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The return generating process of the Arbitrage Pricing Theory : intertemporal stationarity and cross-sectional congruence.

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The Return Generating Process of the Arbitrage Pricing Theory: Intertemporal Stationarity and Cross-sectional Congruence

A Dissertation Presented

By

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DEDICATION

This dissertation is dedicated to my parents, Paul and Greta McBain, my Grandmother, Lucy Gresens, and to the memory of Edward Gresens and Cecil and Virginia McBain. Without their support and encouragement, this dissertation could not have been completed.

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ABSTRACT

The linear returns generating process of the Arbitrage Pricing Theory is examined via a variety of heuristic measures and statistical tests. Interest centers on the intertemporal stationarity and cross-sectional congruence of the parameter estimates obtained from several samples. The time period covered by the analyses is July 1962 through December 1981. Empirical results indicate a significant degree of non-stationarity exists in the linear returns generating process, especially for the smaller dimension models. The evidence of cross-sectional congruence is mixed. The degree of congruence depends upon the subperiod under examination with earlier subperiods exhibiting a greater degree of congruence than later subperiods. Based on the empirical results, several issues regarding interpretation of previous empirical tests of the Theory are discussed and the implications of the results for portfolio management are noted.

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CHAPTER I

INTRODUCTION

Background

Theoretical formulations of the pricing of risky assets have developed rapidly since their introduction in the early 1960's. The development of the now familiar Capital Asset Pricing Model(CAPM) by Sharpe[92], Lintner[63], and Mossin[74] represented the first equilibrium asset pricing theory which rigorously examined the implications of the work of Markowitz[66]. The simplicity of the CAPM formulation and the widely available data on equity returns spawned a plethora of empirical tests of the model in the late 1960's and early 1970's.[1]

Paralleling the evolution of empirical tests of the CAPM was the specification of alternative forms of the model, variants of asset pricing models which were based, in part, on the intuition underlying the CAPM,[2] and applications of the model to corporate finance issues.[3]

Early empirical tests of the model were encouraging but as the number of researchers examining the model increased, so too did the number of empirical anamolies (i.e. the failure of some CAPM conclusions to be supported by the data).[4] In response to the anomalies, researchers derived various models which maintained the basic intuition of the CAPM but proved to be more consistent with observed security returns. An excellent example of such a formulation is Black's[5] zero-beta form of the CAPM.

In 1977, Roll[82] published his now famous critique of empirical tests of the CAPM. Roll's conclusions cast serious doubt on previous empirical tests of the model and have reduced the number of empirical tests of the pricing implications of the CAPM.

Despite the criticisms, the CAPM and its variants remain as powerful tools in the analysis of asset risk and continue to enhance our understanding of among other things, market efficiency, corporate capital budgeting, and portfolio analysis.

In two related articles, Ross[89,90] developed an alternative asset pricing theory based on the process of arbitrage. The Arbitrage Pricing Theory(APT) maintains the general intuition which led to the development of the CAPM and is considered by many to be a more general (and more powerful) model of asset pricing.

The CAPM requires either quadratic utility functions or multivariate normal returns distributions to arrive at its pricing conclusions. In contrast, the APT makes no assumptions regarding the distribution of asset returns and requires only very general assumptions regarding investor utility functions.[5] The lack of overly restrictive assumptions and the appealing intuitive content the APT make it a (potentially) natural successor to the CAPM.

The purpose of this dissertation is not to conduct a test of the pricing relationship of the APT, but rather to rigorously examine the multidimensional linear returns generating process assumed by the theory. In particular, the adequacy of the process to describe observed returns in a given subperiod and the stability of the process over time will be examined using a number of heuristic measures and statistical tests.

Overview of the Arbitrage Pricing Theory

The APT assumes: 1) perfectly competitive and frictionless asset markets; 2) investors prefer more wealth to less with certainty; and 3) investors homogeneously believe the random returns on the securities under consideration are generated by a k-factor model of the form:

$$(1.1) \qquad R_{it} = E_i + b_{i1}\delta_{1t} + \ldots + b_{ij}\delta_{jt} + \varepsilon_{it}$$

where a tilde indicates a random variable, R_{it} is the observed return on the $i^{\underline{th}}$ security in the $t^{\underline{th}}$ time period; E_i is the expected return on security i; b_{ij} is the sensitivity of the $i^{\underline{th}}$ asset's return to the $j^{\underline{th}}$ factor; δ_j is a mean zero factor common to all securities; and ε_{it} is an error term.

There are several assumptions which apply to the error term of the return generating process. Specifically, it is assumed that $E(\varepsilon_{it}|\delta_{jt})=0$ for $i=1,\ldots,n$; $j=1,\ldots,k$ and $t=1,\ldots,T$. In words, this assumption asserts that after accounting for the 'k' sources of systematic risk, the expected value of the error term is zero. The ε_{it} represent truly random, nonsystematic components of returns (i.e. those "shocks" which are security-specific).

Additionally, the following assumption regarding the error terms is also invoked: $E(\varepsilon_i \varepsilon_j) = 0$ for all i=j. This assumption simply states that after accounting for the systematic components of returns, the residual returns are (quite) independent. Too strong a dependence would imply that there are more than 'k' systematic sources of risk.

The above assumptions are those used by Ross in the development of the APT; several other assumptions are necessary before the APT can be examined empirically. These assumptions will be examined in greater detail in Chapter III.

If the return generating process given in Equation 1.1 holds, then Ross[90, p.353] has shown that in equilibrium, the absence of arbitrage opportunities implies:

$$(1.2) \qquad E_i = \lambda_o + \lambda_1 b_{i1} + \ . \ . \ . + \lambda_k b_{ik}$$

where λ_j, $j = 1,2,...,k$ are the factor risk premia. In words, Equation 1.2 asserts that the expected return on any security can be written as a linear combination of its sensitivity coefficients (the b's) and the market determined factor risk premia (the λ's). Equation 1.2 is the pricing relationship postulated by the APT and has been the focus of most of the empirical examinations.

The arguments leading to the development of Equation 1.2 are presented below and are based on those given in Ross[90] and Roll and Ross[83].

Assume an investor is currently holding a large portfolio and is considering altering his holdings. Let x represent the change in security weights in going from the currently held portfolio to the new portfolio. We assume the rebalancing of the portfolio will be such that no new net investment is required, i.e. proceeds from the sale of some securities will be used to increase the holdings of other securities. Algebraically:

$$x'1 = 0 \tag{1.3}$$

where 1 is an n-element vector of ones. A portfolio such as that described by Equation 1.3 is called an arbitrage portfolio since it requires no net additional investment.

From the assumed linear return generating process, the incremental return on this new portfolio is given by:

$$x'R = x'E + x'b_1\delta_1 + . . . + x'b_k\delta_k + x'\varepsilon \tag{1.4}$$

To develop the pricing equation, two conditions are placed on the vector x : 1) assume the portfolio is well diversified, i.e. $x = \pm 1/n$; and 2) choose x such that:

$$x'b_j = 0 \qquad j = 1,...,k \tag{1.5}$$

In other words, the condition given by Equation 1.5

implies that the incremental portfolio has no (net) systematic risk. In terms of the investor being considered here, Equation 1.5 means the original and new portfolio have the same level of systematic risk. Invoking Equation 1.5 on Equation 1.4 gives:

$$x'R = x'E + x'\varepsilon \tag{1.6}$$

The second term on the right hand side(RHS) of (1.6) is eliminated by invoking a strong law of large numbers (SLLN). As an example of how the SLLN works, assume σ^2 is the average variance of the ε_i terms and further that $x_i = \pm 1/n$. Then,

$$\begin{aligned} Var(x'\varepsilon) &= Var(1/n \sum_i \varepsilon_i) \\ &= [Var(\varepsilon_i)]1/n^2 \\ &= \sigma^2/n \end{aligned} \tag{1.7}$$

and as $n \to \infty$, it follows that the variance of ε_i vanishes.

Given that the idiosyncratic variances vanish in large portfolios, Equation 1.6 can be written:

$$x'R = x'E. \tag{1.8}$$

Now x was chosen such that two conditions are satisfied: 1) $x'1 = 0$; and 2) $x'b = 0$. Since the incremental portfolio return required no additional investment and incurs no additional systematic risk, its expected return, on average, must equal zero:

$$x'R = x'E = 0. \tag{1.9}$$

In any large frictionless market, Equation 1.9 must be true. If (1.9) were violated (i.e. if $x'E > 0$), investors would undertake the portfolio rebalancing described above until there were no further arbitrage opportunities, i.e. until (1.9) was true.

The key to deriving the pricing relationship of the APT using the above arguments is expressed most clearly by Roll and Ross[83,p. 1078] "The above conditions are really statements in linear algebra. Any vector (x) which is orthogonal to the constant vector and each of the coefficient vectors b_j ($j=1,...,k$), must also be orthogonal to the vector of expected returns. An algebraic consequence of this statement is that the expected return vector, E, must be a linear combination of the constant vector and the b_j vectors."

In other words, if the above conditions are met, then the n-element vector of expected returns must lie in the space spanned by the constant vector and the coefficient vectors (the b_j). This result is the pricing equation of the APT (Equation 1.2).

Subsequent to the original development of the APT, several researchers have extended Ross' results. Huberman[47], Chen and Ingersoll[16], Chamberlain and Rothschild[13], and Ingersoll[49] are among these. More specifically, Huberman[47] derives a "preference free" version of the APT using a sequence of distinct economies. Ingersoll[49] extends Huberman's results by using a fixed, infinite economy and examining a sequence of nested subsets of assets. Both researchers relax some assumptions on the residuals; specifically, they allow the residuals to be cross-sectionally correlated after accounting for the 'k' sources of systematic risk. Their results are stronger than those of Ross[89,90] where uncorrelated residuals were assumed. Implicit in the methodology herein used to examine the return generating process is the assumption of uncorrelated residuals. Thus, this paper is based on the earlier results of Ross[89,90].

If one were interested in examining empirically the results of Huberman and/or Ingersoll, principal components analysis (PCA) could be used in place of factor analysis to detect the unobservable, systematic components of returns since PCA does not assume uncorrelated residuals.

This approach was suggested by Chamberlain and Rothschild[13].

Chen and Ingersoll[16] present arguments which lead to an APT pricing result without relying on infinite economies or asymptotic mathematics. They also allow the residuals to be cross-sectionally correlated but provide no suggestions concerning empirical tests.

The decision to employ the common factor analytic model (CFAM) in this dissertation rather than another methodology was based largely on the fact that more is known about the properties of sample-based estimates derived from the CFAM and statistical tests of the results are well known. Use of the CFAM places more stringent constraints on the ability of the returns generating process to describe asset returns because of the explicit requirement that the residuals from such a model be cross-sectionally uncorrelated. It should be kept in mind that the results reported herein are, if anything, biased against the linear returns generating process when compared with the results obtained from using an alternative, weaker methodology.

The remainder of this dissertation is organized as follows. Chapter II reviews the body of literature relating to the APT and discusses the main hypotheses to be examined subsequently. Chapter III describes the heuristic measures to be employed, the statistical tests used to examine the hypotheses, and a description of the sample to be used in all the tests. Chapter IV provides empirical results and Chapter V concludes this dissertation with a disscussion of the implications and possible limitations of the findings.

CHAPTER II

LITERATURE REVIEW

Tests of the Pricing Equation

Several studies have examined the pricing relationship postulated by the APT and because of the methodology employed, provide some insight into the dimensionality of the factor space. In a major empirical effort, Roll and Ross[83] examined the pricing relationship (Equation 2.2) using 1260 securities divided alphabetically into forty-two groups each containing thirty securities listed on the NYSE or AMEX on both July 3, 1962 and December 31, 1972. The data was obtained from the CRSP daily return file; the metric used was the simple daily holding period return adjusted for all capital changes and dividends.

A variance-covariance matrix was computed for each of the forty-two groups and maximum likelihood factor analysis was performed providing estimates of the factor loadings and the number of factors. The subsequent cross-sectional regressions utilized the loadings for an hypothesized five factor model. The results were reported as a summary of the forty-two groups.

Roll and Ross conduct two separate analyses: the first specifies λ_o to be 6% (per year); the second allows to be estimated in the cross-sectional regressions. When λ_o is assumed to be a constant 6%, 88.1% of the groups contained at least one significant factor risk premium; 57.1% had at least two significant and 33.1% contained at least three significant factor risk premia. Only 16.7% and 4.8% had at least four and five significant factor risk premia, respectively; a finding which led Roll and Ross to conclude, "...at least three factors are relevent for pricing, but it is unlikely that more than four are present." [83,p.1092]

When λ_o is estimated rather than specified, only two factors seem to be significant but the results are not directly comparable since the augmentation of the loadings matrix with the unit vector negates the statistical tests of significance used when λ_o is assumed to be a known constant. Thus, the treatment of λ_o affects the results of the pricing equation tests but one is unable to ascertain which specification is the more desirable approach for estimating the number of priced factors. Roll and Ross argue for the former approach (λ_o

specified) to maintain independence among the estimated coefficients but this formulation exposes them to criticism regarding the (arbitrary) value chosen for λ_o.[1]

Roll and Ross next test the APT against a specific alternative: the standard deviation of individual returns has incremental explanatory power after accounting for the factor loadings. Within each group, the vector of time series mean returns was regressed on the five factor loadings vectors and on the vector of individual security's standard deviation of returns. The average t-statistic for the coefficient associated with the standard deviation across the forty-two groups was 2.17 and 45.2% of the groups had t-statistics which exceeded the 95% critical level. These results are inconsistent with the APT since according to the Theory, an asset's standard deviation in returns can be diversified away and thus should not be "priced" after accounting for the factor loadings. Roll and Ross are quick to point out however, that the observed significance of standard deviation may be due to: 1) positive dependence across groups which may overstate the true significance of the variable in the cross-sectional regressions; and/or 2) positive skewness in the distribution of individual returns may explain the sample mean's dependence upon security standard deviation. (Miller and Scholes[71] discuss this issue in tests of the CAPM).

To examine the possible effects of skewness, Roll and Ross employ an elegant technique designed to overcome the skewness problem. They use daily observations 1,7,13,... to estimate the expected return, observations 3,9,15,... to estimate the factor loadings, and observations 5,11,17,... to obtain estimates of the asset's own standard deviation. Such a procedure "insulates" the various estimates obtained and thus reduces the cross-sectional effects introduced by positive skewness in the return distributions. Using this procedure, only seven of the forty- two groups (16.7%) display significant effects for standard deviation at the 95% level. This result suggests that positive skewness was at least in part responsible for the observed dependence of mean returns on standard deviation.

Further, Roll and Ross used a methodology similar to that of Fama and MacBeth[34] to estimate the standard deviation of the estimated coefficients and found that just three groups of the forty-two (7.1%) displayed a significant effect of standard deviation on mean returns. Since just two of forty-two would be expected by chance at the 95% level of significance, Roll and Ross conclude that standard deviation seems to have little incremental explanatory power beyond that of the factor loadings vectors. This, of course, is consistent with the APT.

The final test conducted by the authors examined the equality of the intercepts from the cross-sectional regressions across the forty-two groups. Hotelling's T-square statistic was used because the intercept estimates are most likely correlated across groups. The results of the tests indicate, "...there is absolutely no evidence that the intercept terms were different across groups." [83,p.1100]

Hughes[48] tested the pricing relationship of the APT using 220 Canadian securities listed on the Toronto Stock Exchange during the period January 1971 through December 1980 (120 monthly observations). Monthly holding period returns adjusted for stock splits and dividends were used as the metric to be analyzed. Citing singularity of the covariance matrix, the author randomly split the securities into two groups of 110 securities each. Hughes used Harmon's[45] MINRES technique of factor analysis which seeks to minimize the off-diagonal elements of the residual correlation matrix. MINRES requires that the researcher specify in advance the number of common factors; Hughes chose a twelve factor representation based on her intuition that it is unlikely that more than twelve common factors could generate returns for 110 companies. Citing Gibbons[42, p.12] finding a nonstationary covariance structure but stationary correlation structure, Hughes argues for factoring the correlation matrix. If the author had chosen maximum likelihood factor analysis, the choice as to which matrix to factor would not be an issue since maximum likelihood factor analysis is scale-free (i.e. the results will be comparable for correlation and covariance input). The author's choice of MINRES as the factoring technique necessitates the use of correlation input; it would be unfortunate if the stationarity issue prompted her to use this technique mainly because it makes comparisons with other studies difficult.

The results of the factor analysis are very similar, however, to those reported by Kryzanowski and To[60] in terms of the variance accounted for by the common factors. The first factor accounts for roughly 30% of the sample variance while the second through twelth factors approximately account for an additional 20% of the sample variance.

The author next reports the results of the pricing equation tests. The first group of securities contained six statistically significant factor risk premia (at the .05 level of significance); the second group contained three. A claim is then made that by "combining" the results for the two groups, one obtains findings consistent with those reported by Roll and Ross[83]. Exactly how the author arrives at this conclusion is not developed; in my opinion, the results are not consistent with those of Roll and Ross[83]. There are at least three possible reasons why the results of the Hughes study are not comparable to the Roll and Ross findings. First, Roll and Ross use daily return data while Hughes uses monthly returns (daily returns are the preferred metric, see Roll and Ross[83, p.1080]). The use of monthly data has at least two effects upon the analysis. For a given time period, there are fewer observations of the return generating process. For example, using the periods defined herein, each subperiod represents approximately 48 monthly observations while daily return series contain 1,029 observations during the same time period. Thus, use of daily return observations lead to more powerful statistical tests. Also, in factor analytic investigations one requires a large number of observations (T) relative to the number of securities (N) to obtain meaningful estimates of the factor structure. Daily return data meet this requirement more easily than do monthly data. As a rough comparison, to obtain comparable power in the tests of a given subperiod, mothly data requires a subperiod of approximately 90 years! Certainly any assumption of stationarity would be tenuous at best. With daily data, a comparable number of observations are generated in only four years. Of course, the stationarity assumption is more realistic over the shorter time period.

Second, Roll and Ross' sample was drawn from U.S. securities while Hughes used Canadian securities. Of course, one can argue that the factors which generate returns on U.S. equities generate the returns on Canadian equities as well. However, given the differences in the Canadian economy versus the U.S. economy and especially the larger, more active markets in the U.S., one would suspect a priori that the arguments necessary to derive the APT are more nearly approximated in the U.S. equity markets.

Third, the factor analytic estimation techniques differed in the two studies. Roll and Ross used EFAP which is based on Jörekog's[55] method of factor extraction. As indicated previously, Hughes used Harmon's[45] MINRES method. There is a large body of evidence which indicates that in large samples, virtually all factor extraction techniques produce similar results. (See, e.g. McGowan and Tandon[68]). However, the sample size differences in the two studies (Roll and Ross, N=30; Hughes, N=100) suggest the factor analysis results are not directly comparable.

Hughes next reports the results of a test for the equality of intercept terms in the cross-sectional regressions. The null hypothesis of no difference in intercepts across the securities within each group cannot be rejected. These results are consistent with those reported by Roll and Ross[83,p.1100] but are inconsistent with those reported by Brown and Weinstein[9, p.727] although the latter results must be interpreted carefully in light of the authors' discussion of rejection at low numerical values of the F-ratio.

The most insightful part of Hughes' study was her attempt to address the congruence of factors across groups. Recall that one cannot determine that the i^{th} factor extracted in each of the respective groups represents the same economic phenomenon. In an attempt to shed some light on this issue, Hughes first computed the correlation matrix of factor scores between the two groups. If several of the factor scores were found to be highly correlated, this would give tentative support to the assertion that the respective common factors represented the same phenomenon in the two groups. The results were disappointing. Only the first factor had a high correlation with its counterpart in the other group; the remaining factors appeared to be across-group linear combinations (according to Hughes) or were not highly correlated.

A second attempt for assessing factor congruence appears to be a more promising approach. Hughes regressed (cross-sectionally) security returns in group A on monthly estimates of factor risk premia from group B and vice-versa. The results of these regressions seem to indicate that there is some consistency in the factor risk premia in the two groups (i.e. the number of statistically significant factor risk premia in each group was roughly equal). This approach may prove to be helpful in not only assessing factor congruence, but may assist in determining the number of relevant common factors (i.e. the number priced in the market), especially if applied to daily return data for a larger number of disjoint groups.

Brown and Weinstein[9] apply Kruskal's[59] bilinear paradigm to examine the factor structure underlying asset returns. For a complete developement of the paradigm as used in tests of the APT, the reader is referred to the original study (Brown and Weinstein[9, p. 716-719]); only the results of their tests will be examined here.

The sample data used in this study were nearly identical to those used by Roll and Ross[83] so some comparisons across studies can be made. Brown and Weinstein specifically discuss the assumed intertemporal stationarity in the expected returns which is necessary to implement tests of the APT. To my knowledge, this is the first explicit consideration of this issue which appears in an empirical study. If one assumes the ex-ante means of the factors exist and equal zero, and further that the process generating security returns is indeed stationary, then the APT may be tested using differences from mean returns as the dependent variable. This dependent variable is implicitly used by all the studies reviewed herein since when computing the correlation or variance-covariance matrix, the time-series mean return is removed. None of these studies mention the implications of the stability assumption however.

Brown and Weinstein report chi-square tests for the adequacy of a three factor representation using groups of thirty securities which are roughly comparable to the groups used by Roll and Ross[83]. In only three of the forty-two groups does a three factor representation seem to be adequate at the commonly used 50% level of significance. However, when the constancy of factors across groups is assessed via an F-test, in only four of the twenty-one paired groups is the constancy hypothesis rejected. Brown and Weinstein conclude, "...while more than three factors would appear necessary to yield a satisfactory statistical representation of the return generating process, the three factors that best represent the observed variation in the data do not significantly differ across groups." [9,p. 724]. This finding has important implications for empirical tests of the APT. Specifically, a chi-square test of the adequacy of a particular number of factors is not the most important criterion in tests of the APT. Rather the number of priced factors is the critical issue.

Brown and Weinstein go on to test two assertions of the APT: 1) the risk-free (zero-beta) rate should be constant across groups; and 2) the factor prices (risk premia) should be constant across groups at a given point in time. To examine these assertions, the authors consider three tests[9, p. 726]: 1) the implied risk-free rate is constant across groups but potentially different from one period to the next; 2) the implied factor prices and realizations are constant in the cross-section; and 3) both of the above conditions hold.

