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A simulation approach to the investigation of the statistical properties of the rebalancing policy of portfolio adjustment.

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A SIMULATION APPROACH TO THE INVESTIGATION OF THE STATISTICAL PROPERTIES OF THE REBALANCING POLICY OF PORTFOLIO ADJUSTMENT

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This work is dedicated to my mother and father for all they have done for me.

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CHAPTER I

INTRODUCTION

Background

In recent years there has been a growing body of literature subjecting alternative investment strategies to rigorous empirical and analytical tests. While the empirical studies examined the performance of various methods of building or adjusting portfolios by applying each strategy to a sample of ex post stock price data, the analytical approaches required the specification of a well-defined statistical model generating the returns.

Whether empirical or analytical methodology is used, we may conveniently classify an investigation as addressing itself to either economic or statistical hypotheses. The former gives explicit consideration to the costs (e.g., transactions costs and taxes) associated with the actual implementation of each strategy, while the latter ignores them.

A strategy which has received considerable attention is the rebalancing policy of portfolio adjustment (RB). Rebalancing refers to the periodic adjustment of the portfolio so as to maintain a fixed set of weights among the market values of the securities in the portfolio throughout the investment horizon. Thus, if each security i initially accounted for the fraction w_i of the market value of the portfolio, at fixed equal time intervals (e.g., weekly) the portfolio is adjusted so that the current market value of the investment in security i is the original fraction of the current value of the portfolio. The policy requires selling off some part of those securities that have had superior returns during the period just ended, and using the proceeds to purchase additional amounts of those securities that have not performed as well. The result is that each period is begun with the members of the portfolio having the original set of weights.

The usual benchmark against which the outcomes of alternative investment strategies are compared is the buy-and-hold strategy (BH), which is a policy of no adjustment over the investment horizon.

Purpose

Tests comparing returns to RB with returns to BH (both statistical and economic) indicate that the rebalancing strategy can sometimes outperform a buy-and-hold policy. Analytical results have provided some insights into the mechanism of the rebalancing strategy under the assumption that stock price changes conform to a random walk. However, this assumption appears tenuous.

Consequently, the major purpose of this study is to examine the performance of the rebalancing policy versus a policy of buy-and-hold without the imposition of the random walk assumption that a series of stock price changes is a series of independent random variables. It is an attempt to explain when and why RB is superior to BH.

In order to do this, the effects of some of the statistical properties of and between the members of the portfolio on the relative performance of the two strategies is examined. Specifically, this work studies the joint effects of 1) portfolio size, 2) divergence of expected returns between securities in the portfolio, 3) autocorrelation of returns within each stock price series, 4) intercorrelation of returns between series, and 5) the variance of the returns to each security, on the distribution of returns to each strategy.

Because of the complexity of the relationships involved, the orientation is not one of testing fully specified hypotheses. Rather, its objective is to explore the relationships between the relative performance of the two investment policies and certain controllable factors.

The study is restricted to the special case of rebalancing a portfolio whose component securities are weighted equally. In addition, transactions costs and taxes are not considered.

Methods

The method is one of simulating sets of security returns controlled as to the combinations of the levels of the five factors under study. This is the data generating model. The alternative strategies are applied to the simulated data samples and the portfolio performance associated with each is measured.

The relationships between the various factors and the relative performance of the two strategies (the response) are then specified in a mathematical model. This factor-response model is developed through the application of multivariate analysis of variance and nonlinear regression techniques.

Finally, this model is utilized as a basis for drawing inferences concerning the conditions under which RB might give performance superior to BH.

CHAPTER II

LITERATURE REVIEW: REBALANCING

This chapter will provide a summary and unified treatment of the research done with respect to portfolio returns from pursuing a policy of rebalancing, and place it in its proper perspective in the area of portfolio theory.

Define the rate of return plus one, or the value relative, to the i^{th} security in the portfolio at the end of period j as

$$R_{ij} = \frac{P_{ij+1}+D_{ij}}{P_{ij}} \qquad (2\text{-}1)$$

where P_{ij} is the price of the i^{th} security at the beginning of period j, and D_{ij} is the dividend in period j, for $i=1,\ldots,m$ and $j=1,\ldots,n$. In the case of no dividends R_{ij} is referred to as the price relative (PR). Let W_i represent the initial proportion of funds invested in security i so that $\sum_{i=1}^{m} W_i=1$. Also, T is the length of the investment horizon consisting of n periods each of length h. Therefore, T=nh. Define the geometric total return as the dollar value of the portfolio at the end of T for each dollar invested at the beginning of T. Then the geometric total to a buy-and-hold strategy is

$$G^T(BH) = \sum_{i=1}^{m} W_i \prod_{j=1}^{n} R_{ij}. \qquad (2\text{-}2)$$

Under a policy of rebalancing after each of the n periods the geometric total return is

$$G^T(RB) = \prod_{j=1}^{n} \sum_{i=1}^{m} W_i R_{ij}. \tag{2-3}$$

In the case when all securities are weighted equally we have

$$G^T(BH) = \sum_{i=1}^{m} \left(\frac{1}{m} \prod_{j=1}^{n} R_{ij} \right) \tag{2-4}$$

and

$$G^T(RB) = \prod_{j=1}^{n} \left(\frac{1}{m} \sum_{i=1}^{m} R_{ij} \right). \tag{2-5}$$

Empirical Results

Statistical Hypotheses

The earliest empirical evidence indicating that the return to RB may be superior to BH can be garnered from an index number problem which Macauley called "mathematical drift" [137]. Upward drift was particularly evident in an index of railroad stock prices which assumed that equal dollar amounts of money were invested in each stock in January, and each succeeding January the portfolio was rearranged so that again there were equal holdings. This index rose from 100 in 1857 to 1,150 in 1936, while a Dow Jones type index rose only to 241 and a BH type price index weighted by the total number of shares outstanding rose only to 195. Renshaw [178] noted that the mathematical drift was probably the cumulative effect from following a fairly optimal policy. This policy was in fact one of equal weight rebalancing.

A strong upward bias in the arithmetic index computed by Fisher [72] is also relevant. Defining the arithmetic link relative as the arithmetic mean of value relatives in a period (month), an arithmetic investment performance index was computed by multiplying together link relatives for investment horizons of various lengths from 1926 to 1960. When compared with "Rates of Return on Investments in Common Stocks" [75], this index of all NYSE securities was found to have an upward bias. We observe that Fisher's arithmetic index is computed in accordance with a rebalancing strategy, while "Rates of Return" assumed a policy of BH.

Rebalancing was first introduced as an alternative investment strategy by Evans [57]. Based on a sample of the 470 securities from Standard and Poor's 500 for which data was available from January 1, 1958 to July 1, 1967, he compared the geometric mean returns[1] to BH with the returns to a strategy of semi-annual rebalancing. Calculation of these returns for large numbers of random portfolios consisting of up to forty securities weighted equally, indicated that the returns to rebalancing were significantly superior to the returns to BH. Latané and Young [128], however, pointed out that Evans' method of calculating the geometric mean return to BH was biased downward,[2] invalidating his results. Nevertheless, they did consider the rebalancing policy "intuitively appealing." They hypothesized that the relative performance of the RB and BH strategies is

> an evaluation of the tradeoff between (1) the beneficial effects of diversification of the portfolio across time, given by reallocation, and (2) the effects of the divergence of two or more securities with different geometric means (growth rates), given by buy-and-hold [p. 602].

To test their hypothesis, the alternative strategies were applied to portfolios selected according to four different criteria. Their data base consisted of monthly value relatives for 224 securities for the period January, 1953 through December, 1960, overlapping each twelve-month period to increase the number of annual observations for each security from eight to eighty-five.

Their results indicated that for selection criteria that tended to reduce the diversity of price trends, RB was much superior to BH, while for the other selection criteria (including random selection) RB did not appear significantly better. In addition the relative performance of RB was found to improve with the portfolio size.

Despite a statistical bias due to overlapping pointed out by Cheng and Deets [37, 38], which limits the usefulness of some of their conclusions, the direction of the bias does strengthen the evidence that under certain conditions RB can outperform BH.

A study by Evans [56] corrected for the bias in his earlier work and, utilizing the same data base, offers further evidence that the expected return to RB can be greater than the expected return to BH. Calculation of the t statistic for differences within pairs indicated that the expectation of the geometric mean under rebalancing was significantly greater than the expected geometric mean under BH. And the significance level increased with the portfolio size.

In addition, inclusion of the risk factor by forming the reward-to-variability ratio [193] for each strategy resulted in increased significance. The denominator, however, was based upon the period-to-period variability of returns within the investment horizon. While ex ante these fluctuations may be important,[3] once one is committed to a particular portfolio for a given investment horizon, these intermediate fluctuations become irrelevant. What is important is the variability of the return to each strategy about the expected return at the end of the investment horizon. As yet, there have been no empirical investigations of the effect of rebalancing on a measure of this variability.

Cheng and Deets [39] also examined the returns obtained from the application of the two strategies to real stock price data using equal weights and assuming no taxes or transaction costs. Weekly price relatives for the thirty stocks comprising the current Dow Jones Industrials were used. The period covered was from December 31, 1937 to February 21, 1969, yielding 1,625 weekly price relatives for each of the thirty stocks. Although the nature of the study did not permit statistical tests of significance, a comparison of the geometric total returns to the two strategies for random portfolios lent strong support for several observations. First, the expected return under RB is superior to that under BH. Second, the return superiority of rebalancing is an increasing function of the frequency of rebalancing. Third, there appears to be a positive relationship between the degree of RB superiority and the portfolio size. Fourth, the superiority of RB is an increasing function of the length of the investment horizon.[4]

Economic Hypotheses

The actual costs of pursuing a policy of RB are directly related to the frequency of rebalancing. Consequently, RB superiority in the statistical sense may not hold up when these costs are considered.

The first investigation of the effect of rebalancing after taxes and transaction costs was undertaken by Evans in his second paper. To provide a base upon which the costs could be computed, he assumed that the initial amount invested in each security was $1,000, $2,500, or $5,000. He utilized actual "odd lot" commission charges and assumed capital tax rates of 10 per cent, 15 per cent, or 25 per cent. After adjusting his previously described results for these costs, the return superiority of RB was eliminated, substantially modifying his statistical results. As expected, BH was most superior in the case of low initial investment and high tax rate. RB was only superior in the situation of $5,000 initial investment, 10 per cent capital tax rate, and the larger portfolio sizes. Computation of the reward-to-variability ratios as before, further demonstrated the highly situational nature of the determination of the better strategy. For the investor having the high marginal tax rate and investing the lowest amount, the BH strategy was significantly superior in terms of this measure. But, for the situation of a low marginal tax rate coupled with larger investments, the RB strategy performed significantly better.

In a comment concerning the economic interpretation of the Cheng-Deets conclusions, West and Tinic [220] attempted to adjust their data for the incremental transaction costs and the indirect costs of portfolio management associated with the rebalancing strategy. The result of their "rather crude" approximations of these costs was BH superiority.

Additional empirical evidence concerning the relative performance of RB and BH after costs is available as part of a study of alternative portfolio revision procedures undertaken by Smith [203]. He utilized weighted annual historical value relatives for a sample of 132 well-known NYSE stocks to estimate the parameters of Sharpe's diagonal model [195] in each of the thirteen years beginning 1951. This model was then employed to generate an efficient portfolio in each year. Based upon a total investment of $200,000 and exact brokerage fees, he calculated the internal rate of return to each unrevised portfolio, and for the portfolios annually rebalanced to maintain the initial ex ante efficient (not equal) set of weights. All portfolios were held through 1965. The adjustment policy was found to perform poorly compared with the BH policy, especially for the shorter investment horizons.

The conclusions drawn from these last two studies are by no means exhaustive. It should be noted that both samples consisted of large, well-known firms on the NYSE. Since these securities tend to be both high priced and of high quality, there is evidence that they also are the least volatile [84, 100]. RB may require more volatility to beat BH after costs.

Also, we take note of the fact that the Smith study utilized historical parameters for determining efficient portfolios. These portfolios were not necessarily ex post efficient over the period in which the alternative revision procedures were applied.

While there have not yet been any systematic studies of the economic performance of rebalancing when applied to fixed portfolios of highly volatile stocks, a recent article by Pinches and Simon [168] does offer some insight into this question. Their investigation examines two portfolio accumulation strategies utilizing low priced stocks. For both strategies, it is assumed that equal dollar amounts ($1,000) are invested in each of the stocks on the American Stock

Exchange selling at or below $5 per share. All stocks purchased remain in the portfolio until the end of the investment horizon. At quarterly intervals, an amount equal to the average value per security in the existing portfolio is invested in each security not presently held selling at or below $5 per share. The difference between the two strategies is that in the fixed proportion accumulation strategy the portfolio is rebalanced each quarterly period to maintain equal dollar amounts, before any additions are made. In the buy-and-hold accumulation strategy, no such adjustment is made. It was expected that the $5 or less criterion provides a larger proportion of lower quality, and therefore, highly volatile stocks than would a random sample.

The measure of performance was the geometric annual rate of return calculated through the use of geometric "linked relatives" as suggested by Levy [132]. These returns were computed for the two strategies after brokerage commissions for each of the 210 time periods of varying length obtained from the 21 different quarterly observations between January 1, 1965 and January 1, 1970.

The results found the rebalancing type strategy to be generally superior to the buy-and-hold accumulation strategy. The authors also observed that the fixed proportion strategy tends to outperform the BH strategy in rising markets, while BH appeared superior in falling markets. Finally, the longer investment horizons were found to improve the relative performance of the reallocation policy. These results for accumulation strategies have direct implications for the relative economic performance of RB for the "one-time" investments this dissertation is concerned with.

Analytical Results

Independence

The major analytical conclusions regarding the relative performance of the two strategies required the random walk assumption that for any security, price changes are independent, identically distributed random variables. In addition, because they ignored transaction costs and taxes, the conclusions address themselves to statistical, rather than economic hypotheses.

Under these assumptions, Cheng and Deets [39] showed that, for portfolios of securities weighted equally, the expectation of the geometric total return to BH is always at least as great as that under RB. And there is equality only when all securities in the portfolio have the same expected return. Their analysis also demonstrated that

> if the random walk theory holds, the superiority in the [expected geometric total] return of the buy-and-hold strategy over the rebalancing strategy is a decreasing function of the number of securities contained in the portfolio,[5] an increasing function of the length of the investment decision horizon, and an increasing function of the frequency with which the portfolio under the RB strategy is rebalanced [p. 12].

Cheng [33] showed that these results also hold for the general case of portfolios consisting of securities weighted arbitrarily.[6]

With the further assumption of stable covariances between securities, Cheng [33] examined the variance of the geometric totals under both policies for the general case. He found that if a certain condition is met,[7] the variance of the portfolio final return under BH is greater than that under RB. The variance superiority of RB was also shown to be an increasing function of the portfolio size, the length of the investment horizon, and the rebalancing frequency.

The aforementioned condition was also shown to guarantee that

$$\frac{\{V[G^T(BH)]\}^{0.5}}{E[G^T(BH)]} > \frac{\{V[G^T(RB)]\}^{0.5}}{E[G^T(RB)]}, \qquad (2\text{-}6)$$

which is the reciprocal of Sharpe's [193] reward-to-variability ratio[8] except for the absence of the risk-free interest rate. Stating that the subtraction of the risk-free interest rate from the denominators could only make the inequality stronger, Cheng [33] concluded that the RB strategy is superior to the BH strategy in performance.

Actually, subtraction of the risk-free interest rate may, in fact, reverse the inequality.[9] From this we conclude that for a portfolio with given weights, a rebalancing policy may not be superior to a buy-and-hold policy in terms of

Sharpe's measure, even if the condition is met.[10] Defining absolute performance superiority as the situation where one policy has a smaller variance than another with the same expected return, Cheng [33, 34] does, however, demonstrate that equation (2-6) is an approximate test of the absolute performance superiority of RB. That is, if the condition holds, there exists a different set of weights for the rebalancing policy which will raise the RB return to equal the BH return, while still maintaining a lower variance.

Since the original portfolio may be on the Markowitz buy-and-hold efficient frontier [146, 147], the important implication is that mean-variance frontiers beyond the BH frontier can be generated by following a strategy of rebalancing. In [34], Cheng derives the formal necessary and sufficient conditions for the superior frontier to exist, and shows that the superiority of the RB frontier over the BH frontier is maximized when the portfolio is continuously rebalanced.

Once transaction costs are considered, the favorable effect of increasing the frequency of rebalancing must be balanced against the unfavorable effect of the increasing costs of implementation. Because of the highly situational nature of this tradeoff, there have not been any analytical conclusions regarding economic hypotheses. Cheng [33], however, has suggested a model for the determination of the optimal rebalancing frequency in the mean-variance context involving the specification of the cost-adjusted rebalancing frontiers. He concludes that "adaptation to real-world investment management would certainly signal the need of developing a program model of the type proposed, for example, by Pogue [169]."

Without Independence

While many studies[11] have lent support to the independence assumption of the random walk model, the previously discussed analytical results of Cheng-Deets, in conjunction with the results of their empirical investigation utilizing the Dow Jones Industrials, provides strong evidence by contradiction that the independence assumption is not satisfied. They hypothesized the existence of a persistent negative dependence which may not be statistically significant, as the explanation for the expected return superiority of RB over BH.

Therefore, the important implications of Cheng's analytical investigations may not be applicable to real world stock markets which seem to be characterized by dependence in price changes over time. Nevertheless, the direction of the contradiction between the analytical results under a random walk and the empirical evidence is encouraging, for the ex post returns under RB have exceeded those which a random walk would suggest. In Cheng's [34] words, " . . . the market appears to be accommodating for the pursuit of the RB policy."

If stock price changes do not follow a random walk, it behooves us to examine the behavior of the RB policy under various assumptions with regard to dependence. Cheng and Deets [36], in replying to a comment by West and Tinic [220], state that it is not necessary that there be negative serial dependence for RB expected return superiority. They show that the "cross" serial correlation is also important. Let p_i, μ_i, and σ_i be the serial correlation coefficient, expectation, and standard deviation of the series of returns for security i. Now define the cross serial correlation between securities i and j as

$$\bar{p}_{ij} = \frac{\text{Cov }(R_{it}, R_{jt+1})}{\sigma_i \sigma_j}. \tag{2-7}$$

Then the condition for RB superiority in the special case of two securities, two periods, and equal weights can be written as

$$2\bar{p}_{1,2}\sigma_1\sigma_2 - p_1\sigma_1^2 - p_2\sigma_2^2 > (\mu_1 - \mu_2)^2, \tag{2-8}$$

derived by taking the expectation of equations (2-4) and (2-5) expressed as the appropriate inequality. The authors conclude, "It is insufficient to talk about RB superiority in terms of negative serial correlations alone." We also note that in this special case the relative superiority of RB is a decreasing function of the divergence in the expected returns of the securities.

At this time, equation (2-8) offers our only analytical glimpse of the statistical behavior of RB without the restrictive assumptions of the random walk model. But its even more restrictive assumptions necessitate that the reader be wary of any attempt to generalize from it to the multi-security, multi-period case.

A Perspective

Essential to the appreciation of the rebalancing policy of portfolio adjustment is the realization that it is not inconsistent with either the Markowitz single-period portfolio selection model [146, 147], or the intertemporal extensions suggested by Mossin [153], and by Smith [203, 204], Chen, Jen, and Zionts [32], and Pogue [169]. Cheng [33] makes this point quite clear:

> It should be emphasized that this paper points to the need of portfolio adjustment for reasons that are not ad hoc or arbitrary, e.g., expectation revisions, changes in the risk aversion of an investor, etc. It has concerned itself exclusively with the need of periodical portfolio adjustment under the condition of random walk with stationary parameters. In addition, our investigation relates primarily to a single period. Nor did it consider the wealth effect on portfolio selections. Typically, during a given period in a multi-period investment horizon . . . a policy of buy-and-hold prevails . . . If, however, a rebalancing policy . . . is pursued within each period, the mean and variance of final wealth will be superior to those generated under the intra-period BH alternative.

The portfolio composition may be determined according to the Markowitz model when one pursues a policy of intra-period rebalancing just as it is used for the buy-and-hold alternative. However, the appropriate expectational horizon is not the same for the two strategies. Cheng [34] has demonstrated analytically that the relevant parameter sets of the distributions of security returns for determining the Markowitz efficient frontier are those associated with the length of the investment period (T) under BH, and with the rebalancing interval (h) for RB.[12,13]

If, as suggested by Chen, Jen, and Zionts [32], the portfolio should be reviewed, and possibly revised, whenever expectations with regard to the parameter sets have changed, this difference in the appropriate expectational horizon has further significance. In the revision model, the length of the investment period is that portion of the investment horizon during which parameters are stationary, and that also is the relevant length of the expectational horizon for the determination of efficient portfolios under the BH policy. Unfortunately, this period of parameter stationarity is not known ex ante, and so neither is the appropriate expectational horizon. The result can be the selection of an ex post inefficient portfolio even when the means, variances, and correlations of the security returns are known with certainty. It is apparent that a policy of frequent rebalancing at fixed intervals within the investment period can mitigate this problem.

Since the experiment to be undertaken is restricted to the case of equal weight rebalancing, we conclude this chapter by examining the conditions necessary for a policy of equal weight rebalancing to be optimal.

Drawing on the work of Mossin [153] in the multi-period Markowitz context, we state two conditions for the optimality of a rebalancing policy. First, the yield distributions of the securities in the portfolio must be stationary as in the random walk model. Second, complete myopia must be optimal.[14] If the first condition is not met, it is clear that the Markowitz model would dictate a new set of weights as the parameters of the yield distributions changed within the investment period. And unless the second were also met, the change in wealth due to the outcome of the previous period's investment and the approach of the horizon would result in a different optimal portfolio even if the parameters were stationary. The logarithmic utility function is one that allows complete myopia.

Finally, if all securities in the portfolio have the same mean and variance, and all pairs have the same correlation, the optimal mean-variance portfolio has equal weights [18, 144].

With this background, we proceed to the major task of this research--to explore the behavior of the rebalancing policy.

Footnotes to Chapter II

1. The geometric mean return to a portfolio under BH or RB is the n^{th} root of the geometric totals defined in equations (2-4) or (2-5), respectively.

2. Evans computed the BH geometric mean for a portfolio as the average of the geometric means of the securities in the portfolio. Cheng and Deets [37] present proof of the bias in this method.

3. This point, shown by Cheng [34], will be discussed later in this chapter.

4. In terms of the random variables described at the beginning of this chapter, increasing the frequency of RB is equivalent to an increase in n, with h reduced so as to hold T constant. Increasing the length of the investment horizon refers to an increase in T accomplished by increasing n, holding h unchanged. Increasing the portfolio size is an increase in m.

5. The authors qualified this result. It requires that each security added to the portfolio have an expected return sufficiently close to the geometric mean return of the portfolio under RB. For random equal weight portfolios, the expected return of the entering security is equal to the portfolio mean return, and so the result holds.

6. The portfolio size effect depends upon the qualification in footnote 5 as well as the condition that whatever

weight is assigned to the entering security, the weights for the securities already in the portfolio are reduced proportionately.

7. The condition was derived in [33] and [34] where it is argued that it is an easy condition to meet for actual portfolios. The other analytical results with regard to the variance also require the condition to hold.

8. A discussion of this and other performance measures is included in the next chapter.

9. We disprove Cheng's premise by presenting a single contradictory example. Suppose the inequality in equation (2-6) appears as 20/10 > 8/5. Subtracting a risk-free interest rate of 4 from each denominator reverses the inequality.

10. The oversight with regard to the effect of the risk-free interest rate does not affect the validity of the statement that the relative performance of RB in terms of Sharpe's measure is an increasing function of the frequency of rebalancing.

11. See, for example, Fama [63], Kendall [118], Mandelbrot [142], and Osborne [166].

12. This conclusion would also appear to indicate that Smith's previously described empirical results contained a bias in favor of the RB policy by virtue of the fact that he utilized an expectational horizon corresponding to the rebalancing interval (a year). However, we also noted that his parameters were based upon historical returns, not the ex post

returns in the experimental period. A study by Friend and Vickers [82] concluded that there is no reason to assume that such a portfolio would be ex post efficient during the investment period.

13. Except for this paragraph and the next, this paper is concerned with the relative performance of RB in the single, as opposed to multi-period context. Consequently, the investment period T is referred to as the investment horizon. In the intertemporal context of these two paragraphs, however, the investment horizon is assumed to consist of several investment periods of length T, where T is not necessarily of fixed length. In both cases, the alternative policies are to rebalance within T, or to buy-and-hold within T.

14. A single-period decision rule is myopic when it is independent of the possible outcomes in the other periods in a multi-period decision problem.

CHAPTER III

EXPERIMENTAL DESIGN

Consideration of the exploratory nature of the study as well as the possibility of extremely complex relationships resulted in the choice of a full factorial design for the experiment.

Factors

The relative performance (response) of the RB and BH strategies was measured at all combinations of the levels of the following factors:

1. Number of securities in the portfolio
2. Differences in the means of the distributions of returns between securities
3. Autocorrelation within each series of security returns
4. Intercorrelation between series of security returns
5. Variance of the distribution of returns for each security.

The levels selected for each factor are shown in Table 1. Note that at least three levels are examined for each so that quadratic effects would be observed. Because their ranges include both positive and negative values, additional levels were provided for the autocorrelation and intercorrelation factors. The stochastic nature of the response variates (to

TABLE 1

SELECTED FACTOR LEVELS FOR FULL FACTORIAL DESIGN

Factor	Level					
	Number	1	2	3	4	5
1. Portfolio Size	3	2	3	4	-	-
2. Difference in Means	3	0	.001	.002	-	-
3. Autocorrelation	5	-.67	-.33	0	.33	.67
4. Intercorrelation	4	-.25	0	.20[a]	.50	-
5. Variance	3	.004	.005	.006	-	-
Total	540					

[a] Although the computer program utilized for the orthogonal polynomial breakdown (Ch. V) required equal intervals between factor level, this deviation was not considered significant.

be defined in the next section) dictated that the 540 design points be replicated twice. Any additional factor levels would have produced an unmanageably large number of design points.

The factor levels were chosen not to typify any particular set of security return series, but such that the characteristics of many real sets may reasonably be expected to fall within their ranges. The parameters of the distribution of returns for any security, however, depend upon the length of the period, or differencing interval, over which the returns are measured. Similarly, the returns to a policy of RB depend upon the frequency of rebalancing. Since this study does not attempt to examine the effects of the RB frequency or the horizon length, the approach is to specify that the portfolio is rebalanced each simulated "period" of indeterminate length. The effects of the various factors on the relative performance of the rebalancing policy given that frequency, are then examined by varying the statistical properties of the distributions of security returns based upon a differencing interval corresponding to that rebalancing frequency of one simulated "period."

The generation of random variables from distributions whose means differ by a particular amount (factor 2) required the specification of an absolute level of the expected return of each distribution. The average expected return for each period was taken as 0.007.[1] In this case each period may be thought of as being approximately a month.[2] Then, for example, a four security portfolio when factor 2 is at level 1 (see Table 1) would consist of four securities, each with expected return of 1.007. At level 2, securities 1 and 4 would have expected returns equal to 1.0065 and 1.0075, respectively. This difference of .001 will be compounded each period over the investment horizon. And at level 3, securities 1 and 4 would have expected returns of 1.006 and 1.008 (a difference of .002, compounded). Thus, the method of controlling the divergence in the expected returns between securities holds the expected return of the portfolio constant at 1.007 per period. It will be shown later that the major conclusions reached are invariant with respect to the portfolio expected return chosen.

It is clear from the literature on the random walk model of stock price behavior that the range of the levels of the autocorrelation factor indicated in Table 1 includes the values characterizing most actual security time series [10, 45, 62, 118, 166]. Similarly, empirical evidence indicates that the average intercorrelation between securities is approximately 0.5 [119, 128, 144]. Finally, a variance of the distribution of returns of approximately .005 is also empirically justifiable for an expected return of 1.007 [228].

Response Variables

Distributions of Geometric Totals

Geometric total returns were chosen to compare the two strategies. Since this study deals with only a single horizon length, the geometric mean return to each strategy would have been equally suitable for comparisons. The former measure was chosen, however, to avoid the possibility of drawing misleading conclusions if further work were done employing different horizon lengths.[3]

The stochastic nature of security rates of return results in the geometric totals $G^T(BH)$ and $G^T(RB)$ as defined in equations (2-4) and (2-5) also being random variables. Consequently, the comparison of the two strategies requires the inspection of the probability distribution of the geometric total return associated with each. And a determination of the effects of the factors described in the preceding section necessitates that these two probability distributions be calculated and compared for each factor configuration.

