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The location model : discrimination and classification when the data contains both binary and continuous variables.

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THE LOCATION MODEL: DISCRIMINATION AND CLASSIFICATION WHEN THE DATA CONTAINS BOTH BINARY AND CONTINUOUS VARIABLES

A Dissertation Presented

By

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ABSTRACT

The Location Model: Discrimination
And Classification When the Data Contains
Both Binary and Continuous Variables

(February 1979)

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This study was designed to advance a methodology for discriminant analysis and classification when the data set consists of both binary and continuous variaables. By the application of the location model to a real data set, it was shown that non-ratio and ratio data can be combined to classify firms as either liquid or illiquid.

The liquid condition of a firm is of importance to present and future creditors and stockholders.
Firms which are able to maintain a condition of liquidity (financial health) are less likely to create financial difficulties for their providers of debt and equity capital. An ideal framework for evaluating and finan-

cial health of a firm would include performance, leverage, liquidity and cash flow measures in the form of ratios and non-ratio qualitative factors vital to the differentiation between the liquid firm and the illiquid firm.

This study examines a group of firms either downgraded by Standard and Poor's from BBB to BB (BB to B if subordinated) or downgraded by Dun & Bradstreet from 2 to 3. Nineteen firms were downgraded by Dun & Bradstreet and eighteen firms were downgraded by Standard and Poor's. Each of these thirty-seven experimental firms were matched by industry to a control firm whose bond rating or trade credit rating remained constant at BBB (BB if subordinated) or 2.

Twenty financial ratios for each year of a three year period were available for the trade credit rating section, while nineteen financial ratios for each year of a three year period were available for the bond rating section. Two binary variables were obtained for each firm in the two groups of both sections.

The basic hypothesis tested asked the question: Can financial ratio continuous variables be combined with non-ratio binary variables to classify a firm as either liquid or illiquid? Many researchers faced with data composed partly of binary variables have ignored their discrete nature and proceeded with continuous variable techniques. The aim of this study is to derive discriminant functions from the location model for a mixed binary and continuous variables.

For the problem of classifying an observation $w=(x,y)$ to one of two populations, π_1 and π_2, where x is a vector of q binary variables and y is a vector of p continuous variables, the location model assumes that y has a multivariate normal distribution with mean $\mu_i^{(m)}$ in cell m and population π_i $(m=1,2...,k;\ i=1,2)$, with a common dispersion matrix Σ in all cells of both populations. Each distinct response pattern of x uniquely defines a multinomial cell in a table with k cells $(k=2^q)$. Separate discriminant functions are derived for each of the k cells, and the appropriate function is used to classify an observation based on the observed binary variable response pattern.

In general, it was found that the location model provides an alternative to the continuous variable treatment of binary variables. The models developed compared favorably to both an LDF model consisting of only continuous variables and an LDF model consisting of both binary and continuous variables. The utility of the classification rule may be evaluated by the probabilities of misclassifications that it gives rise to. Accordingly, the "Lachenbruch Method" was used to obtain the classification rule for each of the three models tested, and the error rate from each population was estimated by the proportion of observations misclassified in each sample.

Further investigation into the application of the location model is needed. Improved specification of non-ratio binary variables is likely to produce better models. The problems of coefficient stability and variable relationship stationarity need to be addressed, as do the problems of incorporating population prior probabilities and costs of misclassification.

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CHAPTER I

INTRODUCTION

1.1 Statement of the Problem

1.1.1 Statistical and methodological violations. Many of the financial ratio studies that have appeared in the business literature to date have suffered from methodological and statistical problems that have limited the usefulness of their results. The typical study looks for an answer to the question: which of these two groups does this firm resemble most, based on a selected set of financial ratio variables? When it is assumed 1) that the distributions of these ratios are multivariate normal, and 2) that the two groups do not differ in the dispersion matrices, the typical methodology leads to the use of a linear discriminant function which was first derived by Fisher [1936].

When it is assumed 1) that the ratio variables are multivariate normal, and 2) that the dispersion matrices are not equal, the typical methodology then leads to the use of a quadratic discriminant function. In both cases these functions are optimal in the sense of minimizing the probability of misclassification. To make proper use of the significance tests and classification rules, it is only logical that a researcher attempt to determine which of the above assumptions are consistent with his data. It is not uncommon in the applied literature to find studies that appear to ignore the problem of testing for distributional normality and dispersion equality. In 1975, Joy and Tollefson [1975] presented a methodology that was claimed to be better than procedures then used in linear discriminant function (LDF) studies. Shortly after the Joy and Tollefson paper, Eisenbeis [1977] also discussed the problems of application of LDF techniques.

1.1.2 Types of violations. Both of these studies identified problems of different types, among which were difficulties with 1) variable distribution, 2) group dispersion matrices, 3) interpretation of variable significance, 4) group definition, 5) prior probabilities and costs of misclassification, 6) reduction of dimensions, 7) classification error rates, and 8) prediction validity. In 1978, Eisenbeis [1978] added to the list the problem of dealing with a data set that consisted of both binary and continuous variables. Many of the models presented in the literature to date suffer from one or more of these methodological or statistical problems which may affect the interpretation of their results.

<u>1.1.3 Effects of violations</u>. In general, violations of any of these assumptions may 1) bias the significance test for the differences in group means, 2) affect the appropriate form of the classification rules, 3) bias the test of estimated error rates, 4) affect the usefulness of "reduced-space transformations," or 5) bias tests to determine the relative importance of individual variables. The impact of any or all of these violations can be difficult to determine, and in some cases it is possible that any reasonable procedure would have the same relative classification results for that particular set of data. In 1977, Pinches and Trieschmann [1977] were unable to assess the impact of 1) non-multivariate normality, 2) unequal dispersion matrices, and 3) biased estimates of classification error rates on one of their previous studies [1973].

While it is true that these methodological and statistical problems may affect the usefulness of financial ratio studies, there are really two main difficulties encountered in developing models with better classification power. The first is the problem of model specification when the data set consists of mixed binary and continuous variables, and the second is the need to find appropriate qualitative information for use in the model. If the model specification problem can be solved, the use of qualitative information, such as subordination status, could lead to better classification results.

The general purposes of this study were to 1) present a methodology for the use of a model in financial ratio studies when the data set consists of both binary and continuous variables, and 2) consolidate and review the literature dealing with methodological problems and statistical assumption violations in financial ratio studies.

1.2 Binary and Continuous Variables

1.2.1 Mixed variables. The treatment of mixed variables has received very little attention in financial ratio literature. As a result, many researchers either omit binary variables altogether or ignore the discrete nature of these variables and proceed with continuous variables techniques. One method proposed for the treatment of mixed variables is the location model, which has been shown to given improved results in certain cases when compared to Fisher's linear discriminant function (see Krzanowski [1974] or [1975]). This study used the location model as an alternative to the use of the linear discriminant function with binary variables treated as though they were continuous.

1.2.2 Adding variables. Two binary variables were combined with the more traditional financial ratio variables using the location model. The contribution of this study was to use the information contained in both binary and continuous variables without violating many of the assumptions that were violated in previous studies.

<u>1.2.3 An alternative approach</u>. It is possible to construct a model with binary and continuous variables and proceed as though all variables were continuous, pointing out the effects of the different violations on significance tests and classification error rates. This study presented an alternative in the form of the location model which does not violate many of the methodological assumptions while capturing the value of the information in binary variables. Finally, this study demonstrated how the use of the location model on a set of data may be evaluated in a way as to give some idea of its structure and explain the reason for any difference that may be obtained over the linear discriminant function.

1.3 An Overview of the Study

In general, this study consists of the following:

1. A review of the financial ratio literature including an integration of the literature dealing with methodological problems and violations of statistical assumptions.

2. A presentation of the location model for discrimination and classification when the data consists of mixed variables.

3. An application of the location model using ratio data from a previous study and two binary variables.

The remainder of this section will 1) describe the previous study selected, and 2) present a brief introduction to the location model.

1.3.1 The study selected. The study selected (Backer and Gosman [1978]) was one that used financial ratios in a discrimination and classification model as only one part of a major effort dealing with the problem of financial reporting for illiquid firms. An illiquid firm was defined in this study as one which was unable to obtain additional funds from its traditional sources without incurring abnormal interest rates. The 37 firms selected as the experimental group experienced either 1) a reduction of their trade credit rating (19 firms), or 2) a reduction of the rating of their bonds (18 firms). Each experimental firm was matched to a control firm whose trade credit or bond rating remained constant. Twenty financial ratios were compiled for 3 different years for the 37 experimental firms and the matched sample of 37 control firms. The ratio of Inventory to Working Capital was included in the trade credit analysis but not in the bond analysis, leaving only 19 ratios collected for the bond group. The ratios were selected on the basis of an extensive literature search as well as specific inquiries of the two rating agencies.

The study reported 1) trends in mean and median ratios, 2) t-test analysis of the significance of ratio changes over time, and 3) classification results obtained using a linear discriminant function model.

<u>1.3.2 The location model</u>. The location model assumes that each distinct pattern of the binary variables defines a multinomial cell. If there are q binary variables, the number of cells is equal to $k = 2^q$. It further assumes that given the binary variables, the continuous variables have a multivariate normal distribution, and the continuous variables dispersion matrices, given the binary variables, are the same in both groups. It is <u>not</u> assumed that the unconditional dispersion matrices of the continuous variables are equal. The classification rule derived from the location model leads to a different linear discriminant function for each of the multinomial cells, with the cut-off points determined by the binary variables in the model.

Because the only parameter information usually available comes from samples, the location model approach is to replace parameters with estimates obtained from the samples. In practice, unless the samples are large in relation to the number of cells, some of the cells will be either too small for reasonable estimates, or zero. Maximum likelihood estimation of the binary variable parameters as well as the continuous variable parameters leads to a restricted model for the construction of an appropriate classification rule.

Using the Lachenbruch method [1967], each observation of the initial samples is classified using the remaining observations to obtain the appropriate cell classification rule. One of the advantages of the location model is that is allows each continuous variable to have a different mean value depending on the related binary variables. The linear discriminant model requires a single unconditional mean value. Therefore, increasing variability among the mean values should increase the effectiveness of the location model in classification.

The value of Mahalanobis D^2 between all possible pairs of cells in both groups are computed from the restricted model that was used to estimate parameters for the classification rule. Using the matrix of inter-cell D^2 values as a starting point, the relationships among the cells can be examined with principal components analysis. The successive axes of this analysis are interpreted as the directions of the between-cell discrimination in decreasing order of importance. This provides the researcher with a structure to aid in the explanation of any difference in the results obtained by the location model compared to the linear or quadratic discriminant function.

1.4 Outline of the Study

This chapter has presented a statement of the problem that currently exists in the methodology of studies dealing with financial ratios. It also discussed in general terms the purpose of this study, as well as an overview of the study, including a brief introduction to the location model and a description of the data selected for use with that model.

Chapter 2 begins with a brief, general discussion of discrimination and classification. It is followed by a limited review of the financial ratio literature as a means of demonstrating that the need for this study is based on a large body of applied research in diverse subject areas. The literature of the methodological and statistical problems in financial ratio studies is reviewed next, combined with references to the current ratio literature illustrating the most common violations of assumptions. The chapter ends with a review of 1) the literature dealing with the appropriate statistical and methodological procedures for 2-group discrimination and classification, and 2) the location model.

Chapter 3 presents the entire location model in detail along with a concurrent discussion of discrimination and classification.

Chapter 4 is concerned with methodology including definitions, design of model, data collection methods, and data analysis and review.

Chapter 5 is concerned with the results, beginning with a general statistical description of the data. The results of testing for significance as well as classification results are presented.

Chapter 6 concludes this study with a summary of the research. It includes a discussion of its limitations and offers some suggestions for future research.

CHAPTER II

LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to set forth the literature dealing with the statistical and methodological problems in financial ratio research. It begins in Section 2.2 with a brief, general discussion of discriminant analysis and classification, including the assumptions underlying this procedure. This is followed by Section 2.3 which is a review of financial ratio literature in order to demonstrate 1) the types of research problems that are amenable to discriminant analysis, and 2) that the need for methodological and statistical improvement is predicated on a large body of research beginning in 1966 and continuing to the present.

Section 2.4 is a review of the literature that has pointed out different types of statistical and methodological problems, including references to the financial ratio literature as examples of assumption violations. Section 2.5 presents the literature dealing with the appropriate methodological procedures for studies using discriminant analysis techniques on financial ratio variables. In addition, this section includes a discussion of the potential impact of assumption violations. The final section reviews the location model literature for discriminant analysis when the data set consists of mixed binary and continuous variables.

2.2 Discriminant Analysis

2.2.1 Discrimination. Discriminant analysis is used when the basic research problem is to assign an observation of unknown origin to one of two or more groups based on the value of the observation. The first step in discriminant analysis is to establish the groups, as the procedure rests on the assumption that the initial data sample is correctly classified. When the groups have been defined, the next step is to determine the variables that are to be used to describe each observation and to collect the values of these variables, usually from samples taken from each of the well-defined groups. Because the majority of financial ratio studies are concerned with only two groups, the general or k group case will not be considered.

The population, then, consists of two groups, π_1 and π_2. Values of the discriminating variables are obtained and the problem is to assign the observation whose variable values are x to π_1 or π_2. As a criterion for the assignment rule, Fisher [1936] has suggested using a linear combination of the variable values, finding coefficients such that the ratio of mean differences to its variance is maximized.

Maximizing the difference between groups relative to the standard deviation within groups leads to the discriminate function

$$(1) \qquad S^{-1}(\bar{x}_1-\bar{x}_2)$$

where S^{-1} is the inverse of the sample covariance matrix and $\bar{x}_1$ and $\bar{x}_2$ are the mean values from the group samples.

2.2.2 Classification. The next step is to use this function to classify an unknown observation x, to one or another of the groups. The classification rule then becomes: classify to π_1 if

$$(2) \qquad x'S^{-1}(\bar{x}_1-\bar{x}_2)$$

is greater than some constant, and to π_2 otherwise. This rule is distribution free, but it is optimal only if the observations are multivariate normal. The likelihood ratio criteria, if the density functions are multivariate normal distributions with the same covariance matrix, is the same as classifying to π_1 if

$$(3) \qquad D_s = x'S^{-1}(\bar{x}_1-\bar{x}_2)-\tfrac{1}{2}(\bar{x}_1+\bar{x}_2)'S^{-1}(\bar{x}_1-\bar{x}_2)$$

is greater than some constant, 0 if the assumption is made of equal costs of misclassification and equal prior probabilities of belonging to either group. The first part of this function, $x'S^{-1}(\bar{x}_1 - \bar{x}_2)$, is identical to the function derived by Fisher and shown in (2) above. The distribution of D_s (x) is very complicated, so that it is difficult to obtain good estimates of the probabilities of misclassification.

Estimating the mean and standard deviation of D (x) leads to $D^2 = (\bar{x}_1-\bar{x}_2)'S^{-1}(\bar{x}_1-\bar{x}_2)$ where D^2 is the Mahalanobis sample distance. This method is dependent upon the <u>assumption of normality</u>. One procedure used to estimate classification error rates is the so called hold-out or validation sample method where the data is divided into two parts, one of which is used to derive a discriminant function. This function is then used to classify the remaining observations, and the probability of misclassification is estimated by the actual misclassifications observed.

The Lachenbruch method [1967] overcomes many of the difficulties and disadvantages of other methods. In this method, each observation is classified using the remaining observations to obtain the classification rule. Again, the error rate is estimated by the observations misclassified. Lachenbruch [1967] and Lachenbruch and Mickey [1968] compared various methods in use and found many of them to be biased and misleading. They showed that their method yields estimates of misclassification error rates which have a small bias.

In those financial ratio studies that tested for it, the equal covariance assumption was not satisfied. When these matrices are unequal and normality still holds, the classification rule is to assign to π_1 if $Q_s(x) = -\frac{1}{2}[x'(S_1^{-1}-S_2^{-2})x-2x\ (S_1^{-1}\bar{x}_1-S_2^{-1}\bar{x}_2)]$ has the highest value. In other words, the linear discriminant function is used when <u>the normality and dispersion assumptions</u> are met and the quadratic function is used when <u>only the normality assumption</u> has been met.

2.3 Financial Ratio Studies

When William H. Beaver published his study in 1966, he noted that it was offered not as one of the last endeavors in the area, but as one of the first [1966]. His findings made the business research community aware of the potential of ratios. At that time, as Altman pointed out, researchers seemed to be moving toward the elimination of ratio analysis as an analytical technique in assessing the performance of the business firm [1968]. Beaver's study and the remarks made by Altman apparently motivated numerous subsequent research studies dealing with financial ratios. Of course, these studies are not concerned with financial ratios per se, but rather with the accounting data that comprise the ratios. The work done by a large number of researchers over the past decade or so supports the finding that accounting data in the form of financial ratios has predictive ability, and therefore the accounting data is useful. While this section presents some illustrations of the literature which has helped accounting researchers reach this conclusion, the primary purpose of this section will be to provide a background for the present study, not to examine the accounting data implications of these studies. For the sake of convenience, these financial ratio studies are presented using the following taxonomy:

1. Predictions of bond ratings.
2. Prediction of financial impairment.
3. Human information processing.
4. The empirical basis of ratios.
5. Financial ratios and risk.
6. Profile analysis.

2.3.1 Predictions of bond ratings. In 1969, Pogue and Soldofsky [1969] predicted bond ratings using a dichotomous dependent variable regressed on five ratio variables. Three populations were studied, one each in the broad classifications of manufacturing, utilities, and rails during the period from 1961 to 1966. The results of their multiple regression and classification indicated that a leverage ratio and earnings instability were inversely related to a high bond rating while size and profitability were directly related. Leverage and profitability appeared to have the most influence on bond ratings. The authors felt that their model could be used to explain the difference in high and low bond ratings to a significant degree using just five financial ratio variables.

A year later West [1970] published a study which had as its main objective the prediction of the first six corporate bond ratings issued by Moody's. Like Pogue and Soldofsky [1969], West also used a multiple regression with six bond ratings issued by Moody's as the dependent variable regressed on four financial ratios that had been used in other studies and were known to be useful in predicting bond ratings. Two populations were studied, one in 1953 and another in 1961. Separate regressions were used to develop coefficients that were used to predict bond ratings for the two populations. West was able to correctly predict 62% of the 1953 sample and 60% of the 1961 sample.

In 1973, Pinches and Mingo [1973] examined the predictive ability of accounting data in regards to corporate bond ratings. The population studied was that of all industrial bonds rated B or above listed in Moody's Bond Survey for the period from January 1, 1967 to December 31, 1968. A random sample was selected from this population. It consisted of 180 firms with random assignment of firms to a 132 firm analysis sample and a 48 firm hold-out sample.

Twenty-eight financial ratio variables were then factor analyzed into seven factors. One actual financial ratio was then selected from each of the seven factors and used as input into a discriminant analysis model. Using the results of the discriminant analysis, 70% of the firms in the analysis sample were correctly classified, and 65% of the firms in the hold-out sample were correctly classified. Pinches and Mingo were not happy with their results and concluded that their model either omitted quantifiable data or the bond rating process consisted of significant unidentified qualitative factors.

The main objective of a 1975 follow-up study by Pinches and Mingo [1975] was to correct violations of assumptions made in their 1973 study [1973]. According to the authors, these two violations were:

1. The use of a non-multivariate normal independent variable (a 0-1 dummy surrogate for subordination status).

2. The use of a linear combination of variables when a quadratic was appropriate (due to inequality of group dispersion matrices).

The same population was used for this study as the one used in their 1973 study, and the sample drawn from that population was also used.

Two new models were introduced in this 1975 study. The first new model had the same variables as the 1973 study, excluding the dichotomous subordination dummy variable. The second new model involved the calculation of two separate discriminant functions, one for all firms in the sample with subordinated bonds and another for all the firms in the sample with non-subordinated bonds. As a result of tests made for the equality of dispersion matrices, Pinches and Mingo used quadratic discriminant analysis for the classifications made by the two new models described above. The first new model, the one that excluded the subordination variables, was able to achieve only a 65% classification accuracy rate. The second new model was able to achieve a classification accuracy rate of 75%. Omitting the subordination variable produced a greater percentage of misclassification, while two separate discriminant functions were able to achieve a greater degree of classification accuracy.

The main objective of a 1975 study by Ang and Patel [1975] was to compare and validate four of the models that had appeared in the literature and that were designed to predict bond ratings. These four models were developed by Horrigan [1966], West [1970], Pogue and Soldofsky [1969], and Pinches and Mingo [1973]. Ang and Patel had as their goal the determination of the coefficient stability in the models that had been published to date, so the population that they selected to study consisted of all the bonds issued during the five years 1928, 1932, 1934, 1936, and 1938. After excluding all firms with inadequate data, their sample consisted of 424 firms which were randomly assigned to an analysis and to a hold-out sample. Each model of the four studies was developed from the analysis sample and was then used to classify the hold-out sample, maintaining everything as it was in the original studies. The appropriate model, either regression or discriminant analysis, was used to classify the hold-out sample in each of the five years under study. The mean overall predictive correctness for the years selected using the original models as developed from the new analysis sample were as follows:

	Correct Classification
Horrigan	30%
Pogue and Sodolsky	46%
West	35%
Pinches and Mingo	39%

Ang and Patel concluded that all the models tested showed general instability of their coefficients over different time periods. They advised that caution must be used in the practical application of these models, and that there is a decided need to revise and update the model coefficients.

2.3.2 Prediction of financial impairment. The 1966 Beaver study [1966] was designed on a univariate basis to determine the usefulness of financial accounting data in the form of ratios with regard to the particular purpose of predicting corporate bankruptcy. Beaver selected 79 failed firms, and 79 non-failed firms were selected in a matched-pair design with control for industry and asset size. Thirty financial ratios from the various conventional ratio categories were computed for each firm in the sample. The results of this study indicated that the most successful predictor was the cash flow to total debt ratio. The net income to total assets ratio was second best, and the current ratio was among the worst predictors of bankruptcy. Generally, the mixed ratios (i.e., those with income or cash flow in the numerator and assets or liabilities in the denominator) outperformed the short-term solvency ratios which were traditionally believed to be the best predictors of failure.

As did Beaver, Altman [1968] wanted to assess the quality of ratio analysis as an appropriate technique to classify firms as either bankrupt or non-bankrupt. The bankrupt firms' population was that of manufacturers that filed a bankruptcy petition during the period of 1946-1965. Thirty-three bankrupt firms were selected and 33 non-bankrupt firms were selected in a matched-pair design with control for industry and asset size, similar to the design used by Beaver. Using linear discrimination, Altman concluded that 5 ratio variables did the best overall job in the prediction of bankruptcy. Altman's model was able to correctly classify the analysis sample 95% of the time, one year prior to bankruptcy. His function was able to classify a hold-out sample of bankrupt firms 96% correct and a hold-out sample of non-bankrupt firms 79% correct. Altman suggested that his prediction model was an accurate forecaster of failure up to two years prior to bankruptcy. All observed ratios showed a deteriorating trend as bankruptcy neared and the most serious changes took place between the second and third years prior to bankruptcy.

Meyer and Pifer [1970] wanted to develop the ability to predict the failure of banks, so the population studied was all national and state banks in existence during the period from 1948-1965. All 39 banks that closed during that period were selected along with a matched solvent bank with control for 1) geographic location, 2) size, 3) age, 4) identical regulatory requirements, and 5) same period availability of information. Thirty pairs were assigned to the analysis sample and 9 pairs to the hold-out sample. A stepwise multiple regression was used with 32 financial ratios as independent variables and a dichotomous solvent/closed dependent variable. The final stepwise multiple regression with 9 variables included was able to correctly classify 88% of the hold-out sample, and the regression with 5 variables entered was able to classify 75% of the hold-out sample correctly. Meyer and Pifer concluded that their model was an adequate predictor of bank failure. As such, they felt, it should be used because _any_ method of eliminating losses is preferred to bankruptcy from both a social and private viewpoint.

Elam [1975] wondered whether the capitalization of certain leases would enable the user of financial ratio analysis to more accurately predict bankruptcy. At that time many major leases were excluded from both asset and liability classifications and the rental payments were expensed in the accounting period during which they were paid. Elam selected a sample of 48 firms that had adequate disclosure of non-capitalized lease data, and an additional 48 firms were selected, matched by industry, fiscal year end, disclosure, and net sales. Capitalized lease data was added to the financial statements of one of the two paired samples. Ten different discriminant functions were calculated, two for each of five years, one including lease data and one excluding lease data. Elam then compared the classification results obtained with the discriminant functions including lease data to the discriminant functions whose ratios excluded lease data. This test showed an increase in predictive ability in years 2 and 3, but no change whatsoever in predictability for years 1, 4 and 5. He concluded that his study did not support the hypothesis that the addition of lease data would increase the power of financial ratios to predict bankruptcy.

Sinkey [1975] hypothesized that: problem banks, as defined by Federal Banking Regulations, have financial ratios that are significantly different than those of non-problem banks, and that these ratios are useful in predicting problem banks. Sinkey selected all 110 newly identified problem banks during the year 1972, and matched these to an additional 110 banks for geographic area, total deposits, number of offices, and Federal Reserve membership status.

Ten selected financial ratios applicable to banks were obtained for each bank in the sample for the period of 1969-1972. Multiple quadratic discriminant analysis was used and the classification was determined by the Lachenbruch method.

The Sinkey model was able to correctly classify 82% of the analysis sample for the year 1972 and 75% using the Lachenbruch method for the same year.

In a forthcoming study, Backer and Gosman [1978] employed linear discriminant analysis to determine which financial ratios denoted illiquidity, according to their definition of that term. They were seeking to determine whether they could depict the illiquid firm based on the financial ratio criteria used by rating agencies. They found that multiple discriminant analysis revealed that selected ratios themselves predicted a high percentage of actual rating decisions.

2.3.3 Human information processing. Abdel-Khalick [1973] explored the effects of altering the information structure of accounting reports on the quality of the lending decision made by bankers. Abdel-Khalick used two pairs of matched firms, with each pair having one firm that had defaulted on a loan contract. The non-default matched firm was controlled for industry and for size. He then obtained a convenience sample of bank loan officers who were willing to participate in his experiment. These loan officers, using aggregated accounting data in the form of financial ratios, were asked to predict which of the two firms of each pair defaulted on their loan. The prediction results by the bankers were so poor that Abdel-Khalick concluded that his study cast doubts on the usefulness of ratios as predictors of failure by loan officers. He pointed out that the next move after analysis of the ratio and the event was to the analysis of the human decision and the event.