The first hypothesis (equality of risk-free rates) is rejected 21 out of 21 times as are the second and third hypotheses. This evidence indicates a three factor Arbitrage Pricing Model (APM) is inconsistent with the observed sample data. The three hypotheses were then tested using five and seven factor representations. The results indicate that while the five and seven factor models are a better representation of the return generating process (as measured by the chi-square test for goodness-of-fit), the proportion of securities for which the factors are the same across groups decreases as the number of factors is increased. The authors conclude, "Where the factor model is a sufficiently good representation of the data generating process, we find that the expanded factor APM models do no better and sometimes worse than the three factor model in explaining returns. It would appear that if the APM is correct, it is the economy wide factors that are being priced." [9 ,p. 728]

Brown and Weinstein then examine the APM using security groups formed on the basis of two-digit SIC classifications. The results of these industry investigations reveal that whatever the industry group, the chi-square test for goodness-of-fit is worse than that obtained in the alphabetical groupings. The authors conclude, "The similarity of results for the groups organized by industrial classification is yet further evidence that the number of economy wide factors are (sic) small; and the remaining factors are specific firm or industry effects that may be diversified and not priced in an APM scenario." [9 ,p. 731]

Chen[15] extensively examines the assertions of the APT using daily return data for the period 1963-1978 inclusive. The returns were adjusted for all capital changes and included dividends, if any. The securities included in the sample were those that did not have missing data in the four subperiods: 1) 1963-1966; 2) 1967-1970; 3) 1971- 1974; and 4) 1975-1978. At least 1000 firms met the sample selection criterion in each of the four subperiods.

Chen's approach for estimating the factor loadings can be summarized as follows: 1) in each subperiod compute the factor loadings matrix using 180 securities and specifying ten factors; 2) using a linear programming algorithm designed to maximize differences between groups, construct five maximally different portfolios; 3) compute the correlations between the returns of a particular security (not one of the original 180) and the five portfolios; and 4) use the five corrrelation coefficients to solve for the factor loadings associated with the security under consideration. This procedure is repeated for all securities in each subperiod; thus, each security's factor loadings have now been determined and each factor loading corresponds to the same factor for all securities. These computed factor loadings are used in tests of the pricing equation.

Chen provides summary statistics for the cross-sectional factor loadings for each of the four subperiods. In each subperiod only one of the five mean loadings is greater than twice its standard deviation; the other four mean loadings are not significantly different from zero. The generally small mean values of the second through the fifth loadings in each subperiod may help to explain some of the ambiguity found in the later results.

The great bulk of the Chen study focusses on the comparison of the pricing results in the APT versus the CAPM. As a first test, Chen cross-sectionally regressed the returns on either the five factor loadings vectors (for the APT tests) or on beta (for the CAPM tests). The subperiod results for the APT show that in each subperiod, at least two factors are significant in the pricing relationship. This is tentative support for a multi-factor pricing model. The adjusted R-square varied from period to period attaining a maximum of .2874 in the 1963 - 1966 subperiod and a minimum of .0281 in the 1967 - 1970 subperiod.

The CAPM results are also reported on a subperiod basis and Chen used three market proxies to estimate beta: 1) Standard and Poor's 500; 2) CRSP value-weighted index; and 3) CRSP equal-weighted index. The results do not support the CAPM. In only two of the four subperiods (1963 - 1966 and 1975 - 1978) did the cross-sectional regressions result in a significant regression coefficient for beta regardless of the market proxy used. In the latter period (1975 - 1978), the intercept coefficient was also significant. The largest adjusted R-square was .2167 for the equally-weighted index in the 1963-1966 subperiod. In two cases the adjusted R-square was actually negative. Thus, given the fact that at least two factors are priced in each subperiod, the initial cross-section regression results tend to favor the APT over the CAPM pricing equation.

As a further comparative test, Chen regressed the residuals of the CAPM on the factor loadings from the APT and then regressed the APT residuals on beta. If the CAPM is not misspecified the residuals should not contain any "systematic" information and regressing these on the factor loadings should not result in any significant regression coefficients. The results indicate that in each subperiod, at least two of the factor loadings vectors are significant. This demonstrates fairly conclusively that the CAPM is misspecified because after controlling for beta risk, there remains some systematic variation in the returns.

When the residuals from the APT specification are regressed on beta, the resulting regresssion coefficient is never significantly different from zero. These results indicate the factor loadings have more completely accounted for the systematic component of returns than does beta in the CAPM.

As a further test of the adequacy of the APT pricing equation. Chen constructs two portfolios with the same risk profile (i.e. equivalent factor loadings) where one portfolio consists of those securities with a low return variance; the other consists of those securities with a high return variance. The null hypothesis is that the two portfolios should have insignificantly different returns since the factor loadings (systematic risks) for the two portfolios are the same. The t-tests for difference in mean returns is not rejected in any of the four subperiods. Thus, the asset's own variance has no explanatory power after controlling for the factor loadings. This result is in agreement with a similar test conducted by Roll and Ross[83].

Chen's final test of the APT concentrates on the "size effect" anomaly reported in CAPM studies by Banz[3] and Reinganum[79], and in an APT study by Reinganum[80]. Unlike Reinganum[80], Chen finds no size effect after accounting for the factor loadings in three of the four subperiods and the one observed significant difference (subperiod 1975 - 1978) disappears after adjusting the t-test for autoregression over the first ten lagged terms.

In a recent empirical study Cho, Elton, and Gruber[21] conduct tests of the Roll and Ross[83] methodology using both simulated and actual return data. Two types of simulated data were generated: 1) returns generated with exogenously determined fundamental betas; and 2) returns generated using historical betas estimated quarterly using the zero-beta form of the CAPM. Consequently, the first two generated return series are a result of a known, two-factor return generating model. With this approach Cho, Elton, and Gruber were able to assess the methodology proposed and utilized by Roll and

Ross[83]. Factor analysis and subsequent tests of the pricing relationship should indicate that only two factors generate returns since the sample returns were generated by a known two-factor model.

The reported results indicate there is a slight tendency for the Roll and Ross procedure to overstate the number of factors generating security returns. Whether one uses returns generated by the fundamental or historical betas, there are more groups than one would expect by chance which contain three significant factors. Additionally, the results reported in the case of actual return data differ somewhat from those reported by Roll and Ross[83]. Specifically, it appears that Roll and Ross misstated their results concerning the number of factors and the corresponding p-levels. This probable misstatement has been noted by other researchers.[2]

Recently Dhrymes, Friend, and Gultekin[22] have published a paper which reexamines several of the empirical tests of the APT. The authors use a sample of securities which differs only slightly from that used by Roll and Ross[83] and Brown and Weinstein[9]. After explaining the APT and introducing notation, the authors proceed to examine various results reported in previous empirical tests of the APT. Three major conclusions are offered: 1) tests of whether a particular factor is "priced" are generally not permissable - the only issue that can be addressed using the Roll and Ross[83] methodology is whether the set of factors is priced; 2) the common practice of subdividing the universe of securities into smaller groups and then treating these smaller groups as "cross-sections" from a population is inappropriate since it ignores covariation of returns across groups; and 3) using conventional factor analysis results in a larger number of common factors as the group size increases.

In their Reply[86], Roll and Ross strongly disagree with the conclusions of Dhrymes, Friend, and Gultekin[22]. In response to the second criticism above, Roll and Ross argue that if the APT is true, then an appropriate estimate of the factor structure can be obtained using subsets of the universe of securities since the hypothesized factors are concrete entities which affect all security returns.

Roll and Ross agree with Dhrymes, Friend, and Gultekin on the third criticism but do not feel it is an important issue. Certainly, more factors will be extracted when factor analyzing larger groups of securities but the critical issue is not that more factors are extracted but rather, whether the number of priced factors increase as one increases group size. Roll and Ross argue that the number of priced factors will not increase as the number of extracted factors is increased. These issues are unresolved at this time but some results presented herein (Chapter IV) in part address these issues (especially the third criticism given above).

Examination of the Return Generating Process

The empirical studies cited thus far addressed the pricing equation of the APT. Another related body of empirical evidence is concerned with the assumed linear return generating process. Several of these studies (e.g. King[57], Meyers[70], Farrell[35,36], Rosenberg[87], and Livingston[64]) were conducted prior to the formulation of the APT and therefore were not guided by explicit theoretical considerations. Rather, these studies were conducted, in part, because of the results reported by researchers testing the CAPM who found a single-index(factor) return generating process was misspecified (i.e. residuals from market model regressions were not uncorrelated across securities). Correlated residuals after removal of a market index imply that returns must be generated by two or more factors.

Two studies which directly examined the characteristics of the return generating process in the context of the APT are Gibbons[42] and Kryzanowski and To[60].

The Gibbons[42] study suffers from at least two limitations: 1) use of "excess returns", i.e. raw returns less the thiry day T-bill rate; and 2) use of monthly holding period returns. Gibbons found that models up to and including an eight factor representation provided a poor fit to the observed data. These results are most likely due to the use of excess returns. There is some evidence to suggest that one of the latent factors underlying security returns is the asset's sensitivity to changes in the level of interest rates (Roll and Ross[84] and Chen, Roll and Ross[17]). By removing the 30-day Treasury Bill rate prior to estimating the factor structure, Gibbons biased his results against uncovering an interest rate factor (if indeed, one exists). Most studies of the pricing relationship (see e.g., Roll and Ross[83], Hughes[48], Chen[15], Pari and Chen[76], Brown and Weinstein[9], Dhrymes, Friend, and Gultekin[22]) have used raw return data which is the preferred metric.

Both Gibbons[9] and Kryzanowski and To[60] used monthly holding period returns in their empirical examinations. Prior to the formulation of the APT, the use of monthly returns in such investigations was justified since the researchers had no theory to guide their inquiries and monthly return series were readily available from a variety of sources. If one is interested in examining the latent structure of security returns as a preliminary to testing the APT however, monthly return series are ill-suited for this purpose. Roll and Ross specifically state, "The only critical assumption is the returns be generated by Equation 1.1 over the shortest trading period."[83, p.1080]

The use of return data generated over the smallest possible trading interval is by no means a trivial consideration. Indeed, using larger trading intervals (e.g. monthly) may reduce the likelihood of finding support for the APT for the following reason. Monthly data represent a (relatively) limited number of observations of the return generating process; they are less desireable for use in examining a theory based on precise arbitrage arguments. It doesn't seem that even the strongest advocate of efficient markets would argue that the market was always and everywhere efficient. A beginning and/or month-end price may not be the best indicant of security value. Daily return data provide many more observations of the return generating process and by using such data, one is likely to sample more points which are representative of the return generating process.

Partial support for the above assertion can be found by examining the results of Kryzanowski and To[60, p.40]. They found that as one increased the number of time periods (observations) under examination, fewer factors are retained on statistical grounds. Fewer, rather than more, common factors are to be expected. (Brown and Weinstein[9, pp. 728-31] discuss this point and provide some empirical evidence of relatively few common factors).

Another criticism which can be leveled against virtually all of the empirical examinations of the APT and the factor structure is the use of relatively small sample sizes (the exceptions are Chen[15] and Dhrymes, Friend, and Gultekin[22]). Roll and Ross[83] used group sizes of n=30; Kryzanowski and To[60] used group sizes of 10,20,30,40, and 50; Gibbons[9] used 41 stock portfolios; Hughes[48] used two groups of size 110 although with very few time series observations on each security; and Brown and Weinstien[9] used group sizes of 30 and 60. Using relatively small group sizes is motivated by at least two considerations: 1) the APT (if true) allows the estimation of the factor loadings based on a (relatively small) subset of all assets (see discussion in Roll and Ross[86, pp. 348-9); and 2) small group sizes are often dictated by the processing capacity of the computer. (Computing the loadings for, say, 200 securities, requires decomposition of a 40,000 element correlation or covariance matrix. This approaches the processing capacity of all but the largest mainframe computers and efficient algorithms for conducting such a decomposition do not yet exist).

The appropriate number of securities to include in the subset to be examined is not specified by the APT. However, maximizing group size makes it more likely that the full number of truly common factors will be extracted and, ceteris paribus, is more likely to avoid the Heywood cases found and discussed by Brown and Weinstein[9, pp.723-4]. Also, larger group sizes provide better estimates of security response coefficients (loadings).[3]

Testable Hypotheses

An important issue which has been virtually ignored in the empirical literature is the stability of the return generating process over time. The APT makes no assertions regarding the intertemporal stationarity of the process although empirical tests require some stability assumptions in order to provide meaningful results especially when researchers choose dissimilar time periods for their tests. From a practical standpoint, the return generating process (indeed the entire factor structure) must exhibit some degree of stationarity if the implications of the APT can be used by portfolio managers in security selection and portfolio construction. It is this issue which provides the impetus for this research. Specifically, a variety of heuristic and statistical measures are utilized to assess the intertemporal stationarity and cross-sectional congruence of the return generating process underlying security returns.

The heuristic measure which will be used to assess the dimensionality of the factor space is the scree plot suggested by Cattell[11]. The scree plot shows the magnitude of the eigenvalues plotted as a function of the number of factors. An estimate of the number of significant factors is obtained by observing where the plot "breaks". A complete description of this heuristic is given in Chapter III.

The first statistical test to be conducted considers the intertemporal stationarity of the correlation and variance-covariance structure of asset returns. One hundred securities are randomly assigned to each of 'g' groups where 'g' in this study is three. Other researchers have used a different number of groups depending upon the purposes of the study. Investigations of the pricing relationship of the APT are more powerful when 'g' is larger. Roll and Ross[83] use forty-two groups as do Brown and Weinstein[9] and Dhrymes, Freind, and Gultekin[22]. Investigations of the intertemporal nature of the return generating process on the other hand, are more powerful when more subperiods are used. For example, Chen uses a single group of securities in each of four non-overlapping subperiods in his comparison tests of the APT and the CAPM.

Keeping group composition constant, the time period covered by the analysis is split into five non-overlapping subperiods and a correlation and variance-covariance matrix is computed for each group in each subperiod.

The first hypothesis to be examined can be written:

Hypothesis 1A

$$H_{o\Sigma}: \Sigma_{g1} = \Sigma_{g2} = . . . = \Sigma_{gm}$$

where the first subscript refers to the group being examined and the second subscript refers to the subperiod. There is much prior evidence to suggest that Hypothesis 1A will be rejected. Studies by Elton and Gruber[27,28,29], Elton, Gruber, and Urich[31], and Gibbons[9] found the variance-covariance matrix of equity returns to be non-stationary. These results and some further evidence reported by Gibbons[9] regarding the stationarity of the correlation structure lead to an alternative form of the first hypothesis:

Hypothesis 1B

$$H_{oR}: R_{g1} = R_{g2} = . . . = R_{gm}$$

where R is the correlation matrix of returns and the subscripts are the same as under Hypothesis 1A. Tests for the equality of the correlation matrices over time are conducted for a number of reasons. First, since the returns vectors are standardized, all securities possess unit sample variances. Thus, an asset's own variance has no effect upon the tests of stationarity as it does in tests of Hypothesis 1A.

Second, Gibbons[9, p. 12] reports results which indicate a nonstationary covariance structure but a (very) stable correlation structure. His findings are in conflict with those reported by Elton and Gruber[27,28,29]. Therefore, tests of Hypothesis 1B using programming validated on a data set with known properties will provide definitive results concerning the effects of standardization on tests of the intertemporal stationarity hypotheses.

Lastly, modern portfolio theory and related empirical tests argue for the irrelevance of asset-own variance in security pricing relationships. If asset-own variance is indeed irrelevant to investors, then the analysis of correlation structures is sufficient in tests of the APT.

Rejecting either Hypothesis 1A or 1B indicates at least one of the matrices is significantly different from the others. The possibility that the matrices of two adjacent subperiods are equal statistically remains. Suppose for example, that the developement of the CAPM and its subsequent use in security selection has in recent years resulted in securities being priced in a manner more consistent with the predictions of the model. Under such a scenario, the return generating process would exhibit increasing simplicity over time (i.e. fewer factors generating returns). This could lead to a more stable covariance/correlation structure in the later periods than in the earlier periods.

Tests which explicitly consider this possibility are formulated as pair-by-pair tests of adjacent matrices which are similar in form to Hypotheses 1A and 1B:

Hypothesis 1C

$$H_{o\Sigma}: \Sigma_{g1}=\Sigma_{g2} ;. . .; \Sigma_{gm-1} = \Sigma_{gm}$$

Hypothesis 1D

$$H_{oR}: R_{g1}=R_{g2} ;. . .; R_{gm-1} = R_{gm}$$

If all the hypotheses discussed thus far are rejected, one may conclude that the correlation and variance-covariance structures are nonstationary. Such rejections have at least two implications. First, knowledge of the covariance (correlation) structure in prior periods is not useful in estimating the covariance (correlation) structure in later time periods. Second, previous empirical results of APT tests may be valid only for the particular time period covered by the test, i.e. generalization of the test results to other time periods cannot be made with any degree of confidence. Such results are not indicative of the failure of the assumed linear return generating process. Even though the correlation and covariance structures are unstable, the factor structure underlying returns may be stable. A set of tests designed to examine the stability of the factor structure can be written:

Hypothesis 2A

$$H_{ok}: k_1 = k_2 = . . . k_m = k^*$$

where 'k' is the number of factors generating security returns. The value of 'k' is not specified by the APT; indeed many of the empirical tests of the model attempt to establish the value of 'k'. Consequently, Hypothesis 2A will be examined using a reasonable range (based on previous empirical research) of values for 'k'. Hypothesis 2A is also conducted on a pair-by-pair basis as before. These hypotheses will be denoted as Hypothesis 2B:

Hypothesis 2B

$$H_{ok}: k_{g1} = k_{g2}; . . .; k_{gm-1} = k_{gm}$$

In addition to the intertemporal tests described above, several cross-sectional issues are examined in the context of the APT. A serious problem which plagues any attempt to examine empirically the APT is the issue of factor congruency across groups. Currently there is no way to ascertain whether the second factor extracted in one group represents the same phenomenon as the second factor extracted in another group, etc. Roll and Ross[83], Hughes[48], Brown and Weinstein[9], Chen[15], and others provide some indirect evidence of factor congruence through their tests for the equality of the risk-free (zero-beta) rate across groups. Such tests do not provide any information concerning the equivalence of individual factors across groups, however. Rather, this type of test shows only that the set of factors give equivalent intercept estimates in cross-sectional regressions.

In this research, a technique suggested and implemented by Hughes[48] will be used in an attempt to assess factor congruence. The method consists of computing the correlation matix of factor scores between two groups over the same subperiod. Since the factor scores represent the movement of the common factors over time, high correlations between the factor scores across groups would indicate factor congruence. Further details of this procedure are discussed in the next Chapter.

Further, generalized least squares cross-sectional regressions will be conducted each day in each subperiod to estimate the number of priced factors. The results will be compared across groups in the cross-section to ascertain which factor models produce similar estimates of the number of priced factors. The full details of this procedure are discussed in the next Chapter.

Summarizing the discussion thus far, we see that the rapidly expanding body of literature relating to the APT has provided several different means of examining the implications of the Theory. Several issues regarding the ability of researchers to generalize their findings have been discussed and hypotheses designed to examine these issues have been proposed. The implementation of the tests and the sample to be used throughout the remainder of this dissertation are discussed next.

CHAPTER III

SAMPLE DESIGN AND METHODOLOGY

The securities selected for the statistical tests necessarily must be a subset of all securities traded in the equity market. Ideally one would like to decompose a variance-covariance matrix computed using all stock issues traded during a given subperiod. This would provide the best estimate of the factor structure underlying equity returns. Unfortunately, efficient algorithms designed to decompose (factor) a covariance matrix of say, 5,000 securities do not exist. Thus a smaller, more manageable number of securities must be selected. A group size of n=100 securities was chosen since this is the largest number of securities for which all the necessary estimates can be computed by the Control Data Corporation CYBER 175 computer currently in use at the University of Massachusetts-Amherst.

In general, the larger the number of securities in each group, the better will be the estimates of the underlying factor structure. This consideration is especially important for the cross-sectional tests of factor congruence; maximum likelihood factor analysis(MLFA) results are very sensitive to group size.

Consider the results reported by Dhrymes, Freind, and Gultekin[22]. Their Table IV (pp. 342-3) compares loadings estimates obtained for sixty securities when the securities are factor analyzed in two groups of size thirty versus the results when these securities form a subset of a group of n=240 securities. The loadings estimates on all but the first factor differ markedly as group size and composition are changed. In this dissertation this effect is controlled by keeping group size and composition constant through time but it must be kept in mind that the results reported herein are based on a group size of one hundred securities. Larger (or smaller) group sizes than those chosen here may produce different results.

Having established group size, the next sampling consideration is the selection of a data metric. Daily return data is preferred for a number of reasons. First, daily return data currently represent the most frequent measurements of the return generating process. Monthly return data has been used in several studies (e.g. Kryzanowski and To[60], Hughes[48], Gehr[40], etc.) at the cost of having relatively few observations of the return generating process. Roll and Ross[83] suggest the returns be generated over the shortest possible trading interval.

Second, the power of the statistical tests discussed earlier is increased as the number time series observations is increased. Third, MLFA requires that T>>N>>K where T is the number of observations, N is the number of securities, and K is the number of factors. This requirement is difficult to achieve using monthly data, especially when one is designing a study using a large N. Lastly, the intertemporal tests discussed at the end of Chapter II negate the use of monthly data because there simply would be too few observations to implement the tests.

Given the above considerations, daily returns (adjusted for all capital changes and including dividends, if any) are used as the data metric. The data are obtained from the Center for Research in Security Prices (CRSP) 1982 daily returns file.

The actual selection of securities for inclusion in each of the groups must next be addressed. The intertemporal tests to follow place a stringent limitation on sample composition. To be included in the samples, a security must: 1) be continuously listed for the entire sample period (July 3, 1962 through December 31, 1982); and 2) have complete trading data for the entire sample period (i.e. T=5,145 daily returns).

The first requirement is necessary to maintain group composition over time and thereby allow the sequential tests for factoral invariance. The second requirement stems from the fact that simultaneous observations are necessary to compute the sample covariance matrix; missing data are not allowed.