For a given factor configuration, the geometric total return to a strategy over a horizon of 50 periods constitutes one observation from the distribution. In view of the time and storage requirements of the computer program written to perform the simulation, it was decided that 20 such observations would provide sufficient information about each distribution.[4]

A response variable for the factorial experiment can then be defined as the difference between the distributions of the geometric totals for the two strategies in terms of some parameter of the distributions. Denoting the response variable by the particular parameter P selected, the response is defined as

$$P = P[G^T(RB)] - P[G^T(BH)]. \qquad (3\text{-}1)$$

This is consistent with the formulation in the Cheng-Deets study [39] in which P was the expected value of the distribution of geometric totals, and also in the papers by Cheng [33, 34], where in addition to the expected value, the differences between the variances of the distributions were examined.

The factorial experiment was undertaken to provide observations of the response(s) P under controlled conditions with respect to the five factors hypothesized to be functionally related to it.

Performance Measures

With the description of the general form of the response variable, it remains to specify the parameter(s) utilized for the comparison of the distributions of geometric totals. Referring to equation (3-1), five alternative definitions of P were chosen. The five parameters are the geometric mean (G), expected value (E), variance (V), skewness (S), and kurtosis (K).

The geometric mean of a distribution containing n observations of the variable x, $x_i>0$, is defined as

$$G = \left(\prod_{i=1}^{n} x_i\right)^{1/n}. \tag{3-2}$$

The other four measures here refer to the first four sample semi-invariants (cumulants) of the distribution. Defining the r^{th} moment and central moment of the discrete distribution g(x) respectively as

$$m_r = \sum_i x_i^r g(x_i) \tag{3-3}$$

and

$$m_r' = \sum_i (x_i - m_1)^r g(x_i), \tag{3-4}$$

the first four population semi-invariants are

$$\begin{aligned} \kappa_1 &= m_1 \\ \kappa_2 &= m_2' \\ \kappa_3 &= m_3' \\ \kappa_4 &= m_4' - 3m_2'^2. \end{aligned} \tag{3-5}$$

Then the first four sample semi-invariants are given by

$$\begin{aligned} E = k_1 &= S_1/n \\ V = k_2 &= \frac{nS_2 - S_1^2}{n(n-1)} \\ S = k_3 &= \frac{n^2 S_3 - 3nS_2S_1 + 2S_1^3}{n(n-1)(n-2)} \\ K = k_4 &= \frac{(n^3+n^2)S_4 - 4(n^2+n)S_3S_1 - 3(n^2-n)S_2^2 + 12nS_2S_1^2 - 6S_1^4}{n(n-1)(n-2)(n-3)} \end{aligned} \tag{3-6}$$

where n is the number of observations in the frequency distribution f(x) drawn from g(x), and $S_r = \sum_{i=1}^{n} x_i^r f(x_i)$. These sample statistics are unbiased estimators of the population semi-invariants [15, pp. 80-81].

For purposes of comparing the relative performance of RB and BH, it is assumed that the probability distributions of the geometric totals can be characterized either by the multiple response consisting of E, V, S, and K, or simply by the geometric mean (G) of the distributions. The use of these measures is justified in the next section.

Justification of Performance Measures

Multiple Response.--If (1) the members of a set of distributions of returns are each preferred to any distribution under consideration not in that set, and (2) there is indifference between the members of the preferred set, then we shall refer to that set as "efficient." The members of the efficient set are said to "dominate" all inefficient portfolios. The specific criteria for efficiency depends upon the assumptions about the decision maker's utility function.

Levy and Hanoch [129], in empirically comparing the relative effectiveness of various efficiency criteria for portfolio selection, have demonstrated that an inverse relationship exists between the strength of the assumptions about the utility function and the size of the efficient set.

If the only restriction on the utility function is that it is nondecreasing with respect to returns, x, then the criteria for the distribution f(x) to be preferred to g(x) is that

$$F(x) \leq G(x), \text{ for all } x, \qquad (3-7)$$

where a capital letter denotes the cumulative distribution. The efficiency criteria indicated in equation (3-7) is known as the General Efficiency Criterion [93, 97, 129, 133, 176]. While this criterion certainly makes no unreasonable assumption regarding the utility function, it is generally ineffective in limiting the size of the efficient set.

Imposing the further restriction that the utility function is concave (risk aversion) results in the General Concave Efficiency Criterion [97, 129, 133] which indicates that f(x) is preferred to g(x) if

$$\int_{-\infty}^{x}[G(t)-F(t)]dt \geq 0, \text{ for all } x. \qquad (3-8)$$

This criterion is somewhat more effective than the General Efficiency Criterion in reducing the size of the efficient set without introducing any unreasonable assumptions.[5]

Markowitz' monumental contribution to portfolio theory was the introduction of the Mean-Variance Efficiency Criterion [146, 147] under which the condition for f(x) to be preferred to g(x) is

$$E[f(x)] \geq E[g(x)] \text{ and } V[f(x)] \leq V[g(x)]. \qquad (3-9)$$

The wide acceptance of the mean-variance criterion was due to the fact that it is based upon the empirically justifiable assumptions of a nondecreasing and concave utility function, as well as only requiring estimates of the first two moments of the distributions. This contrasts with the General Concave Efficiency criterion which requires that the distributions of returns be completely specified.

The convenience associated with the necessity of only considering the mean and variance of each distribution is obtained, however, only at the expense of imposing some highly restrictive, and perhaps unrealistic, assumptions about either the utility function or about the shape of the distributions of returns. Specifically, for the mean-variance criterion to be optimal, one of the following conditions must hold [129]:

(1) The utility function is quadratic.

(2) Utility is a direct and increasing function of the mean and decreasing function of the variance.

(3) The distributions of returns belong to the same 2-parameter family.

With regard to the form of the utility function, the quadratic assumption has been questioned because at some point the function becomes decreasing, as well as the fact that it is inconsistent with the empirically justifiable notion of decreasing absolute risk aversion.[6] And Arditti [7] has presented evidence that the third moment of the distribution of returns is also considered significant by the investor. This argues against condition (2).

With regard to the shape of the distribution of returns, for our purposes, it appeared unlikely that the distributions of returns to the two strategies would belong to the same 2-parameter family for all factor combinations under study.[7]

In spite of these shortcomings, a huge body of literature, including a theory of capital markets, has been built around the mean-variance criterion. And even Cheng's most significant analytical results with regard to the Markowitz efficient frontier under rebalancing signify the importance of that criterion. Consequently, P was defined as the expected value and as the variance in the responses studied to allow for the application of this efficiency criterion as a means of comparing the relative performance of the two strategies.

But despite the necessity for imposing the highly tenuous assumptions previously described, Levy and Sarnat [131] demonstrated that this criterion is only slightly more effective in reducing the size of the efficient set than the General Concave Efficiency Criterion. Thus it was likely that the experiment would indicate that for certain factor configurations RB is mean-variance superior, for others BH is superior, and for many others neither strategy will dominate. And more significantly, no quantitative measure of superiority will be indicated for any factor configuration. We are faced here with the multiple response problem [173].

The attempt of portfolio theorists to come to grips with the multiple response problem has led to several performance measures which replace the two parameters discussed above with a single quantitative measure of performance. The measure most readily adapted to our needs is Sharpe's [193] reward to variability ratio, $(E-i)/\sqrt{V}$, where i is the risk-free interest rate. This measure, however, is based upon the highly restrictive assumptions of the capital market theory due to the mean-variance efficiency criterion. In addition it requires the specification of the risk-free interest rate. It does, however, combine the E and V response variables into a quantitative measure of performance which can be used for the analysis of the results of applying the alternative investment strategies to portfolios with the statistical properties being studied.

The empirical evidence denying the assumptions underlying the mean-variance criterion and the capital market theory derived from it[8] leads us to the task of examining characteristics of the distributions that are not described by the first two moments.

Defining utility, U, as a function of wealth, W, and income, x, and defining r=x/W as the rate of return, we have

$$U = U(x+W) = U(rW+W). \qquad (3\text{-}10)$$

Expanding U in a Taylor series about W+E(rW), taking expectations of both sides and assuming W and E(rW) constant, Arditti [7] obtained

$$E(U) = U[W+E(rW)] + \frac{W^2}{2!}U''(W+Wm_1)m_2 + \frac{W^3}{3!}U'''(W+Wm_1)m_3 + \ldots \quad (3\text{-}11)$$

where m_r denotes the $r^{\underline{th}}$ moment of the distribution of returns as before. Now assume that the utility function is concave and monotone increasing (as for the General Concave Efficiency Criterion), and that there is decreasing absolute risk aversion. Then an examination of the signs of the coefficients of the moments indicates that for a given expected return, performance is a decreasing function of the variance and an increasing function of the skewness of the distribution.

The lack of a theory to indicate the sign of U'''' does not enable us to reach any rigorous conclusions regarding the sign of the coefficient of the fourth moment, which would appear in the next term of the Taylor expansion. However, Fama [63, p. 94], when describing the economic implications of a highly leptokurtic distribution of stock price changes, stated that the "large number of abrupt changes . . . means that such a market is inherently more risky than a Gaussian market. The variability of a given expected yield is higher . . . and the probability of large losses is greater." Thus, intuitively, we would expect utility, and therefore performance, to be inversely related to the degree of kurtosis.

Consequently, the response variables with P equal to S and K, reflecting the third and fourth moments, deserve consideration along with the E and V responses in evaluating the distributions of returns associated with the two strategies.

The multiple response problem becomes more acute when the number of response variables is increased from two to four, since it is unlikely that either strategy will be dominant with respect to the mean, variance, skewness, and kurtosis. Nor can they be as simply related in a single parameter performance measure as can the mean and variance alone.[9] Therefore, this method cannot be employed to solve the multiple response problem when we specify performance as a function of the first four moments.

Desirous of obtaining meaningful results regarding the relative performances of RB and BH, we turn to the response with P defined as the geometric mean of the distribution to evaluate the strategies.

Geometric Mean Response.--The mean-variance criterion was developed as a means of explicitly considering the variability of returns as a trade-off against expected returns in order to reflect risk aversion and explain diversification. Similarly, skewness and kurtosis were introduced because they affect expected utility in the more general case.

Define the Geometric Mean Criterion as the preference of f(x) over g(x) if

$$G[f(x)] \geq G[g(x)]. \qquad (3\text{-}12)$$

Young and Trent [228] derive the following approximation to the geometric mean based upon the first four moments of the distribution of x_i, i=1,2,...,n; $x_i>0$ and n large:

$$G = m_1 - \frac{m_2}{2m_1} + \frac{m_3}{3m_1^2} - \frac{m_4}{4m_1^3}. \qquad (3\text{-}13)$$

With utility monotone increasing in G, the equation specifies the same type of investor attitudes toward variance and skewness as shown by Arditti using the principle of expected utility maximization (equation 3-11). And the sign of the coefficient of the fourth moment is also consistent with our beliefs as stated by Fama. Thus the geometric mean criterion appears to implicitly include consideration of the higher moments, while having the important property of being a single-parameter performance measure defined on a cardinal scale, rather than just providing an efficient set.

The geometric mean of a distribution may be calculated either directly according to equation (3-2), or accurately approximated by equation (3-13). If the third and fourth moments can be regarded as insignificant, as in the normal distribution, the approximation becomes

$$G = m_1 - \frac{m_2}{2m_1}. \qquad (3\text{-}14)$$

Squaring both sides and assuming the last term is negligible gives

$$G^2 = m_1^2 - m_2. \qquad (3\text{-}15)$$

Despite these favorable properties, the usefulness of the geometric mean criterion depends upon the validity of the utility function which it induces. Since

$$G = \left(\prod_{i=1}^{n} x_i \right)^{1/n} = e^{\frac{1}{n} \sum_{i=1}^{n} \log x_i} = e^{E[\log x_i]} \qquad (3\text{-}16)$$

the criterion of maximizing G is induced by a logarithmic utility function.

Latané [122] argued that the logarithmic function is appropriate for both (1) the wealth holder expected to be faced with repeated risks of similar types and magnitude with cumulative effects when the goal is maximizing wealth at the end of a large number of choices, and (2) the wealth holder to whom each risk is a unique event.

In the first case, a logical subgoal is to choose the strategy that has the greatest probability of leading to as much or more wealth than any other strategy at the end of a large number of choices. Latané showed that the strategy of choosing the portfolio with the maximum geometric mean return will maximize the asymptotic return, and so the geometric mean is a rational criterion for that subgoal. Thus, by taking the goal of portfolio management as long term wealth maximization, the objective geometric mean performance measure is justified without making any assumptions regarding the utility function except that it be monotone increasing.

This argument, however, is not valid for the wealth holder who considers each risk a unique event.

Nevertheless, Bernoulli [17] first proposed the geometric mean criterion for this type of wealth holder on the basis that it maximized utility if the utility resulting from a small increase in wealth varies inversely with the amount of wealth already possessed. Stigler [210] mentioned that the logarithmic utility function was accepted by Laplace and Marshall, and Savage [187, p. 94] said that it best approximates the utility function in moderate ranges. This function may also be defended by appealing to the Weber-Fechner Law of psychology.[10]

Several desirable properties of the logarithmic utility function were noted by Hakansson in his discussions about the growth optimal portfolio [94, 95]. First of all, in addition to being monotone increasing, it is strictly concave throughout, implying risk aversion and hence diversification. Secondly, unlike the quadratic utility function, it has the empirically defensible property of decreasing absolute risk aversion. Finally, because the logarithmic function exhibits constant relative risk aversion[11] (the quadratic does not) a myopic multi-period portfolio policy is consistent with it. As was pointed out in the previous chapter, a necessary condition for an equal weight rebalancing policy to be optimal is that complete myopia be optimal. Thus the assumption of a logarithmic utility function is especially consistent with the strategy being studied.

We conclude that the response variable given by

$$G = G[G^T(RB)] - G[G^T(BH)] \qquad (3\text{-}17)$$

is an appropriate single-parameter performance measure to be used as an alternative to the multiple response measures defined in terms of E, V, S, and K.

The next chapter describes the simulation methodology used to generate a data base consisting of two replicates of the five response variables at each of the 540 factor combinations under study.

Footnotes to Chapter III

1. All parameters of the distribution of security returns refer to the distribution of log price relatives. This point will receive further attention in the next chapter.

2. In a sample of 233 securities from January, 1956 - December, 1960, Young and Trent [228] found the average return to be 1.007; a log return of about 0.007.

3. Cheng and Deets [34] discuss the biases inherent in the use of the geometric mean return.

4. The number of simulated security returns is equal to the product of the number of observations, the length of the investment horizon, the levels of each factor, and the number of replications of the distribution. This is equal to $20 \times 50 \times 540 \times 2 = 1,080,000$.

5. Evidence of risk aversion in the stock market is presented by Sharpe [194].

6. The measure of absolute risk aversion reflected in a utility function $U(x)$ is $-U''(x)/U'(x)$ where the prime denotes the derivative. See Pratt [171].

7. Subsequent examinations of the parameters of the simulated distributions of geometric totals supported this expectation.

8. Friend and Blume [81], for example, show that the one-parameter performance measures based on this theory yield seriously biased estimates of performance.

9. Progress toward extending portfolio theory to three or more parameters is being made. See for example [8, 107, 108].

10. The Weber-Fechner Law states that equal ratios of physical stimulus corresponds to equal intervals of subjective sensations. It is a law of proportionality stating that the just discriminable difference between two quantities is proportional to their level. See [30, 168].

11. For a utility function $U(x)$, Pratt [171] defines a measure of relative risk aversion as $-XU''(x)/U'(x)$.

CHAPTER IV

SIMULATION: THE DATA GENERATING MODEL

Simulation Methodology

Simulation is a technique for experimenting with mathematical models describing a complex system on a digital computer [146]. The method allows controlled experiments since one or more of the values (levels) of variables (factors) may be changed while all others are held constant. This facilitates the study of the inter-relationships between the components.

To study the effects of the various factors on the relative performance of the equal weight rebalancing strategy, it is necessary to control their levels. The impossibility of regulating combinations of these factors within reasonable ranges using actual stock price data makes a simulation methodology imperative. Even if one were able to find sets of stock price series displaying the desired statistical properties, it would not be possible to find enough samples with the same combination of factor levels to obtain a distribution of the geometric totals associated with applying each strategy to a portfolio with that factor configuration.

The simulation methodology solves this problem of insufficient data by enabling the experimenter to generate his own data, controlled as to the factors under study. In this case the data generating model need not be a "truthful" representation of the actual stock price generating mechanism, for the validity of a model should be judged with regard to the purposes and goals for which it is developed [11].

The purpose of this experiment is to examine the properties of RB given various assumptions with regard to portfolio size, divergence of expected returns, autocorrelation, intercorrelation, and variance. While the simulation requires other assumptions in the specification of the data generating model, the intent of this study is not to argue for a particular mechanism generating stock prices. Rather, it is an investigation of the behavior of the RB policy as certain factors are varied given a particular generating process which might appear reasonable.

With this in mind, the conclusions of this study may be put in two different ways. One would begin by qualifying each statement with, "For time series generated according to the model specified in the simulation, then the relative performance of RB will be . . ." The other would draw conclusions regarding the application of the RB strategy to actual stock price series. The qualified statements cannot be denied, except due to technical errors. And while conclusions of this type are certainly of value, it is the conclusions of the second type which make the initial step from the realm of theory to that of application. It is these conclusions which require that the model be a "truthful" representation of reality, or that, in the spirit of positive economics, the implications drawn from it are insensitive to the inaccuracies in its assumptions.

Our intent is to reach conclusions of the latter type as well as the former. Consequently, the data generating model will be based upon assumptions which have strong empirical or theoretical backing where possible, and where there is controversy concerning alternative assumptions the sensitivity of the results to the alternatives will be tested.

Distribution of Stock Price Changes

Gaussian vs. Long-Tailed

The simulation of stochastic variables requires the specification of the general shape of the distribution from which they are generated. In this regard, the literature contains several proposed forms for the distribution of stock price changes.[1] From the point of view of the investor, however, these models may be divided into two main classes: the Gaussian (or normal) distribution and the long-tailed distributions [63, pp. 97-98].

The Gaussian assumption was selected for the data generating model because of the existence of well developed techniques for the generation of multivariate normal variates with arbitrary variance-covariance matrix, and for generating an autocorrelated time series of normally distributed variates.

The substantial body of empirical evidence favoring long-tailed hypotheses, however, would weaken any attempt to generalize the conclusions of this study to actual portfolios of securities without some indication that the results are insensitive to the form of the generating distribution. Consequently, a technique was developed for generating multivariate bilateral exponential random variables while controlling some of the factor levels under study within reasonable ranges.[2] With the bilateral exponential representing the class of long-tailed distributions, the null hypothesis that the response is not affected by the form of the distribution, Gaussian versus long-tailed, was tested.

Experiment

This experiment tests the null hypothesis that the relative performances of the RB and BH policies are unaffected by whether the stock price data is generated from a normal or a long-tailed distribution. A 2^6 factorial design was employed with two replications at each design point. Sets of security returns were simulated with the distribution form and the five factors under study controlled at all combinations of the following levels:

Code	Factor	Level 1	Level 2
1	Distribution	Normal	Bilateral Expon.
2	Number Securities	2	3
3	Δ Means	0	.002
4	Autocorrelation	-.67	0
5	Intercorrelation	0	.50
6	Variance	.004	.006

In all, there were two replications of sixty-four design points. The five responses calculated were G, E, V, S, and K as defined in the preceding chapter, and the statistical method employed to test the null hypothesis was analysis of variance.

Results

The analysis of variance for the five responses is presented in Tables 2 through 6. The tables include all main effects and only those interactions which contain factor 1, the form of the distribution. Each digit in the column labeled "Source of Variation" refers to one of the six factors as coded above. (For example, 14 refers to the distribution × autocorrelation interaction.)

With $F_{1,65}(.05) = 3.99$ and $F_{1,65}(.01) = 7.04$ we note that there are no sources of variation involving factor 1 which test significant at the 0.01 level and only the 1×2×4 interaction for the variance response (Table 4) is significant at the 0.05 level. Thus we do not reject the null hypothesis.

Hence we can proceed to describe the experiments with data simulated from Gaussian distributions with some confidence that the findings are also applicable to time series which demonstrate a better fit to long-tailed distributions.

TABLE 2

ANALYSIS OF VARIANCE FOR GEOMETRIC MEAN RESPONSE FOR 2^6 FACTORIAL DESIGN WITH FACTOR 1 THE DISTRIBUTION FORM

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	1	.00016	.00016	.25
2	1	.00057	.00057	.90
3	1	.00002	.00002	.03
4	1	.08931	.08931	141.76**
5	1	.00971	.00971	15.41**
6	1	.00916	.00916	14.54**
12	1	.00070	.00070	1.11
13	1	.00025	.00025	.40
14	1	.00022	.00022	.35
15	1	.00001	.00001	.02
16	1	.00133	.00133	2.11
123	1	.00079	.00079	1.25
124	1	.00001	.00001	.02
125	1	.00013	.00013	.21
126	1	.00002	.00002	.03
134	1	.00015	.00015	.24
135	1	.00108	.00108	1.71
136	1	.00008	.00008	.13
145	1	.00002	.00002	.03
146	1	.00035	.00035	.56
156	1	.00002	.00002	.03
1234	1	.00068	.00068	1.08
1235	1	.00037	.00037	.59
1236	1	.00057	.00057	.90
1245	1	.00014	.00014	.22
1246	1	.00008	.00008	.13
1256	1	.00031	.00031	.49
1345	1	.00002	.00002	.03
1346	1	.00037	.00037	.59
1356	1	.00159	.00159	2.52
1456	1	.00108	.00108	1.71
12345	1	.00033	.00033	.52
12346	1	.00021	.00021	.33
12356	1	.00091	.00091	1.44
12456	1	.00014	.00014	.22
13456	1	.00010	.00010	.16
123456	1	0	0	0
other	26	.03521	.00135	2.15
within rep	64	.04050	.00063	
total	127	.19670		

[a] * indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 3

ANALYSIS OF VARIANCE FOR EXPECTED VALUE RESPONSE FOR 2^6 FACTORIAL DESIGN WITH FACTOR 1 THE DISTRIBUTION FORM

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	1	.00005	.00005	.06
2	1	.00055	.00055	.62
3	1	0	0	0
4	1	.10397	.10397	118.15 **
5	1	.00753	.00753	8.56 **
6	1	.00842	.00842	9.57 **
12	1	.00017	.00017	.19
13	1	.00046	.00046	.52
14	1	.00002	.00002	.02
15	1	.00002	.00002	.02
16	1	.00158	.00158	1.80
123	1	.00151	.00151	1.72
124	1	.00037	.00037	.42
125	1	.00003	.00003	.03
126	1	0	0	0
134	1	.00037	.00037	.42
135	1	.00125	.00125	1.42
136	1	.00060	.00060	.68
145	1	0	0	0
146	1	.00044	.00044	.50
156	1	.00011	.00011	.13
1234	1	.00025	.00025	.28
1235	1	.00059	.00059	.67
1236	1	.00096	.00096	1.09
1245	1	.00026	.00026	.30
1246	1	.00015	.00015	.17
1256	1	.00027	.00027	.31
1345	1	0	0	0
1346	1	.00118	.00118	1.34
1356	1	.00213	.00213	2.42
1456	1	.00072	.00072	.82
12345	1	.00021	.00021	.24
12346	1	.00007	.00007	.08
12356	1	.00076	.00076	.86
12456	1	.00008	.00008	.09
13456	1	.00021	.00021	.24
123456	1	.00006	.00006	.07
other	26	.04281	.00165	1.87
within rep	64	.05615	.00088	
total	127	.23431		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 4

ANALYSIS OF VARIANCE FOR VARIANCE RESPONSE FOR 2^6 FACTORIAL DESIGN WITH FACTOR 1 THE DISTRIBUTION FORM

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	1	.00660	.00660	1.99
2	1	.00119	.00119	.36
3	1	.00489	.00489	1.47
4	1	.02271	.02271	6.84*
5	1	.00146	.00146	.44
6	1	.00023	.00023	.07
12	1	.01322	.01322	3.98
13	1	.00496	.00496	1.49
14	1	.00971	.00971	2.92
15	1	.00212	.00212	.64
16	1	.00006	.00006	.02
123	1	.01014	.01014	3.05
124	1	.01714	.01714	5.16*
125	1	.00065	.00065	.20
126	1	.00045	.00045	.14
134	1	.00411	.00411	1.24
135	1	.00033	.00033	.10
136	1	.00529	.00529	1.59
145	1	.00037	.00037	.11
146	1	.00089	.00089	.27
156	1	.00598	.00598	1.80
1234	1	.00725	.00725	2.18
1235	1	.00042	.00042	.13
1236	1	.00267	.00267	.80
1245	1	.00146	.00146	.44
1246	1	.00098	.00098	.30
1256	1	.00286	.00286	.86
1345	1	.00003	.00003	.01
1346	1	.00343	.00343	1.03
1356	1	.00001	.00001	.00
1456	1	.00399	.00399	1.20
12345	1	.00003	.00003	.01
12346	1	.00121	.00121	.36
12356	1	.00331	.00331	1.00
12456	1	.00128	.00128	.39
13456	1	.00033	.00033	.10
123456	1	.00540	.00540	1.63
other	26	.07039	.00271	.82
within rep	64	.21244	.00332	
total	127	.42999		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 5

ANALYSIS OF VARIANCE FOR SKEWNESS RESPONSE FOR 2^6 FACTORIAL DESIGN WITH FACTOR 1 THE DISTRIBUTION FORM

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	1	.35596	.35596	3.32
2	1	.17880	.17880	1.67
3	1	.27640	.27640	2.58
4	1	.27122	.27122	2.53
5	1	.00096	.00096	.01
6	1	.00034	.00034	.00
12	1	.36787	.36787	3.43
13	1	.23376	.23376	2.18
14	1	.36231	.36231	3.38
15	1	.00370	.00370	.03
16	1	.00336	.00336	.03
123	1	.27770	.27770	2.59
124	1	.37088	.37088	3.46
125	1	.00673	.00673	.06
126	1	.00003	.00003	.00
134	1	.20496	.20496	1.91
135	1	.00053	.00053	.00
136	1	.00231	.00231	.02
145	1	.00060	.00060	.01
146	1	.00320	.00320	.03
156	1	.31146	.31146	2.91
1234	1	.24763	.24763	2.31
1235	1	.00072	.00072	.01
1236	1	.00099	.00099	.01
1245	1	.01030	.01030	.10
1246	1	.00011	.00011	.00
1256	1	.24588	.24588	2.29
1345	1	.00026	.00026	.00
1346	1	.00018	.00018	.00
1356	1	.20145	.20145	1.88
1456	1	.27547	.27547	2.57
12345	1	.00011	.00011	.00
12346	1	.00001	.00001	.00
12356	1	.30284	.30284	2.83
12456	1	.21995	.21995	2.05
13456	1	.20145	.20145	1.88
123456	1	.31621	.31621	2.95
other	26	3.44363	.13245	1.24
within rep	64	6.86027	.10719	
total	127	15.56054		

[a] * indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 6

ANALYSIS OF VARIANCE FOR KURTOSIS RESPONSE
FOR 2^6 FACTORIAL DESIGN WITH FACTOR 1
THE DISTRIBUTION FORM

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	1	10.66931	10.66931	2.49
2	1	7.37232	7.37232	1.72
3	1	9.20473	9.20473	2.15
4	1	8.23825	8.23825	1.92
5	1	.32916	.32916	.08
6	1	.84679	.84679	.20
12	1	9.36525	9.36525	2.18
13	1	8.26872	8.26872	1.93
14	1	10.55529	10.55529	2.46
15	1	.26527	.26527	.06
16	1	.34851	.34851	.08
123	1	8.32575	8.32575	1.94
124	1	9.23694	9.23694	2.15
125	1	.61480	.61480	.14
126	1	.64454	.64454	.15
134	1	8.08171	8.08171	1.89
135	1	.29347	.29347	.07
136	1	.47106	.47106	.11
145	1	.28285	.28285	.07
146	1	.37465	.37465	.09
156	1	10.32567	10.32567	2.41
1234	1	8.12599	8.12599	1.90
1235	1	.35627	.35627	.08
1236	1	.74131	.74131	.17
1245	1	.62231	.62231	.15
1246	1	.67643	.67643	.16
1256	1	9.05624	9.05624	2.11
1345	1	.29137	.29137	.07
1346	1	.52518	.52518	.12
1356	1	10.00219	10.00219	2.33
1456	1	10.19543	10.19543	2.38
12345	1	.34809	.34809	.08
12346	1	.80502	.80502	.19
12356	1	10.99160	10.99160	2.56
12456	1	8.98933	8.98933	2.10
13456	1	9.97425	9.97425	2.33
123456	1	11.02327	11.02327	2.57
other	26	126.77690	4.87603	1.14
within rep	64	274.33972	4.28656	
total	127	587.95594		

[a] * indicates significance at the .05 level; ** indicates significance at the .01 level.

