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An analysis of selected quantitative techniques for application to process cost accounting systems for planning and control purposes.

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AN ANALYSIS OF SELECTED QUANTITATIVE TECHNIQUES FOR APPLICATION TO PROCESS COST ACCOUNTING SYSTEMS FOR PLANNING AND CONTROL PURPOSES

A Dissertation Presented

By

WAYNE E. LEININGER, JR.

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FOR APPLICATION TO PROCESS COST ACCOUNTING SYSTEMS
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ABSTRACT

An Analysis of Selected Quantitative Techniques for Application to Process Cost Accounting Systems for Planning and Control Purposes

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Process cost accounting systems where historic costs are employed are designed for the expressed purpose of allocating costs among the outputs and inventories of a mass production system on an ex-post basis. The output of this type of system consists of a series of production reports where the physical unit transfers are used as a basis for determining average unit costs. When predetermined or standard costs are employed, a set of price, quantity and overhead differences or variances can also be determined.

The purpose of this study was to identify and evaluate analytical models using standard costs that could serve ex-ante planning models while also meeting the requirements of the traditional process cost accounting models. Models based on Input-Output Analysis and a stochastic transfer matrix were considered in the study. Linear programming

as it related to the input-output model was also considered.

The major conclusions are as follows:

(1) A Process costing system designed as an open static input-output model with modifications to accommodate in-process inventory considerations can be employed as a planning model.

(2) A process costing based on the open static input-output model can be employed for costing output and inventories when standard costs are employed.

(3) The cost differences or variances obtained from an input-output process costing system would better serve the needs of management in the areas of managerial and operational control.

(4) For multiple-product systems, an input-output process costing could be formulated as a linear programming problem.

(5) A cost control system based on the opportunity costs of the linear programming problem would result in a significant data reduction and would provide relevant cost control information to management.

(6) A process costing system can be based upon a stochastic transfer matrix in systems where the process production times are uniform or non-uniform.

(7) Process costing systems based on a stochastic

transfer matrix can be used as an ex-ante planning model or for costing inventories and outputs when standard costs are employed.

(8) The means and variances of the expected outputs can be determined from a stochastic process costing system. Other information of this type can be determined depending on the relationship of the production times of the processes.

(9) Process costing systems based on structured models produce information that would meet more of the needs of management than the information that can be obtained from traditional process costing procedures.

CHAPTER I

INTRODUCTION

Background

Accounting has developed over a period of 1,000 years of trial and error experimentation into a vast complex of procedural knowledge. Cost and financial accounting, the two principal subdivisions of the discipline, developed rather independently of each other and were not merged until about 1920.[1]

Cost accounting is a product of the late eighteen and early nineteen hundreds but its beginning can be traced from the sixteen hundreds when capitalistic forms of production organizations developed.[2] In the past, cost accounting has been concerned with accounting for raw material, direct labor, factory overhead, interdepartment transfers, and the costing of by-products. Traditional cost accounting systems have been oriented toward determining product costs and inventory valuation.

Financial accounting can be traced back to the appearance of trading and merchandising concerns. This branch of accounting is principally concerned with reporting the status of the capital invested and the results of the operations of the enterprise. In spite of the almost random nature of their development, there has always existed a thread of systematization and commonality in both of these subdivisions of accounting.

The Current Status of Accounting

With the economic emphasis in our society, the general function of accounting has been to provide useful information to those individuals charged with making economic decisions. The substantial economic development that the country has experienced has resulted in a significant increase in the complexity of commercial organizations. The accounting systems of these organizations have evolved to become the primary source of quantitative information. This information is employed in the planning and controlling of routine operations, in making nonroutine decisions, and in reporting results to government agencies and other interested parties.[3]

However, the position of accounting as the foremost source of information for economic decision making is being challenged as a result of developments that have taken place in the last twenty-five years in the areas of operations research, industrial engineering and economics. By employing various modeling and measuring techniques, researchers in these disciplines have been working in the areas of budgeting, inventory control, and process cost control. Previously these areas had been the sole domain of the accountant. The challenge to accounting by these disciplines has even extended to questioning the validity of employing accounting information in making economic decisions.

So far accounting has survived the challenge of the other disciplines because in most organizations the accounting system is the only system that provides economic information. Government agencies require accounting information for tax and other regulatory purposes and as long as this situation exists the survival of the discipline seems assured.

The Current Status of Cost or Managerial Accounting

The cost accounting system is the principal source of internal economic information. Traditional cost systems have been oriented toward product costing on an *ex-post* basis with an aim towards allocating costs between outputs and inventories. The distinguishing features between product costing systems is to a large degree a function of an organization's manufacturing technology.

The job-order cost system was the first developed because in the early stages of the Industrial Revolution, production was principally to customer specifications. Today, job order systems are common to firms whose products consist of specialized units or projects. Costs that can be objectively traced to a particular job are assigned to that job and other costs which cannot be related to a particular job are allocated among the jobs on some predetermined basis.

As the technology of mass production developed, accounting needs were met by the evolution of process cost accounting systems. This system is applicable to any operation where standardized items in large quantities are produced. The essential difference between the two systems is that a process system accumulates costs by department or cost center for a period of time and averages the costs among the units processed in that period of time while job-order systems accumulate costs by specific jobs.

Because of the need for cost information for planning and control purposes, standard cost accounting procedures were developed to augment both job order and process cost systems. Standard costs are established in advance of production and are based on engineering specifications and general economic conditions. These costs often serve as the basis for budgets and other decision models of an organization.

Research and development in the area of cost accounting has more or less paralleled industrialization. In <u>The Evaluation of Cost Accounting to 1925</u>, S. Paul Garner made several observations. He wrote:

> By 1900 both the job order and the process cost method had been pretty well described by

several authorities on both sides of the Atlantic.[4]

> Later developments (from 1911) centered around the sharpening of unit cost calculations under the process method, and the adoption of normal burden rates under the job order procedure.[5]

> ... so far as this writer can ascertain very few contributions (to cost accounting) of an original nature were made after 1925 ...[6]

In 1965 L. J. Benninger wrote concerning accounting theory that

> Little of the voluminous writings on accounting theory in recent decades has held specific applicability to the transforming action of cost accounting and its impact upon financial reports.[7]

One possible reason for the practical non-existence of research in the area is that for some period of time product cost accounting has been viewed as a methodology for dealing with accounting data on the level of procedure and therefore has little or no theoretical basis.[8] This point of view relegates cost accounting to a procedural methodology that is only involved with the maintenance of records.

However, in the last ten years there has been a shift in emphasis that has significantly influenced research. In 1970, John T. Wheeler observed

> In the past decade there has been a significant shift from viewing accounting as "what accountants do" to trying to study it analytically and to build theoretical models. There has been more development of accounting theory in the '60s than in the previous 500 years.[9]

This shift in emphasis has influenced cost accounting to such a degree that the title managerial accounting has replaced cost accounting. The scope of managerial accounting has broadened from the emphasis on product costing to include

> ..... the application of appropriate techniques and concepts in processing the historical and projected economic data of an entity to assist management in establishing plans for reasonable economic objectives and in the making of rational decisions with a view toward achieving these objectives.[10]

The use of analytical models has been suggested for the determination of product costs. Chiu and DeCoster (1966) employ correlation analysis in determining the costs of multiple products. Benston (1966) and Jensen (1967) suggest the use of regression analysis for measuring costs. In the area of cost variance analysis, Lev (1969) suggested the use of information theory. Koehler (1968, 1969), Jensen and Thomsen (1968), and Juers (1967) have recommended several methods of utilizing statistical sampling for the purpose of determining the significance of cost variances. Churchill (1964), Corcoran (1966), and Frank and Manes (1967) have represented process cost systems by employing matrix algebra.

Others have suggested analytic techniques for the purpose of constructing planning models of multiple product production systems. Linear programming has been advocated by Onsi (1966), Demski (1967), and Charnes and Cooper (1967). Input-output analysis has been dealt with by Gambling (1968), Ijiri (1968), and Livingstone (1969). Distinguishing these analytic techniques from the more traditional accounting methods is the fact that the emphasis is on the *ex ante* resource allocation decisions rather than *ex-post* analysis of traditional accounting.

Research Problem

Most of the research in the past has been concerned with correcting apparent deficiencies in existing management accounting models in an "ad hoc" manner. In this research study several new process cost accounting systems will be designed based on existing analytic models. These new systems will then be evaluated based on any deficiencies of the traditional method that are overcome. More specifically the new models will be evaluated on their capacity to provide information that would aid management in decision making in the areas of planning and control.

General Hypothesis

> Analytic process cost accounting models will yield more refined predictive information than the existing process costing methodology.

Research Methodology

The common structural and flow characteristics of systems to which process costing methods are applicable make possible the design and evaluation of a general process cost accounting system without relating to a specific application. The advantage of this approach is that the results of research can be evaluated on the merits of general criteria rather than "ad hoc" application to a single system. The methodology that will be employed to test the hypothesis will first involve the establishment of a general framework of managerial planning and control functions. From this framework, criteria will be identified concerning the information required by management from a process cost accounting system.

Then a descriptive model of the current process costing technology will be developed. Emphasis will be placed on the assumptions and outputs of process costing. Both historical and standard cost systems will be entertained. Then process costing models based on input-output analysis, linear programming, and a stochastic transfer matrix will be developed. Emphasis will be placed upon identifying information that could be obtained for planning and cost control.

The concluding chapter will contain an evaluation of all of the process costing models considered in the study. This evaluation will be based upon the criteria from the opening section plus any limitation upon possible application of the models resulting from the capability of the models to handle certain technical problems often encountered.

Comments on Procedure

No attempt will be made at describing the procedures of the existing process costing methodology. Emphasis will be placed upon the assumptions of process costing and the information output. Production reports and the algebraic representation of cost variances based on standard costs will be presented. In the other models, an attempt has been made to employ matrix algebra whereever feasible to minimize descriptive explanation. However, some situations were encountered where detailed information could more directly be obtained by using algebraic expressions rather than engaging in elaborate partitioning and other matrix manipulations. The glossary at the conclusion of the study summarizes the basic notation employed. Examples relating to the models are developed in each of the chapters.

FOOTNOTES

[1]Paul S. Garner, Evolution of Cost Accounting to 1925 (University of Alabama: University of Alabama Press, 1954), p. 340.

[2]Ibid., p. 3.

[3]Charles T. Horngren, Cost Accounting (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967), p. 1-2.

[4]Garner, op. cit., p. 333.

[5]Garner, op. cit., p. 339.

[6]Garner, op. cit., p. X introduction.

[7]Benninger, op. cit., p. 548.

[8]Benninger, op. cit., p. 548.

[9]John T. Wheeler, "Accounting Theory and Research in Perspective," The Accounting Review, XLV (January, 1970), p. 3.

[10]Committee to Prepare a Statement of Basic Accounting Theory, Charles T. Zlatkovich, Chairman. (Evanston, Illinois: American Accounting Association, 1968), p. 39-40.

CHAPTER II

PLANNING AND CONTROL

Introduction

In the most general sense control "is the process of ensuring that what ought to be done is done and of detecting what is not done."[1] Control requires planning to formulate objectives and it also requires methods for evaluating the operation and execution of plans. The group of individuals responsible for procuring and organizing production facilities to accomplish certain objectives is commonly referred to as "management." The function of management does not end with the procurement and organization. The activation of a production system does not guarantee goal achievement. Management is also responsible for controlling the production process to assure the desired results.

Three general types of control systems will be introduced in this chapter. Then planning and control activities will be placed into an organizational framework. An open loop cost control model will then be presented for the purpose of integrating process costing into the planning and control activities of management. The last section of this chapter will involve the identification of criteria for evaluating cost control systems.

These criteria will be used in the conclusion of this study to evaluate the process costing models.

Control Systems

Control systems are customarily described as containing four elements that occur in the same sequence.[2] The first element is the characteristic or condition of the operating system that is measured. This element may be the input or output of any stage of the production cycle. The second element of a control system involves the measurement of performance. Control systems contain some device or procedure for measuring the controlled item.

The control unit of a system encompasses any activity necessary in determining whether there exists a need for correction in the operating system. Deviations would normally be expected and the problem is to assess their significance. The last element of a control system is the implementation of action to correct the operating system. The elements of a control system are linked together by a communication network.

Three types of control systems are possible. In the first, no external control would be exercised. The cost of any control activities would not be justified by the resulting savings. An operating system of this class could be in an undesirable state.

Closed loop control systems are the second type to be considered. The control systems are distinguished by the fact that they are an integral part of the operating system. The measurement of performance is continuous and decisions are altered in light of new information contained in the current state variables.

In contrast, open loop control systems are not an integral part and require energy from a source external to the operating system. In open loop control systems, control action is taken at fixed time intervals. Decisions are made in advance and revisions can only be made after the elapse of a time period.

The mathematical theory of optimal control has reached a sophisticated level of maturity. However, a major shortcoming of theories in mathematical control "is that they assume 'perfect' information for the problem to be solved."[3] This perfect information would include the equation for the operating system, and the statistical properties of any externally or internally generated noise that would affect system performance.

A non-biological production system is a man-machine system.[4] The components of these systems are men and machines working together to achieve a common goal. The creation of man-machine systems results from the interactions of three interrelated subsystems. The first subsystem involves the design and procurement of the required machines and raw materials. The selection and procurement of the human component is the function of the second subsystem. The third subsystem involves the operational management of the man-machine system.

Since men and machines are required for the operation of all mass production systems, no completely automatic production system exists. Men are always required to perform tasks concerning the direction and maintenance of the system. In the chemical and utility industries there exist man-machine production systems that are almost completely automatic. Such "machine-intensive" production system would have closed loop control systems incorporated into their design and the mathematical theory of optimal control would be applicable. Much of the required information would be available from the design of the system.

Another set of mass production systems could be described as "man-intensive". Currently these systems would not lend themselves to mathematical control theory because of a lack of information concerning the variability introduced by the human components. These "man-intensive" production systems are not often amenable to the incorporation of closed loop control systems.

The organization of a man-machine system implies some type of goal or purpose. To achieve a goal or purpose a system must meet certain standards that would be expressed in terms of operational effectiveness. In a "man-intensive" production system, control would be directed towards guiding the operating system so that the goals of the system might be realized. Open loop control systems are the type most often employed in this type of system. The most appropriate standards for evaluating the operation of a production system are profit maximization or cost minimization. Since profits can be greatly influenced by non-controlable factors from the external environment, the control systems in this study will usually be directed towards cost minimization. This study is concerned with the design of open loop cost control systems for "man-intensive mass production systems.

The Organizational Setting

Control requires planning to formulate objectives and it also requires methods for evaluating the execution of plans. Classical management literature abounds with conceptual frameworks where planning and control are viewed as separate functions. Such a classification is intellectually appealing because it facilitates explanation and model building. However, in an organizational context, the distinction between planning and control becomes artificial because the functions overlap. Anthony, in arguing against such classifications has stated

.... although planning and control are definable abstractions and are easily understood as calling for different types of "mental" activity, they do not relate to separate major categories of activities actually carried on in an organization, either at different times or by different people, or for different situations.[5]

He developed a planning and control framework to overcome this deficiency. This framework will also add structure to the analysis of planning and control cost systems. Anthony's framework, with several modifications will be adopted in this study. This framework will be described in the next three sub-sections.

1. Strategic Planning

Within this framework, two types of planning activities are considered. The more abstract planning is concerned with the establishment of the overall organizational goals and policies. The time horizon for this class of planning activity tends to be relatively long and the problems are of an unstructured nature. Most of the information required for this type of planning comes from sources external to the organization and is obtained specifically for the purpose of dealing with the problem area under consideration. Anthony refers to activity as this type of strategic planning and has defined it as:

.... the process of deciding on objectives of the

> organization, on changes in these objectives, on the resources used to attain these objectives, and on the policies that are to govern the acquisition, use, and disposition of these resources.[6]

In general, the use of product cost information for strategic planning purposes would be limited.

2. Management Control

The other class of planning activity within the framework includes those activities that are concerned with the acquisition and utilization of resources within the limits of the organizational goals and policies established as a result of strategic planning. More specifically, activities included in this class would be the formulation of budgets, determination of decision rules for process control, and the evaluation of managerial performance. The time horizon of these activities would be relatively short when compared to strategic planning. The information employed in these activities would generally come from internal sources and much of it would be historical in nature. There are elements of both planning and control in these activities and management control has been defined as

> .... the process by which managers assure that resources are obtained and used effectively and efficiently in the accomplishment of the organization's objectives.[7]

Product cost information would be required in the management control activities of budget preparation, product pricing, determining decision rules concerning cost and inventory control, production scheduling, and the evaluation of management performance. Therefore, when evaluating a process cost system, the relevance of the information generated by the system will be evaluated in light of the needs of various management control activities.

3. Programmed and Non-programmed Operational Control

Tasks, or more generally resource inputs, as they relate to management accounting, fall into two general groupings. The first group includes those inputs that lend themselves to programming where work standards can be developed. The programming of an input involves the creation of work standards that specify procedures to be followed or standard resource inputs necessary under specified conditions. The work standard serves as the basis for standard costs.

However, many resource inputs do not lend themselves to programming but they are controlled by activities similar to operational control. Non-programmed resource inputs are indirectly related to the production process but the costs make up a significant part of the total costs. The focus of operational control is on specific resources and the information is often non-financial. The time horizon for operational control decisions is relatively short and the decisions can often be based on objective information. Product cost information in the context of operational control is generally used in evaluating the cost of resource inputs versus the outputs of the system and cost control by means of follow-up response to feedback.

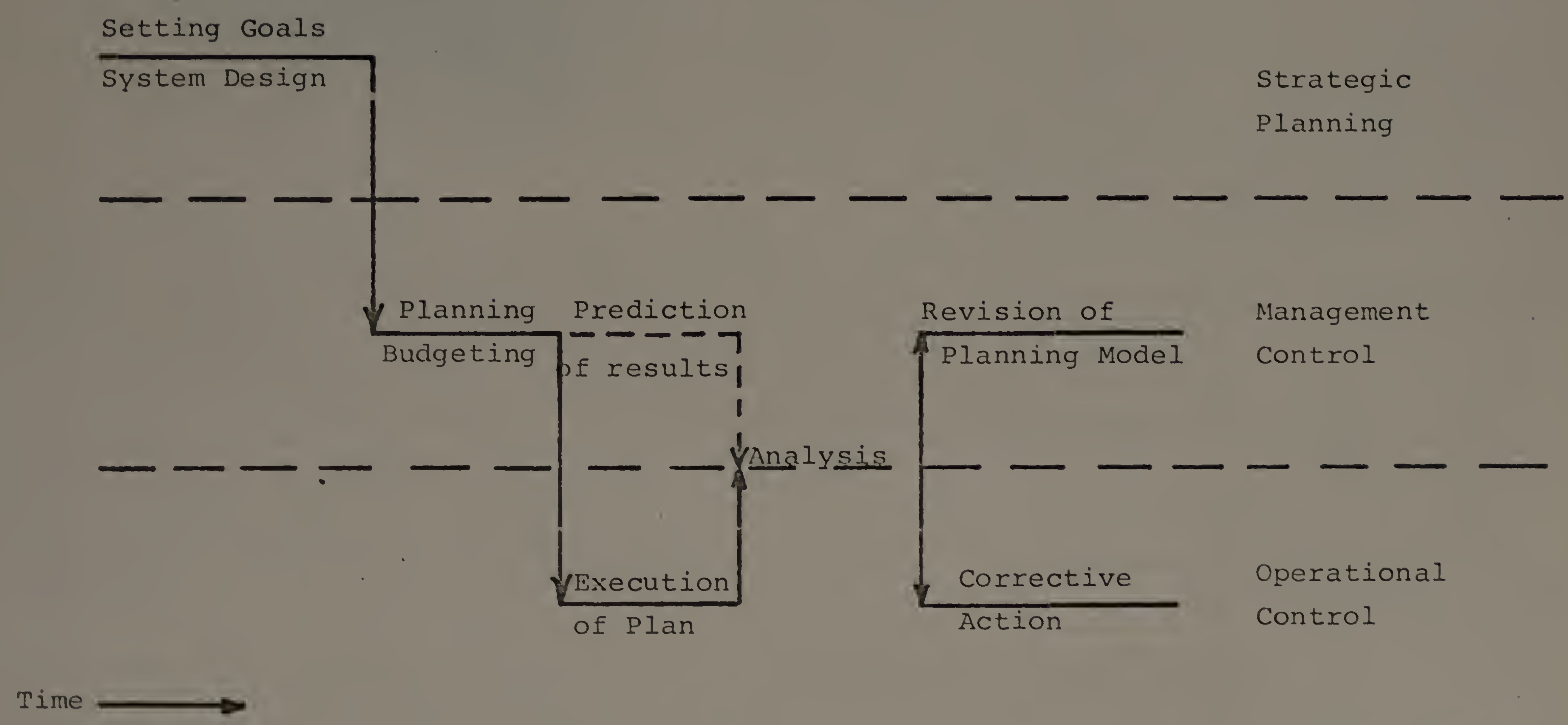
4. Summary

The demand for product cost information by management would be in the areas of management and operational control. The process costing models developed in this study will be evaluated on their capability for providing management with needed information in these areas.

A Cost Control Model

The domain of this cost control model is the production sector of an organization. A schematic diagram of the model is depicted in Figure 2-1. The area of strategic planning would include setting the overall objectives of the organization. The design of the production and control systems would also be activities under the general classification of strategic planning. Almost all of the activity classified as strategic planning would take place prior to the creation of the production system.

Once the production system was operational, the planning activity would involve selecting the production



Schematic of the Cost Control Model

Figure 2-1

alternative for the period that was consistent with the goals established in the strategic planning phase. The set of production alternatives would be limited by the capacity of the production system, the availability of resources, and market conditions. The process costing model could be employed in estimating costs and resource requirements of the alternatives. If the costing model were to be employed in this respect, predetermined resource and cost data would be required. Upon the selection of a plan, the management control activities on the whole would cease in relation to the operating system.

While the production plan was being executed, the accounting system would track the resources employed and their accompanying costs, the transfers between the processes, and the outputs from the system. At the conclusion of the production period, comparisons would be made between the actual and anticipated or predicted results. The analysis of this information would have a two-fold purpose. Decisions would be made determining whether action should be taken with the anticipated result of improving the performance of the operating system. Another area of decision making would involve the determination as to whether the data base or the relationships in the planning model should be altered based on the operating results. Corrective action might also be indicated in the area of resource procurement.

Operational control activities would be concerned with the analysis of the utilization rates of specific resources and the operation of specific processes within the system. The evaluation of the overall performance of the system would be a management control activity. Control in this type of system is primarily achieved through follow-up response to feedback comparisons. Planning serves to control in that the selection of a pattern of resource use influences the operation of the system.

The time span between the initiation of the plan and the analysis of the results is crucial to this type of control system. A lengthy time span could be costly in that the system could be operating out of control for the entire period of time. On the other hand, if the time period was too short, the costs incurred in the planning and analysis activities would be excessive in comparison with possible savings resulting from corrective action. This condition would require sufficient analysis in the design stage of this type of cost control model. In the next section, criteria will be established for the purpose of evaluating open loop cost control systems.

Evaluation Criteria

In this section, the criteria that will serve as the basis for evaluating the process costs models will be presented. They are located at this point in the study so that a reader might better understand the expectations of a process costing model.

As a result of management control activities, budgets are formulated for future production periods. These budgets, or financial plans are statements of the anticipated costs that will be incurred in producing the planned output. Such budgets would serve as the basis for planning the procurement of resources during the production period. However, before selecting a plan for a particular period, management would take under consideration alternative plans. The first criterion that will be employed will concern the predictive ability of the process costing model. Stated in perhaps more operational terms, does the model possess the capability of estimating the expected costs, resource requirements, and process outputs for alternative production plans.

The second criterion will also relate to the management control activity of budgeting. In selecting the production plan for a period, management will evaluate various possibilities in the light of certain goals or objectives. The second criterion will concern the ability of the process costing model to aid management in selecting a production plan. Operationally stated, can management incorporate goals into the process cost planning model? The remaining criteria will relate to operational control.

While discussing cost control models in his book, <u>Decision and Control</u>, Stafford Beer made an observation that will be employed as the third criterion.[8] In summary, Beer states that the control of costs has been considered a function of cost accountants and as a result these people have been considered specialists within the organization. As a result of the specialist designation, cost accountants have generally been regarded as separate from those other people who are concerned with the production system. As a result of the dichotomy, he observes that the model that is employed for cost control is often significantly different from the models used to control production processes.

Beer then argues that production control and cost control are isomorphic systems and should possess a great deal of similarity. In production systems with integrated closed loop control systems, indeed perhaps the only function of a process costing system would be the costing of outputs and inventories. However, in production systems employing open loop control systems, the costing system should serve much broader purposes and similarities should be found between production control systems. The third criterion to be employed will be to evaluate the process cost model on its ability to aid in the control of production.

Most mass production systems are complex and there would be many resource inputs. In many systems, if the inputs were fixed, the output would vary because of random fluctuations within the production processes and quality variations within the input. Conversely, if it were possible to hold the output constant, the input would vary. Therefore, when the costs incurred in the operation of the system are compared with the budgeted costs, deviations are to be expected due to the random characteristics of the inputs, processes, and outputs.

If cost deviations between budgeted and incurred costs are to serve as a basis for identifying conditions to which management should respond to maintain control over the system, then the control model should indicate the source of any deviation. In addition to the source, an ideal control model would also indicate the significance of any deviation. In most open loop control models, it may not be possible to obtain the statistical significance of a cost deviation. However, the significance of the deviation in relation of some optimal plan may be possible to determine.

Once a deviation is made evident, decisions concerning possible investigation and corrective action have to be made. The incorporation of decision rules concerning investigation could also be incorporated into a cost control model. The fourth criterion that will be employed in evaluating the models will encompass the identification of cost deviations, indication of source, indication of significance, and the possibility of incorporating decision rules. The criteria developed in this section will be employed in the evaluation at the conclusion of this study.

Summary

This chapter has served to introduce an open loop cost control model. The next five chapters of this study will present process costing models. The emphasis will be placed upon the outputs of the models as it would relate to the management and operational control activities of management.

Footnotes

[1]Gordon Shillinglaw, "Divisional Performance Review: An Extension of Budgetary Control," in Management Controls New Direction In Basic Research, ed. by Charles P. Bonini, Robert K. Jaedicke, and Harvey M. Wagner (New York: McGraw Hill Book Co., 1964), pp. 149-163. p. IX of Introduction.

[2]Richard A. Johnson, Fremont E. Kast, James E. Rosenzweig, The Theory and Management of Systems (2nd ed.; New York: McGraw Hill Book Co., 1967), p. 72-78.

[3]Harold J. Kushner, Stochastic Stability and Control (New York: Academic Press, 1967), p. 2.

[4]John L. Kennedy, "Psychology and System Development," in Psychological Principles In System Development, ed. by Robert M. Gagne (New York: Holt, Rinehart, and Winston, 1962), p. 14.

[5]Robert N. Anthony, Planning and Control Systems A Framework For Analysis (Boston: Division of Research Graduate School of Business Administration, Harvard University, 1965), p. 10-11.

[6]Ibid. p. 24.

[7]Ibid., p. 17.

[8]Stafford Beer, Decision and Control (London: John Wiley & Sons, 1966), p. 215.

CHAPTER III

THE TRADITIONAL PROCESS COSTING MODEL

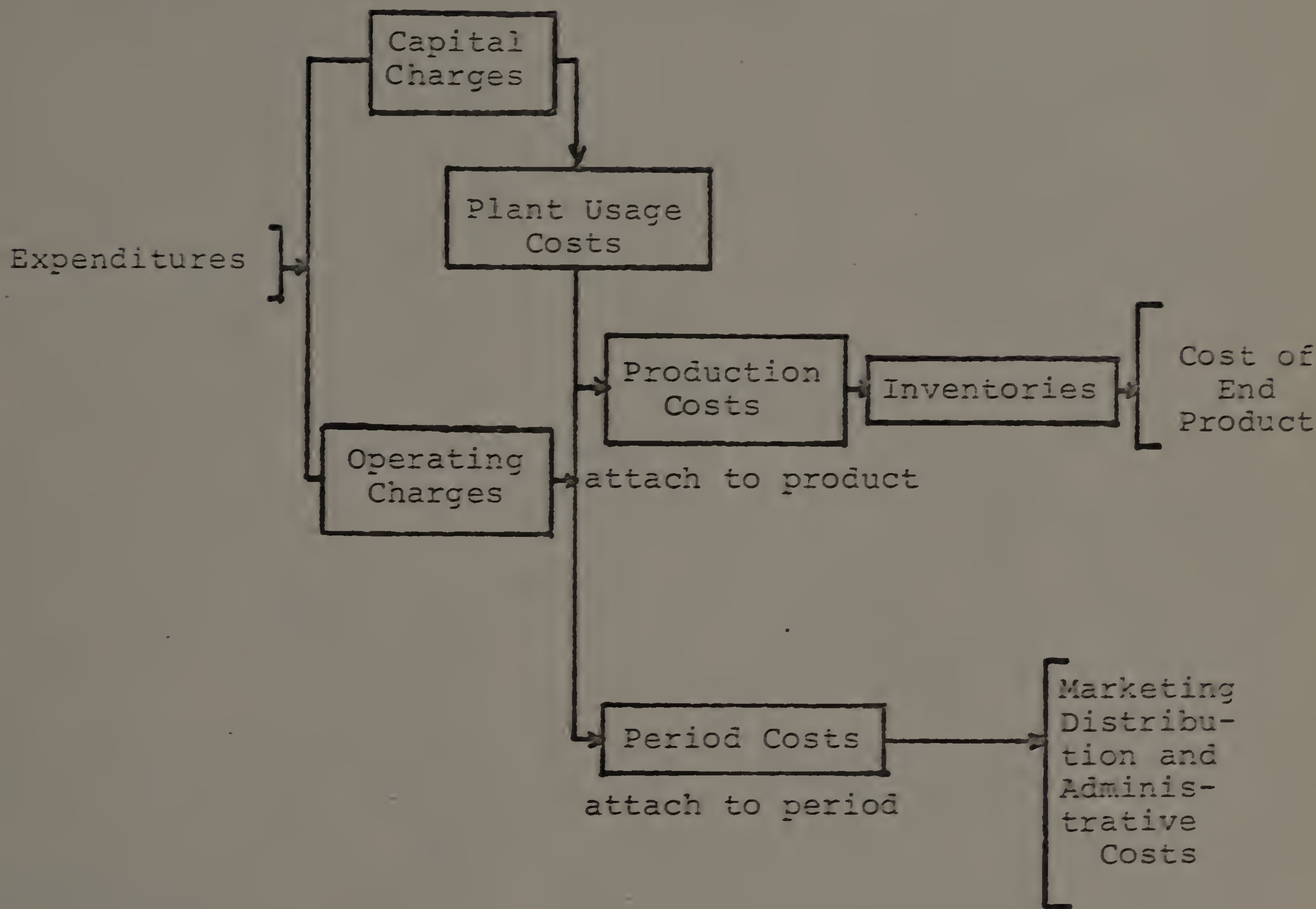
The first section of this chapter will concern the cost terminology and classifications employed in process costing. In the next section detailed consideration will be given to the assumptions which underlie process costing. The detailed presentation is necessitated by the fact that such information is fragmented throughout the accounting literature in varying degrees of completeness.