The CRSP daily returns file contains 360 securities which meet both of the above criteria. Since group size is limited to 100 securities, three groups of one hundred securities each are used in the tests of the hypotheses. The first (alphabetically) one hundred securities are placed in the first group (Group A); the second and third hundred are placed in Groups B and C respectively. The remaining sixty securities which satisfy the data requirements are kept as "alternates" and will be used in place of securities which cause estimation problems. Foremost among these problems is the Heywood[46] case. A Heywood case typically occurs when a security in a sample has a return variance which is very large relative to the return variances of the other securities in the sample. In such a situation, the MLFA procedure extracts a factor which is nearly perfectly correlated with the time series return of the large return variance security. The security in question will load heavily on one factor and have (near) zero loadings on the other k-1 factors.

In the context of the APT, such a result implies that this security can be used as a factor "portfolio"; a result which is clearly inconsistent with the arguments leading to the development of the APT. Recall the development is based on K large, well-diversified portfolios representing the K common sources of risk. A Heywood case result is, of course, inconsistent with those arguments and moreover, is a sample-specific phenomenon, i.e. including the offending security in another group is likely to remove the Heywood case result. In this dissertation, if a Heywood case occurs in any of the groups, the offending security is removed and replaced by one of the alternates and the analysis repeated. In this way the factor structure estimates obtained in the MLFA procedure are large, well diversified portfolios consistent with arguments leading to the development of the APT.

It is quite clear that several potential problems may exist with the samples. Because of the stringent data requirements, the securities tend to be those of large, established companies. (See Appendices A-C for a list of the securities used in the sample). This may introduce at least two often cited biases: 1) "survivorship" bias; and 2) large firm bias, i.e. very few "small" (in market value) companies are included in the sample. As regards the first potential bias, certainly firms which went bankrupt (or were delisted for other reasons) do not qualify for inclusion in the sample. This may or may not bias the estimation of the factor structure. About all that can be said is that the factor structure estimates reported herein are derived from returns of firms which are sucessful and represent a large segment of the U.S. equity markets. By the definition of "common" factors implicit in the arguments of the APT, factors which generated returns for these companies also generated the returns for the now bankrupt companies; the differences between the two are, of course, idiosyncratic and thus there should be little or no bias in the factor structure estimated from these samples.

The omission of small firms from the sample potentially is a serious problem. There exists a large body of empirical literature dealing with the now well-known "small firm" effect in a CAPM framework (e.g. Banz[3], Reinganum[79,81], etc.).

Some empirical evidence reported using the APT however, indicates omitting small firms may not result in biased estimates of the factor structure. For example, Reinganum[80, p. 316] reports correlation coefficients between market values and loadings vectors were between -.09 and .07 for his sample of securities. Also, Chen[15] reports an insignificant size effect in his tests of the APT. Since the extracted factors represent common, economy-wide influences, there is little reason to suspect serious biases being introduced by not including a large number of small firms in the sample.

In fact, the securities of large firms may be more sensitive to the movements of economy-wide factors than those of small firms since larger firms are more closely followed by analysts. In any event, since the samples are restricted to subsets of assets one may argue that the subsets should consist of firms which represent a large segment of the U.S. economy.

Obviously, one cannot ignore other possible systematic biases in the sample. For example, the sample analyzed must represent a wide spectrum of industries lest an important factor be omitted. To exclude utilities for example, may bias the results against finding a systematic interest factor. The distribution of two-digit Standard Industrial Classification (SIC) codes in each sample are presented in Appendix D. There seems to be no systematic exclusion of any particular industry.

Additionally, a breakdown of sample means and variances is presented in Appendix E for each sample group in each subperiod. The distribution of sample means and variances suggests a wide spectrum for these values. Certainly, some small-firm outliers are not included (particularly securities with large return variances) but these tend to wreak havoc in the factor analysis stage (see e.g. Brown and Weinstein[9, p. 723-4]).

To summarize, three groups of one hundred securities each were chosen as the basis for the subsequent analyses. The entire time period (July 3, 1962 through December 31, 1982) was arbitrarily split into five non-overlapping subperiods as follows:

Subperiod	CRSP Days	Dates
1	2 - 1030	07/03/62 - 08/02/66
2	1031 - 2059	08/03/66 - 10/12/70
3	2060 - 3088	10/13/70 - 11/07/74
4	3089 - 4117	11/08/74 - 12/05/78
5	4118 - 5146	12/06/78 - 12/31/82

Each subperiod contains 1,029 daily return observations.

Methodology

In this section of Chapter III the methodology to be used in estimating the various parameters is discussed. In order to keep the discussion consistent throughout, a brief description of notational conventions is warranted.

Notation

The following notational conventions are used throughout the remainder of this paper:

R_{it} = unadjusted (raw) daily holding period return drawn from the 1982 CRSP Daily Return File;

$\bar{R}_i = 1/T \sum_t R_{it}$ = time series mean return on security i;

$r_{it} = R_{it} - \bar{R}_i$ = mean corrected daily return;

E_i = expected return on security i;

δ_j = mean zero common factor;

b_{ij} = loading (sensitivity) coefficient of the i security's return on the j factor;

ε_{ij} = mean zero idiosyncratic error term;

= variance-covariance matrix of returns;

B = loadings matrix;

Ψ = diagonal matrix of idiosyncratic error variances.

Further, the following subscript conventions are used:

$i = 1,\ldots,n$	refers to the i^{th} security;
$j = 1,\ldots,k$	refers to the j^{th} factor;
$m = 1,\ldots,5$	refers to the m^{th} subperiod;
$g = 1,\ldots,3$	refers to the g^{th} group;
$t = 1,\ldots,T$	refers to the t^{th} time period;

k = the number of common factors.

Also, a tilde (~) when used as a subscript refers to a vector or matrix; when used as a superscript the tilde represents a random variable.

Estimating the Factor Structure

The estimation of the factor structure in each group is accomplished through maximum likelihood factor analysis (MLFA) using the International Mathematics and Statistics Library (IMSL) subroutine OFCOMM. OFCOMM requires correlation matrix input rather than the variance-covariance matrix but this is of no concern since MLFA is scale-free (i.e. the results from the correlation input can be rescaled to obtain estimates of factor loadings and idiosyncratic variances consistent with those obtained using variance-covariance input).

In order to assess the adequacy of a particular solution via a chi-square approximation, the vector variables δ and ε are assumed to follow independent multivariate normal distributions. Further, it is assumed that $E(\delta)=E(\varepsilon)=0$ and (without loss of generality) that the variance-covariance matrix of δ is I_k (i.e. the factors are uncorrelated and scaled to have unit variance).

Since the returns vector (r) is written as a linear combination of δ and ε, it follows that r also has an assumed multivariate normal distribution with zero mean vector and variance-covariance matrix Σ. The factor analytic model is written:

$$r = B\delta + \varepsilon \tag{3.1}$$

where B is the (nxk) matrix of loadings. Postmultiplying both sides of Equation 3.1 by its transpose gives:

$$rr' = B\delta\delta'B' + \varepsilon\varepsilon' \tag{3.2}$$

and invoking the assumption of uncorrelated, unit-variance factors gives:

$$\Sigma = BB' + \Psi \tag{3.3}$$

There is an identification problem in the set of equations implied by Equation 3.3. Note that the formulation in 3.3 is satisfied by any loadings matrix (B^*, say) such that $B^* = BM$ where M is any orthogonal transformation matrix of order k (i.e. $MM'=I_k$). In many uses of factor analysis, much effort is spent choosing an orthogonal transformation (rotation) matrix (M) in order to "interpret" the matrix of factor loadings. This approach will not be used directly here since, as Roll and Ross point out, "The APT concludes that excess expected returns lie in the space spanned by the factor loadings. Orthogonal transformations leave that space unchanged, altering only the directions of the defining basis vectors, the column vectors of the loadings". [83 ,p.1084]

Letting Ω denote the set of all matrices Σ that are positive definite and of order n and S denote the sample variance-covariance matrix, the likelihood function reaches its maximum when $\Sigma = S$. The log of the likelihood function is:

$$(3.4) \qquad \ln L_{\Omega} = -1/2T(\ln|\underset{\sim}{S}| + n)$$

where T is the number of observations used to compute S. Letting $\hat{\underset{\sim}{\Sigma}}$ represent the variance-covariance matrix reproduced by the hypothesized factor model (i.e. $\hat{\underset{\sim}{\Sigma}} = BB' + \hat{\underset{\sim}{\Psi}}$ where the circumflex accents denote the maximum likelihood estimates), the likelihood function for this subset (ω) of Ω is:

$$(3.5) \qquad \ln L_{\omega} = -1/2T(\ln|\hat{\underset{\sim}{\Sigma}}| + tr(\underset{\sim}{S}\hat{\underset{\sim}{\Sigma}}^{-1})).$$

The likelihood ratio is $L = L_{\omega} / L_{\Omega}$ and it is well known that $-2\ln(L)$ is distributed approximately as chi-square if the hypothesized factor model is true. The degrees of freedom for the test of a particular k-factor model are $(1/2)[(n-k)^2 - n-k]$. A computed chi-square which is large relative to its degrees of freedom is evidence that a larger number of factors must be extracted to reproduce adequately the sample variance-covariance matrix.

Two points are worth noting. First, the assumption that the returns are distributed as n-dimensional normal is only invoked so the two likelihood functions, (3.4) and (3.5) can be specified. The APT makes no assumptions regarding the underlying returns distributions and therefore, rejection of a particular factor model on the basis of the chi-square statistic is not a rejection of the linear return generating process assumed in the APT. It may well be that the APT is true but significant departures from multivariate normality are present in the data and therefore the likelihood functions are misspecified for these data. In this study, the chi-square statistic is used only as a guide in determining the number of fators generating security returns in each subperiod.

Second, since the value of 'k' is unknown a priori, a sequential approach suggested by Lawley and Maxwell[61 ,pp. 37-8] will be followed. Essentially the approach consists of fitting a k=1 factor model and testing for significance. If the computed chi-square is significant, a k=2 factor model is tried and the resultant chi-square compared with some preselected critical value. The procedure continues until a value of 'k' is found for which the chi-square value is insignificant or the degrees of freedom are nonpositive. The successive chi-square values so computed are not independent since the computation of later values is done only if its predecessor is significant. However, "...use of the significance level of chi-square at each step seems unlikely to cause serious error in practice".[61 ,p.37]

To summarize the estimation of the factor structure, for each group in each subperiod, OFCOMM is used to provide estimates of the loadings matrix ($\underset{\sim}{B}$) and the diagonal matrix of error variances ($\underset{\sim}{\Psi}$). The results of these factor analyses are then examined both cross-sectionally and intertemporally for their ability to provide information regarding the common factors generating returns (i.e. the characteristics of the linear returns generating process).

Intertemporal Tests

The tests for stationarity of the return generating processs follow a sequence of tests described by Joreskog[54]. If it is assumed that the vector variables $\underset{\sim}{\delta}$ and $\underset{\sim}{\varepsilon}$ follow independent normal distributions (consistent with the assumptions in the previous section) then from the assumed return generating process:

$$(3.6) \qquad \tilde{R}_{it} = E_i + b_{i1}\tilde{\delta}_{1t} + \ldots + b_{ik}\tilde{\delta}_{kt} + \tilde{\varepsilon}_{it}$$

or,

$$(3.7) \qquad \tilde{R}_{it} - E_i = b_{i1}\tilde{\delta}_{1t} + \ldots + b_{ik}\tilde{\delta}_{kt} + \tilde{\varepsilon}_{it}$$

it follows that $(R_{it} - E_i)$ is distributed as multivariate normal provided $E_i \equiv R_i$ is non-stochastic.[1]

Given these assumptions on the returns generating process, the first intertemporal test to be considered is:

Hypothesis 1A

$$(3.8) \qquad H_{o\Sigma}: \Sigma_{g1} = \Sigma_{g2} = . . . = \Sigma_{gm}$$

In words, (3.8) is a test of whether the variance-covariance matrix is intertemporally stationary.

To conduct a test of $H_{o\Sigma}$, one first estimates the pooled sample variance-covariance matrix (S_p) for each group defined as:

$$(3.9) \qquad S_p = (1/T) \sum_{m=1}^{5} t_m S_m$$

where:

$T = t_1 + t_2 + t_3 + t_4 + t_5$

t_m = number of observations in the m^{th} subperiod, m=1,...,5;

S_m = sample variance-covariance matrix computed over subperiod m.

Note that S_p computed using (3.9) is simply a weighted average of the variance-covariance matrices computed over the individual subperiods. In the current study, the number of observations (t_m) are the same for each subperiod. Thus, each subperiod is given equal weighting in tests of H_o. The test statistic:

$$(3.10) \qquad T(\ln|S_p| - \sum_{m=1}^{5} t_m(\ln|S_m|))$$

is distributed (approximately) under $H_{o\Sigma}$ as a chi-square with d_{Σ} = [(1/2)(m-1)n(n+1)] degrees of freedom. In the present study, for each of the three samples, m=5, $\dot{t}$ =1029; m=1,...,5, and n=100 so d_{Σ} = 20,200. If $H_{O\Sigma}$ is not rejected, the factor structure can safely be estimated from the pooled sample variance-covariance matrix S_p, i.e. there is no need to analyze each subperiod separately.

If $H_{o\Sigma}$ is rejected, at least one of subperiod sample variance-covariance matrices differs (significantly) from the others. It is quite possible however, that for two adjacent subperiods the variance-covariance matrices are equivalent. To examine this possibility, $H_{o\Sigma}$ can be conducted (m-1) times for each group using adjacent (in time) covariance matrices (Hypothesis 1C). Justification for this procedure was provided at the end of Chapter II. The form of the test is the same as before except that now in each case, m=2 subperiods. The results of these tests can be examined to see if the stationarity hypothesis is tenable in only some of the subperiods.

If $H_{o\Sigma}$ (the simultaneous test using all five subperiods) is rejected, the second test in the sequence examining factoral invariance is a test of an equal number of factors generating returns in each subperiod. This null hypothesis is:

Hypothesis 2A

(3.11) H_k : $k_1 = k_2 = . . . = k_5$ = a specified number k.

To test H_k, one conducts an unrestricted MLFA on the variance-covariance matrix in each subperiod. Assuming the returns are distributed as n-dimensional normal, each subperiod analysis produces a chi-square value with $(1/2)[(n-k)^2 -n-k]$ degrees of freedom. Assuming daily returns are independent over time, the computed chi-square values are also independent. The independence of these chi-square values allows one to add them to obtain a chi-square value with $d_k = (1/2)m[(n-k)^2 -n-k]$ degrees of freedom which is then used to test the overall hypothesis. As before, paired tests of (time) adjacent subperiods can be conducted by setting m=2.

An additional intertemporal test examines the stability of the correlation structure of asset returns (Hypothesis 1B). The test was developed by Jennrich[51] and is designed to examine the effects (if any) of standardizing the variates. The details of the test follow.

To compare two correlation matrices, $\underset{\sim}{R}_1$ and $\underset{\sim}{R}_2$, first define:

T_i = number of observations used to compute $\underset{\sim}{R}_i$;

$R = (n_1\underset{\sim}{R}_1 + n_2\underset{\sim}{R}_2)/(n_1 + n_2)$

$c = n_1 n_2/(n_1 + n_2)$

$S = (\delta_{ij} + \bar{r}_{ij}\bar{r}^{ij})$

where:

δ_{ij} = Kronecker's delta;

$\bar{r}_{ij}$ = (i,j) element of $\underset{\sim}{\bar{R}}$;

r^{ij} = (i,j) element of $\underset{\sim}{\bar{R}}^{-1}$.

Further, define dg($\underset{\sim}{Z}$) to be an n-element vector consisting of the diagonal elements of $\underset{\sim}{Z}$ where:

$$Z = \sqrt{c}\ \underset{\sim}{R}^{-1}(\underset{\sim}{R}_1 - \underset{\sim}{R}_2).$$

Given the above definitions, Jennrich shows that

$$(1/2)tr(\underset{\sim}{Z})^2 - dg(\underset{\sim}{Z})'\underset{\sim}{S}^{-1}dg(\underset{\sim}{Z}) \tag{3.12}$$

is asymptotically distributed as a chi-square variate with n(n-1)/2 degrees of freedom. The quantity given in (3.12) is difficult to compute as Gibbons[42, p. 11] has noted. To ensure the accuracy of the programming written to compute (3.12), the sample correlation matrices used for demonstrating the test in the original article (Jennrich[51]) were used as input. The results reported in Jennrich[51 ,p. 911] were exactly reproduced by the program written to compute (3.12); so we have confidence in the test results reported in Chapter IV.

Cross-sectional Regressions

In addition to computing the cross-correlation matrices of factor scores, cross-sectional regressions of asset returns on the factor loadings vectors are conducted to further examine cross-sectional congruence. To see how this is accomplished, substitute Equation 1.2 into the assumed linear return generating process given by Equation 1.1:

$$(3.13) \qquad \tilde{R}_{it} = \lambda_o + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik} + b_{i1}\tilde{\delta}_{1t} + \ldots + b_{ik}\tilde{\delta}_{kt} + \tilde{\varepsilon}_{it}$$

Equation 3.13 represents the null hypothesis that the APT is true and has formed the basis of most empirical exaiminations of the APT. In this dissertation, Equation 3.13 will be used (with modifications discussed below) to examine the congruence of factor structure estimates across the three sample groups. In other words, explicit tests of the APT implied by Equation 3.13 will not be conducted; rather the empirical form of the APT is used in an effort to assess the cross-sectional congruence of the estimates.

Equation 3.13 can be written:

$$(3.14) \quad \tilde{R}_{it} = \lambda_o + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik} + \zeta_{it}$$

where the error (ζ_{it}) accounts for the intertemporal variation in the factors and the idiosyncratic components of the returns. Note that the error term in Equation 3.14 is simply the hypothesized k-factor model used in estimating the loadings vectors and the idiosyncratic error variances.

Allowing for possibly heteroskedastic error variances, the regression implied by Equation 3.14 can be estimated using generalized least squares (GLS) as follows:

$$(3.15) \quad \underset{\sim}{\lambda}_t = (\underset{\sim}{B}'\hat{\underset{\sim}{\Sigma}}^{-1}\underset{\sim}{B})^{-1}\underset{\sim}{B}'\hat{\underset{\sim}{\Sigma}}^{-1}\underset{\sim}{r}_t$$

where the 't' subscript refers to the vector of coefficients estimated in time period t. Using an approach similar to that suggested by Fama and MacBeth[34], the regression in Equation 3.15 is conducted in each subperiod for each group a total of 1029 times (the number of daily return observations). This results in a time series of each of the k+1 coefficients. The (time series) average of each coefficient and its associated standard error can be used to test whether the associated factor is "priced" in the subperiod. More specifically, the average market risk premium ($\bar{\lambda}_j$) is computed as the time series mean of the (T) coefficients estimated using Equation 3.15 as follows:

$$\bar{\lambda}_j = (1/T)\sum_t \lambda_{jt} \quad (3.16)$$

The associated standard error of the mean coefficient is:

$$\sigma_{\bar{\lambda}_j} = \sigma_{\lambda_j} / \sqrt{T} \quad (3.17)$$

where σ_{λ_j} is the standard deviation of the estimated coefficient computed as follows:

$$\sigma_{\lambda_j} = \{\sum_t(\lambda_{jt} - \bar{\lambda}_j)^2\}/(T-1) \quad (3.18)$$

In this study, there are three groups, five subperiods in each group, 1029 daily observations per group per subperiod, and a total of seven different hypothesized factor models. Thus, a complete examination of the number of priced factors requires: 3(5)(1029)(7) = 108,045 cross-sectional regressions. This, of course, is an enormous task but is necessary to thoroughly examine the issue concerning the number of priced factors.

As is well known, Roll and Ross[83] report three or perhaps four factors are priced in their tests of the APT. Similarly, Brown and Weinstein[9] report evidence indicative of relatively few factors being significant as does Chen[15]. Concerns regarding tests for the number of "priced" factors using the Roll and Ross[83] methodology were expressed by Dhrymes, Freind, and Gultekin[22] and most recently by Dhrymes, Freind, Gultekin and Gultekin[23]. Strictly speaking, if three regression coefficients are significant in Equation 3.15, for example, one cannot conclude that three factors are, in fact, priced in the market. This is due to the indeterminacy in the factor solution discussed in the previous Chapter.

In their Reply[86], Roll and Ross argue that since the factors are extracted in order of their importance in explaining the covariation among returns, regressions such as those in (3.15) are appropriate despite the factor indeterminancy problem. With the large group sizes used herein, there should be less mixing of factors across groups than in other empirical tests, so the arguments of Dhrymes, Freind, and Gultekin[22] are not as relevant here as in empirical tests employing smaller group sizes.

The samples of securities to be used in this study and the specifics of the statistical tests have been discussed. The next Chapter presents and discusses the empirical results.

CHAPTER IV

EMPIRICAL RESULTS

Prior to discussing the empirical results, some problems which were encountered in the first stage of the estimation process must be described and their implications noted.

Preliminary Estimation Issues

It is well known that maximum likelihood factor analysis is very sensitive to the variables (here, securities) used in the estimation. Heywood cases (discussed in Chapter II) tend to occur quite often and some approach must be adopted to deal with them. Several methods which attempt to correct for these "improper" solutions have recently been examined by Dillon, Kumar, and Mulani[26] in the context of structural equation models. Their findings indicate many of the proposed "fixes" to improper solutions are objectionable on statistical grounds and therefore, ill-advised.

When Heywood cases were encountered in past empirical tests of the APT, the offending securities were simply removed from the sample and estimates of the factor structure recomputed using the smaller sample.[1] In this dissertation such an approach would likely result in a different group size in each of the samples which would make cross-sectional comparisons difficult. Thus, to keep group size equal in each of the three samples, the following procedure was used.

The first (alphabetically) 300 securities with complete trading data were split into three groups of 100 securities each and a variance-covariance matrix computed in each subperiod. A twelve factor model was estimated for each of the (3 x 5=) 15 variance-covariance matrices. Within each of the three groups, if a Heywood case occured in any of the subperiods, the offending security was removed and replaced by a security from the set of sixty alternates and a twelve factor model refit. (Recall there were 360 securities which met the data requirements). This process continued until no Heywood cases were encountered in any group during any subperiod.