Libby [1975] followed up on the Abdel-Khalick study when he studied the same decision process by bank loan officers. He used a 60 firm sample consisting of 30 failed and 30 non-failed firms. Starting with 14 ratios, factor analysis was applied to the data and identified 5 factors. One ratio was chosen from each factor. These ratios were then given to the bank loan officers and they were asked to determine which of the firms had failed and which had not. The bankers were able to average 74% prediction correctness. The discriminant analysis classifiction was able to correctly classify 90% of the analysis sample and a reduced set was able to correctly predict 85% of the same analysis sample. Libby pointed out the obvious conclusion that the usseful-ness of ratios is a function of the predictive ability of the ratios and the ability of users to interpret and use these ratios.

Kennedy [1975] used financial ratios to help understand the use of Bayes Theorem as a model of human information processing. Twenty-four loan officers volunteered to participate in his experiment, and Kennedy presented them with 6 pairs of firms, one of which was bankrupt and the other of which was not. Four ratios were calculated for each of the 12 firms for the year prior to the bankruptcy. Firms and ratios were presented to the bankers in random order and they were asked to provide probability judgments of bankruptcy on a scale of 0-1. Kennedy concluded that there was a significant impact caused by the financial ratios on the bankers' decision making process. He also concluded that Bayes Theorem can be used to study the usefulness of ratios and assist in the specification of descriptive decision models.

<u>2.3.4 The empirical basis of ratios</u>. In general, the studies in this area are directed at an understanding of the taxonomy and statistical properties of financial ratios with a view toward improving the external validity of ratio analysis research. Gupta [1969] hypothesized that there are size, growth, and industry correlations among financial ratios. The population he studied was all companies listed in certain statistics published by the Internal Revenue Service for the year 1962. Eighteen ratios were obtained for each of the firms in the selected sample and coefficients of correlation were computed. Activity and leverage ratios were found to be inversely related to sale size, and positively related to growth. Liquidity ratios were found to be positively related to sale size but inversely related to growth. Profitability had no correlation with growth, and was positively related to sale size. Gupta concluded that the financial ratios of manufacturing companies differ in their relationship along sale size, growth rates, and industry classification dimensions.

Pinches and Mingo [1973] had as their objective the development of an empirically based taxonomy of financial ratios to take account of the relationships between and among these ratios. A random sample of 221 firms was selected from the Compustat tape for the period from 1951-1969. Forty-eight ratios in raw data form were log-transformed to improve normality. Factor analysis was performed with the transformed ratios as variables. The factor analysis yielded 7 factors, and only factor loadings greater than .70 were considered. The amount of variance explained in selected years was as follows:

1951	91%
1957	92%
1963	87%
1969	92%

Pinches and Mingo concluded that taxonomies of ratios can be determined by factor analysis and that the groups so identified are reasonably stable over time.

Two years later, Pinches and Mingo [1975] published a study whose objective was to examine the short-term stability of empirically based financial ratio groups. The initial procedure followed in this study was identical to that in their 1973 study reported on above. After determining 7 first order factor groupings, the data of these groups was then further factor analyzed to determine the higher order similarities among these first order groups. The analysis of the first 7 first order groups yielded 3 factors which Pinches and Mingo called Return on Invested Capital, Overall Liquidity, and Short-Term Capital Turnover.

The main objective of the Deakin [1976] study was to investigate the normality of the distribution of financial ratios. The number of firms selected by Deakin for the sample in each of the years studied varied from 1,114 firms in 1973 down to 454 firms in 1955 due to lack of adequate data. Eleven ratios were calculated for each firm for each of the years studied. In addition to working with the raw data two transformations were made, square root and log-normal. For 1973, 10 of the 11 raw data ratios were distributed in a manner that was significantly different from normal. In addition, other tests indicated unstable variances for 5 of the 11 raw data ratios. Deakin concluded that normality could be achieved in certain cases by transformation of the raw data. Deakin also found that there is less ability to reject the normality assumption when using data from a specific industry. It would seem advantageous, then, to use data from only 1 industry if a researcher is at all interested in working with normally distributed data.

2.3.5 Financial ratios and risk. Beaver, Kettler, and Scholes [1970] hypothesized that there is a correlation between the Beta Risk Measure and each of the financial ratios studied. Three hundred and seven firms were selected at random from the Compustat Tape for the period of 1947-1965. Financial ratios were obtained for each firm in the sample for each year. Calculations were made of the Beta Risk and of the accounting-based risk and correlations were computed. The accounting data was found to provide about the same forecast of risk as the Beta Risk measure.

The main objective of Gonedes [1973] was to provide some empirical evidence of the information content of accounting data in the form of financial ratios concerning asset risk. He hypothesized that there is a correlation between accounting-based and market-based estimates of systematic risk. A random sample of 99 firms was selected from the Compustat Tape. Financial ratios used by Gonedes in this study were scaled by dividing by total assets. The conclusion was that there is a statistically significant relationship between the two measures. However, Gonedes did strongly question whether the results reported by Beaver, Kettler and Scholės [1970] were as significant as they reported.

<u>2.3.6 Profile analysis</u>. In general, these types of studies use multivariate statistical techniques with financial ratios to search for a profile of the financial characteristics of any dichotomous relationship. For instance, Simkowitz and Monroe [1971] investigated the financial profile of firms absorbed by conglomerates. The sample studied consisted of 89 non-absorbed firms and 46 absorbed firms for the period studied. These firms were randomly divided into an analysis sample of 25 non-absorbed and 23 absorbed firms, and a hold-out sample of 64 non-absorbed and 23 ab-

sorbed firms. A stepwise discriminant function was derived from the analysis sample only. The derived function was able to correctly classify 77% of the analysis sample and 62% of the hold-out sample. Simkowitz and Monroe concluded that only 7 financial ratios were needed to identify the important characteristics of absorbed companies in comparison to non-absorbed companies.

Stevens [1973] studied the financial characteristics of merged firms during 1966. A sample of 40 merged firms was selected and matched with a 40 firm, non-merged sample, with control for size. A linear discriminant function was derived from the sample after using the results of a factor analysis of selected variables as input into the discriminant function.

The factor analysis produced 6 factors that explained 82% of the variance. Using the function derived from the analysis sample, the model correctly classified 70% of that original analysis sample. Stevens concluded that the ability to identify characteristics of merged firms is very useful for the regulation of anti-trust policy, and for investment analysis, in that potential benefits from mergers can be appraised.

Finnerty [1975] wanted to identify the characteristics of firms engaged in stock issue or repurchase activity. He used a sample of 715 firms, representing all the firms who had either re-purchased or issued stock during the period of 1967-1972. Thirty-two financial ratios were subjected to factor analysis resulting in 6 factors which were used as input to derive a discriminant function. The function derived from the analysis sample was able to correctly classify 83% of the hold-out sample. Finnerty concluded that firms that issued stock were characterized by higher leverage and smaller dividends. Re-purchasing firms appear to be in a position of excess liquidity, with low leverage and higher dividends.

Finnerty [1976] used the same technique to develop a model designed to search for the existence of relationships between insiders trading and the subsequent announcement of financial results. He selected a sample of 854 firms, representing all the firms identified as having had insider transactions for the period 1967-1972. The discriminant function obtained from the analysis sample was able to correctly classify 72% of the hold-out sample. His conclusions were that insiders who have decided to buy are purchasing the securities of companies characterized by smaller size, larger earnings and larger dividends.

2.4 Methodological and Statistical Problems

2.4.1 Pinches and Mingo--1975. The first published criticism of the methodological and statistical problems in financial ratio research was written by Pinches and Mingo

[1975], and the study criticized was one of their own previously published studies that used financial ratios and discriminant analysis to predict industrial bond ratings [1973]. The main objective of their criticism was to correct the violations of assumptions made in their previous study, the two violations being 1) the use of a dichotomous variable to represent the subordination status of the bonds, and 2) the use of a linear combination of variables when a quadratic combination of variables was more appropriate, due to inequality of group dispersion matrices. Pinches and Mingo worked with the same sample in this study as the one that they used earlier. The first thing they did was develop a new model that they called Model II that had the same variables as their model in the 1973 study, excluding the dichotomous subordination variable. This new five variable Model II did not perform as well as their original six variable model, which included the subordination variable. The new Model II misclassified 35% of their original sample as compared with an adjusted 29% for the original six variable linear model. Pinches and Mingo felt because of those results that the subordination variable did have information which was necessary for a more powerful classification of the firms contained in the sample. The next thing they did was to develop still another model called Model III which was really the calculation of two separate discriminant models, one for all of the firms in the sample whose bonds were subordinated, and another for all the firms in the sample whose bonds were not subordinated. Before proceeding to the classification stage with this new Model III, they conducted tests for the equality of group dispersion matrices. The test rejected the equality null hypothesis at the .001 level for the nonsubordinated group in Model III, and rejected the equality null hypothesis at the .10 level for the subordinated group of Model III. As a result of the tests, a quadratic combination was used in the classifications made by both new Models II and III. The new Model II was able to predict 65% of the original sample, and the new Model III was able to predict 75% of the original sample. Pinches and Mingo stated that they were convinced that for bond rating purposes, there were two separate bond populations, one for subordinated bonds and a second for non-subordinated bonds. They again repeated the conclusion that they had arrived at in their original study, that the qualitative factors working to influence the industrial bond raters are responsible for not being able to obtain a higher percentage of correct classifications. The two issues raised by Pinches and Mingo: 1) the use of a dichotomous variable and the subsequent treating of that variable as though it were continuous, and

2) the choice of a quadratic combination rather than a linear combination were not concluded as a result of their paper. Eisenbeis [1978] published a paper shortly after the Pinches and Mingo self-evaluation in which he attempted to clarify some of the issues raised by Pinches and Mingo, including their discussion of 1) the role of their subordination variable classification, 2) the formulation of appropriate classification rules, and 3) the determination of the significance of individual variables.

2.4.2 Eisenbeis reply. Eisenbeis [1978] suggested a method, actually two methods, for measuring the impact of the subordination variable on the overall separation of the groups. He suggested that this impact could be determined by comparing 1) the significance levels of the F-test for the equality of group means and 2) the Chi-Square measures of group overlap. Eisenbeis made these calculations for both the five and six variable models developed by Pinches and Mingo, and found that the test statistics indicated that the addition of the subordination variable increased the overall separation of the group means. He also found that exclusion of the subordination variable decreased the separation among the group means and also decreased the overlaps for all pairs of groups. On this score, Eisenbeis concluded that the values of the other variables in the model and not the subordination variable were responsible for the classification errors obtained by Pinches and Mingo, and that they should have looked elsewhere for the solution to their problem.

Eisenbeis also pointed out that because the sub-ordination variable was constant in 3 of the 5 groups used by Pinches and Mingo, the sample dispersions for these groups were singular and quadratic rules could not be used. In addition, he noted that Model III developed by Pinches and Mingo divided their sample into two groups based upon subordination status. He went on to point out that a quad-ratic combination could not be estimated for all of the sub-groups of subordinated bonds. Noting that there were only two subordinated A bonds Eisenbeis criticized the fact that Pinches and Mingo simply discarded these two sub-ordinated A bonds, eliminating that particular sub-group. In addition, Eisenbeis noted that one non-subordinated bond had a Ba rating which was an event that was excluded from their Model III. Eisenbeis then went on to point out that there still remains a need to devise a method to take into account the subordination status of the bonds and form quadratic classification rules while at the same time re-presenting events as they may occur in the population.

Eisenbeis proposed what he called Model IV, which assumed that the sub-groups within each bond class have common dispersions and differ only in their mean vectors. He then developed separate quadratic models for the sub-ordinated sub-group and the non-subordinated sub-group pointing out that this could be done because the pooled dispersions were nonsingular. The Eisenbeis Model IV was able to classify better than the Pinches and Mingo Model I and II, but performed slightly worse than Model III.

Eisenbeis also developed Model V by again dividing the sample observations into the two sub-groups based upon subordination status. In this model Eisenbeis wanted to be able to include the A and Ba sub-groups that were eliminated from the Pinches and Mingo Model III, and he did this by assuming that the pooled dispersions overall could be used as estimates of the group dispersions for these two sub-groups. He ended up assuming that these two sub-groups had different means, but common dispersions. Model V performed only marginally better than Model IV. Eisenbeis made the point that it was possible to construct models which represent events as they occur in the population, that employ proper quadratic classification rules, and that take advantage of the fact that the subordination variable is a good discriminator among some of the sub-groups. The purpose of this study is to present a model and the methodology for its use which can take into consideration more than one dichotomous variable such as subordination status, in the belief that there are many qualitative variables that contain a great deal of discriminatory power that have been omitted from financial ratio research in the past.

Eisenbeis pointed out that in their original study Pinches and Mingo had conducted a factor analysis on the 35 variables they had collected. He noted that their final model was designed to contain relatively independent variables while at the same time avoiding the problems of multicollinearity. As the purpose of the Pinches and Mingo study was to develop powerful classification accuracy, Eisenbeis noted that it would be inappropriate to discard any variable without first determining what is being given up in terms of classification accuracy by using less than all of the collected variables. He noted that the 7 factors identified by their earlier study accounted for only 63% of the variations in the data. He went on to note that exclusion of the other variables could effect the classification results achieved. Moving on, Eisenbeis discussed the two methods used by Pinches and Mingo for determining the relative discriminatory power of the variables that they finally chose for their variable set. The first method they used was the univariate F method which ranks variables according to the significance of their univariate F-statistics. The second method they used weights the coefficients of the discriminant functions by the square roots of the diagonal elements of the pooled within-groups deviation sums of squares matrix and then ranks variables according to the size of these coefficients. He pointed out that several methods have been discussed in the literature for evaluating the relative importance of variables in discriminant analysis. Other possible methods for evaluation include forwards and backwards selection procedures, and the conditional deletion method, which drops variables in turn with replacement from the set, and ranks them according to their effect on the Wilk's Lambda statistic. Because there is no one generally accepted way to measure the importance of individual variables in discriminant analysis, and because each of the different methods mentioned above may yield different rankings, the only general statements that can be made about individual variables depends entirely upon the extent to which the various methods yield similar rankings in a particular research situation. Eisenbeis used each of the five methods mentioned, that is, 1) the univariate F-test, 2) the scaled and weighted, 3) the stepwise forward, 4) the stepwise backward, and 5) the conditional deletion, to rank the variables in both the five and six variable models developed by Pinches and Mingo [1973] in their original study. He found that for the six variable models, there was strong agreement among the methods that the subordination variable was the most important. The ranks of the remaining variables varied somewhat so that he was not able to draw any conclusions about the ranking of the other variables after the subordination variable. He went on to point out that the variables in the Pinches and Mingo original study had been pre-selected so as to make them nearly independent, and he felt that this was the reason for the close agreement among the various methods he used to rank the variables. He ended by noting that his ranking of the variables confirmed the conclusion he had drawn earlier in his paper [1977] that the subordination variable is a very important discriminator in the model.

2.4.3 Pinches and Mingo reply. Pinches [1978] responded to the criticisms presented by Eisenbeis. In order to assess models IV and V developed by Eisenbeis [1978], Pinches began his reply by first examining the data used to determine if it was likely that chance factors may have influenced the alternative classification results. Although it was not done in the original study, Pinches took this opportunity to test whether or not the variables in his models were distributed multivariate normal. Feeling that it was sufficient to assess whether the variables were univariate normal, he conducted two tests designed to determine the distribution of the variables. After conducting these two tests, the coefficient of skewness and the coefficient of kurtosis, Pinches concluded that the data for both the 132 firm analysis sample and the 48 firm hold-out sample were not univariate normal. He pointed out that non-normality tends to be the rule rather than the exception when discriminant analysis is employed in financial ratio research. He went on to repeat what he had stated earlier [1975] concerning the tests that were conducted for the equality of the dispersion matrices. After pointing out that the data was not normally distributed and the dispersion matrices were unequal, he went on to caution that any classification differences among the Pinches and Mingo models and the Eisenbeis models could be due to problems related to the data itself, rather than differences among the classification models.

With that behind him, Pinches went on to point out that in developing his models IV and V, Eisenbeis had created 3 additional problems. First, Eisenbeis had to pool the dispersion matrices in order to get estimates of the sub-group of A bonds that was not of full rank. Second, Eisenbeis compared his results for models IV and V based on 180 firms as compared to the results of the Pinches and Mingo models I, II, and III, which were estimated on 132 firms. Eisenbeis combined the analysis sample and the hold-out sample to derive the 180 observation function which he then used to classify the observations. Third, Eisenbeis used different *a priori* probabilities for model III. As to the first point, Pinches pointed out that it appeared highly unlikely that the sub-group dispersion matrices could be safely pooled given the demonstrated inequality of all the dispersion matrices he examined. As to the second problem created by Eisenbeis, Pinches repeated the fact that Eisenbeis used all 180 firms in calculating his function while Pinches and Mingo had only used the 132 firm analysis sample. Pinches had no other comments on this particular point. As to the third problem created by Eisenbeis, Pinches recomputed his classification results for his models I, II, and III, using the entire 180 firm sample and the *a priori* probabilities for this larger group. Pinches then compared his recomputed models I, II and III with Eisenbeis' models IV and V and Eisenbeis' calculated Pinches and Mingo models I, II, and III. Pinches reported that models I and III did worse than was indicated by Eisenbeis, but that model II performed better than had been indicated. Pinches concluded that models III, IV, and V all performed at approximately the same level, classifying 131, 132, and 133, respectively, correct. He concluded that given the closeness of the results along with the non-normality and unequal dispersion matrices which appear to influence the quadratic classification rules used, it appeared to him that models III, IV, and V performed equally well in classifying bonds into the five rating groups. However, he pointed out that the Eisenbeis models IV and V were based on the assumption that the dispersion matrices for 3 of the sub-groups were equal and could be pooled. Pinches suggested that this assumption was not appropriate.

Pinches believed that the difficulties encountered in developing models with high discriminatory power were due to 1 of 2 factors. The first is the variable specification problem of which the treatment of the subordination variable is an important component, and is one of the main objectives of this study. The other factor cited by Pinches which impedes development of better models is the qualitative factor in the form of dichotomous variables. This aspect of model development is also one of the objectives of this study.

2.4.4 Joy and Tollefson. In December 1975, Joy and Tollefson [1975] published an article that was designed to discuss the methodology of discriminant analysis, and which included as by-products of these methodological considerations, a number of criticisms leveled at some of the previously published studies dealing with financial ratios. They first point out that the sampling frame should be identical to the populations toward which the research question is directed. The Edmister [1972] study provided an example of inconsistency between purpose and analysis. The Edmister study was designed to assess the usefulness of financial ratio data in predicting failure of small businesses. However, the data that Edmister used was collected from the Small Business Administration loan records, which include only those small businesses that were granted loans. Therefore, Edmister's conclusions could only pertain to firms granted loans. The next problem area that Joy and Tollefson approached was that a separate validation or hold-out sample was needed in order to test the classification accuracy. However, as they point out, if the purpose of the study is to predict, an additional issue arises. The following example to show the difference between the two different kinds of samples required follows the example presented by Joy and Tollefson.

Consider two samples, A and B, of size n_a and n_b, respectively. Each firm in each sample has two parts: a set of m independent variables, such as financial ratios, and a nominal classification status designated as group 1 or group 2, such as whether a firm is bankrupt or not bankrupt. Let the time dimensions for the independent variable set and classification status of sample A firms be t, and t+1, respectively, and let the time dimensions for the independent variable set and classification status of sample B firms be t+1 and t+2, respectively. Now, split A into two separate samples, A1 and A2, of size n_a1 and n_a2, respectively. At this point, the example will refer to A1 as the analysis sample, A2 as the validation or hold-out sample, and B as the inter-temporal validation sample. Joy and Tollefson then referred to Altman [1968] as an example of the use of a validation or hold-out sample for the purpose of predicting the future. They refer to his construction of a discriminant function from sample A1 over the time period 1946-1965 period. In testing his function, Altman used a new set of firms, sample A2, but from the same time interval used to develop the function from sample A1. Joy and Tollefson conclude that Altman could not have predicted bankruptcy, since none of the firms in sample A2 came from years beyond 1965. Joy and Tollefson went back to the Edmister study as well as to the Altman study to point out that after sample A1 has been used to classify the validation or hold-out sample A2, the two samples should be re-combined and a new function derived from the combination of samples A1 and A2. Both Edmister and Altman used only the firms of sample A1.

Joy and Tollefson then discussed the problem of assessing the discriminatory power of individual variables in the set of variables selected to obtain the classification function. He pointed out that many reported financial ratio research studies have used standardized coefficients as indicators of the importance of the variables, but they are not appropriate in assessing the relative discriminatory power of the variables in a discriminant function. They noted that Altman [1968], using the standardized coefficients, concluded that his variable x_5, sales related to total assets, was the second most important discriminator. As calculated by Joy and Tollefson, using a measure of relative discriminating power used by Mosteller and Wallace [1963], this variable was in fact the least important, explaining less than 8% of the average discriminant score separation between groups. Joy and Tollefson concluded that they had presented a methodology that was better than the procedures currently used in financial ratio discriminant analysis studies. Their methodology refrained from presenting meaningless results and emphasized the difference between cross-validation and inter-temporal validation.

<u>2.4.5 Eisenbeis</u>. In a 1977 paper, Eisenbeis [1977] stated that his purpose was to discuss the problems of application of discriminant analysis techniques in financial ratio studies. He identified the problem areas as follows:

1. The distribution of the variables.
2. The group dispersion matrices.

3. The interpretation of the significance of individual variables.

4. The reduction of dimensionality.

5. The definitions of the groups.

6. The choice of the appropriate *a priori* probabilities and/or costs of misclassification.

7. The estimation of classification error rates.

Of course, many of the problems discussed by Eisenbeis in 1977 had been discussed by Joy and Tollefson [1975], and they both served to point to the need for this paper which has as its purpose the presentation of a methodology for discriminant analysis that is able to deal with mixed discrete and continuous variables and violate as few assumptions as possible.

2.4.5.1 Distribution. Eisenbeis begins by pointing out that an optimal discriminant analysis procedure assumes that the variables used to describe the members of the groups are multivariate normally distributed. Like Pinches and Mingo [1975], he points out that deviations from the normality assumption, at least in the financial ratio literature appear more likely to be the rule rather than the exception. Because violations of the normality assumption may bias the test of significance and estimated error rates, it is of interest to determine whether the assumption holds and what effects its relaxation may have on the tests and on the classification. Rather than point to any one study, Eisenbeis merely states that the problem of testing for the distributional assumption has been largely ignored. He also notes the use of certain transformations prior to estimating the discriminant function in some of the financial ratio literature, and refers specifically to a Pinches and Mingo [1973] study. He points out that the effect of the natural log transformation is to make the marginal distribution of the variables more symmetric, but warns that while the marginal distributions of a normal distribution are normal, making a variable's marginal distribution normal may not necessarily make the joint distribution more normal.

2.4.5.2 Dispersions. The second critical assumption of optimal discriminant analysis addressed by Eisenbeis is that the group dispersion matrices are equal across all groups. He refers to a study of inter-locking ownership and directorates among mutual savings banks and commercial banks in New Hampshire by Eisenbeis and McCall [1972]. He pointed out that quadratic procedures significantly improved the classification results over the linear results in this two group, two variable problem. He noted that the linear procedure averaged out the fairly diverse dispersions which were present, whereas the quadratic rule used this additional information resulting in improvement in the classification.

2.4.5.3 Interpretation. The third problem approached by Eisenbeis relates to the determination of the relative importance of individual variables. He singled out Edmister [1972] and Pinches and Mingo [1973] for having excluded highly correlated variables due to their belief that multicollinearity was harmful. These decisions were made on the basis of either univariate F statistics or the standardized coefficient tests. Multicollinearity is largely an irrelevant concern in discriminant analysis except where the correlations are such that it is no longer possible to invert the dispersions matrices. Eisenbeis next pointed out that multiple regression coefficient t tests do not indicate whether a particular coefficient itself is zero because the coefficients are not unique in discriminant analysis. He noted that Meyer and Pifer [1970] made the assumption, in error, that the regression t statistics were valid for determining the significance of individual coefficients in a discriminant function.

2.4.5.4 Dimensionality. The two principal ways for reducing dimensionality in discriminant analysis are to eliminate 1) those variables or 2) those discriminant functions that do not contribute significantly to the overall ability to discriminate among groups. He noted that the dimension-reducing methods used in financial ratio studies have focused solely on determining whether a variable or function contributed significantly to the Wilk's Lambda or related statistics used in testing hypotheses about the equality of group means. These tests are appropriate if the research goal is to maximize the separation among groups while minimizing the number of variables or functions used. If the goal is to construct a classification scheme then use of these methods may not leave the classification results unaffected, even if seemingly insignificant variables are eliminated. If classification accuracy is a primary goal, then the criterion for keeping or deleting variables and dimensions should be related to the overall efficiency of the classification results. It is inappropriate to discard variables in instances such as Pinches and Mingo [1973], Edmister [1972], and Meyer and Pifer [1970]. The researcher should first examine the overall classification results to determine what the effects of dimension reduction really are.

2.4.5.5 Groups. Eisenbeis criticized Altman [1968] for selecting samples where only relatively small firms were included. Joy and Tollefson [1975] had also criticized Edmister [1972] in this regard for having selected only firms whose loans had been granted, and they concluded that the function could only be useful on a population of loans granted. Eisenbeis stressed that the populations sampled to estimate discriminant functions should correspond to the populations generating the new observation to which the model is to be applied. He used as an example the development of credit-scoring systems, where the objective is usually to develop models to discriminate between those new loan applicants who are or are not likely to default on their loans. Loan performance data used to estimate the discriminant functions are not available on the population of all applicants, but rather on only the subset of applicants that were originally granted loans. Omitted are data on that portion of the population who applied for loans but did not receive them, some of which may or may not have defaulted. Eisenbeis also raised another common grouping problem in financial ratio literature which is to take arbitrarily defined groups such as the bond classes used in Pogue and Soldofsky [1969] and Pinches and Mingo [1973]. Another example of the use of arbitrary groupings can be seen in Backer and Gosman [1978] where the two sets of groups are based upon 1) trade credit ratings established by Dun & Bradstreet, and 2) bond ratings developed by Standard and Poor's. The best problems for discriminant analysis are those in which the group definitions are distinct and non-overlapping, and every effort should be made to avoid arbitrary groupings such as the ones mentioned above.