The concluding sections will consider the current accounting technology as applied to the process costing problem. The objectives of this study do not require a detailed narrative of the sometimes complicated procedures which embrace process cost accounting since such documentation is available elsewhere.[1] Rather the outputs of the traditional model that would have implications for planning and control activities will be described. This information will be used in the conclusion of the study when the models under consideration are evaluated.

Cost Terminology and Classification

In general cost implies a sacrifice or foregoing but there is no unique definition that will satisfy all situations.[2] In this study, a cost will be viewed as the result of an expenditure of resources, measured in financial terms, transferred in consideration for goods or services received.[3] In accounting, expenditures are generally classified as either capital or operating charges. The benefits from the assets received as a result of a capital charge are recognized in more than one accounting period whereas the benefits from an operating charge apply to a particular accounting period. Capital charges are recognized as plant usage costs through depreciation and amortization procedures.

The sum of the operating charges and plant usage costs of an organization are then classified as either production or period costs. Production costs are those costs that can be associated with the production activities of the organization. The production costs are allocated between those applicable to inventories within and products exiting the system. Period costs are applicable to the time span of the accounting cycle and are usually related to the marketing, distribution, and administrative functions within the organization. Figure 3-1 depicts the relationships between expenditures and costs. Costing is the process of determining the cost of a product and this study is concerned with process cost models that employ production costs in costing the products of a system.



The Classification of Expenditures

Figure 3-1

A technical jargon has evolved about production cost terminology and some definitions will be established for terms that will be employed in the study. As a point of departure, the economist's and accountant's view of cost patterns will be considered. Although both of these conceptions of cost patterns were originally developed with reference to manufacturing firms, the same patterns are appropriate for any system where a process cost system would be applicable.

1. The Economist's Description Of Cost Patterns

The economist's theoretical description of the cost patterns of a firm assumes a period of time sufficient enough to permit a change in the quantity of products produced but insufficient to permit a change in the capacity of the firm. A fixed input is assumed to be necessary for production and its costs are assumed to be incurred regardless of volume. The required quantity of the variable input and therefore the variable cost is dependent upon the quantity of output produced. The economist's distinction between fixed and variable costs is assumed to be temporal as all costs are considered variable given a sufficient length of time.

The short run total cost function of the economist is generally stated as a function of the volume of output plus the cost of the fixed input.

$$\text{Total Cost} = f(x) + b \qquad (3\text{-}1)$$

A set of cost definitions that are functions of the volume of output can be derived from relationship (3-1).

$$\text{Average Total Unit Cost} = \frac{f(x)+b}{x} \qquad (3\text{-}2)$$

$$\text{Average Variable Unit Cost} = \frac{f(x)}{x} \qquad (3\text{-}3)$$

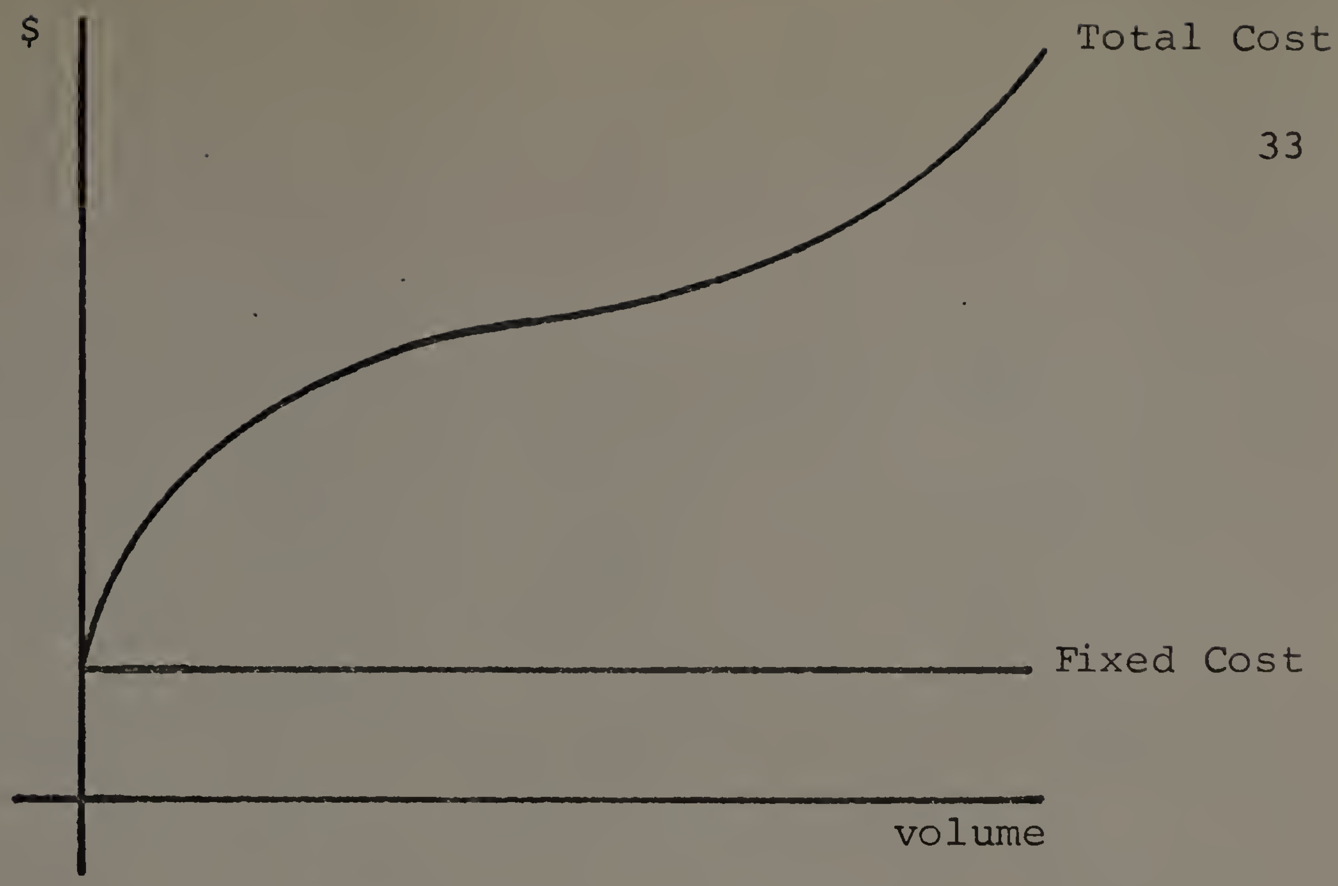
$$\text{Average Fixed Unit Cost} = \frac{b}{x} \qquad (3\text{-}4)$$

Cost functions assume various shapes but the generally accepted description is depicted in Figure 3-2. Fixed costs are represented as a horizontal line that is not influenced by changes in volume. The shape of the total cost curve is explained in terms of economies and diseconomies of the volume of operations. Initially the diseconomies of production dominate, then the economies of mass production are achieved, and finally the diseconomies of production become prevalent and the total cost curve begins to rise at an increasing rate.

Marginal cost is a function of the rate of increased cost relative to an increase in volume. Marginal cost is defined as the first derivative of the total cost function with respect to volume:

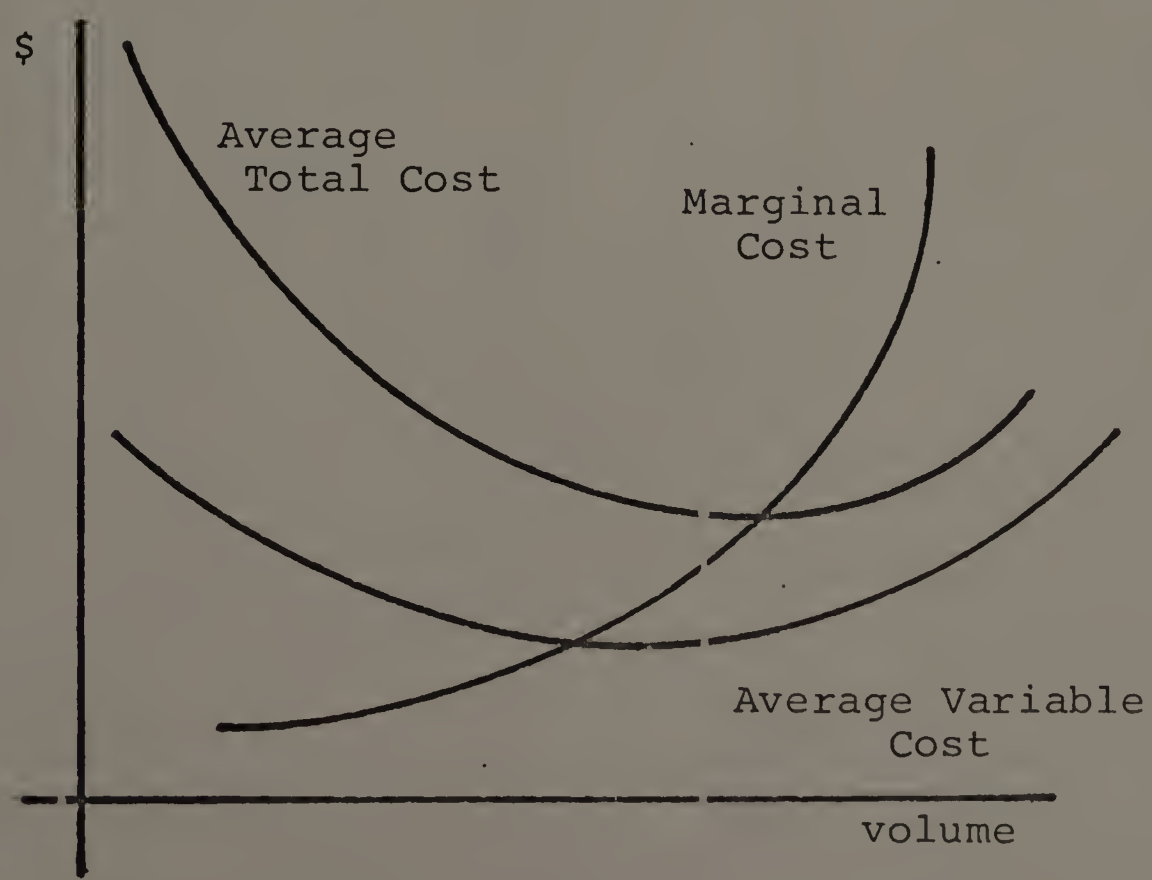
$$\text{Marginal Cost} = f'(x) \qquad (3\text{-}5)$$

Marginal cost is interpreted as the added cost incurred



Economist's Total Cost Curve

Figure 3-2



Economist's Average and Marginal
Cost Curves

Figure 3-3

resulting from the production of one additional unit.

Figure (3-3) illustrates the relationship among the marginal cost, average fixed cost, average variable cost, and average total cost curves. The marginal cost curve is more sensitive to change in volume than the other curves because an average reflects a change more slowly than an individual observation. The marginal cost curve crosses the average variable and the average total cost curves at their minimum points. This can be demonstrated by taking the first derivative of the average variable or the average total cost curve and setting them equal to zero. The equality then can be shown between the volume when the derivative is set equal to zero and the marginal cost at the same volume.

The economist is concerned with developing aggregate relationships in support of economic models and to do so economists make certain simplifying assumptions to reduce the number of variables and facilitate mathematical analysis. For example, economists assume continuous relationships, smooth curves, and that the firm produces a single product and faces a homogenous cost structure.

2. The Accountant's Description Of Cost Patterns

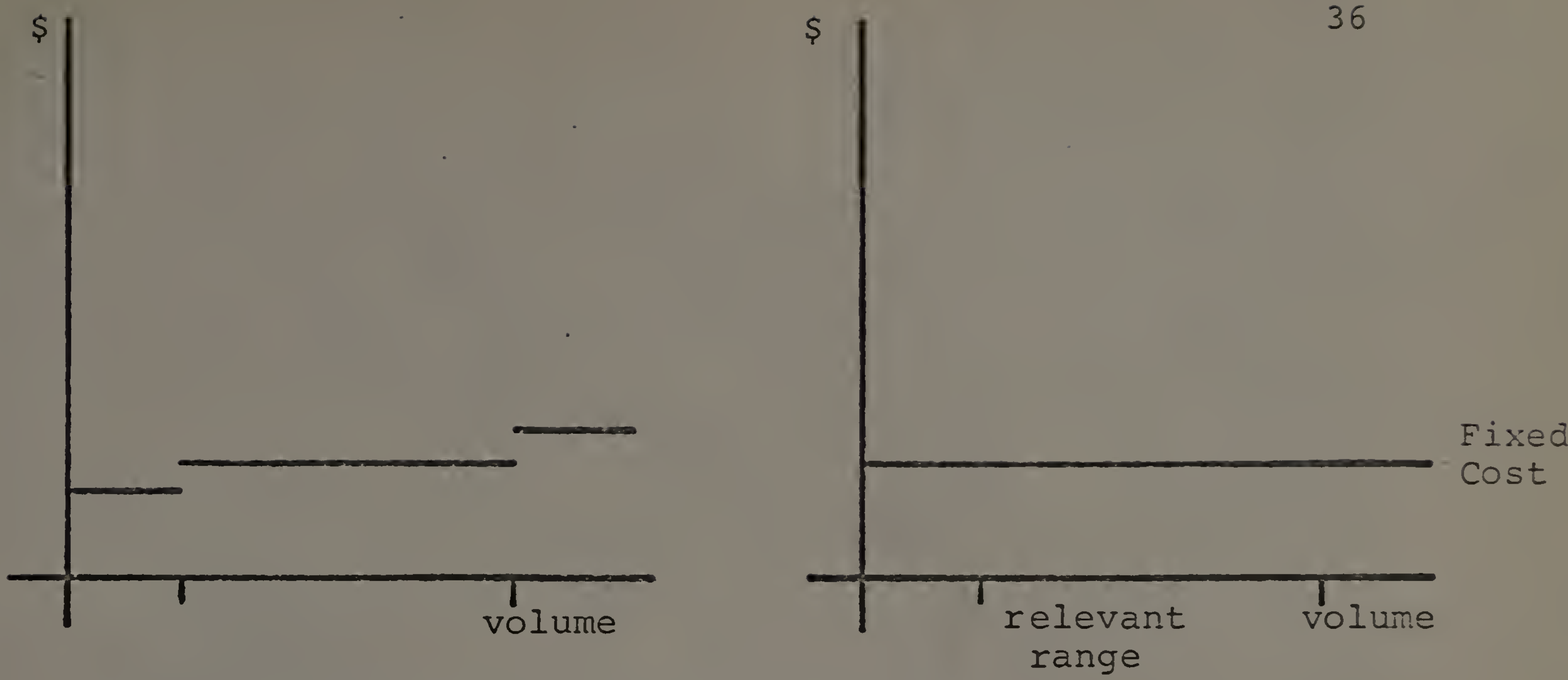
The accountant is faced with the problem of describing the cost patterns of the firm and basing his description on empirical observations. Accountants recognize the complexity of the cost structure of the firm but must make simplifying assumptions to make aggregation and analysis possible. In accounting, a fixed cost is assumed to be fixed in relation to a given period of time and over a given range of activity.[4] Figure (304) represents how fixed costs are viewed over an entire range of activity that is assumed pertinent for the present analysis. Variable costs vary directly with volume and the accounting view of cost-volume relationships is presented in Figure (3-5).

The short run total cost function as assumed by accountants will be employed in this study. The general form of the function is expressed as a function of the volume of output plus the cost of the fixed input that is assumed for some relevant range.

$$\text{Total Cost} = f(x) + b \qquad (3\text{-}6)$$

where f(x) is a linear function. A set of cost definitions can be derived from the accountant's total cost function that are very similar to those derived from the economist's total cost curve.

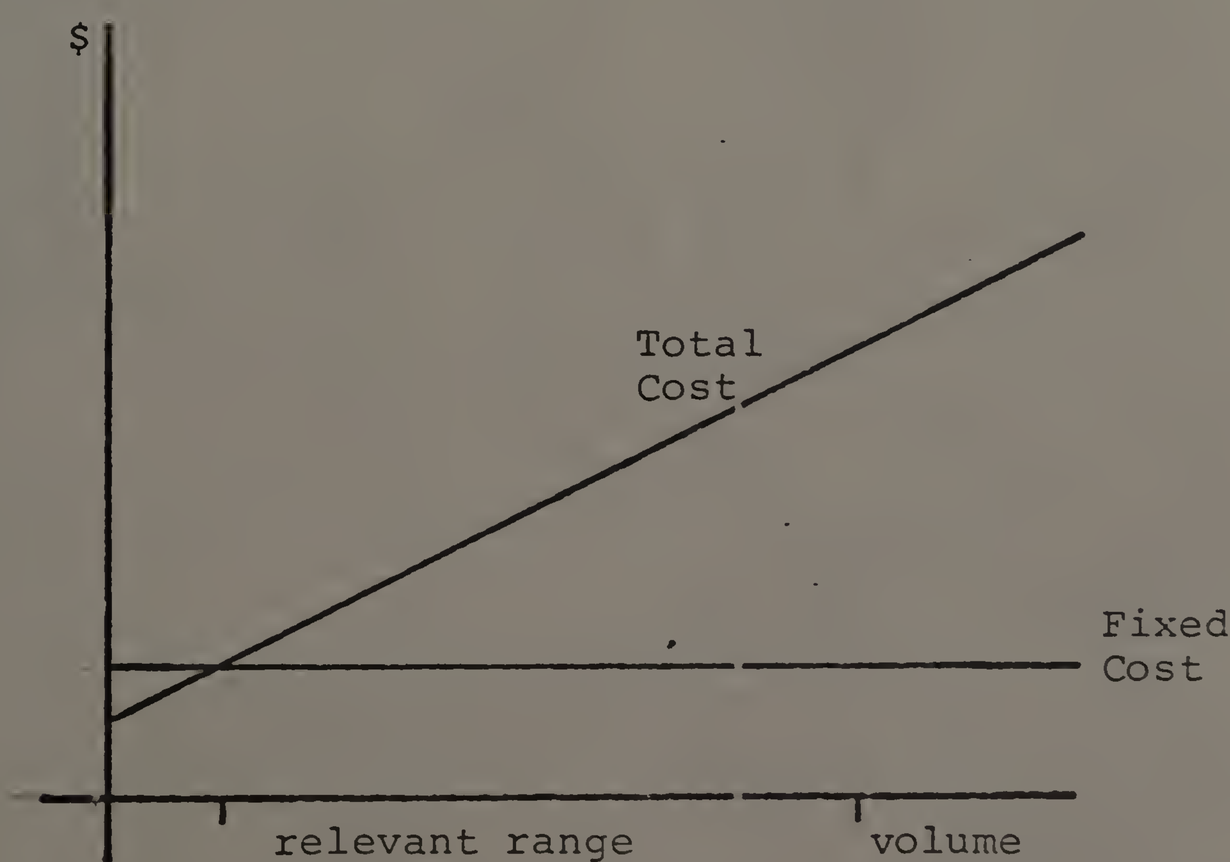
However, as a result of assuming a linear total cost function, the average variable unit cost is constant for all levels of activity within the relevant range. This results in the marginal cost equalling the average variable cost. Although the terms are often used interchange-



Accountant's Fixed Cost Curve

Relevant Range Assumption

Figure 3-4



Accountant's Total Cost Curve

Over A Relevant Range

Figure 3-5

ably by accountants, the practice will be avoided in this study. The average total cost is assumed to decline continuously over the relevant range. The average total cost will always exceed marginal cost under these circumstances because of the equality between average variable cost and marginal cost.

A prime cost is a variable production cost that is associated with a significant resource input into the system. Prime costs are "traceable" directly to the end product and vary proportionately with changes in volume. Overhead costs are not directly "traceable" to the product and are often referred to in the literature as indirect costs. It is generally the case that overhead costs can be segregated into fixed and variable components over a relevant range. The fixed overhead cost component varies in magnitude as a function of the capacity of the system. Fixed costs are not fixed in the sense that they do not fluctuate, but they vary from causes independent of volume. The independence of the level of fixed costs from volume leads to the periodicity assumption in accounting. Simply stated, the periodicity assumption is the basis upon which economic activities (fixed costs) are associated with reporting periods in the determination of product cost.

The variable component of the overhead cost, although not "traceable" to the end product, varies with the volume in the system. The sum of the prime costs and variable overhead costs is referred to as the direct cost. In some applications only the direct costs are employed in the valuation of inventories hence the term "direct costing." Direct costing is not a complete costing system but rather a feature that may be introduced in either process or job-order systems. Full absorption costing, in contrast with direct costing, employs both the fixed and direct costs in determining inventory valuations.

This study will be based on the assumption that a distinction can be made between fixed and variable costs. In applications, this distinction would require arbitrary classifications in certain cases and the output of any system would have to be employed with the realization of these limitations. In this study, the term "direct cost" will be employed in context to identify total per unit variable cost. The term "variable cost" will be used when referring to the unit cost of a particular cost classification. "Fixed cost" will refer to a cost that is assumed to be fixed for an assumed period of time within the limits of a certain range of activity.

"Incremental cost" is generally defined in accounting as the difference between the total costs of two alternatives. "Opportunity cost" is another concept that is prevalent in both accounting and economic literature.

A generally accepted definition of an opportunity cost is that it represents the cost of an opportunity foregone.[5] In this study, the term "incremental cost" will be used when referring to comparisons between alternative in planning. "Opportunity cost" will be employed when comparisons are made between alternatives in the evaluation of performance on an *ex-post* basis.

The terms "historic" and "standard" cost will be used throughout the study. Historic costs result from expenditures and would be determined in the *ex-post* cost analysis of a production system. Historic costs are average costs and result from the allocation and transformation procedures of cost accounting. Standard costs are predetermined costs that would be employed in planning and the *ex-post* analysis of a system. The derivation of standard costs will be described more thoroughly in the development of the descriptive model. The next section will be concerned with the underlying assumptions of the process costing model.

Assumptions of Process Costing

When a process costing system is employed in any organizational context, there exist certain explicit and implicit assumptions concerning costs and the behavior of individuals. The assumptions concerning cost patterns were presented in the first section of this chapter. In this section, assumptions concerning costs-attaching, reporting periods and entities, inventory, and overhead allocation will be discussed. Since all mass production operations are man-machine systems, certain behavioral assumptions would also be required

1. The Costs-Attach Assumption[6]

Resources are possessions that represent service potentials which are acquired through transactions. The cost of a resource is synonymous with the "acquisition price." In accounting, when a service potential is expired, the cost of the resource is treated as an expense in the reporting period in which the expiration takes place. In general, the service potential has expired as a result of a revenue-generating activity when accounting for a merchandising firm. The distinction between assets and expenses is therefore based on the criterion of the time at which service potentials expire.

This criterion is not satisfactory in the context of process costing because the expiration of service potentials is not related to revenue-generating activities but to the transformation of resources into the finished products of the system. The "costs-attach" assumption serves as the justification for the cost transformations undertaken in process costing. Altering a definition posed by David Li to fit the context of this study, costs will be considered to have the "power of cohesion" when figuratively they are brought into contact in mass production systems. The service potentials employed in the processes are not "consumed" but "transformed" and value is added to the product as a result of the transformation. The expired service potentials are recognized as assets in the form of inventories within the system. This emphasis on service-potential transformations is what conceptually distinguishes cost from financial accounting and the "costs-attach" assumption is the theoretical basis for product costing.

2. Reporting Periods and Entities

For all of the costing models constructed in this study it is assumed that an accounting cycle is the span of time over which the predetermined overhead rates or standard costs are relevant. The accounting cycle is often equal to the budget period for an entity. It is assumed that within each accounting cycle there are from one to ψ reporting periods. The length of the reporting periods is contingent upon the needs of management for information for control purposes and the flexibility inherent in the structure of the accounting system. The quality of the information provided by a process cost system for control purposes would be of primary consideration in determining the length of a reporting period. The number of reports per accounting cycle would be a major contributing factor in determining the cost of operating any process cost system.

It is also assumed that processes or cost centers can be identified for purposes of cost allocation, aggregation, and analysis. Each process is considered a reporting entity in that the results of the activity in the process are reported at the conclusion of each period. The processes are also considered responsibility centers in that the management of a process would be responsible for the operating performance.

3. Inventories

When in-process inventories are considered, it is necessary to determine a measure of the productive activity that took place during the reporting period to determine unit costs. This is most generally accomplished by determining the equivalent number of completed units for each prime cost input and an estimate is made for the overhead costs. This procedure takes into consideration the activity necessary to complete the beginning in-process inventory, the units started and completed during the period, and the activity necessary to bring the ending in-process inventory to its current status. The end result of this process is referred to as the equivalent production for the reporting period and is employed in determining the unit costs.

FIFO and moving average are the two most common methods utilized in the literature in determining equivalent production and unit costs. When the FIFO method is employed, beginning of the period in-process cost balances are transferred to the next process as part of the cost of the first units completed during the period. The portion of the unit cost of the current period necessary to complete the in-process inventory and the costs attached to the inventory at the beginning of the period are first transferred to the succeeding process. Units that are started and completed during the period are transferred at the unit cost determined for the reporting period. When the moving average method is employed, all of the costs are averaged together and a single rate is employed for transferring costs.

If the FIFO system is employed, it is always necessary to distinguish between the in-process inventories and any units that are started and completed during a period. If the system were cleared of all in-process inventories during a reporting period, in an n process system, n different transfer costs would have to be determined for the inventories before the rates for the period could be employed. The reason for this complication is that the inventories must partially be costed at the rate for the previous period and partially at the rate applicable to the current period. The more processes in the system, the more burdensome this procedure becomes.

Unless there are significant deviations in the unit costs and a relatively few units involved, this procedure is generally not employed because of the complications that result. The moving average costing method simplifies the computations by employing average unit costs that are "attached" to all units in process during a particular reporting period. The unit costs are determined by averaging the "costs-attached" to the beginning inventory plus the costs incurred during the reporting period.

The differences between the FIFO and moving average methods of costing can be overstressed. The resultant unit costs do not depart significantly unless the length of the reporting period is relatively short and the unit production costs vary significantly between periods. Therefore, the choice between the two methods is generally made on the basis of convenience unless system characteristics dictate the employment of the FIFO method. Theoretically, however, the FIFO method of determining equivalent production results is the most legitimate divisor to employ in calculating unit costs because it more closely approximates the level of activity during the period.

When standard costs are employed, no computations are necessary in determining unit costs for purposes of costing inventories and transferred units. Also the FIFO method of determining the equivalent production can be employed since there will be no variations in the unit costs. Since the main thrust of this study is to evaluate process costing models for planning and cost control purposes, the use of standard costs will be assumed throughout. Therefore, the inventory valuations determined by the models developed in this study would be based on standard costs employing FIFO equivalent production computations. These inventory valuations would be required by the financial accounting system of the organization.[7]

4. Overhead Rates

Perhaps the most perplexing problem encountered when dealing with cost control system is the matter of overhead. There would seem to be advantages and disadvantages to almost all overhead allocation schemes. If historic costs are employed, it is necessary to determine overhead rates for each production period. The utility of such a procedure is questionable for several reasons.

If the production of the system follows seasonal patterns, then the overhead rates will vary depending upon the volume for a particular production period. The situation is further complicated if there are seasonal fluctuations in the overhead costs incurred. Other possible causes of variations in the overhead rates include the incurring of abnormal costs in a particular period, variation in production efficiency between periods, and price changes. Another limitation that results from employing actual costs is that in many cases management decisions cannot be avoided until the actual overhead costs incurred can be determined for a particular production period.

The method employed in most cost systems to overcome the disadvantages of a system based on actual costs is to utilize predetermined overhead rates. In making a case for predetermined overhead rates, several authors have gone so far as to state that:

> A predetermined overhead rate provides the only feasible method of computing product overhead costs promptly enough to serve management's needs and smooth out uncontrollable and somewhat illogical month-to-month fluctuations in unit costs.[8]

Predetermined overhead rates normalize the factory overhead over the reporting periods and tend to eliminate the seasonal fluctuations of costs and the effects of variations in the levels of production.

However the introduction of predetermined overhead rates is not a panacea because many problems arise in determining the rate to be employed. These problems generally fall into one of three classifications. The first problem area is that of determining the measure of volume to be employed in arriving at the overhead rate. The volume may be determined based on the physical capacity of the system or in operational terms based on some measure of anticipated production.

Theoretical capacity is a measure of the volume that is based on the assumption that the system will operate at peak efficiency. Allowances are not made for delays and stoppage for repairs. Another measure of volume based on the physical capacity that is often mentioned in the literature is practical capacity. This measure of volume differs from theoretical capacity in that allowances are made for predictable unproductive time due to operating interruptions. It is based on the engineering capacity of the system with allowances made for normal downtime.

The two common activity measures based on the anticipated production are scheduled production and normal production. When scheduled production is employed as a measure of anticipated activity, the fixed overhead rates vary with changes in the scheduled production. Normal production is the activity measure based on anticipated production. Normal production differs from scheduled production in that the anticipated activity is determined by averaging the scheduled production over the past several accounting cycles. This procedure minimizes the changes in the fixed overhead rates due to changes in scheduled production.