Generation of Stochastic Variates

As previously mentioned, the investigation requires the simulation of series of normally distributed security returns controlled as to the parameters of the distributions, and as to the autocorrelation within each series and intercorrelation between the series. In addition, the experiment just described required the generation of bilateral exponential variates, also controlling these factors. This section outlines the numerical methods used in generating these variates on the digital computer.

Normal Distribution

Naylor *et al*. [158, pp. 97-99] present a method for generating a normal random vector from the multivariate normal distribution with arbitrary mean vector and variance-covariance matrix. In addition they specify a technique for generating an autocorrelated time series of normal variates [pp. 118-120]. They are described below.

Multivariate Normal Distribution.--The probability density for the m-dimensional normal random vector **x** is given by

$$f(\mathbf{x}) = |2\pi\mathbf{V}|^{-0.5}\exp[-1/2(\mathbf{x}-\mathbf{u})'\mathbf{V}^{-1}(\mathbf{x}-\mathbf{u})], \qquad (4\text{-}1)$$

where **u** is the m-component vector of means and **V** is the variance-covariance matrix with elements σ_{ij}, i=1,...,m, j=1,...,m.

The generation of random normal vectors with mean vector **u** and variance-covariance matrix **V** requires the computation of the unique lower triangular matrix **C** such that

$$\mathbf{C} \cdot \mathbf{C}' = \mathbf{V}. \qquad (4\text{-}2)$$

C can be obtained from **V** by the "square root method" which provides the following set of recursive formulas for computing the elements c_{ij} of **C**:

$$c_{i1} = \sigma_{i1}/\sigma_{11}^{0.5}, \quad 1 \le i \le m$$

$$c_{ii} = (\sigma_{ii} - \sum_{k=1}^{i-1} c_{ik}^{2})^{0.5}, \quad 1 < i \le m \qquad (4\text{-}3)$$

$$c_{ij} = (\sigma_{ij} - \sum_{k=1}^{j-1} c_{ik} c_{jk})/c_{jj}, \quad 1 < j < i \le m.$$

After the computation of **C**, the vector **x** can then be calculated by

$$\mathbf{x} = \mathbf{Cz} + \mathbf{u}, \qquad (4\text{-}4)$$

where **z** is a standard normal vector.

Standard normal variates (the elements of **z**) may be generated by any one of many methods [158, pp. 90-97]. The method chosen for the data generating model was the Central Limit Approach which involves taking the sum of N uniformly distributed random variates $r_1, r_2, \ldots r_N$, subtracting the expectation of the sum and dividing by its standard deviation. With N=12, chosen by balancing computational efficiency against accuracy,[3] and the r_i drawn from the computer's uniform random number generator defined on the [0,1] interval, the standard normal variate is calculated as

$$Z = \sum_{i=1}^{12} r_i - 6. \tag{4-5}$$

Autocorrelated Normal Variates.--The technique presented in the above reference for generating a time series of normal variates with a linear autocorrelation function is also based upon the Central Limit Approach. If a standard normal variate in period t is generated as

$$Z_t = \sum_{i=1}^{12} r_i - 6, \tag{4-6}$$

then the next variate is generated as

$$Z_{t+1} = \sum_{i=p+1}^{12+p} r_i - 6, \quad p \leq 12 \tag{4-7}$$

where (12-p) of the r_i are common to successive sums, creating the dependence in the series.

The autocorrelation function with lag k is then

$$\rho(k) = \begin{cases} 1-kp/12 & \text{for } k \leq 12/p \\ 0 & \text{otherwise.}^{4} \end{cases} \tag{4-8}$$

The generation of a negatively autocorrelated time series can be accomplished for a lag of one period by summing $(1-r_i)$

in place of r_i in equation (4-7) for those (12-p) uniform random numbers which are common to successive sums.

Combining Autocorrelation and Intercorrelation.--The techniques described above enable the experimenter to control either the intercorrelation between series of simulated security returns or the autocorrelation within a series.

What remains is to combine these two methods so that the two factors can be jointly controlled. This was accomplished satisfactorily according to the following scheme: Let $x_{ij} = \log R_{ij}$, where R_{ij} is as defined in equation (2-1). Suppose it is desired to generate controlled returns x_{ij}, $i=1,\ldots,m$, $j=1,\ldots,n$, where i is the security and j is the time period.

1. Let j=1. Generate a vector $\mathbf{z}_j$ of m independent standard normal variates z_{ij}, $i=1,\ldots,m$ each according to equation (4-5).

2. Obtain the **C** matrix according to equations (4-3) from the variance-covariance matrix **V** associated with the desired intercorrelation and variance factors.

3. Apply the **C** matrix and desired mean vector **u** to the standard normal vector $\mathbf{z}_j$ according to equation (4-4). The result is the vector $\mathbf{x}_j$ of security returns in period j with components x_{ij}, $i=1,\ldots,m$.

4. Increment the time period j by 1. Generate a new vector $\mathbf{z}_j$ such that each component z_{ij}, $i=1,\ldots,m$ has the desired autocorrelation with the corresponding component

z_{ij-1}, i=1,...,m in the previous period. See equations (4-7) and (4-8) for lag k=1.

5. Repeat steps 3 and 4 until return vectors $\mathbf{x}_j$ are generated for n periods.

Bilateral Exponential Distribution

The method developed here for controlling autocorrelation and intercorrelation in multivariate bilateral exponential series was based upon the following results demonstrated by Agnew [1].

Let **V** be the variance-covariance matrix of an m-dimensional multivariate normal distribution with mean vector **u=0** and the elements of the main diagonal of **V** equal to unity.

Now let θ have the exponential distribution

$$g(\theta) = \exp(-\alpha\theta) \qquad (4-9)$$

with the parameter α=1.

Then mixtures with respect to g of centered m-dimensional multivariate normal variates with variance-covariance matrices θ**V** will be distributed as an m-dimensional standardized bilateral exponential distribution with density

$$f(\mathbf{x}) = 2^{-0.5}\exp(-2^{0.5}|\mathbf{x}|) \qquad (4-10)$$

and variance-covariance matrix **V**.

The inverse transformation technique provides a method for generating random variates from the exponential distribution given by equation (4-9). Let r be drawn from the uniform distribution on the [0,1] interval. Then θ is simulated as

$$\theta = -(1/\alpha) \log r. \tag{4-11}$$

While independent bilateral exponential variates may be generated more efficiently as the convolution of the exponential density with its mirrored image, that procedure does not furnish any means of regulating the autocorrelation or intercorrelation factors. Agnew's results, however, enable us to utilize the techniques for simulating autocorrelated and intercorrelated normal variates as a means of controlling these factors when generating variates from long-tailed bilateral exponential distributions. While Agnew dealt only with the standardized distribution, the extension to the general case is straightforward.

The generating algorithm then required only slight modification to that presented above for "Combining Autocorrelation and Intercorrelation" in the Gaussian case. First, it required the insertion of two steps between steps 1 and 2.

1a. Generate a variate θ from the exponential distribution with $\alpha=1$ according to equation (4-11).

1b. Multiply all elements in the variance-covariance matrix by the scalar θ to obtain a new $\mathbf{V}=\theta\mathbf{V}$.

Second, step 5 was modified to read as follows:

5. Repeat steps 1a through 4 until return vectors $\mathbf{x}_j$ are generated for n periods.

Thus, multivariate normal distributions with variance-covariance matrices $\theta\mathbf{V}$ are mixed by giving θ an exponential distribution so as to generate bilateral exponential variates with the desired parameter configurations. In the special case where instead of being distributed exponentially, $\theta=1$, the variates remain normal.

Validation

The problem of validation is attacked by bearing in mind that (1) our purpose is to obtain insight into the behavior of rebalancing under conditions which might reasonably be considered to include those in real world security markets, and (2) we have specified a data generating model, and an alternative to it, both of which have strong theoretical and empirical documentation in the literature.

At this point then the question is whether the methods previously described for the generation of stock price series on the computer provide satisfactory approximations to the theoretical distributions with the desired parameter configurations. Additional verification of the data generating model can also be inferred from later experimental results.

Distribution Form

Each series of security returns generated was tested to determine whether or not the variates were drawn from the type of distribution intended.

To test the goodness of fit of the distribution of the natural logarithms of simulated returns to the normal distribution, a test developed by Gurland and Dahiya [92] based on generalized minimum chi-square techniques was used.

Let

$$Q = nd'Bd \tag{4-12}$$

where

$$d' = [\log (S_2'),\ S_3',\ \log (S_4'/3)] \tag{4-13}$$

$$B = \begin{vmatrix} 1.5 & 0 & -.75 \\ 0 & 1/(6S_2'^3) & 0 \\ -.75 & 0 & .375 \end{vmatrix} \tag{4-14}$$

and S_r' is the r^{th} central sample moment for the frequency distribution consisting of n observations. Then the asymptotic distribution of Q is chi-square with two degrees of freedom. Thus, to carry out a test of fit for normality at a particular level of significance one refers to the appropriate critical point of the χ_2^2 distribution. This test was chosen for its convenience as well as the fact that it is extremely powerful against the bilateral exponential alternative.[5]

The results of applying the Q test to the simulated data were as hoped. For the overwhelming majority of the time series generated according to the Gaussian model, the null hypothesis of normality could not be rejected at the

0.05 level ($\chi^2_2(.05)=5.991$). And all of those series supposed to be from a bilateral exponential distribution were able to reject the hypothesis of normality at even the 0.005 level ($\chi^2_2(.005)=10.597$).[6]

Because the bilateral exponential distribution was chosen as a representative of the class of long-tailed distributions, a further test was performed to demonstrate that the lack of fit to the normal distribution was due to excessive kurtosis rather than skewness.

Defining the second, third and fourth sample semi-invariants as in equations (3-6), the statistics

$$g_1 = k_3/k_2^{1.5} \qquad (4\text{-}15)$$

and

$$g_2 = k_4/k_2^2$$

may be employed to test the skewness and kurtosis respectively.

Bennet and Franklin [15] provide a table of critical values for $|g_1|$ and lower and upper confidence limits for g_2 when the null distribution is normal. For 1000 degrees of freedom (each series is twenty observations of fifty periods of returns) the 99% confidence intervals are $|g_1|<0.180$ and $-0.31<g_2<0.42$. An inspection of the calculated values indicated that the rejection of the Gaussian hypothesis was in fact due to leptokurtosis.

Controlling Factor Levels

Normal Distribution.--The generation of normal variates was based upon well developed numerical methods. Therefore, we are quite confident in our ability to generate such returns from populations with desired parameters, autocorrelation, and intercorrelation characteristics.

A visual examination of the variance and autocorrelation properties of each series, and of the intercorrelation between series in a portfolio, in all cases showed a close correspondence between the ex post levels of these factors and the population parameters of the distributions from which they were supposed to be generated. The slight deviations in each sample from the population values could easily be attributed to sampling variation. But the sampling variation did not prevent a clear resolution of the different factor levels.

On the other hand, an examination of the mean of the ex post returns in each series did not indicate that the simulation was effective in generating time series that successfully differentiated between expected returns of .006, .0065, .007, .0075, and .008. This is illustrated in Table 7 which is based upon the ex post means of a random sample of fifteen independent time series from each of the five populations. Each time series is of length 1000, consisting of twenty observations of fifty returns. While the t-values in each case do not reject the null hypothesis that

TABLE 7

COMPARISON OF EX POST MEANS OF SIMULATED TIME SERIES WITH THE RESPECTIVE POPULATION MEANS

Sample Statistics[b]	Population Mean[a]				
	.006	.0065	.007	.0075	.008
Mean	.0066	.0060	.0056	.0076	.0138
Standard Deviation	.0015	.0025	.0033	.0029	.0187
T-Value[c]	1.46	-0.75	-1.58	0.08	1.20
Degrees of Freedom	14	14	14	14	14

[a]The population means are specified in the **u** vector of equation (4-4).

[b]Based on the ex post means of a random sample of 15 simulated time series of length 1000 for each population.

[c]The t-value tests the null hypothesis that the sample mean is equal to the population mean. The critical point for the two-tailed test is $t_{14}(.05)=2.145$.

the samples were generated from populations with the desired expected returns, the difficulty in controlling the sample mean returns within reasonable limits relative to one another should be noted. It will require that we be especially careful when drawing inferences regarding the effects of both the divergence in means (factor 2) and the portfolio size (factor 1) on the performance of RB.

Bilateral Exponential Distribution.--Experience with the method developed for the generation of bilateral exponential variates with controlled factor levels indicated that it satisfactorily transfers the parameters associated with the variance-covariance matrix of the multivariate normal distribution to the multivariate bilateral exponential distribution. And the sampling variation in the means was also similar to that discussed for the Gaussian case.

The attempt to impose a specified amount of autocorrelation within each series, however, consistently resulted in a loss of auto-dependence as the Gaussian distributions were mixed to form the double exponential. For example, equation (4-8) indicates that when eight of the twelve uniform variates (or their complements, $1-r_i$) are common to successive sums (p=4), the autocorrelation coefficient within a Gaussian time series for a lag of one period should be ±.67. With p=4, the ex post autocorrelation coefficients for the simulated bilateral exponential time series were found to be consistently and significantly closer to zero than the ±.67 expected.

A trial and error approach found that a value of p=2 (ten of the twelve uniform variates in common) would yield ex post autocorrelation coefficients which approximated ±.67. Along with the generation of bilateral exponential time series characterized by zero autocorrelation, which posed no problem, this approximation to an autocorrelation of -.67 was considered suitably accurate to proceed with the previously described factorial experiment on the sensitivity of the responses to the form of the generating distribution.

The fact that there is as yet no theory predicting the population value of this characteristic for any value of p, nor any other method for accurately controlling the dependence within a series of bilateral exponential variates, was a major consideration in the choice of the Gaussian hypothesis for the data generating model.

A description of the computer program written for the simulation appears in Appendix B.

Footnotes to Chapter IV

1. Although this paper for ease of exposition often speaks of the distribution of stock price changes, we are referring to the distribution of the natural logarithm of the price changes. As in all the literature related to our study, the model is concerned with stock price changes rather than with the prices themselves. Kendall [118] explains that the previous price is the starting point for negotiations in a free market, and so it is the change in price, rather than the absolute level, which constitutes the major element in price negotiations. As pointed out by Roberts [180], the statistical behavior of price changes, which may be independent, is much simpler than that of price levels which certainly are not.

This paper is also in accord with the common practice of defining distributions of security returns in terms of the logarithmic transformation, $\log (P_t/P_{t-1}) = \log P_t - \log P_{t-1}$. As demonstrated by Moore [151], the magnitude of price changes tends to be proportional to the price level. This is in agreement with the common economic assumption that the percentage price change (yield) is important, not the absolute price change [42, p. 82]. The assumption relates to the discussion of the logarithmic utility function in Chapter II. Moore [152] also explains his observation by appealing to Simon and Bonini's [198] law of proportional effect of the size of business firms, and the evidence connecting firm size and security prices. Although other transformations would be satisfactory, the logarithmic transformation is often utilized to neutralize the instability in the mean and variance due to this type of security return behavior. See the Appendix for a brief review of the various proposals offered for the form of this distribution.

2. A description of this technique as well as the method for generating Gaussian variates is presented later in this chapter.

3. Although this value of N truncates the distribution at the $\pm 6\sigma$ limits, it is quite accurate within ± 3 standard deviations. If we must have a bias in the size of the extreme tails, we favor it in this direction since it accentuates the difference between this distribution and the long-tailed distributions represented by the bilateral exponential.

4. The fact that p and k are integers and $p \le 12$ limits the degree of autocorrelation to the thirteen levels 0, 1/12, 2/12, ..., 11/12, 1. In order to obtain intermediate values of the autocorrelation coefficient the Central Limit Approach would require summing more than N=12 uniform random variates.

5. Gurland and Dahiya [92] calculated the power of the Q test for normality against a bilateral exponential alternative for n=100 and a .05 significance level at .879. This compared with a modified form of the Pearson chi-square test which had a power of .800.

6. To increase the power of the test the twenty observations of each of the simulated fifty period security return series were combined to form a series of length 1000 for each stock. The Q test for normality was then applied to these distributions of 1000 returns. The verification of the ability to control the parameter configurations, as discussed in the next section, also was based upon these combined series so as to reduce the sampling variation.

CHAPTER V

STATISTICAL ANALYSIS: THE DEVELOPMENT OF THE FACTOR-RESPONSE MODELS

In the preceding chapters the factors and responses were defined and the data generating model for the simulation was described. The output of the simulation was a data base consisting of two observations of the five responses for each of the 540 portfolios (factor configurations) specified in the data generating model. The factor levels were controlled so that the observations for each of the response variables conformed to a $3^3 \times 4 \times 5$ factorial design replicated twice.

The purpose of this chapter is to trace the development of the mathematical models which predict each response as a function of the factors under study. A stepwise approach to validating the model and evaluating the goodness of fit was employed to provide a reasonable degree of confidence in the inferences drawn from it with regard to the statistical properties of the rebalancing policy.

Assumptions

The factor-response models were developed through the use of multivariate analysis of variance and multiple non-linear regression analysis. The statistical inferences required that the observations of the responses were generated according to a fixed effects regression model with a spherical normal vector of disturbances. The assumptions of the model are that the errors are (1) mutually independent, (2) normally distributed variables with expectation zero and (3) common variance.

Generally, a computer simulation model will satisfy the assumptions required for the data analysis [158, p. 333]. The mutual independence condition is automatically satisfied by the simulation methodology which creates each factor-response observation from a set of security return series generated from a different sequence of pseudorandom numbers. With regard to the other conditions, Scheffé [188, pp. 331-369] concludes that nonnormality has little effect on inferences about fixed main effects or interactions, nor does heteroskedasticity have serious effects so long as the cell numbers are equal.

Analysis of Variance

A simple linear relationship between the response(s) and the independent variables (factors) was not expected. However, the inclusion of all possible high order main effects and interactions (e.g., $x_1^2 x_2^3 x_3$, where the subscript denotes the factor, and the superscript is the power) in the list of regressors was not possible. So as not to arbitrarily exclude a significant term from the regression, an analysis of variance was carried out for each response variable to determine which terms would significantly contribute to the predicting equation.

The Biomedical Computer Program for the Analysis of Variance for Factorial Design [53, pp. 495-510] was employed to partition the total sum of squares. Tables 8 through 12 are the analysis of variance tables for the five response variables G, E, V, S, and K, respectively. The coding of the factors in these tables and in the remainder of this paper is as follows:

Code	Factor	Variable
1	Number Securities	x_1
2	Δ Means	x_2
3	Autocorrelation	x_3
4	Intercorrelation	x_4
5	Variance	x_5

In addition to the information presented in these tables, the output of the analysis of variance included a breakdown of the sum of squares corresponding to each main effect and two-factor interaction into orthogonal polynomial components having a single degree of freedom.

The orthogonal polynomial breakdown provides information as described in the following examples: Table 8 indicates that among the significant sources of variation at the 0.05 level are the main effects of factors 1 and 3. The breakdown (not shown) showed that only the linear component (x_1) was significant for factor 1, while factor 3 had significant

TABLE 8

ANALYSIS OF VARIANCE FOR GEOMETRIC MEAN RESPONSE FOR $3^3 \times 4 \times 5$ FACTORIAL DESIGN

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	2	.08559	.04280	21.29**
2	2	.00254	.00127	.63
3	4	10.50350	2.62588	1306.41**
4	3	.05259	.01753	8.72**
5	2	.02730	.01365	6.79**
12	4	.00915	.00229	1.14
13	8	.66366	.08296	41.27**
14	6	.01247	.00208	1.03
15	4	.00969	.00242	1.21
23	8	.02233	.00279	1.39
24	6	.01377	.00230	1.14
25	4	.00075	.00019	.09
34	12	1.12400	.09367	46.60**
35	8	.39758	.04970	24.73**
45	6	.00579	.00096	.48
123	16	.01101	.00069	.34
124	12	.01473	.00123	.61
125	8	.01951	.00244	1.21
134	24	.08264	.00344	1.71*
135	16	.09896	.00618	3.08**
145	12	.01629	.00136	.68
234	24	.03927	.00164	.81
235	16	.01808	.00113	.56
245	12	.02186	.00182	.91
345	24	.06442	.00268	1.34
1234	48	.06893	.00144	.71
1235	32	.07448	.00233	1.16
1245	24	.03120	.00130	.65
1345	48	.07221	.00150	.75
2345	48	.11226	.00234	1.16
12345	96	.16592	.00173	.86
within rep	540	1.08359	.00201	
total	1079	14.92607		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 9

ANALYSIS OF VARIANCE FOR EXPECTED VALUE RESPONSE FOR $3^3 \times 4 \times 5$ FACTORIAL DESIGN

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	2	.07929	.03965	9.44**
2	2	.00272	.00136	.32
3	4	16.39443	4.09361	975.86**
4	3	.16487	.05496	13.08**
5	2	.11340	.05670	13.50**
12	4	.00627	.00157	.37
13	8	.69039	.08630	20.55**
14	6	.01351	.00225	.54
15	4	.01558	.00389	.93
23	8	.04067	.00508	1.21
24	6	.03478	.00580	1.38
25	4	.00078	.00019	.05
34	12	1.61107	.13426	31.97**
35	8	.78060	.09757	23.23**
45	6	.01353	.00226	.54
123	16	.01078	.00067	.16
124	12	.03030	.00252	.60
125	8	.01788	.00224	.53
134	24	.10658	.00444	1.06
135	16	.12140	.00759	1.81*
145	12	.02943	.00245	.58
234	24	.11921	.00497	1.18
235	16	.03530	.00221	.53
245	12	.04995	.00416	.99
345	24	.10498	.00437	1.04
1234	48	.19654	.00409	.97
1235	32	.10858	.00339	.81
1245	24	.07267	.00303	.72
1345	48	.18616	.00388	.92
2345	48	.25935	.00540	1.29
12345	96	.36370	.00379	.90
within rep	540	2.26917	.00420	
total	1079	24.04387		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 10

ANALYSIS OF VARIANCE FOR VARIANCE RESPONSE FOR $3^3 \times 4 \times 5$ FACTORIAL DESIGN

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	2	.12560	.06280	.38
2	2	.23932	.11966	.72
3	4	62.00656	15.50164	92.94**
4	3	1.03421	.34474	2.07
5	2	3.35219	1.67610	10.05**
12	4	.35120	.08780	.53
13	8	.57048	.07131	.43
14	6	.80358	.13393	.80
15	4	.69280	.17320	1.04
23	8	1.39740	.17467	1.05
24	6	1.18153	.19692	1.18
25	4	.16360	.04090	.25
34	12	1.72252	.14354	.86
35	8	9.52544	1.19068	7.14**
45	6	.41956	.06993	.42
123	16	1.46257	.09141	.55
124	12	3.12194	.26016	1.56
125	8	.43297	.05412	.32
134	24	4.22937	.17622	1.06
135	16	2.82381	.17649	1.06
145	12	1.67764	.13980	.84
234	24	4.96436	.20685	1.24
235	16	.83744	.05234	.31
245	12	2.63863	.21989	1.32
345	24	1.19086	.04962	.30
1234	48	13.46647	.28055	1.68**
1235	32	1.82122	.05691	.34
1245	24	4.34004	.18083	1.08
1345	48	9.00954	.18770	1.13
2345	48	9.16822	.19100	1.15
12345	96	19.66509	.20484	1.23
within rep	540	90.07266	.16680	
total	1079	254.50882		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 11

ANALYSIS OF VARIANCE FOR SKEWNESS RESPONSE FOR $3^3 \times 4 \times 5$ FACTORIAL DESIGN

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	2	16.92676	8.46338	.66
2	2	36.18428	18.09214	1.42
3	4	1302.80922	325.70230	25.53**
4	3	19.49678	6.49893	.51
5	2	143.78494	71.89247	5.63**
12	4	59.59531	14.89883	1.17
13	8	81.56125	10.19516	.80
14	6	87.50001	14.58334	1.14
15	4	32.94566	8.23641	.65
23	8	139.27588	17.40948	1.36
24	6	85.91711	14.31952	1.12
25	4	28.29435	7.07359	.55
34	12	70.62830	5.88569	.46
35	8	517.33369	64.66671	5.07**
45	6	30.30766	5.05128	.40
123	16	222.15313	13.88457	1.09
124	12	252.42916	21.03576	1.65
125	8	68.06687	8.50836	.67
134	24	368.77428	15.36559	1.20
135	16	143.17333	8.94833	.70
145	12	139.51499	11.62625	.91
234	24	343.46969	14.31124	1.12
235	16	140.27824	8.76739	.69
245	12	133.08947	11.09079	.87
345	24	113.31743	4.72156	.37
1234	48	1019.87654	21.24743	1.67**
1235	32	234.84406	7.33888	.58
1245	24	365.83211	15.24300	1.19
1345	48	598.62283	12.47131	.98
2345	48	517.76410	10.78675	.85
12345	96	1482.87369	15.44660	1.21
within rep	540	6890.19386	12.75962	
total	1079	15686.83503		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 12

ANALYSIS OF VARIANCE FOR KURTOSIS RESPONSE
FOR $3^3 \times 4 \times 5$ FACTORIAL DESIGN

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	2	1633.54944	816.77472	.62
2	2	4416.23277	2208.11638	1.68
3	4	59527.29708	14881.82427	11.31**
4	3	634.08086	211.36029	.16
5	2	10205.53210	5102.76605	3.88*
12	4	5853.81575	1463.45394	1.11
13	8	7193.30844	899.16355	.68
14	6	8681.29762	1446.88294	1.10
15	4	3679.70002	919.92501	.70
23	8	17261.32692	2157.66587	1.64
24	6	6574.89477	1095.81579	.83
25	4	5845.10386	1461.27597	1.11
34	12	2742.63664	228.55305	.17
35	8	40231.27184	5028.90898	3.82**
45	6	2443.52429	407.25405	.31
123	16	22880.11577	1430.00724	1.09
124	12	21907.70051	1825.64171	1.39
125	8	6393.02964	799.12870	.61
134	24	35602.57963	1483.44082	1.13
135	16	15385.46790	961.59174	.73
145	12	14933.00460	1244.41705	.95
234	24	26027.56610	1084.48192	.82
235	16	25390.29128	1586.89320	1.21
245	12	13008.76091	1084.06341	.82
345	24	9525.38470	396.89103	.30
1234	48	87233.99822	1817.37496	1.38
1235	32	23935.83376	747.99480	.57
1245	24	37160.48392	1548.35350	1.18
1345	48	59658.47228	1242.88484	.94
2345	48	51687.69584	1076.82700	.82
12345	96	148199.40022	1543.74375	1.17
within rep	540	710623.40564	1315.96927	
total	1079	1486476.76331		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

linear (x_3), quadratic (x_3^2), and fourth degree (x_3^4) components. Similarly, the polynomial breakdown showed that the significant 1×3 interaction of Table 8 was due to the significant components x_1x_3, $x_1x_3^2$, $x_1x_3^3$, and $x_1^2x_3$. Thus, the polynomial breakdown provides guidance as to what terms to include in the regression equations.

The initial step in the sequential validation procedure was the verification of the data generating model as discussed in chapter IV. The comparison of the ANOVA results with some of the analytical conclusions listed in chapter II constitutes the next step in the process. Verification of the experimental evidence at this stage also has the effect of reconfirming the validity of the simulation methodology.