2.4.5.6 *A priori*. Discriminant analysis classification rules that incorporate *a priori* probabilities and misclassification costs have been grossly overlooked in the literature. The paired-sample methods of Beaver [1966], Altman [1968], Meyer and Pifer [1970], and Sinkey [1975] among others are representative of non-random sample procedures that have been used. Non-random methods where certain factors are controlled such as size and industry are appropriate for investigating the importance of certain variables but not for estimating classification error rates. An alternative method would be to draw random samples from the two groups, and to include the control variables along with the other variables. The use of discriminant analysis in a time series context raises additional questions which are addressed by Eisenbeis. For example, in the problem bank study of Sinkey [1975], the number of problem banks varied from year to year from a low of approximately 150 in some years to more than 350 in early 1976. Eisenbeis raises the question, under such circumstances, if it is more appropriate to use the relative group frequencies from a given year as estimates of the <u>a priori</u> probabilities, or to attempt to use some average of past frequencies. The research design and goals of the research should determine the method to be used. In the problem bank study, the expected frequencies of problem and non-problem banks are not independent of the state of the economy, so during unstable times, a simple average of past years' frequencies would tend to understate the expected frequencies of the problem group.

Another aspect of time series problems can be found in studies such as Pinches and Mingo [1973] and Altman [1968], among others, where the data on the groups are obtained by pooling observations from different time periods. Eisenbeis points out that here again it is not clear what the appropriate a priori probabilities are or how they should be estimated. Eisenbeis recommends that the estimation of the priors be geared to the type of classification statements the researcher wishes to make. He offers as an example, that if a 1 period classification is to be made, then it would seem reasonable to use an average of the relative frequencies over several time periods to estimate the priors. On the other hand, if predictions are to be made over several time periods then it may be more appropriate to pool data over the same length of time to estimate frequencies.

2.4.6. Pinches and Trieschmann. The last paper to be reviewed in this section on methodological and statistical problems is one by Pinches and Trieschmann [1977]. The purpose of that paper was to examine the impact of 3 factors influencing classification results, 1) multivariate normality, 2) equality of the dispersion matrices, and 3) misclassification error rates. Data from a previous study by Trieschmann and Pinches [1973] was used in this study to illustrate the impact of these 3 factors on the classification results. In their original study, Trieschmann and Pinches used a six variable model for two groups of insurance firms, solvent and distressed. No tests of normality were employed in that study, and the data is examined in the 1977 study for both univariate and multivariate normality using the coefficient of skewness and the coefficient of kurtosis. The tests on the data for univariate normality showed that the data used in their model was not univariate normal. Multivariate tests of skewness and kurtosis were used on the data and it was found that these multivariate measures indicated that the variables used were not multivariate normal. In order to improve the normality of the data, three different transformations (square, square root, and $\log_{10}$) were applied to the variables. Only slightly greater univariate and multivariate normality could be achieved, and the transformations also failed to influence significantly any of the classification results. Pinches and Trieschmann concluded that nothing could be gained by transforming the data. They also cautioned that all classification results in financial ratio studies may be biased due to the lack of multivariate normality in the continuous variables.

In [1973] Trieschmann and Pinches did not test for the equality of the dispersion matrices. In [1977] they conducted tests to determine whether or not the dispersion matrices were equal. They concluded that the two dispersion matrices were not equal, and that a quadratic classification procedure should be employed instead of the linear one used in their earlier study. They found that the linear classifications results were slightly better overall with 49 of 52 firms correctly classified as opposed to 48 of 52 firms correctly classified by the quadratic rule. Pinches and Trieschmann concluded that given the non-multivariate normality of the data, the quadratic rules are at least as sensitive to non-normality as the linear rules; therefore, it is not clear which classification rule is more appropriate. Given the closeness of the classification results obtained, it probably does not matter which of the results is presented.

Pinches and Trieschmann applied the Lachenbruch [1967] procedure to classify the data from their earlier study using both linear and quadratic classification rules. Their earlier study had used the original observations in the analysis sample to estimate the discriminant function and to classify the original observations. The results of this comparison showed that using the linear rule the percentage of correct classifications using the analysis sample of 94% dropped to 86% using the Lachenbruch method, and using the quadratic rule the 92% correct classification using the analysis sample dropped to 83% using the Lachenbruch method. All four of the classification results were statistically undistinguishable from one another. Pinches and Trieschmann concluded that if they had to report only one classification result, they would report the quadratic Lachenbruch method even though they recognize that the quadratic function can be influenced by non-multivariate normality. Their second choice would be the linear Lachenbruch method results.

They felt that both of these estimates are preferred to the analysis-sample method results used in their original study, which present a biased estimate of the probability of correct classification in the population. They appealed to researchers to provide substantially more information in the future than has been typical in the past. In this way, the audience will have an opportunity to assess the accuracy, reliability and significance of the results.

2.5 Appropriate Application of Discriminant Analysis and Classification

This section presents an appropriate methodology for discriminant analysis and classification using financial ratio variables. It begins with a brief overview of discriminant analysis and classification involving financial ratios. Section 2.5.2 presents 8 common problem areas in this technique and the related literature for the appropriate practical application of the technique taking each one of the eight problem areas into consideration. This section will also include discussions of the potential impact of violations of assumptions.

<u>2.5.1 Overview of the technique</u>. Discriminant analysis begins with the desire of the researcher to distinguish between two or more groups of cases. For purposes of this study the analysis will be conducted from a two-group point of view, as that is what is encountered in the majority of the financial ratio literature. As an example, consider the 1973 study by Stevens [1973], where the main objective was to identify the financial characteristics of two groups of companies; those companies which have merged with other companies, and those companies which have not merged with other companies. The motivation for such a study could be for the regulation of anti-trust policy, or it could be for investment analysis, in that the potential benefits from mergers could be appraised by such knowledge. Regardless of the motivation, discriminant analysis should begin with two clearly defined and non-overlapping groups, such as the ones in the example: merged, or non-merged. Wherever possible, the groups used should be free from any arbitrary or subjective initial assignment to groups.

Consider a study like Pinches and Mingo [1973], where the groups were defined by five bond rating groups which were originally assigned by Moody's or by Standard and Poor's. Because the bond rating process is subjective, and sometimes arbitrary, there always exists the possibility that the original assignment of a firm to one of the five bond rating categories was in error. This type of problem should be avoided if at all possible. Once the research situation has defined the groups, the next step is to determine what data will be used to distinguish between the groups.

In this study, the discriminating data consists of financial ratios and other binary variables representing non-continuous information. Once the researcher has determined what the discriminating variables are to be, he must then go out and collect the values of the discriminating variables for a carefully selected sample, representative of the populations under study. While it is not always possible, it is important that the sampling framework be identicial to the populations toward which the research question is directed. If for example, as in the Edmister [1972] study, the objective is to determine the usefulness of financial ratios as discriminating variables in predicting the failure of small businesses, then the samples should be drawn from the entire population of small businesses. Edmister, however, collected his data from SBA loan records, which consisted of only those small business firms that had been granted SBA loans. Thus, his conclusions should only pertain to small business firms granted SBA loans.

Sample sizes must be chosen with recognition that two populations are being sampled. The key consideration is the need to obtain a sufficiently large number of observations from the smaller group. Many of the research studies dealing with financial ratios have used sample proportions that were approximately in a 1:1 ratio. While it is true that discriminant analysis will work best when the population proportions are not significantly different from 1:1, there are no compelling reasons for any such requirement for the sample. Sample proportions in general should be determined by 1) the cost of sampling, 2) minimal sample size considerations, and 3) data handling facilities. This means that the sample size of the larger population will not be limited by the sample size of the smaller population. If the sample proportions differ from the prior probabilities for the population, inference from the sample to the population requires great care. This issue is discussed in more detail in Section 2.5.2.8 dealing with validation and inter-temporal classification samples.

When the groups have been selected, the discriminating variables decided upon, samples and sample size determined, and the data gathered the next step is to weight and combine the discriminating variables so that the groups are forced to be as statistically distinct as possible. Discriminant analysis does this by forming a linear combination of the discriminating variables in the form of a discriminant function. These functions are formed in such a way as to maximize the separation of the groups. Most researchers use "canned" programs developed to derive the discriminant functions. Discriminant analysis programs in these packages give the researcher various alternatives in the linear combination of his discriminating variables in arriving at a discriminant function. As the maximum number of functions which can be derived is either one less than the number of groups or equal to the number of discriminating variables, most two-group financial ratio research studies therefore end up with just one discriminant function.

The next step consists of the analysis aspect of this technique, which provides tools for the interpretation of the data that has been collected. Generally, the first test that is made is the one that measures the success with which the discriminating variables actually discriminate when combined into the discriminant function. This test is described in some detail in Section 2.5.2.7 that deals with the estimation of classification error rates. The discriminant function can also be used to study the spatial relationships between the groups. Subject to certain precautionary warnings, the coefficients derived in the discriminant function can be interpreted by the researcher in an attempt to identify the discriminating variables which contribute the most to differentiation along the function. In addition, the researcher will test that the observed between-group differences are statistically significant. This determines if there is any hope of classifying future observations using the given variables. If not, then the researcher should try to find better discriminating variables. If the between-group differences observed are greater than would be expected by chance, the researcher asks himself the question, are all of the variables needed? He may wish to reduce the number of variables in the discriminant function. This question is discussed in more detail in Section 2.5.2.4 dealing with reduction of dimensionality.

Because most of the studies using financial ratios are designed with prediction in mind, an important question that faces the researcher is how well the discriminant function will perform in classifying future samples. This problem involves the error rates of the discriminant function when it is used to classify new observations of unknown origin into one of the 2 groups under study. This problem is discussed in more detail in Section 2.5.2.7 dealing with the estimation of classification error rates. Generally, the prediction of the future, such as Altman's [1968] prediction of corporate bankruptcy, is the final goal. The researcher has carefully selected well defined and separate groups, with the initial classifications being made objectively. He has given consideration to the populations under study, and the sampling scheme is conceptually matched to these populations. Sample sizes have been selected, the discriminating variables selected, and the data collected. The discriminant function has been calculated, and the researcher has tested for whether or not the variables selected have sufficiently separated the group means. He has given consideration to a reduction in the number of variables, just in case a fewer number of variables will be able to discriminate as well. Finally, the researcher has determined how well his function is able to classify a new unknown observation correctly into one of the 2 groups under study.

2.5.2 The literature of discriminant analysis problems. Joy and Tollefson [1975] were the first to publish a major critique of the application of discriminant analysis to two-category classification problems in empirical financial research. They noted that these studies had given relatively little attention to design and interpretation difficulties associated with discriminant analysis, and consequently, the conclusions and generalizations that were drawn from such studies were frequently tenuous and questionable.

The purpose of their paper was to discuss the methodology of discriminant analysis, oriented toward financial applications, but not peculiar to finance alone. They first pointed out that if the discriminating variables arise from multivariate normal populations, and have identical dispersion matrices, then linear discriminant analysis provides an optimal solution to the classification problem. When the measurements arise from multivariate normal populations but the dispersion matrices are not equal, quadratic rather than linear multiple discriminant analysis yields the optimal solution. They pointed out that in a majority of financial studies, the use of linear discriminant analysis has not been preceded by tests to determine if the conditions for its optimality have been satisfied. Rather, they note, these studies have applied the technique in the hope that it would yield useful results.

As to classification, they note that financial ratio studies have generally not been explicit concerning the value of the prior probability of group membership and the ratio of the group costs of misclassification. The cut-off value used for classification purposes in these studies would be optimal assuming prior probabilities identical to the sample group frequencies, and the cost of a Type I error being equal to the cost of a Type II error.

Joy and Tollefson point out that the sample selected should be identical to the population toward which the research question is directed. They also point out that the sample size should be determined as mentioned before, and that there are no overriding considerations for a 1:1 sample size ratio. They give extensive treatment to the problem of validating the function derived from an analysis sample, and the kind of validation required in order to claim predictive ability. A more detailed discussion of this problem will follow in Section 2.5.2.8 dealing with validation and prediction problems. Joy and Tollefson discuss a measure of relative discriminating power used by Mosteller and Wallace [1963]. After pointing out that Altman [1968] used a measure that was not appropriate, they calculated the measure of Mosteller and Wallace and compared it to the one used by Altman.

They discussed different measures of classification efficiency, and the importance of prior probabilities and costs of misclassification. They performed Bayesian evaluation using estimated prior probabilities and costs of misclassification and compared their results to those achieved by Altman [1968].

The second study reviewed in this section is the one by Eisenbeis [1977], who noted that many of the studies to date had suffered from problems that limited the usefulness of the results. The purpose of his paper was to discuss these problems of application of discriminant analysis techniques. He had a list of 7 problem areas which he discussed one at a time, as will be done in the remainder of this section. Eisenbeis clearly stated that he considered the selection of the appropriate *a priori* probabilities as being the most important problem related to classification, followed by the selection of the appropriate classification rule (linear vs. quadratic) and the assessment of classification accuracy. In particular, he noted the failure of most studies to relate the estimates of the *a priori* probabilities to the population priors by assuming equal priors, which limits the ability to make any meaningful inferences about the overall performance of the classification rule. He noted that other problems such as non-normality, the selection of subset variables and reduction of dimension, and interpreting the significance of individual variables are not easy to remedy. He urged that research studies include a caution that the reader must temper the conclusions reached by recognizing that the empirical results represent approximations that may be significantly biased in many cases. He concluded by mentioning time series problems which he did not discuss but which he felt were an important class of problems that frequently occur in the business literature. The remainder of this section will deal with the 8 problem areas previously mentioned one at a time, and will include references to the literature which has addressed the problems of the application of discriminant analysis. (See Dillon [1978] for a comprehensive examination of the performance of the LDF in non-optimal situations.)

2.5.2.1 Non-normal data. Optimal discriminant analysis classification procedures assume that the variables used are multivariate normally distributed. A violation of the normality assumption may bias the test of significance and the test of estimated error rates. It is important for the researcher to determine whether the normality assumption holds and what effects its violation may have on these tests and on the classification. In the applied literature, the problem of testing for distributional assumptions has apparently been largely ignored. When the tests have been made (see Pinches and Trieschmann [1977]) the results have indicated that the data are not normally distributed. Neither of the multivariate tests of skewness and kurtosis developed by Mardia [1970] indicated normality when used by Pinches and Trieschmann. Eisenbeis [1977] notes that the major works dealing with normality problems have been of two major types. Some researchers have investigated alternative schemes where specified types of non-normality hold, while others have evaluated the bias introduced in the technique when the normality assumptions are violated in known ways. In the case of the former approach, Chang and Afifi [1974] and Krzanowski [1975] derived and examined classification methods where some of the variables were dichotomous. In the financial ratio literature many of the factors reflecting important characteristics of the groups tend to be categorical in nature, such as subordination status, bond rating, trade credit rating, etc. (see Alves [1978] for the use of a diversification variable).

When continuous and binary variables are mixed, producing a variable set that is known not to be normal, the technique suggested has been to split the samples based on the values of the binary variables, then employ standard discriminant analysis on the subdivided samples. This was the procedure used by Chang and Afifi [1974] in 1974 for the simple two-group case with one binary variable and several continuous variables. Krzanowski [1974] has extended the work of Chang and Afifi to include any number of binary variables and any number of continuous variables. Both these studies concluded that it is appropriate to use these binary variables to split the samples and then construct separate discriminant functions and classification rules for each configuration defined by the values of the binary variables.

In examining the bias introduced when non-normality holds, Gilbert [1968] compared the performance of the linear discriminant function when applied to data where all the variables were discrete, with the performance of two logit models and a model which assumed mutual independence of the variables. The application of the standard techniques when the variables are dichotomous violates the assumption of equality of dispersion matrices, so linear techniques are therefore always inappropriate. Gilbert concluded that there was only a small loss in predictive accuracy using the linear function, and that as the number of variables increased, the results should be quite stable.

Eisenbeis [1977] referred to a study by Lachenbruch, Sneeringer, Revo [1973] that investigated the robustness of both linear and quadratic procedures for three non-multivariate normal distributions. These distributions were transformations of normally distributed variables so that the true classification errors were known. The three distributions were the log normal, the logit normal, and the inverse hyperbolic sine normal. They concluded that standard linear procedures may be quite sensitive to nonmultivariate normality. They found that the <u>overall</u> classification rates were not affected as much as the <u>individual group</u> error rates. They suggested that data should be transformed to approximate normality and then tested for the equality of the dispersion matrices before determining whether linear or quadratic techniques should be used.

Pinches and Mingo [1973] did use transformations in their study. Although the data for the discriminating variables in the population are not normal, the idea of transformation has intuitive appeal because it does adhere more closely to statistical assumptions. As a caution, Joy and Tollefson [1975] noted that transformed variables give less weight to equal percentage changes in a variable when the values are larger than when they are smaller. If, for example, the variable being transformed was firm size, Joy and Tollefson point out that the implication would be that one does not believe that there is as much difference between a $1 billion and a $2 billion size firm as there is between a $1 million and a $2 million size firm. The percentage difference in the log will be greater in the latter than in the former case.

Lachenbruch [1975] points out that relatively little has been done regarding the robustness of the discriminant function to continuous but non-normal distributions. He noted that this stands in sharp contrast to the results for non-normal discrete distributions, which for the most part are fairly robust in the linear discriminate function. Overall, the evidence seems to indicate that non-multivariate normal data may be used in discriminant analysis without significantly biasing the results. More information is available about the implications of using only dichotomous variables, while there is still relatively little information about the effects of non-multivariate normality in continuous variables.

2.5.2.2 Unequal covariance matrices. A second assumption of discriminant analysis is that the group dispersion matrices are equal across all groups. Relaxation of this assumption effects 1) the significance test for the differences in group means, 2) the usefulness of reduced space transformations, and 3) the appropriate form of the classification rule. Eisenbeis [1977] notes that little attention has been given to the effects of unequal dispersion matrices on the test of the equality of group means. He did note that Anderson [1958] has developed a test for the equality of means for the limited case when the sample sizes are equal, and an approximate test for the case when there are unequal sample sizes. Holloway and Dunn [1967] concluded in their study that the robustness of group mean tests depends upon both the number of variables and the relative sample sizes in the groups. With widely different sample sizes, they found the actual significance level is greater than the hypothesized level, and therefore the null hypothesis would be rejected more frequently when the means were in fact equal. When the number of variables increases, the significance level also increases, and the sensitivity to unequal sample sizes increases. Holloway and Dunn concluded that equal sample sizes help in keeping the level of significance close to the supposed level. Lachenbruch [1975] notes that the literature indicates that the linear function is quite satisfactory if the dispersion matrices are not too different. In particular, he notes, the quadratic function is very poor for small sample sizes. When differences between dispersion matrices are quite large, and the samples are also quite large, he recommends the use of the quadratic function.

Reduced-space discriminant analysis (see Tatsuoka [1971]) can be used to reduce the original m dimension variable space to an r dimensional problem, $r < m$. This reduction is possible because the linear transfromation from test-space to reduced-space preserves the relative linear distances among observations and leaves the significance tests and classification results unaffected. Reduced-space transformations hold only if the group dispersion matrices are equal. Eisenbeis [1977] notes that if the dispersions are not equal, then the transformation to reduced-space is no longer distance preserving. The result is a warping of the relative positions of the observations in reduced-space which affects both the significance test and changes the resulting classifications.

Considerably more attention has been given to the effects of unequal dispersion matrices on classification results. Gilbert [1969] has investigated this problem and compared the effects on error rates when a linear rule is used when in fact the dispersions are unequal. The results indicate that significant differences can occur which are directly related to 1) the differences in the dispersions, 2) the number of variables, and 3) the separation among the groups. The further apart the groups are for given dispersions, the less important are the differences between the linear and the quadratic results.

In 1974 Marks and Dunn [1974] investigated the performance in the two-group case of the linear discriminant function and the quadratic function. They concluded that for large samples the quadratic procedures perform better, the closer the groups were to each other, and as the number of variables increased. For small samples, they indicated as Lachenbruch [1975] did that the quadratic performed worse for small numbers of variables and similar dispersion matrices, and this performance deteriorated as the number of variables increased. Eisenbeis [1977] pointed out that these results seemed quite reasonable when consideration is given to what is involved in constructing the discriminant function and classification rule from the samples. For the two-group, m variable linear case, 2m variable means, and m^2 elements of the pooled within-groups dispersion matrix must be estimated. This is a total of m(2+m) parameters while in the quadratic case 2m(1+m) parameters are involved, which is nearly twice the number as in the linear case.

Eisenbeis and Avery [1972] have shown, using the Fisher [1936] Iris Problem that the use of quadratic classification rules yielded identical results to those using linear rules even though they hypothesis of the equality of the dispersion matrices was rejected beyong any reasonable level of significance. They point out that the reason for this was clearly due to the fact that the group means were <u>so far</u> apart and that there was almost no significant overlap among the groups. Eisenbeis [1977] notes that there is little reason to believe that any one application area is likely to be more susceptible to this linear vs. quadratic problem than any other, except to the extent that categorical variables may arise more frequently in certain types of problems than in others. The available evidence indicates that the rejection of the equal dispersion hypothesis may have a significant impact on the test for the equality of group means. In addition, the use of linear rules when quadratic rules are indicated may have drastic effects on the classification results. Logically then, the test for the equality of the dispersion matrices must precede both the test for the equality of group means and the estimation of classification errors. (See Cooley and Lohnes [1962] for a test to determine the equality of dispersion matrices.)

2.5.2.3 Initial group misclassifications. The assumption that the initial samples from π_1 and π_2 are correctly classified may not always hold. Consider as an example the five bond rating categories used by Pinches and Mingo [1973]. Clerical errors may have occurred in assigning bond ratings to the various firms, or the information available to the rater of the bonds was not sufficient for him to make an accurate classification. These types of problems would always have to be present whenever the categories being investigated are subjective in nature, such as bond ratings, or include qualitative data that is not formally incorporated in the rater's decision model. Lachenbruch [1966] studied the effects on the error rate of a linear function if a fraction of the observations supposedly from π_1 were really from π_2 and a fraction of the observations were really from π_1. He found that the behavior for average size samples was about the same as that which would be expected if the observations had been properly classified in the first place. McLachlan [1974] studied the asymptotic theory for the same problem, and his results generally agreed with those of Lachenbruch.

Lachenbruch [1975] points out that these two studies assume that the misclassified observations are a random sample from the parent populations. This is not altogether the case as the firms most likely to be misclassified initially are probably the borderline cases. This problem has also been studied by Lachenbruch [1974], and he found that the actual error rates were unaffected by non-random initial misclassifications. However, he found that the apparent error rates were grossly distorted and totally unreliable for any sample size.

An associated problem is the one dealing with the distinctness of the groups. For example, in the Altman [1968] study it was clear by his definition for the time periods under investigation which firms had failed and which firms were going concerns. In addition to that, his groups did not overlap and they were exhaustive. A firm could belong to only one of the 2 groups and to no other. This type of grouping scheme is the most suitable for discrimination and classification. Eisenbeis and Avery [1972] give an example of a less desirable grouping scheme found in the literature. They cited a study of firms with high and low earnings-price ratios. The sample was divided into 2 groups of firms based upon the distribution of the 3 year earnings-price ratios of the 1955 Fortune 500 firms. The low earnings-price group was defined as those firms in the first quartile of this distribution and the high group as the fourth quartile firms. Eisenbeis and Avery criticized this grouping scheme on two grounds. First, the groups were not truly discrete as in the Altman [1968] study. Secondly, the groups were not exhaustive in that no firms from the second or third quartiles were included, and therefore any classification should be only be applicable to a similar sample. In general, arbitrary grouping schemes should be avoided unless there is sound, theoretical reason for forming groups, and the purpose of the study is to describe the groups rather than to predict group membership.

2.5.2.4 Reduction of dimensionality and number of variables. Two ways to reduce dimensionality in discriminant analysis are to eliminate either those variables or those discriminant functions that do not contribute significantly to the overall ability to discriminate among groups. This procedure is particularly important in financial ratio research where it is possible to generate a large number of ratio variables which need to be reduced to manageable size. So far, the financial ratio literature has focused mainly on determining whether a variable or function contributes significantly to the Wilk's Lambda. This is appropriate if the research goal is to maximize the separation among groups, while minimizing the number of variables or functions used. However, if the goal is to construct a classification scheme, then the use of the Wilk's Lambda may not leave the classification results unaffected.

Eisenbeis and Avery [1972] argue that the existence of statistically significant differences among group means does not convey much information about the ability to construct a successful classification scheme. Following Cooley and Lohnes [1962], they suggest that Chi-Square methods of describing group overlap be used. This method indicates the percentage of Group G means that lie closer to the means of Group H than do a percentage of the members of Group H. If classification accuracy is the primary goal, then the criterion for keeping or deleting variables should be related to the overall efficiency of the classification results. The results using all variables should be compared with those based upon various sub-sets of variables. It does not seem appropriate to discard variables such as was done by Meyer and Pifer [1970] without first examining the overall classification results to determine what the effects of this reduction really are. Lachenbruch [1975] presents an illustration of variable reduction in a study of schizophrenia diagnosis. One thousand one hundred and twenty-one patients were used, with 560 patients in the analysis sample, and 561 patients in the evaluation sample. By design these two samples each contained about 405 schizophrenics and 155 non-schizophrenics. A total of 415 variables was measured on each patient, and the first step was to collect the 150 best variables by means of t-tests. Of these 150, the 69 most statistically significant variables were used in a stepwise discriminant analysis, and then the 12 most discriminating symptoms were chosen. The final rule decided upon by the researcher weighted all 12 symptons equally, and the diagnosis was based on the number of symptoms observed in a given patient.