The determination of an appropriate level of activity for predetermining fixed overhead rates is a matter of judgment. One of the purposes of determining a fixed overhead rate is to obtain an estimate of the utilization of capacity. This point will be clarified in forthcoming discussion of cost variance analysis. Fixed overhead rates are of primary importance for product costing in the long run. However, fixed overhead rates have limited significance in the short-run for control purposes and will be given limited consideration in this study.[9]

The second classification of problems concerning the specification of predetermined overhead rates concerns the question as to whether fixed overhead items should be included when determining product cost. When all system production costs are treated as product costs, the cost system is a full absorption costing system. However, in the past decade direct costing or variable costing systems have been developed. Under direct costing, fixed costs are treated as period costs rather than product costs. Only variable costs are treated as product costs and this results in more stable unit costs over a relevant range irrespective of the volume of production.[10]

Direct costing focuses attention on the contribution margin and facilitates marginal analysis. It offers certain advantages in performance evaluation and in making short-run decisions in the areas of capital budgeting, make-or-buy, pricing, and the selection among alternative uses of production facilities.[11] The appropriateness of the application of direct or absorption costing depends on the type of decisions that are to be made based on the information provided by the cost system. It is not a function of the cost system to specify the decision framework in which particular information should be employed. However, since the relevance of employing either direct costs or full absorption costs has been demonstrated in many problem situations, a process cost accounting system should discriminate between variable and fixed costs. Therefore, the output of a full absorption system could be modified without much effort to provide information in a direct costing format. All of the process costing models developed in this study could be loaded to provide either direct or full absorption costs. The objective function of linear programming is based on variable costing but admustments can be made for fixed costs. (Refer to Chapter V)

The last problem to be considered concerning the employment of predetermined overhead rates is the question of whether a universal rate for the system or individual rates for the individual processes should be employed.

The solution to the problem is contingent upon the characteristics of the system under consideration. If an index of activity is applicable for all processes in the system and rational overhead allocation among the processes is not feasible, then a universal rate is considered to be appropriate. However, in most systems no one index of activity is applicable for all processes and this necessitates the determination of individual rates for each process within the system.

This procedure makes necessary the allocation of some overhead costs that cannot be identified directly with a particular process. The allocation problem exists in almost all applications and to a degree does limit the usefulness of the output. However, if the system discriminates between direct and fixed costs, it is then possible to consider the impact of the allocations upon the decision under consideration. All of the process costing models developed in this study will have the capability of employing either a universal or separate process overhead rates In Chapter V, a method of allocating fixed costs will be developed that will eliminate the distortion in making short-run decisions caused by arbitrary allocations of fixed overhead.

When predetermined overhead rates for the various processes are incorporated into a process cost system, it is necessary to determine the period of time that is to be considered when arriving at the predetermined rates. Generally the period of time is a calendar year and is referred to as the accounting cycle. Reporting periods, it should be recalled, are the assumed time periods between cost reports of the process cost system. In most cases there would be more than one reporting period in an accounting cycle.

After the accounting cycle has been determined, it is then necessary to identify a measure of activity within each process upon which the predetermined overhead rate is to be based. Generally a resource is identified where the input level is correlated with process output over a relevant range of acitvity and the overhead rate is expressed as a linear function of the resource input. Finally, it is necessary to assume an expected level of activity when specifying an overhead rate. The normal production concept of activity level is assumed for the models developed in this study.

5. Behavioral Assumptions

The behavioral assumptions will be presented in the context of a standard cost system. It is the standard cost system with its accompanying work standards and budgets that would directly attempt to influence the behavior of the individual participants in most organizations.

The transition from an actual to a fully predetermined cost model is completed when standards are developed that specify a functional relationship between prime resource inputs and process output. The relationships are generally linear and serve as a basis for standard cost systems. The concept of standard costs can be traced from the scientific management movement in this country starting in the early part of this century and fathered by Fredrick W. Taylor.[12] Time and motion study was used by Taylor and his followers to determine the "one best way" of performing a particular task. This along with other aspects of the scientific management movement brought considerable precision of measurement into organizations concerning production activities.[13] This resulted in the development of work standards that specified resource inputs. Costs are attached to the standard resource inputs and thus the evolution of the term "standard cost".

Standards are generally expressed in physical terms and in many organizational settings are developed by industrial engineers; hence, the term engineering standard is often employed. The standards are transformed to determine standard costs and these costs are transformed in order to specify a budget. In one accounting dictionary a standard has been defined as "a desired attainment: a performance goal; a model."[14] Research has indicated that there are certain behavioral variables that must be considered when standards are incorporated into a cost system.

Stedry (1960) executed a laboratory experiment where he used 108 students as subjects. In the experiment, the subjects had to solve six series of mathematical problems and for each series standards were set for the number of problems to be solved. Small financial rewards and penalties were made based on the performance of the subject as compared with the standard. In some cases the subjects were asked to indicate their anticipated level of performance (their aspiration level as defined by Stedry) either before or after actual performance. The timing of the aspiration question and level of the standards were varied in a 3x4 factorial experimental design.

The results of the experiment would indicate that the formulation of an aspiration level does significantly (1%) influence performance. He found that performance was highest for those subjects who formulated an aspiration level after being made aware of the performance standard. Performance was lowest for those subjects who were not requested to formulate an aspiration level. Stedry found that the influence of the level of the standards on performance was not significant (5%) but that the interaction of the aspiration level and standard level was significant (1%).

In a field experiment, Stedry and Kay (1964) used 17 foremen from a manufacturing plant as subjects. They established two levels of production standards (normal and difficult) and varied these standards in the areas of productivity and quality. The levels of goal difficulty were based on past performance and a normal goal was one that had been achieved 50 percent of the time and a difficult goal only 25 percent of the time. The design was a 2x2 factorial and the foremen were assigned randomly.

None of the hypotheses of the experiment proved to be statistically significant. The small sample size proved to be a limitation. The conclusions were based on tendencies and indicated that difficult goals in one of the areas either lead to very good or very poor performance in comparison with normal standards. This tendency appeared to be especially true if both goals were difficult. As a result of interviewing the subjects before the experiment, it would seem that performance was contingent upon whether the difficult goals were perceived as impossible or challenging.

Morrow, Bowers, and Seashore (1967) have shown that the attitudes of management and the participation of workers are major factors to be considered when work standards are employed. In a field study they were able to demonstrate that productivity could be increased through worker participation in the setting of standards and improved attitudes of management toward the human component of the organization. The change in the attitudes of management is aimed at altering the work environment from exploitive-authoritative to a participative group (Likert 1967). However, Vroom (1959) found that persons with weak independence needs were apparently not affected when given the opportunity to participate in setting work standards. Coch and French (1948) found that group participation in decision-making has a positive influence on productivity, however, in a similar setting in Norway there was no significant change in performance. The conclusion reached was that cultural influences have an influence on worker attitudes and performance.

The research cited thus far is by no means inclusive of all of the evidence that exists concerning the impact and number of behavioral variables that should be considered when work performance standards are integrated into a cost system for control purposes. Enough evidence has been cited however to reach the conclusion that behavioral variables must be considered when work standards are established for purposes of evaluating and controlling performance. Therefore, the designer of any cost control system must either implicitly or explicitly take into consideration certain assumptions concerning human behavior.

Until recently, there was little consideration given to the behavioral implications of accounting systems. However, several authors lately have undertaken to deduce the behavioral assumptions that are implicit in the traditional management accounting models. The work of Caplan (1966) and Hofstede (1967) are examples of these efforts and they will serve as a basis for the behavioral assumptions identified in this study for a process cost accounting system. The assumptions identified by Caplan and Hofstede are general in scope and will be modified to meet the requirements of a standard process cost system.

Perhaps the most general assumption with behavioral implications is that the purpose of a standard process cost system is to aid management in controlling costs. The introduction of performance standards, standard costs, and budgets is based on the assumption that these guidelines act as incentives to motivate individuals to perform at some acceptable level. This assumption is based on a more general premise that concludes that the setting of goals improves the performance of individuals or groups.

The standard process cost system is assumed to be a "goal-allocation" mechanism that makes feasible the subdivision of management objectives among the various processes within the system. This makes possible the evaluation of individual and group performances based on predetermined standards rather than an evaluation based on the performance of the total system.

The organization is assumed to be structured such that the source of control is located at the top of a hierarchy and that the lines of control flow downward. The cost variances determined by the standard cost system should serve as the basis for the control actions taken by management. This implies the assumptions that managers should manage by exception, action will be taken when deviations are reported, and the cost system is capable of identifying undesirable performance. It also is implied that cost systems are neutral in their evaluation and that the systems possess the capability of eliminating personal bias.[15]

When predetermined overhead rates were discussed, the problem of determining the capacity measure to be employed presented a problem that is solved by considering the objectives of the evaluation for which the information produced by the process cost system will serve as a basis. A similar problem arises when resource input standards are to be established. The question is whether the standards should be realistic or idealistic in nature. An idealistic standard would make no allowances for start-up contingencies, rest periods, breakdowns, inconsistencies of resource inputs, and other factors that contribute to the variations in performance. A realistic work standard would take into account the expected time delays due to the above factors.

The concept of realistic work standards has generally been accepted in engineering and standard cost accounting. Li has indicated three reasons for this acceptance. He wrote:

> First, standards may be used as bases for evaluating performances, and realistic standards serve as logical yardsticks. If only idealistic standards are provided, management must "mentally" supply some bases for judging acceptable performance....
>
> Second, standards may be used as bases for computing incentive compensation, and realistic standards serve as a strong motivational force...
>
> Third, standards may be used as building blocks in making plans and co-ordinating activities. The use of realistic standards results in workable plans and meaningful projections; the use of idealistic standards will not.[16]

Once the resource input standard is determined, it is transformed into financial terms by employing the expected cost. Conceptually, there is little difference between controlling input or output in physical units as opposed to financial units. However, the transformation from physical into financial units makes the dollar the common unit in the system and this makes possible the establishment of priorities for management attention.

The Traditional Process Costing Model

In the process costing problem, units pass from one process to the next. The traditional process costing model "attaches" the cost of inputs to units as they pass through the production processes. Equivalent production is the computational basis for determining the unit costs that are attached to production as it flows through the system. The necessity for employing equivalent production is the existence of partially completed units in-process inventories of the system.

Although the concept of process costing as described in the preceding paragraph may seem rather straight-forward, in actuality process costing consists of many complicated procedures.[17] For example, the equivalent production required for costing the output of a process usually requires computations involving the output of the preceding plus several of the resource inputs of the process under consideration. Other problems are often encountered because of variations in the number of units because of shrinkage, scrap, or the addition of materials. The existence of joint or by-products also create situations that will complicate process costing procedures. Any process costing system that would simplify the computational complexity so often encountered would be an improvement as long as the cost of such a system was not excessive.

Either historic, standard, or a combination of these two cost systems are employed in process costing. Considering the two cost systems as a mutually exclusive will result sharpening the distinctions when the systems are evaluated. Initially the traditional model, employing historic costs will be considered followed by a full standard cost system.

In a process costing system employing historic costs and perhaps predetermined overhead rates, the output of the system is generally referred to as a production report.[18] Such production reports would contain the aggregated results of the transactions which influenced each process of the system. These production reports are generally broken down into three sub-sections. The ordering of these sub-sections would appear to be a matter of preference.

One of the sub-sections usually involves the flow of the physical units. The units in any inventory at the beginning of the reporting period plus the units transferred into the process are reconciled with the units that were transferred out plus any in the ending inventory. For the sake of convenience, units between processes are often included in the inventory of the transferring process. Another of the sub-sections of a production report contains the computations of the equivalent productions and the unit costs. These unit costs serve as the basis for the section of the production report where the costs are reconciled. In this section, the costs attached to the beginning inventory plus the costs associated with the resources that were employed or transferred from the process plus the ending inventory. The total unit costs are often determined in this section.

The costs associated with the inventories and output of the system would be employed in financial accounting for reporting purposes. The production reports could serve as the basis for what would appear to be a very crude _ex-post_ analysis of the operations of the production system. Comparisons could be made between the production reports of previous reporting periods. Because of the computational complexities and other temporal factors encountered in determining historic costs, in most cases there would exist a considerable time lag between the termination of a reporting period and the preparation of production reports.

A representative production report employing average costing is contained in Exhibit 3-1. A similar production report is contained in Exhibit 3-2 except that the FIFO costing method was employed. These reports are representative of the information that would be obtained from a process costing system employing historic costs.

Several factors limit the application of a process costing employing historic costs to managerial and operational control activities. Since the expected relationships between the processes are not specified, in multiproduct systems it would be difficult to estimate the costs of alternative outputs. The lack of normative or standard costs would also inhibit the preparation of budgets. In the area of operational control, the use of the cost information would be restricted because of the limited predictive ability of the system. Therefore, process costing employing historic costs can be described as a set of generally accepted procedures of cost allocations and transformations.

To overcome these limitations of a system employing historic costs, standard costs are often substituted. Standard costs are predetermined and are expressed in

EXHIBIT 3-1

Sample Production Report - Average Costing

COST OF PRODUCTION REPORT
FOR THE MONTH OF FEBRUARY, 19--

QUANTITY SCHEDULE:		
Units in process at beginning	3,000	
Units received from preceding department this period	38,000	41,000
Units transferred to next department	36,000	
Units still in process (½ labor and overhead)	4,000	
Units lost in process	1,000	41,000
COST CHARGED TO THE DEPARTMENT:	*Total Cost*	*Unit Cost*
Cost from preceding department		
Work in process — opening inventory (3,000)	$ 5,400	$1.80
Transferred in during this period (38,000)	65,360	1.72
Total (41,000)	$ 70,760	$1.726
Cost added by department		
Work in process — opening inventory		
Labor	$ 910	
Factory overhead	800	
Cost added during period		
Labor	34,050	$ .92
Factory overhead	30,018	.811
Total cost added	$ 65,778	$ 1.731
Adjustment for lost units		.043
Total cost to be accounted for	$136,538	$ 3.500
COST ACCOUNTED FOR AS FOLLOWS:		
Transferred to next department (36,000 × $3.500)		$126,000
Work in process — closing inventory		
Adjusted cost from preceding department [4,000 × ($1.726 + $.043)]	$ 7,076	
Labor (4,000 × ½ × $.92)	1,840	
Factory overhead (4,000 × ½ × $.811)	1,622	10,538
Total cost accounted for		$136,538

ADDITIONAL COMPUTATIONS:

Unit cost from preceding department $= \frac{\$70,760}{41,000} = \$1.726$

Equivalent production

Labor and Overhead $= 36,000 + \frac{4,000}{2} = 38,000$ units

Unit costs

Labor $= \$910 + \$34,050 = \frac{\$34,960}{38,000} = \$.92$

Factory overhead $= \$800 + \$30,018 = \frac{\$30,818}{38,000} = \$.811$

Adjustment for lost units

$\frac{\$70,760}{40,000} = \$1.769$ .... $\$1.769 - \$1.726 = \$.043$

Reproduced from Matz, Curry, and Frank (1967) p. 414 with permission of editor.

EXHIBIT 3-2

Sample Production Report - FIFO Costing

COST OF PRODUCTION REPORT
FOR THE MONTH OF FEBRUARY, 19--

QUANTITY SCHEDULE:			
Units in process at beginning ($\frac{1}{3}$ labor and overhead)		3,000	
Units received from preceding department		38,000	41,000
Units transferred to next department		36,000	
Units still in process ($\frac{1}{2}$ labor and overhead)		4,000	
Units lost in process		1,000	41,000
		Total Cost	*Unit Cost*
COST CHARGED TO THE DEPARTMENT:			
Work in process — opening inventory		$ 7,110	
Cost from preceding department			
Transferred in during the month (38,000)		$ 65,355	$ 1.72
Cost added by department			
Labor		$ 34,050	$ .920
Factory overhead		30,018	.811
Total cost added		$ 64,068	$ 1.731
Adjustment for lost units			.046
Total cost to be accounted for		$136,533	$ 3.497
COST ACCOUNTED FOR AS FOLLOWS:			
Transferred to next department			
From beginning inventory			
Inventory value	$7,110		
Labor added (3,000 × $\frac{2}{3}$ × $.920)	1,840		
Factory overhead added (3,000 × $\frac{2}{3}$ × $.811)	1,622	$ 10,572	
From current production			
Units started and finished (33,000 × $3.497)*		115,435	$126,007
Work in process — closing inventory			
Adjusted cost from preceding department (4,000 × $1.766)		$ 7,064	
Labor (4,000 × $\frac{1}{2}$ × $.920)		1,840	
Factory overhead (4,000 × $\frac{1}{2}$ × $.811)		1,622	10,526
Total cost accounted for			$136,533

ADDITIONAL COMPUTATIONS:

Equivalent production: Labor and Overhead = 2,000 + 33,000 + 2,000 = 37,000 units — or — 36,000 − 1,000 + 2,000 = 37,000 units

Unit costs: Labor = $\frac{\$34{,}050}{37{,}000} = \$.920$ Factory Overhead = $\frac{\$30{,}018}{37{,}000} = \$.811$

Adjustment for lost units

$$\frac{\$65{,}355}{38{,}000 - 1{,}000} = \$1.766 \ldots\ldots \$1.766 - \$1.72 = \$.046$$

*33,000 units × $3.497 = $115,401. Yet, to avoid a decimal discrepancy, the cost transferred from current production is computed as follows: $136,533 − ($10,572 + $10,526) = $115,435.

Reproduced from Matz, Curry, and Frank (1967) p. 422 with permission of editor.

terms of unit inputs.[19] When standard costs are employed the costs of outputs and inventories are based on the sum of the standard allowances for the factors of production without reference to the actual costs incurred.[20]

Although the identification of standard output costs would require the specification of the relationships between the processes, there is no mention in the literature of a general analytic model or set of procedures for specifying these relationships.[21] It would appear that "ad hoc" methods are employed in resolving the standard unit transfer and output costs. Production reports similar to those in Exhibits 3-4 and 3-2 would result from a standard costing system. The unit cost computations would not be included since standard costs are employed.

In addition to the production reports, various differences between the actual and standard results can be determined.[22] These differences relate to either the direct resource inputs or the overhead costs and serve as a basis for evaluating the performance of the processes within the system. These differences also serve a criteria for determining whether the consumption of a resource should be subject to an investigation or review. However, there are no decision rules incorporated in a standard costing system for differentiating between significant and non-significant cost differences.

Two differences are determined in connection with direct resource inputs. At the conclusion of a production period two cost and two quantity values are available with respect to each direct resource input. The observed and standard are the two unit costs and the observed and standard are the two quantity values available. The price difference for resource j would equal

$$\text{Price Difference } j = (c_j - c_j^o)\ell_j^o \qquad (3\text{-}7)$$

where c_j would equal the standard cost of resource j, c_j^o would equal the observed unit cost of resource j, and ℓ_j^o would equal the observed quantity of resource j consumed.

A negative sign would indicate an unfavorable price difference. These differences are employed in evaluating activities concerned with the acquisition of resources. They would also indicate an organization's capability in predicting the cost of resources for a production period. This is important because in most cases the selection of a production plan is at least partially based on the anticipated costs.

The quantity difference for resource j would equal

$$\text{Quantity Difference } j = (\ell_j - \ell_j^o)c_j \qquad (3\text{-}8)$$

where ℓ_j equals the standard resource input for the output obtained from the system. Such differences are used in evaluating the efficiency with which direct resources are employed. It is the standard practice to assign any difference caused by shifts in both the costs and quantities to the price difference. This procedure will be followed throughout this study.

In a standard cost system oberhead absorption is based upon the standard input quantity specified for the realized output. There are several procedures for analyzing the total overhead cost difference between the actual and the overhead cost attached to production. Such procedures are adequately described elsewhere and only selected results will be considered here.[23]

In the two-difference method, any overhead cost difference is segregated into a controllable and a volume difference. The controllable difference indicates any deviation between the actual overhead cost and the budgeted overhead cost based on the standard resource input. The volume difference indicates any deviation between the capacity estimate employed in determining the fixed overhead rate and the standard capacity employed during the production period. The simplicity of the two-difference methods is generally regarded as its main advantage.[24]

Three and four-difference methods are also used for the purpose of identifying the causal reasons for an overhead cost deviation. In the three and four-difference methods, the spending difference equals any deviation between the actual overhead costs and the standard overhead costs based on the actual resource inputs. In the three-difference method, the efficiency difference indicates any deviation caused by differences between the actual and the standard resource utilization rate. This difference only takes into account fixed overhead costs. In the four-difference method, a variable overhead cost efficiency difference is determined. In both methods, the idle capacity difference includes both variable and fixed overhead costs and is caused by differences between the actual resource input and the estimated resource input level used to predetermine the fixed overhead rate. The case most generally made for the three and four-difference methods is that they are more sophisticated and emphasize the causal reasons behind any difference.

The direct resource and overhead differences in addition to the production report represent the output that can be realized from a process costing system employing standard costs. The standard costs, production reports, and cost differences would serve as the basis for the managerial and operational control activities of the management of a production system where this type of managerial accounting system was employed. The objective of the following chapters in this study is to identify procedures that could increase the capability of process costing systems to produce information that will better serve the needs of management.

Summary

In this chapter, the cost terminology employed in process costing was introduced. Then the assumptions of process costing were detailed. The final section included a description of the output of process costing systems when either historic or standard costs are employed. This descriptive model will make possible a comparison and evaluation of the process costing models to be developed in this study and the existing technology. The remaining chapters in this study will concern the development and evaluation of alternative process costing models.

Two Difference Method

Actual Overhead

Budget Based On Standard Inputs

Overhead Cost Attached To Production Based on Standard Inputs

Controllable or Spending Difference

Volume Difference

Three Difference Method

Actual Overhead

Budget Based On Actual Inputs

Budget Based On Standard Inputs

Overhead Attached To Production Based On Standard Inputs

Budget Difference

Capacity Difference

Efficiency Difference

Standard Cost Overhead Differences

Figure 3-6

Footnotes

[1]Process costing is detailed in all of the following texts.

Adolph Matz, Othel J. Curry and George W. Frank, Cost Accounting (Cincinnati: Smith Western Publishing Co., 1967)

Gordon Shillinglaw, Cost Accounting Analysis and Control (Homewood, Illinois: Richard D. Irwin, Inc., Revised Edition, 1967)

David H. Li, Cost Accounting For Management Applications (Columbus, Ohio: Charles E. Merrill Books, Inc., 1966)

Charles T. Horngren, Cost Accounting (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1967)

Morton Backer and Lyle E. Jacobsen, Cost Accounting: A Managerial Approach (New York: McGraw Hill Book Co., 1964).

[2]Charles T. Horngren, Cost Accounting (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1967), p. 17.

[3]Paul Grady, Inventory of Generally Accepted Accounting Principles for Business Enterprises (New York: American Institute of Certified Public Accountants, Inc., 1965), p. 433.

[4]Charles T. Horngren, Cost Accounting, (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1967), p. 20.

[5]Gordon Shillinglaw, Cost Accounting Analysis and Control (Revised Edition; Homewood, Illinois: Richard D. Irwin, Inc., 1967), p. 58.

[6]David H. Li, Cost Accounting For Management Applications, p. 9,10.

[7]Standard costs are accepted for inventory valuation in financial accounting. Refer to: Accounting Research and Terminology Bulletin (New York: American Institute of Certified Public Accountants, 1961), p. 30 and accompanying footnote.

[8]Adolph Matz, Othel J. Curry, and George W. Frank, Cost Accounting (Cincinnati: Smith-Western Publishing Co., 1967), p. 124.

[9]Horngren, Cost Accounting, p. 236.

[10]David H. Li, Cost Accounting for Management Applications (Columbus, Ohio: Charles E. Merrill Books, Inc., 1966), p. 202.

[11]Morton Backer and Lyle E. Jacobsen, Cost Accounting: A Managerial Approach (New York : McGraw-Hill Book Company, 1964), p. 371.

[12]David Solomons, Studies In Cost Analysis (Homewood, Illinois: Richard D. Irwin, Inc., 1968), p. 37.

[13]James G. March and Herbert A. Simon, Organizations (New York: John Wiley & Sons, Inc., 1966), p. 13.

[14]Eric L. Kohler, A Dictionary For Accountants (Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1965), p. 389.

[15]Edwin H. Caplan, "Behavioral Assumptions of Management Accounting," The Accounting Review, Vol. XLI, No. 3 (July, 1966), pp. 446-509.

[16]Li, Cost Accounting, p. 173.

[17]A. Wayne Corcoran, Mathematical Applications in Accounting (New York: Harcourt, Brace & World, 1968), p. 165.

[18]In Exhibits 3-1 and 3-2 are examples of production reports.

[19]Horngren, Cost Accounting, p. 854.

[20]Ibid., p. 854.

[21] In Chapter IV of the study it will be shown that the input-output model could serve as the general analytic model of a process costing system employing standard cost. All technical problems except joint products could be handled in this model.

[22] In the accounting literature the term variance is employed rather than difference. Because statistical variances will be discussed in this study, the term difference has been substituted.

[23] See Shillinglaw, <u>Cost Accounting Analysis and Control</u>, Chapter 15.
Matz, Curry and Frank, <u>Cost Accounting</u>, Chapter 19.

[24] Matz, Curry and Frank, <u>Cost Accounting</u>, p. 611

CHAPTER IV

AN INPUT-OUTPUT PROCESS COSTING MODEL

Indtoruction

The input-output model, which was developed by Wasily W. Leontief,[1] is employed in economics to analyze the interrelationships among industries for purposes of identifying the conditions that equilibrate the supply side with the demand side of an economy.[2] This model has been suggested by several authors in accounting as a possible means for describing the interrelationships among the processes of a mass production system.[3] To this point, no one has elaborated upon the model such that the costing and control of in-process inventories could be accomplished. There has been little consideration given to the control implications of an input-output process costing model.

Before delving into any further elaboration of an input-output process costing model, the general characteristics and assumptions of the Leontief input-output model will be considered. Then aspects of the input-output model will be applied to the process costing problem. Standard costs will be assumed and a method for costing in-process inventories from an input-output process costing model will be presented. A means will be developed for estimating the expected activity levels and resource inputs when in-process inventories are considered.

In the final sections of the chapter, the control implications of an input-output process costing model will be considered. It will be shown how the traditional cost differences could be obtained from an input-output model. Additional differences caused by shifts in the inter-process transfers and resource inputs will be developed. The process costing system based on the input-output model will then be analyzed and contrasted with the other systems discussed in this study in the concluding chapter.

The Input-Output Model

An input-output model may be classified as either static or dynamic. The relationships specified in a static model connect variables that are identified with the same time period. In a dynamic model, the relationships are specified between variables identified with different time periods. At this writing, all of the known applications of input-output analysis to process costing have employed static models. A static or dynamic model can either be open or closed. In an open input-output model, the final demand is determined exogenously while a closed model is self-contained and no variables are determined outside of the system.

Leontief, in his economic analysis, usually employed a static closed model that could be expressed as:

$$x = Bx \tag{4-1}$$

where x is a vector of total output and B is a matrix of technical coefficients. To solve such a system of linear equations when the B matrix is irreducible, the matrix B and one element of x must be known. The solution would be obtained as follows:

$$x = Bx$$

$$x - Bx = \emptyset$$

$$(I-B)x = \emptyset \ .$$

It is necessary that the determinant of the (I-B) matrix equal zero in order that solutions other than a trivial one exist. Most of the economic applications of input-output analysis employ input coefficients that are based on dollar flows. It is then necessary to express all of the input-output data of the system in terms of dollars.

Accounting applications of input-output analysis have extensively employed open static models of the form:

$$x = Bx + y \tag{4-2}$$

where x is a vector of total outputs, B is a matrix of technical coefficients and y is a vector of final demands. In a process costing application, the y vector would contain the expected output from the production system for external consumption. The x vector would contain the gross output necessary to satisfy the internal consumption in the processes necessary to produce the output specified in the y vector.

The final demand vector y is predetermined exogenously. The necessary gross output levels x required to meet the demand can be determined algebraically as follows:

$$x = Bx + y$$

$$x - Bx = y$$

$$(I-B)x = y$$

$$(I-B)^{-1}(I-B)x = (I-B)^{-1}y$$

$$x = (I-B)^{-1}y \ .$$

Most of the suggested accounting applications of input-output models have employed physical output coefficients.[4] The advantage of physical output coefficients is that unit homegeniety is not necessary within the model and outputs can be expressed in different units. This situation is common to many multi-product systems and should prove to be a major advantage of the input-output model when attempting to model a mass production system for costing purposes.

Farag (1968) assumed perfect competition so that any consideration of profits resulting from the transfers between processes could be eliminated. He further assumed that the prices of the inputs and outputs were known. These assumptions were necessary so that he could employ dollar units when determining the technical coefficients. Farag also assumed that the output of each process must be employed in either satisfying the input demands of other processes or in meeting finished goods inventory and sales requirements. Feltham (1970) employing physical output coefficients has relaxed this assumption and has demonstrated how by-products could be handled.

Ijiri (1968), Gambling (1968), Livingstone (1969), and Feltham (1970) employed physical terms in determining the technical coefficients. This eliminates the necessity of making assumptions concerning the market conditions among the processes. The model developed in this chapter will employ physical output coefficients and will be to some degree, based on the work of Ijiri (1968), Gambling (1968), and Feltham (1970).

Model Assumptions

The assumptions underlying the input-output model when applied to process costing will be presented in a summary form similar to that employed when the process cost models were developed in Chapter III. No mention will be made of the behavioral assumptions since they apply for all of the models developed in the study.

1. Linear Cost Functions: The cost of the external inputs can be represented as a linear function of the quantities employed. Prime

costs would take the general form $y=a(x)$ and overhead costs would take the general form $y=a(x)+b$. The overhead costs are divisible into a fixed component that does not vary with volume and a variable component that varies with volume.

