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Testing for a Unit Root in a Process Exhibiting a Structural Break in the Presence of GARCH Errors

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This paper considers the effect of GARCH errors on the tests proposed by Perron (1997) for a unit root in the presence of a structural break. We assess the impact of degeneracy and integratedness of the conditional variance individually and find that, apart from in the limit, the testing procedure is insensitive to the degree of degeneracy but does exhibit an increasing over-sizing as the process becomes more integrated. When we consider the GARCH specifications that we are likely to encounter in empirical research, we find that the Perron tests are reasonably robust to the presence of GARCH and do not suffer from severe over- or under-rejection of a correct null hypothesis.

Keywords:* *Unit roots, structural breaks, Perron Test, GARCH

JEL References: C12, C15, C22

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(1). Introduction

A considerable number of recent empirical studies that have analysed financial data have found them to be characterised by non-stationarity, and their returns by conditional heteroskedasticity. It is now a widely accepted stylised fact that the natural logarithm of almost all asset price series contains a unit root. The pioneering development of techniques designed to test for and model unit root processes was conducted by Dickey and Fuller (Dickey (1976), Dickey and Fuller (1979) and Fuller (1976)).

Another important stylised feature of the natural logarithm of asset prices, in their first differenced form, is that they exhibit volatility clustering or autoregressive conditional heteroskedasticity. The ARCH process, initially suggested by Engle (1982), permits a class of time series models for which the conditional variance is allowed to vary through time as a function of current and past information. Bollerslev (1986) extends this formulation to allow for a more general formulation that allows lags of the conditional variance to influence its current value, termed the generalised auto-regressive conditional heteroskedasticity (GARCH) model. Numerous, other extensions have been proposed such as exponential GARCH (EGARCH), see Nelson (1990), which allows for positive and negative shocks to have an asymmetric effect on the conditional variance, see Bollerslev, Chou and Kroner (1992) for a detailed survey.

Since the unit root in levels, conditionally heteroskedastic differences model has been found to be empirically relevant, there has been some investigation into the impact that the presence of heteroskedasticity has on unit-root testing methodologies. In general, certain types of heteroskedasticity are known to have no influence on unit root tests asymptotically although the impact in finite samples is less clear. Recent studies have considered the impact that heteroskedasticity has on unit root testing methodologies in finite samples. Kim and Schmidt (1993) for example, analysing the Dickey-Fuller tests[1], and Haldrup (1994), analysing the Dickey-Fuller test, Phillips (1987) semi-parametric test and the Dickey-Fuller test using White's (1980) heteroskedastic consistent standard errors, found that these unit root tests were susceptible to GARCH error processes, particularly in certain limiting cases. For example, Haldrup (1994) finds that the Dickey-Fuller t-test and Phillips (1987) semi-parametrically corrected Z-test have similar properties, as the degree of degeneracy and integratedness increases the empirical distributions are shifted leftwards, Kim and Schmidt (1993) confirm the results for the Dickey-Fuller test.

In addition to the problems experienced by the standard unit root methodologies caused by the presence of GARCH, it is further known that the existence of a structural break in the data could also have an impact on the test statistics in finite samples. For example, the most common approach for testing for non-stationarity, the Dickey-Fuller test, has been shown to be biased towards non-rejection of the null hypothesis of a unit root if the series in question exhibits a structural break. As a result, a researcher may incorrectly classify a series that is stationary and exhibits a structural break as a non-stationary process. Incorrectly classifying a series could easily lead to inappropriate modelling and policy decisions and therefore it is of importance to ensure that researchers do not make such errors due to the inappropriateness of the testing methodologies.

Allowing for structural breaks in financial time series is likely to become increasingly important since there have been some major structural changes in the world economy in recent years. For example, the withdrawal of the UK from the European Exchange Rate Mechanism (ERM) in 1992 or the collapse of the Tiger Economies in 1997 and associated currency and stock market turmoil are likely to have had a major impact on the behaviour of financial time-series relating to these countries[2]. Further, the introduction of a single European currency and the convergence to a 'single Europe' are likely to have a significant impact on the time series properties of related financial time series. The existence of a structural break will prevent the traditional Dickey-Fuller approach from being validly used to analyse the existence or otherwise of unit roots in these time series and as such, researchers will have to explicitly allow for the breaks in the testing methodologies.

[1] The procedure is developed in Dickey (1976), Fuller (1976) and Dickey and Fuller (1979).