The approach to validation was based upon the premise that certain insights which may be obtained through the use of analytical techniques may not appear statistically significant in the simulation experiment. Although the methodology employed here may not be sensitive enough to detect certain analytically determined results, there should not be a statistically significant contradiction of them. The approach then was to use the relevant analytical conclusion, stated as an inequality, as the null hypothesis, and determine whether the results developed here reject the null hypothesis in favor of the inequality in the opposite direction.

Four of the analytical observations outlined in chapter II were suitable for this purpose. Since the random walk assumptions were needed for their development, to facilitate the validation the analysis of variance was recomputed only for those factor configurations in which the autocorrelation equaled zero. Under this assumption, Tables 13, 14, and 15 present the analysis of variance for the geometric mean, expected value, and variance responses, respectively.

The analytical results employed for the validation, all under the independence assumption, were:

1. The expected return to RB is less than or equal to the expected return to BH.

2. The expected return superiority of BH over RB is a decreasing function of the number of securities in the portfolio.

3. The expected return superiority of BH over RB requires that not all the securities in the portfolio have the same expected return.

4. The variance of the distribution of returns to RB is less than or equal to the variance of the distribution of returns to BH.

1. $E[G^T(RB)] \leq E[G^T(BH)]$

Since we are only concerned with statistically significant contradictions of the statements, validation required a single-tailed test with the null and alternative hypotheses given by:

TABLE 13

ANALYSIS OF VARIANCE FOR GEOMETRIC MEAN RESPONSE FOR $3^3 \times 4$ FACTORIAL DESIGN WITH AUTOCORRELATION EQUAL TO ZERO

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	2	.00030	.00015	.24
2	2	.00074	.00037	.59
4	3	.00106	.00035	.56
5	2	.00014	.00007	.11
12	4	.00196	.00049	.78
14	6	.00399	.00066	1.06
15	4	.00107	.00027	.42
24	6	.00187	.00031	.49
25	4	.00254	.00063	1.01
45	6	.00226	.00038	.60
124	12	.00996	.00083	1.32
125	8	.00023	.00003	.05
145	12	.00440	.00037	.58
245	12	.00826	.00069	1.09
1245	24	.01274	.00053	.84
within rep	108	.06752	.00063	
total	215	.11904		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 14

ANALYSIS OF VARIANCE FOR EXPECTED VALUE RESPONSE FOR $3^3 \times 4$ FACTORIAL DESIGN WITH AUTOCORRELATION EQUAL TO ZERO

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	2	.00029	.00014	.17
2	2	.00151	.00075	.88
4	3	.00199	.00066	.77
5	2	.00058	.00029	.34
12	4	.00193	.00048	.56
14	6	.00275	.00046	.53
15	4	.00261	.00065	.76
24	6	.00159	.00026	.31
25	4	.00334	.00083	.97
45	6	.00141	.00023	.27
124	12	.01659	.00138	1.61
125	8	.00098	.00012	.14
145	12	.00680	.00057	.66
245	12	.01177	.00098	1.14
1245	24	.01803	.00075	.87
within rep	108	.09339	.00086	
total	215	.16556		

[a] * indicates significance at the .05 level; ** indicates significance at the .01 level.

TABLE 15

ANALYSIS OF VARIANCE FOR VARIANCE RESPONSE FOR $3^3 \times 4$ FACTORIAL DESIGN WITH AUTOCORRELATION EQUAL TO ZERO

source of variation	degrees of freedom	sums of squares	mean squares	f ratio[a]
1	2	.00600	.00300	1.28
2	2	.00420	.00210	.90
4	3	.03019	.01006	4.30**
5	2	.02219	.01110	4.74*
12	4	.00565	.00141	.60
14	6	.00936	.00156	.67
15	4	.00905	.00226	.97
24	6	.00930	.00155	.66
25	4	.01535	.00384	1.64
45	6	.00862	.00144	.61
124	12	.02239	.00191	.82
125	8	.01171	.00146	.63
145	12	.03107	.00259	1.11
245	12	.02144	.00179	.76
1245	24	.08171	.00340	1.45
within rep	108	.25252	.00234	
total	215	.54125		

[a]* indicates significance at the .05 level; ** indicates significance at the .01 level.

$$H_O: \quad E = E[G^T(RB)]-E[G^T(BH)] = 0 \tag{5-1}$$

$$H_A: \quad E = E[G^T(RB)]-E[G^T(BH)] > 0. \tag{5-2}$$

The grand mean of E under the independence assumption was -0.00287. Referring to Table 14, we observe that there are no significant effects at the .05 level. We conclude that E=-0.00287 for all factor configurations, and we cannot reject the null hypothesis.

2. $\{E[G^T(RB)]-E[G^T(BH)]\}_{m+1} \geq \{E[G^T(RB)]-E[G^T(BH)]\}_m$

With m denoting the number of securities in the portfolio, we test the hypotheses

$$H_O: \quad E_{m+1} = E_m \tag{5-3}$$

$$H_A: \quad E_{m+1} < E_m. \tag{5-4}$$

Examination of Table 14 indicates that the number of securities (factor 1) does not significantly effect the expected value response, and so the null hypothesis is not rejected.

3. $\{E[G^T(RB)]-E[G^T(BH)]\}_{\mu_i=\mu_j} > \{E[G^T(RB)]-E[G^T(BH)]\}_{\mu_i\neq\mu_j}$

Let $\mu_i=\mu_j$ refer to the situation where all the securities in the portfolio have the same expected return, and $\mu_i\neq\mu_j$ indicate that at least one pair of securities have different expected returns. Then the test is the null hypothesis

$$H_O: \quad E_{(\mu_i=\mu_j)} = E_{(\mu_i\neq\mu_j)} \tag{5-5}$$

against the alternative hypothesis given by

$$H_A: \; E_{(\mu_i=\mu_j)} < E_{(\mu_i\neq\mu_j)}. \qquad (5\text{-}6)$$

Neither the F-ratio of 0.88 for the main effect of divergent expected returns (factor 2), nor that for any of its interactions (Table 14) is sufficient to reject the null hypothesis in favor of the alternative chosen to contradict the analytical results.

4. $V[G^T(RB)] \leq V[G^T(BH)]$

The null and alternative hypotheses for the validation of our model with respect to this conclusion were specified as

$$H_O: \; V = V[G^T(RB)]-V[G^T(BH)] = 0 \qquad (5\text{-}7)$$

$$H_A: \; V = V[G^T(RB)]-V[G^T(BH)] > 0. \qquad (5\text{-}8)$$

The grandmean of the V response under the independence assumption was -0.01527 which does not reject the null hypothesis. However, the significant F-ratios of 4.30 and 4.74 associated with the intercorrelation and variance factors (Table 15) point to the possibility of a significant positive value for the response variable at certain levels of these factors.

The fact that the marginal means (not shown) for all levels of the latter factor were found to be negative allows us to confine our attention to the intercorrelation factor. Examination of the marginal means for that factor showed a positive value of V equal to 0.00376 when the intercorrelation coefficient was 0.5 (level 4). Computation of the t-value for the twenty-seven observations comprising this positive marginal mean yielded t=0.9894, which was not significantly greater than zero at the .05 level. Therefore, we conclude that at none of the design points is the variance of the distribution of returns to RB significantly greater than the variance under BH.

With these encouraging results, an examination of the "within replicates" and total sum of squares was undertaken for each response (Tables 8 through 12) to determine the amount of variation in the response explained by the five factors under study. The percentages of explained variation were found to be

G	E	V	S	K
94.7%	90.6%	64.6%	56.1%	52.2%.

These figures set an upper limit on the coefficients of determination of the regression equations utilizing the same data base.

With the completion of this step in the validation procedure, the development of the regression model for each of the responses was continued.

Regression

The purpose of the analysis of variance was to reduce the amount of independent variables for inclusion in each regression equation to a manageable number. The list of terms chosen was comprised of all the orthogonal polynomial components which were shown significant or nearly so at the 0.05 level in any of the analyses of variance, as well as the linear components of significant three and four factor interactions (for which an orthogonal polynomial breakdown was not available).

The response variables were each regressed with this list of forty-four independent variables. The regression programs used were part of the Statistical Package for the Social Sciences [161]. A summary of the output is presented in Tables 16 through 20 for the geometric mean, expected value, variance, skewness, and kurtosis responses, respectively. The low coefficients of determination (R^2) for the equations predicting the variance (0.316), skewness (0.155), and kurtosis (0.108) necessitated that these three response variables be dropped from further consideration. Any attempt to respecify these equations so as to bring their coefficients of determination closer to their respective upper limits would have resulted in extreme complexity with at least 35% of the variation still unexplained.

TABLE 16

REGRESSION OF GEOMETRIC MEAN RESPONSE WITH 44 SELECTED VARIABLES

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	44	12,94551	0.29422	153,75337
RESIDUAL	1035	1,98053	0.00191	

MULTIPLE R	0,93129
R SQUARE	0,86731
STANDARD ERROR	0,04374

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
$X_1X_3X_4$	0,04959	0.18169	0,01335	13.79726
X_1	-0,05242	-0.36408	0,03348	2.45221
X_2	-26,85834	-0.18654	31,84272	0.71144
X_3	0,04913	0.19503	0.09018	0.29678
X_4	-0,07653	-0.17883	0,05476	1.95263
X_5	7,06577	0.04907	39,52070	0.03196
X_1^2	0,00991	0,41480	0.00458	4.68434
X_2^2	-1037,07931	-0.01499	3191,62138	0.10558
X_3^2	-0,02400	-0.03720	0,29306	0.00670
X_4^2	0,01487	0.01216	0.09409	0.02499
X_5^2	-541,53586	-0.03767	3808,03805	0.02022
X_1X_2	24,80229	0,56260	20,84562	1.41564
X_1X_3	-0,08146	-1.00544	0.04345	3.51535
X_1X_4	0,00763	0.05572	0,01310	0.33909
X_1X_5	-0,36289	-0.01484	3.01017	0.01453
X_2X_3	-8,42445	-0.04318	12.66953	0.44214
X_2X_4	-22,13931	-0.06899	27.62538	0.64226
X_2X_5	-302,08389	-0.01083	1996,64292	0.02289
X_3X_4	0,09759	0.11501	0.04455	4.79774
X_3X_5	8,81632	0.17731	13,30178	0.43929
X_4X_5	4,85231	0.05757	5,93432	0.66658
X_3^3	0,18558	0,28927	0,08582	4.67601
X_4^3	-0,07795	-0,03705	0,26274	0.08803
$X_1X_3^3$	-0,06841	-0,33152	0,02673	6.54885
$X_1^2 X_3$	0,02266	0.97587	0.00605	14.02894
$X_1X_3^2$	-0,02169	-0.10957	0.02836	0.58510
$X_3^2 X_5$	45,29266	0,36222	55,03117	0.67739
$X_3^2 X_4$	0,23792	0.16325	0,03498	46.26142
$X_1X_3X_5$	-18,48170	-1.15567	4,27830	18.66127
X_3^4	0,67753	0,51402	0,94599	0.51296
$X_2^2 X_3^3$	7808,84652	0.02897	12841,19776	0.36980
$X_3X_4^3$	-0,22275	-0,05581	0,11318	3.87377
$X_2X_4^2$	-43,47116	-0,05303	71,34171	0.37129
$X_1^2 X_4^3$	0,00597	0,03166	0.01156	0.26692
$X_3^4 X_5$	-356,10608	-1,38317	335,92445	1.12377
$X_2^2 X_4$	13850,00892	0.08027	10278,54611	1.81567
$X_2^2 X_3$	-2110,57768	-0.01994	7695,04818	0.07523
$X_2^2 X_3^2$	-1537,81430	-0.00746	4297,70592	0.12804
$X_2X_4^3$	87,36616	0.05649	184,18873	0.22499
$X_1^2 X_2$	-4,60628	-0.39701	3,45830	1.77409
$X_1X_2X_3X_4$	8,46923	0.04006	4,06637	4.33785
$X_3^4 X_5^2$	22203,31459	0,47056	31837,44333	0.48636
$X_1^2 X_3^2 X_4^2$	-0,00592	-0.01793	0,00913	0.41962
$X_1^2 X_3^2 X_5^2$	-285,17682	-0.15276	172,38145	2.73683
Constant	0,05094			

[a]Least squares regression coefficient.

[b]Normalized coefficient. This depends upon the range of factor levels selected.

[c]$F=(B/S)^2$. $F_{1,1000}\ (.01)=6.66$, $F_{1,1000}\ (.05)=3.85$.

TABLE 17

REGRESSION OF EXPECTED VALUE RESPONSE WITH 44 SELECTED VARIABLES

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	44	19,94561	0.45331	114.46101
RESIDUAL	1035	4,09828	0.00396	

MULTIPLE R	0,91080
R SQUARE	0,82955
STANDARD ERROR	0,06293

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
$X_1X_3X_4$	0,04356	0.12574	0,01920	5.14385
X_1	-0,06518	-0.35670	0,04815	1.83239
X_2	-27,19268	-0,14880	45,80573	0.35242
X_3	0,08226	0.25728	0.12972	0.40208
X_4	-0,04900	-0.09022	0,07878	0.38692
X_5	0,78541	0.00430	56.85050	0.00019
X_1^2	0,01168	0,38541	0,00659	3.14819
X_2^2	-2009,32528	-0,02289	4591,14576	0.19154
X_3^2	-0,07822	-0.09553	0,42156	0.03443
X_4^2	0,05923	0.03815	0.13534	0.19153
X_5^2	-508,02281	-0.02785	5477.86084	0.00860
X_1X_2	23,21481	0.41490	29,98642	0.59935
X_1X_3	-0,07612	-0.74028	0.06250	1.48352
X_1X_4	-0,00597	-0,03440	0.01884	0.10058
X_1X_5	0,18689	0.00602	4,33013	0.00186
X_2X_3	-12,28428	-0.04960	18,22511	0.45432
X_2X_4	-67,19196	-0,16498	39.73909	2.85890
X_2X_5	1206,25074	0.03409	2872,16985	0.17638
X_3X_4	0,15510	0.14402	0.06409	5.85639
X_3X_5	-0,40505	-0,00642	19.13460	0.00045
X_4X_5	13,21072	0.12349	8,53652	2.39492
X_3^3	0,14128	0,17351	0,12345	1.30966
X_4^3	-0,31506	-0.11799	0,37795	0.69488
$X_1X_3^3$	-0,08342	-0.31851	0,03845	4.70582
$X_1^2X_3$	0,02497	0.84728	0.00870	8.23260
$X_1X_3^2$	-0,01273	-0.05065	0.04080	0.09731
$X_3^2X_5$	49,31739	0.31075	79,16232	0.38812
$X_3^2X_4$	0,28209	0.15249	0.05032	31.42542
$X_1X_3X_5$	-21,20266	-1.04460	6.15433	11.86913
X_3^4	0,43089	0.25756	1.36081	0.10026
$X_2^2X_3^3$	16649,54790	0.04867	18472,05660	0.81241
$X_3X_4^3$	-0,21350	-0.04215	0.16281	1.71974
$X_2X_4^2$	-154,14687	-0,14817	102.62502	2.25612
$X_1^2X_4^3$	0,01293	0.05398	0,01663	0.60411
$X_3^4X_5$	-248,39466	-0,76017	483,22715	0.26423
$X_2^2X_4$	24333,33058	0.11111	14785,68348	2.70645
$X_2^2X_3$	-4613,73316	-0.03435	11069,32299	0.17373
$X_2^2X_3^2$	-1334,59396	-0,00510	6182,24783	0.04660
$X_2X_4^3$	385,95052	0.19662	264.95540	2.12186
$X_1^2X_2$	-4,26962	-0,29130	4.97476	0.74352
$X_1X_2X_3X_4$	9,77982	0.03645	5,84947	2.79530
$X_3^4X_5^2$	6525,58170	0.10896	45798.14643	0.02030
$X_1^2X_3^2X_4^2$	-0,00185	-0,00442	0.01314	0.01990
$X_1^2X_3^2X_5^2$	-354,98732	-0.14982	247,97063	2.04939
Constant	0,08809			

[a]Least squares regression coefficient.

[b]Normalized coefficient. This depends upon the range of factor levels selected.

[c]$F=(B/S)^2$. $F_{1,1000}(.01)=6.66$, $F_{1,1000}(.05)=3.85$.

TABLE 18

REGRESSION OF VARIANCE RESPONSE WITH 44 SELECTED VARIABLES

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	44	80,52400	1.83009	10,88683
RESIDUAL	1035	173,98486	0.16810	

MULTIPLE R	0,56249
R SQUARE	0,31639
STANDARD ERROR	0,41000

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
$X_1X_3X_4$	-0,09354	-0.08300	0,12513	0.55885
X_1	0,07117	0,11970	0,31376	0.05145
X_2	470,63215	0,79158	298,45221	2.48664
X_3	0,29018	0,27897	0,84522	0.11787
X_4	0,32803	0.18563	0,51329	0.40841
X_5	-45,33593	-0.07625	370,41559	0.01498
X_1^2	-0,00951	-0.09641	0,04291	0.04912
X_2^2	-39456,19473	-0.13815	29914,10613	1.73972
X_3^2	-1,11829	-0,41978	2,74672	0.16576
X_4^2	1,04889	0.20764	0,88185	1.41469
X_5^2	1903,84054	0.03208	35691,59397	0.00285
X_1X_2	-239,53659	-1.31583	195,37976	1.50309
X_1X_3	-0,02746	-0,08208	0,40722	0.00455
X_1X_4	-0,10565	-0.18696	0,12274	0.74094
X_1X_5	2,12232	0.02102	28,21346	0.00566
X_2X_3	211,20317	0.26213	118,74767	3.16338
X_2X_4	-484,05876	-0,36592	258,92433	3.50659
X_2X_5	2204,18068	0.01914	18713,93292	0.01387
X_3X_4	0,40247	0,11486	0,41760	0.92884
X_3X_5	-27,06325	-0.13181	124,67357	0.04712
X_4X_5	60,16012	0.17235	55,62061	1.16989
X_3^3	-0,36289	-0.13698	0,80438	0.20353
X_4^3	-3,19470	-0.36772	2,46256	1.68301
$X_1X_3^3$	-0,19422	-0,22794	0,25054	0,60094
$X_1^2X_3$	0,06322	0.65926	0,05671	1.24272
$X_1X_3^2$	0,14599	0,17858	0,26581	0.30167
$X_3^2X_5$	157,95879	0,30592	515,79062	0.09379
$X_3^2X_4$	0,14705	0.02457	0,32786	0.20337
$X_1X_3X_5$	-55,12313	-0,83473	40,09923	1.88971
X_3^4	0,29175	0.05360	8.86651	0.00108
$X_2^2X_3^3$	11340,41224	0,01019	120356,67999	0.00888
$X_3X_4^3$	1,05498	0.06401	1,06078	0.98910
$X_2X_4^2$	-1445,78846	-0,42715	668,66438	4.67512
$X_1^2X_4^3$	0,03084	0.03958	0,10835	0.08100
$X_3^4X_5$	223,59092	0.21032	3148,51870	0.00504
$X_2^2X_4$	83098,92108	0.11663	96337.71777	0,74404
$X_2^2X_3$	-103569,69318	-0.23702	72123.36958	2.06212
$X_2^2X_3^2$	-6667,36353	-0.00783	40281.10352	0.02740
$X_2X_4^3$	3856,57614	0.60387	1726,34551	4.99055
$X_1^2X_2$	38,22311	0.79781	32,41358	1.39059
$X_1X_2X_3X_4$	-23,13719	-0,02650	38,11205	0.36853
$X_3^4X_5^2$	-97517,59893	-0.50049	298402,76982	0.10680
$X_1^2X_3^2X_4^2$	0,06695	0.04913	0,08560	0.61165
$X_1^2X_3^2X_5^2$	-1466,16296	-0.19020	1615,67942	0.82348
Constant	-0,04694			

[a] Least squares regression coefficient.

[b] Normalized coefficient. This depends upon the range of factor levels selected.

[c] $F=(B/S)^2$. $F_{1,1000}$ (.01)=6.66, $F_{1,1000}$ (.05)=3.85.

TABLE 19

REGRESSION OF SKEWNESS RESPONSE WITH 44 SELECTED VARIABLES

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	44	2433,06474	55.29693	4,31819
RESIDUAL	1035	13253,77033	12.80558	

MULTIPLE R	0,39383
R SQUARE	0,15510
STANDARD ERROR	3,57849

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
$X_1X_3X_4$	-1,27929	-0.14459	1.09210	1.37218
X_1	1,00521	0.21535	2,73845	0.13474
X_2	5738,72053	1.22946	2604,88638	4.85347
X_3	0,60684	0.07431	7.37709	0.00677
X_4	0,83829	0,06043	4,47999	0.03501
X_5	-460,93570	-0.09875	3232,98176	0.02033
X_1^2	-0,12362	-0,15963	0.37450	0.10895
X_2^2	-308462,92563	-0.13757	261089,87064	1,39581
X_3^2	-7,62658	-0,36465	23.97331	0.10121
X_4^2	9,99419	0,25201	7.69681	1.68606
X_5^2	59859,26839	0.12846	311515,69807	0.03692
X_1X_2	-2640,49736	-1,84756	1705,27158	2.39764
X_1X_3	-0,35800	-0,13630	3,55421	0.01015
X_1X_4	0,05448	0,01228	1,07127	0.00259
X_1X_5	-3,34740	-0,00422	246,24666	0.00018
X_2X_3	2013,10268	0.31825	1036,42785	3.77271
X_2X_4	-3295,98801	-0,31684	2259,88771	2.12715
X_2X_5	-146404,20784	-0.16197	163334,92646	0.80343
X_3X_4	3,63928	0.13229	3,64481	0.99697
X_3X_5	488,57538	0,30310	1088.14906	0.20160
X_4X_5	87,40075	0,03199	485,45585	0.03241
X_3^3	1,02708	0,04938	7,02063	0.02140
X_4^3	-11,67133	-0.17112	21,49316	0.29488
$X_1X_3^3$	-1,77104	-0.26475	2.18673	0,65594
$X_1^2X_3$	0,63905	0.84886	0,49495	1.66704
$X_1X_3^2$	1,26241	0,19670	2.31998	0.29610
$X_3^2X_5$	1555,59261	0.38375	4501,81278	0.11940
$X_3^2X_4$	0,13041	0.00276	2,86160	0.00208
$X_1X_3X_5$	-583,21056	-1.12492	349,98547	2.77684
X_3^4	-54,34962	-1,27189	77.38677	0.49324
$X_2^2X_3^3$	-741527,87941	-0.08487	1050471,30179	0.49830
$X_3X_4^3$	13,14041	0.10156	9.25845	2.01438
$X_2X_4^2$	-12815,80612	-0,48229	5836,09267	4.82223
$X_1^2X_4^3$	-0,71627	-0.11710	0,94570	0.57364
$X_3^4X_5$	25512,26140	3,05669	27480,22408	0.86190
$X_2^2X_4$	492516,41007	0.08805	840834,15903	0.34310
$X_2^2X_3$	-780370,34425	-0.22748	629491,69037	1.53681
$X_2^2X_3^2$	-447712,98103	-0,06699	351572.86876	1.62169
$X_2X_4^3$	28803,61081	0.57447	15067,51781	3.65435
$X_1^2X_2$	431,84747	1,14813	282,90520	2,33012
$X_1X_2X_3X_4$	-564,68625	-0.08239	332.64835	2.88167
$X_3^4X_5^2$	-3130486,60596	-2,04650	2604454,90955	1,44474
$X_1^2X_3^2X_4^2$	0,17098	0.01598	0,74712	0.05238
$X_1^2X_3^2X_5^2$	-13815,90761	-0,22829	14101,62582	0,95989
Constant	-1,69009			

[a]Least squares regression coefficient.

[b]Normalized coefficient. This depends upon the range of factor levels selected.

[c]$F=(B/S)^2$. $F_{1,1000}(.01)=6.66$, $F_{1,1000}(.05)=3.85$.

TABLE 20

REGRESSION OF KURTOSIS RESPONSE WITH 44 SELECTED VARIABLES

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	44	160706,30350	3652.41599	2,85136
RESIDUAL	1035	1325770.46042	1280.93764	

MULTIPLE R	0,32880
R SQUARE	0,10811
STANDARD ERROR	35,79019

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
$X_1X_3X_4$	-9,85705	-0.11444	10,92266	0.81440
X_1	10,58284	0.23291	27,38859	0.14930
X_2	61318,11910	1,34951	26052,72973	5.53951
X_3	-31,83553	-0.40047	73,78180	0.18618
X_4	-0,87279	-0,00646	44,80656	0.00038
X_5	-3476,89175	-0,07652	32334,61568	0.01156
X_1^2	-1,51433	-0,20089	3,74554	0,16346
X_2^2	-2247817,16302	-0,10298	2611286,16970	0,74099
X_3^2	-118,88922	-0,58396	239,76867	0.24587
X_4^2	92,77494	0,24032	76,97956	1,45248
X_5^2	548181,53914	0,12085	3115619,30096	0,03096
X_1X_2	-26687,00171	-1,91823	17055,24661	2.44841
X_1X_3	12,90620	0,50478	35,54736	0,13182
X_1X_4	4,49610	0,10411	10,71427	0.17610
X_1X_5	197,58061	0,02560	2462,83206	0.00644
X_2X_3	18999,93995	0,30856	10365,81666	3.35968
X_2X_4	-14792,80117	-0,14608	22602,23104	0.42835
X_2X_5	-2460609,37195	-0.27965	1633591,66909	2.26881
X_3X_4	30,66397	0,11451	36,45347	0.70759
X_3X_5	10417,79268	0,66393	10883,10551	0.91632
X_4X_5	-1145,96663	-0.04308	4855,27894	0.05571
X_3^3	27,02075	0.13346	70,21672	0,14809
X_4^3	-71,17498	-0,10720	214,96349	0.10963
$X_1X_3^3$	-16,91891	-0,25982	21,87055	0,59845
$X_1^2X_3$	4,37487	0,59697	4,95027	0.78104
$X_1X_3^2$	21,73573	0,34790	23,20322	0,87751
$X_3^2X_5$	23340,96163	0,59150	45024,80894	0.26874
$X_3^2X_4$	-2,10453	-0,00458	28,62023	0,00541
$X_1X_3X_5$	-7020,41004	-1,39106	3500,37409	4,02250
X_3^4	-491,15031	-1,18075	773,98253	0.40269
$X_2^2X_3^3$	-10229733,90112	-0,12028	10506272,02222	0.94805
$X_3X_4^3$	62,26238	0,04943	92,59825	0.45211
$X_2X_4^2$	-110329,60016	-0,42652	58369,58808	3.57282
$X_1^2X_4^3$	-8,09838	-0,13601	9,45843	0.73309
$X_3^4X_5$	238663,29827	2,93749	274843,02419	0.75405
$X_2^2X_4$	-182656,78716	-0,00335	8409589,47168	0.00047
$X_2^2X_3$	-6931432,16968	-0,20756	6295851,13220	1.21210
$X_2^2X_3^2$	-5898390,58496	-0.09067	3516250,45673	2.81389
$X_2X_4^3$	224810,68250	0,46061	150697,53976	2.22547
$X_1^2X_2$	4298,74429	1.17406	2829,47190	2.30819
$X_1X_2X_3X_4$	-5169,33525	-0,07748	3326,97718	2.41418
$X_3^4X_5^2$	-29606671,38379	-1.98828	26048414,36572	1.29186
$X_1^2X_3^2X_4^2$	-2,53104	-0.02430	7,47235	0.11473
$X_1^2X_3^2X_5^2$	-189920,80398	-0,32238	141037,18639	1.81333
Constant	-23,25064			

[a]Least squares regression coefficient.

[b]Normalized coefficient. This depends upon the range of factor levels selected.

[c]$F=(B/S)^2$. $F_{1,1000}$ (.01)=6.66, $F_{1,1000}$ (.05)=3.85.

The R^2 values of the equations for the geometric mean response (0.867), and for the prediction of the expected value response (0.830) were assessed as satisfactory. However, due to the complexity of the response surfaces, simpler formulations were tested.

The convenience of quadratic (second-order) response surfaces suggested that the equations be respecified to contain only linear and quadratic terms of the form $b_i x_i$, $b_{ii} x_i^2$, and $b_{ij} x_i x_j$ for $i \neq j$. The result of regressing the geometric mean response with only these variables was an R^2 of 0.832, only slightly less than the value of 0.867 in the more complex formulation. Similarly, the coefficient of determination was only reduced from 0.830 to 0.797 in the case of the expected value response. These regressions are summarized in Tables 21 and 22.