2. System Technology: The production system consists of n processes and each process has a production function that is deterministic, linear, and proportional. Production in one process is assumed to be independent of the activity in the other processes. This assumption eliminates the consideration in the model of the scheduling of the flows of outputs between the processes. The capacities of the processes are assumed to be balanced and it is assumed that management would not schedule production that would exceed the capacity of the system.
3. System Inputs and Outputs: The inputs and outputs of the production system are not restricted to integer amounts. Each process produces at least one measurable product for either internal or external consumption.
4. Endogenous and Exogenous Sectors: The pro-

cesses in the production system are treated as endogenous sectors in the analysis and all relationships among these sectors are assumed to be fixed during the analysis. Net production (the y vector) is treated as an exogenous sector and can be manipulated during the analysis.

5. Matrix of Technical Coefficients: The technical coefficients of the main diagonal of the A matrix must be less than one. If this were not the case, it would be necessary to input one or more units of the same product to get a similar unit of output.[5] All of the principle minors of the (I-A) matrix must be positive if the model is to be employed.[6]

The Input-Output Process Costing Model

A generalized input-output process costing model for an n process mass production system will be developed in this section. The model will be static, open, and physical output coefficients will be assumed. Initially in-process inventories will not be considered. Later sections will consider the costing and control of in-process inventories. A separate section will detail how by-products could be handled with the input-output model. In each section, the initial presentation will be concerned with the modeling of the physical flow of the units among the production processes. Then the discussion will be concerned with the introduction of product cost data for purposes of determining unit costs and the costing of inventories.

1. No In-Process Inventories[7]

Let q_{ij} represent the estimate of the number of units of product i produced by one unit of activity in process j. A unit of activity in process j is defined as that activity that is necessary to produce one unit of output of process j. Initially each process will be limited to one product; therefore, the number of processes n would also equal the number of products. By-products will be considered in a separate section. The output coefficients could be expressed as the matrix:

$$Q = \begin{bmatrix} q_{11} & \cdots & q_{1n} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ q_{n1} & \cdots & q_{nn} \end{bmatrix}$$

with dimensions n x n and where all $q_{ij} \geq 0$. Assuming process i produces product i, the Q matrix would be an identity matrix I.

Let r_{ij} represent the estimate of the number of units of product i consumed in each unit of activity in process j. These input coefficients could be expressed as the matrix:

$$R = \begin{bmatrix} r_{11} & \cdot & \cdot & r_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{n1} & \cdot & \cdot & r_{nn} \end{bmatrix}$$

with dimensions n x n and where all $r_{ij} \geq 0$. All of the elements on the main diagonal of the R matrix must be less than one because of the assumed self consumption limitations.[8]

The input and output coefficients would then be combined into a net input-output or physical technical coefficient matrix A by subtracting the output coefficients (the Q matrix) from the input coefficients (the R matrix). In matrix form the relationship would equal:

$$A = Q - R. \qquad (4\text{-}3)$$

The elements on the main diagonal of the A matrix would be less than or equal to one. These positive elements in the A matrix represent the net number of units of product i produced by one unit of activity in process i. The off diagonal elements in the A matrix would be less than or equal zero and would represent the number of unit of product i estimated to be consumed in the production of one unit of product j.

The input matrix R could be substituted for the technical coefficient matrix B in the general open static input-output model.[9] After substituting, the model would equal:

$$x = Rx + y. \qquad (4\text{-}4)$$

When solving for the gross activity levels x, the expression (I-R) could also be expressed as (Q-R) since Q = I.

When this substitution is made, the model would take the form:

$$x = (Q-R)^{-1} y \tag{4-5}$$

which could be expressed as:

$$x = A^{-1}y \; . \tag{4-6}$$

The elements of the matrix A^{-1} can be interpreted in terms of the gross activity or production in the system. Each element represents the gross production necessary in process i for the system to output one unit of product j. Therefore, when a vector of final or exogenous demand, y, is specified, it is possible to determine the estimated vector of gross outputs, x, using formula (4-6).

When all of the flows in the production system are in one direction, the analysis may not be very complex and it may not prove necessary to determine the inverse of the A matrix. This would be the case if all entries in the R matrix were on or to the right of the main diagonal. However, in most complex mass production systems the "feedback" of the output of a process into proceeding processes would often be found and the size of the system would dictate that the inverse matrix be found so a solution can be determined. When the feedback condition exists, the application of input-output analysis should prove to be especially helpful.

The processes in most production systems would require external inputs besides the inputs received internally from other processes within the system. Let e_{ij} represent the estimated number of units of external input i required by one unit of activity in process j. These input coefficients could be expressed as the matrix:

$$E = \begin{bmatrix} e_{11} & \cdot & \cdot & e_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ e_{z1} & \cdot & \cdot & zn \end{bmatrix}$$

where the E matrix would assume dimensions z by n and $e_{ij} \geq 0$. The first (z-1) elements of each column of the matrix would contain the estimates of the expected resource requirements for a unit of activity in the respective process.

The last (or zth) element of each column of the E matrix would be employed for the purpose of allocating overhead costs to production. Various configurations of the E matrix and the cost vector c would allow for variations in the overhead rates employed in each process.[10] Direct or full-absorption systems could be accommodated.

Let h_{ij} equal the gross amount of external input i required to produce one unit of the output of process j. Then h_{ij} would equal:[11]

$$h_{ij} = \sum_{k=1}^{n} e_{ik} a_{kj}^{-1} . \qquad (4\text{-}7)$$

The general matrix representation of this relationship would be:

$$H = E\,A^{-1} . \qquad (4\text{-}8)$$

If ℓ_i equals the total external units of input required by the system where i=1 to z, then:

$$\ell_i = \sum_{j=1}^{n} e_{ij} x_j \quad . \qquad (4\text{-}9)$$

This relationship in matrix form would be:[12]

$$\ell = Ex \quad . \qquad (4\text{-}10)$$

$$\ell = Hy \quad . \qquad (4\text{-}11)$$

The manipulation of data would also be facilitated employing various aspects of the input-output model are employed. Let c represent a row vector with z elements. The first (z-1) elements would contain the standard resource unit costs. The last (or zth) element would contain the overhead rate being employed. The total costs that would be incurred or allocated to a process j during the production period would equal:

$$\tau_j = \sum_{i=1}^{n} c_i e_{ij} x_j \text{ for } j=1 \text{ to } n \qquad (4\text{-}12)$$

where would be an n element row vector. The unit output cost of product j would equal the unit output cost of process i where j=i. The unit output cost of product j would equal:

$$u_j = \sum_{i=1}^{z} c_i h_{ij} \qquad (4\text{-}13)$$

The generalized expression of the relationship would be:

$$u = c\ H \qquad (4\text{-}14)$$

where **u** is an n element row vector.

The cost of a unit of activity in process j would be determined by the formula

$$s_j = \sum_{k=1}^{z} c_k e_{kj} \quad . \qquad (4\text{-}15)$$

The matrix expression of this relationship is

$$s = c\ E \ . \qquad (4\text{-}16)$$

where the row vector s would contain n elements. The relationship between the direct unit output cost and the direct unit activity cost can be expressed as

$$u = s\ A^{-1} \ . \qquad (4\text{-}17)$$

The standard cost of the output y would equal

$$\text{Standard Cost of Output} = u\ y \ . \qquad (4\text{-}18)$$

The standard cost of production would equal

$$\text{Standard Cost of Production} = c\ \ell. \qquad (4\text{-}19)$$

The expected number of transfers between the processes and the output states could be determined from the input-output model. Let Θ be a partitioned matrix with dimensions (n by n+1). The Θ matrix would assume the general form:

$$\Theta = \left[\begin{array}{ccccc|c} w_{11} & \cdot & \cdot & \cdot & w_{1n} & y_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ w_{n1} & \cdot & \cdot & \cdot & w_{nn} & y_n \end{array}\right]$$

The elements in the W sub-matrix would be determined by the following formula:

$$w_{ij} = r_{ij}x_i \quad \text{for} \quad \begin{matrix} i=1 \text{ to } n \\ j=1 \text{ to } n \end{matrix} \qquad (4\text{-}20)$$

An element θ_{ij} would equal the expected number of units that would be transferred from process i to process j. If j was equal to n+1, then the θ_{ij} element would equal the net output from process i or y_i.[13]

The following relationship would hold for each row of the Θ matrix where:

$$x_j = \sum_{i=1}^{n+1} \theta_{ji} = y_j + \sum_{i=1}^{n} w_{ji} \qquad (4\text{-}21)$$

The interpretation of this relationship would be that the total inter-process transfers plus the net output from process j would equal the gross expected activity level in the process. The Θ matrix can also be interpreted as a mapping of the locations of the consumption during the production period.

The cost of the inter-process transfers during the production can be determined from the W matrix. The cost of the transfers from process i to process j would equal:

$$\delta_{ij} = w_{ij}u_j \quad \text{for} \quad \begin{matrix} i=1 \text{ to } n \\ j=1 \text{ to } n \end{matrix} \qquad (4\text{-}22)$$

where δ is an (n by x) matrix. An element δ_{ij} can be interpreted as the cost of the output of process i transferred into process j. The elements of the transpose of the δ matrix would give output of process j transferred into process i.

By employing the information thus described, the costs of the inputs and outputs of the n processes and the total production could be reconciled as shown in Figure 4-1. The information contained in the reconciliation would be the input required by the financial accounting system of a commercial organization.

This concludes the presentation of an input-output process costing model for a system with no in-process inventories. The next section will elaborate upon the model developed to this point in that in-process inventories will be considered. In addition to determining expected costs, activity levels, and resource requirements, the next section will consider the costing of in-process inventories.

2. Example of a System Without Inventories

The assumptions which serve as a basis for this example are contained in Exhibit 4-1. From formulas 4-3 and 4-6, the inverse of the exchange matrix and the gross output vector are

$$A^{-1} = \begin{bmatrix} 1.6 & 1.1 \\ .8 & 1.8 \end{bmatrix} ; \quad x = \begin{bmatrix} 6 \\ 8 \end{bmatrix} .$$

By formula 4-8, the gross external input matrix would be

	Process 1	Process 2		Process n	Total
Inputs					
Beginning Inventory	g_1^t	g_2^t	. .	g_n^t	$\sum_{i=1}^{n} g_i^t$
Exogenous Inputs	τ_1	τ_2	. .	τ_n	$\sum_{i=1}^{n} \tau_i$
Inter-Process Transfers	$\delta_{1,1}$	$\delta_{1,2}$	. .	$\delta_{1,n}$	$\sum_{i=1}^{n} \delta_{1i}$
	$\delta_{2,1}$	$\delta_{2,2}$	. .	$\delta_{2,n}$	$\sum_{i=1}^{n} \delta_{2i}$
	:	:	: :	:	
	$\delta_{n,1}$	$\delta_{n,2}$	. .	$\delta_{n,n}$	$\sum_{i=1}^{n} \delta_{n,i}$
Total Inputs	$	$		$	$

Outputs					
Inter-Process Transfers	$\delta_{1,1}$	$\delta_{1,2}$	. .	$\delta_{1,n}^{tr}$	$\sum_{i=1}^{n} \delta_{n,i}$
	$\delta_{2,1}$	$\delta_{2,2}$	. .	$\delta_{2,n}^{tr}$	$\sum_{i=1}^{n} \delta_{2,i}^{tr}$
	:	:	: :	:	
	$\delta_{n,1}$	$\delta_{n,2}$	. .	$\delta_{n,n}^{tr}$	$\sum_{i=1}^{n} \delta_{n,6}^{tr}$
Exogenous Outputs	$u_1 y_1$	$u_2 y_2$	. .	$u_n y_n$	uy^*_{t+1}
Ending Inventory	g_1^{t+1}	g_2^{t+1}	. .	g_n^{t+1}	$\sum_{i=1}^{n} {}_{i}^{t+1}$
	$	$		$	$

Figure 4-1

Input-Output Cost Reconciliation

*In a system with in-process inventories, the exogenous output would equal uv.

EXHIBIT 4-1

Assumptions For Input-Output Costing Example

number of processes	$n = 2$
number of direct resource inputs	$z = 4$
output coefficient matrix	$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
input coefficient matrix	$R = \begin{bmatrix} .1 & .55 \\ .4 & .2 \end{bmatrix}$
exogenous input matrix	$E = \begin{bmatrix} 2 & 0 \\ 3 & 3 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$
resource unit cost vector	$c = [\$1.00, \$2.00, \$3.00, \$1.50]$
net output vector	$y = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$$H = \begin{bmatrix} 3.2 & 2.2 \\ 7.2 & 8.7 \\ 3.2 & 4.7 \\ 2.4 & 2.9 \end{bmatrix}$$

and by 4-10 or 4-11, the total required inputs are

$$i = \begin{bmatrix} 12 \\ 42 \\ 22 \\ 14 \end{bmatrix} \quad .$$

The total costs that would be incurred or allocated to each of the processes would equal by 4-12

$$\tau_1 = \$75.00 \qquad \tau_2 = \$108.00 \quad .$$

The standard unit output cost by 4-14 would equal

$$u = [\$30.80, \$38.05]$$

and by 4-16 the standard unit activity costs are

$$s = [\$12.50, \$13.50] \quad .$$

In this case, the standard cost of output would equal the standard cost of production since there are no in-process inventories. By 4-18 and 4-19 they both equal $183.00.

The expected unit transfer matrix θ equals

$$\theta = \left[\begin{array}{cc|c} .6 & 4.4 & 1 \\ 2.4 & 1.6 & 4 \end{array}\right]$$

by 4-20. And in conclusion the cost of the interprocess transfers by formula 4-22 would equal

$$\hat{\delta} = \begin{bmatrix} \$18.48 & \$135.52 \\ \$91.32 & \$\ 60.88 \end{bmatrix} .$$

In Exhibit 4-2 the cost reconciliation of this example is presented.

3. In-Process Inventories

The allocation of costs among finished goods and the in-process inventories is one of the primary reasons for employing the traditional standard process costing model.[14] Therefore, the costing of inventories will be dealt with initially. Then attention will be directed towards determining the expected resource requirements, costs, and unit transfers.

Let π be a diagonalized matrix of dimension n where each element π_{ii} contains the number of units in the in-process inventory of process i. Let α be a z by n matrix where each element α_{ij} would contain a completion estimate with respect to resource i of the in-process of process j. Assume the n element column vector v contains the number of units to be produced for outside consumption as determined by management. Finally, assume the management specified that the in-process inventories at time t+1 should equal π^{t+1} with a related percentage of completion

EXHIBIT 4-2

Input-Output Cost Reconciliation

Inputs	Process 1	Process 2	Total
Beginning Inventory	$ 0.00	$ 0.00	$ 0.00
Cost of Production	75.00	108.00	183.00
Inter-Process Transfer	18.48	135.52	154.00
Inter-Process Transfer	91.32	60.88	152.20
Total Inputs	$184.80	$304.40	$489.20

Outputs			
Inter-Process Transfer	$ 18.48	$ 91.32	$109.80
Inter-Process Transfer	135.52	60.88	196.40
Output	30.80	152.20	183.00
Ending Inventory	0.00	0.00	0.00
Total Outputs	$184.80	$304.40	$489.20

of matrix α^{t+1}.[16]

The costing of any in-process inventory will be more straight-forward if standard transfer costs are determined initially. The standard transfer cost i would equal the standard cost of all of the resources required preceeding a unit of activity in process i. The standard transfer costs would equal:[17]

$$d = s(A^{-1} - I) \qquad (4\text{-}23)$$

where the transfer cost row vector d would contain n elements.

Then, employing standard costs, the in-process inventory of process j would equal:

$$g_j = \pi_{jj}((\sum_{i=1}^{z} \alpha_{ij}c_i e_{ij}) + d_j) \quad \text{for } j=1 \text{ to } n \qquad (4\text{-}24)$$

The in-process inventories of the n processes of the system would be costed at:[18]

$$\text{Total Cost of Inventory} = d\pi a+(c(\alpha\pi E_{tr})_{dg}b) \qquad (4\text{-}25)$$

where a and b are column sum vectors. The expected gross activity levels x for a production period would equal:[19]

$$x = A^{-1} + (A\bar{v}^{1}(\pi^{t+1}-\pi^{t})-\pi^{t+1})a \quad . \qquad (4\text{-}26)$$

The expected external resource input vector ℓ would equal:[20]

$$\ell = Ex + ((\alpha^{t+1}\pi^{t+1}+(N-\alpha^{t})\pi^{t})E_{tr})_{dg}a \qquad (4\text{-}27)$$

where the N matrix contains all ones. The total cost that would be incurred or allocated to a process j would equal

$$\tau_j = \sum_{i=1}^{z} c_i e_{ij} (x_j + \pi_{jj}^{t} (1-\alpha_{ij}^{t} + \pi_{jj}^{t+1} \alpha_{ij}^{t+1})$$

for j=1 to n (4-28)

The standard cost of the output would equal:

Standard Cost of Output = uv. (4-29)

The determination of the expected unit transfers between the processes and the output states can also be determined when in-process inventories exist. However, the equality represented in formula (4-21) will ordinarily not hold because of changes between π^t, π^{t+1} and α^t and α^{t+1}. The last column of the partitioned Θ matrix would contain the output vector v. The elements of the submatrix W would be determined by the formula:[21]

$$w_{ij} = r_{ij} (\sum_{k=1}^{n} a_{jk}^{-1} (v_k + \pi_{kk}^{t+1} - \pi_{kk}^{t})) \quad \text{for } \begin{matrix} i=1 \text{ to } n \\ j=1 \text{ to } n \end{matrix} \quad (4-30)$$

4. Example of a System With Inventories

In this section, examples will be developed with planned increases and decreases in the levels of the in-process inventories. The assumptions, in addition to those in Exhibit 4-1 are summarized in Exhibit 4-3. In both examples, the output vector v equals $[\begin{smallmatrix}1\\4\end{smallmatrix}]$ and the standard transfer costs by formula 4-23 are

d = [\$18.30, \$24.55] .

When the levels of the in-process inventories are increased, the inventories in each process would be costed by formula 4-24 at

$$g_1^t = \$21.55 \qquad g_2^t = \$\ 0.00$$

$$g_1^{t+1} = \$43.10 \qquad g_2^{t+1} = \$102.90\ .$$

Employing formula 4-25, the total inventory at time t would be costed at $21.55 and at t+1 the inventory would be costed at $146.00. The gross activity levels by 4-26 and the expected resource requirements by 4-27 are

$$x = \begin{bmatrix} 8.9 \\ 11.2 \end{bmatrix} \qquad \ell = \begin{bmatrix} 20.8 \\ 67.8 \\ 38.8 \\ 23.1 \end{bmatrix} .$$

The standard cost that would be incurred or allocated to each process by 4-28 would equal

$$\tau_1 = \$127.00, \qquad \tau_2 = \$180.45.$$

The standard cost of the output would equal $183.00 by 4-29 and the standard cost of production would equal $307.45 by formula 4-19. The expected unit transfer matrix Θ equals

$$\Theta = \left[\begin{array}{cc:c} 1.09 & 7.81 & 1 \\ 4.36 & 2.84 & 4 \end{array}\right]$$

by 4-30. The cost of the inter-process transfers by 4-22 are

EXHIBIT 4-3

Inventory Assumptions For Input-Output Costing Example

$$\pi^t = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \pi^{t+1} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\alpha^t = \begin{bmatrix} .5 & 0 \\ 0 & 0 \\ .5 & 0 \\ .5 & 0 \end{bmatrix} \qquad \alpha^{t+1} = \begin{bmatrix} .5 & 0 \\ 0 & .5 \\ .5 & 1 \\ .5 & .5 \end{bmatrix}$$

Increase In Inventories

$$\pi^t = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \qquad \pi^{t+1} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\alpha^t = \begin{bmatrix} .5 & 0 \\ 0 & .5 \\ .5 & 1 \\ .5 & .5 \end{bmatrix} \qquad \alpha^{t+1} = \begin{bmatrix} .5 & 0 \\ 0 & 0 \\ .5 & 0 \\ .5 & 0 \end{bmatrix}$$

Decrease In Inventories

$$\delta = \begin{bmatrix} \$\ 33.572, & \$240.548 \\ \$165.898, & \$108.062 \end{bmatrix}$$

Exhibit 4-4 contains the cost reconciliation of this example.

If the levels of the in-process inventories were <u>decreased</u>, the inventories in each process would be costed by formula 4-24 at

$$g_1^t = \$43.10 \qquad g_2^t = \$102.90$$

$$g_1^{t+1} = \$21.55 \qquad g_2^{t+1} = \$0.00$$

Using formula 4-25, the total inventory at time t would be costed at \$146.00 and at t+1 the inventory would be costed at \$21.55. The gross activity levels by 4-26 and the expected resource requirements by 4-27 are

$$x = \begin{bmatrix} .1 \\ 1.8 \end{bmatrix} ; \qquad \ell = \begin{bmatrix} 3.2 \\ 16.2 \\ 5.2 \\ 4.9 \end{bmatrix}$$

The standard cost that would be incurred or allocated to each process by 4-28 would equal

$$\tau_1 = \$23.00\ , \qquad \tau_2 = \$35.55.$$

The standard cost of the output would equal \$183.00 by 4-29 and the standard cost of production by 4-19 would equal \$58.55. The expected unit transfer matrix Θ equals

EXHIBIT 4-4

Input-Output Cost Reconciliation

Increase in Inventory Levels

Inputs	Process 1	Process 2	Total
Beginning Inventory	$ 21.55	$ 0.00	$ 21.55
Cost of Production	127.00	180.45	307.45
Inter-Process Transfer	33.572	240.548	274.12
Inter-Process Transfer	165.898	108.062	273.96
Total Inputs	$348.020	$529.060	$877.08

Outputs	Process 1	Process 2	Total
Inter-Process Transfer	$ 33.572	$165.898	$199.47
Inter-Process Transfer	240.548	108.062	348.61
Exogenous Output	30.80	152.20	183.00
Ending Inventory	43.10	102.90	146.00
Total Outputs	$348.020	$529.060	$877.08

$$\Theta = \left[\begin{array}{cc|c} .11 & .99 & 1 \\ .44 & .36 & 4 \end{array}\right]$$

by 4-30. The cost of the inter-process transfers by 4-22 are

$$\delta = \begin{bmatrix} \$\ 33.572, & \$240.548 \\ \$165.898, & \$108.062 \end{bmatrix}$$

Exhibit 4-5 contains the cost reconciliation of this example.

5. By-Products

The assumption that limits each process to a single output could greatly limit the applications of the input-output process costing model since many mass production systems output by-products. Feltham (1970) has identified a means by which by-products could be taken into account in the input-output process costing model. Feltham's method involves some special considerations of the resource input coefficient matrix E.

The E matrix would assume dimensions z by n and each element $e_{ij} \geq 0$. The production system would therefore require z-1 direct resource inputs. The zth row of the matrix was reserved for overhead purposes. Assuming the production system produces k by-products, Feltham would add k rows to the E matrix. Then, for every process that produced by-products, the corresponding element in the E

EXHIBIT 4-5

Input-Output Cost Reconciliation

Decrease in Inventory Levels

Inputs	Process 1	Process 2	Total
Beginning Inventory	$ 43.10	$102.90	$146.00
Cost of Production	23.00	35.55	58.55
Inter-Process Transfer	3.888	30.492	34.38
Inter-Process Transfer	16.742	13.698	30.44
Total Inputs	$ 86.730	$186.640	$269.37

Outputs			
Inter-Process Transfer	$ 3.888	$ 16.742	$ 20.63
Inter-Process Transfer	30.492	13.698	44.19
Exogenous Output	30.80	152.20	183.00
Ending Inventory	21.55	0.00	21.55
Total Outputs	$ 86.730	$182.640	$269.37

matrix would be assigned a negative number. The negative number would indicate the expected output of the by-product in units for a unit of activity in the respective process.

The gross external input matrix H could be determined by employing formula (4-8). The negative elements in the H matrix would indicate the by-product resulting from producing one unit of the output in the corresponding process. For example, if h_{ij} were negative, this would indicate that h_{ij} units of by-product i result from producing one unit of net output from process j. The negative elements in the ℓ vector resulting from using formula (4-11) would indicate the expected outputs of the respective by-products. In a system with in-process inventories, the equivalent output vector as determined in formula (4-6) could be used in formula (4-11) to determine the expected by-product outputs.

6. Example Incorporating By-Products

This example is based on the same assumptions as the first example except that the activity in each process results in a by-product. Assuming a resource input matrix of

$$E = \begin{bmatrix} 2 & 0 \\ 3 & 3 \\ 1 & 2 \\ 1 & 1 \\ -.5 & 0 \\ 0 & -2 \end{bmatrix}$$

then by formula 4-8 the gross input matrix would equal

$$H = \begin{bmatrix} 3.2 & 2.2 \\ 7.2 & 8.7 \\ 3.2 & 4.7 \\ 2.4 & 2.9 \\ -.8 & -.55 \\ -1.6 & -3.6 \end{bmatrix}$$

If the net output vector was $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ then the required resource vector by 4-11 would be

$$\ell = \begin{bmatrix} 12 \\ 42 \\ 22 \\ 14 \\ -3 \\ -16 \end{bmatrix}$$

Therefore, the production of the planned output will result in the expected output of 3 units of by-product one and 16 units of by-product two.

<u>Input-Output Cost Analysis Information</u>

The computational procedures for analyzing the cost information from an input-output process costing model will be described in this section. Initially the impact of changes in resource input costs upon unit output costs will be considered. Then it will be shown how the cost differences obtained from the traditional standard process costing model could also be obtained from an input-output model. The material at the conclusion of this section will show how additional information might be obtained from an input-output process costing model.

Standard direct costing will be assumed throughout this section. The modifications necessary to obtain full absorption overhead cost differences will be presented. This would make possible the determination of overhead capacity differences. These differences could not be determined from a direct costing system because fixed costs are treated as period costs.

1. Impact of Changes in Resource Input Costs

The analysis of the impact of changes in the resource input costs upon unit output costs would be relatively straight-forward when an input-output costing model is employed. For example, if the cost of resource j increased by Δc_j, then the direct unit activity and output costs would also change. The increase in the direct unit activity cost in process i would be:

$$\Delta s_i = \Delta c_j e_{ji} \ . \qquad (4\text{-}31)$$

Any change in the direct unit output cost of process k would equal:

$$\Delta u_k = \sum_{i=1}^{n} \Delta s_i a_{ik}^{-1} \ . \qquad (4\text{-}32)$$

If management should wish to determine the maximum direct unit activity costs that would result in specified direct unit output costs, the input-output model would make the computation rather straight-forward. Using formula (4-16) and solving for the direct unit activity cost, the resulting expression would be:

$$s = u\ A \qquad (4\text{-}33)$$

where A is the original technical coefficient matrix. This information should prove especially helpful when management is concerned with a make-or-buy type of decision.

The above will now be demonstrated by assuming that the cost of the first resource increases by $1.00. By formula 4-31 the standard unit activity costs of the two processes would increase by $2.00 and zero respectively. The unit outputs costs from the two processes would increase by $3.20 and $2.20 respectively as determined by formula 4-32. If the direct unit output costs were $50.00 and $40.00 then by 4-33, the unit activity costs would have to equal $29.00 and $4.50 for the system to be balanced.

2. Price and Quantity Differences[22]

The computation procedures for obtaining information for cost control purposes from an input-output process costing model would involve making comparisons between the predicted output of the cost model and the observed output of the production system. The price and quantity differences as determined in the traditional model could also be obtained from an input-output process costing model.

The total price difference for the system during a production period would equal:[23]

$$\text{Total Price Difference} = (c-c^{o})\ell^{0} . \qquad (4\text{-}34)$$

Each term $(c_k-c_k^{o})\ell_k^{0}$ would equal the total price difference of resource input k.[24] Positive numbers would be considered favorable and negative numbers would be considered unfavorable. The price difference for resource input i in process j could also be determined. Let γ^{o} be a z by n matrix where each element contains the observed resource input into the particular process. The process price difference of resource i in process j would equal:

$$\text{Process Price Difference}_{ij} = (c_i-c_{ij}^{o})\gamma_{ij}^{o} . \qquad (4\text{-}35)$$

Resource quantity differences could also be obtained from an input-output model. The total quantity difference for the system during a production period would equal:[25]

$$\text{Total Quantity Difference} = c(\ell-\ell^{o}). \qquad (4\text{-}36)$$

Each operation $c_i(\ell_i-\ell_i^{o})$ would equal the total quantity difference of resource input i. The process quantity difference of resource input i in process j would be

$$\text{Process Quantity Difference}_{ij} = c_i(\gamma_{j.}^{o}-e_{ij}X_j). \qquad (4\text{-}37)$$

Examples of the differences developed in this and the following sections will be shown at the conclusion of the chapter.

3. Full Absorption Overhead Differences

The total spending, efficiency, and capacity difference for a full absorption system using a uniform overhead rate cz are given below:[26]

Full Absorption Total Spending Difference

$$= (c_z^o - c_z)\ell_z^o + p^o - p \qquad (4\text{-}38)$$

Full Absorption Total Efficiency Difference

$$= (c_z(\ell_z^o - \ell_z)] \qquad (4\text{-}39)$$

Full Absorption Total Capacity Difference

$$= (c_z\ell_z + p) - \ell_z^o(c_z + (p/\ell_3)) \qquad (4\text{-}40)$$

4. Transfer Differences

The cost differences obtained in the two previous sections could also have been obtained from a traditional standard process costing system. There exists another set of differences that could be obtained from an input-output process costing system. These differences could not be obtained directly from the traditional model.[27] They are based on the R matrix which specified the expected consumption during production of the various processes. This matrix is the basis upon which the expected unit transfers are determined. (refer to formula 4-20) The information contained in the R matrix is generally not obtainable from a traditional standard process costing system.

At the conclusion of a production period, two R matrices would be available. The first R matrix would contain the standard values used to predict the activity levels and resource requirements for the period. The second, or the R^{o} matrix, would contain the average observed values during the production period.

In determining this set of differences, management might also want to take into account any changes in the external resource input matrix E. If explained differences are observed, then the E^{o} matrix would then be used in determining the differences.