Of course, in the two-group case dimension reduction is quite striking because any large variable problem is transformed to a univariate problem. Classification is then performed with the one discriminant function that obtains in the two-group case, because the number of functions extracted is always equal to one less than the number of groups. Classification then consists of a comparison of the reduced-space variable which is the discriminant score, with a cut-off point. There has been a tendency in the applied literature to misuse the linear reduced-space procedure. In two-group problems the assumption of equal group dispersions in order to use the linear reduced space procedures is very important. When linear reduced-space procedures are used and the dispersions are unequal, the result is to bias the classification results. Given that it is known that the quadratic classification is the appropriate one, use of linear reduced-space classification significantly biases the results and leads to improper conclusions concerning the efficiency of the classification scheme.

A logical extension of the problem of dimension reduction is the one of identifying or comparing the relative discriminatory power of different size sub-sets of variables. Rao [1952] suggests a solution to evaluate the significance of the additon or deletion of a set of variables. Rao's statistic can be used to compare the discriminatory power of any 2 variable sets, even if one is not a sub-set of the other. Many times some variables are not significant discriminators. In these cases the stepwise variable selection procedures may be combined with the Rao test to delete variables from an initial variable set without a significant loss in discriminatory power. Only a complete stepwise process is designed to yield the sub-set that is the smallest and at the same time yields the smallest loss in discriminating power. However, because of the computational difficulties associated with the complete stepwise method, many researchers are satisfied to use the Rao F-test. This test enables the researcher to determine 1) that the reduction in discriminatory power using a set of size i instead of the total m ($i<m$) variable set is less than some

arbitrarily acceptable level, and 2) that a set of size i is the smallest possible sub-set satisfying (1). Eisenbeis and Avery [1972], while stating that the complete stepwise process is clearly superior to the forward and backward techniques, point out that in one of their studies a 12 variable model required 6 minutes of CPU time on an IBM 360-50 to select the optimal sub-set. The time approximately doubles with the addition of another variable.

2.5.2.5 Individidual variable significance. Tests of discriminant coefficients that have a particular non-zero value are not useful in general because the coefficients are determined only up to a constant multiple. If $D_s(x)$ is a discriminant function, $mD_s(x)$, where m is a constant, is an equivalent one. Unlike the coefficients in the classical linear regression model, the discriminant function coefficients are not unique, only their ratios are. It is not possible as is the case with regression analysis to test whether a particular discriminant function coefficient is equal to zero or any other value. A number of methods have been proposed which attempt to determine the relative importance of individual variables in discriminant analysis. Five of these methods were considered by Eisenbeis, Gilbert and Avery [1973]. These five methods were to rank variables on the basis of 1) their univariate

F-statistics, 2) their scaled discriminant function coefficients which were appropriately weighted, 3) stepwise forward methods based on the contribution to the multivariate F-statistic, 4) stepwise backward method as in (3), and 5) a conditional deletion method which removed each variable in turn from the m variable set, with replacement, and ranked variables according to the resulting reduction in overall discriminatory power as measured by the m-1 variable F-test. A sixth method has been suggested in Joy and Tollefson [1975] which weights each coefficient by the difference in the group means divided by the differences in the mean discriminant scores.

The first of these two methods treats the variables independently. However, the unimportant variables on a univariate basis may be very important when combined with other variables. Methods 3, 4, and 5 are conditional models which take into account correlations among the variables. For example, in the stepwise forward method, the second variable to enter is the second most important variable, given that the first variable is already included. The conditional deletion method seems to have the greatest appeal since the relative importance of each variable is conditional based on the inclusion of all other variables. Regression coefficient t-tests do not indicate whether a particular discriminant coefficient itself is zero, and many financial ratio researchers have assumed that the regression t-statistics are valid for determining the significance of individual discriminant coefficients.

The sixth method, attributed to Mosteller and Wallace [1963], gives the change in the linear discriminant function score associated with a change in the discriminatory variable equivalent to movements between groups in mean value, and may be interpreted as the portion of the discriminant score separation between the groups that is attributable to that variable. It must be noted that any method for investigating the relative importance of variables assumes equal dispersion matrices. Rejection of the equality hypothesis implies that these methods are subject to the same limitations as the tests for the significance of the difference in group means (see Section 2.5.2.2).

2.5.2.6 A priori probabilities and misclassification costs. Discriminant analysis classification rules generally in theory incorporate *a priori* probabilities to take into consideration the relative occurrence of observations in the different populations, and misclassification costs to adjust for the fact that some classification errors may be more costly than others. As an example, Eisenbeis [1977] presents a case which compared six-group quadratic classification assuming equal *a priori* probabilities and estimated population *a priori* probabilities. The overall misclassification was 51% for the equal *a priori* probability case as compared with 46% for the unequal *a priori* probability case. More important, he noted, than the overall improvement was the fact that some of the individual group error rates shifted radically. For example, 88% of one particular group was correctly assigned assuming equal priors whereas 100% were misclassified in the unequal priors case. This illustrates that the researcher can be misled about the effectiveness of his classification results if the appropriate priors are not employed. The use of sample proportions as estimates of population priors is appropriate if the pooled data represents a random sample from the population. As most financial ratio studies do not involve a random sample, their resulting classifications only minimize classification errors in the sample rather than providing evidence on the population error rates. The researcher may use estimates of the population priors, which does eliminate the problem of the pooled sample representing a random sample from the population. Sampling methods such as the paired-sample used by Beaver [1966] and Altman [1968] among others are probably appropriate for investigating the importance of certain variables, but not for estimating classification error rates. If the control variables, such as size, industry, regulatory statutes, etc., are not independent of the other variables, the dispersions and group means in the sample will probably differ from a random sample drawn from the population. When this is the case the resulting classification rules would be different, and even use of the appropriate priors would not yield valid estimates of the population error rates. An alternative method for such a study would be to draw random samples from both groups and to include the control variables along with the other variables.

For two-group cases the linear form of the optimal classification rule incorporates misclassification costs into the model. Little or no attempt has been made to incorporate costs into the models that have been developed to date. Sinkey [1975] discussed the development of rules that would maximize the returns from applying his model. However, the discussion was general in nature, and he did not attempt to explicitly specify what models he proposed nor did he include it in the empirical section of his paper. General procedures for incorporating costs of misclassification into the classification procedure have been developed, but they have not yet been explored in the applied literature, probably due to the difficulty in determining the misclassificiation costs of Type I and Type II errors.

<u>2.5.2.7 Classification error rates</u>. The reclassification of the original sample used to construct the classification rule as a means of estimating expected

error rates leads to a biased and overly optimistic prediction of how well the rules would perform in the population. A number of alternative methods to estimate classification errors have been suggested and evaluated. Eisenbeis [1977] notes that the alternatives are basically three types, 1) those using samples to estimate error rates, 2) those using the assumption of normality, and 3) those using jack-knife procedures. The Lachenbruch or leaving-one-out method comes under the general category of a jack-knife method where the discriminant rule is estimated omitting one sample observation and then using the rule to classify the sample omitted. This is done for all observations in the sample and the number of mis-classifications are counted. This gives an almost unbiased estimate of the expected actual error rate.

Another suggested method is to divide the sample into two parts, using one part to construct the classification rule, and the other part to evaluate the rule. This method requires large samples that often are not readily available. Lachenbruch and Mickey [1968] compared a number of methods for estimating error rates and concluded that for moderately large samples, the apparent error rate method may be used, but for small samples, the leaving-one-out method seemed to be preferable. The apparent error rate is defined as the fraction of observations in the initial sample which are misclassified by the sample discriminant function. These methods do not estimate the actual error rate, but rather they estimate the expected actual error rate.

Regardless of the method employed, Joy and Tollefson [1975] point out there are three measures of classification efficiency. Total efficiency shows how well all the observations were classified. Condition efficiency measures pertain to classification efficiency for Type I and Type II errors. Total efficiency is measured by the sum of the correct classifications divided by the total number of observations being classified. Joy and Tollefson raised the question as to the goodness of the observed total classification efficiency. They propose that classification efficiency be compared to some standard of comparison, such as a chance classification scheme. The porportional chance model assigns observations randomly to groups with probabilities equal to group frequency. Joy and Tollefson point out that the expected correct classifications under this scheme for the Altman [1968] study is 60%, while the discriminant function developed by Altman was able to correctly classify 84%, which is significantly better than chance at the .001 level.

There may be research situations in which the analysis of the individual group classification results is warranted.

The researcher may ask the question: given that a company is bankrupt, what is the probability of classifying it as bankrupt? This probability is estimated by dividing the number of correct bankrupt classifications by the total number of bankrupt observations. Joy and Tollefson [1975], using the proportional chance mode, calculated for the Altman study [1968] that the chance probabilities were 28% for correctly predicting bankrupts and 73% for correctly predicting non-bankrupts. The Altman related results are 96% correct bankrupts predicted and 79% correct non-bankrupts predicted. The bankrupts, when compared to the proportional chance model, were classified significantly better than chance at the .001 level, but the non-bankrupt group is not classified significantly better than chance at the .05 level.

2.5.2.8 Validation and prediction. Joy and Tollefson [1975] considered this final problem, and the following analysis is taken from their discussion. Consider two samples, A and B, of size n_a and n_b respectively. The variables that are being used to discriminate will have a time dimension that precedes the classification time dimension. If the ratios are measured for year t, the failure status that is to be predicted is for year t+1. For sample A the variables used are for period t and the predictions are to be made in period t+2. Then split sample A into two parts, A1 and A2 of size n_{a1} and n_{a2}, respectively. Consider sample A1 as the analysis sample, sample A2 as the cross-validation sample, and sample B as the inter-temporal validation sample.

The first step is to fit a discriminant function over sample A1, the analysis sample. Joy and Tollefson point out that not too much importance can be attributed to the results from this first step. The output from two packaged computer programs typically used in financial ratio studies (SPSS [1975] and BMDP [1975]), to estimate the coefficients of the discriminant function obtained from sample A1 normally include the Mahalanobis D^2 which can be used to test the hypothesis that the groups have identical vectors of mean variable values. In addition, these programs normally report the classification results using the function developed from the analysis sample to classify the firms in the analysis sample. Most studies report these results although they do not contain a great deal of information about the discriminating power of the function derived.

The next step is to classify the members of the cross-validation sample A2 using the discriminant function derived from the analysis sample A1. Joy and Tollefson refer to this step as ex-post discrimination rather than prediction, because the cross-validation sample A2 is time-coincident with the analysis sample A1. Successful cross-validation discrimination implies that the researcher's inferences about the importance of the variables in the function is warranted. It is a necessary first step before stating that there is explanatory importance in any of the independent variables. If the function derived from the analysis sample is not successful in classifying members of the cross-validation sample, then no valid inference can be made about the explanatory power of the independent variables.

If the cross-validation classification has been successful, the explanatory power of the variables should be investigated using both the analysis sample and the cross-validation sample recombined into a new discriminant function fitted to the combined sample. The typical approach in many financial ratio studies has been to investigate the importance of the discrimiant coefficients developed by the analysis sample, while the procedures mentioned in Section 2.5.2.5 on the significance of individual variables should only be used in reference to the coefficients developed from the combined analysis and cross-validation sample.

Joy and Tollefson then go on to inter-temporal validation, which involves the ex-ante predictive power of the derived function. They state that ex-ante prediction means using the function derived from the combined analysis and cross-validation sample in an earlier time period to classify observations from sample B from a later time period. A successful predictive model will classify sample B observations significantly better than some alternative chance classification scheme. These alternative classification schemes were discussed in Section 2.5.2.7 dealing with the estimation of classification error rates.

The problem then comes down to: Can a discriminant function derived from sampling time period t, whose classification efficiency has been evaluated using a sample from the same time period, be said to have predictive ability? It would seem that if the researcher wants to state that his model is capable of predicting, then the function derived in time period t should be used to predict group membership of observations drawn from time period t+1. Scott [1978] stated that while the Joy and Tollefson [1975] arguments about inter-temporal validation are valid, the Lachenbruch method gives an almost unbiased and almost efficient estimate of error rates based on all available information. A sufficient statistic utilizes all available data, while the split sample inter-temporal method advocated by Joy and Tollefson used only one-half of the data to generate the statistic.

Moyer [1977] noted that it was important to re-examine some of the critical aspects of the Altman [1968] model. Although Altman tested his model's explanatory power on a cross-validation sample of firms, the data for the sample was drawn from the same years used in the original fit. Moyer used a paired-sample of 27 bankrupt and 27 nonbankrupt firms selected from the 1965-1975 time period. The Altman sample was drawn from the 1946-1965 time period. The asset size of the sample firms used in the Moyer study ranged from $15 million to $1 billion, as compared to the original Altman sample range of up to $30 million. He collected data for each firm in a pair from Moody's for the three years prior to the bankrupt firm's failure. Altman's discriminant function for one year prior to bankruptcy was used to classify 48 of the 54 firms in the Moyer sample for which complete data was available. The original Altman model parameters applied to a data set of larger firms from a different time period had an overall classification success rate of 75%. Type I errors were 39% and Type II errors (classifying nonbankrupt firms as bankrupt), were 12%. On the basis of his replication, Moyer concluded that the original Altman model parameters were sensitive to either the time period used to develop the model or the firm sizes which were represented in the original samples, or both. He warned that practitioners should be aware that the high overall success rate of 96% correct classifications reported for the hold-out sample by Altman in his 1968 paper is considerably higher than the prediction success achieved when the model is applied to a post-1965 sample of larger firms.

2.5.2.9 Summary. Joy and Tollefon [1975] pointed out that the major contention of their paper was that discriminant analysis was being uncritically applied. They felt that future research that employs discriminant analysis should profit from attention to the points raised 1) in their 1975 paper, 2) by subsequent 1978 comments by Altman and Eisenbeis [1978], 3) in a short note dealing with the same problems published by Scott in 1978 [1978], 4) in the paper published by Eisenbeis in 1977 [1977], and 5) by the comment published by Pinches in 1978 [1978]. This study is designed to profit from attention to the points that have been raised in the literature as Joy and Tollefson pointed out. A methodology has been developed for the general application of discriminant analysis and classification to studies using financial ratios as variables. In addition, this paper presents a practical model that can be used when the variable set consists of both binary and continuous variables. The need for an appropriate methodology is clear, as is the need for a model incorporating the powerful information contained in qualitative variables such as subordination which may be dichotomized and used along with the information in the continuous financial ratio variables which have been used exclusively so far in most financial ratio studies (see Alves [1978] for some non-ratio variable models).

2.6 The Location Model

Many scientific experiments have involved the use of multiple measurements on each of many observations. Originally, a separate analysis of each variable was made, but this led to problems of interpretation and inference obtained by the relationships which in theory existed among the measurements for each observation. This problem led to the development of multivariate analysis, where the variables are treated simultaneously. With the development and availability of high-speed electronic computers, these multivariate methods now have attained widespread popularity in applied research. Detailed accounts of these techniques may be found in books such as Anderson [1958] and Morrison [1967].

One assumption underlying many of these multivariate techniques is that the measurements are continuous and come from a population that is distributed multivariate normal.

In reality, in many practical situations this is not the case. The most common departure from this assumption is when a variable takes on only a finite number of discrete values. Such a variable may measure, for example, the presence or absence of a trait or attribute, or it may possibly indicate which one of 2 possible groups the observation belongs to, such as bankrupt or nonbankrupt. If q binary variables are observed, then the vector of values obtained is one of 2q possible outcomes. The sample is usually represented in the form of a contingency table containing 2q cells, each corresponding to one of the possible outcomes.

The analysis of such contingency tables has tended to form a distinct branch of statistical methodology. As a result, a body of techniques exists for analyzing multivariate continuous normal data and another for analyzing multivariate binary data. Many experiments, particularly in the field of psychology, for instance, give rise to data in which some of the variables are continuous while others are binary. The initial research approach to these mixed variables centered on the derivation of coefficients of association among variables. When it is assumed that there exists an underlying continuous multivariate distribution, in which one or more of the variables can be observed only in dichotomized form, the resulting model may be called a dichotomized model.

A great deal of research has been done concerned with the estimation of the points of dichotomy of these underlying continuous multivariate distributions, and estimations of the correlations among these variables. Tate [1955] considered these types of problems for the bivariate case, where one variable is continuous and the other is dichotomized. The underlying assumption there was that both variables had multivariate normal distributions, but that one or more could only be measured in the form of a dichotomized variable. Hannan and Tate [1965] extended this work to the multivariate case with a number of continuous variables and just one variable dichotomized. They considered maximum likelihood estimation, and compared results obtained using such estimators with those from the traditional ones.

An alternative approach to the problem of dealing with mixed variables is to assume certain distributional forms for one set of variables conditional on the other set, together with a marginal distribution for that other set. The location model assumes that the continuous variables have a multivariate normal distribution whose mean depends on the values of the binary variables, but whose dispersion matrix is constant. For the bivariate case of one binary and one continuous variable, this model underlies the idea of point-biserial correlation. Tate [1955] compared the biserial and point-biserial models and discussed the assumptions underlying both of them, and commented on the experimental situations in which one or the other of these coefficients should be used.

So, in general, then, these two approaches have been developed: 1) a dichotomized model based on the concepts of biserial correlation, and 2) the location model based on the concepts underlying point-biserial correlation.

The multivariate form of the point-biserial location model has been studied mainly in conjunction with hypothesis testing. The problem is to test the null hypothesis that the parameters of the two variables are equal in two different populations. Olkin and Tate [1961] studied the canonical correlations between the discrete and continuous variable sets, and investigated these relationships with the conditional means of the continuous variables. They developed hypotheses tests on these coefficients, with emphasis on the null that the coefficient is zero, which corresponds to the hypothesis of equality of conditional means. Afifi and Elashoff [1969] studied the use of Hotelling's T^2 for testing the null hypotheses for the two-group case that the parameters of both the continuous and discrete variables were equal in both groups. They also derived a likelihood ratio test for this location hypothesis and obtained the exact distributions of the test statistic.

Chang and Afifi [1974] used the concepts of point-biserial correlation and the location model to develop a classification scheme based on the special case of one binary and a number of continuous variables. They developed classification rules when the population parameters were known, and unknown. In the case of unknown parameters, when both cells had adequate observations, they showed how maximum likelihood estimates of the parameters are derived, and how they could be used to obtain a sample discriminant function, in either the linear or the quadratic form.

Krzanowski [1975] recognized that in practice, unless the sample sizes were large relative to the number of binary variables, it would be very probable that some of the cells in the contingency table would be either very small or zero. The contribution made by Krzanowski over the work of Chang and Afifi was the development of an approximation which would yield parameter estimates for all possible cells in the contingency matrix. When the parameters can be estimated for all cells, the appropriate classification rule can be constructed by replacing all parameters by their estimates. The work by Krzanowski is the basis for this study. The use of discriminant analysis has been widely applied in financial ratio literature and has been shown to be a very valuable procedure. However, many users of this technique have restricted its application to only continuous variables, while others have applied it in error to all types of variables, including dichotomous ones. This study presents the application of a model based on continuous and binary variables taken together, when the binary variables are not dichotomizations of continuous variables. Krzanowski found when he compared the location model to the logistic model, for instance, that the parameter estimation needed to develop the classification rules using the logistic model required iterative methods which were liable to fluctuate in accuracy and even break down in certain circumstances. Accordingly, he decided to study the location model, which he felt would provide a more tractable solution to the problem of discriminating between two groups.

This study presents the location model in Chapter 3 and then presents the methodology of the application of the location model in empirical research when the data set consists of both binary and continuous variables in Chapter 4. Chapter 5 presents the results, and conclusions are presented in Chapter 6.

CHAPTER III

THE LOCATION MODEL

3.1 Classification

The aim of this paper is to present procedures for the application of discriminant analysis and classification when the data are composed of both continuous and binary variables. To provide a framework for these ideas the following briefly sketches the theory of discriminant functions.

The classification problem arises when a researcher wants to assign an individual observation to one of two groups on the basis of a number of measurements made on that individual. In this paper it is assumed that the number of possible groups is two, although there are no great conceptual difficulties introduced by relaxing this assumption. Denote the two groups by π_1 and π_2. In following a given classification procedure, two kinds of errors may occur. An individual from π_1 may be incorrectly classified to π_2, or vice versa. Denote the cost of the misclassification of an indivudual from π_1 to π_2 as $c(2|1)$ and that of classifying an individual from π_2 to π_1 by $c(1|2)$. An optimal classification procedure is one which minimizes the total cost of misclassification.

Suppose a vector x of observations is available on each individual. Denote the probability density of x in π_1 and π_2 by $p_1(x)$ and $p_2(x)$, respectively, and suppose that prior information is available in the form of probabilities that an observation will come from one or the other of the two groups. Denote the probabilities that an observation comes from π_1, π_2 as q_1, q_2, respectively.

From an historical point of view the common approach to classification utilizes Fisher's rule which finds the linear combination that maximizes the between-to-within variances. Lachenbruch [1975] demonstrates that the coefficients used in the linear combination (discriminant function) are proportional to

(1) $$(\mu_1 - \mu_2)' \Sigma^{-1}$$

where μ_1 and μ_2 are the group means and Σ is the common variance/covariance matrix.

This expression is linear in all the variables that have been used to describe each observation, and is usually known as Fisher's LDF.

The problem now becomes one of choosing regions R_1 and R_2 to minimize the expected loss due to misclassifications. This can be achieved by classifying to R_1 the set of points for which $\frac{p_1(x)}{p_2(x)} \geq \frac{q_2 c(1|2)}{q_1 c(2|1)}$ and as R_2 the set of points for which $\frac{p_1(x)}{p_2(x)} < \frac{q_2 c(1|2)}{q_1 c(2|1)}$. If equal costs of misclassification and equal prior probabilities are assumed, then R_1 is given by the set of points satisfying $p_1(x)/p_2(x) \geq 1$, and R_2 by the remainder of the sample space. In this case region R_1 is composed of those points for which

$$(2) \qquad (\mu_1 - \mu_2)' \Sigma^{-1} x \geq \tfrac{1}{2}(\mu_1 - \mu_2)' \Sigma^{-1} (\mu_1 + \mu_2).$$

The right-hand side of this expression is merely the mid-point of the interval between the scores calculated for each of the two groups using the coefficients developed as set out above.

In most practical applications the population parameters are unknown, but they may be estimated from samples known to come from the two populations. An allocation or classification rule may then be derived by replacing all unknown parameters by their estimates. In the case of multivariate normal populations, estimates x_1, x_2 and S of μ_1, μ_2, and Σ respectively, may be used in place of μ_1, μ_2 and Σ in the classification rule above.

It should be noted that the difference between the two scores,

$$(3) \qquad \bar{Y}_1 - \bar{Y}_2 = (\bar{x}_1 - \bar{x}_2)' S^{-1} \bar{x}_1 - (\bar{x}_1 - \bar{x}_2)' S^{-1} x_2$$

$$= (\bar{x}_1 - \bar{x}_2)' S^{-1} (\bar{x}_1 - \bar{x}_2)$$

where $\overline{Y}_1$ and $\overline{Y}_2$ are the scores using the coefficients from (1), is the Mahalnobis D^2. The distribution of D^2 can be used to test if there are significant differences between the two groups. The variable

$$F = \frac{n_1 n_2 (n_1+n_2-k-1)}{n_1+n_2 \ (n_1+n_2-2)k} \quad D^2 \tag{4}$$

where n_1 and n_2 are the sample sizes from π_1 and π_2, respectively, and k is the number of variables, has an F distribution. This method is distribution free in the sense that it is a reasonable criterion for constructing a linear combination. However, it is only optimal in the sense of minimizing the probability of misclassification if the observations are <u>multivariate</u> <u>normal</u> with equal dispersion matrices.

There is a parallel between the linear discriminant function in the two group case and the multiple linear regression of predictor variables on a dummy-variable indicator of group membership. Tatsuoka [1971] demonstrates that the coefficients determined for the two group discriminant analysis case might be considered as some sort of multiple regression weights, provided that the elements of the column of mean differences on the predictor variables are interpretable as sums-of-products between predictors and criterion, when the criterion variable is a dichotomous one taking the values 1 and 0 for members of group 1 and group 2 respectively. The elements of that column vector of mean differences in discriminant analysis are proportional to the predictor-criterion sum-of-products. While Tatsuoka concludes that in the two group case the discriminant weights are proportional to the regression weights, he emphasizes that this reduction holds only in the two group case, for when there are more than two groups discriminant analysis reduces not to multiple regression but to <u>canonical correlation</u> analysis.

3.2 Theory of the Location Model

The problem considered by the model is that of classifying an individual observation vector $w = (x,y)$, to one of two populations, π_1 and π_2 where x is the vector of binary variables and y is the vector of continuous variables. Assuming p continuous variables and q binary variables means that x has q components, y has p components, and w has $(p + q)$ components.

The q binary variables may be expressed as multinomial $z = (z_1, z_2 \ldots, z_k)$ where $k = 2^q$, i.e., as a contingency table having 2^q cells. Each distinct pattern of the binary variables defines a cell of this table uniquely, with $x = (x_1, x_2, \ldots, x_q)$ falling in cell C, where $C = 1 + \sum_{i=1}^{q} x_i \cdot 2^{(i-1)}$. This is a device for numbering the cells so that they can be readily identified in later analysis stages. Then, following Olkin and Tate[1961], it is assumed, as they do in their discussion of point-biserial correlation, that y has a multivariate normal distribution with mean $\mu_i^{(m)}$ in cell m and population π_i (m = 1, ..., k; i = 1,2), with common dispersion matrix Σ in all cells over both populations. In other words,

(5) $(y|z_m$ is distributed $N(\mu_i^{(m)}, \Sigma)$ in π_i, m = 1,2,..,k and i=1,2)

In addition, the probability of obtaining an observation in cell i is p_{1i} for π_1 and p_{2i} for π_2(i = 1,2,...,k).

An optimum classification rule in the case of known population parameters is derived in the next section, using likelihood ratio theory, and the probabilities of misclassification from each of the populations are also presented. Following that is the procedure for estimating the classification rule when samples are available from π_1 and π_2, as well as the problem of estimating the error rates.

3.3 Known Population Parameters

When the population parameters are known, a vector of observations is classified based on the ratio of the joint density of the continuous and binary variables. In other words, $w = (x,y)$ is classified as belonging to π_1 if the ratio of $p_1(x,y)$ to $p_2(x,y)$ is $\geq k$ (for a suitable k), and otherwise to π_2, where $p_i(x,y)$ is the joint density of x and y in π_i, $(i = 1,2)$.