To further simplify the equations, the quadratic terms were added to the regression in a stepwise manner only so long as the coefficient of the term added was significantly different from zero at the 0.05 level ($F > 3.85$). Tables 23 and 24 present the output of the stepwise regressions after the addition of the last significant term in each equation. Note the R^2 values of 0.831 and 0.796.

The final form of the models predicting the relative performance of the RB and BH policies in terms of the geometric mean and expected value responses was therefore described by the following equations:

TABLE 21

REGRESSION OF GEOMETRIC MEAN RESPONSE WITH FIRST ORDER AND QUADRATIC VARIABLES

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	20	12.42427	0.62121	262.95985
RESIDUAL	1059	2.50177	0.00236	

MULTIPLE R	0.91235
R SQUARE	0.83239
STANDARD ERROR	0.04860

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
X_1	-0.01138	-0.07906	0.02205	0.26650
X_2	6.61865	0.04597	14.51001	0.20807
X_3	0.18152	0.72058	0.02322	61.12922
X_4	-0.04162	-0.09725	0.03981	1.09273
X_5	-0.60743	-0.00422	32.20902	0.00036
X_1^2	0.00377	0.15781	0.00314	1.44349
X_2^2	186.11212	0.00269	3137.40172	0.00352
X_3^2	-0.17131	-0.26554	0.00812	445.50843
X_4^2	-0.01670	-0.01365	0.02273	0.53987
X_5^2	648.65162	0.04513	3137.46724	0.04274
X_1X_2	-2.83542	-0.06432	2.21848	1.63352
X_1X_3	-0.05669	-0.69975	0.00388	213.36022
X_1X_4	0.01214	0.08872	0.00659	3.39033
X_1X_5	-4.08958	-0.16724	2.21848	3.39818
X_2X_3	-4.00463	-0.02052	3.88133	1.06454
X_2X_4	10.22912	0.03188	6.59365	2.40672
X_2X_5	-302.07994	-0.01083	2218.47865	0.01854
X_3X_4	0.23133	0.27261	0.01154	402.11497
X_3X_5	-46.62879	-0.93779	3.88133	144.32686
X_4X_5	4.85231	0.05757	6.59365	0.54156
Constant	0.06119			

[a]Least squares regression coefficient.

[b]Normalized coefficient. This depends upon the range of factor levels selected.

[c]$F=(B/S)^2$. $F_{1,1000}(.01)=6.66$, $F_{1,1000}(.05)=3.85$.

TABLE 22

REGRESSION OF EXPECTED VALUE RESPONSE WITH FIRST ORDER AND QUADRATIC VARIABLES

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	20	19,17140	0.95857	208.33808
RESIDUAL	1059	4,87249	0.00460	
MULTIPLE R	0,89294			
R SQUARE	0,79735			
STANDARD ERROR	0,06783			

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
X_1	-0,02054	-0,11239	0,03077	0.44547
X_2	-2,59741	-0.01421	20.24975	0.01645
X_3	0,22078	0,69056	0.03240	46.43370
X_4	-0,04915	-0.09050	0.05556	0.78267
X_5	5,41218	0.02962	44,94996	0.01450
X_1^2	0,00575	0,18980	0.00438	1.72711
X_2^2	437,50060	0.00498	4378.46580	0.00998
X_3^2	-0,24034	-0.29353	0,01133	450.23885
X_4^2	-0,02779	-0.01790	0.03172	0.76765
X_5^2	-729,14538	-0.03997	4378,55724	0.02773
X_1X_2	-2,52292	-0.04509	3,09604	0.66404
X_1X_3	-0,05719	-0.55612	0.00542	111.46063
X_1X_4	0,00748	0,04307	0.00920	0.66091
X_1X_5	-4,45208	-0.14344	3.09604	2.06782
X_2X_3	-5,88173	-0.02375	5.41667	1.17909
X_2X_4	11,85346	0,02910	9.20190	1.65934
X_2X_5	1206,25137	0,03409	3096.04372	0.15180
X_3X_4	0,27635	0.25659	0.01610	294.65292
X_3X_5	-64,01305	-1.01436	5.41667	139.66007
X_4X_5	13,21072	0,12349	9,20191	2.06109
Constant	0,07735			

[a]Least squares regression coefficient.

[b]Normalized coefficient. This depends upon the range of factor levels selected.

[c]$F=(B/S)^2$. $F_{1,1000}(.01)=6.66$, $F_{1,1000}(.05)=3.85$.

TABLE 23

REGRESSION OF GEOMETRIC MEAN RESPONSE WITH SIGNIFICANT FIRST ORDER AND QUADRATIC VARIABLES AFTER STEPWISE REGRESSION

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	7	12.40087	1.77155	752.06892
RESIDUAL	1072	2,52517	0.00236	

MULTIPLE R	0.91149
R SQUARE	0.83082
STANDARD ERROR	0.04853

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
X_1X_3	-0.05669	-0.69975	0.00388	213.97771
X_3^2	-0.17131	-0.26554	0.00810	446.79777
X_3X_4	0.23133	0.27261	0.01152	403.27873
X_3X_5	-46.62879	-0.93779	3.87573	144.74455
X_3	0.17751	0.70469	0.02286	60.31673
X_1X_5	-2.29050	-0.09367	0.30848	55.13316
X_1X_4	0.00857	0.06266	0.00173	24.67380
Constant	0.04595			

[a]Least squares regression coefficient.

[b]Normalized coefficient. This depends upon the range of factor levels selected.

[c]$F=(B/S)^2$. $F_{1,1000}(.01)=6.66$, $F_{1,1000}(.05)=3.85$.

TABLE 24

REGRESSION OF EXPECTED VALUE RESPONSE WITH SIGNIFICANT FIRST ORDER AND QUADRATIC VARIABLES AFTER STEPWISE REGRESSION

ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
REGRESSION	8	19.13201	2.39150	521.44990
RESIDUAL	1071	4.91188	0.00459	

MULTIPLE R	0.89203
R SQUARE	0.79571
STANDARD ERROR	0.06772

VARIABLE	B[a]	BETA[b]	STD ERROR B	F[c]
X_3X_5	-64.01305	-1.01436	5.40796	140.11006
X_3^2	-0.24034	-0.29353	0.01131	451.68954
X_3X_4	0.27635	0.25659	0.01607	295.60231
X_1X_3	-0.05719	-0.55612	0.00541	111.81976
X_3	0.21490	0.67216	0.03189	45.40336
X_1X_5	-4.31594	-0.13906	0.80229	28.93907
X_1X_4	0.01416	0.08153	0.00241	34.45344
X_1^2	0.00171	0.05639	0.00078	4.74378
Constant	0.05627			

[a]Least squares regression coefficient.

[b]Normalized coefficient. This depends upon the range of factor levels selected.

[c]$F=(B/S)^2$. $F_{1\ 1000}(.01)=6.66$, $F_{1\ 1000}(.05)=3.85$.

$$G = \underset{(.04853)}{0.04595} + \underset{(.02286)}{0.17751x_3} - \underset{(.00810)}{0.17131x_3^2} - \underset{(.00388)}{0.05669x_1x_3} + \underset{(.00173)}{0.00857x_1x_4} - \underset{(.30848)}{2.29050x_1x_5} + \underset{(.01152)}{0.23133x_3x_4} - \underset{(3.87573)}{46.62879x_3x_5} \tag{5-9}$$

$$E = \underset{(.06772)}{0.05627} + \underset{(.03189)}{0.21490x_3} + \underset{(.00078)}{0.00171x_1^2} - \underset{(.01131)}{0.24034x_3^2} - \underset{(.00541)}{0.05719x_1x_3} + \underset{(.00241)}{0.01416x_1x_4} - \underset{(.80229)}{4.31594x_1x_5} + \underset{(.01607)}{0.27635x_3x_4} - \underset{(5.40796)}{64.01305x_3x_5}. \tag{5-10}$$

At this stage validation addresses itself to the question of whether or not the simplification process resulted in models (equations (5-9) and (5-10)) which no longer capture the true nature of the response surfaces within the experimental region. The comparison of the implications of the factor-response model with the previously described analytical conclusions is best deferred until the next chapter. At this point we merely provide a summary of the simplification process in Table 25. The table shows the extent of the trade-off between simplicity and goodness of fit for each equation.

TABLE 25

SUMMARY TABLE: THE DEVELOPMENT OF THE FACTOR-RESPONSE MODELS

	ANOVA	Regression Equation		
		Original	Quadratic	Final
Geometric Mean Response				
Number of Terms[a]	-	44	20	7
Explained Variation (%)	94.7	86.7	83.2	83.1
Expected Value Response				
Number of Terms	-	44	20	8
Explained Variation (%)	90.6	83.0	79.7	79.6
Variance Response				
Number of Terms	-	44	-	-
Explained Variation (%)	64.6	31.6	-	-
Skewness Response				
Number of Terms	-	44	-	-
Explained Variation (%)	56.1	15.5	-	-
Kurtosis Response				
Number of Terms	-	44	-	-
Explained Variation (%)	52.2	10.8	-	-

[a]Number of terms does not include the constant term.

CHAPTER VI

INFERENCES FROM THE FACTOR-RESPONSE MODELS

Response Surface Methodology

The mathematical models developed in the previous chapter furnish the means for investigating the statistical properties of the rebalancing policy. On the assumption that they reasonably depict the true relationships involved, they may be utilized to draw inferences concerning the joint effects of changes in the factors under study on the relative performances of RB and BH.

The exploration of the response surfaces was based upon the computation of the predicted value of the response variables for each factor configuration in the experimental design, and the evaluation of the partial derivatives of the responses with respect to each factor. The definition of the response as the difference between the geometric mean (or expected value) of the distribution of geometric totals to RB and the corresponding parameter under BH made the analysis of the fitted surfaces particularly amenable to significance testing. This is due to the fact that a zero response, indicating equality for the two strategies, provides a meaningful null hypothesis for testing the significance of a predicted response. Similarly, the partial derivatives were tested against a zero null hypothesis as a means of determining the significance of the inferences drawn from them. (For example, to conclude that there exists a maximum with respect to a factor, the relevant partial derivative must range from a statistically significant positive value to a significant negative value within the experimental region.) The t-test was used for these purposes.

Validation

In order to build an acceptable level of confidence in the new inferences drawn from the mathematical models, the stepwise validation procedure was continued with a comparison of certain analytical observations with the implications of the model predicting the expected value response.

With the model predicting E defined in equation (5-10), the partial derivatives of E with respect to the four significant factors are given by

$$\delta E/\delta x_1 = 2\cdot(0.00171)x_1 - 0.05719x_3 + 0.01416x_4 - 4.31594x_5 \quad (6\text{-}1)$$

$$\delta E/\delta x_3 = 0.21490 - 2\cdot(0.24034)x_3 - 0.05719x_1 + 0.27635x_4 - 64.01305x_5 \quad (6\text{-}2)$$

$$\delta E/\delta x_4 = 0.01416x_1 + 0.27635x_3 \quad (6\text{-}3)$$

$$\delta E/\delta x_5 = -4.31594x_1 - 64.01305x_3. \quad (6\text{-}4)$$

The standard errors of the coefficients are in parentheses in the predicting equation.

Table 26 presents the values of these partial derivatives at each point in the experimental design along with the

TABLE 26

PREDICTED RESPONSE, PARTIAL DERIVATIVES, AND T-VALUES FOR EXPECTED VALUE RESPONSE AT EACH DESIGN POINT

Factor Levels[a]				Predicted Response[b]			Partial Derivative (left) and t-Value (right) with Respect to[c]							
Number	Autocor	Intercor	Variance	Forecast	Std Error[d]	t^e Value	Number of Securities		Autocorrelation		Intercorrelation		Variance	
2	-.667	-.250	.004	.064	.068	.944	.024	4.181	.096	2.231	-.156	-13.272	34.043	8.627
2	-.667	-.250	.005	.098	.068	1.445	.020	3.170	.032	.693	-.156	-13.272	34.043	8.627
2	-.667	-.250	.006	.133	.068	1.943	.016	2.283	-.032	-.653	-.156	-13.272	34.043	8.627
2	-.667	0	.004	.025	.068	.374	.028	4.819	.165	3.856	-.156	-13.272	34.043	8.627
2	-.667	0	.005	.059	.068	.875	.023	3.753	.101	2.206	-.156	-13.272	34.043	8.627
2	-.667	0	.006	.094	.068	1.374	.019	2.814	.037	.751	-.156	-13.272	34.043	8.627
2	-.667	.200	.004	-.006	.068	-.084	.031	5.294	.220	5.134	-.156	-13.272	34.043	8.627
2	-.667	.200	.005	.028	.068	.416	.026	4.195	.156	3.406	-.156	-13.272	34.043	8.627
2	-.667	.200	.006	.062	.068	.916	.022	3.224	.092	1.872	-.156	-13.272	34.043	8.627
2	-.667	.500	.004	-.053	.068	-.769	.035	5.922	.303	6.965	-.156	-13.272	34.043	8.627
2	-.667	.500	.005	-.018	.068	-.271	.030	4.800	.239	5.148	-.156	-13.272	34.043	8.627
2	-.667	.500	.006	.016	.068	.228	.026	3.800	.175	3.516	-.156	-13.272	34.043	8.627
2	-.333	-.250	.004	.070	.068	1.026	.005	1.049	-.064	-1.573	-.064	-8.853	12.706	5.265
2	-.333	-.250	.005	.082	.068	1.213	.001	.144	-.128	-2.917	-.064	-8.853	12.706	5.265
2	-.333	-.250	.006	.095	.068	1.400	-.004	-.584	-.192	-4.048	-.064	-8.853	12.706	5.265
2	-.333	0	.004	.054	.068	.791	.009	1.790	.005	.115	-.064	-8.853	12.706	5.265
2	-.333	0	.005	.066	.068	.979	.004	.802	-.059	-1.353	-.064	-8.853	12.706	5.265
2	-.333	0	.006	.079	.068	1.166	.000	.001	-.123	-2.604	-.064	-8.853	12.706	5.265
2	-.333	.200	.004	.041	.068	.603	.011	2.366	.060	1.468	-.064	-8.853	12.706	5.265
2	-.333	.200	.005	.054	.068	.791	.007	1.322	-.004	-.092	-.064	-8.853	12.706	5.265
2	-.333	.200	.006	.066	.068	.978	.003	.471	-.068	-1.434	-.064	-8.853	12.706	5.265
2	-.333	.500	.004	.022	.068	.321	.016	3.161	.143	3.441	-.064	-8.853	12.706	5.265
2	-.333	.500	.005	.035	.068	.509	.011	2.064	.079	1.769	-.064	-8.853	12.706	5.265
2	-.333	.500	.006	.047	.068	.695	.007	1.156	.015	.309	-.064	-8.853	12.706	5.265

[a] See Table 1. Factor 2 is not significant.

[b] See equation (5-10).

[c] See equations (6-1) through (6-4).

[d] Differences between these standard errors of the forecast are beyond the third decimal place.

[e] Single-tailed critical points are $t_\infty(.05)=1.645$ and $t_\infty(.01)=2.326$.

TABLE 26--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
							Number of Securities		Autocorrelation		Intercorrelation		Variance	
Number	Autocor	Intercor	Variance	Forecast	Std Error	t Value								
2	0	-.250	.004	.022	.068	.317	-.014	-3.092	-.225	-5.584	.028	5.876	-8.632	-5.330
2	0	-.250	.005	.013	.068	.190	-.018	-3.572	-.289	-6.654	.028	5.876	-8.632	-5.330
2	0	-.250	.006	.004	.068	.062	-.023	-3.917	-.353	-7.513	.028	5.876	-8.632	-5.330
2	0	0	.004	.029	.068	.421	-.010	-2.329	-.156	-3.886	.028	5.876	-8.632	-5.330
2	0	0	.005	.020	.068	.294	-.015	-2.900	-.220	-5.033	.028	5.876	-8.632	-5.330
2	0	0	.006	.011	.068	.167	-.019	-3.322	-.284	-6.064	.028	5.876	-8.632	-5.330
2	0	.200	.004	.034	.068	.505	-.008	-1.686	-.100	-2.497	.028	5.876	-8.632	-5.330
2	0	.200	.005	.026	.068	.378	-.012	-2.333	-.164	-3.793	.028	5.876	-8.632	-5.330
2	0	.200	.006	.017	.068	.250	-.016	-2.818	-.228	-4.870	.028	5.876	-8.632	-5.330
2	0	.500	.004	.043	.068	.630	-.003	-.721	-.017	-.425	.028	5.876	-8.632	-5.330
2	0	.500	.005	.034	.068	.503	-.008	-1.467	-.031	-1.852	.028	5.876	-8.632	-5.330
2	0	.500	.006	.025	.068	.375	-.012	-2.943	-.145	-3.064	.028	5.876	-8.632	-5.330
2	.333	-.250	.004	-.030	.068	-1.179	-.033	-6.792	-.385	-9.403	.120	16.713	-29.970	-12.418
2	.333	-.250	.005	-.110	.068	-1.621	-.037	-6.882	-.449	-10.196	.120	16.713	-29.970	-12.418
2	.333	-.250	.006	-.140	.068	-2.061	-.042	-6.893	-.513	-10.789	.120	16.713	-29.970	-12.418
2	.333	0	.004	-.050	.068	-.736	-.029	-6.111	-.316	-7.753	.120	16.713	-29.970	-12.418
2	.333	0	.005	-.080	.068	-1.178	-.034	-6.269	-.380	-8.662	.120	16.713	-29.970	-12.418
2	.333	0	.006	-.110	.068	-1.619	-.038	-6.339	-.444	-9.369	.120	16.713	-29.970	-12.418
2	.333	.200	.004	-.026	.068	-.381	-.027	-5.496	-.260	-6.376	.120	16.713	-29.970	-12.418
2	.333	.200	.005	-.056	.068	-.823	-.031	-5.721	-.325	-7.382	.120	16.713	-29.970	-12.418
2	.333	.200	.006	-.086	.068	-1.264	-.035	-5.849	-.389	-8.183	.120	16.713	-29.970	-12.418
2	.333	.500	.004	.010	.068	.151	-.022	-4.505	-.178	-4.278	.120	16.713	-29.970	-12.418
2	.333	.500	.005	-.020	.068	-.290	-.027	-4.836	-.242	-5.420	.120	16.713	-29.970	-12.418
2	.333	.500	.006	-.050	.068	-.731	-.031	-5.061	-.306	-6.361	.120	16.713	-29.970	-12.418
2	.667	-.250	.004	-.235	.068	-3.445	-.052	-9.013	-.545	-12.638	.213	18.093	-51.307	-13.002
2	.667	-.250	.005	-.286	.068	-4.204	-.056	-9.010	-.609	-13.264	.213	18.093	-51.307	-13.002
2	.667	-.250	.006	-.338	.068	-4.950	-.061	-8.926	-.673	-13.653	.213	18.093	-51.307	-13.002
2	.667	0	.004	-.182	.068	-2.671	-.049	-8.446	-.476	-11.129	.213	18.093	-51.307	-13.002
2	.667	0	.005	-.233	.068	-3.430	-.053	-8.483	-.540	-11.805	.213	18.093	-51.307	-13.002
2	.667	0	.006	-.285	.068	-4.179	-.057	-8.439	-.604	-12.293	.213	18.093	-51.307	-13.002
2	.667	.200	.004	-.139	.068	-2.047	-.046	-7.926	-.421	-9.809	.213	18.093	-51.307	-13.002
2	.667	.200	.005	-.191	.068	-2.805	-.050	-8.095	-.485	-10.570	.213	18.093	-51.307	-13.002
2	.667	.200	.006	-.242	.068	-3.555	-.054	-8.001	-.549	-11.144	.213	18.093	-51.307	-13.002
2	.667	.500	.004	-.076	.068	-1.108	-.041	-7.061	-.338	-7.762	.213	18.093	-51.307	-13.002
2	.667	.500	.005	-.127	.068	-1.863	-.046	-7.214	-.402	-8.652	.213	18.093	-51.307	-13.002
2	.667	.500	.006	-.178	.068	-2.613	-.050	-7.280	-.466	-9.356	.213	18.093	-51.307	-13.002

TABLE 26--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Number	Autocor	Intercor	Variance	Forecast	Std Error	t Value	Number of Securities		Autocorrelation		Intercorrelation		Variance	
3	-.667	-.250	.004	.090	.068	1.326	.028	4.086	.039	.866	-.142	-10.968	29.728	6.858
3	-.667	-.250	.005	.120	.068	1.765	.023	3.246	-.025	-.534	-.142	-10.968	29.728	6.858
3	-.667	-.250	.006	.150	.068	2.197	.019	2.479	-.089	-1.761	-.142	-10.968	29.728	6.858
3	-.667	0	.004	.055	.068	.807	.031	4.629	.103	2.424	-.142	-10.968	29.728	6.858
3	-.667	0	.005	.085	.068	1.246	.027	3.754	.044	.924	-.142	-10.968	29.728	6.858
3	-.667	0	.006	.114	.068	1.681	.022	2.951	-.020	-.401	-.142	-10.968	29.728	6.858
3	-.667	.200	.004	.027	.068	.390	.034	5.037	.163	3.658	-.142	-10.968	29.728	6.858
3	-.667	.200	.005	.056	.068	.829	.030	4.141	.099	2.087	-.142	-10.968	29.728	6.858
3	-.667	.200	.006	.086	.068	1.264	.025	3.316	.035	.690	-.142	-10.968	29.728	6.858
3	-.667	.500	.004	-.016	.068	-.235	.038	5.593	.246	5.444	-.142	-10.968	29.728	6.858
3	-.667	.500	.005	.014	.068	.202	.034	4.679	.182	3.790	-.142	-10.968	29.728	6.858
3	-.667	.500	.006	.043	.068	.637	.030	3.832	.118	2.301	-.142	-10.968	29.728	6.858
3	-.333	-.250	.004	.076	.068	1.126	.009	1.424	-.122	-2.849	-.050	-5.516	8.390	2.790
3	-.333	-.250	.005	.085	.068	1.251	.004	.652	-.186	-4.065	-.050	-5.516	8.390	2.790
3	-.333	-.250	.006	.093	.068	1.373	-.000	-.016	-.250	-5.089	-.050	-5.516	8.390	2.790
3	-.333	0	.004	.064	.068	.945	.012	2.025	-.052	-1.236	-.050	-5.516	8.390	2.790
3	-.333	0	.005	.072	.068	1.069	.008	1.206	-.117	-2.562	-.050	-5.516	8.390	2.790
3	-.333	0	.006	.081	.068	1.191	.003	.493	-.181	-3.693	-.050	-5.516	8.390	2.790
3	-.333	.200	.004	.054	.068	.798	.015	2.493	.003	.065	-.050	-5.516	8.390	2.790
3	-.333	.200	.005	.063	.068	.923	.011	1.642	-.061	-1.343	-.050	-5.516	8.390	2.790
3	-.333	.200	.006	.071	.068	1.045	.006	.898	-.125	-2.556	-.050	-5.516	8.390	2.790
3	-.333	.500	.004	.039	.068	.578	.019	3.151	.086	1.981	-.050	-5.516	8.390	2.790
3	-.333	.500	.005	.048	.068	.702	.015	2.269	.022	.469	-.050	-5.516	8.390	2.790
3	-.333	.500	.006	.056	.068	.825	.011	1.489	-.042	-.855	-.050	-5.516	8.390	2.790
3	0	-.250	.004	.009	.068	.136	-.011	-1.848	-.282	-6.709	.042	5.876	-12.948	-5.330
3	0	-.250	.005	-.004	.068	-.055	-.015	-2.399	-.346	-7.680	.042	5.876	-12.948	-5.330
3	0	-.250	.006	-.017	.068	-.245	-.019	-2.845	-.410	-8.455	.042	5.876	-12.948	-5.330
3	0	0	.004	.020	.068	.293	-.007	-1.234	-.213	-5.087	.042	5.876	-12.948	-5.330
3	0	0	.005	.007	.068	.102	-.011	-1.836	-.277	-6.170	.042	5.876	-12.948	-5.330
3	0	0	.006	-.006	.068	-.089	-.016	-2.329	-.341	-7.054	.042	5.876	-12.948	-5.330
3	0	.200	.004	.028	.068	.418	-.004	-.733	-.157	-3.755	.042	5.876	-12.948	-5.330
3	0	.200	.005	.015	.068	.227	-.008	-1.373	-.221	-4.925	.042	5.876	-12.948	-5.330
3	0	.200	.006	.002	.068	.036	-.013	-1.902	-.285	-5.897	.042	5.876	-12.948	-5.330
3	0	.500	.004	.041	.068	.606	.000	.013	-.075	-1.751	.042	5.876	-12.948	-5.330
3	0	.500	.005	.028	.068	.415	-.004	-.675	-.139	-3.041	.042	5.876	-12.948	-5.330
3	0	.500	.006	.015	.068	.224	-.009	-1.254	-.203	-4.137	.042	5.876	-12.948	-5.330

TABLE 26--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Num- ber	Auto- cor	Inter- cor	Vari- ance	Fore- cast	Std Error	t Value	Number of Securities		Autocor- relation		Intercor- relation		Variance	
3	.333	-.250	.004	-.111	.068	-1.640	-.030	-4.947	-.442	-10.358	.135	14.958	-34.286	-11.402
3	.333	-.250	.005	-.146	.068	-2.147	-.034	-5.259	-.506	-11.084	.135	14.958	-34.286	-11.402
3	.333	-.250	.006	-.180	.068	-2.649	-.038	-5.480	-.570	-11.621	.135	14.958	-34.286	-11.402
3	.333	0	.004	-.078	.068	-1.146	-.026	-4.378	-.373	-8.778	.135	14.958	-34.286	-11.402
3	.333	0	.005	-.112	.068	-1.653	-.030	-4.731	-.437	-9.608	.135	14.958	-34.286	-11.402
3	.333	0	.006	-.146	.068	-2.156	-.035	-4.991	-.501	-10.247	.135	14.958	-34.286	-11.402
3	.333	.200	.004	-.051	.068	-.749	-.023	-3.890	-.318	-7.456	.135	14.958	-34.286	-11.402
3	.333	.200	.005	-.085	.068	-1.256	-.028	-4.278	-.382	-8.372	.135	14.958	-34.286	-11.402
3	.333	.200	.006	-.119	.068	-1.759	-.032	-4.573	-.446	-9.097	.135	14.958	-34.286	-11.402
3	.333	.500	.004	-.010	.068	-.154	-.019	-3.125	-.235	-5.429	.135	14.958	-34.286	-11.402
3	.333	.500	.005	-.045	.068	-.659	-.023	-3.566	-.299	-6.470	.135	14.958	-34.286	-11.402
3	.333	.500	.006	-.079	.068	-1.163	-.028	-3.915	-.363	-7.323	.135	14.958	-34.286	-11.402
3	.667	-.250	.004	-.285	.068	-4.190	-.049	-7.210	-.602	-13.494	.227	17.541	-55.623	-12.832
3	.667	-.250	.005	-.341	.068	-5.014	-.053	-7.393	-.666	-14.039	.227	17.541	-55.623	-12.832
3	.667	-.250	.006	-.397	.068	-5.821	-.057	-7.495	-.730	-14.387	.227	17.541	-55.623	-12.832
3	.667	0	.004	-.229	.068	-3.365	-.045	-6.712	-.533	-11.995	.227	17.541	-55.623	-12.832
3	.667	0	.005	-.284	.068	-4.190	-.049	-6.924	-.597	-12.621	.227	17.541	-55.623	-12.832
3	.667	0	.006	-.340	.068	-5.000	-.054	-7.054	-.661	-13.066	.227	17.541	-55.623	-12.832
3	.667	.200	.004	-.183	.068	-2.698	-.042	-6.275	-.478	-10.724	.227	17.541	-55.623	-12.832
3	.667	.200	.005	-.239	.068	-3.522	-.047	-6.512	-.542	-11.426	.227	17.541	-55.623	-12.832
3	.667	.200	.006	-.295	.068	-4.333	-.051	-6.669	-.606	-11.950	.227	17.541	-55.623	-12.832
3	.667	.500	.004	-.115	.068	-1.694	-.038	-5.570	-.395	-8.745	.227	17.541	-55.623	-12.832
3	.667	.500	.005	-.171	.068	-2.514	-.042	-5.850	-.459	-9.564	.227	17.541	-55.623	-12.832
3	.667	.500	.006	-.227	.068	-3.325	-.047	-6.050	-.523	-10.208	.227	17.541	-55.623	-12.832
4	-.667	-.250	.004	.120	.068	1.749	.031	3.918	-.019	-.396	-.128	-8.853	25.412	5.265
4	-.667	-.250	.005	.145	.068	2.126	.027	3.227	-.083	-1.665	-.128	-8.853	25.412	5.265
4	-.667	-.250	.006	.170	.068	2.494	.022	2.575	-.147	-2.779	-.128	-8.853	25.412	5.265
4	-.667	0	.004	.088	.068	1.286	.035	4.378	.051	1.082	-.128	-8.853	25.412	5.265
4	-.667	0	.005	.113	.068	1.663	.030	3.665	-.013	-.273	-.128	-8.853	25.412	5.265
4	-.667	0	.006	.139	.068	2.032	.026	2.990	-.077	-1.473	-.128	-8.853	25.412	5.265
4	-.667	.200	.004	.052	.068	.912	.037	4.728	.106	2.261	-.128	-8.853	25.412	5.265
4	-.667	.200	.005	.088	.068	1.288	.033	4.001	.042	.844	-.128	-8.853	25.412	5.265
4	-.667	.200	.006	.113	.068	1.658	.029	3.311	-.022	-.422	-.128	-8.853	25.412	5.265
4	-.667	.500	.004	.024	.068	.350	.042	5.215	.189	3.983	-.128	-8.853	25.412	5.265
4	-.667	.500	.005	.049	.068	.723	.037	4.475	.125	2.490	-.128	-8.853	25.412	5.265
4	-.667	.500	.006	.075	.068	1.093	.033	3.770	.061	1.141	-.128	-8.853	25.412	5.265