Using the R^{o} matrix, it would be possible to determine the A^{o} matrix.[28] The inverse of the A^{o} matrix would then be determined. Formula (4-6) could then be employed to determine the gross output vector x. Formulas (4-8) and (4-11) could be employed to determine a new H matrix and ℓ' vector. Total Unexplained and Explained quantity resource input differences could then be determined as follows:[29]

Total Unexplained Input Quantity Difference

$$= (c\ell' - \ell^{o}) \qquad (4\text{-}41)$$

Total Explained Input Quantity Difference

$$= c(\ell - \ell') \qquad (4\text{-}42)$$

Each term $c_k(\ell_k' - \ell_k^{o})$ or $c(\ell_k - \ell_k')$ would equal the unexplained or explained quantity difference for resource k.

The differences would be explained in the sense that the increased or decreased consumption was caused by observed shifts in the transfer requirements and the resource input standards. If there were simultaneous shifts in both the transfer requirements and the resource input standards, it would be possible to segregate the explained differences. One of the explained differences could be attributed to the shift in the transfer requirements and the other to changes in the resource input standards.

Once segregated the two differences for resource k would equal:[30]

Explained Quantity Difference (Shift In Inputs)$_k$

$$= (\ell^*_k - \ell_k) c_k \qquad (4\text{-}43)$$

Explained Quantity Difference (Shift In Transfers)$_k$

$$= (\ell^* \ell'_k) c_k \ . \qquad (4\text{-}44)$$

The ℓ^* vector would contain the estimated resource inputs if only the shift in the E matrix were considered.

The Unexplained and Explained quantity differences of resource input k in process j would equal:

Process Unexplained Quantity Difference$_{kj}$

$$= c_k (\gamma^o_{kj} - e^o_{kj} x'_k) \qquad (4\text{-}45)$$

Process Explained Quantity Difference$_{kj}$

$$= c_k (\epsilon^o_{kj} x'_j - e_{kj} x_j) \qquad (4\text{-}46)$$

The explained difference could further be segregated into:

Process Explained Quantity Difference (Shift In Inputs)$_{kj}$ = $c_k(e^o_{kj}x_j - e_{kj}x_k)$ (4-47)

Process Explained Quantity Difference (Shift In Transfers)$_{kj}$ = $c_k(e^o_{kj}x'_j - e^o_{kj}x_k)$. (4-48)

This concludes the presentation of cost analysis based on the input-output standard process costing model. This information and the input-output model will be evaluated in the concluding chapter of this study.

5. Cost Difference Examples

The assumptions for the cost difference examples are summarized in Exhibit 4-6. So that the meaning of the cost differences can be better understood, the computations will be dhown where practical. Note will be made throughout of any relationships that exist between the differences.

EXHIBIT 4-6

Cost Difference Assumptions

observed net output $y^o = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

observed unit cost vector c^o = [\$1.10,\$2.00,\$2.90,\$1.55]

observed input vector $\ell^o = \begin{bmatrix} 15 \\ 56 \\ 33 \\ 15 \end{bmatrix}$

observed process input matrix $\gamma^o = \begin{bmatrix} 15 & 0 \\ 22 & 34 \\ 14 & 19 \\ 20/3 & 25/3 \end{bmatrix}$

standard fixed cost p = \$20.00

observed fixed cost p^o = \$20.50

observed input coefficient matrix $R^o = \begin{bmatrix} .1 & .6 \\ .4 & .2 \end{bmatrix}$

observed input matrix $E^o = \begin{bmatrix} 2 & 0 \\ 3 & 4 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}$

Formula (4-34) Total Price Difference = $(c-c^{o})\ell^{o}$

$$\$1.05=[[\$1,00,\$2,00,\$3.00,\$1.50]-[\$1.10,\$2.00,\$2.90,\$1.55]]\begin{bmatrix}15\\56\\33\\15\end{bmatrix}$$

Formula (4-35) Process Price Difference$_{jk}=(c_j-c_j)\ {}^{o}_{jk}$

$$\text{Process Price Difference} = \begin{bmatrix}\$-1.50 & 0\\ 0 & 0\\ \$\ 1.40 & \$1.90\\ \$-\ .33 & -.42\end{bmatrix}$$

Formula (4-36) Total Quantity Difference - $c(\ell-\ell^{o})$

$$\$-65.50=[\$1.00,\$2.00\$3.00,\$1.50\left[\begin{bmatrix}12\\42\\22\\14\end{bmatrix}-\begin{bmatrix}15\\56\\33\\15\end{bmatrix}\right]$$

Formula (4-37) Process Quantity Difference$_{jk}=c_j(\gamma^{o}_{jk}-e_{jk}x_j)$

$$\text{Process Quantity Difference} = \begin{bmatrix}-\$3.00 & 0\\ -\$8.00 & -\$20.00\\ -\$24.00 & -\$\ 9.00\\ -\$1.00 & -\$\ .50\end{bmatrix}$$

Formula (4-38) Full Absorption Spending Difference

$$= (c^{o}_{z}-c_{z})\ell^{c}_{z}+p^{o}-p$$

Full Absorption Spending Difference = -$1.00

Formula (4-39) Full Absorption Efficiency Difference

$$= c_z(\ell^{o}_{z}-\ell_z)$$

Full Absorption Efficiency Difference=-$1.50

Formula (4-40) Full Absorption Capacity Difference

$$= (c_x \ell_x + p) - \ell_z^o (c_z + p/\ell_z)$$

Full Absorption Capacity Difference=$2.88

Formula (4-6) $X' = A^{o-1} y^o$

$$\begin{bmatrix} 20/3 \\ 25/3 \end{bmatrix} = \begin{bmatrix} 5/3 & 5/4 \\ 5/6 & 15/8 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Formula (4-8) $G' = E^o \; A^{o-1}$

$$\begin{bmatrix} 10/3 & 5/2 \\ 25/3 & 45/4 \\ 5 & 25/4 \\ 5/2 & 25/8 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 3 & 4 \\ 2 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5/3 & 5/4 \\ 5/6 & 15/8 \end{bmatrix}$$

Formula (4-11) $\ell' = G' y^o$

$$\begin{bmatrix} 13\frac{1}{3} \\ 53\frac{1}{3} \\ 30 \\ 15 \end{bmatrix} \begin{bmatrix} 10/3 & 5/2 \\ 25/3 & 45/4 \\ 5 & 25/4 \\ 5/2 & 25/8 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Formula (4-41) Total Unexplained Input Quantity Difference = $c(\ell' - \ell^o)$

$$-\$16.00 = [\$1.00, \$2.00, \$3.00, \$1.50] \left[\begin{bmatrix} 13\frac{1}{3} \\ 53\frac{1}{3} \\ 30 \\ 15 \end{bmatrix} - \begin{bmatrix} 15 \\ 56 \\ 33 \\ 15 \end{bmatrix} \right]$$

Formula (4-42) Total Explained Input Quantity Difference = $c(\ell-\ell')$

$$-\$49.50=[\$1.00,\$2.00,\$3.00,\$1.50]\left[\begin{bmatrix}13\\42\\22\\14\end{bmatrix}-\begin{bmatrix}13\frac{1}{3}\\53\frac{1}{3}\\30\\15\end{bmatrix}\right]$$

<u>Note</u>: The unexplained plus the explained differences total to the quantity difference as determined in the traditional model.

-\$16.00 + (-\$49.50) = -\$65.50

Formula (4-10) $\ell^* = E^o x$

$$\begin{bmatrix}12\\50\\28\\14\end{bmatrix}=\begin{bmatrix}2 & 0\\3 & 4\\2 & 2\\1 & 1\end{bmatrix}\begin{bmatrix}6\\8\end{bmatrix}$$

Formula (4-43) Explained Quantity Difference (Shift In Inputs) = $c(\ell^*-\ell)$

$$-\$34.00=[\$1.00,\$2.00,\$3.00,\$1.50]\left[\begin{bmatrix}12\\50\\28\\14\end{bmatrix}-\begin{bmatrix}12\\42\\22\\14\end{bmatrix}\right]$$

Formula (4-44) Explained Quantity Difference (Shift In Transfers) = $c(\ell^*-\ell')$

$$-\$15.50=[\$1.00,\$2.00,\$3.00,\$1.50\left[\begin{bmatrix}12\\50\\28\\14\end{bmatrix}-\begin{bmatrix}13\frac{1}{3}\\53\frac{1}{3}\\30\\15\end{bmatrix}\right]$$

<u>Note</u>: The sum of the two explained differences equals the total explained difference as determined using formula (4-42)

$$-\$34.00 + (-\$15.50) = -\$49.50$$

Formula (4-45) Process Unexplained Quantity Difference$_{kj}$ $= c_k(Y_{kj}-e^{o}_{kj}X'_j)$

$$\text{Process Unexplained Quantity Difference} = \begin{bmatrix}-\$1.67 & \$\ \ 0\\-\$4.00 & -\$1.33\\-\$2.00 & -\$7.00\\\$\ \ 0 & \$\ \ 0\end{bmatrix}$$

Formula (4-46) Process Explained Quantity Difference$_{jk}$ $= c_k(e^{o}_{kj}X'_j-e_{kj}X_j)$

$$\text{Process Explained Quantity Difference} = \begin{bmatrix}-\$1.33 & \$\ \ 0\\-\$4.00 & -\$18.67\\-\$22.00 & -\$2\text{[illegible]}.00\\-\$1.00 & -\$\ .50\end{bmatrix}$$

<u>Note</u>: The unexplained plus the explained process quantity differences equal the process quantity differences as determined with the traditional model.

$$\begin{bmatrix} -\$3.00 & \$\ 0 \\ -\$8.00 & -\$20.00 \\ -\$24.00 & -\$\ 9.00 \\ -\$1.00 & -\$\ 5.0 \end{bmatrix} = \begin{bmatrix} \$1.67 & \$\ 0 \\ -\$4.00 & -\$1.33 \\ \$2.00 & -\$7.00 \\ \$\ 0 & \$\ 0 \end{bmatrix} + \begin{bmatrix} -\$1.33 & \$\ 0 \\ -\$4.00 & -\$18.67 \\ -\$22.00 & -\$2.00 \\ -\$1.00 & -\$\ .50 \end{bmatrix}$$

Formula (4-47) Explained Process Quantity Difference

$$(\text{Shift in Inputs})_{jk} = c_k(e^o_{kj}x_j - e_{kj}x_j).$$

$$\text{Explained Process Quantity Difference (Shift in Inputs)} = \begin{bmatrix} \$\ 0 & \$\ 0 \\ \$\ 0 & -\$16.00 \\ -\$18.00 & \$\ 0 \\ \$\ 0 & \$\ 0 \end{bmatrix}$$

Formula (4-48) Explained Process Quantity Difference

$$(\text{Shift in Transfers}) = c_k(e^o_{kj}x_j - e^o_{kj}x_j).$$

$$\text{Explained Process Quantity Difference (Shift in Transfers)} = \begin{bmatrix} -\$1.33 & \$\ 0 \\ -\$4.00 & -\$2.67 \\ -\$4.00 & -\$2.00 \\ -\$1.00 & -\$\ .50 \end{bmatrix}$$

Note: The sum of the two explained differences equals the total explained difference as determined by using formula (4-46)

$$\begin{bmatrix} -\$1.33 & \$\ \ 0 \\ -\$4.00 & -\$18.67 \\ -\$22.00 & -\$2.00 \\ -\$1.00 & -\$\ .50 \end{bmatrix} = \begin{bmatrix} \$\ \ 0 & \$\ \ 0 \\ \$\ \ 0 & -\$16.00 \\ \$18.00 & \$\ \ 0 \\ \$\ \ 0 & \$\ \ 0 \end{bmatrix} + \begin{bmatrix} -\$1.33 & \$\ \ 0 \\ -\$4.00 & -\$2.67 \\ -\$4.00 & -\$2.00 \\ -\$1.00 & -\$\ .50 \end{bmatrix}$$

Summary

This chapter has demonstrated that most of the information that could be obtained from a traditional process costing model could also be obtained from an input-output process costing model. It has been demonstrated that outputs and inventories could be costed using an input-output model. This essentially would meet the requirements of a financial accounting system.

It has been shown that the price, quantity, and overhead differences that could be obtained from the traditional model could also be obtained from an input-output model. A method for handling by-products with the input-output model was also discussed. All direct cost allocations in an input-output model are based on the estimates of the physical inputs.

The advantage of the input-output model is in the area of planning. Given the desired outputs and inventory levels, it would be possible to predict the resource requirements, activity levels, and average unit costs. For control purposes, it would be possible to segregate the quantity variances into explained and unexplained components. It would appear that the manipulation of process costing data is more straight-forward when the input-output model is employed.

Footnotes

[1]First introduced in "Quantitative Input-Output Relations in the Economic System of the United States," W. W. Leontiff. Review of Economic Statistics XVIII (August, 1936), 105-125. This study was elaborated upon in The Structure of American Economy 1919-1929 (Cambridge, Mass.: Harvard University Press, 1941).

[2]Shawki M. Farag, Input-Output Analysis: Applications to Business Accounting (Center For International Education and Research in Accounting, University of Illinois, 1967), p. 23.

[3]Suggested applications are found in:

Trevor E. Gambling, "A Technological Model for Use in Input-Output Analysis and Cost Accounting," Management Accounting (December, 1968), p. 33-38.

Yuji Ijiri, "An Application of Input-Output Analysis to Some Problems in Cost Accounting," Management Accounting, XLIX (April, 1968), pp. 49-61

John Leslie Livingstone, "Input-Output Analysis for Cost Accounting, Planning and Control," The Accounting Review, XLIV (January, 1969), pp. 48-64.

Shawki M. Farag, "A Planning Model for the Divisionalized Enterprise," The Accounting Review XLIII (April, 1, 1968), p. 312-320.

Gerald A. Feltham, "Some Quantitative Approaches to Planning for Multiproduct Production Systems," Accounting Review, XLV, No. 1 (January, 1970), 11-26.

[4]Gambling (1968), Livingston (1969), and Feltham (1970) are examples of approaches employing physical output coefficients. Ijiri (1968) employs input, physical output, and dollar input coefficients.

[5]Production would not be economically feasible if more than a unit of output had to be consumed to produce and identical unit that required more than one unit as input.

[6]In "Note: Some Conditions of Macroeconomic Stability," David R. Hawkins and Herbert E. Simon. Econometrica 17:245-248 July-October 1949. In interpreting this condition economically, Hawkins and Simon stated that:

j^{th} ....if this principle minor involving the i^{th} and commodities is negative, this means that the quantity of the i^{th} commodity required to produce one unit of the j^{th} commodity is greater than the quantity of the i commodity that can be produced with an input of one unit of the j commodity. Under these conditions, the production of these two commodities could not be continued for they would exhaust each other in joint production." (p. 298). This condition could be represented in determinant form:

$$\begin{vmatrix} 1-r_{1,1} & -r_{1,2} \\ -r_{2,1} & 1-r_{2,2} \end{vmatrix} > 0$$

[7]Examples are developed at the conclusion of each section.

[8]Refer to assumption 5.

[9]The R matrix would contain the physical input coefficients. The $(Q-R)^{-1}$ matrix would contain the physical output coefficients.

[10]Different overhead rates could be employed for each of the processes within the system. The E matrix would then assume dimensions of $(z-1+n)xn$. The $(z-1+i)$ row of the E matrix would contain a one in the i^{th} column. The c vector (refer to formula 4-11) would then contain $(z-1+n)$ elements. The last n elements would contain the variable overhead rates for the n processes. Another possibility would be to employ a uniform value cz and vary the coefficients in the z^{th} row of the E matrix.

[11]Throughout this section it will be necessary to refer to particular elements of the A^{-1} matrix. These elements will be designated a_{ij}^{-1}. It will always be the case that the element is from the A^{-1} matrix and not the inverse of a particular element of the A matrix.

[12]Formula (4-10) multiplies the net resource input matrix E by the gross output vector x to determine the resource requirements. Formula (4-11) multiplies the gross resource input matrix H by the net output vector y to determine the resource requirements. Formula (4-6) specifies the relationship between x and y.

[13]Another interpretation can be given to the Θ matrix. Each element θ_{ij} indicates the expected number of output units from process i consumed in process j. In the case of $\theta_{i,n+1}$ the element indicates the expected units of output of process i produced for outside consumption.

[14]The cost of units sold and the valuations of the in-process inventories is required by the financial acacounting system. This information is used in determining net income and asset valuations.

[15]In systems with in-process inventories, the v vector will represent the unit produced for outside consumption. The y vector will represent the equivalent net production resulting from x units of net activity. Because of changes in the inventory levels, there would be few cases when the y vector would equal the v vector.

[16]The production period is assumed to begin at time t and be of unit length.

[17]Each column of the A^{-1} matrix contains the number of units in each of the processes required to produce a unit of output from the process represented by the column designation. The subtraction of the I matrix eliminates the unit of activity in the process to which the unit is being transferred. The subtraction of the R_{dg} matrix eliminates the consideration of any resources consumed in the process to which the unit is being transferred. The R_{dg} matrix would only contain the elements on the main diagonal of the R matrix. All other elements would equal zero.

[18]The term d a would equal all of the transfer costs to be inventoried. The term $c(\alpha\pi E_{tr})_{dg}$ b would equal costs attached to the units in the processes where they are partially completed.

[19]The first term $A^{-1}v$ equals the gross activity levels necessary to produce the output v. The second term $A^{-1}(\pi^{t+1}-\pi^{t})-\pi^{t+1}$ equals the activity levels required for any changes in the in-process inventories.

[20] The term Ex equals the resources required for the gross activity in the system. (Refer to Formula 4-10) The term $(\alpha^{t+1}\pi^{t+1}+(N-\alpha^{t})\pi^{t})E_{tr})dg$ a equals the resources required for any adjustments in the in-process inventories.

[21] The term $(\sum_{k=1}^{n} a_{jk}^{-1}(v_k+\pi_{kk}^{t+1}-\pi_{kk}^{t}))$ would equal the gross output from process i if no consideration were given to partially completed in-process inventories.

[22] The term difference has been substituted for variance.

[23] In the term c^o the superscript is used to indicate the observed unit cost during the production period. The c vector would contain the standard costs.

[24] The term $(c_z-c_z^o)\ell_z^o$ would equal the price difference caused by expenditures for direct overhead. In the traditional model the overhead differences would be determined separately. Therefore, the Price Difference attributed to the direct overhead would not be considered and then the differences determined with the traditional and input output models would be equal.

[25] The ℓ vector would contain the resource inputs determined by employing the observed outputs of the system and the <u>ex-ante</u> or standard parameters of the input-output model. The total quantity difference as determined in formula (4-36) would also include the direct overhead difference.

[26] In formula (4-10) it has been assumed that the fixed overhead rate is determined by dividing the total expected fixed costs by the total expected units of activity. The vector p would contain the allocated fixed costs.

[27] Because in a traditional standard process cost system does not specify unit transfer standards.

[28] Formula (4-3) would be used to determine the matrix.

[29]The superscript ℓ' will be used to indicate that the quantity was determined ex-post from the observed R^o and E^o matrices.

[30]The superscript * will be used to indicate that the quantity was determined ex-post from the observed E^o matrix. The explained variance resulting from both shifts in the E^o and R^o matrix combined has been assigned to the explained quantity variance that results from shifts in the transfer standards.

CHAPTER V

LINEAR PROGRAMMING AND PROCESS COSTING

Introduction

When the process costing models that were developed in the previous two chapters are used for planning, the assumption is made that the set of process outputs is given that meet the objectives of the organization. In most multi-product mass production systems there would exist many sets of possible outputs from the production system. A problem faced by management would be to select the "best" or "optimal" set of outputs from all of the feasible sets that could be identified.

The selection of an optimal set of outputs would require the identification of some criterion upon which an evaluation could be made. Perhaps the most common criterion adopted in commercial organizations for planning purposes would be that of profit maximization. However, other measures of utility or goals could be employed as criterion in the evaluation of possible outputs.

Linear programming is a set of techniques that could be employed for the purpose of identifying an optimal set of outputs.[1] Viewed in this context, it is primarily a planning model that would require unit cost estimates from some accounting model. However, the output from a linear programming model could be employed in making control decisions. The opportunity costs developed in a linear programming model could serve as the inputs of a cost control model or as the basis for overhead allocation.

The chapter is devoted to linear programming for two reasons The first reason pertains to the implications of the planning and control information that could be gained from a linear programming model. The second reason behind this effort is that most process standard costing systems possess characteristics that would facilitate the formulation of linear programming problems.

After a brief introduction of linear programming, a review will be presented of linear programming techniques that could be applied to multi-product mass production systems. Then the outputs of a linear programming model for planning and control activities will be considered in relationship to process costing.

Linear Programming and the Simplex Algorithm

Technically speaking, the linear programming problem is one of selecting non-negative values of certain variables to maximize or minimize a linear function subject to a set of linear inequality constraints. A general linear programming problem would take the form

$$\begin{aligned} &\text{Max} \qquad && Z = cx \\ &\text{Subject to} && A\,x \leq b \\ & && x \geq 0 \end{aligned} \qquad (5\text{-}1)$$

The m constraint constants of the problem are summarized by the column vector b. The m by n matrix A summarizes the constant structural coefficients. In a typical process costing primal problem, each column of the A matrix would contain the units of activity and resources required to produce a particular unit of output. The n element row vector c summarizes the criterion elements. In a profit maximization problem, the criterion elements would usually be the marginal contribution of a unit of output. The non-negatively constraints define the Euclidean space so the negative values of the x variable are not considered in the problem.

The dual theorem of linear programming asserts that for every maximization (or minimization) problem there exists a corresponding minimization (or maximization) problem involving the same problem. In general, the dual program, for formula (5-1) could be expressed as:

$$\begin{aligned} &\text{Min} \qquad && G = yb \\ &\text{Subject to} && yA \geq c \\ & && y \geq 0\ . \end{aligned} \qquad (5\text{-}2)$$

The primal problem involves selecting n variables summarized by the vector x whereas the dual problem involves selecting the m variables summarized in the y vector.

The simplex algorithm is an orderly, converging iterative process for determining what variables are in the basis. The algorithm is based on the characteristics of the region (generally referred to as the feasible region) that satisfy all of the linear constratins. The feasible region assumes the shape of a convex polygon. The simplex algorithm is a process for exchanging variables in the basis while continually increasing (or decreasing) the value objective function. The optimal solution will lie at an extreme point or on a face of the feasible region.

There exist three fundamental theorems of linear programming each of which state a necessary and sufficient condition for the existence of a feasible solution.[3] The first theorem usually referred to as the existence theorem states that a necessary and sufficient condition for the existence of a feasible solution is that a feasible region for both the primal and dual problem exists. The second or dual theorem of linear programming states that a necessary and sufficient condition for the existence of a feasible solution is that there exists a feasible solution for the dual problem such that the values of the objective function of both problems would be equal.

The third fundamental theorem of linear programming is concerned with complementary slackness. This theorem states that a necessary and sufficient condition for the vectors x and y to solve the primal and dual problems is that the complementary slackness condition is satisfied. The result of this condition is that if a slack variable is not in the primal solution, the corresponding dual variable will be in solution. In addition, if a slack variable was in the primal solution, the corresponding dual variable would not be in solution.

Process Costing Applications of Linear Programming

The purpose of this section is to present the various classes of constraints that could be incorporated into a linear programming problem that would relate in some way to a multi-product mass production system. Consideration will also be given to "goal programming" which would make possible the incorporation of more than one objective into a linear programming planning model.[4]

In most commercial applications of linear programming to process costing, the objective function would maximize the total expected contribution margin. The first set of constraints would relate to the production capacities of the n processes within the system. A feasible solution would also require that the quantity of the exogenous inputs was less than or equal to the quantities available. Therefore, a second set of constraints could relate to the availability of certain resources. The resource constraints could also relate to the availability of non-production resources within the organizations. Constraints limiting outputs because of restrictions on cash could be incorporated into this type of problem. Sales could also be restricted because of credit policies of the organization.

A third possible set of constraints that could be incorporated into a linear programming problem could relate to market conditions. Constraints could be incorporated because of difficulties or limitations in the quantities of certain outputs that could be sold. Such constraints could also relate to a policy that restricted the output of certain products. A break-even constraint could also be incorporated into the linear programming problem. This constraint would require that the total contribution margin expected to be realized from the output would exceed the total fixed costs.

Goal programming "is distinguished from other types of linear programming problems by virtue of the fact that at least one of the constraints is incorporated in the objective function in such a manner that it becomes a part of the objective for maximization or minimization."[5] Assume a single goal is specified such that the total contribution margin should equal g. The total estimated contribution margin would equal hx where h is a vector of contribution margins and x is the solution to the program.

In goal programming the objective function would take the form:

$$\text{Minimize } z \text{ where } z = y^- + y^+. \qquad (5\text{-}3)$$

The y^- would denote any amount the desired contribution g exceeded the actual contribution. The y^+ would denote any amount by which the actual contribution margin exceeded the desired contribution g. These differences are introduced into the constraints by requiring that the estimated contribution margin plus or minus a "difference" would equal the desired contribution margin g.

In such a case, the contribution margin constraint or goal would take the form:

$$hx - y^+ + y^- = g \qquad (5\text{-}4)$$

A single goal problem could be expressed as:

$$\begin{aligned} \text{Minimize} \quad & y^+ + y^- \\ \text{Subject to} \quad & hx - y^+ + y^- = g \\ & Ax \leq b \\ & x, y^+, y^- \geq 0 \end{aligned} \qquad (5\text{-}5)$$

Slack variables would be introduced and the simplex algorithm would be employed to determine the optimal feasible solution. If either of the artificial variables (y^- and y^+) corresponding to the goal were not in the basis at optimal solution, the solution of the program would result in the goal being exactly realized.

From the single goal problem, goal programming can be extended to a case with multiple goals. A multiple goal problem could be expressed as:[6]

$$\begin{aligned} \text{Minimize} \quad & ey^{+} + ey^{-} \\ \text{Subject to} \quad & Px - Iy^{+} + Iy^{-} = g \\ & Ax \leq b \\ & x, y^{+}, y^{-} \geq 0 \, . \end{aligned} \qquad (5\text{-}6)$$

Assuming m goals, the g column vector would have m elements. The P matrix would have dimensions of m by n and express the relationship between the goals and e would equal an m component vector whose elements all equal one. The multiple goal problem would enable management to incorporate more than one goal into the problem. Such goals as profit level, activity levels in the processes, and output quantities could be considered in this type of linear programming planning model.

This completes the consideration of possible linear programming applications to multi-product mass production systems. From each of the models discussed the expected resource requirements, activity levels, and total contribution margin could be determined along with the optimum production mix. The next section will consider the information that could be obtained from the linear programming that could be employed in cost analysis.

Cost Analysis

If linear programming was employed for planning purposes in a process system, it would be possible to develop cost differences based on opportunity costs. The traditional standard cost model as developed in Chapter III produced cost differences that were based on differences between the standard and actual costs incurred. In the traditional model, the price and quantity differences are based on the levels of the resource inputs. Since the production standards are linear, it would be possible to express the total price and quantity differences as a function of the output. This transformation will be made in this section to facilitate comparisons between the differences of the traditional process cost and linear programming models. The first divisions of this section will compare the traditional differences with those that could be obtained if linear programming were employed.

The remaining sections will be concerned with the analysis of the elements in the final simplex tableau. Initially the analysis will be based on the sensitivity to change of the various parameters of the model. The final sections will be concerned with the control implications of the shadow prices.

1. Price and Quantity Differences[7]

Assume A^a, b^a, and h^a represent the data inputs used to determine the optimal program x^a.[8] This would be the linear programming problem considered in the planning or ex-ante analysis of the system before the production period began. Let A^o, b^o, and h^o represent the data values observed during the production period. Finally, let A^p, b^p, and h^p represent the input values that would have been employed in determining the ex-post program. These ex-post inputs would incorporate all of the additional information acquired during the production period concerning the operation of the production system and market conditions. The ex-post inputs represent what would have been observed if there had been no avoidable deviations in the production activity.

Assuming that the production capacity remained constant, (i.e., those elements in the b^a vector relating to capacity equalled the corresponding elements in the b^p vector) then any deviations from the standard costs or quantities would result in change in the marginal contribution vector h. That is, the h^a vector would differ from the h^o and h^p vectors. If the selling prices remained unchanged during the period, the total price difference as determined by the traditional model would equal:

$$\text{Total Price Difference}^{t} = (h^{a}-h^{o})x^{o}. \quad (5\text{-}7)$$

Employing a linear programming model and allowing the selling prices to vary, the total price difference would equal:

$$\text{Total Price Difference}^{lp} = (h^{a}x^{a})-(h^{o}x^{o}). \quad (5\text{-}8)$$

By adding and subtracting the expression $(h^{p}x^{p})$, the total price difference could be expressed as:

$$\text{Total Price Difference}^{lp} = ((h^{a}x^{a})-(h^{p}x^{p}))+((h^{p}x^{p})-(h^{o}x^{o})) \quad (5\text{-}9)$$

The first expression in formula (5-9) represents any change in the optimum marginal contribution level caused by deviations between the ex-ante and ex-post programs. The absolute value of this term could be employed in evaluating the planning capabilities of the organization. The second expression indicates any difference between the obtainable marginal contribution and the actual marginal contribution realized. The total of the second term would indicate the cost to be associated with avoidable deviations incurred during the production period.

The total of the quantity differences as determined by the traditional cost model would equal:[8]

$$\text{Total Quantity Difference}^{t} = u(x^{o}-x^{a}) \quad (5\text{-}10)$$

When the linear programming model is employed, the total quantity difference would equal:

$$\text{Total Quantity Difference}^{lp} = (h^{a}x^{o})-(h^{a}x^{a}) \quad (5\text{-}11)$$

By adding and subtracting the expression $(h^a x^p)$, the total quantity difference can be expressed as:

$$\text{Total Quantity Difference}^{1p} = [(h^a x^o) - (h^a x^p)] + [h^a x^p) - (h^a x^a)] \quad (5\text{-}12)$$

The first expression in formula (5-12) indicates the change in the contribution margin because of changes in the optimal programs. The second term indicates changes in the contribution margin because of deviations between the optimal *ex-post* program and the observed program. The *ex-ante* contribution margins are used throughout when determining quantity differences. This is in keeping with the accounting practice of assigning all differences due to price changes to the price variance.