To this end, Banerjee, Lumsdaine and Stock (1992) and Zivot and Andrews (1992) develop testing methodologies that allow for a unit root and the possibility of a structural break - a unit root (stationary-structural break) process under the null (alternative) hypotheses. However, Perron (1989) argues that we may want to allow for a break under the null hypothesis and proposes a methodology that allows for a null (alternative) hypothesis of a unit-root with a structural break (stationary with a break). This approach was criticised by Christiano (1992) on the basis that the dates for the structural breaks are chosen such that they are in fact imposed on the data, i.e. the tests assume that the breaks are known *a priori*. If the break dates are chosen in such a manner, the underlying asymptotic distribution is not valid. In response to this, Perron (1997) develops a framework, henceforth the Perron procedure[3], which allows for the choice of the break date to be endogenised and therefore determined by the data.

However, the Perron procedure, whilst allowing for structural breaks, still suffers from the problem that it assumes that the errors in the formulated regressions are white noise. To this end, the purpose of the present paper is to assess the finite sample properties of the Perron procedure in the presence of GARCH processes, so that we can determine how robust the methodology is likely to be in empirical applications where both structural breaks and conditional heteroskedasticity are present. In doing so, we will be able to determine the consequences of using the methodology with high frequency financial data.

The rest of the paper is structured as follows. In Section 2 we review the Perron procedure and outline Bollerslev's (1986) GARCH process, while Section 3 considers the simulation framework that we employ in the analysis. In Section 4 we discuss data generating processes and their accompanying results while in Section 5 we draw our conclusions.

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[2] In a separate paper, we have analysed the impact of the UK resigning from the ERM and found that this event precipitated a significant structural break in short term Euro Sterling interest rates.
[3] Perron develops the necessary testing framework to allow for a break in mean, break in slope or a combination of the two.

(2) Theoretical review

(i) The Perron Model and Test Statistics

The model and testing framework that we consider is based on that proposed by Perron (1997), termed the innovational outlier model. In particular the data generating process (DGP) allows for a unit root process that exhibits a gradual shift in the mean of the series in a way that depends on the correlation of the error process. Perron (1997) and Perron (1989) discuss several alternative specifications that can be considered. As well as the innovational outlier model described in detail below and which allows for a shirt in the intercept only, a second model allows for a combination of a break in the mean and the slope of the DGP. Finally, a third model allows for a change in the slope, but with the segments of the trend function being joined at the time of the break. Perron terms this latter specification the additive outlier model, where the break is assumed to take place quickly. This paper only analyses the mean break or innovational outlier model in order to maintain the paper at a manageable length.

The test equation that is considered in the innovational outlier Perron procedure is given by:

$$y_t = \mu + \theta DU_t(\lambda) + \beta t + \delta D(T_b)_t + \alpha y_{t-1} + \sum_{j=1}^{k} c_j \Delta y_{t-j} + u_t \qquad (1)$$

Where, $DU_t(\lambda) = 1$ if $t > T\lambda$, zero otherwise, $D(T_b)_t = 1$ for $t = T_{b+1}$, zero otherwise, and $u_t \sim$ i.i.d. N(0, 1).

The regression equation is estimated sequentially by ordinary least squares and we are interested in the t-statistic for the test of $\alpha = 1$. The procedure requires a methodology to endogenise the break and Perron proposes two methods[4]. The first approach chooses the minimum value for the t-test on $\alpha = 1$ as we change the date of the hypothesized break. The test statistics are defined as

[4] For a more detailed explanation we direct the reader to Perron (1997)

$t^*_\alpha(1) = t_{\hat{\alpha}}(T^*_b, k)$, where T^*_b is such that $t_\alpha = \mathrm{Min}_{T_b \in (k+1,T)} t_{\hat{\alpha}}(T_b, k)$ [5]. The second method considered involves minimizing the t-statistic on the coefficient for the change in the intercept parameter, θ, denoted $t_{\hat{\theta}}$. This test statistic is denoted $t^*_{\alpha,\theta}(1) = t_{\hat{\theta}}(1, T^*_b, k)$, where T^*_b is such that $t_\theta(T^*_b) = \mathrm{Min}_{T_b \in (k+1,T)} t_{\hat{\theta}}(T_b, k)$. This method assumes that the break is negative, i.e. it imposes a 'mild a priori' that there is a 'crash' where the mean of the series falls. An alternative specification where this restriction is not imposed is where we choose the maximum absolute values of $t_{\hat{\theta}}$, using the same approach to choose the break date as above, for which the test statistics are given by $t^*_{\alpha,|\theta|}(1)$. The analysis that we undertake is based solely on the $t^*_\alpha(1)$ test statistics as Perron remarks that the test statistics $t^*_\alpha(1)$ and $t^*_{\alpha,|\theta|}(1)$ are likely to have the same properties and consequently, we expect that similar conclusions would be made if we considered the other test statistics.

