10,000	$10,000
Output 3	Participation	$^{o}/_{o}$	$^{o}/_{o}$
Output 4	Average New Capital Expenditures per Establishment	$100,000	$100,000
Output 5	Ratio of Number of Establishment to Total Population	#/100,000	#/100,000

Table 5.7 Industry Study: Variables Definitions and Units of Measurement

		Units of Measurement
	Definition	1982
Input 1	Cost of Material Inputs	$1,000,000
Input 2	Wages	$1,000,000
Input 3	Non-Production Wages & Salaries	$1,000,000
Input 4	Number of Hours Worked	1,000,000
Input 5	Number of Production Workers	1,000
Input 6	Number of Non-Production Workers	1,000
Output 1	Total Shipments	$1,000,000
Output 2	Value-Added	$1,000,000
Output 3	Profits	$1,000,000

Table 5.8 Efficient States

1982	1987
AK †	AL *
AR †	AZ *
CA	CA
CT	CT
DC	DC
DE	DE
FL	FL
HI	GA *
IN	HI
IO †	IN
LA	KS *
MA	KY *
MS	LA
NC	MA
NH	MI *
NJ †	MS
NY †	NC
OH †	NH
RI	NV *
SC †	OR *
TN †	RI
TX †	SD *
VA	VA
VT	VT
WI †	

Table 5.9 States 1982: Total Waste
VRS Envelopment/ Global Orientation/ Invariant Pricing

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AL82	0.67	0.47	0.00	0.00	0.18	0.50	2.37	0.00	0.22
AR82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AZ82	1.60	0.94	0.17	0.02	0.32	0.00	6.82	0.00	0.75
CA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO82	1.19	0.68	0.75	0.00	0.02	0.00	5.08	0.52	0.38
CT82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DC82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DE82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FL82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GA82	0.25	0.16	0.00	0.40	0.00	0.00	2.79	0.26	0.11
HI82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ID82	6.01	3.43	0.00	0.00	0.27	0.14	4.99	2.65	0.12
IL82	0.08	0.04	0.97	0.34	0.00	0.00	0.00	0.18	0.24
IN82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
IO82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KS82	1.65	0.81	0.42	0.06	0.00	0.00	0.31	1.93	0.04
KY82	1.11	0.57	0.32	0.00	0.32	0.00	1.23	0.00	0.70
LA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MD82	0.98	0.54	0.72	0.26	0.10	0.00	5.12	0.00	1.09
ME82	2.41	1.65	0.00	0.00	0.15	0.13	2.98	0.00	0.00
MI82	0.11	0.05	2.77	0.46	0.06	0.00	0.00	0.00	0.07
MN82	0.75	0.42	0.55	0.00	0.10	0.24	0.00	0.45	0.15
MO82	0.66	0.36	0.01	0.43	0.22	0.17	0.00	0.00	0.46
MS82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MT82	13.02	9.17	0.78	1.76	0.26	0.16	9.40	3.44	0.00
NC82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ND82	22.39	12.54	0.00	1.10	0.00	0.00	7.75	1.82	0.61
NE82	0.60	0.25	0.00	0.19	0.00	0.00	0.31	2.27	0.00
NH82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NJ82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NM82	6.56	3.94	0.00	0.17	0.00	0.00	4.65	2.12	0.34
NV82	14.49	8.48	0.00	0.00	0.02	0.00	9.00	1.59	0.97
NY82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OH82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OK82	1.04	0.60	0.24	1.36	0.25	0.00	5.90	0.16	0.66
OR82	1.13	0.57	1.31	0.42	0.27	0.00	3.08	0.49	0.00
PA82	0.03	0.03	0.91	0.09	0.04	0.50	0.00	0.15	0.35
RI82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SC82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SD82	12.63	6.97	0.00	0.00	0.08	0.00	6.29	3.00	0.40
TN82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TX82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UT82	3.52	2.08	0.00	0.59	0.35	0.00	6.14	0.82	0.53
VA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
VT82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WA82	0.92	0.54	2.36	0.52	0.04	0.57	0.62	1.41	0.00
WI82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WV82	2.63	1.54	1.39	0.61	0.47	0.00	6.94	0.00	1.08
WY82	32.19	19.18	0.00	2.00	0.00	0.00	9.19	2.68	0.51

Table 5.10 State 1987:Total Waste
VRS Envelopment/ Global Orientation/ Invariant Pricing

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK87	1.57	3.44	0.88	3.28	0.00	0.00	6.23	0.00	0.94
AL87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AR87	0.00	0.84	0.00	0.46	0.10	0.23	1.70	0.00	0.00
AZ87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
CA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO87	0.72	1.13	1.57	0.60	0.01	0.00	4.04	1.98	0.00
CT87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12
DC87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DE87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FL87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HI87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ID87	0.00	1.03	1.04	0.23	0.23	0.07	4.82	1.23	0.00
IL87	1.02	2.15	1.02	1.31	0.07	0.45	1.49	0.00	0.35
IN87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
IO87	0.42	2.98	1.01	1.98	0.31	0.00	2.58	0.00	0.69
KS87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KY87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MD87	0.03	1.48	0.72	1.28	0.00	0.65	5.40	0.00	1.09
ME87	1.08	2.23	0.00	1.03	0.26	1.53	1.83	0.00	0.06
MI87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MN87	0.00	1.16	0.70	0.99	0.03	0.37	0.00	0.00	0.01
MO87	0.00	1.36	0.82	1.31	0.08	0.19	1.33	1.75	0.04
MS87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MT87	0.58	0.31	2.04	1.66	0.09	0.02	9.98	2.56	0.00
NC87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ND87	0.00	0.71	0.00	1.74	0.20	0.00	8.80	1.49	0.69
NE87	0.44	2.05	0.00	1.95	0.15	0.00	4.49	1.69	0.45
NH87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NJ87	0.35	0.80	0.09	0.95	0.00	0.02	2.57	0.18	0.25
NM87	0.06	2.11	1.03	0.80	0.15	0.65	10.84	1.20	0.83
NV87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NY87	0.96	1.66	0.09	0.41	0.00	0.00	3.44	1.51	0.13
OH87	0.28	1.44	2.42	1.39	0.18	0.14	0.00	0.00	0.18
OK87	2.99	2.53	1.05	1.57	0.27	0.00	4.71	2.60	0.27
OR87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PA87	1.69	2.07	0.66	0.85	0.04	0.84	0.87	1.94	0.00
RI87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SC87	0.46	0.33	0.00	0.32	0.13	0.47	1.31	0.00	0.17
SD87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TN87	0.00	0.83	0.00	0.56	0.18	0.53	1.25	0.00	0.29
TX87	1.73	3.05	0.20	2.26	0.35	0.00	4.57	0.00	0.78
UT87	0.00	1.09	0.86	0.00	0.18	0.22	5.75	1.73	0.15
VA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
VT87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WA87	0.00	1.47	2.65	1.52	0.01	0.63	2.73	1.12	0.00
WI87	0.00	1.71	1.55	1.27	0.16	0.49	0.00	0.08	0.00
WV87	1.64	2.18	1.46	1.38	0.63	0.47	5.67	0.00	1.15
WY87	3.05	2.71	2.21	3.17	0.23	0.00	10.46	0.94	0.88

Table 5.11 States 1982: Input Price Ratios of Efficient Units
VRS Envelopment/ Global Orientation/ Invariant Pricing

DMU	ν_1/ν_2	ν_1/ν_3	ν_1/ν_4	ν_2/ν_3	ν_2/ν_4	ν_3/ν_4
AK82	0.60	0.10	0.24	0.16	0.39	2.39
AR82	0.70	0.58	0.48	0.83	0.69	0.83
CA82	0.58	0.07	0.06	0.12	0.10	0.82
CT82	0.55	0.09	0.06	0.17	0.11	0.65
DC82	0.52	0.01	0.47	0.02	0.91	51.24
DE82	0.58	0.01	0.62	0.02	1.06	54.48
FL82	0.57	0.07	0.01	0.13	0.02	0.12
HI82	0.68	1.83	1.39	2.70	2.05	0.76
IN82	0.66	0.06	0.03	0.09	0.05	0.52
IO82	0.67	0.50	0.35	0.75	0.51	0.69
LA82	0.60	3.79	2.29	6.32	3.81	0.60
MA82	0.60	0.01	3.40	0.02	5.63	354.47
MS82	0.57	6.48	3.64	11.38	6.39	0.56
NC82	0.51	6.50	2.88	12.64	5.60	0.44
NH82	0.74	0.76	6.03	1.03	8.14	7.91
NJ82	0.57	9.47	5.73	16.61	10.05	0.61
NY82	0.56	0.02	6.08	0.04	10.88	261.22
OH82	0.26	5.03	0.40	19.25	1.51	0.08
RI82	0.90	23.32	13.53	25.87	15.01	0.58
SC82	2.10	66.44	37.41	31.60	17.79	0.56
TN82	0.69	15.32	13.12	22.17	18.98	0.86
TX82	0.59	18.92	12.17	32.14	20.68	0.64
VA82	0.65	30.97	19.03	47.61	29.26	0.62
VT82	0.55	29.78	17.87	53.98	32.39	0.60
WI82	0.49	26.71	21.43	54.43	43.67	0.80

Table 5.12 States 1987: Input Price Ratios of Efficient Units
VRS Envelopment/ Global Orientation/ Invariant Pricing

DMU	ν_1/ν_2	ν_1/ν_3	ν_1/ν_4	ν_2/ν_3	ν_2/ν_4	ν_3/ν_4
AL87	1.42	8.56	5.18	6.03	3.65	0.61
AZ87	0.83	16.26	7.46	16.67	9.02	0.46
CA87	23.01	119.34	54.15	5.19	2.35	0.45
CT87	0.39	1.83	0.16	4.69	0.40	0.09
DC87	0.20	2.32	0.56	11.65	2.81	0.24
DE87	1.90	60.33	33.09	31.74	17.41	0.55
FL87	3.34	0.41	11.51	0.12	3.45	28.06
GA87	0.94	12.46	7.96	13.27	8.49	0.64
HI87	0.52	0.01	1.04	0.01	2.01	188.27
IN87	0.32	2.43	1.12	7.61	3.51	0.46
KS87	317.84	2198.07	1217.71	6.92	3.83	0.55
KY87	1.68	11.56	6.26	6.88	3.73	0.54
LA87	0.28	1.51	0.90	5.37	3.18	0.59
MA87	0.38	0.03	0.74	0.07	1.96	29.36
MI87	4.63	42.89	18.98	9.26	4.10	0.44
MS87	0.59	0.03	1.37	0.06	2.32	40.86
NC87	0.23	1.81	1.14	7.95	5.00	0.63
NH87	0.02	2.05	0.73	83.92	30.00	0.63
NV87	76.41	172.58	87.27	2.26	1.14	0.51
OR87	4.74	25.82	13.73	5.44	2.90	0.53
RI87	7.66	23.21	13.28	3.03	1.73	0.57
SD87	184.27	1.81	466.32	0.01	2.53	257.20
VA87	9.83	68.83	35.43	7.00	3.60	0.52
VT87	6.75	21.25	4.29	3.15	0.64	0.20

Table 5.13 States 1982: Output Price Ratios of Efficient Units
VRS Envelopment/ Global Orientation/ Invariant Pricing

DMU	μ_1/μ_2	μ_1/μ_3	μ_1/μ_4	μ_1/μ_5	μ_2/μ_3	μ_2/μ_4	μ_2/μ_5	μ_3/μ_4	μ_3/μ_5	μ_4/μ_5
AK82	0.70	1.63	0.84	0.84	2.33	1.20	1.20	0.52	0.52	1.00
AR82	3.50	2.49	0.69	0.37	0.71	0.20	0.11	0.28	0.15	0.53
CA82	0.32	1.86	0.72	0.78	5.90	2.28	2.46	0.39	0.42	1.08
CT82	0.63	3.72	0.84	1.14	5.91	1.34	1.81	0.23	0.31	1.35
DC82	3.26	66.56	25.38	27.36	20.45	7.80	8.41	0.38	0.41	1.08
DE82	0.78	21.75	9.02	8.03	27.97	11.61	10.33	0.42	0.37	0.89
FL82	4.31	8.88	2.47	2.10	2.06	0.57	0.49	0.28	0.24	0.85
HI82	3.04	1.77	0.60	0.88	0.58	0.20	0.29	0.34	0.50	1.46
IN82	3.22	9.77	1.04	1.80	3.04	0.32	0.56	0.11	0.18	1.74
IO82	3.82	10.11	0.48	0.26	2.65	0.13	0.07	0.05	0.03	0.53
LA82	2.80	5.03	1.28	1.14	1.80	0.46	0.41	0.25	0.23	0.89
MA82	0.01	0.19	2.44	1.67	17.45	221.27	151.73	12.68	8.69	0.69
MS82	1.53	4.31	1.34	0.72	2.82	0.87	0.47	0.31	0.17	0.53
NC82	4.44	7.19	1.60	1.02	1.62	0.36	0.23	0.22	0.14	0.64
NH82	3.82	13.10	0.57	1.50	3.43	0.15	0.39	0.04	0.12	2.66
NJ82	3.60	5.67	1.33	0.72	1.58	0.37	0.20	0.24	0.13	0.54
NY82	2.01	40.46	328.07	122.72	20.12	163.18	61.04	8.11	3.03	0.37
OH82	4.19	7.30	0.34	1.49	1.74	0.08	0.36	0.05	0.20	4.32
RI82	0.09	0.49	1.63	0.23	5.38	17.85	2.54	3.32	0.47	0.14
SC82	0.05	0.16	0.17	1.41	3.10	3.39	28.11	1.10	9.08	8.28
TN82	3.84	4.04	1.49	1.81	1.05	0.39	0.47	0.37	0.45	1.21
TX82	82.67	199.87	27.57	40.17	2.42	0.33	0.49	0.14	0.20	1.46
VA82	3.32	2.04	1.66	1.30	0.62	0.50	0.39	0.81	0.64	0.78
VT82	1.83	3.56	6.35	5.71	1.94	3.46	3.12	1.78	1.60	0.90
WI82	4.31	5.89	0.22	1.13	1.37	0.05	0.26	0.04	0.19	5.18

Table 5.14 States 1987: Output Price Ratios of Efficient Units
VRS Envelopment/ Global Orientation/ Invariant Pricing

DMU	μ_1/μ_2	μ_1/μ_3	μ_1/μ_4	μ_1/μ_5	μ_2/μ_3	μ_2/μ_4	μ_2/μ_5	μ_3/μ_4	μ_3/μ_5	μ_4/μ_5
AL87	0.46	7.11	0.57	0.42	15.38	1.24	0.92	0.08	0.06	0.74
AZ87	4.24	3.78	1.40	0.85	0.89	0.33	0.20	0.37	0.23	0.61
CA87	273.3	328.5	73.20	10.30	1.20	0.27	0.04	0.22	0.03	0.14
CT87	30.62	63.69	10.06	11.12	2.08	0.33	0.36	0.16	0.18	1.11
DC87	1.45	10.74	3.53	3.10	7.41	2.43	2.14	0.33	0.29	0.88
DE87	6.12	5.98	4.24	26.07	0.98	0.69	4.26	0.71	4.36	6.15
FL87	147.1	109.1	31.53	34.16	0.74	0.21	0.23	0.29	0.31	1.08
GA87	4.15	2.28	0.98	1.02	0.55	0.24	0.25	0.43	0.45	1.05
HI87	5.75	174.2	85.39	80.27	30.30	14.85	13.96	0.49	0.46	0.94
IN87	0.46	1.44	0.63	1.12	3.15	1.37	2.44	0.44	0.78	1.78
KS87	0.21	212.6	0.38	36.79	1027.1	1.82	177.7	0.00	0.17	97.46
KY87	0.47	5.64	0.76	0.83	12.11	1.63	1.77	0.14	0.15	1.09
LA87	0.50	3.13	3.22	0.75	6.24	6.42	1.49	1.03	0.24	0.23
MA87	20.60	252.8	21.47	47.24	12.27	1.04	2.29	0.09	0.19	2.20
MI87	3.98	2.06	1.93	0.18	0.52	0.49	0.05	0.94	0.09	0.09
MS87	4.63	8.14	1.84	1.22	1.76	0.40	0.26	0.23	0.15	0.66
NC87	4.21	1.63	2.02	1.28	0.39	0.48	0.30	1.24	0.79	0.63
NH87	3.99	5.68	0.83	1.22	1.42	0.21	0.31	0.15	0.22	1.47
NV87	238.0	143.4	66.84	58.94	0.60	0.28	0.25	0.47	0.41	0.88
OR87	4.08	5.38	0.86	0.19	1.32	0.21	0.05	0.16	0.04	0.22
RI87	2.79	7.21	0.57	0.12	2.58	0.21	0.04	0.08	0.02	0.20
SD87	471.9	336.1	87.29	93.40	0.71	0.19	0.20	0.26	0.28	1.07
VA87	188.2	214.3	11.27	30.82	1.14	0.06	0.16	0.05	0.14	2.74
VT87	3.67	6.22	0.65	0.24	1.70	0.18	0.07	0.10	0.04	0.38

Table 5.15 States Study: Limit-Price-Ratio Ranges

	1982		1987	
	MIN	MAX	MIN	MAX
ν_1/ν_2	0.261	2.103	0.024	317.844
ν_1/ν_3	0.011	66.440	0.006	2198.072
ν_1/ν_4	0.056	37.413	0.157	1217.711
ν_2/ν_3	0.018	54.430	0.010	83.917
ν_2/ν_4	0.016	43.671	0.404	30.000
ν_3/ν_4	0.123	354.469	0.202	257.202
μ_1/μ_2	0.011	82.672	0.207	471.985
μ_1/μ_3	0.155	199.886	1.438	336.071
μ_1/μ_4	0.482	328.068	0.377	87.291
μ_1/μ_5	0.232	122.722	0.181	93.402
μ_2/μ_3	0.582	27.970	0.387	1027.102
μ_2/μ_4	0.051	221.272	0.060	6.417
μ_2/μ_5	0.106	151.729	0.042	177.690
μ_3/μ_4	0.037	12.679	0.002	1.028
μ_3/μ_5	0.025	9.081	0.016	4.358
μ_4/μ_5	0.142	8.284	0.094	97.461