Two securities had to be replaced in Group A and one was replaced in Group B. The original one-hundred securities in Group C caused no estimation problems. Once the twelve factor model was successfully estimated in the three groups in all subperiods, we are assured that no

Heywood cases will occur when smaller dimension factor models are estimated.

As previously noted, this replacement process was necessary to maintain an equal number of securities in each group which, in turn, facilitates comparisons among the groups. It should be emphasized that Heywood cases are sample-specific phenomena, i.e. the "offending" security may not prove to be a problem when included in another sample. Estimation problems such as those encountered here are common in factor analytic investigations. These empirical problems will plague any work examining the APT unless alternative methods of estimating the factor structure are utilized.

Eigenstructure Analysis

The eigenvalues of each of the fifteen (standardized) covariance matrices were computed to obtain an initial indication of the number of factors underlying security returns. Table 1 summarizes the results.

Several observations can be made based on these results. First, the results for each of the three groups were markedly similar. This was anticipated since each group had the same number of securities (100) and the assignment of securities to groups was essentially random. Second, in each group over time there is a large decrease in the number of eigenvalues greater than one. If one were using Kaiser's roots-greater-than-one criterion for selecting the number of factors, the evidence suggests that over time, fewer factors are necessary to reproduce the standardized covariance structure.

Since there are one hundred securities in each group, the eigenvalues reported in the body of Table 1 can be viewed as percentage figures. For example, the largest eigenvalue in Group A for the first subperiod (13.39) indicates the first eigenvector accounted for just over 13% of the total variance in the sample. Note that in each group, the largest eigenvalue increases markedly between the second and third subperiod and then tends to stabilize in subperiods four and five. The results indicate the first eigenvector accounted for approximately 18-19% of the total sample variance in the last three subperiods while accounting for only 13-16% in the first two subperiods.

Further, note that in each of the three groups the second eigenvalue increases monotonically over time. This suggests increasing importance of the second eigenvector (in terms of accounted for variance). Eigenvalues three through five tend to be nearly constant over time in each of the groups, indicating the importance of their respective eigenvectors has not changed much over time.

In summary, the eigenstructure of the sample data seems to be becoming simpler over time. As one moves through time, fewer eigenvalues are greater than one and the first two eigenvectors in each group account for more of the observed sample variation. Visual confirmation of the above analysis can be obtained by reviewing the scree plots presented in Figures 1 - 3.

Maximum Likelihood Factor Analysis Results

Maximum likelihood factor analysis (MLFA) was conducted on each of the fifteen standardized covariance matrices for factor values of k=1,...,5,10,12,15 and the associated chi-square values computed. The results are presented in Tables 2 through 9.

With the group size of n=100 securities used in this study, the commonly used 50% level of significance for determining whether another factor needs to be extracted from the data is not attained until k=12 factors are extracted. This result is in sharp contrast to the results reported by Roll and Ross[83 ,p. 1088] where they state, "...the probability level (.980) implied only two chances in 100 that at least six factors were present in the data." The large discrepancy between their results and those reported herein is due to the group size chosen for the factor analysis. Roll and Ross use a group size of n=30 and for this relatively small group size, five factors is probably indeed sufficient. For larger group sizes (i.e. the n=100 securities used herein) a larger number of factors is necessary to reproduce adequately the sample variance-covariance matrix. These observations are consistent with those reported by Dhrymes, Freind and Gultekin[22] and Dhrymes, Freind, Gultekin and Gultekin[23] where the number of statistically significant factors is a positive function of group size. Of course, neither the results reported in Tables 2 through 9 nor those reported by Dhrymes, Freind and Gultekin address the question of how many factors are priced. In other words, is the number of priced factors a positive function of group size? This issue will be examined later in this Chapter.

Another interesting feature found in Tables 2 through 9 regards the intertemporal nature of the factor space. For example, with k=12 factors only in the first two subperiods do twelve factors seem to be an adequate representation of the data. The latter three subperiods in each group seem to require more than twelve factors to obtain an adequate representation of the data. Even with fifteen factors (where two of the subperiods failed to converge) there still remain subperiods for which fifteen factors are not sufficient (in the sense of a chi-square goodness-of-fit test).

Several reasonable explanations can be advanced for these findings. First and foremost, the usual chi-square goodness-of-fit statistic may be an inappropriate measure in examinations of the return generating process assumed by the APT. Surely, more factors will be necessary (in a statistical sense) to reproduce adequately larger covariance structures. However, the central issue of empirical tests of the APT is the number of factors which are priced. The factors beyond the fifth, say, may not be priced in the context of the APT but are obviously necessary to obtain adequate fit statistics.

Second, the data may not be distributed as multivariate normal and hence the likelihood functions (Equations 3.4 and 3.5) and the chi-square values computed from these functions may be misspecified for these data. The findings of Fama[33] are pertinent here. Fama reports that daily return observations are not distributed normally. This fact precludes multivariate normality since univariate normality is a necessary though not sufficient condition for multivariate normality. No attempt was made in this study to assess the degree of departure from multivariate normality in the return data although a technique such as Q-Q plots could be used.

The results presented in Tables 2 through 9 do have some implications for the intertemporal stationarity of the parameters of the APT. For each of the three groups the ten and twelve factor models seem to be reasonable specifications of the structure of the data at least for the first two subperiods. The same cannot be said of the latter three subperiods. Recall that most empirical tests of the APT published to date used data from the period 1962-1972 which corresponds roughly to the first two subperiods used herein. Because of the difference in results for the first two subperiods versus those for the last three subperiods indicated in Tables 8 and 9, past empirical tests of the pricing relationship of the APT must be interpreted carefully. There is no reason to suspect that the results reported by Roll and Ross[83], Brown and Weinstein[9], and others who used the 1962-1972 time period for their tests will be consistent with the results obtained when using more recent time periods.

Test results based on more recent time periods are discussed in the recent paper by Dhrymes, Freind, Gultekin, and Gultekin[23]. Their findings are consistent with those reported herein, i.e. more factors are necessary in the later time periods to obtain comparable fit statistics and associated p-levels.

Intertemporal Tests - Covariance Structure

Test results of Hypothesis 1A are presented in Table 10. Recall that this hypothesis considered the joint equality (stationarity) of covariance structures within each group. As the results in Table 10 indicate, the hypothesis of stationarity of the covariance structure was rejected in all three groups.

As discussed in Chapter II, this test is an extremely stringent one. In words, Hypothesis 1A asserts that all five subperiod covariance structures are (statistically) identical, i.e. have not changed over the approximately twenty-year time period under investigation. As is well known, the chi-square test is very sensitive when applied to large samples which may explain, in part, the consistent rejection of Hypothesis 1A. Such large test statistics as those presented in Table 10 are not at all uncommon, especially when one considers the very large number of degrees of freedom in each test.

One further feature found in Table 10 should be mentioned. Note that all three groups had approximately equal chi-square test statistics. This has two implications: 1) the criteria used to select and assign securities to groups apparently was sufficiently random - no group seems to differ markedly from the others; and 2) a similar degree of (non)stationarity was present in each of the three groups.

Given that Hypothesis 1A is rejected for all groups, the possibility that the covariance structure is stable across adjacent subperiods is examined next as discussed in Chapter II.

The results of testing Hypotheses 1C are presented in Table 11 for all three groups. The hypothesis is rejected for all pairs of subperiods in each group. Thus, using shorter test intervals (approximately four years) does not lead to different conclusions regarding the stationarity hypothesis. Obviously the covariance structure is unstable over time, at least for the group size and time periods used in this study.

Note however, that even though Hypothesis 1C is rejected each time it is tested, there are variations in the magnitude of the chi-square values. For example, when comparing subperiod 2 (8/66-10/70) with subperiod 3 (10/70-11/74), the hypothesis of stationarity is rejected at relatively small chi-square values vis-á-vis the other three comparisons. Thus, one may conclude that the degree of nonstationarity varies with the subperiods being examined. The full implications of these results are discussed in Chapter V.

Intertemporal Tests - Correlation Structure

The results of testing Hypothesis 1B are presented in Table 12. Recall that this hypothesis considered the simultaneous equaltiy (intertemporal stationarity) of the correlation structures within each group using a test developed by Jennrich[51]. This hypothesis is similar to Hypothesis 1A except that it uses the standardized variance-covariance matrix whereas Hypothesis 1A examined the unstandardized covariance structure.

The difference between the two tests, of course, is in the treatment of the asset-specific variances on the main diagonal of the respective matrices. Under

Hypothesis 1A, the asset-specific variances are assumed equal while under Hypothesis 1B they do not enter in the test since they are all equal to unity.

Turning now to the results presented in Table 12, we see that the stationarity hypothesis is rejected each time it is tested. These results are the same as found in the tests of Hypothesis 1A (Table 4.2) and imply that standardizing the variates does not result in a stationary (standardized) covariance structure. The results in Table 12 are in contrast to those reported by Gibbons[42] where standardizing the variates resulted in a (very) stationary covariance structure. (Gibbons' p-levels were all equal to 1.0). It seems quite clear that Gibbons' implementation of the test proposed by Jennrich[51] was in error. The programming for the tests of Hypotheses 1A and 1B used herein was validated on the sample data which accompanied the original Jennrich article.

The implications of the test results are clear. At least for the group size and time periods used herein, knowledge of the correlation structure in a previous time period is of little value in predicting (estimating) the correlation structure in a subsequent time period. The possibility still exists, however, that adjacent (in time) correlation structures may be equivalent as discussed in

Chapter II. This issue is examined next.

The results of the tests of Hypothesis 1D are presented in Table 13. Once again, the hypothesis is rejected at standard levels of significance each time it is tested. Unlike the similar tests using covariance input however, the degree of rejection seems to be roughly equivalent in all subperiod comparisons. Recall that in tests utilizing covariance input the diagonal of each of the covariance matrices (i.e. the asset-specific variances) is compared with the diagonal of the other covariance matrix used in the test. With correlation input of course, such a comparison is not made for it is trivial. Indeed, Jennrich's[51] test explicitly corrects for this through the second term in Equation 3.12.

Given the above observations regarding covariance versus correlation input and the arguments of modern portfolio theory regarding the irrelevance of asset-specific variances in pricing, one may appropriately use correlation inputs in examinations of factor structures. The only qualification which should be kept in mind concerns the methodology employed. Since MLFA is scale-invariant, the choice of input is irrelevant for this methodology. The same cannot be said of some competing methodologies, however. If one accepts the analysis of Chamberlain and Rothschild[13] for example, and uses principal components analysis to extract estimates of the "factor" structure, then covariance input must be used. Principal components analysis of the correlation structure presents difficulties in interpreting the results.[2]

Tests for Equal Number of Factors Generating Returns

Test results pertaining to Hypothesis 2A are presented in Table 14. Recall that this hypothesis is a simultaneous test which asserts an equal number of factors are generating returns in each subperiod. The test results are not encouraging to advocates of the APT. In all groups, the hypothesis is rejected for all factor representations, i.e. the p-level for all tests is <.0001. One may conclude that in a statistical sense, the number of factors generating security returns has not been the same over the roughly twenty-year period under consideration.

As was discussed earlier in the context of the simultaneous test of the equality of covariance(correlation) structures, a twenty-year period is most certainly a very long time over which to expect stationarity in the factor structure. More insight into the degree of (non)stationarity may be gained by examining the tests of stationarity using adjacent subperiods. These hypotheses were denoted as Hypotheses 2B and are discussed next.

The results of tests of Hypotheses 2B are presented in Tables 15 through 21 which correspond to 1,2,3,4,5,10, and 12 factor models, respectively. Considering the simpler models (one through five factors), the hypothesis of an equal number of factors generating returns is rejected each time it is tested. These results are not surprising since the simpler factor models proved to be an inadequate representation of the observed security returns as shown in Tables 2 through 6.

The findings thus far are clear: the simpler factor models are not an adequate representation of the factor structure underlying observed security returns. This is true up to and including a five factor model. Recall that virtually every test of the pricing relationship posited by the APT used a five factor representation in both the first and second stages of the tests following the lead of Roll and Ross[83]. Considering the inadequacy of a five factor model to fit the observed return data and the nonstationarity of the process as indicated by the consistent rejections of Hypotheses 2B, one is led to seriously doubt the validity of the previous empirical tests of the APT. More will be said about this in the discussion of the cross-sectional regression results (below) and in Chapter V.

Turning now to the more complex factor models (ten and twelve factor versions), we see that Hypotheses 2B are not uniformly rejected. In the ten factor representation, a reasonable p-level is attained in Group A in the comparison of the first two subperiods but the same is not true for Groups B and C. Nor is the hypothesis maintained for any of the groups in the latter three comparisons. The twelve factor model results given in Table 21 are similar to the ten factor model representation but with much stronger evidence of stationarity over the first two subperiods. The latter three comparisons are again rejected each time they are tested.

The nature of the results were, in part, anticipated from the results presented in Tables 7 and 8 where in the first two subperiods the respective factor models seen to be adequate representations of the observed return data. The pattern of results are consistent across the two analyses. Specifically, in each group the ten and twelve factor models are reasonable representations of the observed return data and the stationarity hypothesis is maintained in the first two subperiods. In the latter three subperiods the fit of the model is poor and the stationarity hypothesis is rejected. These results have implications in at least two areas: 1) the efficacy of previous empirical tests of the APT; and 2) the ability of the estimates to be applied to portfolio management issues. These implications will be discussed in detail in Chapter V.

Cross-sectional Regression Results

Before examining in detail the cross-sectional regression results, some general comments are in order regarding the value of λ_o in the regressions. First, unlike the values of λ_j ; j=1,...,K, the value of λ_o can be interpreted meaningfully. λ_o is the risk-free or zero-beta rate of return implied by a particular factor representation. Secondly, as noted by Roll and Ross[83], Dhrymes, Freind, and Gultekin[22], Chen[15], and others, λ_o is independent of the particular rotation chosen in the factor analysis stage. Given this information, one may compute annual risk-free (or zero-beta) rates implied by the various model/subperiod combinations. This is done in two ways: 1) assuming 250 trading days per year (which is roughly equivalent to the number of daily return observations each year in this study); and 2) computing a 365 day (calendar year) rate. The estimates of the annualized risk-free or zero-beta rates are simply (one plus) the intercept esimate taken to the 250th or 365th power. The unbiasedness of these types of estimators has recently been examined by Cheng[19].

Obviously, only those model/subperiod combinations which produced an estimate of λ_o which was positive could be used in the computations. The results are presented in Table 22.

The estimates of λ_o and the corresponding implied risk-free rates are inconsistent with the predictions of the APT. Where the intercept term was significantly different from zero, the estimates in all cases appear to be far too large to represent the (annual) return from holding a risk-free or zero-beta asset. More reasonable estimates of the risk-free or zero-beta rate of return are obtained in those model/subperiod combinations where the estimate of λ_o was positive but not significantly different from zero. Such estimates must be interpreted with great caution however, since the λ_o estimate obtained in the GLS cross-sectional regressions is not (statistically) different from zero. Also, one can see the great variability of the estimates which are obtained from the different groups in a given subperiod; consistent, reasonable estimates are the exception rather than the rule.

Recall that Roll and Ross[83], Brown and Weinstein[9], Hughes[48], and others used a test of the terms to assess cross-sectional congruence of factor structures. With the exception of Brown and Weinstein (who used a particularly stringent test), the results of such tests indicate the intercept terms (the λ_o) were not different across groups. What is not reported in their test results is whether they excluded negative or insignificant intercept estimates. Most of the λ_o estimates obtained herein were positive but not significantly different from zero as indicated in Table 22.

Where computations were performed, the implied (annual) returns seem to be unrealistically large. It may well be that other researchers ignored the magnitude and significance of the intercept estimates in their tests. Recently, Dhrymes, Freind, Gultekin, and Gultekin[23] report results of a test which hypothesizes that the intercept estimates(λ_o) are equal to the rate(R_{ft}) available on 30-day Treasury bills during the time period covered by their analysis. They reject the hypothesis for sample sizes roughly equivalent to those used herein. Thus, the "too large" estimates given in Table 22 are not inconsistent with the results reported by Dhrymes, et. al.

Quite obviously the APT does not provide theoretically appealing estimates of the risk-free or zero-beta rate, at least in the group size and factor model dimensions employed herein. The implications of this finding will be discussed further in Chapter V.

Summary results for the cross-sectional regressions are provided in Tables 23 through 29 which correspond to 1,2,3,4,5,10 and 12 factor models, respectively. In each table, the results are those obtained by regressing the (N-element) vector of daily returns on the estimated loadings vectors each day in the corresponding subperiod using Equation 3.15. The mean risk premium during each subperiod, the standard error of the mean, and the associated t-value are computed using Equations 3.16 through 3.18. The tables are constructed to facilitate cross-sectional comparisons. The discussion of the results considers the factor models in increasing order of complexity.

For the case of a single-factor model, several observations can be made. First, the estimate of the risk-free (zero-beta) rate is not significantly different from zero in the first three subperiods for all three groups. This is inconsistent with the APT since one expects the (nominal) risk-free rate to be in excess of zero. The risk-free estimate is significant and positive in the latter two subperiods in each group but as indicated earlier, is of questionable magnitude.

Second, the single factor is significantly different from zero (i.e. "priced") at the .05 level of significance in only the first subperiod in all three groups. In no group is it significant at the .05 level in any of the remaining subperiods. One would like to draw some inference based on the magnitude of the risk premium but as pointed out by Roll and Ross[83], the magnitude and sign of the risk premia are arbitrary. Hence, only statistical significance is of interest.

The fact that the first factor is only priced in one subperiod out of five in each of the groups is not very encouraging, especially if one is an advocate of the zero-beta form of the CAPM which is a close (but not exact) analog of a single-factor APT model. Nevertheless, the results in Table 23 are consistent with the results reported by Chen[15] even though he used slightly different time periods and quite a different estimation technique than used herein.

As regards the cross-sectional congruence of the factor structure, one can (relatively) safely conclude that the one-factor model is consistent across the samples. When the risk-free (zero-beta) rate is (not)significant in one group, it is (not)significant in the other groups. The same is true of the estimated risk premium.

Consider next the two-factor model results reported in Table 24. Again the first factor risk premium is significant at the .05 level in all three groups during the first subperiod. Interestingly, the first factor is significant at the .10 level in two of the groups (B and C) during the second subperiod and in all groups during the third subperiod. Unlike the single-factor model, the risk-free (zero-beta) rate is not significantly different from zero during the fourth subperiod but is significant in the last subperiod for groups B and C.

The second factor is significant in each group in the third and fourth subperiods but only in the fourth subperiod is this factor priced in addition to the first factor. One is led to conclude then, that in only the fourth of the five subperiods does a two-factor model seem to be generating returns. In the first three subperiods a one-factor model seems to be present and in the fifth subperiod a zero-factor model is indicated! These results are only suggestive at this point however; analysis of more complex models must be considered.

Turning now to the three factor model results, the intercept estimate is significant at the .10 level in Group B during the third subperiod and significant at the .05 level in Groups B and C during the fifth subperiod. The general lack of congruence of significant estimates across groups is troubling because the wide variations in magnitude of the coefficients suggest radically different implied risk-free (zero-beta) rates of return.

The pricing of the factors in various group/subperiod combinations also suggests less comparability of the results across groups here than in the smaller dimension models discussed earlier. Some congruence is present however. For example, the first factor is priced in subperiods one and four in all three groups. Similarly, the second factor is priced in the fourth subperiod in all three groups.

Beyond the above observations however, there appears to be little congruence across the three groups. Interestingly, in many of the group/subperiod combinations, more than one factor is significant which suggests a multiple-factor pricing equation. Perhaps the most troubling result is that in each group there is at least one subperiod in which no factors are priced. This result is, of course, inconsistent with the APT.

In the four and five factor specifications, the results become more difficult to interpret due to both the larger dimension of the models and the general incongruity of the results. Note that in the four factor representation, in at least one subperiod in each group, the results indicate none of the factors is priced; a result inconsistent with the APT. Evidence in support of the APT, however, is indicated by the fact that in the first and fourth subperiods in all groups, at least two factors are significant at the .10 level of significance. The congruence of the results across groups however, is generally very poor. There seems to be little consistency both in terms of the number of factors priced in a particular subperiod and the location (i.e. first, second, third, etc.) of the priced factors.

Turning now to the five factor models, one again is faced with a general lack of congruence in both the number and location of the significant factors. Interestingly, in none of the groups is a factor priced in the fifth subperiod; a result in strong conflict with the APT. On the other hand, in those group/subperiod combinations where a factor is priced, typically at least two are priced. This constitutes weak evidence of a multiple factor pricing equation.

Summarizing, we have seen that the larger factor models tend to produce results which are difficult to compare across groups due to the general lack of cross-sectional congruence in the estimates. The smaller factor models (i.e. one and two factor specifications) produce results which are more readily interpreted.

Recall that virtually every previous empirical test of the APT has used a five-factor model in the factor analysis stage and then used the resulting loadings estimates in the second-pass (cross-sectional) regressions. In this study, prompted by the arguments of Dhrymes, Freind and Gultekin[22] and Dhrymes, Freind, Gultekin, and Gultekin[23], ten and twelve factor models were estimated and these larger representations used in the cross-sectional regressions. The main concern here is whether there exist factors beyond the fifth which are relevant in pricing. If there are such priced factors then previous empirical tests of the APT must be interpreted very carefully.

Consider the results in Table 28 for the ten factor specification. In the fifteen group/subperiod combinations, there are no fewer than six instances of a factor beyond the fifth being significant at the .10 level. In the twelve factor version (Table 29), there are again six instances of a factor beyond the fifth being significant in the pricing equation, although the locations of the significant coefficients do not correspond to those of the ten factor model.

The conclusion which may be drawn from these results is clear: previous empirical tests of the APT which used (at most) five factors have ignored factors beyond the fifth which are priced in the cross-sectional regressions.

One very interesting feature in the ten and twelve factor models which should be noted is that even though factors beyond the fifth are priced, the total number of factors priced in any group/subperiod combination never exceeds four. This result is consistent with the assertions of, among others, Roll and Ross[83], Brown and Weinstein[9], Hughes[48], and Chen[15]. They conclude that the number of factors generating returns is relatively small, say three or four. The real problem highlighted by the results presented in Tables 28 and 29 is that the significant factors can be found beyond the fifth factor. Since few previous empirical tests considered these factors, the results presented in those studies and some of the conclusions reached by the authors are suspect.