Finally, Perron considers two methods to select the truncation lag parameter k; a t-test, k(t-sig), and an F-test, k(F-sig), for which the former was shown to have slightly more power and therefore we execute our analysis using this method.

(ii) Allowing the errors to follow a GARCH Process

The extension that we wish to make is to assess the Perron procedure when the DGP of the series in question exhibits GARCH errors. Bollerslev (1986) defines the GARCH model as:

$$h_t = \phi_0 + \sum_{j=1}^{q} \phi_{1,j} u^2_{t-j} + \sum_{i=1}^{p} \phi_{2,i} h_{t-i}, \text{ where } t = 1, \ldots T \quad (2)$$

where ϕ_0, $\phi_{1,j}$ and $\phi_{2,i}$ are non-negative for all i and j. If $\phi_{2,i} = 0$ for all i then h_t collapses to Engle's (1982) ARCH formulation. In particular, the form of heteroskedasticity considered in this paper is that termed GARCH(1, 1) such that the conditional variance is given by:

$$h_t = \phi_0 + \phi_1 u^2_{t-1} + \phi_2 h_{t-1}, \quad (3)$$

[5] It can be seen that the above test is analogous to the sequential test statistic proposed by Banerjee et al (1992), $\tilde{t}^{\min *}_{DF} = \min_{k_0 \le k \le T-k_0} \tilde{t}_{DF}(k/T)$, and to the test of Zivot and Andrews (1992), but is executed on a different model that allows for a structural break under the null.

the implication for u_t in (2) and (3) is that it is now distributed $N(0, h_t)$

If $\phi_0 > 0$ and $\phi_1 + \phi_2 < 1$ then we will have a well defined unconditional variance, given by $\phi_0 /(1-\phi_1-\phi_2)$. Interesting implications arise when these conditions are not satisfied, and there are two specific cases that are typically considered when GARCH modelling is being undertaken – the degenerate and integrated cases. The degenerate case occurs when $\phi_0 = 0$ causing the unconditional variance, $UV(u_t)$, to be zero. In the integrated case, often referred to as IGARCH, and where $\phi_1 + \phi_2 = 1$, and consequently shocks to h_t will persist indefinitely causing the unconditional variance to be infinite. Similarly, we may expect there to be a high degree of persistence, though not infinite, when $\phi_1 + \phi_2$ is close to unity, i.e. near integrated.

Given that $UV(u_t) \rightarrow \infty$ as $\phi_1 + \phi_2 \rightarrow 1$ and $UV(u_t) \rightarrow 0$ as $\phi_0 \rightarrow 0$, then to prevent the limits being approached, therefore the unconditional variance being unbounded, there must be some interaction between ϕ_0 and $(\phi_1 + \phi_2)$. Consider the case where ϕ_0 is large but less than ∞ then if $\phi_1 + \phi_2$ is close to 1, $UV(u_t) \rightarrow \infty$. Similarly, if $\phi_1 + \phi_2 = 0$ and ϕ_0 is close to zero then $UV(u_t) \rightarrow 0$. Thus if one of the limiting conditions is being approached and the other not then the $UV(u_t)$ will tend towards a violation of the existence of an unconditional variance. Consequently, if ϕ_0 or $\phi_1 + \phi_2$ is close to their limits then the non-existence of an unconditional variance is approached. However, if ϕ_0 and $\phi_1 + \phi_2$ are close to their limits simultaneously, then it can be seen that the $UV(u_t)$ is not 'too' small or 'too' large and it will not therefore be approaching the limits of the existence of an unconditional variance. For example if $\phi_0 = 0.00001$ it implies that the $UV(u_t)$ is near degenerate, if $\phi_1 + \phi_2 = 0.99999$ then $UV(u_t)$ is near integrated, but in combination $UV(u_t) = 1$ and hence $UV(u_t)$ is well defined, i.e. there is a mixing of near degeneracy and near integratedness that prevents $UV(u_t)$ from being unbounded.