Table 5.16 States 1982: Total Waste and Substitutions
VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AL82	-0.05	0.10	0.00	0.00	0.12	0.51	2.53	0.36	0.18
AR82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AZ82	-0.02	0.10	0.35	0.01	0.14	0.45	3.13	0.23	0.61
CA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO82	1.03	0.60	1.02	0.62	0.02	-0.08	4.08	0.01	0.20
CT82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DC82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DE82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FL82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GA82	0.01	0.02	1.60	0.49	-0.00	-0.07	2.03	0.02	0.07
HI82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ID82	3.05	1.78	0.28	0.06	0.13	0.52	2.52	2.77	0.19
IL82	0.04	0.01	0.95	0.53	0.04	0.00	-0.04	0.01	0.27
IN82	0.01	0.01	0.83	0.20	0.11	0.00	0.00	-0.07	0.18
IO82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KS82	1.61	0.79	0.45	0.27	0.00	-0.01	0.00	1.65	0.06
KY82	0.09	0.05	0.57	0.38	0.21	-0.02	0.43	-0.10	0.15
LA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MD82	0.21	0.10	1.39	0.62	0.00	-0.19	5.40	-0.13	1.06
ME82	0.38	0.49	0.34	0.24	0.04	0.13	2.00	0.03	0.07
MI82	0.05	0.02	2.05	0.61	0.07	0.05	0.12	0.00	0.02
MN82	0.11	0.07	1.01	0.62	0.06	0.02	0.04	-0.02	0.01
MO82	0.06	0.05	0.67	0.44	0.07	-0.00	0.00	0.28	-0.04
MS82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MT82	3.30	5.82	0.97	1.90	0.40	1.30	4.49	0.72	0.48
NC82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ND82	12.70	7.52	0.15	0.54	0.12	0.45	1.42	0.18	0.19
NE82	0.11	-0.01	0.06	0.20	0.00	-0.02	0.29	1.86	0.03
NH82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NJ82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NM82	1.40	1.30	-0.02	-0.08	0.09	0.22	2.53	0.43	0.29
NV82	16.18	9.42	0.61	0.50	0.03	-0.20	8.89	0.60	0.95
NY82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OH82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OK82	0.23	0.21	1.16	1.15	0.05	0.15	1.16	0.09	0.06
OR82	0.30	0.16	1.54	0.82	0.24	-0.10	3.33	0.50	-0.12
PA82	0.01	0.01	0.94	0.14	0.04	0.48	0.13	0.12	0.35
RI82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SC82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SD82	4.84	2.89	-0.05	-0.27	0.14	0.47	1.05	2.01	0.15
TN82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TX82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UT82	0.00	0.19	-0.00	0.00	0.14	0.61	1.81	1.64	0.27
VA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
VT82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WA82	0.26	0.26	2.50	1.51	0.09	0.31	1.01	0.12	0.22
WI82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WV82	-0.12	0.25	0.84	-0.10	0.22	1.00	2.10	0.53	0.62
WY82	19.98	14.30	-0.48	2.04	0.29	0.83	4.84	0.33	0.42

Table 5.17 States 1987: Total Waste and Substitutions
VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK87	1.99	3.88	1.14	3.57	-0.03	0.01	6.02	0.00	0.92
AL87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AR87	0.00	0.85	-0.01	0.46	0.10	0.23	1.68	0.04	0.01
AZ87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO87	0.52	1.12	0.95	1.29	0.23	0.96	2.55	0.25	0.21
CT87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DC87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DE87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FL87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HI87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ID87	0.00	1.39	-0.02	1.17	0.40	0.69	3.08	0.42	0.16
IL87	0.83	1.60	0.92	1.21	0.10	0.39	0.60	0.14	0.10
IN87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
IO87	0.66	2.39	1.34	0.95	0.12	0.21	0.23	0.07	0.04
KS87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KY87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MD87	0.37	0.34	0.78	0.48	0.00	0.06	3.16	0.02	0.44
ME87	1.02	1.29	0.63	0.51	0.13	0.49	2.06	0.24	0.15
MI87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MN87	0.16	1.01	0.40	0.57	0.00	0.23	0.19	0.00	-0.01
MO87	0.53	1.44	1.16	0.74	0.09	0.34	0.46	0.12	0.08
MS87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MT87	0.03	1.11	0.03	3.14	0.56	1.99	6.76	0.74	0.48
NC87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ND87	0.00	0.60	-0.01	1.72	0.19	0.10	8.33	1.51	0.56
NE87	0.43	1.68	-0.03	2.15	0.18	0.81	2.91	0.28	0.13
NH87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NJ87	0.35	0.79	0.08	0.95	0.00	0.01	2.58	0.18	0.25
NM87	0.00	2.27	-0.03	1.51	0.38	1.69	7.14	0.51	0.67
NV87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NY87	0.77	1.79	-0.03	0.68	0.11	0.38	3.44	0.32	0.44
OH87	0.66	0.92	1.57	0.45	0.10	0.00	-0.16	0.00	0.06
OK87	2.14	2.44	-0.04	1.77	0.40	2.04	2.13	0.48	0.37
OR87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PA87	1.23	1.84	0.18	1.20	0.28	1.04	1.62	0.36	0.45
RI87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SC87	0.47	0.26	0.06	0.22	0.10	0.38	1.24	0.26	0.09
SD87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TN87	0.09	0.67	0.17	0.17	0.11	0.45	0.81	0.23	0.11
TX87	1.22	2.31	-0.04	1.56	0.17	0.90	2.06	0.28	0.16
UT87	0.00	1.44	-0.02	0.30	0.39	1.26	3.15	0.45	0.26
VA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
VT87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WA87	0.60	1.48	1.64	1.77	0.00	0.62	0.71	0.32	-0.07
WI87	0.27	1.20	0.65	0.56	0.07	0.11	0.20	0.04	0.03
WV87	0.43	1.17	0.61	0.13	0.38	2.21	1.48	0.75	0.35
WY87	2.34	2.56	0.34	3.84	0.45	2.25	4.55	0.47	0.37

Table 5.18 States 1982: Efficiency Prices
VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK82	0.01	0.01	0.07	0.00	0.39	0.49	0.02	0.00	0.00
AL82	0.01	0.01	0.09	0.07	0.20	0.10	0.00	0.09	0.14
AR82	0.01	0.02	0.14	0.00	0.00	0.23	0.01	0.00	0.00
AZ82	0.01	0.01	0.02	0.15	0.10	0.17	0.01	0.07	0.01
CA82	0.07	0.08	0.11	0.00	0.55	0.08	0.00	0.03	0.00
CO82	0.00	0.00	0.10	0.02	0.44	0.09	0.00	0.03	0.00
CT82	0.01	0.01	0.12	0.00	0.48	0.02	0.04	0.03	0.00
DC82	0.01	0.00	0.02	0.15	0.00	0.18	0.01	0.00	0.00
DE82	0.01	0.01	0.10	0.00	0.09	0.02	0.04	0.19	0.02
FL82	0.01	0.02	0.14	0.00	0.25	0.17	0.01	0.00	0.00
GA82	0.01	0.00	0.14	0.00	0.24	0.19	0.01	0.00	0.00
HI82	0.01	0.00	0.13	0.00	0.21	0.16	0.01	0.00	0.00
ID82	0.00	0.00	0.09	0.03	0.06	0.21	0.01	0.00	0.01
IL82	0.16	0.63	0.02	0.12	0.01	0.19	0.07	0.01	0.01
IN82	0.95	0.45	0.01	0.03	0.40	2.47	0.82	0.83	0.10
IO82	0.01	0.01	0.10	0.00	0.60	0.70	0.05	0.00	0.01
KS82	0.00	0.00	0.09	0.03	0.10	0.20	0.01	0.00	0.00
KY82	0.00	0.01	0.08	0.05	0.01	0.19	0.01	0.01	0.00
LA82	0.01	0.01	0.10	0.00	0.25	0.06	0.00	0.06	0.01
MA82	0.01	0.02	0.13	0.00	0.52	0.07	0.00	0.00	0.01
MD82	0.01	0.01	0.02	0.15	0.14	0.16	0.01	0.07	0.01
ME82	0.00	0.00	0.10	0.04	0.37	0.05	0.00	0.10	0.10
MI82	0.04	0.16	0.02	0.13	0.05	0.12	0.02	0.10	0.01
MN82	0.01	0.00	0.08	0.05	0.00	0.20	0.01	0.01	0.01
MO82	0.01	0.00	0.09	0.04	0.00	0.20	0.01	0.00	0.01
MS82	0.00	0.00	0.18	0.00	0.55	0.01	0.01	0.13	0.02
MT82	0.00	0.00	0.10	0.00	0.30	0.16	0.01	0.01	0.07
NC82	0.02	0.01	0.15	0.00	0.45	0.02	0.03	0.03	0.00
ND82	0.00	0.00	0.12	0.00	0.14	0.19	0.01	0.01	0.00
NE82	0.00	0.01	0.12	0.00	0.03	0.20	0.01	0.00	0.00
NH82	0.02	0.01	0.28	0.26	0.00	0.24	0.01	0.00	0.00
NJ82	0.01	0.01	0.12	0.00	0.56	0.10	0.00	0.00	0.01
NM82	0.00	0.00	0.13	0.03	0.00	0.23	0.01	0.00	0.00
NV82	0.00	0.00	0.10	0.02	0.47	0.10	0.00	0.00	0.00
NY82	0.04	0.04	0.12	0.00	0.27	0.14	0.01	0.00	0.00
OH82	0.13	0.51	0.08	0.01	0.04	0.97	0.23	0.07	0.03
OK82	0.00	0.00	0.11	0.00	0.37	0.13	0.01	0.01	0.01
OR82	0.01	0.01	0.02	0.15	0.03	0.15	0.01	0.02	0.12
PA82	0.54	0.26	0.02	0.12	0.21	0.04	0.06	0.01	0.01
RI82	0.02	0.02	0.18	0.00	0.74	0.02	0.00	0.00	0.02
SC82	0.02	0.02	0.14	0.00	0.09	0.01	0.02	0.19	0.02
SD82	0.00	0.00	0.11	0.03	0.00	0.22	0.01	0.00	0.00
TN82	0.05	0.18	0.03	0.21	0.08	0.07	0.01	0.17	0.02
TX82	0.01	0.01	0.12	0.00	0.00	0.19	0.01	0.00	0.00
UT82	0.00	0.00	0.12	0.03	0.00	0.23	0.01	0.00	0.00
VA82	0.00	0.00	0.13	0.00	0.37	0.11	0.00	0.02	0.00
VT82	0.01	0.01	0.02	0.19	0.06	0.13	0.01	0.12	0.02
WA82	0.00	0.00	0.07	0.03	0.25	0.15	0.01	0.01	0.00
WI82	0.68	0.79	0.02	0.02	0.28	2.75	0.52	0.17	1.19
WV82	0.01	0.01	0.02	0.14	0.05	0.19	0.01	0.05	0.01
WY82	0.00	0.00	0.11	0.00	0.53	0.08	0.00	0.00	0.00

Table 5.19 States 1987: Efficiency Prices
VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK87	0.00	0.00	0.09	0.00	0.34	0.04	0.00	0.04	0.00
AL87	0.23	0.00	0.00	0.00	0.07	0.02	0.05	0.06	0.01
AR87	0.56	0.00	0.42	0.00	0.06	0.01	0.01	0.16	0.29
AZ87	0.32	0.00	0.00	0.00	0.07	0.12	0.00	0.07	0.00
CA87	0.22	0.00	0.00	0.00	0.83	0.00	0.01	0.01	0.05
CO87	0.00	0.00	0.09	0.00	0.39	0.05	0.00	0.04	0.01
CT87	0.16	0.01	0.00	0.00	0.76	0.00	0.00	0.03	0.01
DC87	0.14	0.00	0.03	0.00	0.74	0.00	0.00	0.01	0.01
DE87	0.17	0.08	0.00	0.00	0.85	0.02	0.04	0.12	0.01
FL87	0.03	0.00	0.13	0.00	0.62	0.00	0.00	0.01	0.01
GA87	0.21	0.20	0.00	0.01	0.05	0.01	0.01	0.14	0.30
HI87	0.00	0.00	0.14	0.00	0.38	0.06	0.00	0.01	0.00
ID87	0.16	0.00	0.06	0.00	0.03	0.13	0.00	0.02	0.12
IL87	0.00	0.00	0.09	0.00	0.37	0.04	0.00	0.04	0.00
IN87	0.14	0.00	0.03	0.00	0.06	0.18	0.04	0.06	0.01
IO87	0.07	0.00	0.06	0.00	0.02	0.11	0.01	0.04	0.05
KS87	0.25	0.00	0.00	0.00	0.03	0.13	0.02	0.07	0.00
KY87	0.21	0.00	0.00	0.00	0.03	0.14	0.00	0.07	0.00
LA87	0.03	0.01	0.07	0.00	0.02	0.08	0.00	0.04	0.00
MA87	0.00	0.00	0.10	0.00	0.36	0.04	0.00	0.03	0.00
MD87	0.10	0.00	0.05	0.00	0.51	0.01	0.00	0.01	0.01
ME87	0.00	0.00	0.10	0.00	0.36	0.04	0.00	0.06	0.08
MI87	0.25	0.00	0.00	0.00	0.31	0.01	0.01	0.08	0.07
MN87	0.17	0.00	0.02	0.00	0.57	0.01	0.00	0.08	0.12
MO87	0.08	0.00	0.06	0.00	0.02	0.12	0.01	0.04	0.05
MS87	0.35	0.00	0.40	0.00	0.03	0.16	0.00	0.09	0.00
MT87	0.06	0.00	0.07	0.00	0.03	0.14	0.00	0.02	0.12
NC87	0.03	0.01	0.11	0.00	0.14	0.11	0.01	0.04	0.00
ND87	0.05	0.00	0.16	0.00	0.03	0.15	0.00	0.02	0.00
NE87	0.00	0.00	0.14	0.00	0.16	0.11	0.00	0.02	0.03
NH87	0.14	0.00	0.03	0.00	0.43	0.00	0.01	0.01	0.06
NJ87	0.00	0.00	0.09	0.00	0.39	0.03	0.00	0.01	0.00
NM87	0.04	0.00	0.15	0.00	0.52	0.06	0.00	0.04	0.01
NV87	0.18	0.00	0.04	0.00	0.60	0.00	0.00	0.01	0.01
NY87	0.00	0.00	0.11	0.00	0.43	0.03	0.00	0.01	0.01
OH87	0.15	0.01	0.01	0.02	0.04	0.10	0.03	0.04	0.01
OK87	0.00	0.00	0.11	0.00	0.24	0.09	0.00	0.05	0.04
OR87	0.24	0.00	0.00	0.00	0.06	0.01	0.01	0.03	0.33
PA87	0.00	0.00	0.10	0.00	0.40	0.05	0.01	0.04	0.00
RI87	0.14	0.00	0.03	0.00	0.31	0.01	0.01	0.01	0.12
SC87	0.00	0.00	0.12	0.00	0.03	0.10	0.00	0.07	0.13
SD87	0.37	0.00	0.41	0.00	0.08	0.15	0.00	0.08	0.00
TN87	0.06	0.00	0.08	0.00	0.10	0.11	0.00	0.05	0.06
TX87	0.00	0.00	0.10	0.00	0.21	0.08	0.00	0.04	0.03
UT87	0.07	0.00	0.11	0.00	0.03	0.14	0.00	0.04	0.10
VA87	0.14	0.00	0.04	0.00	0.60	0.01	0.01	0.01	0.01
VT87	0.21	0.01	0.00	0.00	0.06	0.02	0.01	0.08	0.35
WA87	0.22	0.00	0.00	0.00	0.41	0.01	0.00	0.02	0.22
WI87	0.17	0.00	0.02	0.00	0.03	0.05	0.02	0.03	0.19
WV87	0.00	0.00	0.09	0.00	0.24	0.09	0.01	0.04	0.00
WY87	0.00	0.00	0.09	0.00	0.27	0.09	0.00	0.06	0.05

Table 5.20 States 1982: Output Price Ratios
VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	μ_1/μ_2	μ_1/μ_3	μ_1/μ_4	μ_1/μ_5	μ_2/μ_3	μ_2/μ_4	μ_2/μ_5	μ_3/μ_4	μ_3/μ_5	μ_4/μ_5
AK82	0.81	22.62	178.96	122.71	27.97	221.26	151.71	7.91	5.42	0.69
AL82	2.10	56.91	2.11	1.42	27.09	1.00	0.68	0.04	0.03	0.68
AR82	0.01	0.31	2.43	1.67	27.97	221.31	151.73	7.91	5.43	0.69
AZ82	0.59	16.51	1.43	11.83	27.97	2.42	20.03	0.09	0.72	8.28
CA82	7.15	199.88	22.32	122.71	27.97	3.12	17.17	0.11	0.61	5.50
CO82	5.09	142.47	13.79	114.21	27.97	2.71	22.42	0.10	0.80	8.28
CT82	23.22	13.51	14.81	122.73	0.58	0.64	5.29	1.10	9.08	8.28
DC82	0.01	0.31	2.44	1.67	27.97	221.39	151.73	7.92	5.43	0.69
DE82	4.09	2.38	0.48	3.99	0.58	0.12	0.98	0.20	1.68	8.28
FL82	1.48	41.47	327.96	122.75	27.97	221.20	82.79	7.91	2.96	0.37
GA82	1.24	34.74	274.80	122.75	27.97	221.25	98.80	7.91	3.53	0.45
HI82	1.32	36.92	292.06	122.76	27.97	221.22	92.98	7.91	3.33	0.42
ID82	0.27	7.44	58.89	8.36	27.97	221.27	31.42	7.91	1.12	0.14
IL82	0.06	0.16	1.97	1.41	2.48	31.49	22.55	12.68	9.08	0.72
IN82	0.16	0.49	0.48	3.99	3.02	2.99	24.76	0.99	8.20	8.28
IO82	0.86	13.51	171.40	122.36	15.64	198.31	142.03	12.68	9.08	0.72
KS82	0.52	13.02	115.70	79.36	24.90	221.25	151.75	8.89	6.10	0.69
KY82	0.03	0.74	0.48	3.99	27.97	18.32	151.77	0.66	5.43	8.29
LA82	4.14	115.77	4.28	35.49	27.97	1.04	8.57	0.04	0.31	8.28
MA82	7.15	199.86	328.04	46.59	27.97	45.90	6.52	1.64	0.23	0.14
MD82	0.84	23.57	1.91	15.81	27.97	2.27	18.77	0.08	0.67	8.28
ME82	7.98	99.16	3.67	3.59	12.43	0.46	0.45	0.04	0.04	0.98
MI82	0.40	2.58	0.48	3.99	6.53	1.22	10.10	0.19	1.55	8.28
MN82	0.02	0.31	0.48	0.52	16.09	25.39	27.58	1.58	1.71	1.09
MO82	0.01	0.21	1.70	0.24	19.32	154.67	21.97	8.01	1.14	0.14
MS82	82.67	113.95	4.22	34.93	1.38	0.05	0.42	0.04	0.31	8.28
MT82	1.83	51.13	29.92	4.25	27.97	16.37	2.32	0.59	0.08	0.14
NC82	23.22	13.51	14.81	122.73	0.58	0.64	5.29	1.10	9.08	8.29
ND82	0.74	20.60	10.53	87.27	27.97	14.30	118.49	0.51	4.24	8.29
NE82	0.12	3.48	27.51	18.87	27.97	221.20	151.71	7.91	5.42	0.69
NH82	0.01	0.31	2.43	1.67	27.97	221.29	151.77	7.91	5.43	0.69
NJ82	5.84	163.26	328.04	122.72	27.97	56.19	21.02	2.01	0.75	0.37
NM82	0.01	0.31	2.43	1.67	27.97	221.24	151.72	7.91	5.42	0.69
NV82	5.04	141.01	328.02	122.71	27.97	65.07	24.34	2.33	0.87	0.37
NY82	2.04	56.05	328.07	122.71	27.97	163.72	61.24	5.85	2.19	0.37
OH82	0.04	0.16	0.48	1.41	4.31	13.39	39.11	3.11	9.08	2.92
OK82	2.88	80.48	48.07	58.91	27.97	16.71	20.47	0.60	0.73	1.23
OR82	0.18	4.95	1.63	0.23	27.97	9.23	1.31	0.33	0.05	0.14
PA82	6.01	3.50	44.35	31.76	0.58	7.38	5.29	12.68	9.08	0.72
RI82	30.98	199.88	328.11	46.59	6.45	10.59	1.50	1.64	0.23	0.14
SC82	9.45	5.50	0.48	3.99	0.58	0.05	0.42	0.09	0.73	8.28
SD82	0.01	0.31	2.44	1.67	27.97	221.35	151.72	7.91	5.42	0.69
TN82	1.12	13.03	0.48	3.99	11.61	0.43	3.56	0.04	0.31	8.28
TX82	0.01	0.31	0.68	1.67	27.97	61.71	151.67	2.21	5.42	2.46
UT82	0.02	0.45	3.54	2.43	27.97	221.30	151.70	7.91	5.42	0.69
VA82	3.41	95.50	15.04	122.73	27.97	4.40	35.95	0.16	1.29	8.16
VT82	0.47	13.03	0.48	3.99	27.97	1.04	8.57	0.04	0.31	8.28
WA82	1.70	47.58	18.00	122.71	27.97	10.58	72.12	0.38	2.58	6.82
WI82	0.10	0.54	1.63	0.23	5.33	16.26	2.31	3.05	0.43	0.14
WV82	0.25	6.91	0.88	7.32	27.97	3.58	29.62	0.13	1.06	8.28
WY82	6.50	181.74	328.10	122.08	27.97	50.50	18.89	1.81	0.68	0.37