Correlations of Factors Across Groups

The correlation matrices of factor scores across groups are provided in Tables 30 through 34 which correspond to the first through fifth subperiods, respectively. The (T x K) matrix of factor scores for each group was computed with IMSL subroutines OFCOEF and OFSCOR using the regression method of Harman[45]. The resulting estimates represent the time series behavior of the factors. To facilitate discussion of the results, only a k=10 factor model was used in the estimation process. Also, only those correlations which exceeded 0.25 in absolute value are reported in the Tables. Thus, the size of the Tables depends upon the number of "large" correlation coefficients observed.

Recall that within each group/subperiod combination, the factors are constructed to be orthogonal with unit variance. Thus, the correlation matrix of factor scores within each group is the (k-order) identity matrix and is not reported. Interest centers on the correlation matrices between groups in each of the subperiods.

As was discussed in Chapter II, factor congruence across groups would be indicated by robust elements on the diagonal of the respective matrices and small (near zero) off-diagonal elements. Concerning statistical significance of the correlation coefficients, a correlation coefficient of 0.25 in absolute value has an associated t-statistic of 8.27 in absolute value which is highly significant under the null hypothesis that the correlation coefficient is zero.

The results for Subperiod 1 given in Table 30 are not unambiguous. The factors in each group appear to be linear combinations of the factors in the other groups. No strong congruence is indicated.

The results for Subperiod 2 (Table 31) are much less ambiguous. In each group comparison (i.e. A vs. B, A vs. C, and B vs. C), only two correlation coefficients are larger than 0.25 in absolute value and they appear on the diagonal of the respective correlation matrices. These results indicate that of the ten factors estimated in the factor analysis stage, only the first two in each group were highly correlated with their counterparts in the other groups.

Consider next the results for Subperiod 3 reported in Table 32. Notice that in each group comparison, correlation coefficients for the first four factors are reported. There is some evidence of a lack of congruence when comparing Groups A and B and Groups B and C. This is due to the relatively robust off-diagonal elements in the respective correlation matrices. Much better results are obtained in the Group A vs. Group C comparison. All the robust correlation coefficients are found on the diagonal of the matrix indicating that at least in this comparison, there appears to be a high degree of factor congruence between these two groups.

The results for the fourth subperiod are presented in Table 33 and are indicative of a strong degree of factor congruence. There is only one robust off-diagonal element (Group A vs. Group B comparison). With that one exception, the first three factors in each group appear to represent the same underlying phenomena.

In the fifth subperiod (Table 34), there again appears to be less cross-sectional congruence of the factor estimates indicated by the robust off-diagonal elements in each comparison. Note also that the relatively large number of factors reported in the Group B versus Group C comparison. Some variation in the results is to be expected but the results reported in Table 34 negate any conclusive statements concerning factor congruence in this subperiod.

Some general comments are in order regarding the results reported in Tables 30 through 34. First, in every comparison made, the first factor is (highly) congruent across groups. The smallest correlation between the first factors extracted is .8991 in Table 30 when comparing Groups A and C. The mean correlation coefficient between the first factors extracted from each group computed over all subperiod/group comparison combinations is .9473. This result is certainly indicative of strong congruence of the estimates of the first factor, regardless of the subperiod under examination. Secondly, overall the results presented herein are far more supportive of cross-sectional factor congruence than those reported by Hughes[48]. While Hughes used a group size (n=110 securities) comparable to that used here, far fewer observations (T=120) were used to obtain factor structure estimates in her study. The 1,029 observations used herein to estimate the factor structure obviously result in less ambiguous conclusions regarding cross-sectional congruence.

In summary, the results of the examinations of cross-sectional congruence are quite dependent upon the subperiod under consideration. Relatively strong congruence is indicated in Subperiods 2 and 4; somewhat more ambiguous results are found in the other three subperiods. On a more positive note, the large time series initially used to estimate the variance-covariance matrix for each group/subperiod combination appear to result in factor estimates which are more congruent across groups than when shorter time series are used initially.

The results of all the statistical tests and heuristic examinations have been presented and briefly discussed. The next Chapter discusses the implications of the results, points out some issues relating to applications of the APT to portfolio management problems, and concludes this dissertation with some suggestions for further research.

CHAPTER V

SUMMARY AND CONCLUSIONS

The purpose of this dissertation as set forth at the end of Chapter II was to examine the intertemporal stationarity and cross-sectional congruence of the returns generating process underlying the APT. In keeping with the explicit distinction between the intertemporal and cross-sectional aspects of the analysis, this Chapter will discuss the conclusions based on the empirical results separately, beginning with the intertemporal analysis. At an appropriate juncture, the two analyses will be discussed jointly.

Conclusions Regarding Intertemporal Stationarity

As was discussed in Chapter IV based on the results in Table 1, the factor structure underlying equity returns has become increasingly simpler over the (approximately) twenty-year period covered by the analysis. In all three samples, fewer eigenvalues were larger than one in the later subperiods than in the earlier subperiods. Also, the first two eigenvectors accounted for a larger portion of variance in the samples over time.

This result may reflect the adoption of a security's CAPM beta as a measure of security systematic risk. If an increasing number of investors base their investment decisions on the CAPM risk measure, then the first factor (i.e. the market factor) would show an increasing importance over time in terms of the amount of explained variation. This argument will hold for expected as well as observed returns if we assume that, on average, investor expectations are realized.

At first glance the above line of reasoning may seem to be in conflict with the MLFA results reported in Tables 7 through 9 where it was reported that in only the earlier time periods did the MLFA procedure produce reasonable p-levels. However, there is no conflict. While the MLFA results do reflect an increasing importance of the first factor, factors beyond the first are not trivial in any subperiod.

For the MLFA procedure to produce a single-factor model as an adequate representation of the returns generating process, all factors beyond the first must be trivial (i.e. represent purely random and nonsystematic components of observed returns). The intuition which lies at the heart of the APT is that more than one factor is considered important by investors, however, and therefore priced in the market. Despite the simpler factor structure indicated in the later subperiods, the results reported in Tables 23 through 29 support the intuition underlying the APT.

Turning now to the formal statistical tests of stationarity, recall that the null hypotheses of covariance structure stationarity (Hypotheses 1A and 1B) were rejected each time they were tested. Several conclusions can be reached based on these results. First, and most obvious, the covariance (or correlation) structure of security returns is not intertemporally stationary, at least for the subperiods examined herein. This implies, of course, that any use of an historical covariance (correlation) structure to estimate a structure in a future time period will meet with, at best, limited success.

Further, any security selection and/or portfolio construction criteria based on the covariance (correlation) structure cannot reasonably be expected to be optimal in later time periods. This line of reasoning is consistent with, for example, Merton's[69] analysis of shifts in the efficient frontier and Brenner and Smidt's[8] and Fabozzi and Francis'[32] analysis of the nonstationarity of beta estimates in a CAPM framework.

Second, nearly all previous empirical tests of the APT must be interpreted with due caution. The results reported by other researchers should be viewed only in the context of the particular time period chosen for the analysis. Generalizing the results to a later time period is not valid.

Third, given the rejection of Hypothesis 1B each time it is tested, one may conclude that standardizing the variates does not produce a stationary structure. In other words, the earlier rejection of Hypothesis 1A was not due to the inequality (nonstationarity) of the asset-specific variances (i.e. the diagonal elements of the respective covariance matrix). Thus, the standardized covariances (i.e. correlations) among assets are not stationary. Since modern portfolio theory places such great emphasis on the covariances (correlations) among asset returns, one is led to doubt the ability to apply in practice the selection guidelines suggested by theories such as the CAPM and, at least in its present level of development, the APT.

The above arguments are not altered if one considers the tests of stationarity conducted between adjacent subperiods and reported in Table 11 (covariance input) and in Table 13 (correlation input). In all cases, whether one uses covariance input (Hypothesis 1C) or correlation input (Hypothesis 1D), the null hypothesis of a stationary structure was rejected. So for even shorter and adjacent time periods, the covariance and correlation structures are deemed nonstationary.

Turning now to the less restrictive hypothesis of an equal number of factors generating returns over time, the simultaneous test results of Hypothesis 2A reported in Table 14 indicate nonstationarity of the factor structure. A different number of factors is necessary to reproduce adequately the correlation structure of returns over time. This conclusion is supported (with few exceptions) by the results of testing Hypothesis 2B reported in Tables 15 through 21. In only the twelve-factor model is Hypothesis 2B not rejected and then only when conducting the test between the first two subperiods. Thus, with the exception of the twelve-factor return generating process representation, the results indicate a significant degree of nonstationarity in the return generating process.

Overall then, the tests which posit intertemporal stationarity are overwhelmingly rejected. As noted earlier, these results have several implications. First, the results of tests of the APT must be viewed as specific to the time period chosen for the analysis. Second, using factor structure estimates from one period in an attempt to forecast the structure which holds in a later time period is ill-advised and is not likely to produce a good forecast. Third, asset selection and/or portfolio construction guidelines based on the assumption of a stationary covariance matrix or factor structure are unlikely to produce the (a priori) desired portfolio characteristics and performance.

Several issues regarding the cross-sectional congruence of factor structure estimates were also examined in this dissertation. The results have implications for future empirical tests of the APT and are discussed next.

Conclusions Regarding Cross-sectional Congruence

One of the purposes of this dissertation as set forth in Chapter II was to examine the degree of cross-sectional congruence of the various factor structure estimates among the three groups. Overall, one may conclude that there was not a great degree of cross-sectional congruence among the factor structure estimates.

The results of the GLS cross-sectional regressions presented in Tables 23 through 29 indicate that in most model specifications, the location of the significant market risk premia differed markedly across groups. While some of the significant factors are found in the same location across groups, there appears to be quite a bit of "mixing" of the factor estimates across groups. In other words, the $j^{\underline{th}}$ factor extracted in one group is not generally comparable to the $j^{\underline{th}}$ factor extracted in another group. (The exception to this, of course, is the first factor extracted in each group).

This "mixing" of the factors was noted by Roll and Ross[83], and others, and it was felt that by using a larger group size, less mixing (i.e. more congruence) would result. To some degree, less mixing was accomplished (see Tables 30 through 34) but there still remains enough of this phenomenon to raise serious doubts concerning the application of APT estimates in portfolio management.

If, by using large samples, one could be assured of obtaining congruent loadings estimates across groups, then portfolios of assets could be constructed which had desired sensitivities to the k sources of risk. In other words, one could construct a high-beta portfolio in the spirit of the CAPM if one anticipated a bull market. Further, one may wish to construct the portfolio so it would have a low "beta" on the second factor, etc. Without a greater degree of cross-sectional congruence in the factor structure estimates, however, it is very unlikely that the ex post portfolio performance would conform to the (ex ante) desired performance.

The above discussion of portfolio construction issues presumes two conditions: 1) the factor structure is intertemporally stationary; and 2) the economic phenomena captured by the factors can be identified. As regards the first point, the intertemporal test results presented herein indicate a large degree of nonstationarity in the returns generating process. So even if one found perfect cross-sectional congruence in the factor structure estimates, the lack of stationarity of the factor structure would inhibit applications. Of course, if the factor structure was found to be stationary over shorter time periods than those investigated here, then portfolio construction guidelines may be possible.

The link between the factors extracted from a variance-covariance matrix and the underlying economic phenomena is just beginning to attract the attention of researchers (see, e.g. Chen, Roll, and Ross[17]). The establishment of such a link is critical if the estimates derived in an APT framework are to be applied by portfolio managers in designing portfolios with desired sensitivities to various economic influences. Research in this area requires further refinements and will probably attract increased attention.

Two other areas for possible future research should be noted. First, the intertemporal results showed a nearly monotonic increase in the amount of variance explained over time by the first factor. As was noted at several points in this dissertation, such empirical evidence warrants further attention. It may be that the assertions of the CAPM are a "self-fulfilling prophecy"; a possibility which should be examined in a study specifically designed to investigate this phenomenon.

Secondly, the congruence of factor structure estimates from larger group sizes should be examined. If one is not directly concerned with issues of intertemporal stationarity (as was the case herein), many more securities would have complete trading data within a given subperiod and all such securities could be included in the tests. This area, along with linking the factors to observable economic phenomena, provides numerous avenues for further research.

In summary, the general lack of intertemporal stationarity and cross-sectional congruence of the factor structures reported in this dissertation indicate serious problems exist with past empirical tests of the APT and that application of the APT to portfolio management issues awaits further research results. Whether methodologies can be developed which overcome the difficulties highlighted in this dissertation remains to be seen.

The results presented in this dissertation provide an indication of the plethora of empirical complexities involved when examining the APT. The degree to which the results presented herein guide researchers in designing more definitive tests is, in part, a measure of the success of this dissertation. The more critical measure of this dissertation will be the degree to which it hastens practical applications of the Arbitrage Pricing Theory.

Table 1

Eigenstructure Results

		Subperiod				
		1	2	3	4	5
Eigenvalues > 1		33	31	27	28	27
	E1	13.39	15.58	18.76	18.24	18.72
Group	E2	2.19	2.22	2.51	2.75	3.16
A	E3	1.94	1.61	2.06	1.86	2.10
	E4	1.79	1.57	1.67	1.56	1.61
	E5	1.61	1.47	1.45	1.47	1.48
Eigenvalues > 1		34	30	27	28	25
	E1	13.31	16.10	18.85	18.07	19.12
Group	E2	1.88	1.99	2.34	2.84	2.96
B	E3	1.74	1.63	1.77	1.72	2.27
	E4	1.57	1.50	1.61	1.59	1.73
	E5	1.51	1.47	1.45	1.47	1.51
Eigenvalues > 1		34	30	26	28	26
	E1	13.34	15.25	19.43	19.42	19.12
Group	E2	2.16	2.40	2.72	2.98	3.70
C	E3	1.70	1.75	1.86	1.87	2.30
	E4	1.51	1.51	1.56	1.55	2.09
	E5	1.49	1.48	1.43	1.47	1.45

Table 2

Maximum Likelihood Factor Analysis - Chi-Square Results

Number of Factors = 1
Degrees of Freedom = 4850

	Group	Subperiod	Chi-square	p-level
		1	6981.8	<.0001
		2	6420.9	<.0001
	A	3	8428.4	<.0001
		4	7881.4	<.0001
		5	8564.0	<.0001
1				
		1	6180.2	<.0001
		2	6314.2	<.0001
	B	3	7356.9	<.0001
		4	8114.7	<.0001
		5	8769.4	<.0001
1				
		1	6369.8	<.0001
		2	6743.2	<.0001
	C	3	7752.2	<.0001
		4	8194.0	<.0001
		5	9765.6	<.0001

Table 3

Maximum Likelihood Factor Analysis - Chi-Square Results

Number of Factors = 2
Degrees of Freedom = 4751

Group	Subperiod	Chi-square	p-level
	1	6295.4	<.0001
	2	5748.3	<.0001
A	3	7434.7	<.0001
	4	6643.7	<.0001
	5	6873.2	<.0001
	1	5776.0	<.0001
	2	5779.8	<.0001
B	3	6396.5	<.0001
	4	6738.2	<.0001
	5	7208.1	<.0001
	1	5723.4	<.0001
	2	5903.2	<.0001
C	3	6554.9	<.0001
	4	6743.0	<.0001
	5	7621.7	<.0001

Table 4

Maximum Likelihood Factor Analysis - Chi-Square Results

Number of Factors = 3
Degrees of Freedom = 4653

Group	Subperiod	Chi-square	p-level
	1	5743.1	<.0001
	2	5434.8	<.0001
A	3	6711.3	<.0001
	4	6141.0	<.0001
	5	6157.4	<.0001
	1	5433.3	<.0001
	2	5500.8	<.0001
B	3	5921.5	<.0001
	4	6297.2	<.0001
	5	6112.6	<.0001
	1	5380.9	<.0001
	2	5485.2	<.0001
C	3	6005.0	<.0001
	4	6186.9	<.0001
	5	6523.9	<.0001

Table 5

Maximum Likelihood Factor Analysis - Chi-Square Results

Number of Factors = 4
Degrees of Freedom = 4556

Group	Subperiod	Chi-square	p-level
	1	5249.6	<.0001
	2	5143.3	<.0001
A	3	6283.9	<.0001
	4	5860.8	<.0001
	5	5785.4	<.0001
	1	5171.0	<.0001
	2	5279.2	<.0001
B	3	5606.3	<.0001
	4	5996.6	<.0001
	5	5536.5	<.0001
	1	5108.6	<.0001
	2	5246.4	<.0001
C	3	5643.3	<.0001
	4	5794.0	<.0001
	5	5794.0	<.0001

Table 6

Maximum Likelihood Factor Analysis - Chi-Square Results

Number of Factors = 5
Degrees of Freedom = 4460

Group	Subperiod	Chi-square	p-level
	1	4908.7	<.0001
	2	4927.2	<.0001
A	3	6016.8	<.0001
	4	5621.6	<.0001
	5	5524.8	<.0001
	1	4968.0	<.0001
	2	5070.1	<.0001
B	3	5350.6	<.0001
	4	5718.5	<.0001
	5	5234.5	<.0001
	1	4920.5	<.0001
	2	5026.7	<.0001
C	3	5376.5	<.0001
	4	5484.9	<.0001
	5	5426.1	<.0001

Table 7

Maximum Likelihood Factor Analysis - Chi-Square Results

Number of Factors = 10
Degrees of Freedom = 3995

Group	Subperiod	Chi-square	p-level
	1	4054.8	.2506
	2	4041.9	.2981
A	3	4932.5	<.0001
	4	4616.0	<.0001
	5	4508.9	<.0001
	1	4127.8	.0699
	2	4153.9	.0390
B	3	4353.0	.0001
	4	4696.9	<.0001
	5	4308.4	.0003
	1	4099.7	.1212
	2	4166.1	.0291
C	3	4414.6	<.0001
	4	4480.8	<.0001
	5	4354.4	<.0001

Table 8

Maximum Likelihood Factor Analysis - Chi-Square Results

Number of Factors = 12
Degrees of Freedom = 3816

Group	Subperiod	Chi-square	p-level
	1	3791.6	.6074
	2	3741.1	.8039
A	3	4578.7	<.0001
	4	4284.5	<.0001
	5	4194.4	<.0001
	1	3842.5	.3784
	2	3880.8	.2282
B	3	4050.0	.0042
	4	4360.0	<.0001
	5	3998.2	.0197
	1	3827.8	.4434
	2	3860.5	.3034
C	3	4100.2	.0007
	4	4160.2	.0001
	5	4025.3	.0091

Table 9

Maximum Likelihood Factor Analysis - Chi-Square Results

Number of Factors = 15
Degrees of Freedom = 3555

Group	Subperiod	Chi-square	p-level
	1	*	*
	2	3349.5	.9934
A	3	*	*
	4	3827.7	.0008
	5	3758.7	.0087
	1	3462.3	.8646
	2	3505.7	.7190
B	3	3662.6	.2106
	4	3892.4	.0005
	5	3580.5	.3784
	1	3451.4	.8913
	2	3457.7	.8762
C	3	3680.2	.0700
	4	3700.9	.0432
	5	3624.7	.2033

* indicates subroutine OFCOMM failed to converge for this subperiod/group combination.