TABLE 26--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Num-ber	Auto-cor	Inter-cor	Vari-ance	Fore-cast	Std Error	t Value	Number of Securities		Autocor-relation		Intercor-relation		Variance	
4	-.333	-.250	.004	.087	.068	1.274	.012	1.642	-.179	-3.972	-.035	-3.217	4.074	1.107
4	-.333	-.250	.005	.091	.068	1.336	.008	.996	-.243	-5.074	-.035	-3.217	4.074	1.107
4	-.333	-.250	.006	.095	.068	1.394	.003	.403	-.307	-6.004	-.035	-3.217	4.074	1.107
4	-.333	0	.004	.078	.068	1.145	.015	2.137	-.110	-2.447	-.035	-3.217	4.074	1.107
4	-.333	0	.005	.082	.068	1.207	.011	1.462	-.174	-3.643	-.035	-3.217	4.074	1.107
4	-.333	0	.006	.086	.068	1.265	.007	.847	-.238	-4.666	-.035	-3.217	4.074	1.107
4	-.333	.200	.004	.071	.068	1.041	.018	2.522	-.054	-1.211	-.035	-3.217	4.074	1.107
4	-.333	.200	.005	.075	.068	1.103	.014	1.830	-.118	-2.478	-.035	-3.217	4.074	1.107
4	-.333	.200	.006	.079	.068	1.161	.010	1.195	-.182	-3.574	-.035	-3.217	4.074	1.107
4	-.333	.500	.004	.060	.068	.883	.023	3.072	.028	.625	-.035	-3.217	4.074	1.107
4	-.333	.500	.005	.064	.068	.944	.018	2.360	-.036	-.735	-.035	-3.217	4.074	1.107
4	-.333	.500	.006	.068	.068	1.003	.014	1.704	-.100	-1.930	-.035	-3.217	4.074	1.107
4	0	-.250	.004	.000	.068	.006	-.007	-1.012	-.339	-7.639	.057	5.876	-17.264	-5.330
4	0	-.250	.005	-.017	.068	-.248	-.011	-1.537	-.403	-8.529	.057	5.876	-17.264	-5.330
4	0	-.250	.006	-.034	.068	-.502	-.016	-1.993	-.467	-9.241	.057	5.876	-17.264	-5.330
4	0	0	.004	.015	.068	.215	-.004	-.511	-.270	-6.107	.057	5.876	-17.264	-5.330
4	0	0	.005	-.003	.068	-.040	-.008	-1.065	-.334	-7.093	.057	5.876	-17.264	-5.330
4	0	C	.006	-.020	.068	-.294	-.012	-1.550	-.398	-7.899	.057	5.876	-17.264	-5.330
4	0	.200	.004	.026	.068	.381	-.001	-.107	-.215	-4.844	.057	5.876	-17.264	-5.330
4	0	.200	.005	.009	.068	.127	-.005	-.682	-.279	-5.905	.057	5.876	-17.264	-5.330
4	0	.200	.006	-.009	.068	-.127	-.009	-1.188	-.343	-6.788	.057	5.876	-17.264	-5.330
4	0	.500	.004	.043	.068	.630	.003	.491	-.132	-2.933	.057	5.876	-17.264	-5.330
4	0	.500	.005	.026	.068	.377	-.001	-.109	-.196	-4.099	.057	5.876	-17.264	-5.330
4	0	.500	.006	.008	.068	.123	-.005	-.644	-.260	-5.092	.057	5.876	-17.264	-5.330
4	.333	-.250	.004	-.139	.068	-2.046	-.026	-3.602	-.499	-11.091	.149	13.489	-38.601	-10.437
4	.333	-.250	.005	-.178	.068	-2.617	-.031	-3.983	-.563	-11.771	.149	13.489	-38.601	-10.437
4	.333	-.250	.006	-.216	.068	-3.181	-.035	-4.295	-.627	-12.275	.149	13.489	-38.601	-10.437
4	.333	0	.004	-.102	.068	-1.502	-.023	-3.126	-.430	-9.594	.149	13.489	-38.601	-10.437
4	.333	0	.005	-.141	.068	-2.073	-.027	-3.532	-.494	-10.364	.149	13.489	-38.601	-10.437
4	.333	0	.006	-.179	.068	-2.639	-.031	-3.869	-.558	-10.957	.149	13.489	-38.601	-10.437
4	.333	.200	.004	-.072	.068	-1.065	-.020	-2.729	-.375	-8.340	.149	13.489	-38.601	-10.437
4	.333	.200	.005	-.111	.068	-1.635	-.024	-3.155	-.439	-9.184	.149	13.489	-38.601	-10.437
4	.333	.200	.006	-.150	.068	-2.201	-.028	-3.512	-.503	-9.853	.149	13.489	-38.601	-10.487
4	.333	.500	.004	-.028	.068	-.407	-.016	-2.120	-.292	-6.410	.149	13.489	-38.601	-10.437
4	.333	.500	.005	-.066	.068	-.976	-.020	-2.573	-.356	-7.362	.149	13.489	-38.601	-10.437
4	.333	.500	.006	-.105	.068	-1.541	-.024	-2.960	-.420	-8.144	.149	13.489	-38.601	-10.437

TABLE 26--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Number	Autocor	Intercor	Variance	Forecast	Std Error	t Value	Number of Securities		Autocorrelation		Intercorrelation		Variance	
4	.667	-.250	.004	-.332	.068	-4.862	-.045	-5.719	-.659	-14.070	.241	16.713	-59.939	-12.418
4	.667	-.250	.005	-.392	.068	-5.753	-.050	-5.993	-.723	-14.587	.241	16.713	-59.939	-12.418
4	.667	-.250	.006	-.452	.068	-6.618	-.054	-6.202	-.787	-14.931	.241	16.713	-59.939	-12.418
4	.667	0	.004	-.272	.068	-3.992	-.042	-5.287	-.590	-12.642	.241	16.713	-59.939	-12.418
4	.667	0	.005	-.332	.068	-4.882	-.046	-5.580	-.654	-13.237	.241	16.713	-59.939	-12.418
4	.667	0	.006	-.392	.068	-5.752	-.050	-5.808	-.718	-13.661	.241	16.713	-59.939	-12.418
4	.667	.200	.004	-.224	.068	-3.286	-.039	-4.919	-.535	-11.432	.241	16.713	-59.939	-12.418
4	.667	.200	.005	-.284	.068	-4.175	-.043	-5.228	-.599	-12.094	.241	16.713	-59.939	-12.418
4	.667	.200	.006	-.344	.068	-5.046	-.048	-5.473	-.663	-12.536	.241	16.713	-59.939	-12.418
4	.667	.500	.004	-.152	.068	-2.219	-.035	-4.339	-.452	-9.543	.241	16.713	-59.939	-12.418
4	.667	.500	.005	-.212	.068	-3.103	-.039	-4.672	-.516	-10.307	.241	16.713	-59.939	-12.418
4	.667	.500	.006	-.272	.068	-3.973	-.043	-4.944	-.580	-10.907	.241	16.713	-59.939	-12.418

forecasted value of E at each point. In addition, the t-values testing each forecast and partial derivative against the zero null hypotheses are shown. This information enables us to proceed with the explanation of the validation process deferred from the previous chapter.

Again, the primary interest was to determine that the model does not result in statistically significant contradictions to analytical results. The first result to be considered is the expected return superiority of BH over RB under the assumption of independence. Here the model does not contradict the analytical conclusion. Inspection of the forecasted values and corresponding t-values in the table when the autocorrelation factor is equal to zero indicates that although many of the values are positive (implying RB superiority), none are statistically significant.

A test of the conclusion that under independence the return superiority of BH decreases as securities are added to random portfolios involves an examination of the partial derivative with respect to the number of securities. Here we find that for the smaller portfolio sizes there are negative values that are statistically significant at the 0.01 level, apparently contradicting the analytical conclusion and making the entire model suspect.

This troublesome result can, however, be explained. We first note that despite the fact that the analysis of the simulated data did not show the effect of the divergence in security returns (factor 2) to be significant, there is considerable analytical and empirical evidence in support of the notion that the expected return superiority of BH is an increasing function of the size of this factor. This point is made relevant to this discussion by the Cheng-Deets statement that although the addition of securities improves the relative return to RB for random portfolios, "the relative superiority of the BH strategy can be maintained if the $m^{\underline{th}}$ security is systematically selected such that" its expected return is sufficiently different from the average return in the portfolio [39, p. 17].

Now recall the relatively large sample variation in the simulated ex post values of the mean return of each stock price series. As previously noted, this caused an inability to satisfactorily control the differences between the mean returns realized by the securities in the portfolio (factor 2). Consequently, when the membership of the simulated portfolio was expanded, the additional security did not necessarily have an ex post mean return equal to the average return in the portfolio. Rather, the large sample variation caused a situation where the entering security had returns which could easily have been drawn from a population with a mean return quite different from 1.007. It therefore seems quite plausible that the effect of the divergence of the mean returns (factor 2) among the members of a simulated portfolio is confounded with the portfolio size effect (factor 1).

It can be demonstrated that this divergence due to sampling variation is a decreasing function of the size of the portfolio.[1] This would lead us to expect a decline in the degree to which the portfolio size effect is confounded with the effect of divergent returns, and a consequent increase in the value of the partial derivative in question, as the portfolio size is increased. This expectation is borne out either by an examination of the sign of the coefficient of x_1 in equation (6-1), or by observing in Table 26 that the negative partial derivatives with respect to portfolio size (under independence) in the four security case are no longer statistically significant.

Finally, for series of securities of a given length, the divergence of the ex post mean returns due to sample variation is directly related both to the degree of autocorrelation (factor 3) and to the variance of the returns (factor 5) in each series.[2] If the effect of divergent means can be generalized beyond the random walk case,[3] the confounding that we hypothesize would cause the partial derivative with respect to the number of securities to be a decreasing function of these two factors. The signs of the coefficients of x_3 and x_5 in equation (6-1) are consistent with this hypothesis. With hindsight, it is evident that had the three levels of factor 2 been set at wide enough intervals, the divergence in means would have shown itself as a significant determinant of relative performance beyond any confounding due to sample variation.

These results provide sufficient assurance in the validity of the models to continue to study their implications.

Inferences

The partial derivatives of equation (5-9) predicting G are given by

$$\delta G/\delta x_1 = -0.05669x_3+0.00857x_4-2.29050x_5 \quad (6\text{-}5)$$

$$\delta G/\delta x_3 = 0.017751-2\cdot(0.17131)x_3-0.05669x_1+0.23133x_4-46.62879x_5 \quad (6\text{-}6)$$

$$\delta G/\delta x_4 = 0.00857x_1+0.23133x_3 \quad (6\text{-}7)$$

$$\delta G/\delta x_5 = -2.29050x_1-46.62879x_3. \quad (6\text{-}8)$$

Table 27 presents the predicted response and partial derivatives for each design point as well as the t-values for each.

A comparison of either the predicting equations themselves, or the values in this table with those for the expected value response shows a strong similarity between the two models. In fact, a test of the difference between the forecasts of E and G cannot reject the null hypothesis that they are equal for any of the design points. The most significant difference occurs when the vector of factors is [4, 0.667, -0.250, 0.006] at which point the expected value response is predicted at -0.452 and the forecast of the geometric mean response is -0.352. With a pooled standard error of 0.084, this difference of 0.100 is not significant. This enables us to concentrate our attention on either of the two response equations, with the expectation that the

TABLE 27

PREDICTED RESPONSE, PARTIAL DERIVATIVES, AND T-VALUES FOR GEOMETRIC MEAN RESPONSE AT EACH DESIGN POINT

Factor Levels[a]				Predicted Response[b]			Partial Derivative (left) and t-Value (right) with Respect to[c]							
Number	Autocor	Intercor	Variance	Forecast	Std Error[d]	t[e] Value	Number of Securities		Autocorrelation		Intercorrelation		Variance	
2	-.667	-.250	.004	.067	.049	1.378	.027	9.208	.048	1.565	-.137	-16.274	26.505	9.978
2	-.667	-.250	.005	.094	.049	1.923	.024	8.019	.002	.048	-.137	-16.274	26.505	9.978
2	-.667	-.250	.006	.120	.049	2.464	.022	6.887	-.045	-1.275	-.137	-16.274	26.505	9.978
2	-.667	0	.004	.033	.049	.678	.029	10.060	.106	3.459	-.137	-16.274	26.505	9.978
2	-.667	0	.005	.060	.049	1.223	.027	8.813	.059	1.812	-.137	-16.274	26.505	9.978
2	-.667	0	.006	.086	.049	1.765	.024	7.624	.013	.363	-.137	-16.274	26.505	9.978
2	-.667	.200	.004	.006	.049	.116	.031	10.581	.152	4.954	-.137	-16.274	26.505	9.978
2	-.667	.200	.005	.032	.049	.660	.028	9.321	.106	3.215	-.137	-16.274	26.505	9.978
2	-.667	.200	.006	.059	.049	1.203	.026	8.115	.059	1.673	-.137	-16.274	26.505	9.978
2	-.667	.500	.004	-.035	.049	-.725	.033	11.062	.222	7.107	-.137	-16.274	26.505	9.978
2	-.667	.500	.005	-.009	.049	-.183	.031	9.838	.175	5.259	-.137	-16.274	26.505	9.978
2	-.667	.500	.006	.018	.049	.359	.029	8.657	.128	3.599	-.137	-16.274	26.505	9.978
2	-.333	-.250	.004	.064	.049	1.322	.008	4.099	-.067	-2.290	-.059	-11.516	10.807	7.610
2	-.333	-.250	.005	.075	.049	1.549	.005	2.541	-.114	-3.607	-.059	-11.516	10.807	7.610
2	-.333	-.250	.006	.086	.049	1.774	.003	1.275	-.160	-4.709	-.059	-11.516	10.807	7.610
2	-.333	0	.004	.049	.049	1.015	.010	5.424	-.009	-.319	-.059	-11.516	10.807	7.610
2	-.333	0	.005	.060	.049	1.241	.007	3.669	-.056	-1.781	-.059	-11.516	10.807	7.610
2	-.333	0	.006	.071	.049	1.467	.005	2.250	-.103	-3.022	-.059	-11.516	10.807	7.610
2	-.333	.200	.004	.037	.049	.769	.011	6.270	.037	1.262	-.059	-11.516	10.807	7.610
2	-.333	.200	.005	.048	.049	.995	.009	4.458	-.010	-.307	-.059	-11.516	10.807	7.610
2	-.333	.200	.006	.059	.049	1.220	.007	2.977	-.056	-1.655	-.059	-11.516	10.807	7.610
2	-.333	.500	.004	.019	.049	.398	.014	7.044	.106	3.575	-.059	-11.516	10.807	7.610
2	-.333	.500	.005	.030	.049	.624	.012	5.331	.060	1.870	-.059	-11.516	10.807	7.610
2	-.333	.500	.006	.041	.049	.849	.009	3.877	.013	.380	-.059	-11.516	10.807	7.610

[a] See Table 1. Factor 2 is not significant.

[b] See equation (5-9).

[c] See equations (6-5) through (6-8).

[d] Differences between these standard errors of the forecast are beyond the third decimal place.

[e] Single-tailed critical points are $t_{\infty}(.05)=1.645$ and $t_{\infty}(.01)=2.326$.

TABLE 27--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Number	Autocor	Intercor	Variance	Forecast	Std Error	t Value	Number of Securities		Autocorrelation		Intercorrelation		Variance	
2	0	-.250	.004	.023	.049	.480	-.011	-8.646	-.180	-6.250	.017	4.954	-4.581	-7.425
2	0	-.250	.005	.019	.049	.386	-.014	-8.487	-.227	-7.296	.017	4.954	-4.581	-7.425
2	0	-.250	.006	.014	.049	.292	-.016	-8.358	-.273	-8.129	.017	4.954	-4.581	-7.425
2	0	0	.004	.028	.049	.568	-.009	-7.425	-.122	-4.266	.017	4.954	-4.581	-7.425
2	0	0	.005	.023	.049	.474	-.011	-7.425	-.169	-5.460	.017	4.954	-4.581	-7.425
2	0	0	.006	.018	.049	.380	-.014	-7.425	-.216	-6.433	.017	4.954	-4.581	-7.425
2	0	.200	.004	.031	.049	.639	-.007	-5.812	-.076	-2.645	.017	4.954	-4.581	-7.425
2	0	.200	.005	.026	.049	.545	-.010	-6.161	-.123	-3.954	.017	4.954	-4.581	-7.425
2	0	.200	.006	.022	.049	.451	-.012	-6.388	-.169	-5.041	.017	4.954	-4.581	-7.425
2	0	.500	.004	.036	.049	.744	-.005	-3.236	-.007	-.230	.017	4.954	-4.581	-7.425
2	0	.500	.005	.032	.049	.650	-.007	-4.053	-.053	-1.694	.017	4.954	-4.581	-7.425
2	0	.500	.006	.027	.049	.556	-.009	-4.629	-.100	-2.940	.017	4.954	-4.581	-7.425
2	.333	-.250	.004	-.056	.049	-1.145	-.030	-16.476	-.294	-10.036	.094	18.234	-20.124	-14.056
2	.333	-.250	.005	-.076	.049	-1.560	-.033	-15.830	-.341	-10.808	.094	18.234	-20.124	-14.056
2	.333	-.250	.006	-.096	.049	-1.974	-.035	-15.173	-.388	-11.378	.094	18.234	-20.124	-14.056
2	.333	0	.004	-.032	.049	-.662	-.028	-15.753	-.237	-8.104	.094	18.234	-20.124	-14.056
2	.333	0	.005	-.052	.049	-1.076	-.030	-15.127	-.283	-9.013	.094	18.234	-20.124	-14.056
2	.333	0	.006	-.072	.049	-1.490	-.033	-14.500	-.330	-9.715	.094	18.234	-20.124	-14.056
2	.333	.200	.004	-.013	.049	-.274	-.026	-14.524	-.190	-6.499	.094	18.234	-20.124	-14.056
2	.333	.200	.005	-.033	.049	-.689	-.029	-14.069	-.237	-7.520	.094	18.234	-20.124	-14.056
2	.333	.200	.006	-.054	.049	-1.103	-.031	-13.582	-.284	-8.333	.094	18.234	-20.124	-14.056
2	.333	.500	.004	.015	.049	.306	-.024	-12.022	-.121	-4.064	.094	18.234	-20.124	-14.056
2	.333	.500	.005	-.005	.049	-.107	-.026	-11.942	-.168	-5.245	.094	18.234	-20.124	-14.056
2	.333	.500	.006	-.025	.049	-.521	-.028	-11.768	-.214	-6.220	.094	18.234	-20.124	-14.056
2	.667	-.250	.004	-.173	.049	-3.537	-.049	-17.009	-.409	-13.271	.171	20.343	-35.667	-13.426
2	.667	-.250	.005	-.209	.049	-4.273	-.052	-16.956	-.455	-13.832	.171	20.343	-35.667	-13.426
2	.667	-.250	.006	-.244	.049	-5.000	-.054	-16.785	-.502	-14.204	.171	20.343	-35.667	-13.426
2	.667	0	.004	-.130	.049	-2.666	-.047	-16.454	-.351	-11.443	.171	20.343	-35.667	-13.426
2	.667	0	.005	-.166	.049	-3.402	-.049	-16.418	-.397	-12.122	.171	20.343	-35.667	-13.426
2	.667	0	.006	-.201	.049	-4.130	-.052	-16.266	-.444	-12.609	.171	20.343	-35.667	-13.426
2	.667	.200	.004	-.096	.049	-1.964	-.045	-15.742	-.305	-9.906	.171	20.343	-35.667	-13.426
2	.667	.200	.005	-.131	.049	-2.699	-.048	-15.746	-.351	-10.684	.171	20.343	-35.667	-13.426
2	.667	.200	.006	-.167	.049	-3.428	-.050	-15.635	-.398	-11.271	.171	20.343	-35.667	-13.426
2	.667	.500	.004	-.044	.049	-.908	-.043	-14.321	-.235	-7.538	.171	20.343	-35.667	-13.426
2	.667	.500	.005	-.080	.049	-1.640	-.045	-14.413	-.282	-8.464	.171	20.343	-35.667	-13.426
2	.667	.500	.006	-.116	.049	-2.369	-.047	-14.396	-.328	-9.203	.171	20.343	-35.667	-13.426

TABLE 27--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Num-ber	Auto-cor	Inter-cor	Vari-ance	Fore-cast	Std Error	t Value	Number of Securities		Autocor-relation		Intercor-relation		Variance	
3	-.667	-.250	.004	.094	.049	1.923	.027	9.208	-.008	-.266	-.129	-13.864	24.214	8.823
3	-.667	-.250	.005	.118	.049	2.422	.024	8.019	-.055	-1.619	-.129	-13.864	24.214	8.823
3	-.667	-.250	.006	.142	.049	2.915	.022	6.887	-.102	-2.797	-.129	-13.864	24.214	8.823
3	-.667	0	.004	.062	.049	1.267	.029	10.060	.049	1.549	-.129	-13.864	24.214	8.823
3	-.667	0	.005	.086	.049	1.767	.027	8.813	.003	.080	-.129	-13.864	24.214	8.823
3	-.667	0	.006	.110	.049	2.261	.024	7.624	-.044	-1.211	-.129	-13.864	24.214	8.823
3	-.667	.200	.004	.036	.049	.739	.031	10.581	.096	2.993	-.129	-13.864	24.214	8.823
3	-.667	.200	.005	.060	.049	1.238	.028	9.321	.049	1.441	-.129	-13.864	24.214	8.823
3	-.667	.200	.006	.084	.049	1.734	.026	8.115	.002	.065	-.129	-13.864	24.214	8.823
3	-.667	.500	.004	-.003	.049	-.052	.033	11.062	.165	5.096	-.129	-13.864	24.214	8.823
3	-.667	.500	.005	.022	.049	.444	.031	9.838	.118	3.441	-.129	-13.864	24.214	8.823
3	-.667	.500	.006	.046	.049	.940	.029	8.654	.072	1.954	-.129	-13.864	24.214	8.823
3	-.333	-.250	.004	.072	.049	1.479	.008	4.099	-.124	-4.049	-.051	-7.870	8.516	5.394
3	-.333	-.250	.005	.081	.049	1.658	.005	2.541	-.170	-5.210	-.051	-7.870	8.516	5.394
3	-.333	-.250	.006	.089	.049	1.836	.003	1.275	-.217	-6.176	-.051	-7.870	8.516	5.394
3	-.333	0	.004	.059	.049	1.217	.010	5.424	-.066	-2.168	-.051	-7.870	8.516	5.394
3	-.333	0	.005	.068	.049	1.395	.007	3.669	-.113	-3.456	-.051	-7.870	8.516	5.394
3	-.333	0	.006	.076	.049	1.573	.005	2.250	-.159	-4.546	-.051	-7.870	8.516	5.394
3	-.333	.200	.004	.049	.049	1.005	.011	6.270	-.020	-.647	-.051	-7.870	8.516	5.394
3	-.333	.200	.005	.058	.049	1.184	.009	4.458	-.066	-2.031	-.051	-7.870	8.516	5.394
3	-.333	.200	.006	.066	.049	1.362	.007	2.977	-.113	-3.219	-.051	-7.870	8.516	5.394
3	-.333	.500	.004	.033	.049	.687	.014	7.044	.050	1.602	-.051	-7.870	8.516	5.394
3	-.333	.500	.005	.042	.049	.865	.012	5.331	.003	.091	-.051	-7.870	8.516	5.394
3	-.333	.500	.006	.051	.049	1.043	.009	3.877	-.044	-1.228	-.051	-7.870	8.516	5.394
3	0	-.250	.004	.012	.049	.248	-.011	-8.646	-.237	-7.868	.026	4.954	-6.872	-7.425
3	0	-.250	.005	.005	.049	.106	-.014	-8.487	-.284	-8.734	.026	4.954	-6.872	-7.425
3	0	-.250	.006	-.002	.049	-.035	-.016	-8.358	-.330	-9.503	.026	4.954	-6.872	-7.425
3	0	0	.004	.018	.049	.380	-.009	-7.425	-.179	-5.974	.026	4.954	-6.872	-7.425
3	0	0	.005	.012	.049	.239	-.011	-7.425	-.226	-7.020	.026	4.954	-6.872	-7.425
3	0	0	.006	.005	.049	.097	-.014	-7.425	-.272	-7.865	.026	4.954	-6.872	-7.425
3	0	.200	.004	.024	.049	.486	-.007	-5.812	-.133	-4.418	.026	4.954	-6.872	-7.425
3	0	.200	.005	.017	.049	.344	-.010	-6.161	-.179	-5.567	.026	4.954	-6.872	-7.425
3	0	.200	.006	.010	.049	.203	-.012	-6.388	-.226	-6.515	.026	4.954	-6.872	-7.425
3	0	.500	.004	.031	.049	.644	-.005	-3.236	-.063	-2.078	.026	4.954	-6.872	-7.425
3	0	.500	.005	.024	.049	.503	-.007	-4.053	-.110	-3.369	.026	4.954	-6.872	-7.425
3	0	.500	.006	.018	.049	.361	-.009	-4.629	-.157	-4.463	.026	4.954	-6.872	-7.425