(6) Now, $p_i(x,y) = p_i(x)p_i(y|x)$ $(i = 1,2)$.

Conditioning first on x, assume that the observation falls in cell m, so

(7) $p_i(x,y) = p_i(z_m,y) = p_i z_m \cdot p_i(y|z_m)$.

Recall that $p_i\ (y|z_m)$ is $\sim N(\mu_i^{(m)}, \Sigma)$ in π_i $(i=1,2)$, from (5) so $p_i(y|z_m)$ is

(8) $$2(\pi)^{-\frac{1}{2}p}|\Sigma|^{-\frac{1}{2}}\exp -\tfrac{1}{2}(y-\mu_i^{(m)})'\Sigma^{-1}(y-\mu_i^{(m)}).$$

Now, the ratio of the joint densities is

(9) $$\left\{\frac{p_1(z_m,y)}{p_2(z_m,y)}\right\} = \frac{\exp[-\frac{1}{2}(y-\mu_1^{(m)})'\Sigma^{-1}(y-\mu_1^{(m)})}{\exp[-\frac{1}{2}(y-\mu_2)'\Sigma^{-1}(y-\mu_2)}$$

$$= \exp\{-\tfrac{1}{2}[(y-\mu_1^{(m)})'\Sigma^{-1}(y-\mu_1^{(m)})-(y-\mu_2^{(m)})'\Sigma^{-1}(y-\mu_2^{(m)})]\}.$$

Since the logarithmic function is monotonic increasing, the inequality $\geq k$ can be written in terms of the logarithm of (9) as

(10) $$-\tfrac{1}{2}[(y-\mu_1^{(m)})'\Sigma^{-1}(y-\mu_1^{(m)})-(y-\mu_2^{(m)})'\Sigma^{-1}(y-\mu_2^{(m)})] \geq \log k$$ and

the left hand side of (10) can be expanded and rearranged to obtain

$$(11) \qquad (\mu_1^{(m)} - \mu_2^{(m)})' \Sigma^{-1}\{y - \tfrac{1}{2}(\mu_1^{(m)} + \mu_2^{(m)})\}.$$

In other words, assign to π_1 if

$$(12) \qquad (\mu_1^{(m)} - \mu_2^{(m)})' \Sigma^{-1}\{y-\tfrac{1}{2}(\mu_1^{(m)} + \mu_2^{(m)})\} \geq \log k$$

and to π_2 otherwise.

The probabilities of misclassification from π_1 and π_2 can be obtained by using the Mahalonobis D^2 where

$$(13) \qquad D_m^2 = (\mu_1^{(m)} - \mu_2^{(m)})' \Sigma^{-1}(\mu_1^{(m)} - \mu_2^{(m)})$$

is the distance between populations π_1 and π_2 conditional on the observation being in cell m.

The conditional probabilities of misclassification in cell m, that an observation from π_1 is classified as belonging to π_2 or an observation from π_2 is classified as belonging to π_1, are $p(2|1,z_m)$ and $p(1|2,z_m)$ respectively.

$$(14) \qquad p(2|1,z_m) = \{(\log(p_{2m}/p_{1m})-\tfrac{1}{2}D_m^2)/D_m\}$$

$$(15) \qquad p(1|2,z_m) = \{(\log(p_{1m}/p_{2m})-\tfrac{1}{2}D_m^2)/D_m\}$$

and the unconditional probabilities are

$$(16) \qquad p(2|1) = \sum_{m=1}^{k} p_{1m} \; \Phi\{(\log p_{2m}/p_{1m})-\tfrac{1}{2}D_m^2)/D_m\}$$

$$(17) \qquad p(1|2) = \sum_{m=1}^{k} p_{2m} \; \Phi\{(\log p_{1m}/p_{2m})-\tfrac{1}{2}D_m^2/D_m\}.$$

3.4 The Estimated Classification Rule

The optimum rule for classifying an observation which was given in (12) along with the probabilities of misclassification assumes that the parameters of the two populations are known. Unfortunately, in most realistic settings, population parameters are rarely known and samples of size n_1 and n_2 must be taken. In these cases, the rule for classifying the observation vector to one of the two populations is obtained by replacing all population parameters in the optimum rule by their sample estimates.

Any vector $w = (x,y)$ to be classified has known values of $(x_1,x_2,\ldots,x_q)$ and so its unique position in the multinomial table is fixed. The estimation of the parameters $\mu_1^{(m)}$, $\mu_2^{(m)}$, Σ, and P_{1m} and P_{2m} is the problem to be resolved if the parameters are not known. If all the cells of the multinomial table formed by the samples contained sufficient observations, maximum likelihood estimates of the population parameters could be calculated. Let n_{1m}, n_{2m} be the number of observations in cell m of π_1 and π_2, respectively, and denote by $y_{ij}^{(m)}$ and $\bar{y}_i^{(m)}$ the jth observed vector of continuous variables in cell m of π_i, and the mean continuous vector in cell m of π_i $(i = 1,2;\ m = 1,2\ldots,k;\ j = 1,2\ldots,n_{im})$. Maximum likelihood estimates of the population parameters are given as follows:

$$(18)\quad \hat{p}_{im} = \frac{n_{im}}{n_1} \quad (i = 1,2; m = 1,2,\ldots,k)$$

$$(19)\quad \hat{\mu}_i^{(m)} = \bar{y}_i^{(m)} \quad (i = 1,2; m = 1,2,\ldots,k) \text{ where}$$

$$(20)\quad \bar{y}_i^{(m)} = (1/n_{im}) \sum_{j=1}^{n_{im}} y_{ij}^{(m)}$$

$$(21)\quad \hat{\Sigma} = \frac{1}{(n_1+n_2-2k)} \sum_{i=1}^{2} \sum_{m=1}^{k} \sum_{j=1}^{n_{im}} (y_{ij}^{(m)} - \bar{y}_i^{(m)})(y_{ij}^{(m)} - \bar{y}_i^{(m)})'$$

$$(i=1,2; m=1,2,\ldots,k)$$

When these maximum likelihood estimates have been calculated, they are replaced for the population parameters in the classification rule. So for an observation falling in cell m, the classification rule is the same as the optimum one, but with the cost of misclassification from π_i inversely proportional to the probability of an observation falling in cell m of π_i $(i = 1,2)$. In practice, some of the multinomial cells formed by the samples contain very few observations, or none at all. The method of maximum likelihood estimation of parameters presented above will not provide reliable estimates for some of these parameters. Even if there are no empty cells, some of the cells may have extremely few observations, and then the estimates obtained may not be very reliable. This type of situation becomes very likely as q, the number of binary variables, increases, and may be expected to arise frequently in practice with q even as low as 2. Therefore, maximum likelihood estimation of parameters does not provide a complete solution to the problem and what is needed is a general method of estimation which will allow for some empty cells in the observed multinomial table. A further structuring of the model is therefore necessary.

Estimation of the parameters needed to form the classification rule may be treated in two parts, with the parameters p_{im} relating to the binary variables being estimated separately from the remaining continuous variable parameters. The procedure for each part is to find a reasonable model for the parameters under consideration, where on fitting the full model all of the initial data is recovered, and then approximating it by omitting enough terms from the full model in order to be sure that all cells contain non-zero estimates. The binary variables will be considered first.

In the log-linear model, the logarithms of the expected values of the frequencies in each cell are assumed to satisfy an additive model of main effects and all interactions up to order q. The logarithm of the expected value in the mth cell is expressed as the sum of a number of constants corresponding to the main effects of each of the binary variables taking the value 1 in that cell, together with constants representing interactions of all orders up to q among the variables in the cell. This is called the saturated model, since the expected value in any cell after fitting this model equals the observed value in that cell, What is needed is a reduced model where only terms corresponding to main effects and first order interactions are retained. The use of a reduced model will lead to non-zero expected frequencies in all cells in most practical situations. If the contingency tables are extremely sparse and zero estimates are still obtained, further reduction in the model may be made and only main effects fitted.

Maximum likelihood estimation of the parameters in this reduced model can be effected by a simple iterative procedure, which has been called iterative proportional scaling. If for example only main effects and first order interactions are to be fitted, all possible two-way tables which can be formed from the observed table are regarded as fixed. Starting with a trial solution, frequencies in the table are adjusted by scaling so that each fixed margin in turn contains the correct values. This procedure is repeated until the values in the margins of the fitted table agree with those in the observed table to within a preset level of accuracy. An algorithm for performing this iterative scaling procedure has been given by Haberman [1972] and is the one used in Chapter 4 of this study.

The first part of the problem, that is, the estimation of the expected frequencies p_{im} $(i = 1,2; m = 1, \ldots, k)$, has been solved as follows. The data in the initial samples is used to form two tables for frequency of occurrence of each cell of the multinomial tables in π_1 and π_2. Then, two tables of expected values are constructed from the two observed tables by using Haberman's algorithm. If the expected value in cell m of the table from π_i is denoted by $a_{im}(i = 1,2; m = 1,\ldots,k)$, then estimates of p_{1m} and p_{2m} are given by $a_{1m} | n_1$ and $a_{2m} | n_2$ respectively.

The problem remaining is the estimation of the rest of the parameters needed to form the allocation rule, which all relate to the continuous variables. The simplest structure which can be imposed on the conditional mean vectors of the continuous variables is a linear additive model with components representing the main effects of each binary variable and interactions among the binary variables of all orders up to q. The mean continuous vector μ in cell m of population π_i may be written

(22) $$\hat{\mu}_i^{(m)} = v_i + \sum_{j=1} \alpha_{j,i} x_j^{(m)} + \sum_{j<k}\sum \beta_{jk,i} x_j^{(m)} x_k^{(m)} + \sum_{j<k<b}\sum\sum \gamma_{jkb,i} x_j^{(m)} x_k^{(m)} x_b^{(m)} \ldots + \xi_{1\,2\ldots q} x_1^{(m)} x_2^{(m)} \ldots x_q^{(m)}$$

where $x_j^{(m)}$ is the value of the jth binary variable in cell m $(i = 1,2; j = 1,2\ldots,q; m = 1,2,\ldots,k)$.

This is compatible with the structure imposed on the expected values of the table cells. For consistency with the estimation of the expected frequencies where the main effects and the first order interactions were fitted to the binary variables, only the terms v_i, $\alpha_{j,i}$ and $\beta_{jk,i}$ are retained. If only main effects were fitted to the binary variables, then β_{jk} would also be ignored.

Maximum likelihood estimation of v, α, and β is done using multivariate regression (see Anderson [1958]. Each observation is denoted by

(23) $$w_{\alpha i} = (x_{\alpha i}, y_{\alpha i})(\alpha = 1,\ldots,n_i \text{ for } i = 1,2),$$

and write

(24) $$v'_{\alpha i} = (1, x_{1\alpha i}, x_{2\alpha i}, \ldots x_{q\alpha i}, x_{1\alpha i}x_{2\alpha i}, x_{1\alpha i}\, x_{3\alpha i}, \ldots x_{1\alpha i}x_{q\alpha i}, \ldots, x_{q-1,\alpha i}x_{q\alpha i})$$

(25) $$B_i = (v_i, \alpha_{1i}, \alpha_{2i}, \ldots, \alpha_{qi}, \beta_{12i}, \beta_{13i}, \ldots, \beta_{1qi}, \ldots \beta_{q-1,qi})$$

with μ_i equal to $B_i \, v_i$.

Following Anderson [1958], estimates $\hat{B}_i$ of B_i $(i = 1,2)$ are given by

$$(26) \quad \hat{B}_i = C_i \, A_i^{-1}, \quad \text{where}$$

$$(27) \quad C_i = \sum_{\alpha=1}^{n_i} y_{\alpha i} \, v'_{\alpha i}$$

and

$$(28) \quad A_i = \sum_{\alpha=1}^{n_i} v_{\alpha i} \, v'_{\alpha i}.$$

The final estimate needed, assuming that the dispersion matrices are the same for both populations, is (see Anderson [1958]) obtained by pooling, i.e.,

$$(29) \quad (n_1+n_2)\hat{\Sigma} = \sum_{\alpha=1}^{n_1} y_{\alpha 1} \, y'_{\alpha 1} + \sum_{\alpha=1}^{n_2} y_{\alpha 2} \, y'_{\alpha 2} - \hat{B}_1 \, A_1 \, \hat{B}'_1 - \hat{B}_2 - A_2 - \hat{B}'_2.$$

So estimates of all the parameters needed for the classification rule are available under the restricted model and this rule is constructed by replacing all parameters by their estimates. The entire procedure may be summarized as follows. Data from the original samples are used to form two incidence tables of observed frequencies. Haberman's algorithm is then applied to form two tables of expected frequencies. The next stage is multivariate regression using the appropriate constant s as indicated above, and the estimation of the mean and variance/covariance matrix. Having estimated all the parameters, the data to be classified is considered and the allocation of each observation is made on the basis of the classification rule presented earlier.

3.5 Estimation of Error Rates

The value of a classification rule can be assessed by the probabilities of misclassification. In the case of the location model with known parameter values, these probabilities have already been given in section 3.3, and they represent the optimum error rates in that situation. When the parameter values are unknown and the estimation procedure of section 3.4 is followed, a method of estimating the error rates from this procedure is needed in order to assess how useful it will be in practice. Estimation of error rates has been considered by Lachenbruch [1967] and Lachenbruch and Mickey [1968]. They examined some of the more common methods of estimating error rates in current use and highlighted their strengths and deficiencies. The two most common methods seen in empirical financial ratio research are the cross-validation sample method as described by Joy and Tollefson [1975] and the analysis sample method. The cross-validation sample method requires splitting the initial sample from the two populations randomly into two parts. One part is used to compute the discriminant function, while the other is used to estimate the error rate. Following the method of Joy and Tollefson presented in Chapter 2 [1975], the discriminant function is then recomputed using the entire combined sample. Lachenbruch and Mickey [1968] point out three drawbacks to this method:

1) Large samples are required, which may not be available in practical situations.

2) The discriminant function that is evaluated is not the one which is used in practice.

3) If the cross-validation sample is large, a good estimate of the performance of the discriminant function is obtained but the function itself is poor, and vice versa when the cross-validation sample is small. Also, the method is very uneconomical with data.

The analysis sample method does not split the initial samples, but uses the same data to compute the discriminant function and evaluate its performance. Lachenbruch and Mickey [1968] find this method to be misleading, yielding an estimate which can be badly biased. The assessment of classification performance is generally too high, particularly if either of the samples used is small, as they are many times in financial ratio research.

Lachenbruch and Mickey [1968] have developed an empirical method of estimating the error rates which uses all the data but removes some of the objections to the two methods presented above. It is usually refered to as the leaving-one-out method. To begin with, one observation is omitted from the data, and the classification rule is then computed from the remaining observations and then the omitted observation is classified by this rule. This is repeated for each item in the entire sample, omitting each observation once and classifying it by means of the rule computed from the remaining observations. Lachenbruch [1967] demonstrated that this method was unbiased for samples of total size $n_1 + n_2-1$ on any type of data, with generally superior performance relative to other empirical methods.

The primary problem in the application of Lachenbruch's leaving-one-out method to the location model is whether or not it is computationally feasible. The following discussion follows the lines of the arguments developed by Krzanowski [1974]. Considering the continuous variables first, assume that estimates $\hat{B}_1,\hat{B}_2,\hat{\Sigma}$ have been obtained as given in (26) and (29) from the entire initial samples of size n_1 from π_1 and n_2 from π_2. The question is, how will these estimates be altered when one of the observations is omitted from the intial sample? Suppose that the mth observation is omitted from the sample from π_1. Then $\hat{B}_2$ remains the same, while $\hat{B}_1$ and $\hat{\Sigma}$ will be changed.

Now, recall that

(30) $\hat{B}_1 = C_1 A_1^{-1}$ from (26)

and

(31) $(n_1 + n_2)\hat{\Sigma} = \sum_{\alpha=1}^{n_1} y_{\alpha 1} y'_{\alpha 1} + \sum_{\alpha=1}^{n_2} y_{\alpha 2} y'_{\alpha 2} - \hat{B}_1 A_1 \hat{B}'_1 - \hat{B}_2 A_2 \hat{B}'_2$

from (29) where

(32) $C_1 = \sum_{\alpha=1}^{n_1} y_{\alpha 1} v'_{\alpha 1}$ from (27)

and

(33) $A_1 = \sum_{\alpha=1}^{n_1} v_{\alpha 1} v'_{\alpha 1}$ from (28)

while $v'_{\alpha 1}$ is the vector of constants depending on the binary variables.

(34) Denote by $\tilde{C}_1$, $\tilde{A}_1$, $\tilde{B}_1$, and $\tilde{\Sigma}$ from the new matrices and estimates when $w_{m1} = (x_{m1}, y_{m1})$ is omitted from the initial sample. Then if v_{m1} is the vector of constants corresponding to x_{m1}, then

(35) $\tilde{C}_1 = C_1 - y_{m1} v'_{m1}$

and

(35) $\tilde{A}_1 = A_1 - v_{m1} v'_{m1}.$

Lachenbruch and Mickey [1968] used the following identity:

(37) If $B = A + u\ v$, then $B^{-1} = A^{-1} - \dfrac{A^{-1} u\ v' A^{-1}}{1 + v' A^{-1}}$

so

$$(38) \quad \tilde{A}_1^{-1} = A_1^{-1} + \frac{A_1^{-1} \, v_{m1} \, v_{m1}' \, A_1^{-1}}{1 - v_{m1}' \, A_1^{-1} \, v_{m1}} .$$

This can be written as

$$(39) \quad \tilde{A}_1^{-1} = A_1^{-1} + \frac{1}{k} s_{m1} \, s_{m1}'$$

$$(40) \quad \text{where } s_{m1} = A_1^{-1} \, v_{m1}$$

$$(41) \quad \text{and} \qquad k = 1 - s_{m1}' \, v_{m1}.$$

If there are samples of size n_1 and n_2 from π_1 and π_2, the quantities C_1, C_2, A_1^{-1}, A_2^{-1}, $\hat{B}_1, \hat{B}_2$, and $\hat{\Sigma}$ are calculated to obtain estimates of the continuous variable parameters for use in the classification rules. In order to update these estimates when the mth observation is omitted from the sample from π_1, the additional quantities

$$(42) \quad s_{m1} = A_1^{-1} \, v_{m1}$$

$$(43) \quad k = 1 - s_{m1}' \, v_{m1}$$

$$(44) \quad t_{m1} = \frac{1}{k} \{C_1 \, s_{m1} - y_{m1}\}$$

are calculated and

$$(45) \quad \tilde{B}_1 = \hat{B}_1 + t_{m1} \, s_{m1}'$$

along with

$$(46) \quad (n_1 + n_2 - 1)\tilde{\Sigma} = (n_1 + n_2)\hat{\Sigma} - k t_{m1} \, t_{m1}'$$

only require matrix addition and vector multiplication. Analogous expressions are used if the unit omitted is from π_2. For a detailed explanation of this procedure, see Krzanowski [1974].

The updated parameter estimates are found for the continuous variable parameters. The two parameters which remain are those corresponding to the multinomial cell frequencies, p_{1m} and p_{2m}, and since estimates of these are obtained by an iterative process, no convenient method of updating is available, so they must be re-estimated using the iterative procedure each time an observation is omitted. In conclusion, Lachenbruch's method for estimating the error rates is computationally feasible. The stages and the procedure can be summarized as follows:

1) Form incidence tables for the multinomial cells from the binary variables for the two samples. Use the iterative scaling procedure to determine the expected frequencies.

2) Form C_1, C_2, A_1^{-1}, A_2^{-1}, $\hat{B}_1$, $\hat{B}_2$, and $\hat{\Sigma}$ using complete samples.

3) For each cell in the multinomial table, use the iterative scaling procedure to compute the new expected frequency table when the frequency has been reduced by 1, and use this new table for subsequent units in the same cell. Then omitting each observation in the cell in turn, calculate $\tilde{B}_1$ and $\tilde{\Sigma}$, classify the omitted observation and note whether or not it is misclassified.

Krzanowski [1974] noted that two drawbacks had been encountered with the method in practice. Omitting an observation occasionally resulted in zero cell estimates and the need for a reduced order of fitting. In such circumstances the orginal estimates of p_{1m} and p_{2m} from the complete samples have been taken. Krzanowski noted that this would introduce some bias, but the circumstances were sufficiently rare in practice for this bias to be negligible. He also found that after omitting an observation in the estimate of A_1^{-1} that the inverse could not be found. He noted that this corresponded to a singular A matrix in the reduced sample, and stated that a lower order model was needed. In his work this occurred twice out of 456 observations analyzed and he classified these two units as unknown.

Chapter 4 presents the research methodology for this study.

CHAPTER IV

METHODOLOGY OF THIS STUDY

4.1 Introduction

The objectives of this study are twofold: 1) to review an appropriate methodology for financial ratio research using discriminant analysis and classification, and 2) to present a practical application of the location model to discriminant analysis and classification when the data is composed of both continuous and binary variables. This chapter presents the research methodology.

This study takes the data from a previous research study and uses it in the location model - the continuous variables used in the location model are financial ratios from the previously published study, and the binary variables in the location model have been added in order to determine if any difference will obtain when both binary and continuous variables are used in a discriminant analysis and classification procedure. Accordingly, Section 4.2 presents the research methodology of the previously published study, while Section 4.3 presents the research methodology for the present study. The selected study is "Financial Reporting and Business Liquidity" by Morton Backer and Martin L. Gosman (forthcoming). This selected study will be referred to as B and G for the remainder of this present study.

4.2 Selected study methodology

4.2.1 Purpose. The primary goal of the B and G study was to measure the debt capacity of firms. In the study, illiquidity was defined as the inability of a business to obtain additional funds from its traditional borrowing sources without having to incur abnormal interest rates. B and G wanted to determine the point or points at which a company's borrowing capacity becomes impaired, and to do this they needed criteria. Three time horizons and three forms of credit were selected and investigated in their study: 1) short-term, 2) intermediate term, and 3) long-term. In order to measure illiquidity in these three different credit situations, B and G chose the following surrogates: 1) Dun & Bradstreet trade credit rating for short-term, 2) substandard loan evaluation used by bank examiners for intermediate term, and 3) Standard and Poor's bond ratings for long-term. For each of the three illiquidity situations, they wanted to identify the relevant financial ratios indicative of the surrogate used. For purposes of this present study, only the short-term trade credit rating and the long-term bond ratings are to be considered, due to the lack of a control group of firms in the intermediate or bank loan area.

In their study, B & G were concerned with those financial ratios which were reflective of change, from 2 to 3 for trade credit and from BBB to BB (BB to B if subordinated) for bond ratings.

The purpose of the B and G study is not to be confused with predictions of bankruptcy, as many illiquid firms are able to overcome their financial difficulties through changes in management, cost control, improved marketing programs, etc. An awareness of illiquidity is essential to investors, creditors, and management. Apart from the potential of future bankruptcy, the illiquid firm is severely constrained in its ability to finance capital expansions and replacements, pay dividends, meet loan obligations, and generate sufficient cash to assure profitable operations.

B and G were seeking to determine whether they could depict the illiquid firm based on the financial ratio criteria used by Dun & Bradstreet and Standard and Poor's.

Bonds downgraded by Standard and Poor's from BBB to BB (BB to B if subordinated) become non-investment grade, and accordingly many major prior sources of capital are cut off in the future. Firms rated below 2 by Dun & Bradstreet often must make many efforts to improve their profitability and liquidity since a continued deterioration of their trade credit rating can lead to severe financial difficulties.

The present study will use the trade credit and bond rating sections of the B and G study as the vehicle for the demonstration of the use of the location model and the development of an appropriate methodology for financial ratio research.

4.2.2 Bond ratings design. Most prior studies dealing with bond ratings, except for the one by Horrigan [1966], focused on bond ratings assigned by Moody's. Horrigan's study focused on both bond rating agencies and found absolutely no difference in the ability of his model to gauge the ratings of both organizations. That is probably the reason why subsequent studies used only the ratings assigned by Moody's. The B and G study used the ratings of Standard and Poor's for the following reasons:

1. Horrigan's research lent strong support to the idea that no real bias was likely to be introduced by a choice of one or the other of the two rating agencies.

2. Financial writers in the past have attributed to Standard and Poor's certain fairly rigid relationships between the levels of key financial ratios and the resulting bond rating.
   The authors wanted to test whether the actual data would confirm the existence of such rigid relationships.

The 18 downgraded firms that comprised the bond experimental group were matched to 18 control group firms by their SIC (Standard Industrial Classification) numbers, and potential bias due to industry differences were therefore greatly minimized.

Pinches and Mingo [1973] made an effort to incorporate the subordination status of a bond into their model. They did this by using a 0-1 variable, with 0 representing subordinated ratings. Two years later, Pinches and Mingo [1975] changed this approach by trying to 1) ignore the subordination status completely, and 2) specify two models, one for subordinated bonds and another for non-subordinated bonds. B and G overcame the subordination problem by making certain that any subordinated bond in their sample carried a rating one notch lower than any non-subordinated bond. The emphasis in the B and G study was on rating changes; they compared the ratio levels of the 18 experimental firms downgraded by Standard and Poor's from BBB to BB (BB to B if subordinated) with the ratio levels of a matched sample of 18 firms which Standard and Poor's chose to maintain at BBB (BB if subordinated).

The 18 experimental firms whose bonds were downgraded were selected from the monthly issues of Bond Guide published by Standard and Poor's from January 1972 through December 1976. After downgrades of bonds of non-manufacturing firms were eliminated, 23 downgraded firms remained. An examination of the Bond Guide revealed that approximately 150 manufacturing company bonds had been rated BBB at some point during the 1972-1976 period. These control group candidates were matched to the experimental firms on the basis of the first two digits of their respective SIC numbers. After the potential matches were arrived at on the basis of SIC numbers, B and G checked to insure that each control group firm had carried a consistent Standard and Poor rating of BBB for the entire four-year period ranging from three years prior to its matched firm's downgrade to one year following such downgrade. This selection process resulted in the 18 matched firms in the control group sample.