Table 10

Hypothesis 1A

Intertemporal Stationarity - Simultaneous Test

Covariance Input

Group	Chi-square	Degrees of Freedom	P-level
A	54249.5	20,200	<.001
B	58435.4	20,200	<.001
C	55823.6	20,200	<.001

Table 11

Hypothesis 1C

Intertemporal Stationarity - Adjacent Subperiods

Covariance Input

	Subperiod Comparisons			
	1-2	2-3	3-4	4-5
Group A	15144.3	9942.2	10176.0	11058.5
Group B	13968.3	9736.9	9936.1	12830.3
Group C	15486.9	9124.7	10213.1	10378.9

All test statistics have 5050 degrees of freedom

P-level for all tests is <.0001

Table 12

Hypothesis 1B

Intertemporal Stationarity - Simultaneous Test

Correlation Input

Group	Chi quare	Degrees of Freedom	P-level
A	49954.3	19,800	<.001
B	49224.0	19,800	<.001
C	48581.6	19,800	<.001

Table 13

Hypothesis 1D

Intertemporal Stationarity - Adjacent Subperiods

Correlation Input

	Subperiod Comparisons			
	1-2	2-3	3-4	4-5
Group A	6684.6	7259.9	6463.1	6425.7
Group B	5990.6	6255.2	6200.2	6555.9
Group C	6171.9	6314.0	6168.5	6364.2

All tests have 4950 degrees of freedom

p-level for all tests is <.0001

Table 14

Hypothesis 2A

Simultaneous Tests - Equal Number of Factors

Number of Factors	Group A	Group B	Group C	Degrees of Freedom
1	38276.58	36735.41	38824.76	24,250
2	32995.43	31898.58	32545.27	23,775
3	30187.56	29265.55	29581.95	23,265
4	28323.09	27589.56	27495.51	22,780
5	26999.14	26341.66	26234.61	22,300
10	22153.99	21640.00	21515.55	19,975
12	20590.20	20031.42	19974.03	19,080

Table entries are chi-square values

p-level for all tests is <.0001

Table 15

Hypothesis 2B

Equal Number of Factors - Tests of Adjacent Subperiods

Number of Factors = 1

Degrees of Freedom = 9,700

	Subperiod Comparison			
Group	1-2	2-3	3-4	4-5
A	13402.74 (<.0001)	14849.31 (<.0001)	16309.82 (<.0001)	16445.45 (<.0001)
B	12494.39 (<.0001)	13671.08 (<.0001)	15471.58 (<.0001)	16884.17 (<.0001)
C	13113.00 (<.0001)	14495.41 (<.0001)	15946.19 (<.0001)	17959.57 (<.0001)

Table entries are chi-square values

p-level in parentheses

Table 16

Hypothesis 2B

Equal Number of Factors - Tests of Adjacent Subperiods

Number of Factors = 2

Degrees of Freedom = 9,502

	Subperiod Comparison			
Group	1-2	2-3	3-4	4-5
A	12043.70 (<.0001)	13183.04 (<.0001)	14078.49 (<.0001)	13516.99 (<.0001)
B	11555.87 (<.0001)	12176.29 (<.0001)	13134.64 (<.0001)	13946.25 (<.0001)
C	11626.61 (<.0001)	12458.18 (<.0001)	13297.96 (<.0001)	14364.73 (<.0001)

Table entries are chi-square values

p-level in parentheses

Table 17

Hypothesis 2B

Equal Number of Factors - Tests of Adjacent Subperiods

Number of Factors = 3

Degrees of Freedom = 9,306

	Subperiod Comparison			
Group	1-2	2-3	3-4	4-5
A	11177.85 (<.0001)	12146.05 (<.0001)	12852.30 (<.0001)	12298.42 (<.0001)
B	10934.15 (<.0001)	11422.34 (<.0001)	12218.75 (<.0001)	12409.90 (<.0001)
C	10866.07 (<.0001)	11490.19 (<.0001)	12191.97 (<.0001)	12710.84 (<.0001)

Table entries are chi-square values

p-level in parentheses

Table 18

Hypothesis 2B

Equal Number of Factors - Tests of Adjacent Subperiods

Number of Factors = 4

Degrees of Freedom = 9,112

	Subperiod Comparison			
Group	1-2	2-3	3-4	4-5
A	10392.98 (<.0001)	11427.22 (<.0001)	12144.71 (<.0001)	11646.23 (<.0001)
B	10450.22 (<.0001)	10885.46 (<.0001)	11602.80 (<.0001)	11533.09 (<.0001)
C	10354.94 (<.0001)	10889.72 (<.0001)	11437.32 (<.0001)	11497.23 (<.0001)

Table entries are chi-square values

p-level in parentheses

Table 19

Hypothesis 2B

Equal Number of Factors - Tests of Adjacent Subperiods

Number of Factors = 5

Degrees of Freedom = 8,920

Group	Subperiod Comparison 1-2	2-3	3-4	4-5
A	9835.90 (<.0001)	10944.00 (<.0001)	11638.42 (<.0001)	11146.40 (<.0001)
B	10038.06 (<.0001)	10420.62 (<.0001)	11069.10 (<.0001)	10953.05 (<.0001)
C	9947.16 (<.0001)	10403.14 (<.0001)	10861.36 (<.0001)	10910.98 (<.0001)

Table entries are chi-square values

p-level in parentheses

Table 20

Hypothesis 2B

Equal Number of Factors - Tests of Adjacent Subperiods

Number of Factors = 10

Degrees of Freedom = 7,990

Group	Subperiod Comparison 1-2	2-3	3-4	4-5
A	8096.65 (.1990)	8974.34 (<.0001)	9548.48 (<.0001)	9124.89 (<.0001)
B	8281.74 (.0111)	8506.93 (<.0001)	9049.85 (<.0001)	9005.28 (<.0001)
C	8265.78 (.0153)	8580.62 (<.0001)	8895.33 (<.0001)	8835.21 (<.0001)

Table entries are chi-square values

p-level in parentheses

Table 21

Hypothesis 2B

Equal Number of Factors - Tests of Adjacent Subperiods

Number of Factors = 12

Degrees of Freedom = 7,632

	Subperiod Comparison			
Group	1-2	2-3	3-4	4-5
A	7532.67 (.7888)	8319.79 (<.0001)	8763.15 (<.0001)	8478.83 (<.0001)
B	7723.23 (.2294)	7930.74 (.0084)	8409.97 (<.0001)	8358.22 (<.0001)
C	7688.25 (.3229)	7960.71 (.0093)	8260.45 (<.0001)	8185.54 (<.0001)

Table entries are chi-square values

p-level in parentheses

Table 22

Implied Risk-Free Rates of Return - Percent

Computed Using 250 and 365 Day Years

		Subperiod									
		1		2		3		4		5	
Group	K	250	365	250	365	250	365	250	365	250	365
A	1	4.21	6.21	*1.56	*2.29	**	**	17.76	26.95	11.54	17.29
B	1	*1.94	*2.85	**	**	**	**	16.44	24.88	17.32	26.26
C	1	*3.90	*5.74	*1.79	*2.63	**	**	15.95	24.11	16.06	24.29
A	2	*3.28	*4.82	*3.05	*4.48	*5.94	*8.80	*3.77	*5.55	*8.60	*12.80
B	2	*2.63	*3.87	**	**	*6.96	*10.32	*2.66	*3.91	16.44	24.88
C	2	*3.23	*4.74	**	**	*4.73	*6.99	*4.08	*6.01	16.50	24.98
A	3	*1.06	*1.54	*2.56	*3.76	*6.72	*9.95	*3.15	*4.63	*9.36	*13.96
B	3	*4.94	*7.30	**	**	7.81	11.61	*3.05	*4.48	16.61	25.16
C	3	*2.10	*3.08	*1.92	*2.81	**	**	*4.39	*6.48	15.98	24.16
A	4	6.24	9.23	*0.90	*1.32	*5.07	*7.49	*3.85	*5.67	*9.66	*14.41
B	4	*4.86	*7.18	**	**	17.52	26.58	*5.94	*8.80	18.32	27.83
C	4	*2.22	*3.26	*0.18	*0.26	**	**	*5.36	*7.93	17.20	26.07
A	5	6.93	9.23	*1.23	*1.80	*3.77	*5.55	*3.90	*5.74	12.18	18.28
B	5	*4.29	*6.32	**	**	17.08	25.89	*5.89	*8.72	19.47	29.67
C	5	*2.40	*3.53	*0.63	*0.92	**	**	7.71	11.45	17.93	27.23
A	10	*5.05	*7.45	*0.90	*1.32	*1.87	*2.74	*4.37	*6.44	*2.12	*3.11
B	10	*3.59	*5.28	*2.84	*4.17	17.67	26.81	9.44	14.08	23.26	35.71
C	10	**	**	*1.82	*2.66	**	**	10.82	16.18	19.59	29.85
A	12	*4.92	*7.26	*0.63	*0.92	*3.51	*5.17	*4.81	*7.10	*4.76	*7.02
B	12	*3.72	*5.47	*2.94	*4.32	17.87	27.14	9.47	14.12	22.13	33.89
C	12	**	**	*2.48	*3.64	**	**	10.76	16.10	19.68	30.00

'*' indicates intercept estimate was not significantly different from zero.

'**' indicates intercept estimate was negative.

Table 23

Cross-sectional Regression Results

Number of Factors = 1

Subperiod	Group	λ_o	λ_1
	A	.000165*	-.097330**
1	B	.000077	-.110759**
	C	.000153	-.090904**
	A	.000062	-.036868
2	B	-.000061	-.067030*
	C	.000071	-.041481
	A	-.000148	-.036235
3	B	-.000032	-.024810
	C	-.000183	-.048131
	A	.000654**	-.017067
4	B	.000609**	-.012500
	C	.000592**	-.016231
	A	.000437**	-.036905
5	B	.000639**	-.012093
	C	.000596**	-.014601

* Coefficient significant at .10 level

** Coefficient significant at .05 level

Table 24

Cross-sectional Regression Results

Number of Factors = 2

Subperiod	Group	λ_o	λ_1	λ_2
	A	.000129	-.103651**	.039185
1	B	.000104	-.106006**	-.011609
	C	.000127	-.095499**	-.010696
	A	.000120	-.029160	.018931
2	B	-.000070	-.068202*	.002623
	C	-.000157	-.072935*	-.052351
	A	.000231	.006854	-.080834*
3	B	.000269	.009310	-.073389*
	C	.000185	-.005616	-.074362*
	A	.000148	-.083280**	-.117817**
4	B	.000105	-.076385**	-.146883**
	C	.000160	-.071282*	.103722**
	A	.000330	-.050530	-.017014
5	B	.000609**	-.015524	.005523
	C	.000611**	-.012800	-.002104

* Coefficient significant at .10 level

** Coefficient significant at .05 level

Table 25

Cross-sectional Regression Results

Number of Factors = 3

Subperiod	Group	λ_o	λ_1	λ_2	λ_3
	A	.000042	-.118851**	.045543	-.036810
1	B	.000193	-.089930**	-.031815	.106425**
	C	.000083	-.103216**	-.023225	.059390
	A	.000101	-.031672	.016695	.042113
2	B	-.000103	-.072504*	.008125	.060626
	C	.000076	-.040817	-.015923	-.104420**
	A	.000260	.010204	-.083683*	.006328
3	B	.000301*	.012989	-.076598*	.139369**
	C	-.000010	-.028105	-.054724	-.042829
	A	.000124	-.086426**	-.120269**	.076201*
4	B	.000120	-.074494**	-.145610**	.035686
	C	.000172	-.069786*	.102234**	.009398
	A	.000358	-.046881	-.014255	.036946
5	B	.000615**	-.014834	.004968	-.016546
	C	.000593**	-.014985	-.000339	-.003397

* Coefficient Significant at .10 level

** Coefficient Significant at .05 level

Table 26

Cross-sectional Regression Results

Number of Factors = 4

Subperiod	Group	λ_o	λ_1	λ_2	λ_3	λ_4
	A	.000242*	-.083406**	.028764	-.001343	.097181**
1	B	.000190	-.090495**	-.032002	.105615**	-.012171
	C	.000088	-.102985**	-.021402	.053731	.072713*
	A	.000036	-.040286	.013104	.055349*	-.048912
2	B	-.000103	-.072698*	.007803	.065989	.033768
	C	.000007	-.050248	-.026045	-.101690**	-.047197
	A	.000198	.003145	-.078060	.003239	.047397
3	B	.000646**	.051817	-.109662	.147429**	.122932**
	C	-.000111	-.039533	-.043783	.050813	-.043705
	A	.000151	-.082674**	-.117024	.077005**	-.011284
4	B	.000231	-.060645*	-.136875	.034155	-.079139*
	C	.000209	-.065338*	.098609	.018215	.064381
	A	.000369	-.045570	-.013019	.037496	.005367
5	B	.000673**	-.008119	-.000554	-.016959	.036285
	C	.000635**	-.009662	-.004241	-.000820	-.040567

* Coefficient Significant at .10 level
** Coefficient Significant at .05 level

Table 27

Cross-sectional Regression Results

Number of Factors = 5

Subperiod	Group	λ_0	λ_1	λ_2	λ_3	λ_4	λ_5
	A	.000268**	-.079217**	.028965	.005763	.101622**	-.132238**
1	B	.000168	-.094419**	-.026173	.105285**	.010458	.055472
	C	.000095**	-.101120	-.019822	.059019	.067278	.009581
	A	.000049	-.038507	.010609	.043084	-.057629*	.061689*
2	B	-.000017	-.061266	-.006248	.059121	.011320	.060695
	C	.000025	-.047831	-.023877	-.102253**	-.049063	-.016023
	A	.000148	-.002568	-.072818*	-.001439	.045406	-.061503
3	B	.000631**	.050312	-.109151**	.141161**	.116981**	.040576
	C	-.000086	-.036905	-.047039	-.049227	.043827	-.037459
	A	.000153	-.082598**	-.117286**	.075962*	.013950	.002998
4	B	.000229	-.060709	-.136329**	.037429	-.078778*	-.001107
	C	.000297*	-.053815	.086344**	.013930	-.070822*	.045716
	A	.000460*	-.034023	-.004534	.037746	.006471	-.059125
5	B	.000712**	-.003589	-.004089	-.016975	.037821	-.049110
	C	.000660**	-.006711	-.007023	-.000718	-.041177	-.071884

* Coefficient Significant at .10 level
** Coefficient Significant at .05 level

Table [illegible]

Cross-Sectional Regression Results

Number of Factors = 10

Subperiod	Group	λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
1	A	.000197	-.091473**	.038754	.001033	.101477**	-.125109**	.018405	.006896	.071620	.036050	-.035168
	B	.000141	-.098978**	-.013483	.102860**	-.001957	.050484	-.001213	.001076	.021090	-.041452	.039999
	C	-.000005	-.118236**	-.044383	.064745	-.061608	-.050077	.076277	-.044270	-.060474	.070050	.001338
2	A	.000036	-.039636	.003891	.042000	.055101	-.075164**	.044779	-.043975	-.034975	-.005180	-.034338
	B	.000112	-.043960	-.026805	.061804	-.015401	.056543	-.076784*	.006797	-.059863	-.027584	-.040051
	C	.000072	-.040703	-.017708	-.102021**	.044344	-.014560	.015421	.018591	.019606	.027026	.015417
3	A	.000074	-.011016	-.065134	-.006127	.037428	.071321	-.010400	.062678	-.024945	-.003409	-.062180
	B	.000651**	.052092	-.107232**	.142401**	.119312**	.033319	.039150	-.032060	-.016057	.095864**	-.064169
	C	-.000147	-.043580	-.039533	-.054675	.043320	-.044005	-.067821	-.043445	.023048	-.023413	-.048383
4	A	.000171	-.078636**	-.117614**	.075378*	.006716	-.010682	.006064	-.007990	.010313	.016595	-.052971
	B	.000361**	-.043647	-.122962**	.033388	-.086503**	-.005452	-.067539	-.046165	.039358	-.065033	-.005093
	C	.000411**	-.038758	.073530*	.029140	.063632	-.041348	-.137951**	-.032303	-.022268	-.004596	.015100
5	A	.000084	-.081353*	-.042349	-.034963	-.005749	.003496	-.085398*	.083760*	.023897	-.049619	-.080608*
	B	.000837**	.011057	-.016066	-.015682	.041838	-.052201	.058765	.034666	.065403	-.034302	.013220
	C	.000716**	.000358	-.012656	.001362	-.040389	-.007570	-.111176**	.002517	-.012739	.013504	.015432

* Coefficient Significant at .10 level
** Coefficient Significant at .05 level

Table 29

Cross-sectional Regression Results

Number of Factors = 12

Subperiod	Group	λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}
1	A	.000192	-.029963	-.093877**	-.030232	.004540	-.122507**	-.097837**	.004452	.071977	.009917	-.063784	.040479	.033204
	B	.000146	-.098044**	-.012287	.103476**	.003759	.045299	-.009889	-.005710	-.030734	-.011465	-.054299	.010851	.029827
	C	-.000014	-.120111**	-.048523	.062607	.063857	.053423	.071189	-.041689	.059104	.066715	-.021870	.040940	.000323
2	A	.000025	-.040951	.001982	.046832	-.026174	.082820**	.038408	-.053895	-.030585	.011260	-.030469	.030293	-.015254
	B	.000116	-.043541	-.026671	.058523	.018605	.063012	-.066588	.027297	.055954	-.019274	.053419	.020890	.033016
	C	.000098	-.020733	-.030562	.015120	-.103270**	.045053	.014949	-.012337	.031038	-.023853	-.014226	.020037	.022013
3	A	.000138	-.003812	-.068991	-.007557	-.038647	-.053019	-.028931	.064063	-.024299	.023772	.064163	-.002014	.042992
	B	.000658**	.052914	-.108826**	.140674**	.118656**	-.037011	-.042652	.026404	.006739	-.089099*	-.078830	.002618	-.040877
	C	-.000128	-.041386	-.041745	-.052092	-.041189	.043196	-.069131	.032080	.034115	-.017111	.049118	-.006811	-.032536
4	A	.000188	-.076350*	-.113255**	.005495	-.083353**	-.010615	-.008816	-.007167	-.013433	-.007088	-.027827	-.058973	.019847
	B	.000362**	-.043611	-.122275**	.031005	.085610*	.022489	.064866	-.052836	-.035500	-.058880	-.007313	.010996	.009139
	C	.000409**	-.039081	.073998*	.030160	-.062517	.043265	-.128102**	-.030881	-.029290	.023150	-.024430	-.014246	.002324
5	A	.000186	-.068195	-.032987	.034576	.001734	.009997	-.094003**	.076559*	.013481	-.028278	-.075265	.077735	-.033204
	B	.000800**	.006749	-.012574	-.016009	.040230	-.047942	.057996	.021841	.070896	.025447	.034479	.018295	.029827
	C	.000719**	.000957	-.013298	-.002681	.038561	-.007180	.110538**	.000684	-.001534	.017821	.008346	.014612	.000323

* Coefficient Significant at .10 level
** Coefficient Significant at .05 level

Table 30

Between Group Correlations of Factor Scores

Subperiod 1

	F1A	F2A	F3A	F4A	F5A
F1B	.9366				
F2B			-.4015		
F3B				.2784	-.2607
F4B				-.2752	

	F1A
F1C	.8991
F2C	.7038
F3C	
F4C	.4410
F5C	
F6C	.2557
F7C	.2880
F8C	
F9C	-.3283

	FB1
F1C	.9809
F2C	.7153
F3C	
F4C	.4299
F5C	
F6C	
F7C	.2588
F8C	
F9C	-.3447

Note: For clarity, only those coefficients greater than 0.25 (in absolute value) are reported.

Table 31

Between Group Correlations of Factor Scores

Subperiod 2

	F1A	F2A
F1B	.9476	
F2B		-.3507

	F1A	F2A
F1C	.9456	
F2C		.3994

	F1B	F2B
F1C	.9467	
F2C		-.5777

Note: For clarity, only those coefficients greater than 0.25 (in absolute value) are reported.

Table 32

Between Group Correlations of Factor Scores

Subperiod 3

	F1A	F2A	F3A	F4A
F1B	.9595			
F2B		.5299		.2651
F3B			.2881	
F4B			.3440	

	F1A	F2A	F3A	F4A
F1C	.9598			
F2C		.6720		
F3C			.5194	
F4C				.4376

	F1B	F2B	F3B	F4B
F1C	.9612			
F2C		.5593		
F3C				.3465
F4C		.2648	.2727	

Note: For clarity, only those coefficients greater than 0.25 (in absolute value) are reported.

Table 33

Between Group Correlations of Factor Scores

Subperiod 4

	F1A	F2A	F3A
F1B	.9605		
F2B		.5941	-.2918
F3B			.3664

	F1A	F2A	F3A
F1C	.9600		
F2C		-.6960	
F3C			.4607

	F1B	F2B	F3B
F1C	.9615		
F2C		-.6915	
F3C			.4730

Note: For clarity, only those coefficients greater than 0.25 (in absolute value) are reported.

Table 34

Between Group Correlations of Factor Scores

Subperiod 5

	F1A	F2A	F3A	F4A
F1B	.9491			
F2B		-.5933		
F3B			.5348	.3009
F4B		.3433		

	F1A	F2A	F3A	F4A
F1C	.9536			
F2C		-.6730		
F3C		.2531	.4772	
F4C			.3304	
F5C				.3055

	F1B	F2B	F3B	F4B	F5B
F1C	.9599				
F2C		.7411			
F3C			.5066	.3238	
F4C			.4017	-.4090	
F5C					
F6C					
F7C					-.2607

Note: For clarity, only those coefficients greater than 0.25 (in absolute value) are reported.