Table 5.21 States 1987: Output Price Ratios
VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	μ_1/μ_2	μ_1/μ_3	μ_1/μ_4	μ_1/μ_5	μ_2/μ_3	μ_2/μ_4	μ_2/μ_5	μ_3/μ_4	μ_3/μ_5	μ_4/μ_5
AK87	8.18	336.10	9.78	93.41	41.08	1.20	11.42	0.03	0.28	9.55
AL87	3.72	1.44	1.11	6.27	0.39	0.30	1.69	0.77	4.36	5.65
AR87	4.89	12.84	0.38	0.21	2.63	0.08	0.04	0.03	0.02	0.55
AZ87	0.53	21.43	0.96	93.34	40.78	1.82	177.57	0.05	4.36	97.40
CA87	407.28	157.62	87.29	17.11	0.39	0.21	0.04	0.55	0.11	0.20
CO87	8.54	335.95	9.11	44.14	39.32	1.07	5.17	0.03	0.13	4.85
CT87	471.94	182.65	28.32	93.40	0.39	0.06	0.20	0.16	0.51	3.30
DC87	472.09	336.01	87.29	93.40	0.71	0.19	0.20	0.26	0.28	1.07
DE87	50.60	19.58	7.12	85.33	0.39	0.14	1.69	0.36	4.36	11.98
FL87	471.96	336.02	87.29	93.40	0.71	0.19	0.20	0.26	0.28	1.07
GA87	4.31	10.65	0.40	0.18	2.47	0.09	0.04	0.04	0.02	0.45
HI87	6.16	336.14	39.52	93.40	54.58	6.42	15.17	0.12	0.28	2.36
ID87	0.21	13.38	1.33	0.21	64.61	6.42	1.03	0.10	0.02	0.16
IL87	8.62	335.96	9.73	88.05	38.99	1.13	10.22	0.03	0.26	9.05
IN87	0.31	1.44	0.95	6.27	4.58	3.04	19.96	0.66	4.36	6.58
IO87	0.21	2.60	0.64	0.45	12.54	3.09	2.19	0.25	0.18	0.71
KS87	0.21	1.44	0.38	6.27	6.95	1.82	30.27	0.26	4.36	16.62
KY87	0.21	188.64	0.38	36.79	911.28	1.82	177.72	0.00	0.20	97.47
LA87	0.21	212.82	0.43	36.79	1028.20	2.05	177.73	0.00	0.17	86.52
MA87	9.23	336.19	11.14	93.41	36.43	1.21	10.12	0.03	0.28	8.39
MD87	110.64	336.06	44.86	93.40	3.04	0.41	0.84	0.13	0.28	2.08
ME87	9.37	277.28	6.40	4.44	29.60	0.68	0.47	0.02	0.02	0.69
MI87	64.97	25.14	3.90	4.72	0.39	0.06	0.07	0.16	0.19	1.21
MN87	110.06	288.87	6.96	4.62	2.63	0.06	0.04	0.02	0.02	0.66
MO87	0.21	2.60	0.64	0.45	12.54	3.09	2.19	0.25	0.18	0.71
MS87	0.21	8.44	0.38	36.78	40.77	1.82	177.70	0.05	4.36	97.47
MT87	0.21	14.41	1.33	0.23	69.59	6.42	1.11	0.09	0.02	0.17
NC87	1.28	21.43	3.54	93.39	16.70	2.76	72.75	0.17	4.36	26.40
ND87	0.21	212.98	1.33	36.78	1028.90	6.42	177.70	0.01	0.17	27.69
NE87	1.39	335.75	8.93	5.38	241.31	6.42	3.87	0.03	0.02	0.60
NH87	159.65	69.39	71.34	6.71	0.44	0.45	0.04	1.03	0.10	0.09
NJ87	12.57	336.01	80.64	93.40	26.74	6.42	7.43	0.24	0.28	1.16
NM87	8.05	336.06	16.67	93.40	41.75	1.70	11.60	0.04	0.28	6.83
NV87	471.80	336.13	87.29	93.40	0.71	0.19	0.20	0.26	0.28	1.07
NY87	13.10	336.07	84.05	93.40	25.66	6.42	7.13	0.25	0.28	1.11
OH87	0.42	1.44	0.98	6.27	3.42	2.33	14.92	0.68	4.36	6.42
OK87	2.61	336.09	5.23	6.33	128.63	2.00	2.42	0.02	0.02	1.21
OR87	4.31	11.31	1.93	0.18	2.63	0.45	0.04	0.17	0.02	0.09
PA87	8.61	80.17	9.44	93.40	9.31	1.10	10.84	0.12	1.17	9.90
RI87	60.43	26.28	27.00	2.54	0.44	0.45	0.04	1.03	0.10	0.09
SC87	0.25	12.11	0.38	0.19	47.86	1.49	0.77	0.03	0.02	0.51
SD87	0.53	336.45	0.96	93.41	640.07	1.82	177.71	0.00	0.28	97.47
TN87	0.86	24.89	2.08	1.59	28.96	2.42	1.85	0.08	0.06	0.76
TX87	2.61	336.03	5.23	6.33	128.61	2.00	2.42	0.02	0.02	1.21
UT87	0.21	19.45	0.85	0.31	93.95	4.11	1.50	0.04	0.02	0.37
VA87	133.24	84.91	87.29	93.40	0.64	0.66	0.70	1.03	1.10	1.07
VT87	4.31	11.31	0.76	0.18	2.63	0.18	0.04	0.07	0.02	0.24
WA87	45.13	118.46	20.16	1.90	2.63	0.45	0.04	0.17	0.02	0.09
WI87	0.62	1.83	1.13	0.18	2.96	1.83	0.29	0.62	0.10	0.16
WV87	2.70	21.36	5.49	93.07	7.92	2.04	34.51	0.26	4.36	16.95
WY87	3.22	336.09	4.99	5.83	104.48	1.55	1.81	0.02	0.02	1.17

Table 5.22 States 1982: Input Price Ratios and Efficiency Scores
VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	ν_1/ν_2	ν_1/ν_3	ν_1/ν_4	ν_2/ν_3	ν_2/ν_4	ν_3/ν_4	θ	ϕ	E
AK82	0.86	0.11	37.43	0.12	43.70	354.67	1.00	1.00	1.00
AL82	2.10	0.15	0.19	0.07	0.09	1.26	1.00	1.14	0.88
AR82	0.26	0.03	11.39	0.12	43.64	354.25	1.00	1.00	1.00
AZ82	2.10	0.64	0.08	0.30	0.04	0.12	0.99	1.13	0.88
CA82	0.86	0.60	37.41	0.70	43.66	62.86	1.00	1.00	1.00
CO82	0.61	0.01	0.06	0.02	0.09	5.09	0.89	1.02	0.87
CT82	0.86	0.11	37.40	0.12	43.65	354.32	1.00	1.00	1.00
DC82	2.10	0.46	0.06	0.22	0.03	0.12	1.00	1.00	1.00
DE82	0.86	0.11	37.43	0.12	43.69	354.64	1.00	1.00	1.00
FL82	0.26	0.03	11.39	0.12	43.64	354.18	1.00	1.00	1.00
GA82	2.10	0.04	14.26	0.02	6.78	354.25	0.98	1.00	0.98
HI82	2.10	0.04	13.42	0.02	6.38	354.38	1.00	1.00	1.00
ID82	0.93	0.02	0.06	0.02	0.06	3.34	0.96	1.14	0.85
IL82	0.26	10.89	1.34	41.71	5.13	0.12	0.91	1.00	0.91
IN82	2.10	66.44	37.41	31.59	17.79	0.56	0.97	1.00	0.97
IO82	0.86	0.11	37.39	0.12	43.65	354.31	1.00	1.00	1.00
KS82	1.21	0.02	0.06	0.02	0.05	2.58	0.95	1.00	0.95
KY82	0.43	0.04	0.06	0.09	0.13	1.41	0.94	1.00	0.94
LA82	0.86	0.11	37.40	0.12	43.65	354.31	1.00	1.00	1.00
MA82	0.86	0.11	37.40	0.12	43.66	354.38	1.00	1.00	1.00
MD82	0.89	0.46	0.06	0.51	0.06	0.12	0.88	1.00	0.88
ME82	1.30	0.02	0.06	0.02	0.04	2.40	0.95	1.04	0.92
MI82	0.26	2.58	0.32	9.87	1.21	0.12	0.88	1.01	0.87
MN82	2.10	0.08	0.15	0.04	0.07	1.83	0.89	1.00	0.89
MO82	2.10	0.07	0.17	0.03	0.08	2.52	0.92	1.00	0.92
MS82	0.61	0.01	3.90	0.02	6.38	354.47	1.00	1.00	1.00
MT82	0.61	0.01	0.90	0.02	6.37	354.15	0.89	1.40	0.64
NC82	2.10	0.11	37.45	0.05	17.81	354.83	1.00	1.00	1.00
ND82	0.61	0.01	3.90	0.02	6.38	354.48	0.95	1.11	0.85
NE82	0.26	0.01	4.40	0.05	16.86	354.72	0.99	1.00	0.99
NH82	2.10	0.05	0.06	0.03	0.03	1.06	1.00	1.00	1.00
NJ82	0.86	0.11	37.45	0.12	43.71	354.79	1.00	1.00	1.00
NM82	0.75	0.01	0.06	0.02	0.08	4.18	1.00	1.07	0.93
NV82	0.61	0.01	0.06	0.02	0.09	5.09	0.89	1.03	0.87
NY82	0.86	0.32	37.40	0.37	43.65	117.56	1.00	1.00	1.00
OH82	0.26	1.70	11.40	6.51	43.67	6.71	1.00	1.00	1.00
OK82	1.85	0.03	11.78	0.02	6.39	354.67	0.87	1.04	0.83
OR82	1.04	0.46	0.06	0.44	0.05	0.12	0.85	1.00	0.85
PA82	2.10	36.15	4.45	17.19	2.11	0.12	0.96	1.04	0.93
RI82	0.86	0.11	37.39	0.12	43.65	354.27	1.00	1.00	1.00
SC82	0.86	0.11	37.44	0.12	43.70	354.69	1.00	1.00	1.00
SD82	0.79	0.01	0.06	0.02	0.07	3.96	1.00	1.11	0.90
TN82	0.26	1.86	0.23	7.14	0.88	0.12	1.00	1.00	1.00
TX82	2.10	0.11	37.38	0.05	17.78	354.16	1.00	1.00	1.00
UT82	1.11	0.02	0.09	0.02	0.08	4.35	1.00	1.16	0.86
VA82	0.73	0.01	4.66	0.02	6.39	354.83	1.00	1.00	1.00
VT82	2.10	0.46	0.06	0.22	0.03	0.12	1.00	1.00	1.00
WA82	1.75	0.03	0.07	0.02	0.04	2.18	0.77	1.08	0.72
WI82	0.86	46.63	37.41	54.43	43.67	0.80	1.00	1.00	1.00
WV82	2.10	0.61	0.08	0.29	0.04	0.12	1.00	1.24	0.80
WY82	0.61	0.01	3.90	0.02	6.39	354.69	1.00	1.24	0.81

Table 5.23 States 1987: Input Price Ratios and Efficiency Scores
VRS Envelopment/ Frontier Model: Limit-Price-Ratios Derived from Limit Price Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	ν_1/ν_2	ν_1/ν_3	ν_1/ν_4	ν_2/ν_3	ν_2/ν_4	ν_3/ν_4	θ	ϕ	E
AK87	0.60	0.01	1.54	0.01	2.57	256.98	0.90	1.00	0.90
AL87	319.89	2206.52	1216.57	6.94	3.83	0.55	1.00	1.00	1.00
AR87	134.82	1.35	346.77	0.01	2.57	257.23	1.00	1.03	0.98
AZ87	317.84	2196.39	1215.56	6.91	3.82	0.55	1.00	1.00	1.00
CA87	317.76	2192.82	1216.90	6.91	3.83	0.56	1.00	1.15	1.00
CO87	0.60	0.01	1.54	0.01	2.57	257.20	0.91	1.00	0.80
CT87	26.19	2189.69	785.78	83.60	30.00	0.36	1.00	1.00	1.00
DC87	40.59	4.73	1217.10	0.12	29.98	257.07	1.00	1.00	1.00
DE87	2.13	178.39	63.80	83.90	30.01	0.36	1.00	1.00	1.00
FL87	26.73	0.27	68.75	0.01	2.57	257.25	1.00	1.00	1.00
GA87	1.03	86.29	30.85	83.91	30.00	0.36	1.00	1.00	1.00
HI87	0.60	0.01	1.54	0.01	2.57	257.28	1.00	1.00	1.00
ID87	270.26	2.70	693.75	0.01	2.57	256.81	1.00	1.13	0.88
IL87	0.60	0.01	1.54	0.01	2.57	257.03	0.91	1.06	0.86
IN87	40.60	4.73	1218.59	0.12	30.02	257.39	1.00	1.00	1.00
IO87	119.75	1.20	308.78	0.01	2.58	257.67	0.88	1.03	0.85
KS87	317.80	2207.56	1217.97	6.95	3.83	0.55	1.00	1.00	1.00
KY87	317.78	2191.03	1217.24	6.90	3.83	0.56	1.00	1.00	1.00
LA87	3.68	0.43	110.17	0.12	29.98	256.99	1.00	1.00	1.00
MA87	0.60	0.01	1.55	0.01	2.58	257.44	1.00	1.00	1.00
MD87	218.61	2.19	561.42	0.01	2.57	257.77	0.93	1.01	0.92
ME87	0.60	0.01	1.54	0.01	2.57	257.16	0.93	1.09	0.85
MI87	317.65	2203.89	1214.82	6.94	3.82	0.55	1.00	1.00	1.00
MN87	317.89	7.67	1218.22	0.02	3.83	158.77	0.96	1.00	0.96
MO87	119.94	1.20	308.28	0.01	2.57	257.20	0.89	1.06	0.84
MS87	87.45	0.87	224.97	0.01	2.57	257.28	1.00	1.00	1.00
MT87	95.38	0.95	245.32	0.01	2.57	257.23	1.00	1.38	0.72
NC87	2.20	0.26	65.92	0.12	29.98	257.02	1.00	1.00	1.00
ND87	34.45	0.34	88.56	0.01	2.57	257.08	1.00	1.06	0.95
NE87	0.60	0.01	1.54	0.01	2.57	257.17	1.00	1.13	0.89
NH87	40.59	4.73	1213.56	0.12	29.90	256.33	1.00	1.00	1.00
NJ87	0.60	0.01	1.54	0.01	2.57	257.21	0.99	1.01	0.99
NM87	27.94	0.28	71.90	0.01	2.57	257.41	1.00	1.34	0.75
NV87	318.12	4.73	1215.42	0.02	3.82	256.72	1.00	1.00	1.00
NY87	0.60	0.01	1.54	0.01	2.57	256.97	1.00	1.07	0.94
OH87	17.87	13.34	7.22	0.75	0.40	0.54	0.87	1.00	0.87
OK87	0.60	0.01	1.54	0.01	2.57	257.28	1.00	1.32	0.76
OR87	318.01	2194.54	1217.97	6.90	3.83	0.56	1.00	1.00	1.00
PA87	0.60	0.01	1.54	0.01	2.57	257.31	0.98	1.19	0.83
RI87	40.60	4.74	1220.32	0.12	30.06	257.75	1.00	1.00	1.00
SC87	0.60	0.01	1.55	0.01	2.57	257.43	0.99	1.07	0.92
SD87	90.35	0.90	232.44	0.01	2.57	257.27	1.00	1.00	1.00
TN87	66.14	0.66	169.89	0.01	2.57	256.85	0.98	1.08	0.91
TX87	0.60	0.01	1.55	0.01	2.58	257.48	1.00	1.13	0.89
UT87	66.81	0.67	172.00	0.01	2.57	257.44	1.00	1.24	0.81
VA87	317.62	3.18	817.28	0.01	2.57	257.13	1.00	1.00	1.00
VT87	40.59	2187.76	1219.62	53.90	30.05	0.56	1.00	1.00	1.00
WA87	317.69	53.54	1218.70	0.17	3.84	22.76	0.86	1.00	0.86
WI87	317.80	10.05	1215.25	0.03	3.82	120.92	0.94	1.02	0.92
WV87	0.60	0.01	0.24	0.01	0.40	40.40	0.95	1.34	0.71
WY87	0.60	0.01	1.54	0.01	2.57	257.03	0.97	1.36	0.71

Table 5.24 States Study: Limit-Value-Ratio Ranges

	1982		1987	
	MIN	MAX	MIN	MAX
ν_1/ν_2	0.41	0.74	0.02	1.19
ν_1/ν_3	0.40	52.18	1.51	3.33
ν_1/ν_4	0.26	31.18	0.56	1.56
ν_2/ν_3	0.97	99.66	2.26	83.92
ν_2/ν_4	0.64	59.65	1.14	30.00
ν_3/ν_4	0.37	0.74	0.24	0.77
μ_1/μ_2	0.18	0.37	0.11	0.42
μ_1/μ_3	0.07	0.61	0.12	0.76
μ_1/μ_4	0.17	1.86	0.17	2.17
μ_1/μ_5	0.37	1.79	0.54	2.47
μ_2/μ_3	0.28	2.09	0.39	3.34
μ_2/μ_4	0.71	5.56	0.75	10.18
μ_2/μ_5	1.11	6.78	2.27	11.93
μ_3/μ_4	0.74	19.90	1.21	12.56
μ_3/μ_5	2.43	10.74	2.17	16.78
μ_4/μ_5	0.20	6.60	0.31	13.92