In addition, Nelson (1990, 1992) defines a theoretical framework for showing that degeneracy and integratedness are likely to occur together by considering GARCH as an approximation to a continuous time diffusion process. For a detailed explanation we direct the reader to the original articles. In brief, Nelson (1992) shows that GARCH can be seen to be a consistent approximation to a continuous time diffusion process in the limit as $\gamma \rightarrow 0$ if:

$$\phi_0 = \omega\gamma,\ \phi_1 = \alpha\gamma^{(1/2)},\ \phi_2 = 1 - \phi_1 \qquad (4)$$

where γ is the time interval of the data, i.e. the frequency, and ω and α are fixed. If we consider high frequency data then γ is small and hence we would expect that ϕ_0 and ϕ_1 would be small and ϕ_2 would be approximately equal to 1, considering the limit $\gamma = 0$ then $\phi_0 = 0$, $\phi_1 = 0$ and $\phi_2 = 1$. Consequently, if we accept Nelson's theoretical framework, then it can be seen that we would necessarily expect degeneracy and integratedness to exist in union.

The empirical evidence supporting Nelson's framework is somewhat mixed. Baillie and Bollerslev (1989), in a comprehensive analysis of exchange rate volatility, provide a large amount of evidence that can be used to assess the applicability of the framework. They consider six exchange rates sampled daily, weekly, fortnightly and monthly from which log-returns are constructed. In the monthly data no ARCH effects are detected, whilst the fortnightly results indicate that ARCH is present. Once the frequency reaches the weekly and daily levels, GARCH(1,1) is found to be present. Concentrating, therefore on the weekly and daily data, the results indicate that $\phi_1 + \phi_2 \approx 0.9$ for four of the currencies whilst the Yen showed strong persistence with a value for $\phi_1 + \phi_2 = 0.999$. According to Nelson's framework, we would expect the Yen to exhibit a small value for the intercept, ϕ_0 and for ϕ_1 and this was indeed the case, with the smallest ϕ_0 and second smallest ϕ_1 belonging to the Yen. If we consider the daily data then we can state that we would expect persistence to be greater relative to the weekly data if Nelson's framework is to be supported. Baillie and Bollerslev (1989) indeed found that the persistence was generally stronger for the daily data with $\phi_1 + \phi_2 > 0.94$ for each of the six currencies. Further if Nelson's framework is accurate then ϕ_0 and $\phi_1$should be smaller than in the weekly data analysis and indeed this was the case. In summary, the daily results found that the largest persistence was accompanied by the smallest intercept and volatility and vice versa.

Consequently, there is a theoretical grounding and empirical support for the existence of (near-) degeneracy and (near-) integratedness simultaneously. The implication of this is that we will need to investigate the effect of integratedness and degeneracy as well as each one separately on the Perron Tests.

To summarise, we have discussed the implication of degeneracy and integratedness and showed that the unconditional variance does not exist in these limiting cases. We then reviewed theoretical and empirical evidence suggesting that degeneracy and integratedness could generally be expected to occur together and if this was found to be the case then it will prove beneficial to separate the effects out to see the impact of each on the Perron procedure as well as investigating the simultaneous presence of both. We will now proceed to discuss the framework of the simulations.

(3) The Simulation Framework

The purpose of this paper is to consider the impact that an error structure of GARCH(1,1) has on the Perron Tests. Consequently, we need to redefine the DGP that Perron considered such that we explicitly allow the error structure to follow a GARCH(1,1). Perron defined the DGP for his tests as:

$$y_t = \theta DU_t + \delta D(T_b)_t + \alpha y_{t-1} + \sum \phi(i)\Delta y_{t-i} + (1+\psi L)e_t \qquad (5)$$

From this specification of the DGP, it is easy to consider different error structures by varying the parameters $\phi(i)$ and ψ[6]. To allow for the GARCH(1,1) process we replace the error structure components $\sum \phi(i)\Delta y_{t-i} + (1+\psi L)e_t$ with u_t, where u_t follows a conditionally normal GARCH(1,1) giving $u_t \sim N(0,h_t)$. Consequently, the DGP that we will consider is given by:

$$y_t = \theta DU_t + \delta D(T_b)_t + \alpha y_{t-1} + u_t \qquad (6)$$

where $u_t \sim N(0,h_t)$.