2. Overhead Differences

The allocation of fixed overhead costs among the products of a system is generally based upon some activity level that has been found to be correlated with system output. An overhead budget is then prepared and an overhead rate is determined by dividing the budget total by an estimate of the correlated activity level. the two and three variance methods developed in the traditional model are based on identifying differences caused by variations in the activity levels on the overhead costs.

When either the *ex-ante* or *ex-post* programs are solved, b-Ax units of capacity are idle. The dual evaluators or the opportunity costs $(z_j - c_j)$ of the idle capacity will equal zero at an optimal solution. A capacity difference expressed as an opportunity cost would be the difference between the contribution margin obtained employing the <u>ex-ante</u> program and the actual contribution margin realized. This capacity difference would equal:

$$\text{Total Capacity Difference}^{1p} = (h^p x^o) - (h^p x^p) \qquad (5\text{-}13)$$

This capacity difference represents the marginal contribution foregone by producing x^o rather than x^p units. This is the opportunity cost resulting from not employing the available capacity optimally. The linear programming model does not lend itself to determining versions of the spending and efficiency overhead variances that can be obtained from the traditional model.

3. Post Optimality Analysis

In the final simplex tableau, the opportunity costs $(z_j - c_j)$ with zero values will correspond with the slack variables in the basis. The slack variables in the basis indicate the unused capacity in the system. The zero valued opportunity costs indicate that the total contribution margin would not be altered with either a slight increase or decrease in the availability of the corresponding resource. The opportunity costs will assume positive values for the slack variables that assume zero values at solution. These opportunity costs represent the increase or decrease in the contribution margin that would be realized if one additional unit of the corresponding resource were either added to or subtracted from the available capacity.

If a slack variable is not in the basis, the availability of the corresponding resource may be altered over a specified range before it would become necessary to substitute a vector in the basis to assure a feasible solution. Let $a^*_{i,r}$ represent the i^{th} element in the final simplex tableau corresponding to the slack variable r that has a zero value. The upper limit for adding additional units of resource r before a change would take place in the basis would equal the minimum positive value of the ratio $b^*_i/(-1)a^*_{i,r}$ for $i=1$ to m. Assuming the minimum positive value of ratio occurs at $i=j$, this then indicates that all of the remaining slack of resource j has now been utilized. Any additional units of resource r added beyond this limit would result in slack variable j exiting from the basis because it would now assume a zero value.

If n units of resource r were removed from the available capacity, the contribution margin would be reduced by $n(z_r-c_r)$. The limit of the number of units of resource r that could be removed before the basis would change could also be determined. The limit would be determined by the minimum positive value of the ratio $b_i^*/a_{i,r}^*$ for i=1 to m. If the minimum value occured at i=j, then this would indicate the limit of the reduction before variable j would go out of solution.

4. A Cost Control System

Most mass production systems would require a significant number of resource inputs. It would be expected that the employment of some of the resources would be more critical in evaluating the performance of the system. The cost control models discussed to this point have not possessed the capability of discriminating between the relative importance of the resource inputs. For example, quantity differences are determined for each resource input in the traditional process standard costing model. However, the output from the model would give no indication as to the implication of any difference upon the output of the system.

A cost control system based on the opportunity costs as determined in linear programming would overcome this deficiency. It should be recalled that the opportunity costs are determined for those resources that constrain the output of the system. In this control system, the only costs that would be attached to the outputs would be the opportunity costs of those factors of production which constrain the output of the system. The total opportunity costs that would be attached to a unit of output specified in the ex-ante program would equal the expected marginal contribution of a unit of output from that process.

If a particular class of output was not included in the ex-ante solution, the opportunity cost which would be attached to the output would exceed the expected marginal contribution. The difference between the attached opportunity cost and the marginal contribution would be the profit foregone if a unit of this class of output was produced for outside consumption. The goal of management employing this type of control system would be to break-even with the ex-ante plan.

Operationally, this control system would be described as follows. Employing the primal solution of the ex-ante program, let p equal an n element row vector determined by[9]

$$p = qA. \qquad (5\text{-}14)$$

The q vector is an m element row vector that contains the $(z_j - c_j)$ elements associated with the slack variables in the final simplex tableau. This vector would contain the elements in the $(z_j - c_j)$ row from $(z_{n+1} - c_{n+1})$ to $(z_{n+m} - c_{n+m})$. The i^{th} element of the p vector would contain the opportunity cost that would be charged to a unit of output from process i.

During the production period, every unit of output would be charged with the appropriate cost from the p vector. The _ex-post_ analysis could either concern the entire system or the individual processes. When considering the entire system, if $h^a x^a$ equalled px^o, the determination would be made that all resources that constrain the output of the system were employed. If $h^a x^a$ was less than px^o, this would indicate that more capacity was available than was indicated in the constraints of the _ex-ante_ program. In a case where $h^a x^a$ was greater than px^o, the constraining resources would not have been fully employed during the period. The difference would also indicate the profit foregone by the underutilization of capacity.

When $h^a x^a$ equalled $h^c x^o$, the conclusion drawn would be that the output of the system was in accordance with the _ex-ante_ plan. Furthermore, it could be concluded that the optimal planned output would not have been altered by either market or production conditions encountered during the period. Shifts in either of these conditions normally would influence the observed contribution margin.

If $h^a x^a$ did not equal $h^o x^o$ then the system would not have operated according to the optimal ex-ante plan. Further analysis could lead to several conclusions. A solution to an ex-post program would be required. If the solution of the ex-post program equalled the solution to the ex-ante program, this would indicate that the foregone profit resulted from controllable conditions within the system. The difference between $(p-h^o)x^o$ would indicate the amount of the foregone profit. The analysis of the individual processes would locate the source of the deviation.

If the solutions of the ex-post and ex-ante programs were not equal the shift would have most likely been caused by factors external to the organization. The solution of the ex-post program would permit the determination of any profit foregone because of the inability of the system to adapt to the conditions encountered during the period.

The analysis of a system employing this type of control system is therefore two-fold. Initial comparisons are made between the results and the ex-ante program. The objective of management at the beginning of any period would be to break-even with the ex-ante program. If is were determined that an ex-post program should be solved, then the adaptive capability of the system would also be evaluated. Under such conditions, management would seek to break-even with the ex-post program.

There are several advantages to the control system previously described. Only the critical resources are considered and this could result in a significant data reduction in the requirements for the system. Rather than considering all resource inputs, only those that constrain the system would be necessary. This should result in the control activities of management being focused on areas where immediate and perhaps more profitable results would be realized. And perhaps a less obvious advantage is that the analysis would be shifted from what could have been based on an ex-ante program to what could have been based on an ex-post program.

5. Allocation of Fixed Overhead[10]

The allocation of fixed overhead to products has been a perplexing problem to accountants since the development of cost accounting. Generally fixed overhead is allocated on the basis of some activity levels that varies with the output of the system. The full absorption costs resulting from such allocations have proved to be of dubious value in the context of many decisions. The shortcomings of such allocations are quite apparent in making decisions concerning temporary shut-downs or the acceptance of special orders.

If the allocation of fixed overhead were based on the opportunity costs attached to the outputs would not be distorted. Assume total fixed costs of w and that the total marginal contribution exceeds the total fixed costs. Let the constant k be defined by

$$k = w/hx. \qquad (5\text{-}15)$$

The constant k would be less than one because of the assumption that hx>w. The fixed overhead allocated to product j would equal kp_i. If the full absorption costs were incorporated into the linear programming problem, it would take the form:

$$\text{Max } (h-kp)x$$
$$\text{Subject to } Ax \leq b$$
$$x \geq 0. \qquad (5\text{-}16)$$

This method of allocating fixed overhead would not distort the optimal program.

In some cases there may be a negative entry in the A matrix. Such a situation might exist if one of the constraints in the program required a minimum output of a certain output. It would be possible under such a condition to incur a negative overhead allocation which could be interpreted as a subsidy. An example of fixed overhead allocation based on the opportunity cost is shown in the following examples.

This concludes the discussion of cost analysis based on the linear programming model. This material will be considered again in the analysis chapter where the models will be evaluated.

Linear Programming Examples

In this section a comprehensive example will be presented to demonstrate the uses of linear programming in process costing. The problem is an extension of the sample problem developed in the previous chapter where the input-output model was presented.

The initial simplex tableau is shown in Exhibit 5-1. The elements relating to the capacity and resource input constraints are taken directly from the input-output example. The elements relating to the capacity constraints are taken from the inverse of the technological coefficient matrix as determined by formula 4-5. The element relating to the resource input constraints are taken from the gross input matrix as determined by formula 4-8.

The final simplex tableau is obtained after two iterations and is shown in Exhibit 5-2. The optimal solution indicates that 5.5 units of product x_1 should be produced. The expected contribution margin is $22.00 and the expected profit equals $2.00. The solution also indicates that the capacity of the first process is constraining the output of the system. Based on this problem, the remaining sections will demonstrate cost analysis procedures

EXHIBIT 5-1

Initial Simplex Tableau

	c_j			4	2	0	0	0	0	0	0	0	0	0	-m
	c_i	Basis	b	x_1	x_2	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
capa-city	0	s_1	8.8	1.6	1.1	1	0	0	0	0	0	0	0	0	0
	0	s_2	10	.8	1.8	0	1	0	0	0	0	0	0	0	0
input	0	s_3	25	3.2	2.2	0	0	1	0	0	0	0	0	0	0
	0	s_4	50	7.2	8.7	0	0	0	1	0	0	0	0	0	0
	0	s_5	30	3.2	4.7	0	0	0	0	1	0	0	0	0	0
mar-ket	0	s_6	50	1.6	3.6	0	0	0	0	0	1	0	0	0	0
	0	s_7	8	1	0	0	0	0	0	0	0	1	0	0	0
Break-even	0	s_8	8	0	1	0	0	0	0	0	0	0	1	0	0
	-m	s_{10}	20	4	2	0	0	0	0	0	0	0	0	-1	1
	Z		-m	-4m	-2m	0	0	0	0	0	0	0	0	m	-m
	z_j-c_j			-4m4	-2m-2	0	0	0	0	0	0	0	0	m	0

EXHIBIT 5-2

Final Simplex Tableau

		c_j	4	4	2	0	0	0	0	0	0	0	0	0	-m
	c_i	Basis	b	x_4	x_2	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
capacity	0	s_9	s	0	3/4	5/2	0	0	0	0	0	0	0	1	-1
	0	s_2	5.6	0	3/5	-1/2	1	0	0	0	0	0	0	0	0
inputs	0	s_3	7.4	0	0	-2	0	1	0	0	0	0	0	0	0
	0	s_4	10.4	0	15/4	-9/2	0	0	1	0	0	0	0	0	0
	0	s_5	12.4	0	9/10	-2	0	0	0	1	0	0	0	0	0
	0	s_6	41.2	0	5/4	-5/8	0	0	0	0	1	0	0	0	0
market	0	s_7	2.5	0	-11/16	-9/8	0	0	0	0	0	1	0	0	0
	0	s_8	8	0	0	0	0	0	0	0	0	0	1	0	0
Break-even	4	x_1	5.5	1	5/16	5/8	0	0	0	0	0	0	0	0	0
		Z	22	4	5/4	5/2	0	0	0	0	0	0	0	0	0
		$z_k - c_j$		0	3/4	5/2	0	0	0	0	0	0	0	0	m

based on the linear programming model.

1. Price, Quantity, and Capacity Differences

The assumptions for this example are shown in Exhibit 5-3. The computations are shown to clarify the computations of the various sets of differences.

EXHIBIT 5-3

Cost Analysis Assumptions

observed unit costs c^o = [$1.00,$2.00,$1.00,$1.50]

observed marginal contribution h = [$10.60, $11.35]

observed resource input $\ell^o \begin{bmatrix} 17.6 \\ 39.6 \\ 17.6 \\ 39.6 \end{bmatrix}$

standard resource input $\ell^{std} = \begin{bmatrix} 17.92 \\ 40.32 \\ 17.92 \\ 39.84 \end{bmatrix}$
(By formula 4-11)

observed unit output cost u^o = [$24.40, $28.65]
(By formula 4-14)

solution to <u>ex-post</u> program employing h^o and c^o

$$x^p = \begin{bmatrix} 0 \\ 5.56 \end{bmatrix}$$

Formula (5-7) Total Price Difference^t = $(h^a - h^o)x^o$

$$\$-36.96 = [[\$4.00, \$2.00] - [\$10.60, \$11.35]] \begin{bmatrix} 5.6 \\ 0 \end{bmatrix}$$

Formula (5-9) Total Price Difference^{lp}

$$= ((h^a x^a) - h^p x^p)) + ((h^p x^p) - (h^o x^o))$$

$$\text{Price Difference} = \left[[\$4.00, \$2.00] \begin{bmatrix} 5.5 \\ 0 \end{bmatrix} \right] - \left[[\$10.60, \$11.35] \begin{bmatrix} 0 \\ 5.56 \end{bmatrix} \right]$$

$$+ \left[[\$10.60, \$11.35] \begin{bmatrix} 0 \\ 5.5 \end{bmatrix} \right] - \left[[\$10.60, \$11.35] \begin{bmatrix} 5.6 \\ 0 \end{bmatrix} \right]$$

$$[-\$37.36] = [\$41.11] + [\$3.75]$$

In this case, when planning under-estimated profits by \$41.11. An additional \$3.75 could have been earned based on the ex-post program when compared with the observed results.

Formula (5-10) Total Quantity Difference^T = $u(x^o - x^a)$

$$\$3.08 = [\$30.80, \$38.05] \left[\begin{bmatrix} 5.6 \\ 0 \end{bmatrix} - \begin{bmatrix} 5.5 \\ 0 \end{bmatrix} \right]$$

Formula (5-12) Total Quantity Difference^{lp}

$$= [(h^q x^o) - (h^q x^p)] - [h^a x^p) - (h^a x^a)]$$

Quantity Difference

$$= \left[[\$4.00, \$2.00] \begin{bmatrix} 5.6 \\ 0 \end{bmatrix} \right] - \left[[\$4.00, \$2.00] \begin{bmatrix} 0 \\ 5.56 \end{bmatrix} \right]$$

$$+ \left[\$4.00, \$2.00] \begin{bmatrix} 0 \\ 5.56 \end{bmatrix} \right] - \left[[\$4.00, \$2.00] \begin{bmatrix} 5.5 \\ 0 \end{bmatrix} \right]$$

$$\$.40 \quad = \quad [\$11.28] \quad + \quad [\$-10.88]$$

The \$10.88 would be attributed to changes in the optimal programs. The \$11.28 quantity variance is attributed to differences between the optimal <u>ex-post</u> and the observed results.

Formula (5-13) Total Capacity Difference = $(h^p x^o) - (h^p x^p)$

$$-\$3.75 = \left[[\$10.60, \$11.35] \begin{bmatrix} 5.6 \\ 0 \end{bmatrix} \right] - \left[[\$10.60, \$11.35] \begin{bmatrix} 0 \\ 5.56 \end{bmatrix} \right]$$

2. Post Optimality Analysis

The post optimality analysis in this section is based on the first example shown in Exhibits 5-1 and 5-2. In the final simplex tableau, slack variable s_1 is not in the basis and its corresponding opportunity cost $(z_j - c_j)$ equals \$2.50. Assume an additional unit of capacity was made available in process 1.

The optimum output would then equal

$$b_1/a^{-1}_{1,1} = 9.8/1.6 = 6.125 \text{ units of } x_1$$

Total Contribution Margin = 6.125 x \$4.00	=	\$24.50
Less original total contribution margin	=	22.00
Increase in contribution margin		\$ 2.50

The increase in the contribution margin equals the opportunity cost associated with the slack variable s_1.

The range over which the \$2.50 opportunity cost would be relevant while increasing the availability of capacity in process 1 would equal:

minimum positive value of the ratio of $b^*_i/(-1)y^*_{ir}$

which in this case is

$$= 10.4/(-1)4.5 = 2.31 .$$

This means that 2.31 units of capacity in process 1 could be added and the contribution margin would increase by \$2.50 for every unit added. After 2.31 units are added, the resource corresponding to slack would exit from the basis.

If additional units of capacity were removed from process 1, the total contribution margin would decrease by $2.50 for each unit removed. Since there is no positive value of the ratio $b_i^*/y_{i,r}$, then the relevant range of the opportunity cost has no lower limit.

The opportunity cost $(z_j - c_j)$ associated with the v_2 vector equals $.75. It equals the contribution margin that would be foregone if a unit of v_2 were produced. For example, if one unit of v_2 is produced, then 4.8125 units of v_1 can be produced. The total contribution margin would equal

4.8125	x	$4	=	$19.25
1	x	$2	=	2.00
Total Contribution Margin			=	$21.25

The total contribution margin has decreased by $.75 which equals the opportunity cost associated with the v vector.

3. Control System Example

In this example, a control system employing the opportunity costs provided by a linear programming model will be demonstrated. The assumptions are all taken from the first example in Exhibit 5-1. Three cases will be considered and the data is summarized in Exhibit 5-4.

Formula (5-14) $p = q^A$

$p = [\$4.00, \$2.75]$

Case One Assume the observed output of the system equalled $\begin{bmatrix} 5.5 \\ 0 \end{bmatrix}$. The output from the control system is summarized in Exhibit 5-4. Since all of the column totals are equal, the conclusion would be that the system was operated in an optimal manner.

Case Two Assume the observed output of the system equalled $\begin{bmatrix} 4.8125 \\ 1 \end{bmatrix}$. The output of the control system is summariced in Exhibit 5-4. Since all of the column totals are not equal, the system was not operated according to the optimal plan. Since the total of column one equals the total of column two, all of the resources that constrain the output of the system were used. Since the total of column three does not equal the total of column three, the output of the system was not in accordance with the optimal plan. The seventy-five cent reduction was caused by the output from process two.

Case Three Assume the observed output of the system equalled $\begin{bmatrix} 5.5 \\ 0 \end{bmatrix}$. Since the observed output equalled the expected outputs and the columns are not in balance, there must have been a shift in the marginal contribution. Therefore an ex-post program would be solved. The results of the ex-post program indicate the \$4.70 was lost because of a failure of the system to adjust to the operating con-

EXHIBIT 5-4

Summary of Control System Information

	Expected Marginal Contribution $h_i \; x_k^a$ (1)	Opportunity Cost Charged to Output $p_i \; x_i$ (2)	Marginal Contribution $x_i^b \; h_i^o$ (3)	Difference $(p_i-h_i^o)x_i$ (2)-(3) (4)
Case 1				
Process 1	$22	$22	$22	$ 0
Process 2	0	0	0	0
Total	$22	$22	$22	$ 0
Cast 2				
Process 1	$22	$19.25	$19.25	$0.00
Process 2	$ 0	2.75	2.00	.75
Total	$22	$22.00	$21.75	$ .75
Case 3 - Ex-Ante				
Process 1	$22	$22	$58.30	-$36.30
Process 2	0	0	0	0
Total	$22	$22	$58.30	$36.30
Case 3 - Ex-Post				
Process 1	$ 0	$63	$58.30	$4.70
Process 2	63	0	0	0
Total	$63	$63	$58.30	$4.70

ditions encountered during the production period.

4. Allocation of Fixed Overhead

In this example, fixed overhead costs will be allocated based on the opportunity costs generated by a linear programming model. The assumptions are identical to those in the first example of this chapter.

Formula (5-15) $k = w/hx$

$k = 10/11$

Formula (5-14) $p = q^A$

$p = [\$4.00, \$2.75]$

Allocated Overhead product one = \$3.64

product two = \$2.50

Output of $\begin{bmatrix} 5.5 \\ 0 \end{bmatrix}$, allocated overhead would equal

5.5 x \$3.64 = \$20.00

Solution of (5-16), x^a would equal $\begin{bmatrix} 5.5 \\ 0 \end{bmatrix}$, therefore allocation has not influenced <u>ex-ante</u> optimal solution.

Summary

The purpose of this chapter was to present the information that could be obtained for planning and control purposes from a linear programming model. As a planning model, linear programming determines the output that will maximize the contribution margin within the constraints of teh mass production system. The opportunity costs of the constraints that limit the output of the system are also determined.

For control purposes, linear programming would make possible the evaluation of management planning. The opportunity cost of what "should" have been the program as opposed to the actual production program for the period could be determined. The cost differences that could be obtained from a linear programming model are in some respects quite similar to the differences obtwined from the traditional costing model. However, the emphasis in the linear programming differences is in making comparisons between the ex-ante and ex-post optimal production programs.

However, linear programming is not a product costing model. It requires estimates of unit output costs so that an optimal program can be determined. The similarity of the information required by the input-output and linear programming models is quite pronounced. The two models could form a rather sophisticated process cost accounting system. Applications of linear programming would however, be limited to multi-product systems. Systems with only one output certainly would not require linear programming to determine the optimal output.

FOOTNOTES

[1]First described in:
George B. Dantzig, "The Programming of Interdependent Activities II Mathematical Model" Econometrica Vol. 17, No's 3 and 4 (July, October 1949), pp. 200-211.

[2]The y variables of the opportunity costs of employing the various resources. The costs are not related to conventional accounting costs.

[3]The discussion of the three fundamental theorems of linear programming is based on material in:
Michael D. Intriligator, Mathematical Optimization and Economic Theory (Englewood Cliffs, New Jersey: Prentice-Hall, 1971), p. 120-135.

[4]The first accounting application of goal programming was suggested in: A. Charnes, W.W. Cooper and Y. Ijiri, "Breakeven Budgeting and Programming to Goals," Journal of Accounting Research (Spring, 1963) pp. 16-43.

[5]Yuji Ijiri, Management Goals and Accounting for Control (Amsterdam: North-Holland Publishing Co., 1965) p. 36.

[6]Ibid., pp. 44-50.

[7]Much of the material presented in this section is based on:
Joel Demski, "Variance Analysis Using A Constant L-P Model," in Studies In Cost Analysis, ed. by David Solomons (Homewood, Illinois: Richard D. Irwin, Inc., 1968), pp. 526-551.

[8]The vector u equals the standard unit output costs. A quantity variance as normally determined in accounting would equal the difference between the actual and standard quantities of a resource input times the standard unit cost. (refer to formula (3-8) in Chapter III). The total quantity variance for a production period would equal the sum of the quantity variances for the individual resource inputs. The quantity variance as determined in formula (4-36) equals the foregone contribution margin because of quantity deviations in the output.

[9] The elements in the p vector would equal the elements in the $(z_j - c_j)$ row in the final simplex tableau from $(z_1 - c_1)$ to $(z_n - c_n)$.

[10] The original concept was first described in:

Robert S. Kaplan and Gerald L. Thompson, "Overhead Allocation via Mathematical Programming Models," <u>Accounting Review</u> Vol. XLVI No. 2 (April 1971), pp. 352-364.

The model developed by Kaplan and Thompson is based on what generally would be considered the dual solution of the linear programming problem. The model developed here is based on the primal solution.

CHAPTER VI

A STOCHASTIC PROCESS COSTING MODEL

Introduction

The pattern of the cost flows within a process costing system is a function of the product flows within the corresponding production system. In a deterministic production system, all units would pass in a common sequence through the processes and exit from the system as final product. However, most mass production systems could not be classified as deterministic.

Any unit entering a production system could generally be expected to exit the system in one of several forms. Even units that exit the system in the same form may not have cycled through the production processes in identical sequences. Often additional processing or recycling is employed to salvage partially completed but defective units. The process costing model that will be developed in this chapter is based on the premise that product flows in a mass production system could be described stochastically.

The model and assumptions will be described in the initial sections of the chapter. The standard costs will be integrated into the model for the purpose of costing output and inventory. The final sections of the chapter will treat cost analysis that would be possible based on the stochastic model.

The Stochastic Model

Assume an n process production system with m ways in which a unit could exit from the system. The physical input coefficients can only take on values of zero and one and the unit transfers within the production system are described by a stochastic matrix P.[1] The matrix would be square with dimensions (n+m) and each element p_{ij} would indicate the probability of a unit transferring to state j given that it has completed processing in state i.

The states include the production processes and the ways by which a unit could exit from the system. The probability of a transition of a unit from one state to another state is dependent only upon the completion of the production activity in the most recent process. The elements of each row in the P matrix sum to one because they exhaust all possibilities. If a state i represented a way of exiting from the system, the element p_{ii} would equal one. This would be an absorbing state and once entered, a unit could no longer cycle through the system. The P matrix would take the general form:

$$P = \left[\begin{array}{cccc|cccc} P_{1,1} & P_{1,2} & \cdot\;\cdot & P_{1,n} & P_{1,n+1} & P_{1,n+2} & \cdot & P_{1,n+m} \\ P_{2,1} & P_{2,2} & \cdot\;\cdot & P_{2,n} & P_{2,n+1} & P_{2,n+2} & \cdot & P_{2,n+m} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ P_{n,1} & P_{n,2} & \cdot\;\cdot & P_{n,n} & P_{n,n+2} & P_{n,n+2} & \cdot & P_{n,n+m} \\ \hline 0 & 0 & \cdot\;\cdot & 0 & 1 & 0 & \cdot & 0 \\ 0 & 0 & \cdot\;\cdot & 0 & 0 & 1 & \cdot & 0 \\ \vdots & \vdots & \vdots\;\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdot\;\cdot & 0 & 0 & 0 & \cdot & 1 \end{array}\right]$$

The matrix could also be relabelled as:[2]

$$P = \begin{array}{c} \begin{array}{cc} n & m \end{array} \\ \left[\begin{array}{c|c} Q & R \\ \hline \emptyset & I \end{array}\right] \end{array} \begin{array}{c} \\ n \\ m \end{array}$$

The elements of the Q submatrix would indicate the expected transfers among the production processes. In most systems, a majority of the non-zero elements in the Q matrix would be located just to the right of the main diagonal. The basis for this observation is that the output of most mass production systems is sequentially processed. If the elements of any row in the Q matrix summed to one, this would indicate that the corresponding process i did not produce for outside consumption. In a case where only one row did not sum to one, it normally would be the n^{th} row because of sequential processing. In most rows of the Q matrix the sum of the elements to the right of the main diagonal would be expected to be greater than the sum of the remaining probabilities. If this were not the situation, the indication would be that a significant amount of recycling was anticipated within the system.

The elements in the R sub-matrix indicate which production processes communicate directly with the absorbing states.[3] Since by-products and the final product would be expected to exit the system from a particular process, most of the columns in the R matrix would have only one non-zero element. If separate states were established for defective units exiting from each process, then all of the columns in the R matrix would contain only one non-zero element. Unless the activity in a particular process resulted in more than one class of exiting unit, there would be only one non-zero element in each row of the R matrix. If a process did not communicate with any of the absorbing states, all of the elements in the corresponding row of the R matrix would equal zero.

The I submatrix would be an identity matrix. Once a unit entered one of the absorbing states, it could not be transferred to any state within the system. All of the elements in the Ø submatrix would equal zero indicating that the absorbing states do not communicate with the production processes.

The mean number of times a unit would pass through a process before exiting the system given the entering process was known is given by[4]

$$F = (I-Q)^{-1}. \tag{6-1}$$

An element f_{ij}, of the F or fundamental matrix, would equal the average number of times a unit would be in process j before exiting from the system given that the unit entered the system in process i. The probability that a unit starting in a particular process will exit from the system in a particular state is given by a matrix B where[5]

$$B = FR. \tag{6-2}$$

Elements of the i^{th} row of the B matrix would contain the probabilities that a unit would exit the production system as final product or some other form given that the unit entered the system through process i.

The stochastic model will initially be considered as a planning model and time will be taken into account in two distinct ways. In one case, termed the production period, the time dimension will be a production period of any specified length. It will be assumed that the beginning inventory levels are known and that the ending inventory levels and desired outputs are prescribed by management. When production is completed on a unit, the unit will be transferred and the tranfers will be governed by some stochastic matrix. The assumption implicit to this situation is that sufficient capacity is available in each process to handle the expected production plus any variations that might occur during the period. Under these conditions, it will not be possible to discuss any probable inventory levels during the production period because the state of the system is indeterminant. This same situation exists when the linear-programming and input-output models are employed.

However, with the stochastic model, time can be handled in another manner termed the transfer period. The assumption made in this case is that all transfers take place at uniform time intervals during the production period. The duration of the time interval would depend on the characteristics of the production system. In a school it would be the length of a semester, in a hospital, it might be a day, and in production systems it could vary from several minutes to days, weeks, or even months. One primary consideration would be the expected production times in each of the processes. It would not be necessary that they all be equal in that dummy or holding states could be introduced to balance the system.

If the same number of units enter the production system at the beginning of each production period, then after several periods the inventories and outputs of the system would reach a steady state. The derivation of this steady state information will be central when the transfer period model is developed. The transfer and production period models will be considered in separate sections where the emphasis will be placed on planning information. Then the costing of inventories and other analysis based on the stochastic model will be entertained in the remaining sections.

Assumptions of the Stochastic Model

The assumptions underlying the stochastic process costing model will be presented in summary form. No mention will be made of the behavioral assumptions identified in Chapter III since they would universally apply for all of the models.

1. Linear Cost Functions: The cost of the external inputs can be represented as a linear function of the quantities employed. Prime costs would take the general form y=a(x) and overhead costs would take the general form y=a(x)+b. The overhead costs are divisible into a fixed component that does not vary with volume and a variable component that varies with volume.

2. System Technology: The production system consists of n processes and each process has a production function that is linear and propor-

tional. The probability distribution that describes the expected outputs from a process is discrete and is assumed to be stationary over the production period. These distributions are assumed to be dependent only on the activity in the process whose output they describe. Production in one process is assumed to be independent of the activity in the other processes. This condition eliminates the consideration in the model of the scheduling of the flows of outputs between the processes. The capacities of the processes are assumed to be balanced and it is assumed that management would not schedule production that would exceed the capacity of the system.

3. System Inputs and Outputs: The inputs and outputs of the production system are not restricted to integer quantities. Each process produces at least one measurable product for either internal or external consumption. If a process receives material from another process, the flow is tracked in the transfer matrix as a single unit of completed product rather than as a compilation of various components. When said detail is needed, a parts explosion matrix may be introduced separate from the Markov chain analysis.

4. Stochastic Transfer Matrix: Because of the stochastic production functions the expected unit transfers within the system can be described by a stochastic matrix. The stochastic matrix can be assumed to be stationary under normal conditions over the production period. The transfer probabilities of a unit are dependent upon the process from which the unit is being transferred.
5. Products: The mass production system has one principal product and all other outputs would be considered by-products.