Table 5.25 States 1982: Total Waste and Substitutions
VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK82	25.91	10.46	2.58	2.21	-0.39	-2.21	10.63	0.47	1.05
AL82	0.59	0.39	0.86	0.35	0.07	-0.07	4.93	-0.35	0.30
AR82	1.37	0.79	0.38	0.50	0.07	-0.52	5.05	0.40	0.24
AZ82	1.71	0.99	1.35	-0.04	0.09	-0.44	8.19	-0.03	0.67
CA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO82	1.17	0.64	1.51	0.39	-0.08	-0.50	7.18	0.06	0.48
CT82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DC82	19.64	8.55	2.00	-0.75	-0.05	-1.72	10.88	0.55	1.30
DE82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FL82	0.22	0.11	-0.11	-0.03	-0.06	-0.34	9.01	0.85	0.50
GA82	0.25	0.14	0.36	0.50	-0.06	-0.25	4.37	0.20	0.17
HI82	13.65	7.00	-1.28	2.20	0.27	-0.89	11.20	0.70	1.17
ID82	6.36	3.54	0.16	1.40	0.52	-0.50	8.69	0.60	0.70
IL82	0.05	0.01	1.53	0.29	-0.06	-0.20	0.77	0.27	0.21
IN82	0.16	0.03	3.75	0.20	-0.10	-0.83	2.60	-0.30	0.23
IO82	0.93	0.26	2.39	1.24	0.17	-1.92	6.13	0.02	0.74
KS82	1.48	0.65	1.93	1.46	-0.12	-1.30	6.26	0.37	0.46
KY82	0.80	0.33	1.62	0.81	0.21	-1.11	6.72	-0.67	0.92
LA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MA82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MD82	0.74	0.37	1.36	0.46	0.05	-0.53	8.00	0.01	1.12
ME82	2.67	1.79	0.92	-0.20	-0.03	-0.10	3.58	0.00	-0.09
MI82	0.10	0.03	3.68	0.56	0.03	-0.25	0.80	-0.18	0.15
MN82	0.29	0.10	1.21	0.68	0.13	-0.63	5.00	0.09	0.35
MO82	0.21	0.04	1.16	0.75	0.13	-0.79	5.22	0.11	0.50
MS82	1.26	0.78	0.07	0.30	0.13	-0.31	5.41	-1.20	0.46
MT82	16.08	11.27	1.26	3.50	0.44	0.33	11.05	0.90	0.79
NC82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ND82	22.61	12.60	1.30	1.80	-0.12	-0.84	11.07	0.80	0.81
NE82	3.01	1.42	-0.04	2.23	0.25	-1.07	7.78	0.46	0.85
NH82	2.48	1.65	-0.70	-0.18	0.14	0.05	2.15	0.66	-0.08
NJ82	0.06	0.00	1.65	-0.05	-0.41	-0.46	2.33	0.99	-0.32
NM82	9.65	5.57	-0.83	1.49	0.34	-0.47	11.06	0.88	1.11
NV82	15.95	9.26	0.12	0.39	0.14	-0.40	11.20	0.80	1.12
NY82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OH82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OK82	1.27	0.72	2.07	1.00	-0.16	-0.54	7.21	0.41	0.43
OR82	1.04	0.54	1.26	0.90	0.39	-0.45	6.63	0.50	0.02
PA82	0.01	0.01	1.06	0.07	0.01	0.43	0.33	0.19	0.33
RI82	2.57	1.92	0.14	-0.60	-0.19	0.24	1.33	1.90	-1.32
SC82	0.50	0.39	0.31	0.00	0.06	0.24	1.86	-1.10	0.37
SD82	13.12	7.13	-0.52	1.70	0.50	-0.63	10.03	0.80	1.06
TN82	0.24	0.14	0.40	0.18	0.10	-0.24	3.43	-0.87	0.39
TX82	0.03	-0.01	-1.58	1.42	-0.05	-0.53	3.18	-1.58	0.29
UT82	3.66	2.13	1.55	0.52	0.04	-0.58	7.94	0.78	0.43
VA82	0.28	0.10	0.35	0.13	-0.01	-0.72	6.26	-0.58	0.87
VT82	6.86	3.83	1.18	-1.10	-0.08	-0.78	4.29	0.00	-0.44
WA82	0.47	0.22	3.10	1.82	0.10	-0.55	6.67	0.26	0.42
WI82	0.16	0.02	2.58	0.26	-0.04	-0.90	2.89	0.46	-0.03
WV82	3.02	1.72	3.01	0.50	0.18	-0.57	8.48	-0.17	0.97
WY82	33.79	20.19	0.96	3.11	0.02	-0.37	11.51	0.95	0.96

Table 5.26 States 1987: Total Waste and Substitutions
VRS Envelopment/ Frontier Model:Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK87	1.94	4.18	1.40	3.40	-0.04	-0.45	8.21	-0.10	1.40
AL87	-0.05	0.46	0.82	0.30	0.13	0.30	4.59	-4.40	1.20
AR87	-0.46	1.76	-0.92	1.54	0.43	1.27	3.12	-0.49	0.54
AZ87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CA87	-0.41	1.92	0.51	1.20	0.03	0.87	2.64	-0.20	0.40
CO87	0.53	1.66	1.13	1.30	0.24	0.74	4.70	-0.20	0.78
CT87	1.36	1.62	2.15	-0.49	-0.63	0.38	0.14	0.39	-0.22
DC87	-0.09	0.84	1.11	-0.90	-0.15	-1.95	7.53	0.60	1.42
DE87	0.42	0.49	3.04	1.70	-0.17	0.24	1.93	-1.62	0.80
FL87	-0.81	1.36	-1.57	1.50	0.25	1.61	6.12	0.30	0.91
GA87	-0.91	0.25	-0.67	1.51	0.22	0.72	2.10	-0.81	0.58
HI87	0.77	2.81	-1.29	2.30	0.25	0.83	8.23	0.50	1.27
ID87	-0.88	1.68	-0.43	2.10	0.63	1.42	5.00	-0.10	0.71
IL87	0.96	2.34	0.92	1.54	0.15	0.69	1.87	-0.85	0.60
IN87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
IO87	0.42	3.21	0.89	2.30	0.41	0.18	2.97	-0.80	0.95
KS87	-0.88	1.42	0.58	2.30	0.30	0.38	2.64	-1.60	0.89
KY87	-0.12	1.41	0.46	2.10	0.61	0.03	3.50	-3.20	1.22
LA87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MA87	1.14	1.86	0.64	0.35	-0.25	0.66	1.07	-0.14	0.18
MD87	0.22	2.08	0.80	1.40	-0.01	1.13	5.17	-0.60	1.27
ME87	1.15	2.14	0.47	0.81	0.19	1.29	3.03	-0.45	0.15
MI87	-0.68	0.85	4.29	1.51	0.00	0.19	1.12	-0.87	0.21
MN87	-0.48	1.33	0.78	1.35	0.15	0.72	1.95	-0.80	0.45
MO87	-0.20	1.92	0.39	2.05	0.33	0.96	2.18	-0.66	0.76
MS87	-0.52	1.89	-1.32	1.35	0.54	1.64	3.34	0.20	0.70
MT87	0.14	1.28	0.20	3.20	0.58	1.66	7.79	0.70	0.68
NC87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ND87	-0.30	1.50	-1.22	2.70	0.53	0.81	7.97	0.70	1.28
NE87	0.23	2.59	-0.61	2.70	0.40	0.80	4.64	-0.20	1.04
NH87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NJ87	1.11	2.32	0.71	1.28	-0.23	0.99	1.39	-0.08	0.23
NM87	-0.37	3.01	-0.93	2.30	0.62	2.24	7.96	0.00	1.33
NV87	-1.30	4.17	-0.38	1.30	0.17	3.30	7.91	0.40	1.24
NY87	1.26	2.81	0.16	1.00	0.01	0.97	3.15	0.10	0.55
OH87	0.29	1.51	2.67	1.39	0.19	0.14	1.01	-0.90	0.42
OK87	2.77	3.16	0.52	2.40	0.55	0.88	5.50	0.00	1.06
OR87	-0.82	1.78	0.27	1.90	0.42	1.54	2.79	0.30	-0.12
PA87	1.60	2.42	0.67	1.23	0.17	1.24	3.32	-0.10	0.59
RI87	0.74	1.75	0.28	-0.30	-0.23	1.23	1.85	1.80	-1.21
SC87	0.44	0.42	0.15	0.39	0.16	0.53	2.20	-0.87	0.41
SD87	-0.94	2.33	-2.03	2.60	0.57	1.81	6.42	0.50	1.14
TN87	-0.13	0.92	-0.01	0.80	0.23	0.67	2.02	-0.58	0.50
TX87	1.69	3.23	0.07	2.50	0.42	0.21	4.83	-0.70	1.00
UT87	-0.61	1.99	-0.75	1.70	0.66	1.78	4.94	-0.40	0.97
VA87	-0.76	1.24	-0.55	1.30	0.15	0.86	3.04	-1.00	1.17
VT87	-0.04	2.39	0.49	0.17	0.12	1.09	2.99	-0.53	-0.38
WA87	-0.51	1.87	1.95	2.60	0.25	1.40	3.44	-0.10	0.53
WI87	-0.03	1.65	1.80	1.23	0.13	0.37	0.89	-0.16	0.08
WV87	1.58	2.35	1.32	1.60	0.71	0.71	5.88	-0.70	1.36
WY87	2.78	3.27	1.00	4.10	0.53	0.99	8.68	0.20	1.19

Table 5.27 States 1982: Efficiency Prices
VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK82	0.01	0.02	0.02	0.04	0.03	0.11	0.05	0.06	0.02
AL82	0.03	0.07	0.04	0.10	0.03	0.10	0.05	0.06	0.02
AR82	0.02	0.05	0.06	0.08	0.03	0.10	0.05	0.07	0.02
AZ82	0.03	0.06	0.04	0.10	0.03	0.12	0.06	0.08	0.02
CA82	1.47	2.81	0.03	0.05	0.02	0.06	0.06	0.08	0.01
CO82	0.02	0.06	0.04	0.10	0.03	0.11	0.05	0.07	0.02
CT82	0.47	0.90	0.01	0.02	0.21	0.80	0.38	0.14	0.12
DC82	0.01	0.03	0.03	0.05	0.04	0.13	0.06	0.06	0.02
DE82	0.08	0.16	0.00	0.00	0.20	0.55	0.57	0.77	0.12
FL82	0.03	0.06	0.06	0.10	0.04	0.14	0.07	0.04	0.03
GA82	0.03	0.06	0.06	0.09	0.03	0.10	0.05	0.06	0.02
HI82	0.01	0.03	0.03	0.05	0.04	0.15	0.07	0.03	0.02
ID82	0.02	0.04	0.04	0.06	0.02	0.14	0.07	0.02	0.03
IL82	0.24	0.59	0.03	0.09	0.03	0.10	0.05	0.02	0.02
IN82	0.03	0.08	0.04	0.10	0.02	0.08	0.04	0.05	0.01
IO82	0.02	0.06	0.03	0.09	0.02	0.09	0.04	0.06	0.01
KS82	0.02	0.05	0.05	0.07	0.03	0.11	0.05	0.03	0.02
KY82	0.03	0.06	0.04	0.10	0.02	0.10	0.05	0.06	0.01
LA82	0.25	0.61	0.01	0.01	0.27	1.51	0.72	0.98	0.22
MA82	0.07	0.18	0.10	0.28	0.02	0.07	0.05	0.07	0.01
MD82	0.03	0.06	0.04	0.10	0.03	0.12	0.06	0.07	0.02
ME82	0.02	0.05	0.05	0.08	0.03	0.09	0.05	0.06	0.02
MI82	0.26	0.63	0.03	0.07	0.02	0.09	0.04	0.06	0.01
MN82	0.03	0.06	0.04	0.10	0.03	0.10	0.05	0.06	0.02
MO82	0.03	0.06	0.04	0.10	0.03	0.10	0.05	0.06	0.01
MS82	0.02	0.06	0.06	0.09	0.02	0.10	0.05	0.06	0.01
MT82	0.01	0.02	0.02	0.04	0.05	0.18	0.09	0.03	0.04
NC82	0.75	1.44	0.02	0.02	0.01	0.03	0.09	0.00	0.01
ND82	0.01	0.02	0.02	0.04	0.04	0.14	0.07	0.09	0.02
NE82	0.02	0.05	0.05	0.06	0.03	0.12	0.06	0.03	0.02
NH82	0.02	0.05	0.06	0.09	0.02	0.10	0.05	0.02	0.02
NJ82	0.23	0.57	0.03	0.09	0.03	0.10	0.05	0.02	0.02
NM82	0.01	0.04	0.04	0.06	0.05	0.16	0.08	0.08	0.03
NV82	0.01	0.03	0.03	0.05	0.05	0.16	0.08	0.09	0.03
NY82	0.28	0.68	0.05	0.13	0.03	0.11	0.05	0.02	0.02
OH82	0.75	1.82	0.02	0.05	0.02	0.08	0.11	0.12	0.02
OK82	0.02	0.06	0.04	0.09	0.03	0.11	0.05	0.07	0.02
OR82	0.02	0.05	0.05	0.08	0.02	0.12	0.06	0.02	0.02
PA82	0.86	1.16	0.02	0.06	0.01	0.02	0.09	0.00	0.01
RI82	0.02	0.06	0.06	0.09	0.01	0.03	0.07	0.01	0.03
SC82	0.02	0.06	0.06	0.09	0.01	0.05	0.05	0.07	0.01
SD82	0.01	0.03	0.03	0.05	0.04	0.15	0.07	0.03	0.02
TN82	0.03	0.07	0.04	0.11	0.02	0.09	0.04	0.06	0.01
TX82	0.54	1.33	0.03	0.05	0.06	0.21	0.10	0.14	0.03
UT82	0.02	0.05	0.05	0.07	0.04	0.13	0.06	0.05	0.02
VA82	0.03	0.07	0.04	0.11	0.03	0.10	0.05	0.06	0.01
VT82	0.02	0.05	0.05	0.07	0.03	0.09	0.04	0.06	0.02
WA82	0.02	0.05	0.04	0.08	0.03	0.11	0.05	0.07	0.02
WI82	0.08	0.20	0.04	0.10	0.03	0.09	0.05	0.02	0.02
WV82	0.02	0.05	0.03	0.08	0.03	0.12	0.06	0.08	0.02
WY82	0.01	0.02	0.02	0.03	0.05	0.16	0.08	0.10	0.03

Table 5.28 States 1987: Efficiency Prices
VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	Output 5
AK87	0.05	0.04	0.02	0.03	0.02	0.11	0.03	0.03	0.01
AL87	0.08	0.07	0.02	0.06	0.01	0.04	0.05	0.05	0.00
AR87	0.08	0.07	0.03	0.05	0.01	0.09	0.05	0.04	0.01
AZ87	0.37	0.31	0.11	0.02	0.01	0.12	0.04	0.03	0.01
CA87	0.08	0.06	0.03	0.05	0.02	0.10	0.03	0.03	0.01
CO87	0.07	0.06	0.03	0.05	0.02	0.11	0.03	0.03	0.01
CT87	0.04	0.08	0.03	0.07	0.02	0.04	0.06	0.01	0.02
DC87	0.08	0.11	0.02	0.10	0.02	0.10	0.03	0.01	0.01
DE87	0.07	0.06	0.02	0.05	0.01	0.07	0.04	0.04	0.01
FL87	0.09	0.07	0.03	0.05	0.03	0.13	0.04	0.03	0.01
GA87	0.08	0.09	0.04	0.05	0.01	0.08	0.05	0.04	0.01
HI87	0.07	0.06	0.02	0.04	0.03	0.13	0.04	0.01	0.01
ID87	0.08	0.07	0.03	0.05	0.01	0.13	0.04	0.03	0.01
IL87	0.07	0.06	0.02	0.04	0.02	0.08	0.04	0.04	0.01
IN87	0.05	0.08	0.04	0.05	0.14	0.68	0.29	0.24	0.06
IO87	0.06	0.05	0.02	0.04	0.01	0.10	0.03	0.03	0.01
KS87	0.08	0.06	0.03	0.05	0.01	0.10	0.03	0.02	0.01
KY87	0.07	0.06	0.03	0.05	0.01	0.09	0.03	0.02	0.01
LA87	0.04	0.06	0.03	0.04	0.03	0.27	0.08	0.07	0.02
MA87	0.07	0.06	0.03	0.05	0.03	0.07	0.05	0.04	0.01
MD87	0.07	0.06	0.03	0.05	0.03	0.12	0.04	0.03	0.01
ME87	0.07	0.05	0.02	0.05	0.03	0.08	0.05	0.04	0.02
MI87	0.08	0.07	0.02	0.05	0.01	0.07	0.04	0.04	0.01
MN87	0.08	0.07	0.03	0.05	0.02	0.08	0.05	0.04	0.01
MO87	0.07	0.06	0.03	0.05	0.01	0.08	0.05	0.04	0.01
MS87	0.08	0.07	0.03	0.05	0.01	0.10	0.05	0.04	0.01
MT87	0.06	0.07	0.03	0.04	0.03	0.15	0.04	0.02	0.02
NC87	0.08	0.09	0.04	0.05	0.01	0.03	0.05	0.04	0.00
ND87	0.08	0.06	0.03	0.05	0.03	0.13	0.04	0.01	0.01
NE87	0.07	0.06	0.03	0.04	0.01	0.11	0.03	0.03	0.01
NH87	0.06	1.45	0.04	0.05	0.02	0.09	0.03	0.01	0.01
NJ87	0.06	0.05	0.02	0.05	0.03	0.07	0.05	0.04	0.01
NM87	0.07	0.06	0.03	0.05	0.04	0.16	0.05	0.04	0.01
NV87	0.07	0.06	0.03	0.05	0.04	0.19	0.06	0.05	0.02
NY87	0.06	0.05	0.02	0.05	0.03	0.11	0.03	0.03	0.01
OH87	0.07	0.06	0.03	0.04	0.01	0.07	0.04	0.03	0.01
OK87	0.06	0.05	0.02	0.04	0.01	0.12	0.04	0.03	0.01
OR87	0.08	0.06	0.03	0.05	0.01	0.12	0.03	0.03	0.02
PA87	0.06	0.05	0.02	0.05	0.03	0.08	0.05	0.04	0.01
RI87	0.07	0.06	0.03	0.05	0.02	0.06	0.06	0.01	0.03
SC87	0.05	0.08	0.03	0.07	0.01	0.06	0.05	0.04	0.01
SD87	0.08	0.07	0.03	0.05	0.03	0.14	0.04	0.04	0.01
TN87	0.08	0.07	0.03	0.05	0.01	0.08	0.05	0.04	0.01
TX87	0.06	0.05	0.02	0.04	0.01	0.11	0.03	0.03	0.01
UT87	0.08	0.07	0.03	0.05	0.01	0.13	0.04	0.03	0.01
VA87	0.08	0.07	0.03	0.05	0.02	0.11	0.03	0.03	0.01
VT87	0.07	0.06	0.03	0.05	0.03	0.07	0.05	0.04	0.02
WA87	0.07	0.06	0.03	0.04	0.03	0.11	0.03	0.03	0.01
WI87	0.07	0.06	0.03	0.05	0.01	0.07	0.04	0.04	0.02
WV87	0.06	0.05	0.02	0.04	0.01	0.12	0.04	0.03	0.01
WY87	0.05	0.04	0.02	0.03	0.03	0.14	0.04	0.03	0.01