Nineteen ratios were selected for study in the bond rating section, and are listed in Table 4-1. These

TABLE 4-1

Ratios Examined in Statistical Studies

	Trade Credit	Bonds
OPERATING RATIOS:		
Return on Sales	x	x
Return on Tangible Net Worth	x	x
Percentage Sales Change	x	x
Percentage Profit Change	x	x
Return on Working Capital	x	
Net Sales to Tangible Net Worth	x	
Net Sales to Working Capital	x	
Fixed Assets to Tangible Net Worth	x	
Return on Total Assets		x
Gross Margin		
Effective Tax Rate		
Fixed Asset Turnover		
LEVERAGE (DEBT COVERAGE) RATIOS:		
Fixed Charge Coverage	x	x
Long-Term Debt to Working Capital	x	x
Total Debt to Tangible Net Worth	x	
Current Liabilities to Tangible Net Worth	x	
Long-Term Debt to Capitalization		x
Net Tangible Assets to Long-Term Debt		x
Long-Term Debt to Property, Plant, & Equipment		x
Senior Debt Leverage		
Interest Coverage		
LIQUIDITY RATIOS:		
Current Ratio	x	x
Quick Ratio	x	x
Accounts Receivable Turnover	x	x
Inventory Turnover	x	x
Current Liabilities to Inventory	x	
Inventory to Working Capital	x	
Liquidity Ratio		x
Working Capital Turnover		x
CASH FLOW RATIOS:		
Percentage Cash Flow Change	x	x
Cash Flow to Long-Term Debt	x	x
Cash Flow to Senior Debt		x
Cash Flow to Total Liabilities		
Total Number of Ratios	20	19

SOURCE: Backer and Gosman, Forthcoming.

ratios were selected by B and G because they met at least one of the following conditions:

1. It was stressed in their interviews with investment bankers.

2. It had been publicly stated that particular ratios had rigid cut-off levels which were used by Standard and Poor's for each of the four highest bond grades.

3. It was mentioned as an important ratio in the rating of corporate bonds. In addition to the above, the authors of the selected study included ratios that had been mentioned in previous studies, that had been stated as important by the bond rating agency, and conformed to the selection of ratios in the other section of their study.

The data for the calculation of the 19 selected ratios were gathered from a combination of either 1) Form 10K filed with the Securities and Exchange Commission, 2) annual reports, or 3) Moody's Industrial Manuals. The years of analysis were held as constant as possible, given the difference in the fiscal years of the experimental and control firms. This was done to alleviate the potential effect of changing general economic conditions. The data was gathered for the three most recent fiscal years ended prior to the date of the downgrade for the experimental firms and for the three most recent fiscal years most closely corresponding for the control firms.

As an example, Sprague Electric's bonds were downgraded by Standard and Poor's from BBB to BB in August 1972. Data was gathered on the 19 ratios for the three years ending December 31, 1971, 1970, and 1969. Sola Basic was selected as the matched control firm for Sprague Electric, and their fiscal year ends on March 31st. Data was collected for this control firm for the years ending March 31, 1972, 1971, and 1970, since these were less removed in time from Sprague's statements than any other three years.

Table 4-2 presents the means of the 19 financial ratios for both the experimental and control firms. The means data presented in this Table form the basis for a test of the significance of ratio trends. Table 4-3 presents the significance levels resulting from these tests. B and G selected 5% as the cut-off for significance and with that level of significance concluded that the following three distinct patterns could be observed:

1. None of the control firms ratio levels exhibited any significant (5%) deterioration during the years preceding the downgrades received by their experimental partners.

2. Eight of the 19 financial ratios for the experimental firms did exhibit significant (5%) deterioration prior to the bond downgrades received by these firms. Those ratios that deteriorated were as follows:

TABLE 4-2

Means of Ratio-Levels for Bond Sample Companies

	18 Control Firms			18 Experimental Firms		
	Yr. 2	Yr. 1	Yr. 0*	Yr. 2	Yr. 1	Yr. 0*
<u>OPERATING RATIOS</u>						
1. Return on Sales	5.3%	3.9	3.6	3.9%	2.0	(2.0)
2. Return on Total Assets	8.6%	7.8	7.3	7.2%	5.1	2.2
3. Return on Tangible Net Worth	14.1%	12.7	11.1	13.8%	6.1	(0.1)
4. Percentage Profit Change	(4.3%)	9.6	(6.7)	8.1%	(5.4)	(62.1)
5. Percentage Sales Change	7.2%	9.3	12.7	12.5%	7.8	8.5
<u>LEVERAGE (DEBT COVERAGE) RATIOS</u>						
6. Long-Term Debt to Capitalization	31.8%	32.0	32.9	36.3%	40.8	47.6
7. Net Tangible Assets to Long-Term Debt	3.3x	3.3	3.3	3.2x	2.6	2.2
8. Long-Term to Prop., Pl., & Eq.	1.0x	0.9	1.0	1.0x	1.1	1.4
9. Working Capital to Long-Term Debt	1.8x	1.8	1.8	1.7x	1.3	1.1
10. Fixed Charge Coverage	2.4x	2.2	2.0	1.5x	1.4	0.5
<u>LIQUIDITY RATIOS</u>						
11. Current Ratio	2.7x	2.5	2.4	2.5x	2.7	2.6
12. Quick (Acid-Test) Ratio	1.2x	1.1	1.1	1.1x	1.2	1.1
13. Liquidity Ratio	20.0x	11.3	7.9	7.6x	10.1	11.4
14. Working Capital Turnover	3.6x	3.7	4.0	3.5x	3.4	3.6
15. Accounts Receivable Turnover	6.1x	6.0	6.1	5.7x	5.5	5.6
16. Inventory Turnover	3.0x	3.0	3.1	2.7x	2.6	2.7

TABLE 4-2
(CONT)

	18 Control Firms			18 Experimental Firms		
	Yr. 2	Yr. 1	Yr. 0*	Yr. 2	Yr. 1	Yr. 0*
CASH FLOW RATIOS						
17. Cash Flow to Long-Term Debt	34.6%	34.0	32.7	28.7%	19.5	10.0
18. Cash Flow to Senior Debt	66.0%	65.5	46.3	74.7%	56.3	12.3
19. Percentage Cash Flow Change	(1.0%)	19.9	13.2	53.7%	5.1	(42.9)

*Year 0 is most recent fiscal year ended prior to the date of the experimental firm's receipt of the bond downgrade.

SOURCE: Backer and Gosman, Forthcoming.

TABLE 4-3

Results of Test of Significance Conducted on Ratio-Level Means

	Significance Levels (%)					
	18 Control Firms			18 Experimental Firms		
	Yr. 2 vs. Yr. 1	Yr. 1 vs. Yr. 0	Yr. 2 vs. Yr. 0	Yr. 2 vs. Yr. 1	Yr. 1 vs. Yr. 0	Yr. 2 vs. Yr. 0
<u>OPERATING RATIOS</u>						
1. Return on Sales	50.9	84.4	30.3	19.4	10.5	3.0
2. Return on Total Assets	68.3	79.3	50.2	24.0	13.8	3.8
3. Return on Tangible Net Worth	73.9	67.6	48.2	13.7	17.5	2.8
4. Percentage Profit Change	55.9	47.2	92.2	80.3	65.3	56.4
5. Percentage Sales Change	70.6	58.7	30.8	24.1	88.3	30.0
<u>LEVERAGE (DEBT COVERAGE) RATIOS</u>						
6. Long-Term Debt to Capitalization	96.2	79.9	75.8	25.4	10.4	1.2
7. Net Tangible Assets to Long-Term Debt	95.5	92.0	96.0	16.2	9.8	2.6
8. Long-Term Debt to Prop., Pl., & Eq.	85.6	85.6	100.0	31.1	16.8	4.9
9. Working Capital to Long-Term Debt	88.5	91.4	80.7	22.7	35.5	8.8
10. Fixed Charge Coverage	77.1	64.4	45.8	71.1	8.2	6.6
<u>LIQUIDITY RATIOS</u>						
11. Current Ratio	54.8	62.0	27.5	43.2	71.5	68.0
12. Quick (Acid-Test) Ratio	45.0	68.2	24.0	79.4	69.9	86.7
13. Liquidity Ratio	23.9	42.5	9.2	53.3	80.6	42.4
14. Working Capital Turnover	76.4	59.5	40.0	81.7	62.5	78.0
15. Accounts Receivable Turnover	91.1	93.1	97.1	82.1	90.8	91.5
16. Inventory Turnover	90.3	78.4	69.9	81.5	67.4	81.4

TABLE 4-3
(CONT)

	Significance Levels (%)					
	18 Control Firms			18 Experimental Firms		
	Yr. 2 vs. Yr. 1	Yr. 1 vs. Yr. 0	Yr. 2 vs. Yr. 0	Yr. 2 vs. Yr. 1	Yr. 1 vs. Yr. 0	Yr. 2 vs. Yr. 0
CASH FLOW RATIOS						
17. Cash Flow to Long-Term Debt	93.6	86.1	81.9	17.7	0.6	1.0
18. Cash Flow to Senior Debt	98.8	47.1	42.8	68.5	18.1	4.1
19. Percentage Cash Flow Change	7.8	79.7	57.7	18.8	29.6	25.1

SOURCE: Backer and Gosman, Forthcoming.

(a) Return on sales.

(b) Return on total assets.

(c) Return on tangible net worth.

(d) Long-term debt to capitalization.

(e) Net tangible assets to long-term debt.

(f) Long-term debt to fixed assets.

(g) Cash flow to long-term debt.

(h) Cash flow to senior debt.

3. None of the so-called liquidity ratios (Nos. 11-16 in Table 4-1) had a statistically significant deterioration for the experimental group.

As these tests of significance only test for mean ratio difference between years, B and G also used linear discriminant analysis to test for mean ratio differences between groups. Three methods were used for the determination of the ratio variables that comprised the three discriminant functions calculated: 1) stepwise selection of variables, 2) factor scores derived from factor analysis, and 3) the actual ratio level for the highest loading variable on each factor. Table 4-4 presents the ratio variables selected in the stepwise procedure for each of the three years, while Table 4-8 presents the factors and selected ratios. The Lachenbruch classification results for each of the three discriminant functions used for each of the three years in question are

TABLE 4-4

Ratio Variables Selected in Stepwise Discriminant Procedure

No.	Ratios Description	Periods for which Variable was Selected Year 2	Year 1	Year 0
1	Return on Sales	x		x
2	Return on Total Assets	x		x
3	Return on Tangible Net Worth	x		
5*	Percentage Sales Change	x	x	x
6	Long-Term Debt to Capitalization	x		
7	Net Tangible Assets to Long-Term Debt	x		x
8	Long-Term Debt to Prop., Pl., & Eq.	x	x	
9	Working Capital to Long-Term Debt	x	x	x
10	Fixed Charge Coverage	x		x
11	Current Ratio	x		x
12	Quick (Acid Test) Ratio	x	x	x
13	Liquidity Ratio	x		x
14	Working Capital Turnover	x	x	
15	Accounts Receivable Turnover	x	x	x
16	Inventory Turnover	x	x	x
17	Cash Flow to Long-Term Debt	x	x	x
18	Cash Flow to Senior Debt	x	x	x
19	Percentage Cash Flow Change	x	x	
	Yearly Totals of Ratios	18	10	13

*Ratio No. 4, Percentage Profit Change, is not listed as it was not selected for any year.

SOURCE: Backer and Gosman, Forthcoming.

presented in Table 4-10. Table 4-10 indicates that the classification results for years 2 and 1 for all 3 methods of discrimination are not significantly better than chance. All three discriminant functions did classify significantly better than chance for year 0. B and G pointed out that the classification results achieved were higher than any hold-out sample levels of classification success achieved in previous bond empirical work. Pinches and Mingo [1973] arrived at classification accuracy of only 59-65%. B and G also noted that in the Pinches and Mingo [1973] study their model was only able to correctly classify 35% of Moody's Baa bonds, which are equivalent to Standard and Poor's BBB bond ratings.

4.2.3 Trade credit design. B and G worked with a downgraded experimental sample of 19 firms whose trade credit rating was reduced by Dun & Bradstreet, and a matched sample of 19 control group firms whose trade credit rating remained constant. They requested and received from Dun & Bradstreet a list of 60 firms which met certain criteria and that had been downgraded from a rating of 2 to a rating of 3. This list of 60 potential experimental firms was reduced to 19 after the requirement was added that such firms must have been consistently rated at 2 during the three-year period preceding the downgrade, and consistently rated at 3 for the one-year period following the downgrade. When the experimental group had been established at 19 firms, reference was made to the rating books published bi-monthly by Dun & Bradstreet in order to determine which companies had received a 2 rating. The first 80 such companies found were selected, and this group was reduced to the 19 finally selected for the control group by requiring 1) that the control group firm matched the experimental group firm on at least the first two digits of the SIC number, 2) were consistently rated at 2 during the three-year period preceding its matched partner's downgrade, and 3) were consistently rated at 2 during the one-year period following its matched partner's downgrade.

The 20 financial ratios selected for the study of the trade credit decision are presented in Table 4-1. These ratios, like the ones used for the bond rating decision section are divided into the four main categories of Operating, Leverage, Liquidity, and Cash Flow. These ratios were selected by B and G as a result of their literature review, their interviews with executives of Dun & Bradstreet, the publication of the ratio in the Dun & Bradstreet _Key Business Ratios_, and the ratios being stressed in interviews at Standard and Poor's. The data for the calculation of the ratios was collected from the 10K forms filed by these firms with the S.E.C.

The procedure followed for the determination of the three years used in this section was identical to that used and described in the bond rating section. That is, for the experimental or downgraded firm information was gathered for the three most recent fiscal years ended prior to the date of the downgrade. Information was gathered for the control firm for the three years that most closely matched the three years used for their matched experimental partner.

Table 4-6 presents the means of the 20 ratios for the trade credit control and experimental firms for each of the three years involved. The results of significance tests between years are presented in Table 4-7. As with the bond rating section, 5% was chosen as a cut-off for significance and as can be seen from Table 4-7, all 20 ratios exhibited one of only two distinct patterns over time.

1. None of the control group's ratio levels exhibited any significant deterioration during the years immediately preceding the trade credit downgrades received by the experimental firms.

2. Four of the 20 ratios examined for the experimental firms did exhibit statistically significant deterioration

prior to the trade credit downgrades.

(a) Return on tangible net worth.

(b) Return on working capital.

(c) Percentage profit change.

(d) Percentage cash flow change.

3. Both the Leverage group of ratios (Nos. 9-12 in Table 4-1) and the Liquidity group (Nos. 13-18 in Table 4-1) showed significant deterioration over the period studied. The authors of this selected study found this to be surprising, particularly as to the liquidity ratios in view of the short-term nature of trade credit decisions.

As the tests of significance present only results for ratio differences between years, it was necessary to use linear discriminant analysis in order to directly test for ratio differences between groups.

As with the bond rating section, three different methods of discrimination were used, 1) stepwise selection, 2) factor scores, and 3) selected variables from derived factors. Table 4-5 presents the ratios selected in the stepwise discriminant procedure for each of the three years. Table 4-9 presents the factors derived and the ratios selected as most important and representative. Table 4-10 presents the results of classifying the entire 38 firm sample for each of the three years involved using each of the three different

TABLE 4-5

Ratio Variables Selected in Stepwise Discriminant Procedure

No.	Ratios Description	Periods for which Variable was Selected Year 2	Year 1	Year 0
1	Return on Sales	x	x	
2	Return on Tangible New Worth	x	x	x
3	Return on Working Capital		x	x
4	Net Sales to Tangible Net Worth	x	x	
5	Net Sales to Working Capital	x	x	x
6	Fixed Assets to Tangible Net Worth		x	x
7	Percentage Sales Change		x	
8	Percentage Profit Change		x	x
9	Current Liabilities to Tangible Net Worth	x		x
10	Total Debt to Tangible Net Worth		x	x
11	Long-Term Debt to Working Capital	x	x	
12	Fixed Charge Coverge		x	
13	Current Ratio		x	x
14	Quick Ratio	x	x	x
15	Current Liabilities to Inventory	x	x	
16	Inventory to Working Capital	x	x	x
17	Accounts Receivable Turnover	x		
18	Inventory Turnover	x	x	x
19	Cash Flow to Long-Term Debt	x	x	x
20	Percentage Cash Flow Change	x	x	x
	Yearly Totals of Ratios	13	18	13

SOURCE: Backer and Gosman, Forthcoming.

TABLE 4-6

Means of Ratio-Levels for Trade Credit Sample Companies

	19 Control Firms			19 Experimental Firms		
OPERATING RATIOS	Year 2	Year 1	Year 0*	Year 2	Year 1	Year 0*
1. Return on Sales	4.5%	3.9%	3.6	3.6%	3.6	(2.3
2. Return on Tangible Net Worth	12.7%	10.7	12.3	12.3%	12.0	(0.9)
3. Return on Working Capital	29.4%	22.3	34.4	19.9%	20.6	(2.9)
4. Net Sales to Tangible Net Worth	3.4x	3.7	4.1	3.5x	3.6	3.9
5. Net Sales to Working Capital	6.1%	6.1	6.5	4.7x	4.9	5.9
6. Fixed Assets to Tangible Net Worth	71.8%	73.2	77.6	65.0%	65.6	72.7
7. Percentage Sales Change	19.9%	18.7	15.5	17.1%	18.4%	7.5
8. Percentage Profit Change	58.8%	130.4	22.9	47.9%	88.5	(163.4)
LEVERAGE (DEBT COVERAGE) RATIOS						
9. Current Liabilities to Tangible Net Worth	70.9%	75.7	78.8	66.2%	72.5	88.3
10. Total Debt to Tangible Net Worth	1.2x	1.3	1.4	1.2x	1.3	1.5
11. Long-Term Debt to Working Capital	123.9%	129.1	145.8	66.2%	69.9	91.5
12. Fixed Charge Coverage	3.0x	3.2	2.9	3.5x	2.7	0.4

TABLE 4-6
(CONT)

	19 Control Firms			19 Experimental Firms		
LIQUIDITY RATIOS	Year 2	Year 1	Year 0*	Year 2	Year 1	Year 0*
13. Current Ratio	2.3x	2.4	2.9	2.9x	2.6	2.5
14. Quick Ratio	1.2x	1.1	1.5	1.6x	1.3	1.1
15. Current Liabilities to Inventory	122.9%	115.0	103.0	99.4%	102.5	98.4
16. Inventory to Working Capital	109.3%	107.2	111.5	87.3%	96.9	117.0
17. Accounts Receivable Turnover	10.2x	9.4	7.5	6.5x	8.4	8.5
18. Inventory Turnover	7.0x	6.6	6.5	4.9x	4.7	4.7
CASH FLOW RATIOS						
19. Cash Flow to Long-Term Debt	90.0%	118.8	397.8	224.7%	164.5	53.4
20. Percentage Cash Flow Change	61.1%	30.2	19.6	38.7%	13.8	(74.9)

*Year 0 is most recent fiscal year ended prior to the date of the experimental firm's receipt of the trade-credit downgrade.

SOURCE: Backer and Gosman, Forthcoming.

TABLE 4-7

Results of Test of Significance Conducted on Ratio-Level Means

	Significance Levels (%)					
	19 Control Firms			19 Experimental Firms		
	Year 2 vs. Year 1	Year 1 vs. Year 0	Year 2 vs. Year 0	Year 2 vs. Year 1	Year 1 vs. Year 0	Year 2 vs. Year 0
OPERATING RATIOS						
1. Return on Sales	65.7	84.7	50.7	93.8	5.2	5.1
2. Return on Tangible Net Worth	63.6	72.4	91.7	89.9	0.1	0.1
3. Return on Working Capital	55.0	59.3	82.5	92.2	0.6	0.6
4. Net Sales to Tangible Net Worth	67.5	63.9	41.7	85.4	69.0	57.3
5. Net Sales to Working Capital	99.0	79.5	80.7	84.6	59.0	51.5
6. Fixed Assets to Tangible Net Worth	93.1	78.5	70.7	95.4	54.7	51.9
7. Percentage Sales Change	88.9	67.8	59.4	80.8	6.4	12.4
8. Percentage Profit Change	47.1	28.2	31.4	46.8	1.3	1.7
LEVERAGE (DEBT COVERAGE) RATIOS						
9. Current Liabilities to Tangible Net Worth	83.4	90.0	73.2	60.6	30.0	13.9
10. Total Debt to Tangible Net Worth	86.2	84.1	70.4	63.6	34.7	16.4
11. Long-Term Debt to Working Capital	75.9	92.5	71.0	84.2	40.5	32.2
12. Fixed Charge Coverage	86.7	79.7	88.7	58.2	7.0	6.6

TABLE 4-7
(CONT)

	Significance Levels (%)					
	19 Control Firms			19 Experimental Firms		
LIQUIDITY RATIOS	Year 2 vs. Year 1	Year 1 vs. Year 0	Year 2 vs. Year 0	Year 2 vs. Year 1	Year 1 vs. Year 0	Year 2 vs. Year 0
13. Current Ratio	81.0	35.6	27.1	64.1	86.0	56.4
14. Quick Ratio	86.2	26.5	31.6	57.7	56.3	32.9
15. Current Liabilities to Inventory	70.5	50.3	33.7	84.4	78.8	94.5
16. Inventory to Working Capital	94.7	88.0	94.2	50.2	32.0	11.7
17. Accounts Receivable Turnover	86.8	41.4	47.7	43.0	98.1	29.1
18. Inventory Turnover	80.4	91.9	74.4	78.6	98.5	78.1
CASH FLOW RATIOS						
19. Cash Flow to Long-Term Debt	50.3	35.5	30.7	63.1	23.9	9.4
20. Percentage Cash Flow Change	28.5	68.8	10.1	16.7	0.4	0.1

SOURCE: Backer and Gosman, Forthcoming.

TABLE 4-8

Most Important Components of Derived Factors

Year	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
	Factors and Ratios*					
2	1	11	6	16	4	14
	2	12	7	18	19	15
	3	13	9			
	10	14	17			
	17					
1	1	6	11	14	4	8
	2	7	12	15		18
	3	9	13	16		
	5	17				
	10					
0	(1)	14	5	12	(11)	6
	2	(15)	14	(13)	19	(7)
	3		(16)			9
	4					17
	10					18

*Ratios:

Operating	Leverage	Liquidity	Cash Flow
1. Return on Sales	6. LT Debt to Capitalization	11. Current Ratio	17. Cash Flow to LT Debt
2. Return on Tot. Assets	7. Net Tangible Assets to LT Debt	12. Quick Ratio	18. Cash Flow to Senior Debt
3. Return on Tangible Net Worth	8. LT Debt to Prop., Pl., & Eq.	13. Liquidity Ratio	19. Percentage Cash Flow Change
4. Percentage Profit Change	9. Working Capital to LT Debt	14. WC Turnover	
5. Percentage Sales Change	10. Fixed Charge Coverage	15. AR Turnover	
		16. Inv. Turnover	

Note: Ratio circled under each factor is the one which loaded (took on the greatest importance) in that factor's specification.

SOURCE: Backer and Gosman, Forthcoming.

TABLE 4-9

Most Important Components of Derived Factors

Year	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
	Factors and Ratios*					
2	2	9	6	8	13	1
	3	10	11	20	14	12
	5	16	15			19
	17					
1	3	4	1	12	18	5
	5	9	2	19		17
	6	10	7			
	11	13	20			
	15	14				
0	(5)	(1)	4	(3)	(18)	(13)
	16	2	9	7		14
	17	8	(10)	11		19
		12				
		20				

*Ratios:

Operating
1. Return on Sales
2. Return on Tangible Net Worth
3. Return on Working Capital
4. Net Sales to Tangible Net Worth
5. Net Sales to Working Capital
6. Fixed Assets to Tangible Net Worth
7. Percentage Sales Change
8. Percentage Profit Change

Leverage
9. Current Liabilities to Tangible Net Worth
10. Total Debt to Tangible Net Worth
11. Long-Term Debt to Working Capital
12. Fixed Charge Coverage

Liquidity
13. Current Ratio
14. Quick Ratio
15. Current Liabilities to Inventory
16. Inventory to Working Capital
17. Accts. Rec. Turnover
18. Inventory Turnover

Cash Flow
19. Cash Flow to Long-Term Debt
20. Percentage Cash Flow Change

Note: Ratio circled under each factor is the one which loaded highest (took on the greatest importance in the factor's specification.

SOURCE: Backer and Gosman, Forthcoming.

TABLE 4-10

Percentage of Sample Firms Correctly Classified Into Nondowngrade/Downgrade Categories by Discriminant Functions

	Prediction Percentages*					
	Trade Credit			Bonds		
Discrimination Basis	Yr. 2	Yr. 1	Yr. 0	Yr. 2	Yr. 1	Yr. 0
1. Stepwise selection of variables	65.8	52.6	65.8	61.1	58.3	72.2
2. Factors derived from factor analysis	42.1	42.1	73.7	55.6	50.0	80.6
3. Selected variables taken from factors derived from factor analysis	26.3	50.0	65.8	55.6	55.6	72.2

*Prediction Percentages are nonbiased in the sense that they resulted from a computer program that treated the original data in such a manner as to effectively approximate a holdout-sample test.

SOURCE: Backer and Gosman, Forthcoming.

methods of discrimination. Again, the Lachenbruch classification accuracy for years 2 and 1 was not significantly different from chance. This indicates that the ratios used possessed little ability to discriminate between the groups for those particular years. For year 0, the 3 discriminant functions were all able to classify significantly better than chance.

4.3 Methodology of the Present Study

4.3.1 An appropriate methodology

4.3.1.1 Group definition and initial misclassification. The group definitions for this study are the same as the ones in the B and G study. For the trade credit rating section, the groups are defined as: 1) those firms whose trade credit rating was downgraded from a 2 to a 3 by Dun & Bradstreet, and 2) those firms whose trade credit rating remained constant at 2. For the bond rating section, the groups are defined as: 1) those firms who had their bonds downgraded from BBB to BB (BB to B if subordinated) by Standard and Poor's, and 2) those firms whose bonds remained constant at a rating of BBB (BB if subordinated). The definition of the groups in both sections is subjective and not as objective as would be optimal for procedures employing discriminant analysis and classification. However, the researcher must always take into consideration the problem under study, and if need be, employ subjective group criteria if that is necessary. The definition of groups is of course not all inclusive as it excludes from any consideration whatsoever all firms who had trade credit ratings of 1 or 4, and all firms whose bonds were rated higher or lower than the BBB or BB designation used in that section. Therefore, classification is applicable to only these populations.