TABLE 27--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Number	Autocor	Intercor	Variance	Forecast	Std Error	t Value	Number of Securities		Autocorrelation		Intercorrelation		Variance	
3	.333	-.250	.004	-.086	.049	-1.767	-.030	-16.476	-.351	-11.477	.103	15.926	-22.414	-14.104
3	.333	-.250	.005	-.108	.049	-2.229	-.033	-15.830	-.398	-12.153	.103	15.926	-22.414	-14.104
3	.333	-.250	.006	-.131	.049	-2.688	-.035	-15.173	-.444	-12.638	.103	15.926	-22.414	-14.104
3	.333	0	.004	-.060	.049	-1.240	-.028	-15.753	-.293	-9.630	.103	15.926	-22.414	-14.104
3	.333	0	.005	-.083	.049	-1.702	-.030	-15.127	-.340	-10.427	.103	15.926	-22.414	-14.104
3	.333	0	.006	-.105	.049	-2.162	-.033	-14.500	-.387	-11.031	.103	15.926	-22.414	-14.104
3	.333	.200	.004	-.040	.049	-.817	-.026	-14.524	-.247	-8.037	.103	15.926	-22.414	-14.104
3	.333	.200	.005	-.062	.049	-1.279	-.029	-14.069	-.294	-8.985	.103	15.926	-22.414	-14.104
3	.333	.200	.006	-.085	.049	-1.739	-.031	-13.582	-.340	-9.689	.103	15.926	-22.414	-14.104
3	.333	.500	.004	-.009	.049	-.132	-.024	-12.022	-.178	-5.730	.103	15.926	-22.414	-14.104
3	.333	.500	.005	-.031	.049	-.643	-.026	-11.942	-.224	-6.774	.103	15.926	-22.414	-14.104
3	.333	.500	.006	-.054	.049	-1.103	-.028	-11.768	-.271	-7.627	.103	15.926	-22.414	-14.104
3	.667	-.250	.004	-.222	.049	-4.550	-.049	-17.009	-.465	-14.546	.180	19.412	-37.957	-13.330
3	.667	-.250	.005	-.260	.049	-5.334	-.052	-16.956	-.512	-15.041	.180	19.412	-37.957	-13.330
3	.667	-.250	.006	-.298	.049	-6.104	-.054	-16.785	-.559	-15.352	.180	19.412	-37.957	-13.330
3	.667	0	.004	-.177	.049	-3.636	-.047	-16.454	-.407	-12.790	.180	19.412	-37.957	-13.330
3	.667	0	.005	-.215	.049	-4.420	-.049	-16.418	-.454	-13.390	.180	19.412	-37.957	-13.330
3	.667	0	.006	-.253	.049	-5.194	-.052	-16.266	-.501	-13.306	.180	19.412	-37.957	-13.330
3	.667	.200	.004	-.141	.049	-2.897	-.045	-15.742	-.361	-11.308	.180	19.412	-37.957	-13.330
3	.667	.200	.005	-.179	.049	-3.681	-.048	-15.746	-.408	-11.993	.180	19.412	-37.957	-13.330
3	.667	.200	.006	-.217	.049	-4.456	-.050	-15.635	-.454	-12.505	.180	19.412	-37.957	-13.330
3	.667	.500	.004	-.087	.049	-1.784	-.043	-14.321	-.292	-9.013	.180	19.412	-37.957	-13.330
3	.667	.500	.005	-.125	.049	-2.565	-.045	-14.413	-.338	-9.833	.180	19.412	-37.957	-13.330
3	.667	.500	.006	-.163	.049	-3.339	-.047	-14.396	-.385	-10.486	.180	19.412	-37.957	-13.330
4	-.667	-.250	.004	.120	.049	2.461	.027	9.208	-.065	-1.940	-.120	-11.602	21.924	7.657
4	-.667	-.250	.005	.142	.049	2.911	.024	8.019	-.112	-3.145	-.120	-11.602	21.924	7.657
4	-.667	-.250	.006	.164	.049	3.352	.022	6.887	-.158	-4.191	-.120	-11.602	21.924	7.657
4	-.667	0	.004	.090	.049	1.853	.029	10.060	-.007	-.220	-.120	-11.602	21.924	7.657
4	-.667	0	.005	.112	.049	2.304	.027	8.813	-.054	-1.523	-.120	-11.602	21.924	7.657
4	-.667	0	.006	.134	.049	2.747	.024	7.624	-.101	-2.669	-.120	-11.602	21.924	7.657
4	-.667	.200	.004	.066	.049	1.361	.031	10.581	.039	1.160	-.120	-11.602	21.924	7.657
4	-.667	.200	.005	.088	.049	1.812	.028	9.321	-.008	-.217	-.120	-11.602	21.924	7.657
4	-.667	.200	.006	.110	.049	2.257	.026	8.115	-.054	-1.439	-.120	-11.602	21.924	7.657
4	-.667	.500	.004	.030	.049	.621	.033	11.062	.108	3.189	-.120	-11.602	21.924	7.657
4	-.667	.500	.005	.052	.049	1.070	.031	9.838	.062	1.718	-.120	-11.602	21.924	7.657
4	-.667	.500	.006	.074	.049	1.515	.029	8.657	.015	.395	-.120	-11.602	21.924	7.657

TABLE 27--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Num-ber	Auto-cor	Inter-cor	Vari-ance	Fore-cast	Std Error	t Value	Number of Securities		Autocor-relation		Intercor-relation		Variance	
4	-.333	-.250	.004	.080	.049	1.634	.008	4.099	-.181	-5.596	-.042	-5.327	6.226	3.503
4	-.333	-.250	.005	.086	.049	1.765	.005	2.541	-.227	-6.625	-.042	-5.327	6.226	3.503
4	-.333	-.250	.006	.092	.049	1.893	.003	1.275	-.274	-7.476	-.042	-5.327	6.226	3.503
4	-.333	0	.004	.069	.049	1.417	.010	5.424	-.123	-3.819	-.042	-5.327	6.226	3.503
4	-.333	0	.005	.075	.049	1.547	.007	3.669	-.169	-4.956	-.042	-5.327	6.226	3.503
4	-.333	0	.006	.082	.049	1.676	.005	2.250	-.216	-5.915	-.042	-5.327	6.226	3.503
4	-.333	.200	.004	.060	.049	1.240	.011	6.270	-.076	-2.373	-.042	-5.327	6.226	3.503
4	-.333	.200	.005	.067	.049	1.371	.009	4.458	-.123	-3.593	-.042	-5.327	6.226	3.503
4	-.333	.200	.006	.073	.049	1.501	.007	2.977	-.170	-4.639	-.042	-5.327	6.226	3.503
4	-.333	.500	.004	.047	.049	.974	.014	7.044	-.007	-.216	-.042	-5.327	6.226	3.503
4	-.333	.500	.005	.054	.049	1.105	.012	5.331	-.054	-1.549	-.042	-5.327	6.226	3.503
4	-.333	.500	.006	.060	.049	1.234	.009	3.877	-.100	-2.714	-.042	-5.327	6.226	3.503
4	0	-.250	.004	.001	.049	.015	-.011	-8.646	-.294	-9.229	.034	4.954	-9.162	-7.425
4	0	-.250	.005	-.008	.049	-.173	-.014	-8.487	-.340	-10.045	.034	4.954	-9.162	-7.425
4	0	-.250	.006	-.018	.049	-.361	-.016	-8.358	-.387	-10.678	.034	4.954	-9.162	-7.425
4	0	0	.004	.009	.049	.191	-.009	-7.425	-.236	-7.441	.034	4.954	-9.162	-7.425
4	0	0	.005	.000	.049	.003	-.011	-7.425	-.282	-8.368	.034	4.954	-9.162	-7.425
4	0	0	.006	-.009	.049	-.185	-.014	-7.425	-.329	-9.111	.034	4.954	-9.162	-7.425
4	0	.200	.004	.016	.049	.333	-.007	-5.812	-.189	-5.965	.034	4.954	-9.162	-7.425
4	0	.200	.005	.007	.049	.144	-.010	-6.161	-.236	-6.980	.034	4.954	-9.162	-7.425
4	0	.200	.006	-.002	.049	-.045	-.012	-6.388	-.283	-7.814	.034	4.954	-9.162	-7.425
4	0	.500	.004	.026	.049	.543	-.005	-3.236	-.120	-3.730	.034	4.954	-9.162	-7.425
4	0	.500	.005	.017	.049	.355	-.007	-4.053	-.167	-4.870	.034	4.954	-9.162	-7.425
4	0	.500	.006	.008	.049	.167	-.009	-4.629	-.213	-5.834	.034	4.954	-9.162	-7.425
4	.333	-.250	.004	-.116	.049	-2.386	-.030	-16.476	-.408	-12.638	.111	14.075	-24.705	-13.829
4	.333	-.250	.005	-.141	.049	-2.893	-.033	-15.830	-.454	-13.249	.111	14.075	-24.705	-13.829
4	.333	-.250	.006	-.166	.049	-3.395	-.035	-15.173	-.501	-13.679	.111	14.075	-24.705	-13.829
4	.333	0	.004	-.088	.049	-1.817	-.028	-15.753	-.350	-10.889	.111	14.075	-24.705	-13.829
4	.333	0	.005	-.113	.049	-2.325	-.030	-15.127	-.397	-11.604	.111	14.075	-24.705	-13.829
4	.333	0	.006	-.138	.049	-2.829	-.033	-14.500	-.443	-12.138	.111	14.075	-24.705	-13.829
4	.333	.200	.004	-.066	.049	-1.359	-.026	-14.524	-.304	-9.425	.111	14.075	-24.705	-13.829
4	.333	.200	.005	-.091	.049	-1.867	-.029	-14.069	-.350	-10.227	.111	14.075	-24.705	-13.829
4	.333	.200	.006	-.115	.049	-2.371	-.031	-13.582	-.397	-10.350	.111	14.075	-24.705	-13.829
4	.333	.500	.004	-.033	.049	-.670	-.024	-12.022	-.234	-7.176	.111	14.075	-24.705	-13.829
4	.333	.500	.005	-.057	.049	-1.177	-.026	-11.942	-.281	-8.105	.111	14.075	-24.705	-13.829
4	.333	.500	.006	-.082	.049	-1.682	-.028	-11.768	-.328	-8.861	.111	14.075	-24.705	-13.829

TABLE 27--Continued

Factor Levels				Predicted Response			Partial Derivative (left) and t-Value (right) with Respect to							
Number	Autocor	Intercor	Variance	Forecast	Std Error	t Value	Number of Securities		Autocorrelation		Intercorrelation		Variance	
4	.667	-.250	.004	-.271	.049	-5.546	-.049	-17.009	-.522	-15.538	.189	18.234	-40.248	-14.056
4	.667	-.250	.005	-.311	.049	-6.373	-.052	-16.956	-.569	-15.995	.189	18.234	-40.248	-14.056
4	.667	-.250	.006	-.352	.049	-7.180	-.054	-16.785	-.615	-16.275	.189	18.234	-40.248	-14.056
4	.667	0	.004	-.224	.049	-4.594	-.047	-16.454	-.464	-13.367	.189	18.234	-40.248	-14.056
4	.667	0	.005	-.264	.049	-5.423	-.049	-16.418	-.511	-14.415	.189	18.234	-40.248	-14.056
4	.667	0	.006	-.305	.049	-6.234	-.052	-16.266	-.557	-14.783	.189	18.234	-40.248	-14.056
4	.667	.200	.004	-.186	.049	-3.821	-.045	-15.742	-.418	-12.456	.189	18.234	-40.248	-14.056
4	.667	.200	.005	-.227	.049	-4.649	-.048	-15.746	-.465	-13.082	.189	18.234	-40.248	-14.056
4	.667	.200	.006	-.267	.049	-5.463	-.050	-15.635	-.511	-13.536	.189	18.234	-40.248	-14.056
4	.667	.500	.004	-.130	.049	-2.652	-.043	-14.321	-.349	-10.261	.189	18.234	-40.248	-14.056
4	.667	.500	.005	-.170	.049	-3.478	-.045	-14.413	-.395	-11.007	.189	18.234	-40.248	-14.056
4	.667	.500	.006	-.210	.049	-4.292	-.047	-14.396	-.442	-11.585	.189	18.234	-40.248	-14.056

characteristics observed are applicable to the other with little modification.

While an examination of the differences in the expected returns to the two strategies under different factor configurations is valuable in its own right, evidence of investor risk aversion implies that a good performance measure must consider at least the first two moments of return distributions. Because we were unable to obtain a good equation for V to accompany the equation for E, we focus our attention on the geometric mean response as a measure of the relative performances of RB and BH. This choice was also influenced by the fact that the equation for G had a slightly better fit to the simulated data.

The most striking implication of the model is the importance of the autocorrelation factor. Of the seven terms (excluding the constant) in equation (5-9), five include the variable x_3. And these five are the most significant factors in the regression, together accounting for 81.9 of the 83.1 per cent of the variation in the response that is explained by the regression.

A look at the forecasted values of G in Table 27 shows the autocorrelation factor to have the most powerful influence in determining which strategy is superior. Note the overwhelming majority of positive forecasts (RB superiority) when the autocorrelation coefficient is negative, and the complete dominance of negative responses (BH superiority) predicted for positive levels of that factor. The general conclusion that high positive autocorrelation is favorable to BH and high negative autocorrelation is favorable to RB appears unaffected by either the number of securities, the intercorrelation, or the variance. There are no statistically significant forecasts which refute this observation.

Examining the partial derivatives with respect to this factor, we observe that the relative performance of RB is a decreasing function of the autocorrelation coefficient at all positive levels of autocorrelation. However, the existence of significant positive values of this derivative at certain high negative levels of autocorrelation indicates that G can achieve a maximum with respect to this factor. For example, when the factor configuration is [2, -0.667, 0.200, 0.005], the partial derivative of 0.106 (t=3.215) indicates that any further decrease in the autocorrelation of the return series, ceteris paribus, would reduce the superiority of RB. The exact value of the autocorrelation coefficient at which the response is maximized given levels of the other factors, may be estimated by setting equation (6-6) equal to zero and solving for x_3. For those factor configurations in which the number of securities is sufficiently high and the intercorrelation sufficiently low, the relative performance of RB will always be improved by making the autocorrelation more negative (within the range of the observations).

With regard to the effect of the portfolio size, we also study Table 27. The partial derivatives with respect to the number of securities indicate that RB performance is an increasing function of the portfolio size for autocorrelation sufficiently negative, especially for low levels of the variance factor. For zero and positive autocorrelation, however, and especially for high levels of variance, the addition of securities to the portfolio is unfavorable to RB.

The similarity between the equations predicting E and G makes the previous discussion of the effect of portfolio size from the validation section pertinent, for the explanation for those results also applies here. It is hypothesized that there is no interaction between the portfolio size alone and the degree of autocorrelation. Rather, ceteris paribus, increasing the portfolio size is probably always favorable to RB. However, the aforementioned sampling variation causes the mean return of the entering security to differ from the existing portfolio mean. And this sampling variation is magnified by high positive autocorrelation and high variance factors. It appears that for portfolios of two, three, and four securities, at higher levels of autocorrelation the unfavorable side effect of divergent means is sufficiently strong to overcome the favorable effect of expanding the portfolio membership.

The equations for the partial derivatives with respect to the intercorrelation and the variance factors ((6-7) and

(6-8)) are the simplest in form. It is apparent, either from the values in the table or from the equations themselves, that for high negative autocorrelation, G is a decreasing function of the intercorrelation between securities, and an increasing function of the variance of the returns to each security. And for high positive autocorrelation, the relative performance of BH is improved by lower intercorrelation and higher variance factors.

Solving the inequality $\delta G/\delta x_4 < 0$ we estimate that a decrease in the intercorrelation coefficient improves the relative performance of RB so long as the ratio x_3/x_1 is less than -0.037. For the two, three, and four security cases, G is then a decreasing function of intercorrelation so long as the autocorrelation coefficient is less than -0.074, -0.111, and -0.148, respectively. Similarly, the relative performance of RB is an increasing function of the variance factor so long as $x_3/x_1 < -0.049$. For the two, three, and four security cases, this requires that the autocorrelation coefficient be less than -0.098, -0.147, and -0.196, respectively.

It is noteworthy that the point estimates of these ratios defining stationary ridges in the x_1, x_3 plane are quite similar. In fact, the size of the standard errors of the coefficients from which they were computed does not allow the rejection of the hypothesis that they are the same. Consequently, it appears that when low intercorrelation is favorable to RB, so is high variance. Or in the case when a higher intercorrelation would improve the relative performance of RB, so would a lower variance.

Furthermore, a comparison of the signs of the forecasted values with the signs of the partial derivatives with respect to the intercorrelation factor indicates that for either positive or negative autocorrelation (not zero), a decline in this factor would significantly magnify the superiority of whichever strategy is superior at that factor configuration. Similarly, an increase in variance would also tend to magnify any difference in the geometric means of the return distributions to the two strategies. Although these magnification effects are not displayed in the case of zero autocorrelation, the positive forecasts predicted there are not statistically significant, and so the generality of these effects is not refuted.

Finally, the fact that these two partial derivatives are of opposite sign at every design point provides strong evidence that the <u>portfolio</u> variance does not influence the value of the response. Rather, the contributions to the portfolio variance must be looked at. For example, let us assume that all security series exhibit strong negative autocorrelations. Then RB will probably be superior to BH. Now if two <u>portfolios</u> have the same inter-period variance of returns, with one the result of low security variances (factor 5) and high intercorrelations between securities (factor 4), and the other having higher security variances but lower intercorrelations, then the latter portfolio would display a greater RB superiority than the former.

Since these results are also applicable to the prediction of the expected value response, they appear to be consistent with the Cheng-Deets condition (2-8) for RB expected return superiority in the two-security, two-period case. Under the assumption that the two securities have equal variances σ^2, and equal autocorrelations p_0, we rewrite the condition as

$$(2\bar{p}_{1,2}-2p_0)\sigma^2 > (\mu_1-\mu_2)^2. \tag{6-9}$$

Assuming the absolute superiority of either strategy to be directly related to the strength of the inequality, the condition indicates that E is a decreasing function of the autocorrelation factor and of the divergence in means.

In the case of RB superiority, the sign of the left hand side (LHS) is positive, while in the case of BH superiority it is indeterminate. Assuming the RHS is relatively small, if BH is sufficiently superior the LHS will be negative. Then the strength of the inequality in either direction is an increasing function of the variance factor just as our model predicts.

With some reflection it is clear that for strong negative autocorrelation, the cross covariance is a decreasing function of the intercorrelation.[4] Since RB superiority is most likely in this case, a decrease in the intercorrelation will tend to magnify RB superiority by increasing the cross covariance term in the LHS. In the situation where there is strong positive autocorrelation, and BH superiority is most likely, the cross covariance is an increasing function of the intercorrelation. Consequently a reduction in the level of this factor will reduce the cross correlation term and magnify the BH superiority.

Since the implications of this inequality are consistent with the results of this study, an attempt to generalize this condition for RB superiority to the multi-period, multi-security situation appears to offer promise. This study, however, offers no further progress in that direction.

Although not statistically significant, the forecast of positive values of E when the autocorrelation equals zero (Table 26) does suggest as did Cheng and Deets, that RB expected return superiority does not require negative autocorrelation in stock price series. In an attempt to confirm this possibility, distributions of returns to the two strategies were simulated with a low positive autocorrelation of 0.083.[5] The models indicated that with positive autocorrelation, both E and G are decreasing functions of the portfolio size and variance factors, and increasing functions of the intercorrelation factor. In accordance with these observations, the simulated portfolios consisted of two securities with variances equal to 0.001, an intercorrelation of 0.90, and equal expected returns of 1.007. Fifteen ex post values of E and G were simulated (as described in chapter IV) with this factor configuration. The result was a mean E response of -0.0003 and a mean G of -0.0004. The respective t-values of -1.7838 and -1.8708 (fourteen degrees of freedom) both were significant at the 0.05 level, rejecting the hypothesis of RB superiority. Although this test is not conclusive, it does appear that expected return superiority of RB requires negative autocorrelation.

The inconsistency of this conclusion with the observation of positive values of E forecasted for autocorrelation equal to zero may be reconciled. The marginal mean of the simulated values of E for that autocorrelation was in fact -0.00287, indicating a slight upward bias in the expected value response model for that factor level. On the other hand, the positive value (0.00148) of the marginal mean of the simulated values of the G response does not allow one to draw the same conclusion for the geometric mean response. With the appropriate combination of the other factors, RB may in fact be geometric mean superior to BH for positive autocorrelations less than 0.083. Recalling the geometric mean approximations in equations (3-13) through (3-15), these results are consistent with Cheng's analytical conclusions with regard to the variance response (V).

This chapter is concluded by noting that the set of conditions under which the rebalancing strategy is most superior to a policy of buy-and-hold is the combination of a large portfolio size, small differences between security expected returns, strong negative autocorrelation, strong negative intercorrelation, and high variance.

Footnotes to Chapter VI

1. See Appendix C, part 1.

2. See Appendix C, part 2.

3. If we accept the Cheng-Deets [39] conclusion that stock price series are not a random walk, then the empirical results of Latané and Young [128] provide some evidence that the effect of divergent returns can be generalized. Equation (2-8) indicates that it does hold in the two security, two period case.

4. To see this let a "+" refer to a return greater than the expected return in a series, and let a "-" refer to a return below average. For the two security situation we present four pair of time series for illustrative purposes.

(a)

Security	Time
1	+ - + - + -
2	+ - + - + -

(b)

Security	Time
1	+ - + - + -
2	- + - + - +

(c)

Security	Time
1	+ + + - - -
2	+ + + - - -

(d)

Security	Time
1	+ + + - - -
2	- - - + + +

Observe that for the two cases (a) and (b) exhibiting negative autocorrelation, only case (b), which also is characterized by negative intercorrelation, exhibits positive cross correlation. And for the two cases (c) and (d) showing positive autocorrelation, the positive cross correlation in (c) requires positive intercorrelation.

5. With N=12 in the Central Limit Approach to generating random normal variates, 0.083 was the lowest level of positive autocorrelation that could be simulated. It is obtained by setting p=11. See equation (4-8) and footnote 4 to chapter IV.

CHAPTER VII

SUMMARY AND CONCLUDING COMMENTS

We began by simulating sets of security returns characterized by predetermined levels of five factors thought to influence the relative performances of the rebalancing and buy-and-hold strategies. All portfolios had expected returns of 1.007 in each of the fifty periods over which the geometric totals to the alternative strategies were computed. The distribution of twenty such geometric totals was obtained for each policy for each of the 540 factor combinations. The procedure was replicated twice.

Responses were defined as the differences between certain parameters of the respective distributions for the two strategies. Multivariate analysis of variance and nonlinear regression analysis were utilized to obtain models to predict the responses from the levels of the five factors under study. While satisfactory functions of the factors could not be found to predict the responses associated with the second, third, and fourth moments of the distributions, quadratic functions of the factors were obtained which explained approximately 80% of the variation in the expected value and geometric mean responses.

Taking the geometric mean of the distribution of geometric totals as a single-parameter performance measure, it was found that the autocorrelation factor is the most important determinant of which strategy is superior. As the returns to the individual securities in the portfolio diverge from one another, whether due to differences in the first moment of the generating distribution or to sampling variation, a greater degree of negative autocorrelation is necessary for RB superiority.

It was also found that the effect of a decrease in the intercorrelation between securities, and/or an increase in the variance of the distribution of returns to each security is to magnify the superiority of either strategy.

Finally, except where the accompanying effect of increasing divergence in the ex post security returns is large, the relative performance of RB is an increasing function of the portfolio size. As the portfolio size increases, and especially with negative autocorrelation, the unfavorable side effect becomes negligible.

These observations pointed to the conclusion that the most favorable set of conditions for RB is a large portfolio of uncorrelated, highly volatile securities, each having the same expected return and exhibiting strong negative autocorrelation.

Since there were no significant differences between the predicted values of the geometric mean response and the corresponding forecasts of the expected value response, these conclusions are also applicable to performance defined strictly in terms of the expected value.

These returns are not peculiar to the selection of a mean portfolio return of 1.007 for the simulation. It can be demonstrated that in the fifty period case, to predict either the expected value or geometric mean response for an expected portfolio return of μ, the forecasts of our model should be multiplied by $(\mu/1.007)^{50}$. The basic relationships between the variables, however, are undisturbed. Nor would these results be invalidated by the assumption of a horizon length of other than fifty periods. Again, the magnitudes of the responses would be altered (though not as simply as in the case of a different mean), but the general conclusions would still hold.

This paper has investigated the statistical properties of the rebalancing policy of portfolio adjustment without the restrictions of a random walk. It addressed itself to the question of when a policy of rebalancing at a given frequency will result in performance superior to a BH policy for a particular portfolio of securities held in equal amounts. It did not consider taxes or transaction costs.

In actuality, the investor who plans to hold securities in equal amounts must make the choice between RB and BH in conjunction with two other decisions--the rebalancing frequency and the portfolio composition--based upon the distributions of returns after adjustment for the implementation costs peculiar to his investment situation.

Once the random walk assumption is removed, the relevant factor configuration for the choice between RB and BH for a particular portfolio, depends upon the specific rebalancing frequency considered. While the random walk case implies that performance before costs is a monotone increasing function of this frequency, in the general case this is not so. Reducing the rebalancing interval may now, for example, change the relevant autocorrelation coefficient in a series of returns from negative to positive. Our statistical results indicate that this increase in frequency would then have an adverse effect on RB performance even before the adjustment for costs.

The implementation costs also effect the portfolio composition decision given a particular rebalancing frequency. Suppose, for example, that two portfolios are the same with respect to all factors except the intercorrelations between securities. Our statistical results indicate that at certain levels of the other factors, the one characterized by low intercorrelations will have the greater RB superiority over BH. However, it is apparent that rebalancing this portfolio will necessitate transactions of greater magnitude than will rebalancing the other. Consequently, a characteristic making one portfolio more suitable for rebalancing in the statistical sense, may at the same time reduce its suitability for rebalancing when implementation costs are considered.

Once one allows the securities to be held in unequal amounts, the determination of the cost-adjusted optimal feasible distribution of geometric totals becomes even more complex.

Given an investor's cost functions, the search for this optimum with respect to the portfolio composition and the rebalancing frequency (with BH as a limiting case) does not presently lend itself to either an analytical or simulated solution. These methods await further knowledge as to the nature of the mechanism generating stock price changes. Indeed, this research especially points to the need for further investigations of the autocorrelation properties of stock price series. We see the need for determining the types of functions underlying any dependencies, as well as examining their stability. In the meantime we must rely on ex post emperical investigations to approximate this optimum for different types of investors.

APPENDIX A

THE DISTRIBUTION OF STOCK PRICE CHANGES

The literature contains several proposed forms for the distribution of log price relatives. The earliest assumption, known as the Gaussian hypothesis, states that the distribution of stock price changes over fixed time intervals is approximately normal. This hypothesis was put forth by Bachelier [10] and Osborne [166] who appealed to the Central Limit Theorem for theoretical justification. Empirical evidence in support of the model was presented by Kendall [118] and Moore [151].

Observed departures from normality in the form of leptokurtosis, led to the stable Paretian hypothesis, first introduced by Mandelbrot [142]. While the Gaussian distribution is the limiting case of the symmetric stable Paretian family, Mandelbrot contends that stock price changes come from the non-Gaussian, long-tailed members of this family characterized by infinite variance. Extensive theoretical and empirical arguments favoring this hypothesis over the Gaussian model may be found in papers by Mandelbrot [139, 142], Fama [63, 64], Mandelbrot and Taylor [143], and Fisher and Lorie [74].

The evidence indicating that the observed distributions have more weight in their extreme tails than do normal distributions, is overwhelming. The key point in the controversy, however, is whether the tails are so long as to indicate an infinite variance and the consequent impairment of all statistical techniques utilizing this parameter.

Alternative long-tailed distributions having finite second moment have been proposed by Press [172], Praetz [170], and Agnew [1]. All three suggest compound distributions obtained by mixing normal distributions. The "compound events model" hypothesized by Press indicates that log price relatives are determined as the sum of a sequence of identically distributed, Gaussian price-changing events occurring randomly in time (hence the number of such variables has the Poisson distribution), plus an independent Gaussian disturbance variable with zero mean.

Praetz also notes that the information which affects prices comes in "bursts of activity," and cites evidence that the variance of the distribution changes over time. Consequently, he modifies the Gaussian model by specifying that the variance of the normal distribution itself is distributed as an inverted gamma variate. The resulting long-tailed compound distribution is a scaled t-distribution. He provides empirical evidence that, at least for stock price indices, this model gives a better fit than either the Gaussian, stable Paretian, or "compound events" models.

Agnew's alternative to the stable Paretian model was also obtained by mixing normal distributions according to a prior placed on the variance. In this case the variance is distributed exponentially, and the resulting distribution is bilateral exponential. This model is generalized to the multivariate case, with arbitrary intercorrelations between securities. Like the Gaussian and stable Paretian hypotheses, each of these three alternatives exhibits the important property of stability under addition.

Finally, we take note of empirical evidence recently presented by Hsu, Miller, and Wichern [103] which suggests that stable Paretian distributions have tails which are too fat, and a top which is not peaked enough to adequately describe daily stock price changes. They find that a Gaussian process with an unstable variance may be a more suitable explanation for the analysis of stock price changes, both daily and monthly.

APPENDIX B

COMPUTER PROGRAM FOR THE SIMULATION

The simulation was carried out on the UMASS time-sharing system which allows remote access to a CDC 3800 computer located at the Research Computing Center at the University of Massachusetts. The computer program was written in the UMASS FORTRAN language which is compatible in most respects with FORTRAN IV. A listing of the program is presented at the end of this appendix.