Table 5.29 States 1982: Output Price Ratios
VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	μ_1/μ_2	μ_1/μ_3	μ_1/μ_4	μ_1/μ_5	μ_2/μ_3	μ_2/μ_4	μ_2/μ_5	μ_3/μ_4	μ_3/μ_5	μ_4/μ_5
AK82	0.29	0.61	0.55	1.79	2.09	1.88	6.13	0.90	2.93	3.26
AL82	0.29	0.61	0.45	1.79	2.09	1.55	6.13	0.74	2.93	3.97
AR82	0.29	0.61	0.45	1.48	2.09	1.55	5.08	0.74	2.43	3.28
AZ82	0.29	0.61	0.45	1.79	2.09	1.55	6.13	0.74	2.93	3.97
CA82	0.37	0.37	0.27	1.79	0.99	0.73	4.84	0.74	4.88	6.60
CO82	0.29	0.61	0.45	1.48	2.09	1.55	5.08	0.74	2.43	3.28
CT82	0.26	0.55	1.47	1.79	2.09	5.56	6.78	2.66	3.24	1.22
DC82	0.29	0.61	0.58	1.79	2.09	1.99	6.13	0.95	2.93	3.09
DE82	0.37	0.35	0.26	1.73	0.96	0.71	4.69	0.74	4.88	6.60
FL82	0.29	0.61	0.92	1.48	2.09	3.16	5.08	1.51	2.43	1.61
GA82	0.29	0.61	0.45	1.48	2.09	1.55	5.08	0.74	2.43	3.28
HI82	0.29	0.61	1.62	1.79	2.09	5.56	6.13	2.66	2.93	1.10
ID82	0.18	0.38	1.00	0.91	2.09	5.56	5.08	2.66	2.43	0.91
IL82	0.26	0.55	1.47	1.79	2.09	5.56	6.78	2.66	3.24	1.22
IN82	0.29	0.61	0.45	1.79	2.09	1.55	6.13	0.74	2.93	3.97
IO82	0.18	0.38	0.28	1.22	2.09	1.55	6.78	0.74	3.24	4.38
KS82	0.29	0.61	0.96	1.79	2.09	3.28	6.13	1.57	2.93	1.87
KY82	0.18	0.38	0.28	1.22	2.09	1.55	6.78	0.74	3.24	4.38
LA82	0.18	0.38	0.28	1.22	2.09	1.55	6.78	0.74	3.24	4.38
MA82	0.26	0.37	0.27	1.79	1.39	1.03	6.78	0.74	4.88	6.60
MD82	0.29	0.61	0.45	1.79	2.09	1.55	6.13	0.74	2.93	3.97
ME82	0.29	0.61	0.45	1.48	2.09	1.55	5.08	0.74	2.43	3.28
MI82	0.18	0.38	0.28	1.22	2.09	1.55	6.78	0.74	3.24	4.38
MN82	0.29	0.61	0.45	1.79	2.09	1.55	6.13	0.74	2.93	3.97
MO82	0.26	0.55	0.41	1.79	2.09	1.55	6.78	0.74	3.24	4.38
MS82	0.18	0.38	0.28	1.22	2.09	1.55	6.78	0.74	3.24	4.38
MT82	0.29	0.61	1.62	1.48	2.09	5.56	5.08	2.66	2.43	0.91
NC82	0.25	0.07	1.39	0.75	0.28	5.56	3.01	19.86	10.74	0.54
ND82	0.29	0.61	0.45	1.79	2.09	1.55	6.13	0.74	2.93	3.97
NE82	0.26	0.55	1.06	1.79	2.09	4.03	6.78	1.93	3.24	1.68
NH82	0.25	0.52	1.39	1.27	2.09	5.56	5.08	2.66	2.43	0.91
NJ82	0.29	0.61	1.62	1.48	2.09	5.56	5.08	2.66	2.43	0.91
NM82	0.29	0.61	0.58	1.79	2.09	1.99	6.13	0.95	2.93	3.09
NV82	0.29	0.61	0.54	1.79	2.09	1.85	6.13	0.89	2.93	3.32
NY82	0.29	0.61	1.62	1.48	2.09	5.56	5.08	2.66	2.43	0.91
OH82	0.24	0.18	0.17	1.12	0.77	0.71	4.69	0.93	6.11	6.60
OK82	0.29	0.61	0.45	1.48	2.09	1.55	5.08	0.74	2.43	3.28
OR82	0.18	0.38	1.00	0.91	2.09	5.56	5.08	2.66	2.43	0.91
PA82	0.33	0.09	1.86	1.01	0.28	5.56	3.01	19.86	10.74	0.54
RI82	0.34	0.15	1.86	0.37	0.46	5.55	1.11	12.15	2.43	0.20
SC82	0.24	0.23	0.17	1.12	0.96	0.71	4.69	0.74	4.88	6.60
SD82	0.26	0.55	1.47	1.79	2.09	5.56	6.78	2.66	3.24	1.22
TN82	0.18	0.38	0.28	1.22	2.09	1.55	6.78	0.74	3.24	4.38
TX82	0.26	0.55	0.41	1.79	2.09	1.55	6.78	0.74	3.24	4.38
UT82	0.29	0.61	0.73	1.48	2.09	2.52	5.08	1.20	2.43	2.02
VA82	0.26	0.55	0.41	1.79	2.09	1.55	6.78	0.74	3.24	4.38
VT82	0.29	0.61	0.45	1.48	2.09	1.55	5.08	0.74	2.43	3.28
WA82	0.29	0.61	0.45	1.48	2.09	1.55	5.08	0.74	2.43	3.28
WI82	0.29	0.61	1.62	1.48	2.09	5.56	5.08	2.66	2.43	0.91
WV82	0.26	0.55	0.41	1.79	2.09	1.55	6.78	0.74	3.24	4.38
WY82	0.29	0.61	0.47	1.79	2.09	1.61	6.13	0.77	2.93	3.80

Table 5.30 States 1987: Output Price Ratios
VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	$\mu1/\mu2$	$\mu1/\mu3$	$\mu1/\mu4$	$\mu1/\mu5$	$\mu2/\mu3$	$\mu2/\mu4$	$\mu2/\mu5$	$\mu3/\mu4$	$\mu3/\mu5$	$\mu4/\mu5$
AK87	0.21	0.69	0.84	2.47	3.34	4.04	11.93	1.21	3.57	2.95
AL87	0.20	0.14	0.17	2.36	0.71	0.86	11.93	1.21	16.78	13.87
AR87	0.11	0.20	0.24	1.31	1.77	2.15	11.93	1.21	6.73	5.56
AZ87	0.23	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
CA87	0.21	0.76	0.92	2.47	3.34	4.04	10.86	1.21	3.25	2.69
CO87	0.42	0.69	0.84	2.47	3.34	4.04	11.93	1.21	3.57	2.95
CT87	0.21	0.25	2.17	0.95	0.59	5.17	2.27	8.83	3.88	0.44
DC87	0.21	0.69	2.11	2.47	3.34	10.18	11.93	3.05	3.57	1.17
DE87	0.23	0.32	0.39	2.47	1.54	1.87	11.93	1.21	7.74	6.40
FL87	0.11	0.76	0.92	2.47	3.34	4.04	10.86	1.21	3.25	2.69
GA87	0.23	0.19	0.23	1.31	1.17	2.06	11.93	1.21	7.02	5.80
HI87	0.11	0.76	2.17	2.47	3.34	9.54	10.86	2.86	3.25	1.14
ID87	0.21	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
IL87	0.21	0.36	0.44	2.47	1.75	2.12	11.93	1.21	6.80	5.62
IN87	0.11	0.48	0.59	2.47	2.34	2.83	11.93	1.21	5.11	4.22
IO87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
KS87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
KY87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
LA87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
MA87	0.42	0.63	0.76	2.47	1.49	1.81	5.88	1.21	3.94	2.95
MD87	0.23	0.76	0.92	2.47	3.34	4.04	10.86	1.21	3.25	3.26
ME87	0.42	0.64	0.77	1.38	1.52	1.83	3.29	1.21	2.17	2.69
MI87	0.21	0.31	0.38	2.47	1.51	1.83	11.93	1.21	7.90	1.79
MN87	0.21	0.35	0.42	2.47	1.68	2.03	11.93	1.21	7.10	6.53
MO87	0.11	0.20	0.24	1.31	1.78	2.15	11.93	1.21	6.70	5.87
MS87	0.11	0.20	0.24	1.31	1.80	2.18	11.93	1.21	6.63	5.54
MT87	0.23	0.76	2.17	1.65	3.34	9.54	7.25	2.86	2.17	0.76
NC87	0.24	0.15	0.18	2.47	0.62	0.75	10.40	1.21	16.78	13.87
ND87	0.21	0.69	2.11	2.47	3.34	10.18	11.93	3.05	3.57	1.17
NE87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
NH87	0.21	0.69	2.11	2.47	3.34	10.18	11.93	3.05	3.57	1.17
NJ87	0.42	0.65	0.78	2.47	1.54	1.87	5.88	1.21	3.81	3.15
NM87	0.23	0.76	0.92	2.47	3.34	4.04	10.86	1.21	3.25	2.69
NV87	0.23	0.76	0.92	2.47	3.34	4.04	10.86	1.21	3.25	2.69
NY87	0.23	0.76	0.92	2.47	3.34	4.04	10.86	1.21	3.25	2.69
OH87	0.11	0.19	0.23	1.31	1.74	2.11	11.93	1.21	6.85	5.66
OK87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
OR87	0.11	0.37	0.45	0.80	3.34	4.04	7.25	1.21	2.17	1.79
PA87	0.42	0.67	0.81	2.47	1.59	1.92	5.88	1.21	3.70	3.06
RI87	0.30	0.31	2.17	0.67	1.05	7.32	2.27	7.00	2.17	0.31
SC87	0.11	0.14	0.17	1.31	1.30	1.57	11.93	1.21	9.18	7.58
SD87	0.21	0.69	0.84	2.47	3.34	4.04	11.93	1.21	3.57	2.95
TN87	0.11	0.19	0.23	1.31	1.70	2.06	11.93	1.21	7.01	5.79
TX87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
UT87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
VA87	0.21	0.69	0.84	2.47	3.34	4.04	11.93	1.21	3.57	2.95
VT87	0.42	0.61	0.74	1.32	1.45	1.76	3.15	1.21	2.17	1.79
WA87	0.23	0.76	0.92	2.47	3.34	4.04	10.86	1.21	3.25	2.69
WI87	0.15	0.25	0.30	0.54	1.62	1.96	3.52	1.21	2.17	1.79
WV87	0.11	0.37	0.45	1.31	3.34	4.04	11.93	1.21	3.57	2.95
WY87	0.21	0.69	0.84	2.47	3.34	4.04	11.93	1.21	3.57	2.95

Table 5.31 States 1982: Input Price Ratios and Efficiency Scores
VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	ν_1/ν_2	ν_1/ν_3	ν_1/ν_4	ν_2/ν_3	ν_2/ν_4	ν_3/ν_4	θ	ϕ	E
AK82	0.41	0.40	0.26	0.97	0.64	0.66	0.38	1.36	0.28
AL82	0.41	0.63	0.26	1.54	0.64	0.41	0.89	1.21	0.73
AR82	0.41	0.40	0.26	0.97	0.64	0.66	0.86	1.22	0.71
AZ82	0.41	0.62	0.26	1.51	0.64	0.42	0.84	1.42	0.59
CA82	0.52	52.18	31.18	99.66	59.55	0.60	1.00	1.00	1.00
CO82	0.41	0.61	0.26	1.49	0.64	0.43	0.83	1.33	0.62
CT82	0.52	42.13	31.18	80.47	59.55	0.74	1.00	1.00	1.00
DC82	0.41	0.40	0.26	0.98	0.64	0.66	0.49	1.50	0.33
DE82	0.52	42.14	31.19	80.48	59.56	0.74	1.00	1.00	1.00
FL82	0.41	0.40	0.26	0.97	0.64	0.66	1.00	1.61	0.62
GA82	0.41	0.40	0.30	0.97	0.72	0.74	0.92	1.20	0.77
HI82	0.41	0.40	0.26	0.97	0.64	0.66	0.53	1.74	0.31
ID82	0.41	0.40	0.26	0.97	0.64	0.66	0.66	1.55	0.43
IL82	0.41	7.42	2.74	18.09	6.69	0.37	0.91	1.02	0.89
IN82	0.41	0.95	0.35	2.31	0.86	0.37	0.83	1.02	0.82
IO82	0.41	0.69	0.26	1.67	0.64	0.38	0.77	1.11	0.69
KS82	0.41	0.40	0.26	0.98	0.64	0.66	0.75	1.19	0.62
KY82	0.41	0.69	0.26	1.67	0.64	0.38	0.82	1.18	0.69
LA82	0.41	40.86	24.42	99.65	59.55	0.60	1.00	1.00	1.00
MA82	0.41	0.71	0.26	1.73	0.64	0.37	1.00	1.00	1.00
MD82	0.41	0.62	0.26	1.52	0.64	0.42	0.86	1.41	0.61
ME82	0.41	0.40	0.26	0.98	0.64	0.66	0.82	1.15	0.71
MI82	0.41	9.29	3.44	22.67	8.39	0.37	0.81	1.00	0.81
MN82	0.41	0.63	0.26	1.54	0.64	0.41	0.87	1.19	0.73
MO82	0.41	0.65	0.26	1.59	0.64	0.40	0.87	1.19	0.73
MS82	0.41	0.40	0.26	0.97	0.64	0.66	0.90	1.15	0.78
MT82	0.41	0.40	0.26	0.97	0.64	0.66	0.41	2.12	0.19
NC82	0.52	42.13	31.18	80.47	59.55	0.74	1.00	1.00	1.00
ND82	0.41	0.40	0.26	0.97	0.64	0.66	0.38	1.72	0.22
NE82	0.41	0.40	0.30	0.97	0.72	0.74	0.74	1.35	0.54
NH82	0.41	0.40	0.26	0.97	0.64	0.66	0.91	1.12	0.81
NJ82	0.41	7.34	2.71	17.89	6.62	0.37	0.94	1.06	0.88
NM82	0.41	0.40	0.26	0.97	0.64	0.66	0.60	1.88	0.32
NV82	0.41	0.40	0.26	0.97	0.64	0.66	0.50	1.88	0.27
NY82	0.41	5.95	2.20	14.50	5.37	0.37	1.00	1.00	1.00
OH82	0.41	40.86	15.12	99.66	36.87	0.37	1.00	1.00	1.00
OK82	0.41	0.60	0.26	1.47	0.64	0.44	0.77	1.36	0.56
OR82	0.41	0.40	0.26	0.97	0.64	0.66	0.82	1.34	0.61
PA82	0.74	38.47	14.23	51.98	19.23	0.37	0.95	1.04	0.91
RI82	0.41	0.40	0.26	0.98	0.64	0.66	0.88	1.07	0.82
SC82	0.41	0.40	0.26	0.98	0.64	0.66	0.95	1.03	0.91
SD82	0.41	0.40	0.26	0.97	0.64	0.66	0.53	1.68	0.32
TN82	0.41	0.69	0.26	1.67	0.64	0.38	0.95	1.08	0.87
TX82	0.41	15.88	11.75	38.73	28.66	0.74	0.98	1.00	0.98
UT82	0.41	0.40	0.26	0.98	0.64	0.66	0.73	1.46	0.50
VA82	0.41	0.65	0.26	1.59	0.64	0.40	0.95	1.20	0.79
VT82	0.41	0.40	0.26	0.98	0.64	0.66	0.72	1.11	0.65
WA82	0.41	0.60	0.26	1.46	0.64	0.44	0.72	1.31	0.55
WI82	0.41	2.27	0.84	5.54	2.05	0.37	0.87	1.05	0.82
WV82	0.41	0.63	0.26	1.54	0.64	0.42	0.70	1.42	0.50
WY82	0.41	0.40	0.26	0.97	0.64	0.66	0.29	1.95	0.15

Table 5.32 States 1987: Input Price Ratios and Efficiency Scores
VRS Envelopment/ Frontier Model: Limit-Value-Ratios Derived from Limit Value Ratios of Units Efficient under Global Orientation and Invariant Pricing Evaluation