There is a possibility that there have been initial group misclassifications on the part of either one of the two rating agencies involved. By this is meant that Dun & Bradstreet might have initially classified a firm with a true but unknown trade credit rating of 3 into the trade credit rating 2 classification in error. In addition, Standard and Poor's may have incorrectly assigned a bond to the BBB rating subclassification, when in actuality that company's bond may belong to the A subclassification of bond ratings. Lachenbruch [1974] has studied the effects on the error rate of a linear function if some of the observations are initially misclassified. He found that the behavior for average size samples was about the same as that which would be expected if the observations had been properly classified in the first place. However, as he pointed out, his conclusions are based on a random sample from the parent populations. This is not the case with the selected study. While the actual error rates may be unaffected by an initial misclassification, he found that the apparent error rates were grossly distorted and totally unreliable for any sample size. Whenever possible, then, the groups should be based on objective definitions of group distinction, and subjective group definitions should be avoided when the research problem permits it.

4.3.1.2 Sample size. Both the experimental groups in the trade credit and bond rating sections were selected as a result of restrictions placed upon the total populations by B and G. In the case of the trade credit section, only those firms who had 1) been downgraded from a 2 to a 3, 2) a 2 rating for the three years preceding the downgrade, and 3) maintained the 3 rating for one year after the change were selected. This set of restrictions limited the sample size to 19 experimental firms.

In the bond rating section, the restrictions again reduced the available population to the 18 firms finally selected, which required that the downgraded firms had maintained a BBB rating for the three years preceding the downgrade, and that they maintained the BB rating for one year after the downgrade. The control group sample was done on a matching basis for both sections, although there is no overriding need for a 1:1 sample size ratio. Sample size should be a function of the researcher's ability to obtain the data needed for the sample selected, the cost of sampling, and the computational problems that might be associated with an extremely large sample.

The sample selected should always be identical to the population toward which the research question is directed. In this instance because of the manner in which the samples were selected, the only research question that should be addressed would involve only firms who have trade credit ratings of 2 or 3, or only firms who have bond ratings of BBB or BB. The discriminant function derived from these restrictive samples should only be used in classifying observations from a population consisting of firms with a 2 or 3 trade credit rating or a BBB or BB bond rating. This study will use discriminant functions derived from the two sets of data being used, that is the trade credit section and the bond rating section, to classify the combined experimental and control group sample. The classification results achieved are applicable only to firms in those particular trade credit or bond rating classifications.

4.3.1.3 Variable distributions. This study will conduct tests of skewness and kurtosis in order to determine if the ratio data from the B and G study is distributed multivariate normal. It is assumed in the location model that, conditional on the values of the binary variables, the continuous variables are distributed multivariate normal. A violation of this assumption may bias the test of significance of differences in group means and the estimated error rates. See Section 2.5.2.1 on non-normal data presented in Chapter 2 for full discussion of the potential effects of a violation of the normality assumption.

4.3.1.4 Dispersion matrices. The B and G study used a linear discriminant function to classify. Optimal use of a linear function requires that the group dispersion matrices are equal over all groups. When the group dispersion matrices are not equal, then a quadratic discriminant function is appropriate, as opposed to a linear. There is a test for the equality of the dispersion matrices between the experimental and control groups for both the trade credit and bond rating sections (See Section 2.5.2.2). Logically, this test for the equality of dispersion matrices should precede both the test for the equality of group means and the estimation of classification error rates.

4.3.1.5 Variable reduction. The two principal ways for reducing the dimensions in discriminant analysis are to eliminate 1) those variables or 2) those discriminant functions that do not contribute significantly to the ability to discriminate among groups. In the two group case, there is only one function so the only way that dimensionality can be reduced in the typical two group study is to eliminate variables. In the B and G study, variables were included or excluded depending upon their contribution to the Wilk's Lambda. In many studies, which have as a goal the construction of a classification scheme, variables will be eliminated only if they do not affect the overall efficiency of the classification results. Because classification accuracy is the primary goal, the criteria for keeping or deleting variables is related to the classification results.

4.3.1.6 Relative importance of variables. The B and G study did not identify the relative importance of the variables that were included in any of the three discriminant functions that were used in that study. Because there is no one generally accepted way to measure the importance of individual variables, the only general statements that can be made depend upon the extent to which the various methods used yield similar rankings. Five such methods are 1) the univariate F-test, 2) the scaled and weighted, 3) the stepwise forward, 4) the stepwise backward, and 5) the conditional deletion. No attempt is made in this study to preselect or predetermine variables that are independent, because of the fact that the variables *are* correlated and a fictitious attempt to use only uncorrelated variables begs the research question. Eisenbeis [1978] found that the subordination variable was the most important one in the Pinches and Mingo [1973] study by calculating all five of the methods mentioned above.

4.3.1.7 Prior probabilities and costs of misclassification. Classification rules incorporate *a priori* probabilities and costs of misclassification to adjust for the fact that some classification errors may occur more frequently than others and may be more serious than others in terms of cost. Discriminant analysis is most efficient when the population *a priori* probabilities are approximately 1:1; however, this is not the case for the sample probabilities, which are 1:1 in the matched pair design of the B and G study. While a random sample drawn from the related populations and the inclusion of the industry variable along with the other variables might be an alternative approach, the present study will use the paired sample design used in the B and G study. Nothing is known at this time about the costs of Type I or Type II errors as they apply to trade credit or bond ratings, so the present study will also assume equal misclassification costs as was assumed in the B and G study, although intuitively this would not seem to be the case.

4.3.1.8 Estimating error rates. In many of the earlier studies dealing with financial ratios, cross-validation samples were used to estimate classification error rates. This is very costly in terms of sample size, and financial ratio studies are known for their small samples. This is due to the fact that the observations in one of the two populations usually studied, such as failed firms or bankrupt firms, is very small because the number of failed or bankrupt firms is very small in relationship to the total number of firms in existence. In the B and G study, the Lachenbruch method was used to estimate the sample classification error rates. In this study, the same method is used. As was pointed out in Section 2.5.2.7, the Lachenbruch method has been empirically tested and shown to be an efficient method of estimating classification error rates. The Lachenbruch method gives an unbiased estimate of the expected actual error rate, and therefore it was the method selected to be used in this study.

4.3.2 Research design of this study. The data for this study was gathered for the B and G study for 74 firms, divided into two sections, one of which had 36 firms, and the other 38 firms. The 36 firm section represents a matched pair sample consisting of an equal number of firms whose bonds were downgraded by Standard and Poor's from BBB to BB (BB to B if the bonds were subordinated) and an equal number of firms whose bond ratings remained constant during the period under investigation. The second section consisted of a matched pair sample of 38 firms; 19 firms whose Dun & Bradstreet trade credit rating was downgraded from 2 to 3, and an equal number of firms whose trade credit rating was maintained at 2 for the period under investigation. These two sections were selected as examples of the inability of these firms to secure future capital from sources which had provided it with funds in the past. When a company's bonds are downgraded from BBB to BB, they become non-investment grade, and therefore many major prior sources of capital are cut off in the future. Firms rated below 2 by Dun and Bradstreet usually must make efforts to improve their liquidity, since a continued deterioration may lead to severe short-term financial difficulties. Financial ratio variables and non-financial ratio variables are used in this study to distinguish between the downgraded firms and the non-downgraded firms, and to provide a basis for the timely identification of illiquid companies so that the companies may take appropriate steps to correct the situation. For both sections, information was gathered on the levels of selected financial ratios for the most recent fiscal year ended prior to the date of the downgrade received by the firm and for the two preceding years. Information of a binary nature was also gathered for all firms in both sections. Nineteen financial ratios were gathered for the bond rating section and 20 financial ratios were gathered for the trade credit section. Binary information on 2 non-ratio variables was also gathered for both sections.

The binary variables selected for this study represent two time measures of liquidity.

These two binary variables represent the relative position of each firm in the samples to the national average of all U.S. manufacturing companies for current ratio and for fixed charge coverage.

The current ratio is an indicator of short-term liquidity while fixed charge coverage is an indicator of long-term liquidity. (See Backer and Gosman [1978]). Using the Quarterly Financial Reports, U.S. Federal Trade Commission, the mean current ratio for all manufacturing companies for the period 1971-1975 is 2.00, while the mean fixed charge coverage for the same period is 3.54 times. These relative measures were selected to determine if the relative position of a firm vis-à-vis the national average current ratio and fixed charge coverage resulted in significantly different mean ratio vectors. If these relative measures do result in different mean vectors conditional on current ratio and fixed charge coverage within the experimental and control groups, then the location model should result in an improved rate of classification accuracy over that obtained using the LDF as in the B and G study.

The hypothesis as a result of including the non-ratio binary variables is:

> Stratification by the relative measures of current ratio and fixed charge coverage leads to greater within group cell mean vector separation and therefore to improved classification results.

To summarize, financial ratios for each of the 74 firms were obtained, with 19 ratios gathered for each of 3 years for the bond rating group and 20 ratios gathered for each of 3 years for the trade credit group. This makes a total of 4,332 ratios for the combined sections. In addition, 2 non-ratio variables were obtained for all the firms in both sections for 3 years, representing a total of 464 non-ratio variables. The total pieces of variable information used in this study totals approximately 5,000.

All the financial ratio variables were obtained from the financial statements of the firms selected for the various samples and were transformed into percentages which were used in the remainder of the study. For a complete discussion of the manner of selecting the firms contained in the samples for the two sections, see the discussion in Section 4.2 concerning sample consideration. The selection of the firms included in the study coincides with those used in the B and G study, and was limited by the nature of the population from which they were drawn, and the conditions imposed upon their selection by the original researchers.

The research design consists of deriving a separate discriminant function for each unique cell in a contingency table formed from the 2 binary variables. The contingency table consists of 4 cells for each of the control and experimental groups, with each cell representing a unique pattern of responses to the binary variables. The location model and the related computer program called Locat were used to derive these separate cell specific discriminant functions. Briefly, Locat computes the expected frequencies for all the cells of the contingency tables, and then calculates estimates of group means and a common variance which are used in the standard classification rule. The separate discriminant functions derived from the combined samples are then used to classify each member of these samples using the Lackenbruch leave-one-out method.

The statistics calculated consist of tests for skewness and kurtosis, tests for the equality of within group cell mean vectors, and determination of the classification error rate in the respective populations.

The results of the tests listed above as well as the results of the classification using the separate discriminant functions are reported and analyzed in Chapter 5.

Chapter 5 also includes results obtained at various points in the process of obtaining the discriminant functions and the subsequent classification using these separate functions. This is done to give the reader a better understanding of the location model. The underlying assumption of the location model enables the researcher to work with both binary and continuous variables, without the usual violation of assumptions, and with a hypothesized difference in the classification accuracy obtained. It is hypothesized that the use of the location model to add non-ratio binary variables will increase predictive efficiency in the form of improved classification error rates. Chapter 6 presents a summary of this study, the conclusions drawn from the results of the research design, and discusses areas for future research involving continuous financial ratio variables and binary non-ratio variables.

CHAPTER V

RESULTS

5.1 Introduction

This chapter presents the results of the data analysis. Section 5.2 presents a discussion of the data used and the results of various tests. Section 5.3 presents the classification results obtained using each of the different models tested.

5.2 Data Description

5.2.1 Distributions. Generally, the first step in data analysis is to determine the basic distributional characteristics of each of the variables. The SPSS [1975] program Condescriptive was used to examine means, variance, standard deviation, standard error, minimum and maximum, range, kurtosis, and skewness. The distributions of the variables do not appear to follow the normal distribution. The majority of the trade credit group ratios are more peaked (narrow) than would be for a normal distribution. In years 1 and 2, all but one of these ratios has clusters of values more to the left of the mean. In year 0, five of the variables have clusters more to the right of the mean, an indication of the change taking place in resource allocation during the year preceding the downgrade.

The bond rating group ratios are mostly clustered to the left of the mean and are more peaked than would be for a normal distribution. In year 2, four of the variables are less peaked than in a normal distribution, and four are clustered to the right of the mean. In years 1 and 0, the corresponding results are 5 ratios clustered to the right and three ratios flatter than normal in both years.

In general, neither section appeared to have ratios that are normally distributed, with the bond rating section appearing slightly less non-normal than the trade credit section in all three years.

Many of the ratios in both sections have standard deviations that are large relative to the mean, although the trade credit section standard deviations are smaller as indicated by their coefficients of variation.

There are some extreme outliers (beyond three standard deviations) in both data sets. These outliers were not removed nor were they replaced with a substituted value.

5.2.2 t-tests. A complete discussion of the results of t-tests on all the variables in both sections has been presented in Chapter 4. In general, these observations can be made based on the results:

1. The various ratios did not change in unison in either section as the experimental firms approached the date of downgrade.

2. The various ratios did not show statistically significant deterioration for each control firm during the three years preceding the downgrade received by their experimental partners.

3. Some of the ratios of the downgraded firms did exhibit significant deterioration prior to the date of the downgrade. In the bond rating section, eight ratios deteriorated, and four ratios deteriorated in the trade credit section. Ratios from the Operating, Leverage, and Cash Flow groupings in the bond rating section showed deterioration, while only ratios that are influenced by profits exhibited deterioration in the trade credit section.

5.2.3 Significance. The tests for significance were made to determine if the cell mean vectors within each group are statistically different. In a regular two-group discriminant function, the continuous variables are averaged within each group. In the location model, the continuous variable means within each group are conditional on their cell configuration. Thus, when there are interactions between the binary variables and the groups, this should be evidenced by different cell mean vectors within groups.

The test used was one with the F distribution in the form of

$$\frac{n_1\ n_2}{n_n + n_2} \quad \frac{(n_1+n_2-k-1)}{(n_1+n_2-2)k} \quad D^2$$

where D^2 is the Mahalanobis sample distance measure, n_1 and n_2 are the number of observations in the two cells tested, and k is the number of variables in the cell mean vector. The degrees of freedom are k, and n_1+n_2-k-1.

A total of 216 significance tests were made, and only 18 of these tests indicate within-group cell mean vector differences at the .05 level. Less than 10% of the cell comparisons indicated significant differences. In the trade credit section, the majority of cell differences that are significant are between the cell representing firms below both average Current Ratio and Fixed Charge Coverage, and the cell representing firms above the Current Ratio average but below the Fixed Charge Coverage average.

In the bond rating section, the majority of cell differences that are significant are between the cell representing firms above the average Current Ratio but below the average Fixed Charge Coverage, and the cell representing firms below the average Current Ratio but above the average Fixed Charge Coverage.

It would seem that the greatest cell differences should be between the cell representing firms below both the average Current Ratio and Fixed Charge Coverage, and the cell representing firms above both the average Current Ratio and Fixed Charge Coverage. One of the conclusions of the B and G study offers an explanation for these findings. They concluded that there is no definitive cutoff point for individual ratios whereby it could be said that firms with ratios below such points would be downgraded. While the ratios of the experimental firms did change just prior to the downgrade, certain experimental firm ratio levels were less deteriorated than some control firms for the same year. For example, in the bond rating section, the range for Return on Sales in year 0 for control firms was (0.3)% to 7.6%, while the experimental firm range for the same year was 0.5% to 8.2%.

5.3 Classification

5.3.1 The models. Continuous ratio variables are combined with binary variables in the location model to derive linear discriminant functions for each cell. Each distinct pattern of the binary variables defines a multinomial cell uniquely, and in the case of two binary variables, the number of cells is equal to 2^2 or four cells for each of the two groups in both sections. The two binary variables in the models are as follows:

1. Current Ratio. The average Current Ratio for all U.S. manufacturing firms for the period 1971-1975 was 2.00. Coding for this variable is 1 if the firm's Current Ratio is greater than or equal to 2.00, and 0 if less.

2. Fixed Charge Coverage. The average Fixed Charge Coverage for all U.S. manufacturing firms for the period 1971-1975 was 3.54 times. Coding for this variable is 1 if the firm's Fixed Charge Coverage is greater than or equal to 3.54, and 0 if less.

The possible response patterns from these two binary variables are as follows:

Cell No.	Pattern	Explanation
1	0,0	Below the average for both Current Ratio and Fixed Charge Coverage.
2	1,0	Above the average Current Ratio and below the Fixed Charge Coverage.
3	0,1	Below the average Current Ratio and above the Fixed Charge Coverage.
4	1,1	Above the average for both Current Ratio and Fixed Charge Coverage.

The above two binary variables are included in each of the three models developed. Each of the next three sections presents one of the models used to derive the various discriminant functions.

5.3.1.1 Model I. Model I includes the two binary variables Current Ratio and Fixed Charge Coverage as well as the following continuous ratio variables:

1. Trade credit section.

   a) Return on Sales.
   b) Return on Working Capital.
   c) Net Sales to Working Capital.
   d) Total Debt to Tangible Net Worth.
   e) Current Ratio.
   f) Inventory Turnover.

2. Bond rating section.

   a) Return on Sales.
   b) Net Tangible Assets to Long-Term Debt.
   c) Current Ratio.
   d) Liquidity Ratio.
   e) Accounts Receivable Turnover.
   f) Inventory Turnover.

The above ratios were selected for Model I because they are the highest "scoring" ratio in each of the six factors derived from factor analysis. B and G undertook factor analysis in order to reduce the number of variables in their discriminant function. Each of the six factors developed can be thought of as relatively homogeneous groups, each depicting a different dimension. The actual values of the selected ratios were used in Model I.

Model I consists of the two binary and the six continuous variables listed above. Separate calculations are made for each of the three years preceding downgrade in both the trade credit and bond rating sections. For identification purposes, each model is given a subscript indicating the section (TC, BR) and the applicable year preceding the downgrade (0,1,2). For example, the model for the trade credit section for year 0 would be written as Model $I_{TC,0}$.

5.3.1.2 Model II. Model II includes the two binary variables Current Ratio and Fixed Charge Coverage, as well as the six factor scores for each of the firms. As explained in Section 5.3.1.1, B and G undertook factor analysis of the ratio data for all three years in both sections.

In factor analysis (see Cooley and Lohnes [1962] for a complete explanation of factor analysis) the structure matrix consists of correlation coefficients between each variable and the applicable factor. Consider the following brief example:

	Factor A	Factor B
Variable 1	.97652	.08240

The correlation between variable 1 and factor A in this example is .97652. The total variance of variable 1 accounted for by factor A is $(.97652)^2$.

The factor score for a firm is calculated from the factor score coefficient matrix and the standardized values of the variables, and represents the dimension associated with a particular factor. The factor score coefficient matrix in this case is derived from the factor structure matrix illustrated above and the correlation matrix. There is a separate factor score for each factor, and only the six most significant factors are considered in Model II (see Backer and Gosman [1978] for details).

5.3.1.3 Model III. Model III includes the two binary variables Current Ratio and Fixed Charge Coverage, and the following continuous ratio variables:

1. Trade credit section.
   a) Current Liabilities to Tangible Net Worth.
   b) Total Liabilities to Tangible Net Worth.
   c) Inventory to Net Working Capital.
   d) Long-Term Liabilities to Net Working Capital.
   e) Current Ratio.
   f) Quick Ratio.
2. Bond rating section.
   a) Long-Term Debt to Capitalization.
   b) Fixed Charge Coverage.
   c) Cash Flow to Long-Term Debt.

The B and G study included in-depth interviews with the two rating agencies, Dun & Bradstreet and Standard and Poor's. One of the questions asked during these interviews was: Which financial ratios take on the greatest importance in your decision process? The ratios cited in response to this question are the ratios used in Model III. Since the rating decision makers attach importance to these specific ratios, it is felt that they should be included in one of the models used in this study.

5.3.2 Classification results. The inclusion of a constant in the discriminant function results in a classification cutoff score (Z) equal to zero. Any firm with a score greater than zero is classified as liquid (non-downgraded) and any firm with a score less than zero is classified as illiquid (downgraded). All classification results are obtained using the leaving-one-out method (see Lachenbruch [1967] and Lachenbruch and Mickey [1968]).

The results are presented in a confusion matrix, and an example of such a matrix follows:

Actual Group		% Correct	Classified into I	II
I		45.0	18	22
II		50.0	20	20
	Total	47.5	38	42

The correct classification for group I is 18/40, or 45.0%. The correct classification for group II is 20/40, or 50.0%. The total correct classification is

18 + 20/80, or 47.5%.

Tables 5-1, 5-2, 5-3, 5-4, 5-5, and 5-6 present the classification results, and Table 5-7 presents a summary of correct classifications.

Table 5-7 indicates that in every instance, all the models for the trade credit section classified the downgraded group better than the non-downgraded group. With just two exceptions, all the models for the bond rating section classified the non-downgraded group better than the downgraded group.

Table 5-8 presents a comparison of the classification results of this study for Models I and II to the classification results of the B and G study. The models are different in that in this study they include two binary variables which are not included in the B and G study. An objective of this study is to present an example of the location model for discrimination and classification when the variables are mixed. The purpose was not to compare the location model with two binary and six continuous variables to a linear discriminant model with two binary variables and six continuous variables all treated as continuous. The comparison of the location model with binary variables included to the linear discriminant model with no binary variables is compatible with the research objective of this study.

Chapter 6 discusses the results reported in this chapter. It presents a summary of the models developed and details of their classification effectiveness. The best models in this study in terms of classification accuracy are also discussed, and the chapter ends with comments relative to the application of the location model and suggestions for future research.

TABLE 5-1

Trade Credit Section - Model I

Classification Results

Actual Classification	% Correct	Classified into Group: Non-downgraded	Down-graded
Year 2			
Non-downgraded	31.6	6	13
Downgraded	68.4	6	13
Total	50.0	12	26
Year 1			
Non-downgraded	47.4	9	10
Downgraded	79.0	4	15
Total	63.2	13	25
Year 0			
Non-downgraded	42.1	8	11
Downgraded	63.2	7	12
Total	52.6	15	23

TABLE 5-2

Trade Credit Section - Model II

Classification Results

Actual Classification	% Correct	Classified into Group: Non-downgraded	Classified into Group: Down-graded
Year 2			
Non-downgraded	15.8	3	16
Downgraded	68.4	6	13
Total	42.1	9	29
Year 1			
Non-downgraded	31.6	6	13
Downgraded	63.2	7	12
Total	47.4	13	25
Year 0			
Non-downgraded	47.4	9	10
Downgraded	63.2	7	12
Total	55.3	16	22

TABLE 5-3

Trade Credit Section - Model III

Classification Results

		Classified into Group	
Actual Classification	% Correct	Non-downgraded	Down-graded
Year 2			
Non-downgraded	52.6	10	9
Downgraded	68.4	6	13
Total	60.5	16	22
Year 1			
Non-downgraded	47.4	9	10
Downgraded	73.7	5	14
Total	60.5	14	24
Year 0			
Non-downgraded	21.1	4	15
Downgraded	42.1	11	8
Total	31.6	15	23

TABLE 5-4

Bond Rating Section - Model I

Classification Results

Actual Classification	% Correct	Classified into Group: Non-downgraded	Down-graded
Year 2			
Non-downgraded	39.0	7	11
Downgraded	61.1	7	11
Total	50.0	14	22
Year 1			
Non-downgraded	66.7	12	6
Downgraded	61.1	7	11
Total	63.9	19	17
Year 0			
Non-downgraded	83.3	15	3
Downgraded	66.7	6	12
Total	75.0	21	15

TABLE 5-5

Bond Rating Section - Model II

Classification Results

Actual Classification	% Correct	Classified into Group: Non-downgraded	Down-graded
Year 2			
Non-downgraded	50.0	9	9
Downgraded	44.4	10	8
Total	47.2	19	17
Year 1			
Non-downgraded	76.1	12	6
Downgraded	61.1	7	11
Total	63.9	19	17
Year 0			
Non-downgraded	72.2	13	5
Downgraded	66.7	6	12
Total	69.4	19	17

TABLE 5-6

Bond Rating Section - Model III

Classification Results

Actual Classification	% Correct	Classified into Group: Non-downgraded	Down-graded
Year 2			
Non-downgraded	83.3	15	3
Downgraded	44.4	10	8
Total	63.9	25	11
Year 1			
Non-downgraded	66.7	12	6
Downgraded	77.8	4	14
Total	72.2	16	20
Year 0			
Non-downgraded	77.8	14	4
Downgraded	72.2	5	13
Total	75.0	19	17

TABLE 5-7

Summary of Correct Classifications

	% Correct					
	Trade credit			Bond rating		
Model I	Total	Non	Down	Total	Non	Down
Year 2	50.0	31.6	68.4	50.0	39.0	61.1
Year 1	63.2	47.4	79.0	63.9	66.7	61.1
Year 0	52.6	42.1	63.2	75.0	83.3	66.7
Model II						
Year 2	42.1	15.8	68.4	47.2	50.0	44.4
Year 1	47.4	31.6	63.2	63.9	76.1	61.1
Year 0	55.3	47.4	63.2	69.4	72.2	66.7
Model III						
Year 2	60.5	52.6	68.4	63.9	83.3	44.4
Year 1	60.5	47.4	73.7	72.2	66.7	77.8
Year 0	31.6	21.1	42.1	75.0	77.8	72.2

TABLE 5-8

Comparison of Classification Results

	% Correct Model					
	I			II		
	Year			Year		
	2	1	0	2	1	0
Trade credit						
B and G	26.3	50.0	65.8	42.1	42.1	73.7
Demotses	50.0	63.2	52.6	42.1	47.4	55.3
Bond rating						
B and G	55.6	55.6	72.2	55.6	50.0	80.6
Demotses	50.0	63.9	75.0	47.2	63.9	69.4

CHAPTER VI

CONCLUSIONS

6.1 Introduction

Chapter 5 presented an example of the location model for discrimination and classification when the data set consists of both binary and continuous variables. The location model is an alternative to the use of an LDF model including both binary and continuous variables. Researchers faced with such a data set may be tempted to ignore the discrete nature of the binary variables and proceed with continuous variable techniques.

A question of major practical importance is how well the location model will perform on real data, and whether it will give different results than the LDF. To investigate this, a set of financial ratio data was obtained and two binary variables added to construct a model for the mixed data set. The results obtained using the location model are reported in Chapter 5 in detail.

In this Chapter, Section 6.2 presents a summary of the models, Section 6.3 discusses the most successful classification models, and Section 6.4 concludes the chapter with comments relative to the application of the location model, model weaknesses, and areas for further study.