Figure 1

Representative Scree Plot

Group A - Subperiod 3

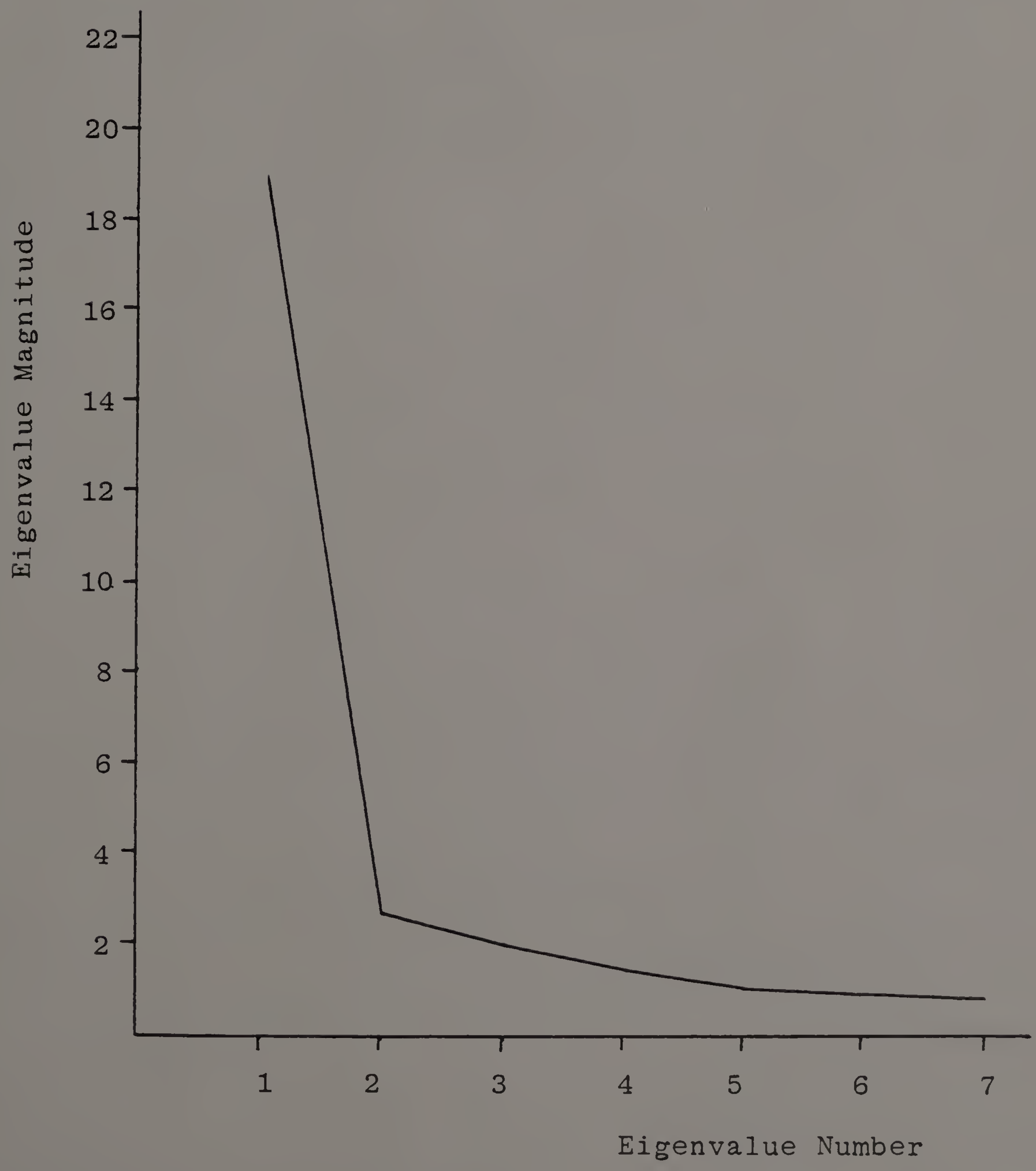


Figure 2

Scree Plot

Average of All Groups
Subperiod 1

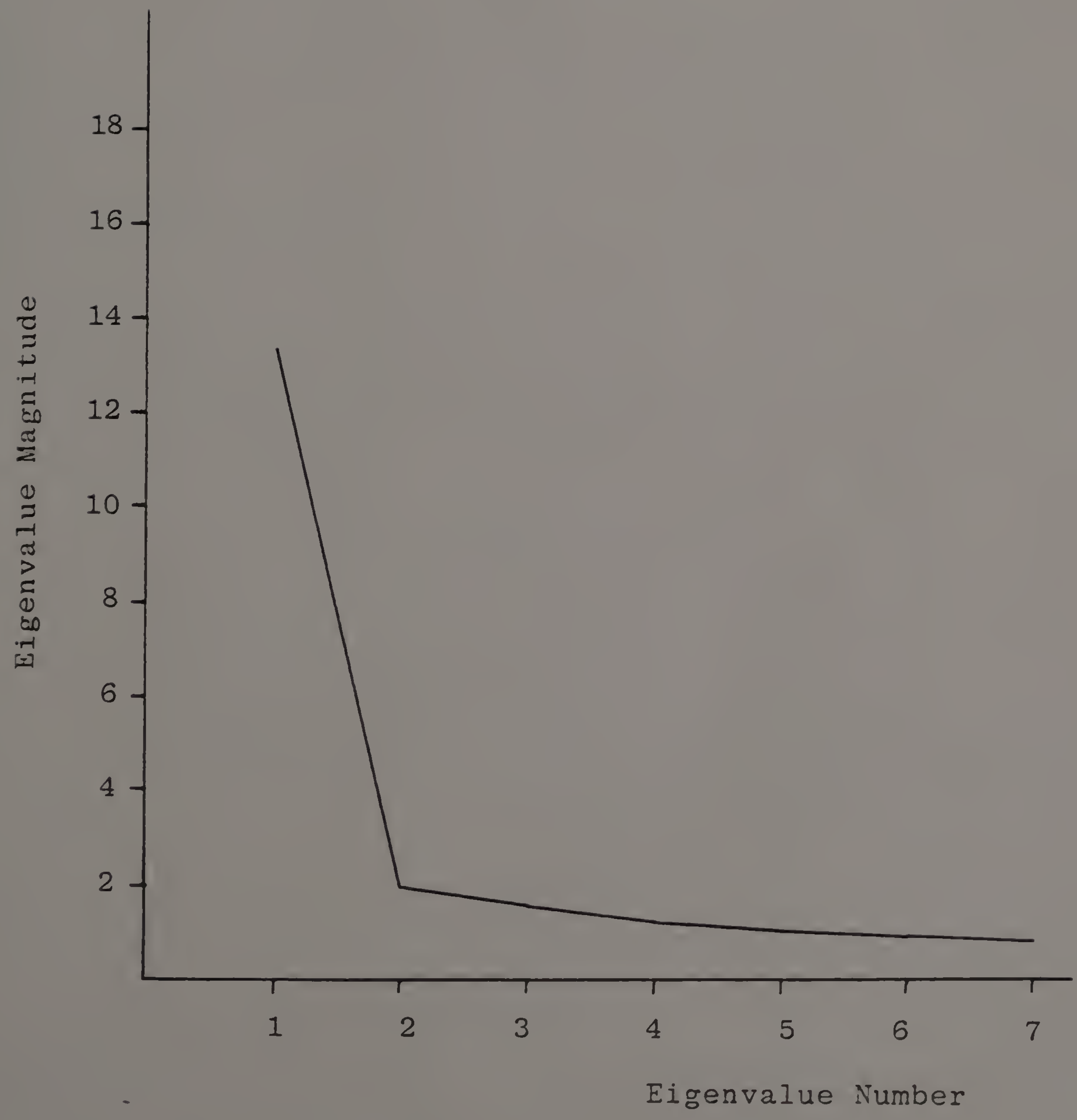
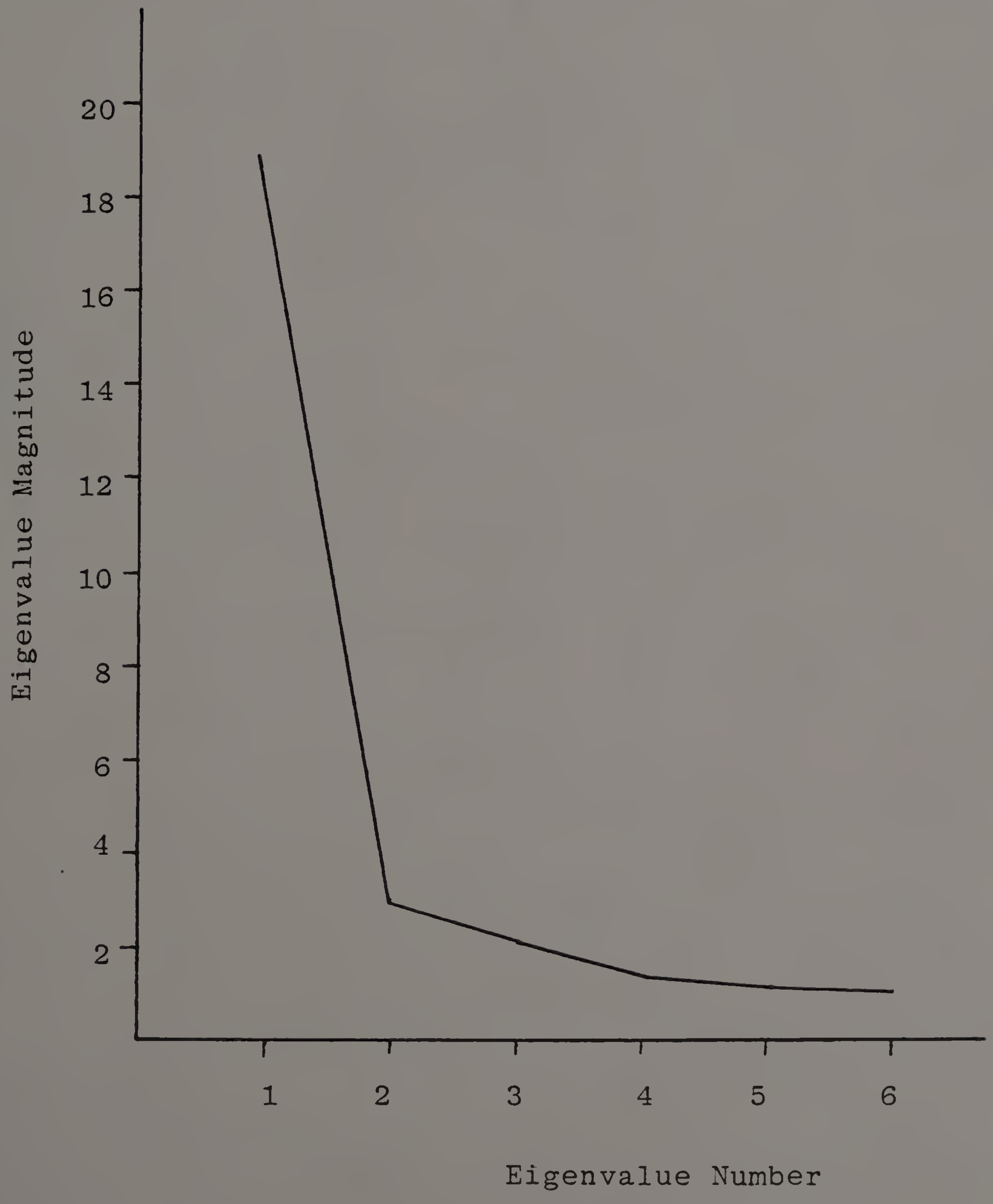


Figure 3

Scree Plot

Average of All Groups
Subperiod 5



ENDNOTES

Chapter I

1. Due to the very large number of empirical tests of the CAPM, no single article can be considered a comprehensive review. A review which discusses some of the problems of testing the CAPM is Roll[82].

2. Examples of models which are variants of the Sharpe[92], Lintner[63], and Mossin[74] version of the CAPM can be found in Black[5], Merton[69], and Kraus and Litzenberger[58].

3. See, for example, Hamada[44].

4. Anamolies relating to firm size are discussed in Banz[3] and Reinganum[79,81]. The price/earnings ratio anomaly is discussed in Ball[2], Reinganum[79], and Basu[2].

5. Specifically, the investor's utility function is assumed to be monotonically increasing and strictly concave. This type of utility function includes, but is certainly not limited to, the quadratic.

Chapter II

1. One could examine the sensitivity of the test results to the choice of the assumed risk-free or zero-beta rate. Most of the recent tests of the APT have estimated this rate by augmenting the factor loadings matrix with a column vector of ones. The GLS regressions then provide an estimate of the risk-free or zero-beta rate. This approach is followed in this dissertation.

2. The results of Cho, Elton, and Gruber[21] and Dhrymes, Friend, and Gultekin[22] indicate Roll and Ross[83] may have reported incorrectly the distribution of p-levels in their results.

3. The "best" estimate of the factor structure, of course, would be obtained by factoring the variance-covariance matrix computed using all risky assets. Obviously, this cannot be accomplished in practice.

Chapter III

1. See discussion in Brown and Weinstein[9] for a detailed treatment.

Chapter IV

1. This procedure was followed by Roll and Ross[83], Brown and Weinstein[9], Cho, Elton, and Gruber[21], Dhrymes, Friend, and Gultekin[22], and Dhrymes, Friend, Gultekin, and Gultekin[23]. Brown and Weinstein[9] discuss in some detail the causes of Heywood cases in their samples.

2. See, for example, Dillon and Goldstein[25, p.36].

BIBLIOGRAPHY

1. Anderson, T. and H. Rubin, "Statistical Inference in Factor Analysis", Proceedings of the Third Berkeley Symposium on Mathematics, Statistics, and Probability, Vol. 5,(1956), pp. 111-150.

2. Ball, R. "Anamolies in Relationships between Securities' Yields and Yield Surrogates", Journal of Financial Economics, Vol. 6 (1978), pp. 103-126.

3. Banz, R. "The Relationship between Return and Market Value of Common Stocks", Journal of Financial Economics, Vol. 9 (1981), pp. 3-18.

4. Basu, S. "Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of Market Efficiency", Journal of Finance, Vol. 32 (June, 1977), pp. 663-682.

5. Black, F. "Capital Market Equilibrium with Restricted Borrowing", Journal of Business, Vol. 45, No. 3 (July 1977), pp. 444-455.

6. Brennan, M.J. "Discussion" Journal of Finance, Vol. 36, No. 2 (May 1981), pp. 352-353.

7. Bower, D.H., R.S. Bower and D.H. Logue, "Arbitrage Pricing Theory and Utility Stock Returns" Journal of Finance, Vol. 39, No. 4 (September 1984), pp. 1041-54.

8. Brenner, M. and S. Smidt "A Simple Model of Non-stationarity of Systematic Risk" Journal of Finance, Vol. 32, No. 4 (September 1977), pp. 1081-1092.

9. Brown, S. and M. Weinstein "A New Approach to Testing Asset Pricing Models: The Bilinear Paradigm", Journal of Finance, Vol. 38, No. 3 (June 1983), pp.711-743.

10. Box, G."A General Distribution Theory for a Class of Likelihood Criteria", Biometrika, 36, 1949.

11. Cattell, R. "The Scree Test for the Number of Factors", Multivariate Behavioral Research, Vol. 1 (April 1966), pp.245-275.

12. Chamberlain, G. "Funds, Factors, and Diversification in Arbitrage Pricing Models" Econometrica, Vol. 51, No. 5(September 1983), pp. 1305- 1323.

13. Chamberlain, G. and M. Rothschild "Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets", Econometrica, Vol. 51, No. 5(September 1983), pp. 1281-1304.

14. Chen, N. "The Arbitrage Pricing Theory: Estimation and Applications", U.C.L.A. Working Paper No. 4-80, 1980.

15. Chen, N. "Some Empirical Tests of the Theory of Arbitrage Pricing", Journal of Finance, Vol. 38, No. 5(December 1983), pp. 1393-1414.

16. Chen, N. and J. Ingersoll, Jr. "Exact Pricing in Linear Factor Models with Finitely Many Assets: A Note", Journal of Finance, Vol. 38, No. 3 (June 1983), pp. 985-988.

17. Chen, N., R. Roll and S. Ross "Economic Forces and the Stock Market: Testing the APT and Alternative Asset Pricing Theories", Working Paper No. 119, Center for Research in Security Prices, University of Chicago, December, 1983.

18. Chen, S. "Multi-period Asset Valuation Under Uncertain Inflation: An APT Framework" Unpublished Working Paper, University of Maryland at College Park, 1984.

19. Cheng, P. "Unbiased Estimates of Long-Run Expected Returns Revisited", Journal of Fanancial and Quantitative Analysis Vol. 19, No. 4(December,1984), pp. 375-393.

20. Cho, D.C. "On Testing the Arbitrage Pricing Theory: Inter-Battery Factor Analysis", Journal of Finance, Vol. 40, No.5(December 1984). .

21. Cho, D.C., E.J. Elton, and M.J.Gruber "On the Robustness of the Roll and Ross Arbitrage Pricing Theory", Journal of Financial and Quantitative Analysis, Vol. 19 No. 1 (March 1984), pp. 1-10.

22. Dhrymes, P.J., I. Freind, and N.B. Gultekin "A Critical Reexamination of the Empirical Evidence on the Arbitrage Pricing Theory", Journal of Finance, Vol. 39 No. 2 (June 1984), pp. 323-346.

23. Dhrymes, P., I. Friend, M. Gultekin, and N. Gultekin "New Tests of the APT and Their Implications", Working Paper, Columbia University, 1985.

24. Dickinson, J. "The Reliability of Estimation Procedures in Portfolio Analysis" Journal of Financial and Quantitative Analysis, June 1974, pp. 447-462.

25. Dillon, W. and M. Goldstein, Multivariate Analysis: Methods and Applications, (New York:John Wiley, 1984).

26. Dillon, W. A. Kumar, and N. Mulani "Offending Estimates in Covariance Structure Analysis: Comments on the Causes of and Solutions to Heywood Cases", Working Paper No. 84-16, University of Massachusetts-Amherst, 1984.

27. Elton, E., and M. Gruber "Homogeneous Groups and the Testing of Economic Hypotheses", Journal of Financial and Quantitative Analysis, Vol. 4, No. 5, (January, 1970), pp. 581-602.

28. Elton, E. and M. Gruber "Improved Forecasting Through the Design of Homogeneous Groups", Journal of Business, Vol. 44, No. 4 (October 1971), pp. 432-450.

29. Elton, E. and M. Gruber "Estimating the Dependence of Share Prices - Implications for Portfolio Selection", Journal of Finance, Vol. 8, No. 5 (December, 1973), pp. 1203-1232.

30. Elton, E., M. Gruber, and J. Rentzler "The Arbitrage Pricing Model and Returns on Assets Under Uncertain Inflation", Journal of Finance, Vol. 38, No. 2 (May 1983), pp. 525-537.

31. Elton E.J., M.J. Gruber, and T.J. Urich "Are Betas Best?", Journal of Finance, Vol. 33, No. 5 (December 1978), pp. 1375-1384.

32. Fabozzi, F. and J. Francis "Stability Tests for Alphas and Betas Over Bull and Bear Market Conditions" Journal of Finance, Vol. 32, No. 4 (September 1977), pp. 1093-1099.

33. Fama, E. Foundations of Finance, (New York:Basic Books,1976).

34. Fama, E. and J. MacBeth, "Risk, Return, and Equilibrium: Some Empirical Tests", Journal of Political Economy, May 1973, pp. 607-636.

35. Farrell, J. "Analyzing Covariation of Returns to Determine Homogeneous Stock Groupings", Journal of Business, Vol. 47, No. 4 (April 1974), pp.186-207.

36. Farrell, J. "Homogeneous Stock Groupings", Financial Analysts Journal, May-June 1975, pp. 50-62.

37. Fogler, H. "Common Sense on CAPM, APT, and Correlated Residuals", Journal of Portfolio Management, Summer 1982, pp. 20-28.

38. Fogler, H., John, K., and Tipton, L. "Three Factors, Interest Rate Differentials and Stock Groups", Journal of Finance, Vol. 36, No. 2 (May 1981), pp. 323-335.

39. Freind, I. "Discussion" Journal of Finance, Vol. 36, No. 2, (May 1981), pp. 350-352.

40. Gehr, A.K.,Jr., "Some Tests of The Arbitrage Pricing Theory", Journal of The Midwest Finance Association, 1975.

41. Gehr, A.K.,Jr., "Linearity, Non-linearity and the Arbitrage Pricing Theory" Unpublished Working Paper, University of Missouri, October 1984.

42. Gibbons, M. "Empirical Examination of the Return Generating Process of the Arbitrage Pricing Theory", Stanford University Working Paper, May 1981.

43. Gnanadesikan, R. Statistical Data Analysis of Multivariate Observations, (New York:John Wiley, 1977).

44. Hamada, R. "Portfolio Analysis, Market Equilibrium and Corporate Finance" Journal of Finance, Vol. 24, No. 1 (March 1969), pp. 13-31.

45. Harmon, H. Modern Factor Analysis, (Chicago:University of Chicago Press), 1976.

46. Heywood, H. "On Finite Sequences of Real Numbers", Proceedings of the Royal Society of London, Vol. 134, 1931, pp. 486-501.

47. Huberman, G. "A Simple Approach to Arbitrage Pricing Theory" Journal of Economic Theory, Vol. 28 (1982), pp. 182-191.

48. Hughes, P. "A Test of The Arbitrage Pricing Theory", Working Paper, University of British Columbia, (August, 1981;rev. April, 1982).

49. Ingersoll,Jr., J.E. "Some Results in the Theory of Arbitrage Pricing" Journal of Finance, Vol. 39, No. 4 (September 1984), pp. 1021-1039.

50. Jarrow, R. and A. Rudd "Real Assets, Technology and the Arbitrage Pricing Theorem" Working Papewr No. 80-13, Graduate School of Business and Public Administration, Cornell University, October, 1980.

51. Jennrich, R. "An Asymptotic χ^2 Test For the Equality of Two Correlation Matrices", Journal of The American Statistical Association, Vol. 65 (June 1970), pp. 904-912.

52. Jobson, J.D. "A Multivariate Linear Regression Test for the Arbitrage Pricing Theory" Journal of Finance, Vol. 37, No. 4 (September 1982), pp. 1037-1042.

53. Jöreskog, K. "A General Approach to Confirmatory Maximum Likelihood Factor Analysis", Psychometrika, Vol. 34, No. 2 (June 1969).

54. Jöreskog, K. "Simultaneous Factor Analysis in Several Populations", in J. Magidson, Ed., Advances in Factor Analysis and Structural Equation Models, (Cambridge, Mass.:ABT Books), 1979.

55. Jöreskog, K. "A General Method for Analysis of Covariance Structures", Biometrika, Vol.57, No. 2 (1970).

56. Kaiser, H. "The Application of Electronic Computers to Factor Analysis", Educational and Psychological Measurement, Vol. 20 (1960).

57. King, B. "Market and Industry Factors in Stock Price Behavior", Journal of Business, Vol. 36 (1966), pp. 139-190.

58. Kraus, A. and R. Litzenberger "Skewness Preference and the Valuation of Risky Assets" Journal of Finance, Vol. 31 (September 1976), pp. 1085-1100. .

59. Kruskal, J. "Factor Analysis and Principal Components", In W. Kruskal and J. Tanur, International Encyclopedia of Statistics, The Free Press, 1978, pp. 307-328.

60. Kryzanowski, L. and M. To "General Factor Models and The Structure of Security Returns", Journal of Financial and Quantitative Analysis, Vol. 18, No. 1, pp. 31-52.

61. Lawley, D. and Maxwell, A. "Factor Analysis as a Statistical Method, (New York:American Elsevier, 1971).

62. Lee, C.F. and J.K.C. Wei "The Interrelationship Among the APT, the Multi-factor CAPM and the CAPM: Theory and Empirical Evidence" Paper presented at the 1984 Annual Meeting of the Financial Management Association, October 1984.

63. Lintner, J. "The Valuation of Risky Assets and Selection of Risky Investments in Stock Portfolios and Capital Budgets", Review of Economics and Statistics, February 1975, pp. 13-37.

64. Livingston, M. "Industry Movements of Common Stocks", Journal of Finance, Vol. 32, No. 3 (June 1977), pp. 861-874.

65. Lloyd, W.P. and R.A. Shick "A Test of Stone's Two-index Model of Returns" Journal of Financial and Quantitative Analysis, September 1977, pp. 363-376.

66. Markowitz, H. "Portfolio Selection", Journal of Finance, Vol. 12, No. (March 1952), pp. 77-91.

67. Markowitz, H. and A. Perold "Portfolio Analysis with Factors and Scenarios", Journal of Finance, Vol. 36, No. 14 (September,1981), pp. 871-877.

68. McGowan, C.B., Jr. and K. Tandon "A Test of the Cross-sectional Robustness of the Arbitrage Pricing Model Using Foreign Exchange Rates" Paper presented at the 1984 Annual Meeting of the Financial Management Association, October, 1984.

69. Merton, R. "An Intertemporal Capital Asset Pricing Model" Econometrica, Vol. 40, (September,1973), pp. 867-887.

70. Meyers, S. "A Re-examination of Market and Industry Factors in Stock Price Behavior", Journal of Finance, Vol. 28 (June 1973), pp. 695-706.

71. Miller, M. and M. Scholes "Rate of Return in Relation to Risk: A Re-examination of Some Recent Findings", in: M. Jensen, ed., Studies in the Theory of Capital Markets (Praeger, New York), 1972.