The program accomplishes the following functions:

1. Specification of the data generating model and factor configuration.--An example of the input data is:

```
line
8000  0 0 0 0 0 0 0 0 0  [Optional output (no=0, yes=1)]
8010  0  [Distribution form (0=normal, 1=bilateral exponential)]
8020  2 8 -1 2 20 50  [See below]
8030  .004 .002 )
8040  .002 .004 }  [Variance-Covariance matrix]
8050  .0065 .0075  [Mean return for each security, a diver-
                    gence of .001]
```

The data on line 8020, from left to right, specifies:

2 -- the number of securities in the portfolio

8 -- p in equation (4-7) to determine the autocorrelation coefficient (±.67)

-1 -- the sign of the autocorrelation coefficient (-.67)

2 -- the number of times the simulation is replicated

20 -- the number of observations of the return to each strategy which determine the distribution (in each replication)

50 -- the horizon length (T) or number of periods of simulated returns which determines the return to each strategy (in each observation).

2. Generation of time series of (50) log price relatives (and price relatives) for each security (2) in the portfolio according to the model specified in 1.--Optional output.

3. Computation of geometric total returns to RB and BH.--This is one observation of the distribution of returns to each strategy. Optional output.

4. Computation of ex post statistical characteristics of the simulated series of log price relatives.--This includes the first four sample semi-invariants, g_1, g_2 and Q statistics, and the autocorrelation coefficient for each series of log price relatives in the portfolio. Also calculated are the intercovariance and intercorrelation matrices and the "cross" covariance and correlation matrices between the sets of returns. Optional output.

5. Repetition of steps 2, 3, and 4 for the number of observations indicated (20) in 1.

6. Computation of geometric mean, sample semi-invariants, and g_1, g_2 and Q statistics for the distribution of geometric total returns to each strategy.--Also computed are the five relative performance measures based upon these parameters. Output.

7. Computation of the statistical characteristics (as in 4) of the entire series (20 observations × 50 periods = 1,000) of log price relatives for each security.--Output.

8. Repetition of steps 2 through 7 for the number of replications (2) specified in 1.

9. Repetition of steps 1 through 8, inputting a new factor configuration (lines 8020-8050) until program out of data.

An example of the output (for steps 6 and 7, not including the optional output) for the first replication, given the input data in 1, is presented in Table 28. After the listing of the input data, the first block of output data is that described in step 6. The lines GTRB and GTBH refer to the distributions of geometric totals to the respective strategies, and the line GTDF is the difference between the two, hence the performance measures.

The other data blocks are the ex post characteristics of the simulated series as indicated in step 7. Since the factor levels used in the analysis are the population characteristics specified in the data generating model (step 1), these ex post measures are utilized only for purposes of validation.

TABLE 28

SAMPLE OUTPUT OF SIMULATION

```
0
 2   8  -1   2  20  50
.00400  .00200
.00200  .00400
.00650  .00750
```

replicate 1

parameters of distributions of geom totals

	gmean	amean	variance	skew	kurt	g1	g2	q
gtrb	1.437	1.460	.062	-.008	.010	-.507	2.739	4.014
gtbh	1.417	1.439	.061	-.006	.009	-.406	2.521	3.340
gtdf	.020	.020	.001	-.002	.001			

parameters of distrib of log pr by security

gmean	amean	variance	skew	kurt	g1	g2	q
	.0074	.0041	-.0000	.0000	-.1026	.007	1.757
	.0062	.0040	.0000	-.0000	.0437	-.162	1.469

covariance matrix of log pr between securities

.00407	.00215
.00215	.00395

correlation matrix of log pr between securit

1.00000	.53638
.53638	1.00000

cross covariance of log pr between securities

-.00134	-.00082
-.00066	-.00135

cross correlations of log pr between securit

-.32967	-.20417
-.16349	-.34068

autocorrelation of log pr by security, lag 1

security	autocovariance	variance	autocorrelation
1	-.00134	.00407	-.32967
2	-.00135	.00395	-.34068

```
LIST
PROGRAM RBBR
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DIMENSION GMEAN(3),SUM1(3),SUM2(3),SUM3(3),SUM4(3)
DIMENSION XCOV(5,5)
A=TIMEF(1.)
CALL RANFSET(A)
READ,NA,NB,NC,ND,NE,NF,NG,NH,NI
READ,NORM
NAA=NA  $  NBB=NB  $  NCC=NC  $  NDD=ND
NEE=NE  $  NFF=NF  $  NGG=NG
DO 90 J6=1,100
READ,M,KA,NSIGN,KR,N,KT
READ,((COV(I,J),J=1,M),I=1,M)
READ,(XMEAN(I),I=1,M)
DO 10 I=1,12
DO 10 J=1,M
10 RX(I,J)=RANF(-1)
IF(NORM .EQ. 0)GO TO 13
DO 12 J=1,M
RAN1(J)=RANF(-1)
12 RAN2(J)=RANF(-1)
13 CONTINUE
CALL INTCOR
WRITE(61,834) NORM
WRITE(61,831)M,KA,NSIGN,KR,N,KT
DO 14 I=1,M
14 WRITE(61,832) (COV(I,J),J=1,M)
WRITE(61,833) (XMEAN(I),I=1,M)
DO 60 K1=1,KR
ICODE=0
DO 16 I=1,M
DO 16 J=1,M
16 XCOV(I,J)=COV(I,J)
WRITE(61,801)
WRITE(61,830)
WRITE(61,830)
WRITE(61,808)K1
WRITE(61,830)
WRITE(61,830)
DO 5 L=1,3
GMEAN(L)=1.0  $  SUM1(L)=0.0  $  SUM2(L)=0.0
SUM4(L)=0.0
5 SUM3(L)=0.0
NA=NAA  $  NB=NBB  $  NC=NCC  $  ND=NDD
NE=NEE  $  NF=NFF  $  NG=NGG
DO 50 JD=1,N
IF(NH .EQ. 0)GO TO 15
```

```
      WRITE(61,830)
      WRITE(61,830)
      WRITE(61,800)K1,JD
15    CONTINUE
      DO 20 J=1,KT
      CALL MASTER(ACOV)
      DO 20 I=1,M
      RSUML(I,J)=RSUM(I)
20    PRX(I,J)=PR(I)
      CALL TOTAL(JD)
      IF(NA .EQ. 0)GO TO 45
      WRITE(61,801)
      WRITE(61,801)
      WRITE(61,802)
      WRITE(61,801)
      DO 30 J=1,KT
30    WRITE(61,803)(RSUML(I,J),I=1,M)
      WRITE(61,801)
      WRITE(61,804)
      WRITE(61,801)
      DO 40 J=1,KT
40    WRITE(61,803)(PRX(I,J),I=1,M)
      WRITE(61,801)
      WRITE(61,801)
45    CONTINUE
      CALL STAT(ICODE,N)
      CALL GRB(GTRB)
      CALL GBH(GTBH)
      GTDIF=GTRB/GTBH
      IF(NH .EQ. 0)GO TO 25
      WRITE(61,830)
      WRITE(61,805)GTRB
      WRITE(61,806)GTBH
      WRITE(61,807)GTDIF
      WRITE(61,801)
25    IF(NH .NE. 0 .OR. NI .EQ. 0)GO TO 26
      IF(JD .NE. 1)GO TO 27
      WRITE(61,810)
      WRITE(61,801)
27    WRITE(61,811)JD,GTRB,GTBH,GTDIF
26    CONTINUE
      DO 50 L=1,2
      IF(L .EQ. 1)H=GTRB
      IF(L .EQ. 2)H=GTBH
      GMEAN(L)=GMEAN(L)*H
      SUM1(L)=SUM1(L)+H
      SUM2(L)=SUM2(L)+H**2
      SUM3(L)=SUM3(L)+H**3
50    SUM4(L)=SUM4(L)+H**4
      L1=3
      WRITE(61,830)
      WRITE(61,830)
      WRITE(61,808)K1
      WRITE(61,801)
      WRITE(61,801)
      WRITE(61,809)
      I=1
      CALL PARAM(GMEAN,SUM1,SUM2,SUM3,SUM4,L1,N,I,ICODE)
      ICODE=1
      WRITE(61,801)
```

```
WRITE(61,801)
CALL STAT(ICODE,N)
60 CONTINUE
WRITE(61,830)
WRITE(61,830)
90 CONTINUE
800 FORMAT(1H ,1X,*REPLICATION-*,I3,8X,*OBSERVATION-*,I4)
801 FORMAT(1H )
802 FORMAT(1H ,1X,*LOG PRICE RELATIVES, ROW=PERIOD, COL=SECURITY*)
803 FORMAT(1H ,1X,5F12.5)
804 FORMAT(1H ,1X,*PRICE RELATIVES, ROW=PERIOD, COL=SECURITY*)
805 FORMAT(1H ,1X,*GEOMETRIC TOTAL TO RB = *,F12.5)
806 FORMAT(1H ,1X,*GEOMETRIC TOTAL TO BH = *,F12.5)
807 FORMAT(1H ,1X,*       DIFFERENCE        = *,F12.5)
808 FORMAT(1H ,10X,*REPLICATE*,I4)
809 FORMAT(1H ,1X,*PARAMETERS OF DISTRIBUTIONS OF GEOM TOTALS*)
810 FORMAT(1H ,1X,*OBS*,10X,*GTRB*,10X,*GTBH*,10X,*GTDIF*)
834 FORMAT(1H ,1X,I3)
811 FORMAT(1H ,1X,I4,3F16.5)
830 FORMAT(1H0)
831 FORMAT(1H ,1X,6I4)
832 FORMAT(1H ,1X,5F8.5)
833 FORMAT(1H ,1X,5F8.5)
END
SUBROUTINE MASTER(XCOV)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DIMENSION XCOV(5,5)
DO 20 J=1,M
CALL AUTOCR(J)
20 CONTINUE
IF(NORM .EQ. 0)GO TO 35
CALL EXPENT(EXPEN)
DO 25 I=1,M
DO 25 J=1,M
25 COV(I,J)=EXPEN*XCOV(I,J)
CALL INTCOR
35 CONTINUE
CALL COMBINE
DO 30 I=1,M
30 PR(I)=EXP(RSUM(I))
RETURN
END
SUBROUTINE INTCOR
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DO 10 I=1,M
DO 10 J=1,M
10 C(I,J)=0.0
DO 20 I=1,M
20 C(I,1)=COV(I,1)/(COV(1,1)**.5)
C(2,2)=(COV(2,2)-(C(2,1)**2))**.5
DO 60 I=3,M
IJ=I-1
```

```
DO 50 J=2,IJ
KJ=J-1
SUM1=0.0
DO 30 K=1,KJ
30 SUM1=SUM1+(C(I,K)*C(J,K))
50 C(I,J)=(COV(I,J)-SUM1)/C(J,J)
SUM1=0.0
DO 40 K=1,IJ
40 SUM1=SUM1+(C(I,K)**2)
60 C(I,I)=(COV(I,I)-SUM1)**.5
RETURN
END
SUBROUTINE AUTOCR(J)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
L=12-KA
X(J)=0.0
DO 10 I=1,L
RX(I,J)=RX(I+KA,J)
IF(NSIGN .LT. 0)RX(I,J)=1.0-RX(I,J)
10 X(J)=X(J)+RX(I,J)
L=L+1
DO 20 I=L,12
RX(I,J)=RANF(-1)
20 X(J)=X(J)+RX(I,J)
X(J)=X(J)-6.0
RETURN
END
SUBROUTINE COMBINE
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DO 30 I=1,M
RSUM(I)=0.0
DO 10 J=1,M
10 RSUM(I)=RSUM(I)+(C(I,J)*X(J))
20 RSUM(I)=RSUM(I)+XMEAN(I)
30 CONTINUE
RETURN
END
SUBROUTINE STAT(ICODE,N)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DIMENSION GMEAN(1),SUM1(1),SUM2(1),SUM3(1),SUM4(1)
L1=1
KSAVE=KT
DO 20 I=1,M
IF(ICODE .EQ. 1)KT=KT*N
GMEAN(1)=1.0  $  SUM1(1)=0.0  $  SUM2(1)=0.0
SUM3(1)=0.0  $  SUM4(1)=0.0
DO 10 J=1,KT
IF(ICODE .EQ. 1)RSUML(I,J)=TSUML(I,J)
```

```
GMEAN(1)=GMEAN(1)*RSUML(I,J)
SUM1(1)=SUM1(1)+RSUML(I,J)
SUM2(1)=SUM2(1)+(RSUML(I,J)**2)
SUM3(1)=SUM3(1)+(RSUML(I,J)**3)
10 SUM4(1)=SUM4(1)+(RSUML(I,J)**4)
KT=KSAVE
20 CALL PARAM(GMEAN,SUM1,SUM2,SUM3,SUM4,L1,N,I,ICODE)
CALL CORINT(ICODE,N)
CALL CHSCH(ICODE,N)
CALL CORAUT(ICODE,N)
RETURN
END
SUBROUTINE PARAM(GMEAN,SUM1,SUM2,SUM3,SUM4,L1,N,I,ICODE)
COMMON M,COV(5,5),KA,NSIGN,AMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DIMENSION GMEAN(3),SUM1(3),SUM2(3),SUM3(3),SUM4(3)
REAL KUR1,KUR2,KUR3,KUR
T=KT
IF(L1 .EQ. 3)T=N
IF(ICODE .EQ. 1)T=KT*N
IF(ICODE .EQ. 1)NB=1
IF(NB .EQ. 0 .AND. L1 .EQ. 1)GO TO 5
IF(L1 .EQ. 1 .AND. I .EQ. 1)WRITE(61,814)
IF(I .EQ. 1)WRITE(61,801)
IF(I .EQ. 1)WRITE(61,810)
IF(I .EQ. 1)WRITE(61,801)
5 CONTINUE
DO 10 L=1,L1
IF(L .EQ. 3)GO TO 35
IF(L1 .EQ. 3)GMEAN1=GMEAN(L)**(1/T)
AVE=SUM1(L)/T
VAR=(T*SUM2(L)-SUM1(L)**2)/(T*(T-1.0))
SKEW1=(T**2*SUM3(L)-3.*T*SUM2(L)*SUM1(L)+2.*SUM1(L)**3)
SKEW2=T*(T-1.0)*(T-2.0)
SKEW=SKEW1/SKEW2
KUR1=(T**3*SUM4(L))-4.*T**2*SUM3(L)*SUM1(L)
KUR2=6.*T*SUM1(L)**2*SUM2(L)-3.*SUM1(L)**4
KUR=(KUR1+KUR2)/(SKEW2*(T-3.))-3.*VAR**2
G1=SKEW/(VAR**1.5)
G2=KUR/(VAR**2)
H1=LOG(VAR)
H2=SKEW
H3=LOG((KUR+3.*VAR**2)/3.)
Q1=T*(1.5*H1**2-1.5*H3*H1+H2**2/(6.*VAR**3))
Q2=T*(.375*H3**2)
Q=Q1+Q2
IF(L1 .EQ. 1)GO TO 50
IF(L1 .EQ. 3 .AND. L .EQ. 1)GO TO 20
IF(L .EQ. 2)GO TO 30
20 WRITE(61,811)GMEAN1,AVE,VAR,SKEW,KUR,G1,G2,Q
GMRB=GMEAN1   $   AVRB=AVE
SKEWRB=SKEW   $   RBKUR=KUR
VARRB=VAR
GO TO 60
30 WRITE(61,812)GMEAN1,AVE,VAR,SKEW,KUR,G1,G2,Q
GMBR=GMEAN1   $   AVBR=AVE
SKEWBR=SKEW   $   BRKUR=KUR
```

```
VARBH=VAR
GO TO 60
35 GMEAN1=GMRB-GMBH
AVE=AVRB-AVBH
SKEW=SKEWRB-SKEWBH
KUR=RBKUR-BHKUR
VAR=VARRB-VARBH
WRITE(61,813) GMEAN1,AVE,VAR,SKEW,KUR
GO TO 60
50 CONTINUE
IF(NB .EQ. 0)GO TO 55
WRITE(61,815)AVE,VAR,SKEW,KUR,G1,G2,Q
55 CONTINUE
60 CONTINUE
10 CONTINUE
810 FORMAT(1H ,7X,*GMEAN*,1X,*AMEAN*,2X,*VARIANCE*,2X,
C *SKEW*,2X,*KURT*,6X,*G1*,6X,*G2*,9X,*Q*)
811 FORMAT(1H ,1X,*GTRB*,6F8.3,2F9.3)
812 FORMAT(1H ,1X,*GTEH*,6F8.3,2F9.3)
813 FORMAT(1H ,1X,*GTDF*,6F8.3,2F9.3)
814 FORMAT(1H ,1X,*PARAMETERS OF DISTRIB OF LOG PR BY SECURITY*)
815 FORMAT(1H ,9X,5F8.4,2F9.3)
801 FORMAT(1H )
RETURN
END
SUBROUTINE CORINT(ICODE,N)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5)
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DIMENSION CV(5,5),COR(5,5)
KSAVE=KT
IF(ICODE .EQ. 1)KT=KT*N
T=KT
IF(ICODE .EQ. 1)NC=1
IF(ICODE .EQ. 1)ND=1
DO 20 I=1,M
DO 20 K=1,M
SUM1=0.0  $  SUM2=0.0  $  SUM3=0.0
DO 10 J=1,KT
IF(ICODE .EQ. 1)RSUML(K,J)=TSUML(K,J)
IF(ICODE .EQ. 1)RSUML(I,J)=TSUML(I,J)
SUM1=SUM1+(RSUML(I,J)*RSUML(K,J))
SUM2=SUM2+RSUML(I,J)
10 SUM3=SUM3+RSUML(K,J)
20 CV(I,K)=(1./(T-1.))*(SUM1-(SUM2*SUM3)/T)
IF(NC .EQ. 0)GO TO 35
WRITE(61,830)
WRITE(61,816)
WRITE(61,801)
DO 30 I=1,M
30 WRITE(61,817)(CV(I,K),K=1,M)
35 CONTINUE
IF(ND .EQ. 0)GO TO 45
WRITE(61,801)
WRITE(61,818)
DO 40 I=1,M
DO 40 K=1,M
40 COR(I,K)=CV(I,K)/(SQRT(CV(I,I)*CV(K,K)))
```

```
WRITE(61,801)
DO 50 I=1,M
50 WRITE(61,817)(COR(I,K),K=1,M)
45 CONTINUE
KT=KSAVE
801 FORMAT(1H )
816 FORMAT(1H ,1X,*COVARIANCE MATRIX OF LOG PR BETWEEN SECURITIES*)
817 FORMAT(1H ,1X,5F14.5)
818 FORMAT(1H ,1X,*CORRELATION MATRIX OF LOG PR BETWEEN SECURIT*)
830 FORMAT(1H0)
RETURN
END
SUBROUTINE CRSCR(ICODE,N)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DIMENSION CV(5,5),COR(5,5),V(5)
IF(ICODE .EQ. 1)NE=1
IF(ICODE .EQ. 1)NF=1
KSAVE=KT
IF(ICODE .EQ. 1)KT=KT*N
T=KT  $  KT1=KT-1
DO 20 I=1,M
DO 20 K=1,M
SUM1=0.0  $  SUM2=0.0  $  SUM3=0.0
SUM4=0.0
DO 10 J=1,KT1
IF(ICODE .EQ. 1)RSUML(I,J)=TSUML(I,J)
IF(ICODE .EQ. 1)RSUML(K,J+1)=TSUML(K,J+1)
SUM1=SUM1+(RSUML(I,J)*RSUML(K,J+1))
SUM2=SUM2+RSUML(I,J)
SUM3=SUM3+RSUML(K,J+1)
10 SUM4=SUM4+(RSUML(I,J)**2)
SUM1=SUM1+(RSUML(I,KT)*RSUML(K,1))
SUM2=SUM2+RSUML(I,KT)
SUM3=SUM3+RSUML(K,1)
SUM4=SUM4+(RSUML(I,KT)**2)
CV(I,K)=(1./(T-1.))*(SUM1-(SUM2*SUM3)/T)
20 V(I)=(1./(T-1.))*(SUM4-(SUM2**2)/T)
IF(NE .EQ. 0)GO TO 45
WRITE(61,801)
WRITE(61,801)
WRITE(61,820)
WRITE(61,801)
DO 40 I=1,M
40 WRITE(61,821)(CV(I,K),K=1,M)
WRITE(61,801)
45 CONTINUE
IF(NF .EQ. 0)GO TO 55
DO 50 I=1,M
DO 50 K=1,M
50 COR(I,K)=CV(I,K)/(SQRT(V(I)*V(K)))
WRITE(61,822)
WRITE(61,801)
DO 60 I=1,M
60 WRITE(61,821)(COR(I,K),K=1,M)
55 CONTINUE
KT=KSAVE
```

```
801 FORMAT(1H )
820 FORMAT(1H ,1X,*CROSS COVARIANCE OF LOG PR BETWEEN SECURITIES*)
821 FORMAT(1H ,1X,5F14.5)
822 FORMAT(1H ,1X,*CROSS CORRELATIONS OF LOG PR BETWEEN SECURIT*)
RETURN
END
SUBROUTINE CORAUT(ICODE,N)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
KSAVE=KT
IF(ICODE .EQ. 1)NG=1
IF(ICODE .EQ. 1)NH=1
IF(ICODE .EQ. 1)KT=KT*N
T=KT $ KT1=KT-1
IF(NG .EQ. 0)GO TO 25
WRITE(61,801)
WRITE(61,801)
WRITE(61,823)
WRITE(61,801)
WRITE(61,824)
WRITE(61,801)
DO 20 I=1,M
SUM1=0.0 $ SUM2=0.0 $ SUM3=0.0
DO 10 J=1,KT1
IF(ICODE .EQ. 1)RSUML(I,J)=TSUML(I,J)
IF(ICODE .EQ. 1)RSUML(I,J+1)=TSUML(I,J+1)
SUM1=SUM1+(RSUML(I,J)*RSUML(I,J+1))
SUM2=SUM2+RSUML(I,J)
10 SUM3=SUM3+(RSUML(I,J)**2)
SUM1=SUM1+(RSUML(I,KT)*RSUML(I,1))
SUM2=SUM2+RSUML(I,KT)
SUM3=SUM3+(RSUML(I,KT)**2)
ACOV=(1./(T-1.))*(SUM1-(SUM2**2)/T)
AVAR=(1./(T-1.))*(SUM3-(SUM2**2)/T)
ACOR=ACOV/AVAR
20 WRITE(61,825)I,ACOV,AVAR,ACOR
25 CONTINUE
KT=KSAVE
801 FORMAT(1H )
823 FORMAT(1H ,1X,*AUTOCORRELATION OF LOG PR BY SECURITY, LAG 1*)
824 FORMAT(1H ,1X,*SECURITY*,4X,*AUTOCOVARIANCE*,8X,*VARIANCE*,
C 8X,*AUTOCORRELATION*)
825 FORMAT(1H ,1X,I5,6X,3F15.5)
RETURN
END
SUBROUTINE GRB(GTRB)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5)
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
SM=M
PROD=1.0
DO 20 J=1,KT
SUM=0.0
DO 10 I=1,M
10 SUM=SUM+PRX(I,J)
```

```
SUM=SUM/SM
20 PROD=PROD*SUM
GTBB=PROD
RETURN
END
SUBROUTINE GBH(GTBH)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
SM=M
SUM=0.0
DO 20 I=1,M
PROD=1.0
DO 10 J=1,KT
10 PROD=PROD*PRX(I,J)
20 SUM=SUM+PROD
GTBH=SUM/SM
RETURN
END
SUBROUTINE TOTAL(JD)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5)
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
DO 10 K=1,KT
J=K+(KT*(JD-1))
DO 10 I=1,M
10 TSUML(I,J)=RSUML(I,K)
RETURN
END
SUBROUTINE EXPENT(EXPEN)
COMMON M,COV(5,5),KA,NSIGN,XMEAN(5),C(5,5),RX(12,5),X(5),
C RSUM(5),PR(5),PRX(5,100),KT
COMMON RSUML(5,1000),TSUML(5,1000)
COMMON NA,NB,NC,ND,NE,NF,NG
COMMON NORM,RAN1(5),RAN2(5)
R=RANF(-1)
EXPEN=-LOG(R)
RETURN
END
ENDPROG
```

APPENDIX C

SAMPLING VARIATION AND DIVERGENCE IN MEANS

1. Let

$$R_i = \mu + e_i \qquad \text{(C-1)}$$

be the ex post arithmetic mean return of a single series of returns to the $i^{\underline{th}}$ security, where μ is the population mean and e_i is a disturbance variable causing the sampling variation such that $E(e_i)=0$.

Then the divergence in means associated with the addition of the $m^{\underline{th}}$ security to a portfolio of m-1 securities is defined as

$$D = \bar{R}_{m-1} - R_m, \qquad \text{(C-2)}$$

where $\bar{R}_{m-1}$ is the portfolio mean prior to the entry of the $m^{\underline{th}}$ security.

To show that the expected divergence in means due to sampling variation is a decreasing function of the portfolio size it must be shown that the expectation of the absolute value of D is a decreasing function of m. For mathematical convenience, we prove that the equivalent statement

$$E(D^2) = E[(\bar{R}_{m-1}-R_m)^2] \qquad \text{(C-3)}$$

is a decreasing function of m.

From the definitions of $\bar{R}$ and R_i we have

$$D = \bar{R}_{m-1} - R_m = \frac{1}{m-1}\sum_{i=1}^{m-1}(\mu+e_i)-(\mu+e_m)$$

$$= \frac{1}{m-1}\left(\sum_{i=1}^{m-1} e_i\right) - e_m. \tag{C-4}$$

Taking the expected value of both sides we have E(D)=0. Subtracting E[D] from the right hand side, squaring both sides and taking the expectation gives

$$E[D^2] = E\{[D-E(D)]^2\}. \tag{C-5}$$

But the term on the right is the variance of D. Therefore, to show that $E[D^2]$ is a decreasing function of m we need to prove that V[D] is a decreasing function of m. The proof follows under the assumption that the variance of the disturbance is the same for each security, i.e., $V(e_i)=V(e)$, i=1,m. Under this assumption

$$V[D] = V[\frac{1}{m-1}\left(\sum_{i=1}^{m-1} e_i\right) - e_m]$$

$$= \frac{1}{m-1}V[e] + V[e] + 2 \cdot [\text{sum of distinct covariances}]. \tag{C-6}$$

Since there are (m-2)(m-1)/2 distinct covariances of the form $[1/(m-1)^2]$ Cov (e_i, e_k) associated with the m-1 securities already in the portfolio, and m-1 distinct covariances of the form [1/(m-1)] Cov (e_i, e_m) between the entering security m and the m-1 securities in the existing portfolio, we substitute for the last term in equation (C-6) to obtain

$$V[D] = \frac{1}{m-1}V[e] + V[e] + \frac{m-2}{m-1}[\text{average Cov}(e_i, e_k)] + 2[\text{average Cov}(e_i, e_m)]. \tag{C-7}$$

Differentiating with respect to m, we have

$$\frac{\delta V[D]}{\delta m} = \frac{-V[e]}{(m-1)^2} + \frac{\text{average Cov}(e_i, e_k)}{(m-1)^2}. \tag{C-8}$$

The value of the derivative is zero when the average covariance between the $e^{'s}$ is equal to the variance of e. So long as the m-1 disturbances are not perfectly correlated with each other, the average covariance must be less than V[e]. And since V[e] and the denominators must be positive, we conclude that the derivative is less than zero.

Thus we have shown that (C-3) is a decreasing function of m, which implies that the divergence in means due to sampling variation is a decreasing function of the portfolio size.

2. To show that the divergence in means due to sampling variation is an increasing function of both the autocorrelation factor and the variance factor, we note that the disturbance variable as previously defined is the average of the disturbances a_j, j=1,n associated with each of the n periods in the time series for a security. That is,

$$e = \frac{1}{n}\sum_{j=1}^{n} a_j . \qquad \text{(C-9)}$$

Assuming (as in the simulation) that all securities in the portfolio have the same variance factor, we have $V[a_j]=V[a]$ for all j, and

$$V[e] = \frac{1}{n}V[a] + \frac{n-1}{n}[\text{average } Cov(a_j,a_{j+1})]. \qquad \text{(C-10)}$$

Since V[a] is the variance factor (factor 5), and the covariance term is recognizable as the autocovariance, V[e] is an increasing function of both the variance and autocorrelation factors. And, equation (C-7) indicates that V[D] is an increasing function of V[e]. Consequently the divergence in means due to sampling variation is an increasing function of the variance and the autocorrelation factors.

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