6.2 Summary of the Models

Three models were used to demonstrate the applicability of the location model to real data. The three models for the trade credit section included six financial ratio continuous variables and two binary variables. Model I consisted of the actual values of the six ratios that "scored" the highest on each of the six factors derived from a factor analysis of the entire ratio variable set, and the two binary variables. Model II consisted of the six factor scores for each observation, and the two binary variables. Model III consisted of the six ratios preferred by Dun & Bradstreet and the two binary variables.

The three models for the bond rating section consisted of:

Model I — Two binary variables and the actual values of the six highest "scoring" ratio variables derived from a factor analysis of the entire data set.

Model II — Two binary variables and the six factor scores for each observation derived from factor analysis.

Model III Two binary variables and the three ratios preferred by Standard and Poor's.

The factor scores used in the above models were computed by the SPSS [1975] Factor Analysis program. Rao's canonical procedure was used for factor extraction, and the direct oblimin procedure was used for factor rotation (see Backer and Gosman [1978]).

Each of these models were used for each of the three years involved, making a total of 18 separate models. The two binary variables result in a four cell configuration, so that there are a total of 72 separate discriminant functions. In general, the classification functions using binary variables outperformed the LDF classification functions without any binary variables. The major exception was in the year just preceding the experimental firm downgrade. In that year, the cell mean vectors within each group were not separated enough, which leads to decreased classification accuracy when compared to the LDF classifications for the same period.

The two binary variables, relative Current Ratio and Fixed Charge Coverage, were selected on the assumption that the downgraded group would have mean ratio vectors that are considerably different from the mean ratio vectors of the non-downgraded group. As the experimental firms approach the downgrade date, their ratio levels are assumed to be considerably below the national average for Current Ratio and Fixed Charge Coverage. This is not the case.

The B and G study pointed out that no different absolute levels of ratio values were observed in the experimental and control groups. They concluded that the downgrade decision, for the most part, is based on the relative deterioration of ratio values rather than on any absolute minimum level of ratio levels. This is again demonstrated by the absence of cell mean vector differences between the experimental and control groups as reported in Chapter 5. It appears that the absolute level of illiquid firm ratios does not differ greatly from the absolute level of liquid firm ratios.

Some of the variables included in the models did not differentiate between the downgraded and non-downgraded firms on a univariate basis. However, when they are combined with other variables in the multi-variate analysis stage, they do have the information content needed to discriminate between the two groups. In general, the models presented here did differentiate between the two groups.

Table 5-7 presents a summary of the classification results of all the models tested. Overall, the models in the bond rating section have higher classification success rates than do the models in the trade credit section. For example, averaged over the three models, the bond rating section percentage of correct classifications of the non-downgraded group compared to the trade credit section is as follows:

	% Correct Classifications	
	Trade	Bond
Year 2	33.3	57.4
Year 1	42.1	69.8
Year 0	36.9	77.8

There is not as much difference in correct classifications for the downgraded group:

	% Correct Classifications	
	Trade	Bond
Year 2	68.4	50.0
Year 1	72.0	66.7
Year 0	56.2	68.5

Overall or total correct classifications averaged over models is as follows:

	% Correct Classifications	
	Trade	Bond
Year 2	50.9	53.7
Year 1	57.0	66.7
Year 0	46.5	73.1

Averaged over all models, the bond rating section has better correct classification results than does the trade credit section. The same results were found in the B and G study. Averaged over Model I and Model II, the total correct classifications in the B and G study by section were as follows:

	% Correct Classifications	
	Trade	Bond
Year 2	34.2	55.6
Year 1	46.1	52.8
Year 0	69.8	76.4

Table 5-8 compares the classification results of Models I and II of this study to the two comparable LDF models from the B and G study. As noted in Chapter 5, these models differ in that the models from this study include two binary variables, while the models from the B and G have only ratio variables. Except for the addition of the binary variables, the models of the two studies are comparable.

Generally speaking, the models from this study obtained higher classification results in years 2 and 1, while the B and G models were higher in year 0. The only model from this study that has a higher classification accuracy in year 0 is Model $I_{BR,0}$. As noted earlier in this chapter, it appears that the level of illiquid firm ratios does not differ greatly from the level of liquid firm ratios in the year just preceding the experimental firm downgrade.

In order to examine another model in comparison to the models developed in this study, an LDF model was developed using the actual values of the six highest "scoring" ratios as determined by factor analysis and the two binary variables, relative-to-average Current Ratio and Fixed Charge Coverage.

In this model, all eight variables, including the two binary variables, were treated as though they were continuous, a technique used in Pinches and Mingo [1973]. The classification results of this model are as follows:

	% Correct Classifications	
	Trade	Bond
Year 2	44.7	47.2
Year 1	55.3	63.9
Year 0	63.2	72.2

A comparison of Model I from this study, the comparable B and G model excluding binary variables, and the above model is as follows:

	% Correct Classifications		
		Year	
	2	1	0
Trade			
B and G	26.3	50.0	65.8

	% Correct Classifications		
	Year		
	2	1	0
Model I	50.0	63.2	52.6
New Model	44.7	55.3	63.2
Bond			
B and G	55.6	55.6	72.2
Model I	50.0	63.9	75.0
New Model	47.2	63.9	72.2

The inclusion of the binary variables as continuous in an LDF model resulted in approximately the same classification results for the bond rating section, but very different results for the trade credit section. These results are another indication that the two sections came from two different populations, and that the classification results in the bond rating section are higher than those in the trade credit section.

6.3 Most Successful Models

The most effective models in terms of total correct classifications are Model $I_{BR,0}$ and Model $III_{BR,0}$. These two models alone account for 22% of all the statistically significant cell mean vector differences. In both these models Cell 2 was statistically different from Cell 3 in both the downgraded and non-downgraded groups. Cell 2 represents those firms that are above the average Current Ratio and below the average Fixed Charge Coverage, while Cell 3 represents just the opposite configuration. Twelve of eighteen firms are in Cells 2 and 3 for the non-downgraded group, and fourteen of eighteen firms are in Cells 2 and 3 for the downgraded group. When the number of observations in a cell is very small, as it is for Cells 1 and 4 in this case, the resulting LDF coefficients and constant tend to be unstable and therefore do not classify effectively.

Model $III_{BR,0}$ has the Fixed Charge Coverage ratio as one of three in the model, which is correlated with the binary variable representing relative-to-average Fixed Charge Coverage. Model $I_{BR,0}$ has the Current Ratio as one of the six in the model, which is correlated with the binary variable representing relative-to-average Current Ratio.

The combination of a large number of observations in two cells and the correlation between a continuous and binary variable appears to be the cause of these two models having the highest correct classification rate.

The two best models developed in this study do not compare favorably, in terms of correct classification results, when compared to some recent models developed by others, as can be seen in the following comparison:

Model	% Correct Classifications
Altman [1977] LDF	91
Altman [1977] QDF	87
Alves [1978]	91
Backer and Gosman [1978]	81
Demotses [1978]	75

Both Altman models and the Alves model are designed to predict bankruptcy, while the Backer and Gosman and Demotses models are designed to distinguish among liquid and illiquid firms as defined by Backer and Gosman [1978]. All the studies reported above used the leaving-one-out method to classify observations. An earlier model developed by Altman [1968] classified 88% correctly using the hold-out sample method. At that time, Altman concluded that his model had predictive ability. As pointed out by Joy and Tollefson [1975], Altman had validation accuracy, but not predictive ability. Altman and Eisenbeis [1978] argued that their assumption of coefficient stability and stationarity among the variable relationships observed during validation (the hold-out sample method or the leaving-one-out method) is tantamount to prediction ability. Alves [1978] noted that the issue is an empirical one, and that his study revealed that the variable relationships were not stationary between his two sampling periods. Any claim of predictive ability should be supported by the use of an inter-temporal sample (see Alves [1978] for a good example of the use of an inter-temporal sample).

6.4 Comments and Recommendations

6.4.1 Comments. The location model has been presented here as an alternative to: 1) ignoring binary variables completely, 2) ignoring the discrete nature of binary variables and proceding with continuous variable techniques, and 3) dichotomizing the continuous variables and proceding with discrete variable techniques (see Dillon [1978]). One objective of this study is to derive discriminant functions from a location model for mixed binary and continuous variables. The model is introduced in Chapter 3, and the results of the application of the model to a practical example are presented in Chapter 5. The applicability of the location model to other situations must take a number of factors into consideration.

The models presented here are based on equal sample sizes of non-downgraded and downgraded firms. The prior probability of group membership is 50%. If the research goal is to minimize error rates, prior probabilities of population group membership can be taken into consideration.

If the research goal is to minimize the cost of misclassification, the costs of: 1) classifying a non-downgraded firm as a downgraded firm, and 2) classifying a downgraded firm as a non-downgraded firm must be estimated and included in the estimation of classification accuracy. To date, no studies have estimated and used these costs.

Another area of concern is the possibility that the samples selected do not represent the population. The use of hold-out samples, inter-temporal samples, or the leaving-one-out method may reduce but not eliminate this problem.

Liquidity and illiquidity were defined by B and G based on their definition of trade credit and bond rating downgrades. It may be possible to develop different concepts of liquidity based on other criteria.

The selection of non-ratio information in the form of binary variables that result in significant within-group cell mean vector differences is a very important consideration for the specification of the location model. The two binary variables used in this study may be poorly specified and may not represent important non-ratio characteristics of the firm. For example, managerial competence may be a much better selection as a binary variable, or economic variables such as dichotomized interest rates or levels of economic activity may prove to be better.

The location model can be generalized to handle discrete variables with any number of levels. In light of the large number of very sparse cells occurring in this study using only two binary variables, the move to discrete variables of more than two levels may prove to be a practical problem.

<u>6.4.2 Recommendations</u>. Many areas have to be investigated in any research that is based on an empirical evaluation of an unsupported theory such as the difference between a liquid and an illiquid firm. The use of inter-temporal samples to determine if predictive ability can be claimed for the location model is an area that could prove to be very helpful in the validation of the model.

The majority of the ratio research to date has not addressed the trend problems inherent in the use of financial ratios. Both Moyer [1977] and Joy and Tollefson [1975] indicate that time series analysis of financial ratios may reveal structural shifts in relationships that are not considered by recent models.

Additional testing of new and existing models should be conducted to determine the stability of the

LDF coefficients and variable relationship stationarity. It is not sufficient for the researcher to state he is of the opinion that the coefficients are stable and the relationships stationary.

Population prior probabilities and costs of misclassification need to be incorporated into models. In many situations the relative costs of Type I and Type II errors may be approximately 1:1, but in areas such as credit-scoring models, the costs may be very disproportionate.

Moyer [1977] demonstrated that the original Altman [1968] model did not perform as well in terms of classification accuracy when applied to a sample of larger firms from a subsequent time period. Future research can be directed to determine the continued classification effectiveness over time.

Additional study is needed in the area of group definition. Alves [1978] noted that some of the observations in his study of small business failure had in excess of 1,000 employees. Backer and Gosman [1978] noted that while liquidity is a continuum, it was necessary for them to define a sharp distinction between liquid and illiquid firms. Regardless of the research objective, future research is needed of a definitional nature.

The last area for further study to be discussed is the need for improved model specification to more accurately reflect non-ratio information important to the research question. If the research is directed at bankruptcy, then efforts are needed to find the non-ratio information important to the continuation of the firm. If the research is directed at illiquidity, efforts are needed to find the non-ratio information important to the financial health of the firm. Regardless of the role of ratios, non-ratio information considerations are also important. The problem with non-ratio information is not in specifying the data. The problem is in finding the non-ratio information once that specification has been made.

LITERATURE CITED

Abate, Robert P., "Numerical Scoring Systems for Commercial Loans, Bankers Monthly (January 1969).

Abdel-Khalik, A. Rashad, "Detailing Financial Information and the Loan Decision: An Emperical Study," Journal of Commercial Bank Lending (April 1973).

Afifi, A. A., and R. M. Elashoff, "Multivariate Two Sample Tests with Dichotomous and Continuous Variables. I. The Location Model," The Annals of Mathematical Statistics (February 1969).

Altman, E. I., "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," The Journal of Finance (September 1968).

Altman, Edward I., "Corporate Bankruptcy Prediction and its Implications for Commercial Loan Evaluation," Journal of Commercial Bank Lending (December 1970).

________. Corporate Bankruptcy in America, Heath Lexington Books, 1971.

Altman, E. I., M. Margaine, M. Schlosser, and P. Vernimmen, "Financial and Statistical Analysis for Commercial Loan Evaluation: A French Experience," Journal of Financial and Quantitative Analysis (March 1974).

Altman, E. I., R. G. Haldeman, and P. Narayanan, "Zeta Analysis. A New Model to Identify Bankruptcy Risk of Corporations," Journal of Banking and Finance (June 1977).

Altman, Edward I. and Robert A. Eisenbeis, "Financial Applications of Discriminant Analysis: A Clarification," Journal of Financial and Quantitative Analysis (March 1978).

Alves, Jeffrey R., "The Prediction of Small Business Failure Utilizing Financial and Nonfinancial Data," Unpublished Ph.D. Dissertation, Amherst, MA: University of Masscahusetts, September 1978.

Anderson, T. W., An Introduction to Multivariate Statistical Analysis, John Wiley and Sons, 1958.

Ang, James S., and Kiritkumar A. Patel, "Bond Rating Methods: Comparison and Validation," The Journal of Finance (May 1975).

Apilado, Vincent P., D. C. Warner, and J. J. Dauten, "Evaluative Techniques in Consumer Finance--Experimental Results and Policy Implications for Financial Institutions," Journal of Financial and Quantitative Analysis (March 1974).

Backer, Morton, and Martin L. Gosman, Financial Reporting and Business Liquidity, N. A. A., 1978, Forthcoming.

Beaver, William H., "Financial Ratios as Predictors of Failure," Journal of Accounting Research (Supplement 1966).

Beaver, William H., Paul Kettler, and Myron Scholes, "The Association Between Market Determined and Accounting Determined Risk Measures," The Accounting Review (October 1970).

Biborosch, R. A., "Credit Scoring Systems have Built-In Bonuses," Bankers Monthly (March 1967).

Blalock, Hubert M., Jr., Causal Inferences in Nonexperimental Research, W. W. Norton & Company, 1964.

BMDP: Biomedical Computer Programs, University of California Press, 1975.

Burnham, F. W., "Loan Review: A Necessary Exercise in 20-20 Hindsight," Journal of Comercial Bank Lending (June 1973).

Carleton, W. T., and E. M. Lerner, "Statistical Credit Scoring of Municipal Bonds," Journal of Money, Credit, and Banking (November 1969).

Chang, D. C. and A. A. Afifi, "Classification Based on Dichotomous and Continuous Variables," Journal of the American Statistical Association (June 1974).

Chesser, Delton L., "Predicting Loan Noncompliance," Journal of Commercial Bank Lending (August 1974).

Cochran, W. G., "On the Performance of the Linear Discriminant Function," Technometrics (May 1964).

Cohen, Kalman J., and F. S. Hammer (eds.), Analytical Methods in Banking, Richard D. Irwin, 1966.

Cooley, William W. and Paul R. Lohnes, Multivariate Procedures for the Behavioral Sciences, John Wiley and Sons, 1962.

Croxton, Frederick E., D. J. Cowden, and Sidney Klein, Applied General Statistics, Prentice-Hall, 1967.

Deakin, Edward B., "A Discriminant Analysis of Predictors of Business Failure," Journal of Accounting Research (Spring 1972).

________. "Distributions of Financial Accounting Ratios," The Accounting Review (January 1976).

Dillon, William R., "The Performance of the Linear Discriminant Function in Non-optimal Situations and the Estimation of Classification Error Rates--A Review of Recent Findings," Journal of Marketing Research, Forthcoming.

Edmister, Robert O., "Financial Ratios and Credit Scoring for Small Business Loans," Journal of Commercial Bank Lending (September 1971).

________. "An Emperical Test of Financial Ratio Analysis for Small Business Failure Prediction," Journal of Financial and Quantitative Analysis (March 1972).

Eilon, S., and T. R. Fowkes (eds.), Applications of Management Science in Banking and Finance, Gower Press, 1972.

Eisenbeis, Robert A., "Pitfalls in the Application of Discriminant Analysis in Business, Finance and Economics," The Journal of Finance (June 1977).

Eisenbeis, Robert A., "A Multivariate Analysis of Industrial Bond Ratings and the Role of Subordination: A Comment," The Journal of Finance (March 1978).

Eisenbeis, Robert A. and Robert B. Avery, Discriminant Analysis and Classification Procedures: Theory and Applications, D. C. Heath and Co., 1972.

Eisenbeis, Robert A. and Alan S. McCall, "Some Effects of Affiliations among Mutual Savings and Commercial Banks," The Journal of Finance (September 1972).

Eisenbeis, Robert A., Gary G. Gilbert and Robert B. Avery, "Investigating the Relative Importance of Individual Variables and Variable Subsets in Discriminant Analysis," Communications in Statistics (September 1973).

Elam, Rick, "The Effect of Lease Data on the Prediction Ability of Financial Ratios," The Accounting Review (January 1975).

Federal Reserve Bulletin, July 1977.

Finnerty, Joseph E., "Corporate Stock Issue and Repurchase," Financial Management (Autumn 1975).

________. "Insider's Activity and Inside Information: A Multivariate Analysis," Journal of Financial and Quantitative Analysis (June 1976).

Fisher, R. A., "The Use of Multiple Measurements in Taxonomic Problems," Annals of Eugenics (September 1936).

Gilbert, E. S., "On Discrimination Using Qualitative Variables," Journal of the American Statistical Association (December 1968).

________. "The Effect of Unequal Variance-Covariance Matrices on Fisher's Linear Discriminant Function," Biometrics (September 1969).

Goldberger, Arthur S., Econometric Theory, John Wiley and Sons, 1964.

Goldberger, Arthur S., Topics in Regression Analysis, Macmillan Company, 1968.

Gonedes, Nicholas J., "Evidence on the Information Content of Accounting Numbers: Accounting-Based and Market-Based Estimates of Systematic Risk," Journal of Financial and Quantitative Analysis (June 1973).

Greer, Carl C., "The Optimal Credit Acceptance Policy," Journal of Financial and Quantitative Analysis (December 1967).

Gupta, Manak C., "The Effect of Size, Growth, and Industry on the Financial Structure of Manufacturing Companies," The Journal of Finance (June 1969).

Haberman, S. J., "Log Linear Fit for Contingency Tables," Applied Statistics, 21 (1972).

Hammer, Frederick S., and Yair E. Orgler, "Developments in Credit Scoring for Commercial Loans, "Journal of Commercial Bank Lending (July 1969).

Hannan, J. F. and R. R. Tate, "Estimation of the Parameters for a Multivariate Normal Distribution when One is Dichotomized," Biometrika (December 1965).

Hettenhouse, George W., and J. R. Wentworth, "Credit Analysis Model-A New Look for Credit Scoring," Journal of Commercial Bank Lending (December 1971).

Holloway, L. N. and O. J. Dunn, "The Robustness of Hotellings T^2," Journal of the Americal Statistical Association (March 1967).

Horrigan, James O., "The Determination of Long-Term Credit Standing with Financial Ratios," Journal of Accounting Research (Supplement 1966).

Hoskin, G. J., "Some aids to Establishing a Grading System for Commercial Loans," Journal of Commercial Bank Lending (May 1968).

Jacobsen, R. A., "A Bank Examiner's Loan Review," Journal of Commercial Bank Lending (May 1973).

Jaffee, Dwight M., Credit Rationing and the Commercial Loan Market, John Wiley & Sons, 1971.

Joy, O. Maurice, and John O. Tollefson, "On the Financial Applications of Discriminant Analysis," Journal of Financial and Quantitative Analysis (December 1975).

Kennedy, Henry A., "A Behavioral Study of the Usefulness of Four Financial Ratios," Journal of Accounting Research (Spring 1975).

Kmenta, Jan, Elements of Econometrics, Macmillan Publishing Co., 1971.

Krzanowski, W. J., "Discrimination and Classification Using Both Binary and Continuous Variables," Unpublished Ph.D. Dissertation, Reading, Eng.: University of Reading, September 1974.

________. "Discrimination and Classification Using Both Binary and Continuous Variables," Journal of the American Statistical Association (December 1975).

Lachenbruch, Peter A., "Discriminant Analysis When the Initial Samples are Misclassified," Technometrics (February 1966).

________. "An Almost Unbiased Method of Obtaining Confidence Intervals for the Probability of Misclassification in Discriminant Analysis," Biometrics (December 1967).

________. "Discriminant Analysis Where the Initial Samples are Misclassified," Technometrics (May 1974).

________. Discriminant Analysis, Hafner Press, 1975.

Lachenbruch, Peter A., and M. Ray Mickey, "Estimation of Error Rates in Discriminant Analysis," Technometrics (February 1968).

Lachenbruch, Peter A., C. Sneeringer and L. T. Revo, "Robustness of the Linear and Quadratic Discriminant Function to Certain Tyes of Non-Normality," Communications in Statistics, 1 (1973).

Lev, Baruch, Financial Statement Analysis: A New Approach, Prentice-Hall, 1974.

Libby, Robert, "Accounting Ratios and the Prediction of Failure: Some Behavioral Evidence," Journal of Accounting Research (Spring 1975).

Mardia, K. V., "Measures of Multivariate Skewness and Kurtosis with Applications," Biometrika (December 1970).

Marks, Sidney and Olive Jean Dunn, "Discriminant Functions when Covariance Matrices are Unequal," Journal of the American Statistical Association (June 1974).

McLachlan, G. J., "Estimation of the Errors of Misclassification on the Criterion of Asymptotic Mean Square Error," Technometrics (May 1974).

Meyer, Paul A., and Howard W. Pifer, "Predictions of Bank Failures," The Journal of Finance (September 1970).

Morris, R. A., "Credit Analysis: An O. R. Approach," RMA Bulletin (July 1966).

Morrison, Donald F., Multivariate Statistical Methods, McGraw-Hill Book Company, 1967.

Mosteller, F., and D. L. Wallace, "Inference in an Authorship Problem," Journal of the Americal Statistical Association (June 1963).

Moyer, R. Charles, "Forecasting Financial Failure: A Re-Examination," Financial Management (Spring 1977).

Myers, J. H., "Numerical Scoring Systems in Retail Credit Evaluation," Credit World (April 1962).

Myers, J. H., and E. W. Forgy, "The Development of Numerical Credit Evaluation Systems," Journal of the American Statistical Association (September 1963).

Olkin, I. and R. F. Tate, "Multivariate Correlation Models with Mixed Discrete and Continuous Variables," Annals of Mathematical Statistics (June 1961).

Orgler, Yair E., "Selection of Bank Loans for Evaluation: An Analytical Approach," The Journal of Finance (March 1969).

________. "A Credit-Scoring Model for Commercial Loans," Journal of Money, Credit and Banking (November 1970).

________. "Evaluation of Consumer Loans with Credit-Scoring Models," Journal of Bank Research (Spring 1971).

________. Analytical Methods in Loan Evaluation, Lexington Books, 1975.

Pinches, George E., " A Multivariate Analysis of Industrial Bond Ratings and the Role of Subordination: Reply," The Journal of Finance (March 1978).

Pinches, George E., and Kent A. Mingo, "A Multivariate Analysis of Industrial Bond Ratings," The Journal of Finance (March 1973).

________. "The Stability of Financial Patterns In Industrial Organizations," The Journal of Finance (May 1973).

________. "The Role of Subordination and Industrial Bond Ratings," The Journal of Finance (March 1975).

________. "The Hierarchical Classification of Financial Ratios," Journal of Business Research (October 1975).

Pinches, George E., and James S. Trieschmann, "Discriminant Analysis, Classification Results, and Financially Distressed P-L Insurers," The Journal of Risk and Insurance (June 1977).

Pogue, Thomas F., and Robert M. Soldofsky, "What's In A Bond Rating," Journal of Financial and Quantitative Analysis (June 1969).

Rao, C. R., Advanced Statistical Methods in Biometric Research, John Wiley and Sons, 1952.

Scott, Elton, "On the Financial Applications of Discriminant Analysis: Comment," Journal of Financial and Quantitative Analysis (March 1978).

Simkowitz, Michael, and Robert J. Monroe, "A Discriminant Analysis Function for Conglomerate Targets," Southern Journal of Business (November 1971).

Sinkey, Joseph F., "A Multivariate Statistical Analysis of the Characteristics of Problem Banks," The Journal of Finance (March 1975).

Smith, P. F., "Measuring Risk on Consumer Installment Credit," Management Science (November 1964).

SPSS: Statistical Package for the Social Sciences, McGraw-Hill Book Company, 1975.

Stevens, Donald L., "Financial Characteristics of Merged Firms: A Multivariate Analysis," Journal of Financial and Quantitative Analysis (March 1973).

Tate, R. F., "Applications of Correlation Models for Biserial Data," American Statistical Association Journal (December 1955).

________. "The Theory of Correlation Between Two Continuous Variables When One is Dichotomized," Biometrika (December 1955).

Tatsuoka, Maurice M., Techniques for Educational and Psychological Research, John Wiley and Sons, 1971.

Trieschmann, James S., and George E. Pinches, "A Multivariate Model for Predicting Financially Distressed P-L Insurers," The Journal of Risk and Insurance (September 1973).

Walker, Strother H., and D. B. Duncan, "Estimation of the Probability of an Event as a Function of Several Independent Variables," Biometrika (June 1967).

Weingartner, M. H., "Concepts and Utilization of Credit-Scoring Techniques," Banking (February 1966).

West, Richard R., "An Alternative Approach to Predicting Corporate Bond Ratings," Journal of Accounting Research (Spring 1970).

Williams, S., "Survey Shows Increasing Importance of Mathematical Aids to Managements," Journal of Bank Research (Summer 1970).

Wonnacott, Ronald J., and T. H. Wonnacott, Econometrics, John Wiley and Sons, 1969.

Wood, J. H., Commercial Bank Loan and Investment Behavior, John Wiley and Sons, 1975.

Wrean, William H., The Demand for Business Loan Credit, Lexington Books, 1974.

Wu, H. K., "Bank-Examiner Criticisms, Bank Loan Defaults, and Bank Loan Quality," Journal of Finance (September 1969).