Transfer Period Model[6]

Let there by ψ transfer periods in each production period and let the n element row vector k contain the new units started into production at the beginning of each transfer period. In many applications all of the units would likely enter the system through the first process, but situations might be encountered where the units would enter in several processes. If the superscript ss designated steady state information, then let the n element column vector π^{ss} equal the steady-state in-process inventories where

$$\pi_{tr}^{ss} = kF. \tag{6-3}$$

The variance of the steady state in-process inventory levels can be determined. Let the vector η be the initial probability vector of a Markov chain where $\eta = (1/\sum_{i=1}^{n} k_i)k$. In a case where all of the units entered the initial process, the first element of the probability vector would equal one. The upper bound formula for the variances of the steady state inventories is[7]

$$Var(\pi^{ss}) \leq ka(\eta F - \eta_{sq}(I-Q_{sq})^{-1}) \qquad (6-4)$$

where a is an n element sum vector and the subscript sq indicates that each element is the appropriate matrix is squared.

The means and variances of the expected outputs from the steady state in process inventories can also be identified. The expected output that would be realized if production was completed on the inventories would equal

$$y^{\pi} = \pi^{ss}B \ . \qquad (6-5)$$

To determine the variances of the expected outputs from the steady-state in process inventories, the n element probability vector σ is required where $\sigma = (1/\sum_{i=1}^{n} \pi_i^{ss})\pi^{ss}$. The variances of the expected outputs from the inventories are

$$Var(y^{\pi}) = \pi^{ss}a(\sigma B-(\sigma B)_{sq}) . \qquad (6-6)$$

The steady state outputs for each transfer period are

$$y^{ss} = kR \qquad (6-7)$$

and the upper bound formula for the variances of the steady state outputs is

$$Var(y^{ss} \leq ka(\eta B-\eta_{sq}(I-Q_{sq})^{-1}R_{sq}) \ . \qquad (6\text{-}8)$$

The expected unit transfers at the conclusion of each period can be determined from the P matrix. The expected transfers could be determined employing matrix algebra by altering the inventory vector; however, a more direct means using algebra will be specified Let Θ be an n by (n+m) matrix where each element θ_{ij} would equal the expected unit transfers from state i to state j. Each element of the matrix would be determined by

$$\theta_{ij}=\pi_i^{ss}P_{ij} \qquad \begin{array}{l} i = 1 \text{ to } n \\ j = 1 \text{ to } n+m \end{array} \qquad (6\text{-}9)$$

When the Θ matrix is partitioned as shown below, the mapping of the expected transfers can be better understood.

$$\begin{array}{cc} n & m \end{array}$$

$$\left[\begin{array}{c|c} \text{Expected} & \text{Expected} \\ \text{Transfers} & \text{Outputs} \end{array}\right] n$$

The vector of expected resource requirements from a transfer period equals

$$\ell = E\pi^{ss} \tag{6-10}$$

and the standard cost of manufacturing would be

$$\text{Standard Cost of Manufacturing} = s\pi^{ss} \ . \tag{6-11}$$

where the activity cost vector s is determined by (4-16).

Costs could be attached to either or all of the outputs or only the principle product. The value of the transfer cost vector d would vary depending upon which condition was encountered. In both cases, the standard cost of the steady state inventories would equal

$$g = d\pi^{ss} \ . \tag{6-12}$$

In conclusion, if δ_{ij} equalled the expected cost of the units transferred from state i to state j and costs were attached to all outputs, then

$$\delta_{ij} = \theta_{ij}\, d_i \quad \text{for } \begin{array}{l} i=1 \text{ to } n \\ j=1 \text{ to } n+\pi \end{array} \ . \tag{6-13}$$

If costs were only attached to the final product, then

$$\delta_{ij} = \theta_{ij}\, d_i \quad \text{for } \begin{array}{l} i=1 \text{ to } n \\ j=1 \text{ to } n \end{array} \tag{6-14}$$

and

$$\delta_{n,n+1} = \theta_{n,n+1} u_n \tag{6-15}$$

where u_n is the standard output cost of the final product.

Therefore, from the transfer period model, the steady state inventory levels, outputs, production costs and unit transfers could be obtained. The variances and, of course, the standard deviations of the inventories and outputs could be determined. An example will now be presented to clarify the material discussed to this point.

Example of a Transfer Period Model

The assumptions which serve as a basis for this example are contained in Exhibit 6-1. From 6-1 and 6-2, the F and B matrices are

$$F = \begin{bmatrix} 1 & 1 & 3/4 \\ 0 & 1 & 3/4 \\ 0 & 0 & 1 \end{bmatrix} ; \qquad B = \begin{bmatrix} 9/16 & 7/16 \\ 9/16 & 7/16 \\ 3/4 & 1/4 \end{bmatrix} .$$

The steady state inventories, variances, and standard deviations by (6-3) and (6-4) are

$$\pi^{ss} = \begin{bmatrix} 100 \\ 100 \\ 75 \end{bmatrix} ; \quad \text{Var}(\pi^{ss}) \leq \begin{bmatrix} 0 \\ 0 \\ 75/4 \end{bmatrix} ; \quad \text{Std.Dev.}(\pi^{ss}) \leq \begin{bmatrix} 0 \\ 0 \\ 4.33 \end{bmatrix} .$$

EXHIBIT 6-1

Assumptions of Transfer Period Model Example

number of production processes $n = 3$

number of exiting states $m = 2$

number of direct resource inputs $z = 4$

resource input matrix $E = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 3 & 0 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

standard unit cost vector c = [\$1.00,\$2.00,\$3.00,\$1.50]

By (4-16) unit activity cost vector s = [\$12.50,\$13.50,\$10.50]

By (6-34) standard transfer costs d = [\$12.50,\$26.00,\$36.50]

By (6-35) standard transfer costs d = [\$12.50,\$26.00,\$45.18]

By (6-36) standard output costs u = [\$12.50,\$34.67,\$60.22]

starting unit vector k = [100,0,0]

stochastic matrix $P = \left[\begin{array}{ccc|cc} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3/4 & 0 & 1/4 \\ 0 & 0 & 0 & 3/4 & 1/4 \\ \hline 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$

The expected outputs, variances, and standard deviations of the in-process inventories by (6-5) and (6-6) are

$$y^{\pi} = \begin{bmatrix} 168.75 \\ 106.25 \end{bmatrix}; \ \mathrm{Var}(y^{\pi}) = \begin{bmatrix} 66.275 \\ 66.275 \end{bmatrix}; \ \mathrm{Std\ Dev}\ (y^{\pi}) = \begin{bmatrix} 8.1 \\ 8.1 \end{bmatrix}$$

The steady state outputs, variances, and standard deviations by (6-7) and (6-8) are

$$y^{ss} = \begin{bmatrix} 56.25 \\ 43.75 \end{bmatrix}; \ \mathrm{Var}(y^{ss}) \leq \begin{bmatrix} 111.1 \\ 111.1 \end{bmatrix}; \ \mathrm{Std\ Dev}\ (y^{ss}) \leq \begin{bmatrix} 10.5 \\ 10.5 \end{bmatrix}$$

The steady-state transfer matrix by (6-12) would be

$$\Theta = \begin{bmatrix} 0 & 1000 & 0 & 0 & 0 \\ 0 & 0 & 75 & 0 & 25 \\ 0 & 0 & 0 & 56.25 & 18.75 \end{bmatrix} .$$

The vector of expected resources by (6-10) would equal

$$\ell = \begin{bmatrix} 200 \\ 600 \\ 525 \\ 275 \end{bmatrix}$$

and by (6-11) the standard cost of manufacturing is $3,387.50.

If costs were attached to all the outputs, the activity cost vector d equalling [$12.50, $26.00, $36.50] would be used and by (6-12) the inventories would be costed at $6,587.50. The expected transfer cost matrix by (6-13) would equal

$$\delta = \left[\begin{array}{ccc|cc} 0 & \$1250 & 0 & 0 & 0 \\ 0 & 0 & \$1950 & 0 & \$650 \\ 0 & 0 & 0 & \$2053.125 & \$684.375 \end{array}\right].$$

The total cost of the outputs equals the cost of production which would be expected at the steady state. The cost reconciliation is shown in Exhibit (6-2).

If costs were only attached to the final product, the activity cost vector d equalling [$12.50, $26.00, $45.18] would be used and by (6-12) the inventories would be costed at $7,238.50. The expected transfer cost matrix by (6-13) and (6-14) would equal

$$\delta = \left[\begin{array}{ccc|cc} 0 & \$1250 & 0 & 0 & 0 \\ 0 & 0 & \$2600 & 0 & 0 \\ 0 & 0 & 0 & \$3387.50 & 0 \end{array}\right].$$

As would be expected, the cost of the output equals the cost of production. The cost reconciliation is shown in Exhibit (6-3).[8] The production period model will be presented in the next section.

EXHIBIT 6-2

Transfer Period Reconciliation

	Process 1	Process 2	Process 3	Total
Inputs				
Beginning Inventory	$1,250.00	$2,600.00	$2,737.50	$6,587.50
Exogenous Inputs	1,250.00	1,350.00	787.50	3,387.50
Inter-Process Transfer	0.00	1,250.00	0.00	1,250.00
Inter-Process Transfer	0.00	0.00	1,950.00	1,950.00
Inter-Process Transfer	0.00	0.00	0.00	0.00
Total Inputs	$2,500.00	$5,200.00	$5,475.00	$13,175.00
Outputs				
Inter-Process Transfer	$ 0.00	$ 0.00	$ 0.00	$ 0.00
Inter-Process Transfer	1,250.00	0.00	0.00	1,250.00
Inter-Process Transfer	0.00	1,950.00	0.00	1,950.00
Product 1	0.00	0.00	2,553.125	2,553.125
Product 2	0.00	650.00	684.375	1,334.375
Ending Inventory	1,250.00	2,600.00	2,737.50	6,587.50
Total Outputs	$2,500.00	$5,200.00	$5,475.00	$13,175.00

Only final product costed

EXHIBIT 6-3

Transfer Period Cost Reconciliation

	Process 1	Process 2	Process 3	Total
Inputs				
Beginning Inventory	$1,250.00	$2,600.00	$3,388.50	$7,238.50
Exogenous Inputs	1,250.00	1,350.00	787.50	3,387.50
Inter-Process Transfer	0.00	1,250.00	0.00	1,250.00
Inter-Process Transfer	0.00	0.00	2,600.00	2,600.00
Inter-Process Transfer	0.00	0.00	0.00	0.00
Total Inputs	$2,500.00	$5,200.00	$6,776.00	$14,476.00
Outputs				
Inter-Process Transfer	$ 0.00	$ 0.00	$ 0.00	$ 0.00
Inter-Process Transfer	1,250.00	0.00	0.00	1,250.00
Inter-Process Transfer	0.00	2,600.00	0.00	2,600.00
Product 1	0.00	0.00	3,387.50	3,387.50
Ending Inventory	1,250.00	2,600.00	3,387.50	7,238.50
Total Outputs	$2,500.00	$5,200.00	$6,775.00	$14,476.00

All outputs costed

Production Period Model

In the production period model, it is required that the beginning inventory levels are known and that the ending inventory levels and desired outputs are prescribed by management. The transfers are assumed to be governed by a stochastic matrix P and when a unit is completed in a process it is immediately transferred. The capacities of each process are assumed to be sufficient to handle the planned production. Another dimension is added to the problem with the production period model. Since the ending inventories levels and outputs are specified, the expected number of units to be started into production must be determined.

Two cases relating to planned changes in the levels of the production period model will not involve inventories. They will be studied in a subsequent section.

1. No In-Process Inventories

The elements of row λ in the fundamental matrix F contain the expected number of units of activity necessary in each process to produce a unit of final product with a probability of $b_{\lambda,1}$ given that the unit entered the system in process λ. Let H be a matrix with diagonal entries b_{ij}. If v units of final product were required during a production period, then the expected number of units that would have to be cycled through the system would equal

$$k = v\eta H^{-1} \tag{6-16}$$

Let x be an n element row vector where each element would contain the expected number of units processed in the process while producing v units of final product. The x vector would equal

$$x = kF. \tag{6-17}$$

The variances of the expected activity levels are[9]

$$\text{Var}(x) = ka(nF(2F_{dg}-I)-(nF)_{sq}). \tag{6-18}$$

The expected outputs for the production period would be

$$y = xR \tag{6-19}$$

The first element of the y vector would equal the expected output of final product (i.e., v). The variances of the expected outputs are[10]

$$\text{Var}(y) = ka(nB-(nB)_{sq}). \tag{6-20}$$

The expected inter-process transfers would be determined by the algebraic expression

$$\theta_{ij} = x_i p_{ij}. \tag{6-21}$$

The expected resource requirements for the production period could be determined by employing formula (4-10).

2. In-Process Inventories

Two cases concerning planned changes in the levels of in-process inventories will be studied. They can be represented as

Case 1 $\pi^t \geq \pi^{t+1}$

Case 2 $\pi^t \leq \pi^{t+1}$.

Instances where there are planned increases and decreases in the inventory levels would require more detailed analysis than will be demonstrated here. As has been indicated, when no change in the inventory levels is planned, the formulas for case 1 or case 2 could be used.

The expected number of units to be started into production to realize v units of final product and the planned inventory change equals[11]

Case 1 $k = (v\eta - (\pi^{t} - \pi^{t+1})H)H^{-1}$. (6-22)

No general expression was found for the case when the inventories are incerased. Each element of the k vector could be determined algebraically by

Case 2 $k_j = vn_j/b_{j,1} + \sum_{i=1}^{n} ((\pi_i^{t+1} - \pi_i^{t})/f_{\lambda i})$ for j=1 to n (6-23)

when all units causing the increase in the inventories enter process λ. The expected number of units to be transferred from the processes during the production period is[12]

Case 1 $x = (k + (\pi^{t} - \pi^{t+1}))F$ (6-24)

Case 2 $x_j = (vn_j/b_{j,1})f_{\lambda j} + (\sum_{i=j+1} (\pi_i^{t+1} - \pi_i^{t})/f_{\lambda i})f_{\lambda j}$

for j = 1 to n. (6-25)

Again, no general expression was found for the case when the inventories are increased.

To determine the variances of the expected transfers, a probability vector ρ and a variable q must be determined The variable q indicates the total number of units that would be processed in the system. In summary the values of q would be

Case 1 $\pi^t \geq \pi^{t+1}$ $q = ka + \sum_{i=1}^{n} \pi_i^t - \pi_i^{t+1}$

Case 2 $\pi^t \leq \pi^{t+1}$ $q = ka.$

The probability vector ρ refers to the source of the activity within the system. Summarizing, the values of ρ are

Case 1 $\pi^t \geq \pi^{t+1}$ $\rho_i = (k_i + \pi_i^t - \pi_i^{t+1})/k_q$

Case 2 $\pi^t \leq \pi^{t+1}$ $\rho_i = k_i/q$.

The variances of the expected transfers during the production period are

$$Var(x) = q(\rho F)2F_{dg} - I) - (\rho F)_{sq}). \qquad (6-26)$$

When the inventories are increased, the variances in some cases could be overstated. This is caused by the fact that the units entering the system for the purpose of increasing the inventories were treated as if they were completely processed through the system.

The expected outputs would be determined by formula (6-19). The variances of the expected outputs equal

$$Var\ y) = g(\rho B - (\rho B)_{sq}). \qquad (6-27)$$

In the case where there is a planned increase in the in-process inventories, the variances would be overstated. This results from the fact that k includes the units necessary to realize the expected outputs plus the increases in the inventories. Since no general expression was found to eliminate this condition, the variances would be overstated.

The expected requirements of resource j equal

Case 1 $$\ell_j = \sum_{i=1}^{n} x_i e_{ji} - ((\pi_i^t - \pi_i^{t+1}) e_{ji} \alpha_{ji}^t) + \pi_i^{t+1} (\alpha_{ji}^{t+1} - \alpha_{ji}^t) e_{ji} \quad \text{for } j = 1 \text{ to } z \qquad (6\text{-}28)$$

Case 2 $$\ell_j = \sum_{i=1}^{n} x_i e_{ji} + ((\pi_i^{t+1} - \pi_i^t) e_{ji} \alpha^{t+1}) + \pi^{t+1} (\alpha_{ji}^{t+1} - \alpha_{ji}^t) e_{ji}$$

$$\text{for } j = 1 \text{ to } z \qquad (6\text{-}29)$$

The expected cost incurred or allocated to process j during the production period equals

Case 1 $$\tau_j = \sum_{i=1}^{z} c_i (e_{ij} x_j + ((\pi_j^t - \pi_j^{t+1}) e_{ij} \alpha_{ij}^t) + \pi_j^{t+1} (\alpha_{ij}^{t+1} - \alpha_{ij}^t) e_{ij})$$

$$\text{for } j = 1 \text{ to } n \qquad (6\text{-}30)$$

Case 2 $$\tau_j = \sum_{i=1}^{z} c_i (e_{ij} x_j + ((\pi_j^{t+1} - \pi_j^t) e_{ij} \alpha_{ij}^{t+1}) + \pi_j^{t+1} (\alpha_{ij}^{t+1} - \alpha_{ij}^t) e_{ij})$$

$$\text{for } j = 1 \text{ to } n \qquad (6\text{-}31)$$

The in-process inventories would be costed by

$$g = d\pi^t \quad + \quad (\pi_j^t \sum_{i-1}^{z} \alpha_{ji}^t c_i e_i) \qquad (6\text{-}32)$$

and the expected unit transfers would be determined by the formula

$$\epsilon_{ij} = x_k P_{ij} \quad \text{for } \begin{matrix} i = 1 \text{ to } n \\ j = 1 \text{ to } n+m \end{matrix} . \qquad (6\text{-}33)$$

The expected transfer costs would be determined by formulas (6-13) or (6-14) and (6-15) depending on the costing of the outputs.

From the production period model the expected outputs, activity levels, resource requirements, production costs and unit transfers can be obtained. The variances of the activity levels and the outputs can be determined. Another example will be developed to demonstrate the production period model.

Example of a Production Period Model

The assumptions which serve as a basis for this example are contained in Exhibit (6-1) except that the k and d vectors do not pertain. It is assumed that 9 units of final product are to be produced during the production period and that all costs are allocated to the final product. The assumptions relating to the inventories are contained in Exhibit (6-4).

By (6-22) k, the expected number of units that would be started in process equals [37/3, 0, 0]. Formulas (6-24) and (6-26) give the expected transfers and the respectives variances that equal

$$x = \begin{bmatrix} 37/3 \\ 40/3 \\ 12 \end{bmatrix}; \quad \mathrm{Var}(x) = \begin{bmatrix} 2.45 \\ 1.73 \\ 2.56 \end{bmatrix}; \ \mathrm{Std.Dev.}(x) = \begin{bmatrix} 1.55 \\ 1.3 \\ 1.6 \end{bmatrix}.$$

The means, variances, and standard deviations of the expected outputs are given by (6-19) and (6-27)

$$y = \begin{bmatrix} 9 \\ 19/3 \end{bmatrix}; \ \mathrm{Var}(y) = \begin{bmatrix} 3.709 \\ 3.709 \end{bmatrix}; \ \mathrm{Std.\ Dev}(y) = \begin{bmatrix} 1.93 \\ 1.93 \end{bmatrix}.$$

The ℓ vector of expected resource requirements is given by (6-28) and the expected cost by process is given by (6-30).

EXHIBIT 6-4

Assumptions of Production Period Model Example

starting process for all units $\lambda = 1$

units of output of final product $v = 9$

in-process inventories $\pi^t = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$; $\pi^{t+1} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

completion estimates $\alpha^t = \begin{bmatrix} .5 & 0 & 0 \\ .5 & .33 & 0 \\ .5 & .5 & .33 \\ .5 & .5 & .5 \end{bmatrix}$; $\alpha^{t+1} = \begin{bmatrix} .5 & 0 & 0 \\ .5 & .67 & 0 \\ .5 & .5 & .67 \\ .5 & .5 & .5 \end{bmatrix}$

By (6-37) standard transfer costs d = [\$0, \$12.50,\$34.67]

By (6-36) standard output costs u = [\$12.50,\$34.67,\$60.22]

$$\ell = \begin{bmatrix} 74/3 \\ 77 \\ 73 \\ 217/6 \end{bmatrix} ; \quad \tau = [\$154.17, \$176.25, \$121.50]$$

By (6-32) the beginning and ending in-process inventories would be costed at $164.25 and $74.17 respectively. The expected unit transfers by (6-33) would equal

$$\Theta = \left[\begin{array}{ccc|cc} 0 & 37/3 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 10/3 \\ 0 & 0 & 0 & 9 & 3 \end{array}\right].$$

By (6-14) and (6-15), the expected transfer costs would equal

$$\delta = \left[\begin{array}{ccc|cc} 0 & \$154.17 & 0 & 0 & 0 \\ 0 & 0 & \$346.67 & 0 & 0 \\ 0 & 0 & 0 & \$542 & 0 \end{array}\right].$$

A cost reconciliation is shown in Exhibit (6-5).

The in-process inventories in Exhibit (6-4) will be reversed and a case with a decrease in the inventories will be demonstrated. By (6-23) k, the expected number of units that would be started in process [59/3, 0, 0]. Formulas (6-23) and (6-24) give the expected transfers and the respective variances that equal

$$x = \begin{bmatrix} 59/3 \\ 56/3 \\ 12 \end{bmatrix} ; \; \mathrm{Var}(x) = \begin{bmatrix} 0 \\ 0 \\ 59/16 \end{bmatrix} ; \; \mathrm{Std.Dev}(x) = \begin{bmatrix} 0 \\ 0 \\ 1.975 \end{bmatrix}.$$

EXHIBIT 6-5

Production Period Cost Reconciliation

Decrease In Inventories

	Process 1	Process 2	Process 3	Total
Inputs				
Beginning Inventory	$ 12.50	$ 36.50	$115.25	$164.25
Exogenous Inputs	154.17	176.25	121.50	451.92
Inter-Process Transfer	0.00	154.17	0.00	154.17
Inter-Process Transfer	0.00	0.00	346.67	346.67
Inter-Process Transfer	0.00	0.00	0.00	0.00
Total Inputs	$166.67	$366.92	$583.42	$1,117.01
Outputs				
Inter-Process Transfer	$ 0.00	$ 0.00	$ 0.00	$ 0.00
Inter-Process Transfer	154.17	0.00	0.00	154.17
Inter-Process Transfer	0.00	346.67	0.00	346.67
Product 1	0.00	0.00	542.00	542.00
Ending Inventory	12.50	20.25	41.42	74.17
Total Outputs	$166.67	$366.92	$583.42	$1,117.01

The means, variances, and standard deviations of the expected outputs are given by (6-19) and (6-27).

$$y = \begin{bmatrix} 9 \\ 23/3 \end{bmatrix}; \quad Var(y) = \begin{bmatrix} 1239/256 \\ 1239/256 \end{bmatrix}; \quad Std.Dev.y = \begin{bmatrix} 2.2 \\ 2.2 \end{bmatrix}.$$

The ℓ vector of expected resource requirements is given by (6-29) and the expected cost by process is given by (6-31)

$$\ell = \begin{bmatrix} 118/3 \\ 115 \\ 95 \\ 311/6 \end{bmatrix}; \qquad \tau = [\$245.83, \$255.75, \$130.50]$$

By (6-32) the beginning and ending in-process inventories would be costed at \$74.17 and \$164.25 respectively. The expected unit transfers by (6-33) would equal

$$\Theta = \left[\begin{array}{ccc|cc} 0 & 59/3 & 0 & 0 & 0 \\ 0 & 0 & 14 & 0 & 14/3 \\ 0 & 0 & 0 & 9 & 3 \end{array}\right]$$

By (6-14) and (6-15), the expected transfer costs would equal

$$\delta \left[\begin{array}{ccc|cc} 0 & \$245.83 & 0 & 0 & 0 \\ 0 & 0 & \$485.38 & 0 & 0 \\ 0 & 0 & 0 & \text{\$54[illegible]} & 0 \end{array}\right]$$

A cost reconciliation is shown in Exhibit (6-6).

EXHIBIT 6-6

Production Period Cost Reconciliation

Increase In Inventories

Inputs	Process 1	Process 2	Process 3	Total
Beginning Inventory	$ 12.50	$ 20.25	$441.42	$ 74.17
Exogenous Inputs	245.83	255.75	130.50	632.08
Inter-Process Transfer	0.00	245.83	0.00	245.83
Inter-Process Transfer	0.00	0.00	485.33	485.33
Inter-Process Transfer	0.00	0.00	0.00	0.00
Total Inputs	$258.33	$521.83	$657.25	$1,437.41

Outputs	Process 1	Process 2	Process 3	Total
Inter-Process Transfer	$ 0.00	$ 0.00	$ 0.00	$ 0.00
Inter-Process Transfer	245.83	0.00	0.00	245.83
Inter-Process Transfer	0.00	485.33	0.00	485.33
Product 1	0.00	0.00	542.00	542.00
Ending Inventory	12.50	36.50	115.25	164.25
Total Outputs	$258.33	$521.83	$657.25	$1,437.41

Product Costing

The costing of outputs and inventories with the stochastic model can be accomplished in various ways. Two methods will be considered in this section. In the first method, costs will be allocated among all of the outputs of the system whereas in the second method, costs will only be allocated to the final product. The possibility of non-sequential transfers and multiple entry points are conditions that would complicate the allocation of costs. Initially systems with only one entry point and sequential transfers will be considered. Then the implications of multiple entry points and non-sequential transfers will be explored.

1. Transfer Period Model

In a sequential transfer system where all units enter through process λ and all outputs are costed, the vector of standard transfer costs would equal[13]

$$d_j = \sum_{i=\lambda}^{j} s_i \quad \text{for } j = \lambda \text{ to } n. \qquad (6\text{-}34)$$

In commercial systems, the attaching of costs to all outputs would be rare, but such information may be desired in a non-commercial application.

In most situations, costs are attached only to the final product of the system. In a single entry system with sequential transfers and only the final product costed, the standard transfer costs equal[14]

$$d_j = \sum_{i=\lambda}^{j} s_i / \prod_{i=\lambda}^{j} p_{i+1,i} \quad \text{for } j = 1 \text{ to } n. \qquad (6\text{-}35)$$

These costs would be used for costing inventories and inter-process transfers. The output of the system would be costed at

$$u_j = \left(\sum_{i=\lambda}^{j} f_{\lambda i} s_i \right) / \sum_{i=\lambda}^{j} p_{i,i+1} \quad \text{for } j = 1 \text{ to } n. \qquad (6\text{-}36)$$

These costs would be attached to the output of final products from process j. In most cases the output of final product would come from the last or n^{th} process.

2. Production Period Model

In a single entry system with sequential transfers where all of the outputs were costed, the standard transfer costs would equal[15]

$$d_j = \sum_{i=\lambda}^{j-1} s_i \quad \text{for } j = 2 \text{ to } n. \qquad (6\text{-}37)$$

The first term would equal zero since no units could be transferred into the first process. The output costs would equal

$$u_j = \sum_{i=\lambda}^{j} s_i \quad \text{for } i = 1 \text{ to } n. \qquad (6\text{-}38)$$

If costs were only attached to the final product of the system, the standard transfer costs would be[16]

$$d_j = \left(\sum_{i=1}^{j-1} f_{\ i} s_i \right) / f_{\lambda j} \quad \text{for } j = 2 \text{ to } n. \qquad (6\text{-}39)$$

The standard output costs would be determined by formula (6-36).

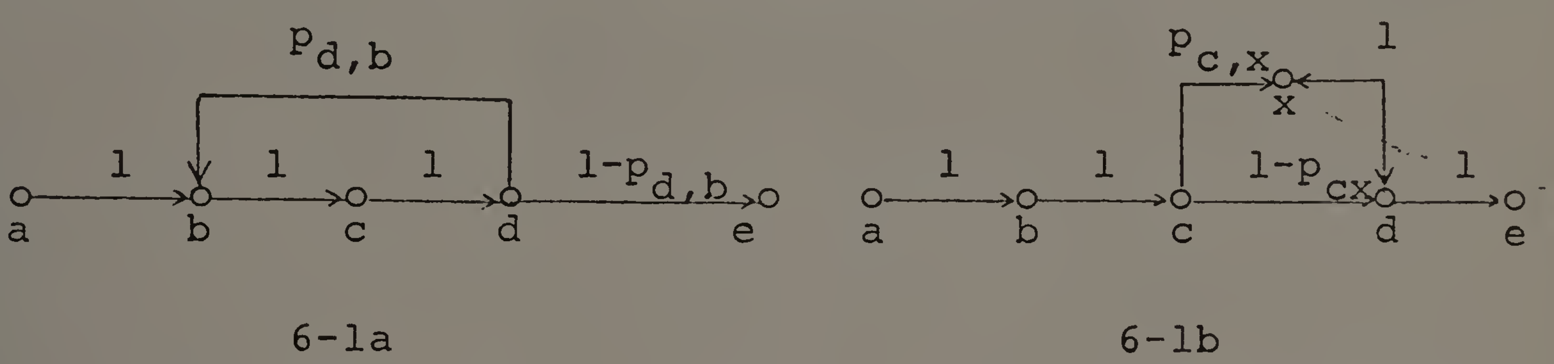
3. Multiple Entry Points

The costing in a system with multiple entry points would be complex but not beyond the capabilities of the stochastic model. In a system with multiple entry points, units could enter the system through any production process. All of the formulation of the stochastic model dealing with unit transfers has taken into account the possibility of multiple entries. The costing phase of the model has only been designed for a single entry system. For a multiple entry system, the transfer and output cost vectors d and u would become matrices because a different set of standard costs would have to be determined for each entry point. To cost the output and inventories, it would be necessary to identify the entry point.

The description of such a model would result in complexity that is not required by the objectives of this study. However, such a system would lend itself without much difficulty to a computer application of the stochastic model.