72. Morgan, G. "Grouping Procedures for Portfolio Formation" Journal of Finance, Vol. 32, No. 5 (December 1977), pp. 1759-1765.

73. Morrison, D. Multivariate Statistical Methods, 2nd. Ed., (New York:McGraw-Hill, 1976).

74. Mossin, J. "Equilibrium in a Capital Asset Market", Econometrica, October 1966, pp. 768-783.

75. Oldfield, G. and Rogalski, R. "Treasury Bill Factors and Common Stock Returns", Journal of Finance, Vol. 36, No. 2 (May 1981), pp. 337-350.

76. Pari, R. and Chen, S. "An Empirical Test of the Arbitrage Pricing Theory", Unpublished Working Paper, 1982.

77. Rao, C. "Estimation and Tests of Significance in Factor Analysis", Psychometrika, June 1955.

78. Reilly, F. and E. Drzycimski "Alternative Industry Performance and Risk" Journal of Financial and Quantitative Analysis, June 1974, pp. 423-446.

79. Reinganum, M. "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings' Yields and Market Values", Journal of Financial Economics, Vol. 9 (1981a), pp. 19-46.

80. Reinganum, M. "The Arbitrage Pricing Theory: Some Empirical Results", Journal of Finance, Vol. 36, No. 2 (May 1981), pp. 313-321.

81. Reinganum, M. "A Direct Test of Roll's Conjecture on the Firm Size Effect", Journal of Finance, Vol. 37, No. 1 (March, 1982), pp. 27-35.

82. Roll, R. "A Critique of the Asset Pricing Theory's Tests, Part I", Journal of Financial Economics, Vol. 4, (May 1977), pp. 129-76.

83. Roll, R. and Ross, S. "An Empirical Examination of the Arbitrage Pricing Theory", Journal of Finance, Vol. 35, No. 5 (December 1980), pp. 1073-1103.

84. Roll, R. and S. Ross "Regulation, The Capital Asset Pricing Model and The Arbitrage Pricing Theory", Public Utilities Fortnightly, May 26, 1983, pp. 22-28.

85. Roll, R. and S. Ross "The Arbitrage Pricing Theory Approach to Strategic Portfolio Planning", Financial Analysts Journal, May-June, 1984, pp. 14-26.

86. Roll, R. and S. Ross "A Critical Reexamination of the Empirical Evidence on the Arbitrage Pricing Theory: A Reply", Journal of Finance, Vol. 39, No. 2, (June 1984), pp. 347-350.

87. Rosenburg, B. "Extra-Market Components of Covariance in Security Returns", Journal of Financial and Quantitative Analysis, March 1974, pp. 263-274.

88. Rosenberg, B. "The Capital Asset Pricing Model and the Market Model" Journal of Portfolio Management, Winter 1981, pp. 5-16.

89. Ross, S. "Return, Risk and Arbitrage", in: Freind and Bicksler, Eds., Risk and Return in Finance, (Ballinger:Cambridge,Ma.), 1975.

90. Ross, S. "The Arbitrage Theory of Capital Asset Pricing", Journal of Economic Theory, Vol. 19 (December 1976), pp. 341-360.

91. Shanken, J. "The Arbitrage Pricing Theory: Is It Testable?", Journal of Finance, Vol. 37, No. 5 (December, 1982), pp. 1129-1140.

92. Sharpe, W. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk", Journal of Finance, Vol. 19 (September 1964), pp. 425-442.

93. Sharpe, W. "Factors in New York Stock Exchange Security Returns, 1931-1979" Journal of Portfolio Management, Summer 1982, pp. 5-19.

94. Stone, B.K. "Systematic Interest Rate Risk in a Two-index Model of Returns" Journal of Financial and Quantitative Analysis, 1974 Proceedings (November 1974), pp. 709-725.

95. Thurstone, L. Multiple Factor Analysis, (Chicago:University of Chicago Press, 1947.

Appendix A

List of Companies - Sample A

Number	Cusip	Name
1	168810	AMERICAN MACH & FDRY CO
2	176510	AMERICAN AIRLS INC
3	282410	ABBOTT LABS
4	621210	ADAMS EXPRESS CO
5	915810	AIR PROD & CHEMS INC
6	1371610	ALUMINIUM LTD
7	1717610	ALLEGHANY CORP
8	1741110	ALLEGHENY PWR SYS INC
9	1951910	ALLIED STORES CORP
10	1964510	ALLIS CHALMERS MFG CO
11	2224910	ALUMINUM CO AMER
12	2406910	AMERICAN BAKERIES CO
13	2470310	AMERICAN TOB CO
14	2473510	AMERICAN BROADCASTING PA
15	2553710	AMERICAN ELEC PWR INC
16	2660910	AMERICAN HOME PRODS CORP
17	2668110	AMERICAN HOSP SUPPLY COR
18	2860910	AMERICAN NAT GAS CO
19	2960910	AMERICAN SHIP BLDG CO
20	3110510	AMETEK INC
21	3217710	AMSTED INDS INC
22	3948310	ARCHER DANIELS MIDLAND C
23	4055510	ARIZONA PUB SVC CO
24	4123710	ARKANSAS LA GAS CO
25	4217010	ARMCO STL CORP
26	4246510	ARMSTRONG RUBR CO
27	4341310	AMERICAN SMLT & REFNG CO
28	4454010	ASHLAND OIL & REFNG CO
29	4557310	ASSOCIATED DRY GOODS COR
30	4830310	ATLANTIC CITY ELEC CO
31	4926730	ATLAS CORP
32	5350110	AVCO CORP
33	5380710	AVNET ELECTRS CORP
34	5916510	BALTIMORE GAS & ELEC CO
35	7189210	BAXTER LABS INC
36	7741910	BELCO PETE CORP
37	8172110	BENEFICIAL FIN CO
38	8750910	BETHLEHEM STL CORP
39	9179710	BLACK & DECKER MFG CO
40	9702310	BOEING CO

Appendix A (cont.)

List of Companies - Sample A

Number	Cusip	Name
41	9959910	BORDEN CO
42	9972510	BORG WARNER CORP
43	10059910	BOSTON EDISON CO
44	11009710	BRISTOL MYERS CO
45	11425910	BROOKLYN UN GAS CO
46	11565710	BROWN SHOE INC
47	11874510	BUCYRUS ERIE CO
48	12278110	BURROUGHS CORP
49	12484510	COLUMBIA BROADCASTING SY
50	12500510	CONTINENTAL COPPER & STL
51	12614910	CORN PRODS CO
52	12650110	C T S CORP
53	13106910	CALLAHAN MNG CORP
54	13442910	CAMPBELL SOUP CO
55	13986110	CAPITAL CITIES BROADCAST
56	14233910	CARLISLE CORP
57	14414110	CAROLINA PWR & LT CO
58	14628510	CARTER PRODS INC
59	14912310	CATERPILLAR TRACTOR CO
60	15003310	CECO CORP
61	15084310	CELANESE CORP
62	15235710	CENTRAL & SOUTH WEST COR
63	15366310	CENTRAL ILL PUB SVC CO
64	15517710	CENTRAL SOYA INC
65	15717710	CESSNA AIRCRAFT CO
66	15852510	UNITED STS PLYWOOD CORP
67	16533910	CHESEBROUGH PONDS INC
68	16789810	CHICAGO PNEUMATIC TOOL C
69	17026810	CHOCK FULL O NUTS CORP
70	17110610	CHROMALLOY CORP
71	17119610	CHRYSLER CORP
72	17207010	CINCINNATI GAS & ELEC CO
73	17784610	CITY INVESTING CO
74	18139610	CLARK EQUIP CO
75	18600010	CLEVELAND CLIFFS IRON CO
76	18948610	CLUETT PEABODY & CO INC
77	19121610	COCA COLA CO
78	19416210	COLGATE PALMOLIVE CO
79	19482810	COLLINS & AIKMAN CORP
80	19764810	COLUMBIA GAS SYS INC

Appendix A (cont.)

List of Companies - Sample A

Number	Cusip	Name
81	20279510	COMMONWEALTH EDISON CO
82	20681310	CONE MLS CORP
83	20911110	CONSOLIDATED EDISON CO N
84	20921910	CONSOLIDATED FOODS CORP
85	20961510	CONSOLIDATED NAT GAS CO
86	21061510	CONSUMERS PWR CO
87	21161520	CONTINENTAL MATLS CORP
88	22439910	CRANE CO
89	22825510	CROWN CORK & SEAL INC
90	22866910	CROWN ZELLERBACH CORP
91	22966910	CUBIC CORP
92	22989010	GENERAL CIGAR INC
93	23252510	UNIVERSAL CYCLOPS STL CO
94	23957710	DAYCO CORP
95	24001910	DAYTON PWR & LT CO
96	24419910	DEERE & CO
97	24710910	DELAWARE PWR & LT CO
98	24736110	DELTA AIR LINES INC DEL
99	83186510	SMITH A O CORP
100	83541510	SOUTHERN NAT GAS CO

Appendix B

List of Companies - Sample B

Number	Cusip	Name
1	24788310	C K P DEVELOPMENTS INC
2	25084710	DETROIT EDISON CO
3	25243510	DI GIORGIO FRUIT CORP
4	25468710	DISNEY WALT PRODTNS INC
5	26000310	DOVER CORP
6	26054310	DOW CHEM CO
7	26159710	DRESSER INDS INC
8	26622810	DUQUESNE LT CO
9	26781310	DYNALECTRON CORP
10	27746110	EASTMAN KODAK CO
11	28336210	EL PASO NAT GAS CO
12	29101110	EMERSON ELEC MFG CO
13	29110110	EMERY AIR FGHT CORP
14	29121010	AMERICAN HARDWARE CORP
15	29356710	LONE STAR GAS CO
16	29449710	EQUITABLE GAS CO
17	29665910	ESQUIRE INC
18	29669510	ESSEX CHEM CORP
19	29920910	EVANS PRODS CO
20	30058710	EX CELL O CORP
21	30229010	STANDARD OIL CO N J
22	30371110	FAIRCHILD STRATOS CORP
23	31313510	FEDDERS CORP
24	31354910	FEDERAL MOGUL BOWER BEAR
25	31409910	FEDERATED DEPT STORES IN
26	31438710	FELMONT PETE CORP
27	31540510	FERRO CORP
28	31831510	FIRESTONE TIRE & RUBR CO
29	32054810	WESTERN BANCORPORATION
30	33769310	FISCHER & PORTER CO
31	34108110	FLORIDA PWR & LT CO
32	34110910	FLORIDA PWR CORP
33	34551410	FOREMOST DAIRIES INC
34	35024410	FOSTER WHEELER CORP
35	36144810	GENERAL AMERN TRANSN COR
36	36232010	GENERAL TEL & ELECTRS CO
37	36960410	GENERAL ELEC CO
38	36985610	GENERAL FOODS CORP
39	37033410	GENERAL MLS INC
40	37083810	GENERAL RY SIGNAL CO

Appendix B (cont.)

List of Companies - Sample B

Number	Cusip	Name
41	37153210	GENESCO INC
42	37329810	GEORGIA PAC CORP
43	37428010	GETTY OIL CO
44	37453210	GIANT PORTLAND CEM CO
45	37465810	GIBRALTAR FINL CORP CALI
46	38255010	GOODYEAR TIRE & RUBR CO
47	38274810	GORDON JEWELRY CORP
48	38747810	GRANITEVILLE CO
49	39006410	GREAT ATLANTIC & PAC TEA
50	39106410	GREAT NORTHN IRON ORE PP
51	39109010	GREAT NORTHN PAPER CO
52	39802810	GREYHOUND CORP
53	40018110	GRUMMAN AIRCRAFT ENGR CO
54	40206410	GULF & WESTN INDS INC
55	40255010	GULF STS UTILS CO
56	40278410	GULTON INDS INC
57	40621610	HALLIBURTON CO
58	41387510	HARRIS INTERTYPE CORP
59	41586410	HARSCO CORP
60	42075810	HAYES INDS INC
61	42159610	HAZELTINE CORP
62	42270410	HECLA MNG CO
63	42307410	HEINZ H J CO
64	42323610	HELENE CURTIS INDS INC
65	42345210	HELMERICH & PAYNE INC
66	42705610	HERCULES POWDER CO
67	42786610	HERSHEY CHOCOLATE CORP
68	42823610	HEWLETT PACKARD CO
69	43575810	HOLLY CORP
70	43850610	MINNEAPOLIS HONEYWELL RE
71	44181510	HOUSEHOLD FIN CORP
72	45138010	IDAHO PWR CO
73	45209210	ILLINOIS PWR CO
74	45325840	INTERNATIONAL NICKEL CO
75	45543410	INDIANAPOLIS PWR & LT CO
76	45686610	INGERSOLL RAND CO
77	45747010	INLAND STL CO
78	45765910	INTERNATIONAL SILVER CO
79	45850610	INTERNATIONAL SHOE CO
80	45957810	INTERNATIONAL HARVESTER

Appendix B (cont.)

List of Companies - Sample B

Number	Cusip	Name
81	45988410	INTERNATIONAL MINERALS &
82	46014610	INTERNATIONAL PAPER CO
83	46025410	INTERNATIONAL RECTIFIER
84	46057510	NORTHERN NAT GAS CO
85	46107410	INTERSTATE PWR CO
86	46253710	IOWA PWR & LT CO
87	47816010	JOHNSON & JOHNSON
88	48119610	JOY MFG CO
89	48258410	KREGSE S S CO
90	48517010	KANSAS CITY SOUTHN RY CO
91	49238610	KERR MCGEE CORP
92	49436810	KIMBERLY CLARK CORP
93	50060210	KOPPERS INC
94	50558810	LACLEDE GAS CO
95	52517410	LEHMAN CORP
96	53000010	LIBBEY OWENS FORD GLASS
97	53802110	LITTON INDS INC
98	54042410	LOEWS THEATRES INC
99	54229010	LONE STAR CEM CORP
100	83571610	SOO LINE RR CO

Appendix C

List of Companies - Sample C

Number	Cusip	Name
1	54267110	LONG ISLAND LTG CO
2	54385910	LORAL ELECTRS CORP
3	54626810	LOUISIANA LD & EXPL CO
4	54777910	LOWENSTEIN M & SONS INC
5	55261810	MICROWAVE ASSOC INC
6	55265310	M C A INC
7	55479010	CROWELL COLLIER PUBG CO
8	55613910	MACY R H & CO INC
9	56828710	MARINE MIDLAND CORP
10	57777810	MAY DEPT STORES CO
11	57859210	MAYTAG CO
12	58003310	MC DERMOTT J RAY & CO IN
13	58016910	MC DONNELL AIRCRAFT CORP
14	58256210	MC NEIL MACH & ENGR CORP
15	58283410	MEAD CORP
16	58574510	MELVILLE SHOE CORP
17	58933110	MERCK & CO INC
18	59067210	MESABI TR
19	59583210	MIDDLE SOUTH UTILS INC
20	60405910	MINNESOTA MNG & MFG CO
21	60624910	MISSOURI PUB SVC CO
22	60705910	SOCONY MOBIL OIL INC
23	60803010	MOHASCO INDS INC
24	60976210	MONOGRAM PRECISION INDS
25	61166210	MONSANTO CHEM CO
26	61201710	MONTANA DAKOTA UTILS CO
27	62007610	MOTOROLA INC
28	62664310	MURPHY G C & CO
29	62671710	MURPHY CORP
30	62715110	MURRAY OHIO MFG CO
31	62886210	NATIONAL CASH REGISTER C
32	63512810	NATIONAL CAN CORP
33	63565510	NATIONAL DISTILLERS & CH
34	63618010	NATIONAL FUEL GAS CO N J
35	63631610	NATIONAL GYPSUM CO
36	63784410	NATIONAL STL CORP
37	64400110	NEW ENGLAND ELEC SYS
38	64984010	NEW YORK ST ELEC & GAS C
39	65163910	NEWMONT MNG CORP
40	65352210	NIAGARA MOHAWK PWR CORP

Appendix C (cont.)

List of Companies - Sample C

Number	Cusip	Name
41	66577210	NORTHERN STS PWR CO MINN
42	66728110	NORTHWEST AIRLS INC
43	67034610	NUCLEAR CORP AMER
44	67459910	OCCIDENTAL PETE CORP
45	67634610	OGDEN CORP
46	67734710	OHIO EDISON CO
47	68066520	OLIN MATHIESON CHEM CORP
48	68406510	ORANGE & ROCKLAND UTILS
49	69002010	OUTBOARD MARINE CORP
50	69073410	OWENS CORNING FIBERGLAS
51	69076810	OWENS ILL GLASS CO
52	69430810	PACIFIC GAS & ELEC CO
53	69447810	PACIFIC LTG CORP
54	69846210	PANHANDLE EASTN PIPE LIN
55	70816010	PENNEY J C INC
56	70905110	PENNSYLVANIA PWR & LT CO
57	71103010	PEOPLES GAS LT & COKE CO
58	71344810	PEPSI COLA CO
59	71404110	PERKIN ELMER CORP
60	71654910	PETROLEUM CORP AMER
61	71708110	PFIZER CHAS & CO INC
62	71726510	PHELPS DODGE CORP
63	71753710	PHILADELPHIA ELEC CO
64	71816710	PHILIP MORRIS INC
65	71850710	PHILLIPS PETE CO
66	71859210	PHILLIPS VAN HEUSEN CORP
67	72151010	PILLSBURY CO
68	72447910	PITNEY BOWES INC
69	72570110	PITTSTON CO
70	73109510	POLAROID CORP
71	73620210	POOR & CO
72	73767910	POTOMAC ELEC PWR CO
73	74271810	PROCTER & GAMBLE CO
74	74446510	PUBLIC SVC CO IND INC
75	74456710	PUBLIC SVC ELEC & GAS CO
76	74533210	PUGET SOUND PWR & LT CO
77	74740210	QUAKER OATS CO
78	74928510	RADIO CORP AMER
79	75127710	RALSTON PURINA CO
80	75472110	RAYMOND INTL INC NJ

Appendix C (cont.)

List of Companies - Sample C

Number	Cusip	Name
81	75511110	RAYTHEON CO
82	75920010	REICHHOLD CHEMS INC
83	76077910	REPUBLIC STL CORP
84	76152510	REVLON INC
85	77051910	ROBERTSHAW FULTON CTLS C
86	77175810	ROCHESTER TEL CORP
87	77434710	NORTH AMERN AVIATION INC
88	77537110	ROHM & HAAS CO
89	78354910	RYDER SYS INC
90	78462610	STANDARD PRESSED STL CO
91	78651410	SAFEWAY STORES INC
92	79345310	ST REGIS PAPER CO
93	79744010	SAN DIEGO GAS & ELEC CO
94	80660510	SCHERING CORP
95	80685710	SCHLUMBERGER LTD
96	81064010	SCOVILL MFG CO
97	81238710	SEARS ROEBUCK & CO
98	82263510	SHELL OIL CO
99	82930210	SINGER MFG CO
100	83237710	SMITH KLINE & FRENCH LAB

Appendix D

Distribution of SIC Codes

Two-digit SIC Codes	Sample A	Sample B	Sample C
10 - 19	5	7	5
20 - 29	25	24	27
30 - 39	37	36	31
40 - 49	25	20	25
50 - 59	3	5	7
60 - 69	5	6	4
70 - 79	0	2	1

Table entries are the number of firms in the samples with the associated SIC Code.

APPENDIX E

Distribution of Mean Returns

Subperiod	Group	Minimum	Quintile Means 1	2	3	4	5	Maximum
	A	-.0003102	.0001518	.0004013	.0006352	.0009820	.0016403	.0028648
1	B	-.0004403	.0000557	.0003654	.0006214	.0009889	.0016281	.0025247
	C	-.0000573	.0001840	.0003836	.0005365	.0007595	.0014931	.0030366
	A	-.0008793	-.0001831	.0000559	.0003328	.0006229	.0010019	.0020826
2	B	-.0006458	-.0000954	.0002038	.0003367	.0005727	.0014384	.0026857
	C	-.0005955	-.0001706	.0000923	.0003255	.0005412	.0010689	.0017476
	A	-.0011073	-.0005125	-.0001137	.0000994	.0003221	.0007655	.0011888
3	B	-.0016125	-.0007355	-.0001899	.0000550	.0003151	.0009654	.0018119
	C	-.0012350	-.0004642	-.0000764	.0001338	.0003887	.0008372	.0015726
	A	-.0002528	.0002186	.0006122	.0007725	.0010647	.0017372	.0027235
4	B	-.0008477	.0000717	.0005525	.0007971	.0011384	.0018724	.0025104
	C	-.0003851	.0001719	.0005704	.0007591	.0010172	.0016008	.0029877
	A	-.0007356	.0001102	.0005151	.0007470	.0010272	.0014569	.0021076
5	B	-.0013347	.0001816	.0005496	.0007140	.0008792	.0014676	.0026485
	C	-.0002465	.0002469	.0006129	.0007369	.0008534	.0012082	.0018663

APPENDIX E (cont.)

Distribution of Return Standard Deviations

			Quintile Means					
Subperiod	Group	Minimum	1	2	3	4	5	Maximum
	A	.006625	.009727	.011776	.013715	.017724	.028118	.075520
1	B	.007458	.009985	.011974	.014144	.017695	.029545	.071910
	C	.008318	.009368	.011096	.012834	.016302	.027003	.055422
	A	.009527	.012867	.015600	.018021	.021920	.031687	.057706
2	B	.009733	.012257	.015653	.018369	.022196	.033716	.057154
	C	.011153	.012742	.014901	.016987	.020492	.028803	.042308
	A	.010866	.014156	.016996	.019478	.022345	.032141	.060161
3	B	.009585	.013640	.017082	.019154	.022869	.035415	.064046
	C	.011250	.013768	.015912	.018503	.022179	.030090	.045822
	A	.010268	.012226	.014865	.017322	.020218	.030965	.073534
4	B	.008389	.012179	.015057	.017372	.020620	.033914	.048044
	C	.009465	.011379	.014271	.017097	.020772	.026028	.033941
	A	.010528	.012572	.015543	.018592	.021864	.029113	.040809
5	B	.010564	.012786	.016429	.019006	.022736	.031803	.044043
	C	.010851	.012742	.016018	.017913	.021524	.025485	.031684