DMU	$\nu1/\nu2$	$\nu1/\nu3$	$\nu1/\nu4$	$\nu2/\nu3$	$\nu2/\nu4$	$\nu3/\nu4$	θ	ϕ	E
AK87	1.19	2.69	1.56	2.26	1.31	0.58	0.58	1.23	0.47
AL87	1.19	3.33	1.36	2.80	1.14	0.41	0.94	1.07	0.88
AR87	1.19	2.69	1.56	2.26	1.31	0.58	0.87	1.25	0.70
AZ87	1.19	3.33	1.56	2.80	1.31	0.47	1.00	1.00	1.00
CA87	1.19	2.69	1.56	2.26	1.31	0.58	0.84	1.17	0.71
CO87	1.19	2.69	1.56	2.26	1.31	0.58	0.78	1.25	0.62
CT87	0.49	1.51	0.56	3.07	1.14	0.37	0.79	1.01	0.79
DC87	0.70	3.33	0.80	4.75	1.14	0.24	0.98	1.04	0.94
DE87	1.19	3.33	1.56	2.80	1.31	0.47	0.79	1.04	0.76
FL87	1.19	2.69	1.56	2.26	1.31	0.58	0.94	1.48	0.64
GA87	0.90	2.03	1.56	2.26	1.74	0.77	1.00	1.13	0.89
HI87	1.19	2.69	1.56	2.26	1.31	0.58	0.73	1.47	0.50
ID87	1.19	2.69	1.56	2.26	1.31	0.58	0.87	1.38	0.63
IL87	1.19	2.69	1.56	2.26	1.31	0.58	0.72	1.11	0.65
IN87	0.67	1.51	1.16	2.26	1.74	0.77	1.00	1.00	1.00
IO87	1.19	2.69	1.56	2.26	1.31	0.58	0.69	1.10	0.63
KS87	1.19	2.69	1.56	2.26	1.31	0.58	0.85	1.09	0.78
KY87	1.19	2.69	1.56	2.26	1.31	0.58	0.81	1.04	0.78
LA87	0.67	1.51	1.16	2.26	1.74	0.77	1.00	1.00	1.00
MA87	1.19	2.69	1.36	2.26	1.14	0.50	0.78	1.08	0.73
MD87	1.19	2.69	1.56	2.26	1.31	0.58	0.78	1.31	0.60
ME87	1.19	2.69	1.36	2.26	1.14	0.50	0.76	1.24	0.61
MI87	1.19	3.33	1.56	2.80	1.31	0.47	0.82	1.03	0.80
MN87	1.19	2.69	1.56	2.26	1.31	0.58	0.86	1.12	0.77
MO87	1.19	2.69	1.56	2.26	1.31	0.58	0.79	1.16	0.68
MS87	1.19	2.69	1.56	2.26	1.31	0.58	0.89	1.35	0.66
MT87	0.90	2.03	1.56	2.26	1.74	0.77	0.77	1.63	0.47
NC87	0.90	2.03	1.56	2.26	1.74	0.77	1.00	1.00	1.00
ND87	1.19	2.69	1.56	2.26	1.31	0.58	0.83	1.46	0.57
NE87	1.19	2.69	1.56	2.26	1.31	0.58	0.74	1.26	0.59
NH87	0.04	1.51	1.16	38.96	30.00	0.77	1.00	1.00	1.00
NJ87	1.19	2.69	1.36	2.26	1.14	0.50	0.73	1.13	0.65
NM87	1.19	2.69	1.56	2.26	1.31	0.58	0.77	1.77	0.44
NV87	1.19	2.69	1.56	2.26	1.31	0.58	0.79	2.10	0.38
NY87	1.19	2.69	1.36	2.26	1.14	0.50	0.72	1.22	0.59
OH87	1.19	2.69	1.56	2.26	1.31	0.58	0.77	1.03	0.74
OK87	1.19	2.69	1.56	2.26	1.31	0.58	0.61	1.32	0.46
OR87	1.19	2.69	1.56	2.26	1.31	0.58	0.85	1.29	0.66
PA87	1.19	2.69	1.36	2.26	1.14	0.50	0.71	1.22	0.58
RI87	1.19	2.69	1.36	2.26	1.14	0.50	0.85	1.16	0.74
SC87	0.67	1.51	0.76	2.26	1.14	0.50	0.91	1.11	0.83
SD87	1.19	2.69	1.56	2.26	1.31	0.58	0.85	1.57	0.54
TN87	1.19	2.69	1.56	2.26	1.31	0.58	0.91	1.13	0.81
TX87	1.19	2.69	1.56	2.26	1.31	0.58	0.65	1.17	0.55
UT87	1.19	2.69	1.56	2.26	1.31	0.58	0.86	1.43	0.60
VA87	1.19	2.69	1.56	2.26	1.31	0.58	0.92	1.17	0.79
VT87	1.19	2.69	1.36	2.26	1.14	0.50	0.84	1.20	0.70
WA87	1.19	2.69	1.56	2.26	1.31	0.58	0.76	1.29	0.59
WI87	1.19	2.69	1.56	2.26	1.31	0.58	0.80	1.06	0.76
WV87	1.19	2.69	1.36	2.26	1.14	0.50	0.69	1.29	0.53
WY87	1.19	2.69	1.56	2.26	1.31	0.58	0.67	1.52	0.37

Table 5.33 SIC367 Total Waste 1982 Data
VRS Envelopment/ Global Orientation/ Invariant Pricing

DMU	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6	Output 1	Output 2	Output 3
AL367	41.56	1.61	6.49	2.56	1.00	0.14	40.10	0.00	32.00
AZ367	7.92	29.52	144.18	4.03	3.00	4.33	181.31	0.00	7.54
CA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO367	0.00	0.00	0.00	1.60	0.77	0.03	125.32	124.83	125.32
CT367	21.10	24.40	31.63	7.23	3.25	1.40	80.31	0.00	24.28
FL367	0.00	45.46	28.55	9.83	4.89	2.00	99.99	0.00	25.98
GA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
IL367	79.50	11.62	23.70	9.00	4.78	1.84	102.67	0.00	67.36
IN367	67.39	36.33	13.30	6.15	3.32	0.84	116.57	0.00	66.94
IO367	0.00	1.79	0.00	0.33	0.09	0.17	37.82	35.65	36.03
KS367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MA367	0.00	83.15	12.96	15.72	8.04	1.45	96.11	7.50	0.00
MD367	0.00	0.14	0.00	0.20	0.17	0.01	52.86	52.15	52.72
ME367	0.00	6.66	1.36	2.27	1.14	0.09	8.02	7.83	0.00
MI367	0.00	0.00	4.65	0.37	0.15	0.19	19.15	14.98	14.50
MN367	0.00	21.47	1.43	4.20	2.15	0.21	146.97	117.67	124.07
MO367	12.09	44.30	19.05	2.25	1.39	0.46	84.15	0.00	20.80
NC367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NE367	4.81	0.00	0.88	1.11	0.48	0.36	32.29	26.62	31.41
NH367	0.00	18.86	10.66	4.92	2.80	0.57	76.17	46.40	46.65
NJ367	29.36	41.16	48.13	8.63	4.64	2.02	126.21	0.00	36.92
NM367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NY367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OH367	20.89	30.98	20.32	4.25	2.26	1.02	74.30	0.00	23.00
OK367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OR367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RI367	0.00	0.00	0.00	0.66	0.37	0.37	35.65	32.23	35.65
SC367	14.00	25.00	17.20	5.70	2.60	1.10	162.20	111.30	120.00
TN367	0.00	0.00	1.55	0.11	0.17	0.00	52.85	53.69	51.31
TX367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UT367	0.00	7.09	4.07	1.46	0.92	0.00	108.99	97.86	97.83
VA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 5.34 Industry Study: Limit-Value-Ratio Ranges

	1982	
	MIN	MAX
ν_1/ν_2	0.106	0.686
ν_1/ν_3	0.145	1.474
ν_1/ν_4	0.015	0.118
ν_1/ν_5	0.008	0.059
ν_1/ν_6	0.006	0.059
ν_2/ν_3	0.501	1.887
ν_2/ν_4	0.109	0.180
ν_2/ν_5	0.059	0.092
ν_2/ν_6	0.021	0.097
ν_3/ν_4	0.058	0.338
ν_3/ν_5	0.030	0.182
ν_3/ν_6	0.032	0.055
ν_4/ν_5	0.497	0.568
ν_4/ν_6	0.162	0.690
ν_5/ν_6	0.214	1.318
μ_1/μ_2	0.227	0.591
μ_1/μ_3	0.446	0.743
μ_2/μ_3	1.145	2.053

Table 5.35 SIC367 Total Waste and Substitution/ Frontier Model
Limit-Value-Ratios derived from Limit Value Ratios of DEA-Efficient Units

DMU	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6	Output 1	Output 2	Output 3
AL367	12.67	-9.84	0.94	1.15	0.28	-0.08	94.72	99.69	103.63
AZ367	-33.23	-12.79	67.14	-2.27	0.14	1.40	298.42	278.68	244.06
CA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO367	-8.43	-4.89	-0.22	0.96	0.46	0.01	143.67	156.49	148.79
CT367	-56.69	-1.74	-3.13	3.89	1.56	0.17	153.81	210.22	158.68
FL367	-113.07	11.32	-11.43	5.89	2.75	0.63	190.76	275.15	190.87
GA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
IL367	-22.47	-32.10	-10.95	4.13	2.17	0.60	193.27	271.35	236.32
IN367	-11.12	-0.77	-9.42	2.22	1.16	0.02	184.24	207.42	194.43
IO367	-2.95	0.66	-1.78	0.19	0.02	0.10	45.55	49.10	46.67
KS367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MA367	-166.19	14.64	-11.44	8.81	4.14	0.63	212.45	385.27	209.25
MD367	-2.29	-0.56	-0.07	0.12	0.13	0.00	55.82	58.12	56.45
ME367	-23.04	-0.53	-0.41	1.42	0.70	0.02	40.27	71.62	41.21
MI367	0.00	-1.47	-0.09	0.17	0.05	0.01	32.78	34.53	34.33
MN367	-49.38	6.18	-2.06	2.35	1.21	0.11	179.65	218.09	175.53
MO367	18.52	50.42	17.36	2.73	1.70	0.40	80.48	-15.62	12.70
NC367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NE367	-4.86	-3.51	-3.47	0.69	0.26	0.19	54.20	65.68	61.19
NH367	-53.69	0.98	-1.69	2.78	1.69	0.09	170.08	222.74	170.79
NJ367	-74.77	3.00	-3.40	3.91	2.20	0.19	229.04	294.49	229.44
NM367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NY367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OH367	-39.06	8.10	-3.31	1.54	0.84	0.18	129.27	160.79	124.49
OK367	-13.02	1.87	0.32	0.65	0.37	-0.04	14.87	27.35	12.68
OR367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RI367	0.00	-3.06	-3.82	0.23	0.17	0.21	52.76	55.93	59.64
SC367	-45.98	1.21	-5.40	2.94	1.14	0.30	216.66	271.76	220.85
TN367	0.00	-0.96	0.89	-0.03	0.11	-0.03	57.20	59.59	57.28
TX367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UT367	-18.20	1.35	2.24	0.78	0.57	-0.07	135.47	149.73	131.88
VA367	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WA367	58.97	-5.73	6.64	-0.50	-0.04	0.00	38.27	-9.62	37.37

Table 5.36 SIC367 Price Ratios VRS Envelopment/ Frontier Model
Limit-Value-Ratios derived from Limit Value Ratios of DEA-Efficient Units

DMU	μ_1/μ_2	μ_1/μ_2	μ_2/μ_3	ν_1/ν_2	ν_1/ν_3	ν_1/ν_4	ν_1/ν_5	ν_1/ν_6	ν_2/ν_3
AL367	0.23	0.47	2.05	0.11	0.19	0.02	0.01	0.01	1.77
AZ367	0.23	0.46	2.05	0.69	1.29	0.08	0.05	0.06	1.89
CA367	0.37	0.75	2.03	0.64	0.32	0.10	0.06	0.02	0.51
CO367	0.23	0.47	2.05	0.66	0.61	0.12	0.06	0.03	0.94
CT367	0.23	0.47	2.06	0.66	0.52	0.12	0.06	0.03	0.80
FL367	0.23	0.45	1.97	0.69	0.86	0.12	0.06	0.05	1.26
GA367	0.23	0.47	2.05	0.11	0.19	0.02	0.01	0.01	1.76
IL367	0.23	0.47	2.06	0.64	0.51	0.12	0.06	0.03	0.80
IN367	0.23	0.47	2.06	0.64	0.32	0.11	0.06	0.02	0.51
IO367	0.23	0.47	2.05	0.69	0.62	0.12	0.06	0.03	0.90
KS367	0.59	0.74	1.26	0.11	0.19	0.02	0.01	0.01	1.76
MA367	0.39	0.45	1.15	0.69	0.51	0.12	0.06	0.03	0.74
MD367	0.23	0.47	2.05	0.66	0.61	0.12	0.06	0.03	0.94
ME367	0.39	0.45	1.15	0.66	0.57	0.12	0.06	0.03	0.87
MI367	0.23	0.47	2.05	0.60	0.59	0.11	0.06	0.03	0.99
MN367	0.23	0.47	2.05	0.69	0.87	0.12	0.06	0.05	1.27
MO367	0.23	0.47	2.05	0.69	0.81	0.10	0.06	0.03	1.18
NC367	0.23	0.47	2.05	0.11	0.20	0.02	0.01	0.01	1.89
NE367	0.23	0.47	2.05	0.66	0.61	0.12	0.06	0.03	0.94
NH367	0.23	0.47	2.05	0.69	0.87	0.12	0.06	0.05	1.27
NJ367	0.23	0.47	2.05	0.69	0.87	0.12	0.06	0.05	1.27
NM367	0.23	0.47	2.05	0.61	1.07	0.09	0.05	0.06	1.76
NY367	0.23	0.47	2.07	0.42	0.79	0.05	0.03	0.03	1.89
OH367	0.23	0.47	2.06	0.69	0.53	0.12	0.06	0.03	0.78
OK367	0.23	0.45	1.97	0.69	0.60	0.12	0.06	0.02	0.87
OR367	0.59	0.68	1.15	0.69	1.30	0.12	0.06	0.06	1.89
PA367	0.23	0.46	2.05	0.64	0.32	0.10	0.06	0.02	0.51
RI367	0.23	0.47	2.05	0.60	0.59	0.11	0.06	0.03	0.99
SC367	0.23	0.47	2.05	0.69	0.53	0.12	0.06	0.03	0.78
TN367	0.23	0.47	2.05	0.64	0.78	0.10	0.06	0.03	1.22
TX367	0.39	0.44	1.14	0.61	1.15	0.09	0.05	0.06	1.89
UT367	0.23	0.47	2.05	0.69	0.80	0.12	0.06	0.03	1.17
VA367	0.53	0.74	1.39	0.14	0.19	0.02	0.01	0.01	1.41
WA367	0.23	0.47	2.05	0.11	0.20	0.02	0.01	0.01	1.89

DMU	ν_2/ν_4	ν_2/ν_5	ν_2/ν_6	ν_3/ν_4	ν_2/ν_5	ν_3/ν_6	ν_4/ν_5	ν_4/ν_6	ν_5/ν_6
AL367	0.18	0.09	0.06	0.10	0.05	0.03	0.51	0.31	0.62
AZ367	0.12	0.07	0.08	0.06	0.04	0.04	0.57	0.69	1.22
CA367	0.16	0.09	0.03	0.32	0.18	0.05	0.57	0.16	0.29
CO367	0.18	0.09	0.05	0.19	0.10	0.06	0.50	0.29	0.57
CT367	0.18	0.09	0.04	0.23	0.11	0.06	0.50	0.24	0.49
FL367	0.17	0.09	0.07	0.14	0.07	0.06	0.50	0.40	0.81
GA367	0.14	0.08	0.10	0.08	0.04	0.06	0.53	0.69	1.29
IL367	0.18	0.09	0.04	0.23	0.12	0.06	0.51	0.25	0.48
IN367	0.17	0.09	0.03	0.34	0.18	0.06	0.54	0.16	0.30
IO367	0.17	0.09	0.05	0.19	0.10	0.06	0.50	0.29	0.58
KS367	0.14	0.07	0.10	0.08	0.04	0.06	0.52	0.69	1.32
MA367	0.17	0.09	0.04	0.23	0.12	0.06	0.50	0.24	0.48
MD367	0.18	0.09	0.05	0.19	0.10	0.06	0.50	0.29	0.57
ME367	0.18	0.09	0.05	0.21	0.10	0.06	0.50	0.27	0.53
MI367	0.18	0.09	0.05	0.18	0.09	0.06	0.51	0.30	0.59
MN367	0.17	0.09	0.07	0.14	0.07	0.06	0.50	0.41	0.81
MO367	0.15	0.09	0.04	0.13	0.07	0.03	0.57	0.25	0.44
NC367	0.14	0.08	0.06	0.08	0.04	0.03	0.57	0.43	0.75
NE367	0.18	0.09	0.05	0.19	0.10	0.06	0.50	0.29	0.57
NH367	0.17	0.09	0.07	0.14	0.07	0.06	0.50	0.41	0.81
NJ367	0.17	0.09	0.07	0.14	0.07	0.06	0.50	0.41	0.81
NM367	0.14	0.07	0.10	0.08	0.04	0.06	0.52	0.69	1.32
NY367	0.12	0.06	0.08	0.06	0.03	0.04	0.50	0.66	1.32
OH367	0.17	0.09	0.04	0.22	0.11	0.06	0.50	0.25	0.50
OK367	0.17	0.09	0.03	0.20	0.10	0.03	0.50	0.16	0.32
OR367	0.17	0.09	0.09	0.09	0.05	0.05	0.50	0.50	1.01
PA367	0.16	0.09	0.03	0.32	0.18	0.05	0.57	0.16	0.29
RI367	0.18	0.09	0.05	0.18	0.09	0.06	0.51	0.30	0.59
SC367	0.17	0.09	0.04	0.22	0.11	0.06	0.50	0.25	0.50
TN367	0.16	0.09	0.04	0.13	0.08	0.03	0.57	0.24	0.43
TX367	0.14	0.07	0.10	0.08	0.04	0.05	0.52	0.69	1.32
UT367	0.17	0.09	0.04	0.15	0.07	0.03	0.50	0.22	0.44
VA367	0.11	0.06	0.08	0.08	0.04	0.06	0.52	0.69	1.32
WA367	0.14	0.08	0.08	0.08	0.04	0.04	0.53	0.58	1.09

Table 5.37 SIC 367 1982 Data: Comparative Advantage

DMU	θ	ϕ	Closest Competitors
CA367	1.00	0.47	NY367
GA367	1.15	1.00	KS367 GA367
KS367	1.09	1.00	NM367
NC367	1.00	0.88	VA367 NY367
NM367	1.10	1.00	KS367 GA367 OR367
NY367	1.00	0.91	CA367 NC367
OR367	1.00	0.74	NM367 VA367
PA367	1.00	0.95	CA367 NC367 VA367
TX367	1.00	0.98	CA367 OR367
VA367	1.00	0.69	GA367 NC367 OR367

Table 5.38 SIC367 1982 Data:
Comparative Advantage

	DATA SET								
DMU	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6	Output 1	Output 2	Output 3
CA367	3615.70	1290.20	1489.80	159.40	82.20	52.90	3469.60	9797.80	6249.60
GA367	44.20	11.30	6.40	1.70	0.90	0.30	28.60	91.10	46.30
KS367	25.10	8.90	5.50	1.60	0.80	0.30	11.60	51.20	26.00
NC367	886.00	94.20	160.70	13.20	6.90	5.10	424.50	1417.10	679.40
NM367	15.30	10.50	5.70	1.80	0.90	0.30	17.70	50.00	33.90
NY367	2679.70	428.00	807.80	57.40	28.70	28.40	1297.30	5201.10	2533.10
OR367	49.40	30.00	72.80	4.20	2.20	2.90	215.10	364.10	317.90
PA367	824.10	371.00	220.70	40.60	222.00	8.10	820.70	2245.80	1412.40
TX367	601.50	276.80	404.90	36.60	18.20	15.00	866.70	2140.00	1548.40
VA367	160.70	57.50	28.80	7.40	3.80	1.20	248.90	493.20	335.20

	TOTAL WASTE and SUSTITUTIONS								
DMU	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6	Output 1	Output 2	Output 3
CA367	936.00	862.20	682.00	102.00	53.50	24.50	-2172.30	-4596.70	-3716.50
GA367	7.85	-1.63	-1.03	-0.38	-0.15	-0.08	2.69	-3.22	5.36
KS367	9.80	-1.60	-0.20	-0.20	-0.10	0.00	6.10	-1.20	7.90
NC367	344.54	-19.30	14.15	-1.76	-0.66	-0.21	-17.13	-212.28	-11.98
NM367	-10.58	1.31	-0.56	0.17	0.08	-0.03	-3.43	5.68	-4.18
NY367	804.56	-99.58	165.49	-8.78	-5.49	5.98	230.63	-747.16	164.72
OR367	-48.96	-7.35	53.90	-0.80	-0.36	2.09	-65.32	-60.91	-111.87
PA367	0.00	107.19	-64.72	7.59	4.97	-2.08	-21.86	-106.83	-64.33
TX367	-131.23	5.34	60.59	2.66	0.67	2.52	-28.01	31.67	-93.94
VA367	0.00	28.54	-21.23	3.28	1.64	-0.64	-102.86	-128.45	-110.16