4. Non-Sequential Transfers

When non-sequential transfers are permitted in a production system, units may enter a process from more than one source. This condition would result from the two reprocessing or recycling possibilities depicted in Figure 6-1. In the case described in 6-1, a unit exiting process d would have a probability of $p_{d,b}$ of being re-cycled back into process b. In 6-1b, the normal sequence would be that a unit would be transferred from process c to process d; however, a unit exiting from process c would have a probability of $p_{c,x}$ of being recycled through process x before being transferred to process d.



Reprocessing In The Stochastic Model

Figure 6-1

Often the cost of reprocessing is charged to an over-head account and then allocated to the production. If this procedure is followed, then reprocessing probabilities would not be incorporated into the P matrix and the stan-dard costs could be determined based on the appropriate formulas from the previous sections.

However, the stochastic model would make possible cost analysis concerning the decision as to whether units should be recycled. The description of the analysis will be carried out by means of an example. Referring to the stochastic matrix in Exhibit (6-1) assume the problem is to decide whether to recycle the discarded output from process three back through process two. If the recycling was undertaken the new stochastic matrix would take the form

$$P = \left[\begin{array}{ccc|cc} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3/4 & 0 & 1/4 \\ 0 & 1/4 & 0 & 3/4 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

and the F and B matrices by (6-1) and (6-2) would equal

$$F = \begin{bmatrix} 1 & 16/13 & 12/13 \\ 0 & 16/13 & 12/13 \\ 0 & 4/13 & 16/13 \end{bmatrix}; \quad B = \begin{bmatrix} 9/13 & 4/13 \\ 9/13 & 4/13 \\ 12/13 & 1/13 \end{bmatrix}.$$

Assuming that the full activity cost is incurred in the second process and that the units are then transferred like all other units exiting from process two, the standard unit cost with recycling can be compared with the unit cost without recycling. The standard output cost with the recycling would be determined by $(\sum_{i=1}^{n} f_{\lambda i} s_i)/b_{\lambda,1}$ to equal \$56.06. Since the standard cost of a unit of final product without recycling was \$60.22, by recycling the average cost of a unit of final product is reduced \$4.16. The analysis would be quite similar if the question concerned the type of recycling as depicted in

Figure 6-1b.

The recycling question is only one area where the stochastic model could provide additional cost information. The next section will present other areas where the information obtained from the stochastic model could aid in decision making.

Cost Analysis

The material in the following sub-sections will concern several methods of analyzing cost information based on the stochastic model. Each sub-section will be followed by an example to clarify the material in the test. The first two topics will deal with the analysis of the unit cost of final product. The last two topics will relate to control and the *ex-ante* and *ex-post* analysis of the system.

1. Analysis of Unit Costs

In a system where non-sequential transfers are possible, additional information related to the unit costs of final output may be obtained by further analyzing the stochastic matrix P. The minimum cost of a unit of final product results from a unit passing through the processes in the sequence 1,2,3,...n and then exiting to state n+1. Any unit that followed this sequence would not be recycled.

The probability of a unit following the minimum cost route and exiting the system as final product would be

$$p^* = p_{1,2}p_{2,3}p_{3,4} \cdot \cdot p_{n-1,n}p_{n,n+1}. \qquad (6-40)$$

The expected unit cost of a unit passing through the system in the least cost system equals

$$u_n^* = \sum_{i=1}^{n} s_i \ . \qquad (6-41)$$

Any difference between the expected unit cost u and the minimum (superscript*) unit cost is caused by the existence of defects, by-products, or recycling within the system. Any combination of the aforementioned characteristics causes an increase in the estimated average unit cost.

Under some circumstances it may be advantageous to segregate estimates of the unit costs. The expected number of minimum cost units would equal p*k and they could assume a unit cost of u*. The remaining (v-p*k) units of final product could be assigned an average unit cost of (sx-ku*)/(v-p*k).

The above will now be demonstrated by assuming the stochastic matrix that was used in the non-sequential transfer example. The probability of a unit following the minimum cost route would equal 9/16 from (6-40). The minimum unit cost would equal $36.50 from (6-41). If v equalled 9, then k_λ would equal 13 by (6-16) and the total cost of producing 9 units would equal $504.54 by (6-36). The segregated unit costs would equal

7.3125 units at $36.50

1.6875 units at $140.82

The impact of a change in the cost of a resource input upon the estimated unit cost of final product may be obtained directly from the stochastic costing model. Assuming the cost of resource j shifts Δc_j, then the change in the estimated unit of final product equals

$$u_n = \sum_{i=1}^{n} f_{\lambda i} e_{ji} \Delta c_j / b_{\lambda i} \; . \qquad (6\text{-}42)$$

In the example with recycling, if the cost of resource 2 was increased by one dollar, the estimated unit output cost by (6-36) would increase by $9.67.

2. Cost Analysis Employing Conditional Probabilities

Employing conditional probabilities, it would be possible to determine all of the probabilities for a unit given that the unit exited from the production system in a particular state. Assuming that the unit exits from the system as final product, let H be a matrix with diagonal entries $b_{1,k}$. The conditional $\hat{Q}$ matrix would equal[17]

$$\hat{Q} = H^{-1} Q H \; . \qquad (6\text{-}43)$$

The conditional fundamental matrix would be

$$\hat{F} = H^{-1} F H \; . \qquad (6\text{-}44)$$

With the conditional fundamental matrix, the conditional standard cost of a unit of final product could be determined. This cost would be less than the non-conditional cost because there is only one possible way of exiting from the system. The difference between the two standard unit costs is the maximum amount a person would be willing to pay for information concerning the outcome of a unit entering the system.

Employing the stochastic matrix with recycling, by formula (6-4) the conditional stochastic matrix would be

$$\hat{P} = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 3/16 & 0 & 13/16 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] .$$

The conditional fundamental matrix would equal

$$\hat{F} = \begin{bmatrix} 1 & 16/13 & 16/13 \\ 0 & 16/13 & 16/13 \\ 0 & 3/13 & 16/13 \end{bmatrix} .$$

By formula (6-36) the conditional unit cost would equal \$42.10. The difference between the expected cost of \$56.06 and conditional cost of \$42.10 is the maximum that one would be willing to pay for the information that a unit entering the production system would exit as final product.

Additional information may also be obtained from the conditional fundamental matrix. Employing formula (6-7) the conditional activity levels in each process could be determined. The difference between the conditional activity levels and those normally estimated would indicate the production activity that would be committed to units that would not exit the system as final product. In the example, the two expected activity levels would equal

$$x = \begin{bmatrix} 13 \\ 16 \\ 12 \end{bmatrix}; \qquad \hat{x} = \begin{bmatrix} 9 \\ 144/13 \\ 144/13 \end{bmatrix}.$$

The difference between the two of

$$\begin{bmatrix} 4 \\ 64/13 \\ 12/13 \end{bmatrix}$$

would equal the activity on units that would not exit as final product.

3. Analysis of Attached Overhead Costs

If a uniform rate was employed throughout the system, the total expected overhead cost that would be attached could be determined directly. The vector g would give the mean number of times a unit would be processed given the entering process was known. The g vector equals[18]

$$g = Fa. \tag{6-45}$$

The expected attached overhead during the period in a system without inventories would be

$$\text{Expected Attached Overhead} = c_z kg. \tag{6-46}$$

In a system with inventories, the total expected overhead that would be attached during a production period would equal

Expected Attached Overhead

$$= c_z\left(\sum_{i=1}^{n} x_i + \pi_i^t(1-\alpha_{zi}^t) + \pi_i^{t+1}\alpha_{xi}^{t+1}\right) \quad (6\text{-}47)$$

The variance of the attached overhead equals[19]

$$\text{Var(Attached Overhead)} = c_z^2 h[\rho(2F-I)Fa-(\rho Fa)_{sq}]. \quad (6\text{-}48)$$

The values of h and ρ would be equal to those used in determining the output variances. The vector a is an n element sum vector.

In the production period example where the inventories were increased, the expected attached overhead was found to equal $87.50 with a standard deviation of $7.07. In the case where the in-process inventories were decreased, the expected attached overhead was found to equal $68.50 with a standard deviation of $6.89. If different overhead rates were used in each process, this same type of information could be determined by employing the expected activity level and standard deviation of the activity level for each process.

4. Ex-Post and Ex-Ante Analysis

Price and quantity difference analysis could be undertaken based on the stochastic process costing model. This analysis could be extended in the same manner as the quantity difference analysis presented with the input-output process costing model. Rather than reiterate that material, the discussion in this section will include an additional procedure that could be employed in *ex-ante* analysis.

The standard deviations of the activity levels and outputs could be used in evaluation of the observed activity levels and outputs. This information could be employed when determining the number of units to be started into production. If the output of final product were critical, management may determine to start more units into production because of the expected variation in the output. For example, if a decision maker wanted to be 95 percent certain of producing v units of final product, the number of units to be started into production in process λ is

$$b_1(k_1+2 \quad Var(y_1)k_2) = v \ . \qquad (6\text{-}48)$$

If the recycling example v would equal 17.2 units as compared with the mean estimate in starting 13 units. Of course when 13 units are started into production, fifty percent of the time the output would be expected to be less than 9 units of final product. By starting 17.2 units, the expected output of final product would be greater than 9 units 95 percent of the time.

This completes the presentation dealing with cost analysis based on the stochastic model. The intention was not to cover all possibilities but rather to highlight the advantages of the stochastic model.

Summary

The stochastic process costing models developed in the chapter are capable of being employed to achieve results similar to those of a traditional process costing model. In addition, variations in the outputs and activity levels and therefore, costs could be accounted for based on analysis of the stochastic transition matrix. When employed as a planning model, the stochastic costing models would provide the same information as the input-output model.

That is, a unit of activity results in a single unit of output. The resource inputs in any process can assume any value of zero or greater.

FOOTNOTES

[1]In a stochastic matrix the values of all the elements are in the range $0 \le p_{ij} \le 1$. The sum of the elements on each row of the matrix equals one. The expected flows in the system would be determined by the expected outputs of each process. For example, if it was expected that eighty percent of the output of process n met quality standards, then such units would be transferred to process n+1. It might be expected that the remaining twenty percent of the units would be recycled through process x.

[2]Based on Kemeny and Snell (John G. Kemeny and Laurie J. Snell, <u>Finite Markov Chains</u> (Princeton, New Jersey: D. Van Nostrand Co., Inc., 1960), p. 44. The P matrix was transposed so that the states would be more in line with the way one would conceive of a production system.

[3]The first absorbing state will always be the final product of the system. The final product is considered to be the primary output of the production system.

[4]A proof that the means are finite is that once a unit enters one of the set of absorbing states it can never return to any of the production processes. The probability tends to be that a unit entering at any production process will exit from the production system after n transitions among the production processes as n tends towards infinity. Assume it is possible to enter the set of exiting states in x transitions. Let p equal the probability of entering the exiting set of states. Then (1-p) would equal the probability that the unit would not have exited the system after x transitions. The probability of not exiting the system after jx transitions would equal $(1-p)^j$. The term $(1-p)^j$ tends towards infinity, the probability of exiting the system tends toward one.

Proof of formula 6-1

Let q^x_{ij} equal the probability that a unit would be in process j after x transitions given that the unit started in process i. Let $m_{i,j}$ equal the mean number of times a unit would be in process j given that it entered the system in process i.

$$m_{i,j} = \sum_{x=0}^{\infty} ((1-q^x_{i,j})\cdot 0 + q^x_{i,j}\cdot 1)$$

$$m_{i,j} = \sum_{x=0}^{\infty} q_{ij}^{x}$$

Since both i and j are production process, the general expression for determining the means would equal:

$$m = \sum_{x=0}^{\infty} Q^{x}$$

Hadley (1961, p. 118) has shown that

$$(I-A)^{-1} = \sum_{x=0}^{\infty} A^{K} = I + A^{1} + A^{2} + A^{3} + \ldots$$

if the A matrix has the following properties

(1) $0 \leq a_{ij} < 1$ for all i, j

(2) $\sum_{i=1}^{n} a_{ij} < 1$

These are the properties of the Q matrix, therefore

$$m = \sum_{x=0}^{\infty} Q^{x} = (I-Q)^{-1} = F$$

[5]For proof see Kemeny and Snell, Finite Markov Chains, p. 52.

[6]The transfer period model is based on: R.M. Cyert, H.J. Davidson, and G.L. Thompson, "Estimation Of The Allowance For Doubtful Accounts By Markov Chains," Management Science (April, 1962), pp. 287-303. The proofs of the formulas in this study are similar to the proofs of Cyert, Davidson, and Thompson.

[7]Ibid., p. 296.

[8]Refer to Chapter IV for a more detailed discussion of the reconciliation.

[9]The variance of F equals:

$$Var(x) = Mean\ (F^{2}) - Mean\ (F)^{2}.$$

The term Mean $(F)^{2}$ equals:

$$F_{sq}.$$

A unit starting in process i can go to process j with a probability p_{ik}. If the new state is exiting, then it would not be possible to reach state j. Let d_{ij} be a constant function that equals one if i=j. If the new state were not exiting the unit will be in state j d_{ij}

times from the original position and f_j times from the other processes. The function f_j gives the total number of times a unit is in process j

$$\text{Mean } (f_j^2) = \sum_{k=n+1}^{n+m} p_{ik} d_{ij}^2 + \sum_{k=1}^{n} p_{ik} \text{ Mean}(f_j + d_{ij})^2$$

$$= \sum_{k=1}^{n} p_{ik} (\text{Mean } f_j^2 + 2 \text{ Mean } f_j d_{ij} + d_{ij})$$

$$= Q(\text{Mean } f_j^2) + 2(QF)_{dg} + I$$

$$= (I-Q^{-1})(2(AF)_{dg} + I)$$

substitute for $Q = (I-F^{-1})$

$$= F(2((I-F^{-1})F)_{dg} + I$$

$$\text{Mean}(f_j^2) = F(2F_{dg} - I)$$

Therefore: $\text{Var}(x) = F(2F_{dg} - I) - F_{sq}$

Since ρ is the probability vector that contains the probabilities of where a unit starts, the variance of one unit would equal:

$$\text{Var}(x) = 7F(2F_{dg} - I) - (nF).$$

Since ka units are cycling through the system the variance for the ka units would equal:

$$\text{Var}(x) = ka(nF(2F_{dg} - I) - (nF)_{sg})$$

[10] Let $(nB)_{sg}$ be a matrix obtained by squaring each element in the vector (nB). The components in the P vector give the probabilities of where an exiting unit started in the production system. The probability of a unit exiting from the system as final product is given by the first component of the vector (nB). The other components would give the probabilities of a unit exiting in another state. If ka units exit from the system, then ka(ρB) gives the expected units exiting in each state. Let f be a function that takes the value of 1 when a unit exits as final product, and zero otherwise. The variance of the function f would equal:

$$\text{Var}(f) = m(f)^2 - [m(f)]^2.$$

Since f can only assume values of zero and one, the $f=f^2$. Thus $m(f) = m(f)^2$. Hence the variance of f is given the first term of:

$$nB - (nB)_{sq}.$$

Since the process is started h times, the variance of y would equal

$$Var(y)\ ka(nB - (nB)_{sq})$$

[11] $k = (vn-(\pi^{t}_{t}-\pi^{t+1}_{t+1})H)\ H^{-1}$

The expression $(\pi^{t}-\pi^{t+1})H$ gives the vector of expected output of final product that results from the decrease in the inventory of the processes. Subtracting this from the vector vn results in the number of units of final product that must come from units being started into production. The multiplication by H^{-1} gives the expected units that would have to be started into production.

[12] $x = (k + (\pi^{t}-\pi^{t+1}))F$

The expression $k+\pi^{t}-\pi^{t+1}$ gives the vector of the units that will be cycling through the system. Then multiplying by F gives the expected activity in each process.

[13] Formula (6-13) would be used in conjunction with formula (6-34) for costing all transfers and outputs. In cases where $\lambda>1$ the element of the d matrix from 1 to $\lambda-1$ would equal zero.

[14] Formulas (6-14) and (6-15) would be used in conjunction with formulas (6-35) and (6-36) when only the final product is costed.

[15] Formulas (6-14) and (6-15) would be used in conjunction with formulas (6-37) and (6-38) in a production period system when all outputs are costed.

[16] Formulas (6-14) and (6-15) would be used in conjunction with formulas (6-36) and (6-38) in a production period system when only the final product is costed.

[17] Kemeny and Snell, Finite Markov Chains, p. 64, 65.

[18] Ibid., p. 51.

[19] Ibid., p. 51 and 52.

CHAPTER VII

EVALUATION AND CONCLUSIONS

Introduction

In this chapter the process costing models entertained in this study will be evaluated based on the criteria identified in the second chapter. Attention will also be directed to certain questions concerning the validity of the models. The last section of the chapter will concern the conclusions drawn from the study, limitations of the study, and possible areas for future research.

Evaluation

Each process costing model considered in this study will be evaluated individually. The conclusions reached based on the evaluation will be presented in the last subsection of this chapter.

1. Process Costing With Historic Costs

Process costing employing historic costs can best be described as a set of rules and procedures for the allocation of costs on an *ex-post* basis among the inventories and outputs of a mass production system. No specification of the relationships among the processes is necessary because the cost allocations are handled on an *ad hoc* basis at the conclusion of each period.

This class of costing system would not require any specifications of the relationships between expected activity levels and the corresponding resource requirements.

Budgets could be prepared based on the unit costs and relationships determined for a preceeding production period, but this type of accounting system lacks any control mechanism for assuring that the results determined from previous production periods represent an efficient or normal operation of the production system. The incorporation of a process costing system employing historical costs into an open loop control system is precluded because of the lack of standards. Such standards would serve as the norm to which control activities would be directed at influencing the operation of the system.

In evaluating process costing systems employing historic costs based on the criteria established in Chapter II, these conclusions were reached. This method is not structured in a manner which allows it to be used as a planning model. Moreover, it was also determined that there is no obvious means for incorporating management goals into the planning model to permit the identification of feasible alternatives. The output from a historic cost model would be of little use in the area of operation control.

In summary, process costing with historic costs does not meet any of the evaluation criteria. It is concluded that process costing with historic costs is a methodology for ex-post cost allocation and the use of information obtained from such a system for purposes other than financial reporting is of dubious value.

2. Process Costing With Standard Costs

Process costing employing standard costs implies the existence of a structured model of a mass production system. However, the literature contains only detailed descriptions of process costing methods employing standard costs. Ad hoc procedures are described when such technical problems as, consumption within the system and non-homogenous physical output coefficients are encountered. The absence of an analytic formulation of the standard cost process costing model would inhibit its application in the area of planning. The lack of structure would complicate the computational procedures necessary for evaluating various production alternatives.

Based on the criteria established in Chapter II, these conclusions were reached concerning process costing with standard costs. The evaluation of production alternatives in planning with a standard costing system would be possible for relatively non-involved systems. However, such evaluation could become cumbersome if the system were complex or if many alternatives were to be considered. The lack of a general analytic structure of the standard costing model would preclude the incorporation of management goals in the model for planning.

Since standard costs are based on standard resource inputs, some of the output from a standard costing system would be useful in production control. The resource quantity differences would indicate deviations from the expected levels of consumption. Such information would meet the requirements of an open loop control system. The several sets of overhead differences would highlight deviations caused by spending and capacity utilization variations. Such information should prove useful to management in the area of operational control.

However, variation in the consumption of resources is to be expected. The standard costing model has the capacity to indicate the source of resource consumption deviations, but not the statistical significance of the variation because the appropriate probability distributions are not incorporated into the model.

Except for the second evaluation criterion concerning the incorporation of management goals in the planning model, the standard process costing model to varying degrees meets the other three criteria. However, the lack of a generalized analytic model of the standard process costing procedure would appear to limit its application and the information obtained for managerial and operation control activities. The lack of structure would also impair the integration of the costing model with most other planning and control models of an organization.

3. Input-Output Model With Standard Costs

The input-output process costing model would appear to overcome the most evident deficiency of the standard process costing model. By employing physical output coefficients, the input-output model provides the structure which was lacking in the standard process cost model. When employed as a planning model, various production alternatives can be entertained and the expected costs and resource requirements can be determined. Price, quantity, and overhead differences can also be obtained from the input-output model for use in an open loop control system.

Based on the input-output model, differences can be developed based on the consumption of inter-process resource transfers. Also, the expected unit transfers and standard transfer costs can be determined based on the input-output model. Perhaps the significant advantage to be associated with the input-output model is the requirement that the entire production system be considered in the model. The resource inputs and the relationships among the processes must be considered in the construction of an input-output model.

The domain of the input-output model is considerably broadened when the system is formulated as a linear programming problem. The application of linear programming would be appropriate in multiple product systems and would permit the incorporation of management goals into the planning model. With linear programming, the constraints which limit the output of the system are identified with their corresponding opportunity costs. A cost control system based on these opportunity costs could lead to a reduction of the data manipulation requirements. Such a system would also indicate any foregone profits resulting from less than optimal operations of the system.

The solution of the _ex-post_ linear programming problem permits the determination of the optimal output under the conditions encountered during a production period. The analysis of the _ex-post_ solution and the observed results would make possible the evaluation of a management's capacity to adapt to operating conditions during a production period. A set of price and quantity differences based on the linear programming solutions can be determined to indicate the opportunity costs resulting from various operating decisions.

In evaluation the input-output model based on the criteria from Chapter II, the following observations are made. The input-output model has the capacity to serve as an ex ante planning model in that estimates can be obtained for the expected costs, resource requirements, and inter-process transfers. In a multiple product system, the model could be formulated as a linear programming problem. As has been previously mentioned, this would permit the incorporation of management goals into the planning model. Therefore, an input-output costing model would meet the first two evaluation criteria concerning the estimation of costs on an ex ante basis and the selection of a production alternative.

Concerning the last two criteria, the necessity of modeling the entire production system to implement an input-output system, could result in isomorphic production and cost control systems. Since linear programming would identify the resources constraining the output of the system, the significance of a quantity difference of a constraining resource would be more apparent. The statistical significance of any of the price or quantity differences could not be determined since the model is deterministic. However, the segregation of the total quantity differences between that caused by shifts in the consumption pattern between the processes and any variation of consumption within a process should aid in operational control activities.

Therefore, based on the last two criteria, the input-output model would appear to be a significant improvement over the existing technology. The necessity for modeling the entire system would results in information that would aid in the control of production. This model would also permit more detailed analysis of deviations between observed and planned results.

4. The Stochastic Process Costing Model

The stochastic model is the most sophisticated of the models considered in this study. With the potential of determining the means and variances of the expected outputs and activity levels, this costing model perhaps could be the most useful to management. The capability of the transfer period model to estimate the steady-state in process inventory would provide management with information that could not be obtained from the other models.

Lost units can be incorporated into the stochastic model by assigning them an exiting state. The model can also handle by-products but joint products would present problems unless they were produced in the terminal process. In the input-output model, each process is limited to a single output. If the joint product occurred in the last process and the unit output could be considered to consist of certain percentages of the joint product. This same method could be employed with the stochastic model. If any process except the n^{th} produced joint products that required additional processing, neither the input-output or stochastic models could be employed.

Both the transfer and production period models could be employed as ex ante planning models. With either of the models, costs could be allocated among the inventories and outputs if standard costs were employed. Since the stochastic model is limited to one primary output, it would lend itself to linear programming for the purpose of selecting a production alternative. However, the parameters of the stochastic models could be incorporated into a general linear programming model of a firm. The stochastic model would therefore meet the first two evaluation criteria.

The stochastic models would appear to be more than adequate as production control models. The variances of the outputs and activity levels would permit the determination of the statistical significance of variations within the system. The identification of this type of information is not common to most cost accounting systems. The stochastic models are judged to be superior to both the historic and standard costing systesm.

Validation

Except for the traditional process costing models, the analytic models in the study have been constructed to simulate the behavior of abstract mass production The problem of validating these models would center around the assumption of linear production functions. The classification of costs into fixed and variable components is based on the assumption of a linear production function over some relevant range of activity.

The accountant, by segregating costs into fixed and variable components, has divided a series of complex interrelationships into components whose performance can be understood. A trade-off is made between simplicity of construction and the degree of accuracy necessary and the departure has been tolerated in the interest of simplicity and economy. Ijiri and Jaedicke (1966) have emphasized the reliability of the output of accounting systems. When considering reliability, the important question is whether the system works as it is supposed to work. This system does indicate estimates of the incremental costs between alternatives, and when fixed overhead cost allocations are treated discretely, these models would appear to provide useful information for decision making.

Another validation question would be encountered were any of the models operationalized. Since the predictive output of the model would serve as a basis for selecting a production alternative and the establishment of the norms upon which the <u>ex-post</u> analysis would be based. The validation question would be concerned with whether the model is "a correct or valid inference for the actual process."[1] Two factors would have to be considered. The first, dealt with in Chapter III would concern the managerial philosophy of determining standard costs. If motivational factors are considered in setting "tight" standards, all inferences drawn from the model would have to take this into consideration.

In a more general sense, the validation question would be directed at the goodness of fit between the model output and the observed output. The operational control problem of the investigation of cost differences should also consider the validity of the parameter settings in the costing model. The characteristics of the production system would be expected to shift through time, and therefore the costing model would have to be maintained so that it would remain a reasonable representation of the production system. The use of historical data, trend analysis, and statistical goodness of fit tests would be employed in maintaining the costing model.

Conclusions

Process costing employing historic costs is a methodology cloaked in a technical jargon for the allocation of costs. The information obtained from employing the methodology certainly meets the requirements of financial reporting but would have dubious value in the areas of management and operations control. Since few structural relationships must be specified when employing the methodology, significant manipulation would be required to employ the costing information for purposes of evaluating production alternatives. Since norms are not explicitly required by the methodology any decisions in the area of operations control, based on process cost information, would rely heavily on a decision maker's intuition. The lack of structure and the use of arbitrary cost allocations successfully eliminate any technical problems that might limit applications. However, applications are limited because of the unproven value of the information output and the lack of a precise and lucid description of the model that might make non-commercial applications more apparent.

The introduction of standard costs into process costing adds structure in that expected resource inputs are specified for given outputs. This structure should theoretically make possible the ex-ante costing of production alternatives. The price, quantity, and overhead differences generated from standard costs also should aid management in making decisions relating to operational control. But process costing procedures employing standard costs have not been formalized into a structured model. They rather exist in detailed procedural descriptions which are encumbered with a technical jargon.

In this study, it has been shown that process costing procedures employing standard costs can be structured as a Leontief input-output model employing physical output coefficients. The costing of outputs and inventories is possible with this model and therefore, the requirements of financial reporting are met. With the input-output costing model it is possible to determine directly ex-ante estimates of the cost and resource requirements of output and inventory alternatives. In the ex-post analysis of the operation of the system, it is possible to analyze the consumption of outputs of the various processes by other processes within the system.

An input-output process costing model can be formulated directly into a linear programming problem. Linear programming techniques permit the incorporation of objectives or goals in one form or another into the model and identify the production alternative or set of alternatives that would result in the realization of the objectives. A linear programming model will also identify those resources which constrain the output of the system and determine the related opportunity cost. A process cost system employing these opportunity costs would result in considerable data reduction. A budgetary control system based on these opportunity costs would highlight deviations from planned activity and the foregone profit resulting from such deviations. The solution of an *ex-post* linear programming problem would permit the determination of what could have been accomplished based on the constraints encountered during a production period. This would permit analytic *ex-post* analysis of the ability of management to adapt to unforeseen contingencies.

As a result of structuring process costing as an input-output model, the relationship to the linear programming problem became apparent with the resulting additional information for planning and control. The joint-product condition was the only technical problem encountered that could not be incorporated directly into the input-output model.

The process costing systems based on the stochastic transfer matrix are the most sophisticated costing models considered in the study. They offer several added advantages over the input-output costing model. In the transfer period model, the steady state in-process inventories can be determined. In both the transfer and production period models, the means and variances of the expected outputs and activity levels in each process can be determined. This information would be of value in both ex-ante and ex-post analysis. With the stochastic models it is also possible to cost the inventories, outputs, and determine the cost and resource requirements of various production alternatives.

As a result of formalizing the structure of the process costing methodology the information that can be obtained from the models will better meet the needs of management in the areas of managerial and operations control. By structuring the process costing model, the existing technology is presented in a general comprehensive format such that other applications could become more apparent. In the context of non-commercial organizations there exists a need for managerial accounting systems. Perhaps methods employed in commercial organizations would be suitable and these applications will become more evident if the techniques are presented in a structured rather than a descriptive fashion.

The structuring of any managerial accounting system should foster the integration of the planning and control models of an organization. For example, linear programming requires unit contribution margins, Bayesian decision models require various profit and loss estimates, and cash budgeting models require estimates of future expenditures. This information can be provided from a structured process costing system.

In conclusion, it would appear that future research might be directed toward the integration of Bayesian decision models into process costing systems. The non-structured information that is often available to management could then be incorporated into the model. Queuing theory might find application in the costing of mass production systems where the expected service times of the processes and waiting times between processes were available.

The stochastic aspects of mass production systems could possibly be incorporated in a stochastic linear programming model. All of the models considered in this assumed linear cost functions. Future research might be directed towards employing non-linear programming and other models where non-linear cost functions could be considered.

FOOTNOTES

[1] Richard L. Van Horn, "Validation of Simulation Results," Management Science Vol. 17, No. 5 (January, 1971), pp. 247-250.

GLOSSARY

Basic Notation

A = a_{ij} = matrix with elements a_{ij}

a_{ij}^{-1} = the ij element of the inverse of the A matrix

b = b_i = vector with elements b_i

A_{tr} = the transpose of matrix A

A_{sq} = results from squaring each element in the H matrix

A_{dg} = results from A by setting off diagonal elements equal to zero

Var(x) = variance of the mean values given by the vector x

Std.Dev.(x) = standard deviation of the mean values given by the vector x

Variable Names

c = n element vector of standard costs

d = n element vector of standard transfer costs

E = z by n matrix of resource requirements

F = fundamental matrix of stochastic process

g = inventory valuation vector

I = identity matrix

ℓℓ= z element vector of resource requirements

m = number of production system outputs

n = number of processes in system

P = stochastic matrix

u = n element vector of standard output costs

x = n element vector of units of activity

y = m element vector of outputs

z = number of resource requirements

δ = n by n matrix of standard inter-process cost transfers

Θ = n by m+n matrix of unit transfers

α = z by n matrix of completion estimates

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