	PRICE RATIOS								
DMU	μ_1/μ_2	μ_1/μ_3	μ_2/μ_3	v_1/v_2	v_1/v_3	v_1/v_4	v_1/v_5	v_1/v_6	v_2/v_3
CA367	0.59	0.68	1.16	-.--	-.--	-.--	-.--	-.--	-.--
GA367	0.23	0.47	2.05	0.14	0.15	0.02	0.01	0.01	1.07
KS367	-.--	-.--	-.--	0.11	0.20	0.01	0.01	0.01	1.89
NC367	0.23	0.47	2.05	0.11	0.20	0.01	0.01	0.01	1.89
NM367	0.59	0.68	1.14	0.69	0.68	0.12	0.06	0.04	0.99
NY367	0.23	0.47	2.07	0.11	0.20	0.02	0.01	0.01	1.89
OR367	0.59	0.68	1.15	0.69	1.29	0.09	0.04	0.06	1.89
PA367	0.23	0.47	2.06	0.42	0.21	0.07	0.04	0.01	0.50
TX367	0.59	0.68	1.15	0.69	1.30	0.02	0.06	0.06	1.89
VA367	0.59	0.68	1.15	0.30	0.15	0.05	0.03	0.01	0.50
DMU	v_2/v_4	v_2/v_5	v_2/v_6	v_3/v_4	v_3/v_5	v_3/v_6	v_4/v_5	v_4/v_6	v_5/v_6
CA367	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
GA367	0.11	0.06	0.04	0.10	0.06	0.04	0.53	0.40	0.75
KS367	0.14	0.08	0.10	0.07	0.04	0.05	0.53	0.69	1.29
NC367	0.14	0.08	0.06	0.08	0.04	0.03	0.53	0.43	0.80
NM367	0.17	0.09	0.05	0.17	0.09	0.06	0.50	0.32	0.63
NY367	0.14	0.07	0.10	0.07	0.04	0.05	0.53	0.69	1.29
OR367	0.12	0.07	0.09	0.07	0.03	0.05	0.52	0.69	1.32
PA367	0.16	0.09	0.03	0.32	0.18	0.05	0.57	0.16	0.29
TX367	0.17	0.09	0.09	0.09	0.05	0.05	0.50	0.50	1.00
VA367	0.16	0.09	0.03	0.32	0.18	0.05	0.57	0.16	0.29

CHAPTER 6

CONCLUSIONS

Over the past decades the explosion of information technology has altered the dynamics of economic competition by accelerating the pace toward equilibria in all economic and geographic spheres. Adequate and fast processing of information is a requisite to develop strategies and policies that will maintain and foster economic health and wealth. However, preliminary to continuous improvement, the phrase so often construed as the key to guaranteed productivity and competitiveness in industrial circles, are continuous assessment of one's competitive environment and of one's relative performance within that environment. The contributions this dissertation made relate to the latter, closely intertwined tasks. These contributions are summarized and put into perspective in this chapter.

The performance of an economic or decision-making unit is assessed by means of comparison with the performance of other economic units. An immediate difficulty stems from the fact that any reasonable and constructive assessment is likely to demand **simultaneous** consideration of **multiple** indicators of performance. Moreover, an important characteristic of the conclusions of such assessments is their relativity. It is recognized, at the outset, that there is no single, absolute, normative best performance. Instead there is an array of acknowledged best, or declared efficient, performances which, together, constitute an empirical frontier of feasible achievements. The concept of frontier conveys the message that extremal levels are sought for the indicators of performance. In particular, when higher levels of an indicator are preferred, that indicator is generically referred to as an "output" measure. Similarly, when lower levels of an indicator are preferred, that indicator is generically referred to as an "input" measure. It follows that the first step toward performance assessment is the selection of relevant input and output measures. Once this is done the actual performance assessment is effected through a three-point procedure:

i) identify the empirical frontier defined by the selected measures. Depending on the nature of the selected indicators, specific characteristics may be expected regarding the shape of the frontier.

ii) identify, for each economic unit, an ideal performance on that frontier. The statement of this point is intended to stress the fact that there cannot be a value-free assessment. Tradeoffs across the selected measures are reflected in the selection of an efficient, i.e. frontier defining, performance and these tradeoffs have to be recognized.

iii) gauge each economic unit's distance between their current and identified ideal performances, thereby measuring their relative efficiency.

Each component of the assessment procedure, namely the choice of performance indicators, the characteristics of the envelopment surface, the valuation mechanism, has to be reflected upon and designed carefully to ensure that the results of the assessment and their associated recommendations match the concerns and objectives of the user of such assessments.

To this effect Chapter 2 took an indepth look at concepts and measures of efficiency. In dealing with measures of efficiency, necessary properties, that entail some fairness in the comparison of the efficiency score across assessed units, were considered and served to clarify and contrast the purpose of existing efficiency measures as well as define the expression of a new measure that satisfies all these necessary/desirable properties. This new measure incorporates recommendations to the evaluated units on how to gain efficiency. Indeed the measure gauges the distance between a unit's current position and its ideal position on the frontier of feasible and acceptable achievements, thereby indicating what this ideal position is.

Chapter 3 dealt with the methodological aspects of efficiency measurement and showed how highly effective a tool Data Envelopement Analysis (DEA) can be to assess the relative efficiency of decision-making units. In particular Chapter 3 offered a unifying perspective of DEA models. A taxonomy for the models was developed that stresses the differences across evaluation procedures. These differ in terms of embedded economic principles and implicit managerial objectives. The taxonomy affords systematic connections between the various models and production theory, hence providing a consistent interpretation of all models along with their limitations within the context of production theory. In particular the taxonomy differentiates the various DEA models according to the type of envelopment surface that is assumed, the prioritization of the search for waste across types of measures (input, output, or no distinction), and the pricing mechanism applied to the measures and their associated amounts of waste. That is, given a type of envelopment surface: CRS (i.e. constant-returns-to-scale or polyhedral cone) or VRS (i.e. variable-returns-to-scale or convex polyhedron), DEA models, in the process of identifying excessive input consumption and deficient output production, may focus primarily on controlling input consumption or primarily on output production, or on avoiding waste without distinction between inputs and outputs. Moreover, the models account for the identified waste via a pricing mechanism that may offer various degrees of flexibility regarding the relative tradeoffs across inputs and across outputs that the estimated prices imply. The viability of the DEA methodology hinges on the realism that the implemented pricing mechanism imparts the evaluations. Hence, the central role of the pricing mechanism was emphasized. The identification of relative values (efficiency prices) for the various measures allows the aggregation of non-commensurable inputs and outputs necessary to compute efficiency scores and rank/compare the observed behaviors. The proposed taxonomy further distinguished between two classes of pricing mechanisms: explicit pricing mechanism and constrained implicit pricing mechanisms. An explicit pricing mechanism attributes explicit values to the measures for which waste has been identified but it allows free determination of prices when no waste is present. A constrained implicit pricing mechanism recognizes that price ratios across measures represent tradeoffs across measures and that these tradeoffs are constrained to some acceptable ranges, either by market conditions, by societal valuations, or by the conscious choice of decision-making units. Consequently this latter mechanism allows prices to be determined freely across measures subject to the condition that they evaluate to permissible tradeoffs. A new model, called the Frontier model, was developed with the specific purpose to strengthen the bridge between DEA and economics by allowing the measurement of economic efficiency. That is, this new model ensures that pricing is consistently effected across all units and completely avoids unrealistic implicit tradeoff values across inputs and across inputs. The Frontier model is built on the assumption that a DMU's current input ratios and output ratios are characteristic of the economic environment and supply and demand conditions in which the DMU operates. It follows that, in dealing with its main concerns of gauging inefficiency and identifying sources of inefficiency, the model seeks to identify waste proportionately across all input and across all output measures.

Chapter 4 built on the results of Chapter 3 and formalized the application of Data Envelopment Analysis to strategic planning. A new mathematical model, called the Comparative Advantage model, was developed that adapts the methodology of Data Envelopment to identify a DMU's most direct competitors and, hence, derive information regarding the DMU's comparative strengths and weaknesses. In this type of analysis input measures are aggregated to form a universal "cost" function, and, analogously, output measures are aggregated to form a universal "revenue" function. The identified competitors threaten the unit's efficiency by means of similar low cost operations or high revenue output mixes. The assessment of its current competitive environment represents critical information to assist the unit in formulating its strategy to help it maintain its efficiency and eventually sharpen its competitive edge. The Comparative Advantage model also allows the extent of the unit's comparative advantage to be gauged providing the unit with a means to control the progress and success of its strategy, defined in terms of choice of technique of production (described by its input ratios) and its choice of output mix. Finally, central to the formulation of a strategy are the identification of strategic prices which define tradeoffs across inputs and across outputs. The Comparative Advantage model by evaluating such strategic prices allows the derivation of these tradeoffs. An efficient unit can maintain its efficient status and protect its comparative advantage as long as substitutions, effected according to these tradeoffs, keep the unit within the facet of the frontier where these tradeoffs are valid.

An application to regional economics was presented in Chapter 5. A comparative statics analysis was performed to evaluate the relative efficiency of the economy of states in the U.S. This illustrative study showed that frontier analysis can be used to gauge the effect of changes over time and that the Frontier model, geared toward the identification of sources of comparative disadvantage, offers ample flexibility through the specification of ranges for allowable tradeoffs across measures. The specification of these ranges is critical to the evaluation and allows filtering of performances. A second study was aimed at identifying, for a particular industry, which states were the leaders and what their comparative advantages were. The Comparative Advantage model was applied to the states found to exhibit frontier behavior and revealed which measures contributed to building comparative advantage. Such information is valuable to help a state design assistance programs and target incentives that will promote growth and strength of the state's economy. Moreover, these studies revealed the acute need for identifying and collecting relevant data to support decision-making. Economic data series which are produced annually, while providing current information, are typically limited in scope (geographic and industry detail). Relevant data, unfortunately, is often delayed. For effective use of such analyses in regional planning or any other area of application, the timeliness of the collection of such data is crucial.

Data Envelopment Analysis stands as a viable methodology to investigate any type of frontier behavior. The host of available models are as many testimonies of the versatility of the methodology. Beyond the common principles of identifying an envelopment surface, it should be clearly understood that each of these available models effect evaluations and offer recommendations that are answers to different questions. A lot can be learned by "evaluating the evaluations". A wrong evaluation might be the best indicator that the wrong question was asked, i.e. that an inadequate model was selected. Recognizing similarities across areas of research in terms of the types of questions asked will contribute to the dissemination of the methodology to new areas of applications. Alternatively, recognizing new questions will lead to the development of new models and the advancement of the theory of Data Envelopment Analysis.

APPENDIX A

MONOTONICITY OF DEBREU-FARRELL MEASURE OF EFFICIENCY

Lemma: Given the class of technologies described by (IC1)-(IC4) and the assumption of strong disposability of essential inputs,

$E_{DF}(x,y) = \text{Min}\{\ \theta: \theta x \in L(y)\}$ is monotonically decreasing in inputs:

$x^* \leq x \Rightarrow E_{DF}(x^*,y) \geq E_{DF}(x,y)$.

Proof: Let x, $x' \in L(y)$ and $\theta^* = \text{Min}\{\ \theta: \theta x \in L(y)\}$

$x' \geq x \Rightarrow \theta^* x' \geq \theta^* x$

$\theta^* x' \in L(y)$ by weak disposability of inputs,

hence, $\theta' = \text{Min}\{\ \theta: \theta x' \in L(y)\}$ is such that $\theta' \leq \theta^*$ ■

APPENDIX B

STRICT MONOTONICITY OF FÄRE-LOVELL MEASURE OF EFFICIENCY

Lemma: Given the class of technologies described by (IC1)-(IC4) and the assumption of strong disposability of essential inputs,

$$E_{FL}(x,y) = \mathrm{Min}_{x^e \in Eff(y)} \left\{ \frac{1}{m} \sum_{i=1}^{m} \frac{x_i^e}{x_i} \right\}$$

is monotonically stricly decreasing in inputs:

$x^* \leq x \Rightarrow E_{FL}(x^*,y) > E_{FL}(x,y).$

Proof: Let $x, x' \in L(y)$, $x' \geq x$, $x' = x + \Delta x$ with $\Delta x \neq 0$

$$E_{FL}(x,y) = \frac{1}{m} \sum_{i=1}^{m} \frac{x_i^*}{x_i}$$

$$\frac{1}{m} \sum_{i=1}^{m} \frac{x_i^*}{x_i'} = \frac{1}{m} \sum_{i=1}^{m} \frac{x_i^*}{x_i + \Delta x_i} < \frac{1}{m} \sum_{i=1}^{m} \frac{x_i^*}{x_i} \quad \text{since } x_i^* > 0 \ \forall \ i$$

It follows:

$$E_{FL}(x',y) = \mathrm{Min}_{x^e \in Eff(y)} \left\{ \frac{1}{m} \sum_{i=1}^{m} \frac{x_i^e}{x_i} \right\} \leq \frac{1}{m} \sum_{i=1}^{m} \frac{x_i^*}{x_i'} < E_{FL}(x,y)$$

■

APPENDIX C

THE INVARIANT ADDITIVE DEA MODEL

The invariant additive model determines which of n decision-making units (DMUs) determine an envelopment surface when considering m input variables and s output variables. For each decision-making unit $j = 1,\ldots,n$, x_{ij} denote the i^{th} input measure and y_{rj} denote the r^{th} output measure. For each decision-making unit, l, the following linear program is solved.

$$\mathrm{Min}_{\lambda,s,e} \; - \left(\sum_{r=1}^{s} \frac{1}{y_{rl}} s_r + \sum_{i=1}^{m} \frac{1}{x_{il}} e_i \right)$$

$$\text{st} \begin{cases} \sum_{j=1}^{m} y_{rj}\lambda_j - s_r = y_{rl} & r = 1,\ldots,s \\ -\sum_{j=1}^{n} x_{ij}\lambda_j - e_i = -x_{il} & i = 1,\ldots,m \\ \sum_{j=1}^{n} \lambda_j = 1 & \\ \lambda_j \geq 0 & j = 1,\ldots,n \\ s_r,\, e_i \geq 0 & r = 1,\ldots,s \;\; i = 1,\ldots,m \end{cases} \qquad \text{(ADD Primal)}$$

An optimal solution to the above linear program identifies an n-vector λ, an s-vector **s**, and an m-vector **e**. The convex combination of DMUs j such that $\lambda_j > 0$ represents a point on the envelopment surface against which DMU_l's efficiency is gauged. Sources of inefficiency for DMU_l are given by **s** which represents slack output quantities for DMU_l, and by **e** which represents excess input consumptions for DMU_l.

The corresponding dual program, whose formulation follows, identifies an s-vector $\boldsymbol{\mu}$, an s-vector $\boldsymbol{\nu}$, and a scalar ω_l.

$$\text{Max}_{\mu,\nu,\omega_l} \sum_{r=1}^{s} \mu_r y_{rl} - \sum_{i=1}^{m} \nu_i x_{il} + \omega_l$$

$$\text{st} \begin{cases} \displaystyle\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} + \omega_l \leq 0 & j = 1,\ldots,n \\ \mu_r \geq \dfrac{1}{y_{rl}} & r = 1,\ldots,s \\ \nu_i \geq \dfrac{1}{x_{il}} & i = 1,\ldots,m \\ \omega_l \ \text{unrestricted} & \end{cases} \qquad \text{(ADD Dual)}$$

The vectors μ and ν are efficiency price vectors which serve to define a revenue function and a cost function respectively. The scalar ω_l at optimality represents the negative of the maximum profit level attainable by any DMU given the prices μ and ν.

APPENDIX D

THE FRONTIER MODEL

The frontier model determines which of n decision-making units (DMUs) determine an envelopment surface when considering m input variables and s output variables. For each decision-making unit $j = 1,\ldots,n$, x_{ij} denote the i^{th} input measure and y_{rj} denote the r^{th} output measure. x_j and y_j denote, respectively, the vectors of input and output measures and **X** and **Y** respectively denote the m x n matrix of inputs and s x n matrix of outputs.

The vectors μ and ν are efficiency price vectors which serve to define a revenue function and a cost function respectively. The scalar ω_l at optimality represents the negative of the maximum profit level attainable by any DMU given the prices μ and ν.

The frontier model makes the role of prices explicit in the evaluation of efficiency. This is accomplished by restricting the range of rates of substitution across inputs and across outputs that are implied by the ratios of identified efficiency prices. For any two inputs i, j, and any two outputs k, l, lower and upper bounds for their respective rates of substitution are predefined that translates into the following constraints:

$$\underline{r}^{ij} \leq \frac{\nu_i}{\nu_j} \leq \overline{r}_{ij}$$

$$\underline{r}^{kl} \leq \frac{\mu_k}{\mu_l} \leq \overline{r}_{kl}$$

These conditions are easily converted into linear constraints on the efficiency prices to be estimated:

$$-\nu_i + \underline{r}^{ij}\nu_j \le 0 \qquad \nu_i - \overline{r}_{ij}\nu_j \le 0$$
$$-\mu_k + \underline{r}^{kl}\mu_l \le 0 \qquad \mu_k - \overline{r}_{kl}\mu_l \le 0$$

Such constraints summarize to a global matrix format:

$$\nu R_i \le 0 \qquad \mu R_o \le 0$$

where $\mathbf{R}_i$ and $\mathbf{R}_o$ are respectively m x $2\binom{m}{2}$ and s x $2\binom{s}{2}$. These constraints, added to the dual model, translate into the inclusion of substitution variables, summarized by the $2\binom{m}{2}$-vector σ_i, and the $2\binom{s}{2}$-vector σ_o, in the primal formulation.

The scalars θ and ϕ represent respectively the proportional input reduction from DMU$_l$'s levels and the proportional output augmentation from DMU$_l$'s levels that would render DMU$_l$ efficient. In particular DMU$_l$ is efficient iff $\theta = 1$ and $\phi = 1$.

The mathematical statement of the primal model is given by:

$$\mathrm{Min}_{\theta,\phi,\lambda,\sigma_i,\sigma_o} \; \theta - \phi$$
$$st \left\{ \begin{array}{rl} -\phi y_l + Y\lambda + \mathbf{R}_o\sigma_o &= 0 \\ \theta x_l - X\lambda + \mathbf{R}_i\,\sigma_i &= 0 \\ 1.\lambda &= 1 \\ \phi &\ge 1 \\ -\theta &\ge -1 \\ \lambda,\ \sigma_o,\ \sigma_i &\ge 0 \end{array} \right. \qquad \text{(Frontier Primal)}$$

The corresponding dual is given by:

$$
\begin{aligned}
&\text{Max}_{\mu,\nu,\omega_1,R,C} \quad \omega_1 + R - C \\
&st \begin{cases}
\mu\mathbf{Y} - \nu\mathbf{X} + \omega_1 \leq 0 \\
\nu\mathbf{x}_1 - C \leq 1 \\
-\mu\mathbf{y}_1 + R \leq -1 \\
\nu\mathbf{R}_i \leq 0 \\
\mu\mathbf{R}_o \leq 0 \\
R,\ C \geq 0 \\
\mu,\ \nu,\ \omega_1 \text{ unrestricted}
\end{cases}
\end{aligned}
\qquad \text{(Frontier Dual)}
$$

The scalars R and C are such that R - C evaluates to the profit level attained by DMU_1 given the identified prices μ and ν.

APPENDIX E

THE COMPARATIVE ADVANTAGE MODEL

The comparative advantage model effects a relative assessment of patterns of input consumptions and output productions across n decision-making units (DMUs) when considering m input variables and s output variables. For each decision-making unit $j = 1,\ldots,n$, x_{ij} denote the i^{th} input measure and y_{rj} denote the r^{th} output measure. $\mathbf{x}_j$ and $\mathbf{y}_j$ denote, respectively, the vectors of input and output measures and $\mathbf{X}$ and $\mathbf{Y}$ respectively denote the m x (n-1) matrix of inputs and s x (n-1) matrix of outputs across all DMUs except DMU_l being evaluated.

The vectors μ and ν are efficiency price vectors which serve to define a revenue function and a cost function respectively. The scalar ω_l at optimality represents the negative of the maximum profit level attainable by any DMU other than DMU_l being evaluated, given the prices μ and ν.

The comparative advantage model makes the role of prices explicit in the evaluation. This is accomplished by restricting the range of rates of substitution across inputs and across outputs that are implied by the ratios of identified efficiency prices. For any two inputs i, j, and any two outputs k, l, lower and upper bounds for their respective rates of substitution are predefined that translates into the following constraints:

$$\underline{r}^{ij} \leq \frac{\nu_i}{\nu_j} \leq \overline{r}_{ij}$$

$$\underline{r}^{kl} \leq \frac{\mu_k}{\mu_l} \leq \overline{r}_{kl}$$

These conditions are easily converted into linear constraints on the efficiency prices to be estimated:

$$-\nu_i + \underline{r}^{ij}\nu_j \le 0 \qquad \nu_i - \overline{r}_{ij}\nu_j \le 0$$
$$-\mu_k + \underline{r}^{kl}\mu_l \le 0 \qquad \mu_k - \overline{r}_{kl}\mu_l \le 0$$

Such constraints summarize to a global matrix format:

$$\nu R_i \le 0 \qquad \mu R_o \le 0$$

where $\mathbf{R}_i$ and $\mathbf{R}_o$ are respectively m x $2\binom{m}{2}$ and s x $2\binom{s}{2}$. These constraints, added to the dual model, translate into the inclusion of substitution variables, summarized by the $2\binom{m}{2}$-vector σ_i, and the $2\binom{s}{2}$-vector σ_o, in the primal formulation.

The scalars θ and ϕ represent respectively the proportional input augmentation from DMU_l's levels and the proportional output reduction from DMU_l's levels that would match the levels of the best convex combination of other DMUs to match DMU_l in terms of revenues and costs. In particular DMU_l has a comparative advantage in inputs if $\theta > 1$ and it has a comparative advantage in outputs if $\phi < 1$ at optimality.

The mathematical statement of the primal model is given by:

$$\text{Min}_{\theta,\phi,\lambda,\sigma_i,\sigma_o} \ \theta - \phi$$
$$st \begin{cases} -\phi y_l + Y\lambda + \mathbf{R}_o\sigma_o = 0 \\ \theta x_l - X\lambda + \mathbf{R}_i\sigma_i = 0 \\ 1.\lambda = 1 \\ -\phi \ge -1 \\ \theta \ge 1 \\ \lambda, \sigma_o, \sigma_i \ge 0 \end{cases} \qquad \text{(C-A Primal)}$$

The corresponding dual is given by:

$$\mathrm{Max}_{\mu,\nu,\omega_1,R,C} \quad \omega_1 - (R - C)$$

$$\text{st} \begin{cases} \mu Y - \nu X + \omega_1 \leq 0 \\ \nu X_1 + C \leq 1 \\ \mu Y_1 + R \geq 1 \\ \nu R_i \leq 0 \\ \mu R_o \leq 0 \\ R,\ C \geq 0 \\ \mu,\ \nu,\ \omega_1 \text{ unrestricted} \end{cases} \qquad \text{(C-A Dual)}$$

The scalars R and C are such that R - C evaluates to the profit level attained by DMU_1 given the identified prices μ and ν.

APPENDIX F

STANDARD INDUSTRIAL CLASSIFICATION
MANUFACTURING DIVISION

20 FOOD AND KINDRED PRODUCTS
201 Meat Products
202 Dairy Products
203 Preserved Fruit & Vegetables
204 Grain Mill Products
205 Bakery Goods
206 Sugar & Confectionery Products
207 Fats & Oils
208 Beverages
209 Misc. Food & Kindred Products

21 TOBACCO PRODUCTS
211 Cigarettes
212 Cigars
213 Chewing and Smoking Tobacco
214 Tobacco Stemming and Redrying

22 TEXTILE MILL PRODUCTS
221 Broadwoven Fabric Mills, Cotton
222 Broadwoven Fabric Mills, Manmade
223 Broadwoven Fabric Mills, Wool
224 Narrow Fabric Mills
225 Knitting Mills
226 Textile Finishing, Except Wool
227 Carpet and Rugs
228 Yarn and Thread Mills
229 Misc. Textile Goods

23 APPAREL AND OTHER TEXTILE PRODUCTS
231 Men's and Boy's Suits and Coats
232 Men's and Boy's Furnishings
233 Women's and Misses' Outerwear
234 Women's and Misses' Undergarments
235 Hats, Caps, and Millinery
236 Girl's and Children's Outerwear
237 Fur Goods
238 Misc. Apparel and Accessories
239 Misc. Fabricates Textile Products

24 LUMBER AND WOOD PRODUCTS
241 Logging
242 Sawmills and Planing Mills
243 Millwork, Plywood, & Struc. Members
244 Wood Containers
245 Wood Building and Mobile Homes
249 Misc. Wood Products

25 FURNITURE AND FIXTURES
251 Household Furniture
252 Office Furniture
253 Public Building & Related Furniture
254 Partitions and Fixtures
259 Misc. Furniture and Fixtures

26 PAPER AND ALLIED PRODUCTS
261 Pulp Mills
262 Paper Mills
263 Paperboard Mills
265 Paperboard Containers & Boxes
267 Misc. Converted Paper Products

27 PRINTING AND PUBLISHING
271 Newspapers
272 Periodicals
273 Books
274 Misc. Publishing
275 Commercial Printing
276 Manifold Business Forms
277 Greeting Cards
278 Blankbooks and Bookbinding
279 Printing Trade Services

28 CHEMICALS AND ALLIED PRODUCTS
281 Industrial Inorganic Chemicals
282 Plastic Materials and Synthetics
283 Drugs
284 Soap, Cleansers, and Toilet Goods
285 Paint and Allied Products
286 Industrial Inorganic Chemicals
287 Agricultural Chemicals
289 Misc. Chemical Products

29 PETROLEUM AND COAL PRODUCTS
291 Petroleum Refining
299 Misc. Petroleum & Coal Products

295 Asphalt Paving & Roofing Materials

30 RUBBER AND MISC. PLASTIC PRODUCTS
301 Tires and Inner Tubes
302 Rubber and Plastics Footwear
305 Hose & Belting & Gaskets & Packing
306 Fabricated Rubber Products, NEC
308 Misc. Plastics Products, NEC

31 LEATHER AND LEATHER PRODUCTS
311 Leather Tanning and Finishing
313 Footwear Cut Stock
314 Footwear, except rubber
315 Leather Gloves and Mittens
316 Luggage
317 Handbags & Pers. Leath. Products
319 Leather Goods, NEC

32 STONE, CLAY, AND GLASS PRODUCTS
321 Flat Glass
322 Glass & Glassware, Pressed, Blown
323 Products of Purchases Glass
324 Cement, Hydraulic
325 Structural Clay Products
326 Pottery and Related Products
327 Concrete, Gyps., & Plaster Prod.
328 Cut Stone and Stone Products
329 Misc. Nonmetal. Mineral Products

33 PRIMARY METAL INDUSTRIES
331 Blast Furnace & Basic Steel Products
332 Iron and Steel Foundries
333 Primary Nonferrous Metals
334 Secondary Nonferrous Metals
335 Nonferrous Rolling and Drawing
336 Nonferrous Foundries (Castings)
339 Misc. Primary Metal Products

34 FABRICATED METAL PRODUCTS
341 Metal Cans and Shipping Containers
342 Cutlery, Handtools, and Hardware
343 Plumbing and Heating, exc. Electrics
344 Fabricated Structural Metal Products
345 Screw Machine Prod., Bolts, etc.
346 Metal Forgings and Stampings
347 Metal Services, NEC
348 Ordnance and Accessories, NEC
349 Misc. Fabricated Metal Products

35 INDUSTRIAL MACHINERY AND EQUIPMENT
351 Engines and Turbines
352 Farm and Garden Equipment
353 Construction and Related Machinery
354 Metalworking Machinery
355 Special Industry Machinery
356 General Industrial Machinery
357 Computer and Office Equipment
358 Refriger. and Service Machinery
359 Industrial Machinery, NEC

36 ELECTRONIC & OTHER ELECTRIC EQUIPMENT
361 Electric Distribution Equipment
362 Electrical Industrial Apparatus
363 Household Appliances
364 Electrical Lighting & Wiring Equipment
365 Househ. Audio & Video Equipmt
366 Communications Equipment
367 Electronic Compon. & Accessor.
369 Misc, Electrical Equip. & Supplies

37 TRANSPORTATION EQUIPMENT
371 Motor Vehicles and Equipment
372 Aircraft and Parts
373 Ship & Boat Building & Repairing
374 Railroad Equipment
375 Motorcycles, Bicycles, and Parts
376 Guided Miss., Space Vehic., Parts
379 Misc. Transportation Equipment

38 INSTRUMENTS AND RELATED EQUIPMENT
381 Search & Navigation Equipment
382 Measuring and Controlling Devices
384 Medical Instruments & Supplies
385 Ophthalmic Goods
386 Photographic Equip. & Supplies
387 Watches, Clocks, Watchcas., Parts

39 MISC. MANUFACTURING INDUSTRIES
391 Jewelry, Silverware & Plated Ware
393 Musical Instruments
394 Toys and Sporting Goods
395 Pens, Pencils, Off. & Art Supplies
396 Costume Jewelry and Notions
399 Misc. Manufactures

REFERENCES

Ali, A.I., 1991, "IDEAS: Integrated Data Envelopment Analysis System", Technical report, Department of General Business and Finance, University of Massachusetts at Amherst, MA.

Ali A.I., C.S. Lerme, and L. Seiford, 1992, "Data Envelopment Analysis: a Unifying Perspective", Technical Report, Deapartment of General Business and Finance, School of Management, University of Massachusetts, Amherst.

Banker R.D., 1984, "Estimating Most Productive Scale Size using Data Envelopment Analysis", European Journal of Operational Research, **17**, pp 35-44.

Banker R.D., A. Charnes, and W.W. Cooper, 1984, "Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis", Management Science, **30**, pp 1078-1092.

Banker R.D., A. Charnes, W.W. Cooper, J. Swarts, and D.A. Thomas, 1989, "An Introduction to Data Envelopment Analysis with some of its Models and their Uses", Research in Governmental and Nonprofit Accounting, **5**, pp 125-163.

Banker R.D., and A. Maindiratta, 1988, "Non-parametric Analysis of Technical and Allocative Efficiencies in Production", Econometrica, **56**, pp 1315-1332.

Banker R.D., and R.C. Morey, 1989, "Incorporating Value Judgments in Efficiency Analysis", Research in Governmental and Nonprofit Accounting, **5**, pp 245-267.

Banker R.D., and R.M. Thrall, 1990, "Estimation of Returns to Scale Using Data Envelopment Analysis", Working Paper No 81, Rice University.

Boyd, and R. Färe, 1984, "Measuring the Efficiency of Decision-Making Units: A Comment", European Journal of Operational Research, **15**, pp 331-332.

Charnes A., and W.W. Cooper, 1984, "The Non-Archimedean CCR Ratio for Efficiency Analysis: A Rejoinder to Boyd and Färe", European Journal of Operational Research, **15**, pp 333-334.

Charnes A., and W.W. Cooper, 1985, "Preface to Topics in Data Envelopment Analysis", Annals of Operations Research, **2**, pp 59-94.

Charnes A., W.W. Cooper, Z.M. Huang, and D.B. Sun, 1990, "Polyhedral Cone-Ratio DEA Models with an Illustrative Application to Large Commercial Banks", Journal of Econometrics, **46**, pp 73-91.

Charnes A., W.W. Cooper, Z.M. Huang, and D.B. Sun, 1991, "Relations between Half-Space and Finitely Generated Cones in Polyhedral Cone-Ratio DEA Models", International Journal of Systems Science, **22**, pp 2057-2077.

Charnes A., W.W. Cooper, B. Golany, L. Seiford, and J. Stutz, 1985, "Foundations of Data Envelopment Analysis for Pareto-Koopmans Efficient Empirical Production Functions", Journal of Econometrics, **30**, pp 91-107.

Charnes A., W.W. Cooper, and E. Rhodes, 1978, "Mesuring the Efficiency of Decision Making Units", European Journal of Operational Research, **2**, pp 429-444.

Charnes A., W.W. Cooper, and R.M. Thrall, 1986, "Classifying and Characterizing Efficiencies and Inefficiencies in Data Envelopment Analysis", Operations Research Letters, **5**, pp 105-110.

Cline W.R., 1989, "Macroeconomic Influences on Trade Policy", AEA papers and Proceedings, 79(2), pp 123-127.

Day, D.L., J.U. Farley, and J. Wind, 1990, "New Perspectives on Strategy Research: a View from the Management Sciences", Management Science, **36**, pp 0001-0006.

Drhymes, P.J., 1990, "The Structure of Production Technology: Evidence from the LRD Sample I", Proceedings of Annual Research Conference, Bureau of the Census, March.

Dyson R.G., and E. Thanassoulis, 1988, "Reducing Weight Flexibility in Data Envelopment Analysis", Journal of the Operational Research Society, **39**, pp 563-576.

Erickson, R.A., 1989, "Export Performance and State Industrial Growth", Economic Geography, **65**, pp 280-292.

Färe R., 1983, "On Strictly Monotonic Production Correspondences", <u>Quantitative Studies on Production and Prices</u>, Physica-Verlag, Würzburg-Wien.

Färe R., and C.A. Knox Lovell, 1978, "Measuring the Technical Efficiency of Production", Journal of Economic Theory, **19**, 150-162.

Färe R., C.A. Knox Lovell, and K. Zieschang, 1983, "Measuring the Technical Efficiency of Multiple Output Production Technologies", in <u>Quantitative Studies on Production and Prices</u>, Physica-Verlag, Würzburg-Wien.

Farrell M.J., 1957, "The Measurement of Productive Efficiency", Journal of the Royal Statistics Society, Ser. A, **120**, pp 253-290.

Fuss M., and D. McFadden (editors), 1978, Production Economics: A Dual Approach to Theory and Applications, North Holland.

Henderson J.M., and R. E. Quandt, 1980, Microeconomic Theory: A Mathematical Approach, McGraw-Hill Book Company, third edition.

Knox Lovell C.A., and P. Schmidt, 1988, "A Comparison of Alternative Approaches to the Measurement of Productive Efficiency" in Applications of Modern Production Theory: Efficiency and Productivity, A. Dogramaci, R. Fare editors, Kluwer Academic Publishers.

Lawrence R.Z., 1989, "Protection: Is there a better Way?", AEA papers and Proceedings, 79(2), pp 118-122.

Mansfield E., 1990, Managerial Economics: Theory, Applications, and Cases, W.W. Norton & Company, first edition.

Petersen N.C., 1990, "Data Envelopment Analysis on a Relaxed Set of Assumptions", Management Science, **36**, pp 305-314.

Plaut T.R., and J.E. Pluta, 1983, "Business Climate, Taxes and Expenditures, and State Industrial Growth in the U.S.", Southern Economic Journal, **50**, pp 99-119.

Roll Y., W.D. Cook, and B. Golany, 1991, "Controlling Factor Weights in Data Envelopment Analysis", IIE Transactions, **23**, pp 2-9.

Russell R.R., 1985, "Measures of Technical Efficiency", Journal of Economic Theory, **35**, pp 109-126.

Russell R.R., 1990, "Continuity of Measures of Technical Efficiency", Journal of Economic Theory, **51**, pp 255-267.

Seiford L.M., 1990, "A Bibliography of Data Envelopment Analysis (1978 - 1990)", Version 5.0, April.

Shephard R.W., 1953, Cost and Production Functions, Princeton University Press, Princeton, N.J..

Shephard R.W., 1970, Theory of Costs and Production Functions, Princeton University Press, Princeton, New Jersey.

Sexton T.R., 1986, "The methodology of Data Envelopment Analysis" in Measuring Efficiency: An Assessment of Data Envelopment Analysis, R. H. Silkman (ed.), no. 32, San Francisco: Jossey Bass, Winter.

Sexton T.R., R. H. Silkman, and A. J. Hogan, 1986, " Data Envelopment Analysis: Critique and Extensions" in Measuring Efficiency: An Assessment of Data Envelopment Analysis, R. H. Silkman (ed.), no. 32, San Francisco: Jossey Bass, Winter.

Thompson R.G., L.N. Langemeier, C.-T. Lee, E. Lee, and R.M. Thrall, 1990, "The Role of Multiplier Bounds in Efficiency Analysis with Apllication to Kansas Farming", Journal of Econometrics, **46**, pp 93-108.

Tulkens H., 1990, "Non-Parametric Efficiency Analysis in Four Service Activities: Retail Banking, Municipalities, Courts and Urban Transit", C.O.R.E. Discussion Paper, Université Catholique de Louvain, Belgium.

U.S. Bureau of the Census, 1989, Statistical Abstract of the United States: 1989, (109th edition), Washington, DC.

Wasylenko M., and T. McGuire, 1985, "Jobs and Taxes: The Effect of Business Climate on States' Employment Growth Rates", National Tax Journal, **38**, pp 497-511.

Wong Y.-H.B., and J.E. Beasley, 1990, "Restricting Weight Flexibility in Data Envelopment Analysis", Journal of the Operational Research Society, **41**, pp 829-835.

Zieschang K.,1984, "An Extended Farrell Technical Efficiency Measure", Journal of Economic Theory, **33**, 387-396.