For transparency of results, the specification of the GARCH process follows that of Kim and Schmidt (1993), i.e. we incorporate the same coefficient values for ϕ_1, ϕ_2, ϕ_3 and h_0. In particularly, we consider varying ϕ_1 and ϕ_2 to assess the impact that the degree of integratedness, degeneracy and volatility of the variance process has on the tests. Consequently, the values of ϕ_1 are held constant and ϕ_2 is varied and vice versa. In all cases, to simplify the analysis, $h_0 = 1$, however this does not adversely affect the results as Kim and Schmidt (1993), pp. 292 note that "the results are invariant to changes in h_0, so long as ϕ_0 is changed proportionally, so that ϕ_0/h_0 is held constant". In this first set of tests Kim and Schmidt set the initial variance equal to the long-run variance (i.e. $\phi_0 = h_0(1 - \phi_1 - \phi_2)$). Whilst, this prevents us from distinguishing whether or not degeneracy or integratedness has the greatest impact on the tests, it is at least consistent with the theoretical and empirical work of Nelson (1989) and Baillie and Bollerslev (1989). Further, once we have discovered how degeneracy and integratedness impact the Perron Tests, we can then consider the individual components. To do this we set $\phi_1 + \phi_2 = 1$, thus imposing the degree of integratedness and then vary ϕ_0. Therefore we are no longer imposing the earlier relationship between ϕ_0 and h_0, rather we are directly investigating the impact of ϕ_0 on the Perron Tests and hence the impact of degeneracy on the tests. Similarly, we will consider fixing ϕ_0, h_0 and ϕ_1 whilst varying ϕ_2. Additional experiments are included to assess the individual impact of ϕ_1 and ϕ_2.

We will consider sample sizes that are consistent with Perron's size and power tests and also with Kim and Schmidt (1993) and consequently we consider samples of 100, 500 and 1000 with 10,000 simulations. In doing so, we can compare the results with those of the other error structures considered by Perron and also have a direct comparison with the analysis of Kim and Schmidt. In the

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[6] Perron explicitly considers (i) iid errors; (ii) positive autocorrelation; (3) negative autocorrelation; (4) two specifications of higher-order correlation; (5) two specifications of MA(1) errors

extra experiments, due to the computational intensity of the simulations we only consider sample sizes of 100.

4. Description of the Data Generating Processes and Results

Due to space constraints we only report the full analysis for the experiments based on the statistic t^*_α. Further, since Perron (1997) shows that the procedure exhibits good size and power properties when the magnitude of the structural break is less than 5 standard deviations, which is typically the case in the data series analysed, the following analysis considers $\theta = \delta = 0$ unless otherwise stated.

We split the following section in to separate sub-sections that deal with separate characteristics with respect to the GARCH process being analysed however in each case the numbers in the tables refer to the proportion of rejections under the null hypothesis for 1%, 2.5%, 5% and 10% tests.

(i) The Impact on the Perron Procedure as $\phi_1 + \phi_2 \rightarrow 1$ when $\phi_0 = 1 - \phi_1 - \phi_2$, $\alpha = 1$

In the first set of experiments we vary the value of $\phi_1+\phi_2$ from a non-integrated case, $\phi_1+\phi_2 = 0.9$, to an integrated case, $\phi_1+\phi_2 = 1$, where $\phi_0 = 1 - \phi_1 - \phi_2$. Firstly we fix the value of ϕ_1 and vary the value of ϕ_2 such that the process becomes increasingly integrated. We then repeat the experiment for a fixed ϕ_2 and vary the value of ϕ_1. The initial variance is set to 1, therefore $h_0 = 1$, hence the initial variance is equal to the long run variance, i.e. $\phi_0 = h_0(1-\phi_1-\phi_2)$ and consequently as we increase the degree of integratedness the process also becomes degenerate.

Table 1 summarises the results obtained from the experiments for the three samples sizes when ϕ_1 is fixed and ϕ_2 is varied and Table 2 for when ϕ_2 is fixed and ϕ_1 is varied. The GARCH coefficients reported are (0.1, 0.3, 0.6), (0.05, 0.3, 0.65) and (0, 0.3, 0.7) in the first instance and (0.1, 0.6, 0.3), (0.05, 0.65, 0.3) and (0, 0.7, 0.3) for Table 2.

It can be seen that for a given sample size, as the GARCH process approaches integratedness and degeneracy the Perron procedure becomes increasingly over sized. In the limit, when the GARCH process is both integrated and degenerate the procedure suffers from a very serious over-sizing problem. If we consider the effect as we change the sample size we can see that in the integrated and degenerate case the proportion of rejections reported actually increases as the sample size increases. Consequently, we can see that the standard asymptotic theory is inapplicable in this case, however we have not isolated whether it is the degree of integratedness of the degree of degeneracy that is driving the over-rejection problem. Further, even when the process is not degenerate/integrated, not only is there still a serious over sizing of the test but it is not clear that the test statistics are converging to their asymptotic distributions as $T \rightarrow \infty$, therefore it appears that with certain non-degenerate/non-integrated GARCH specifications the standard asymptotic distributions are also not applicable.

It can also be seen that for given values of $\phi_1 + \phi_2$ and ϕ_0, the over rejection is higher in the case where ϕ_1 is higher and ϕ_2 is lower, i.e. the over-rejection is greater for the simulations reported in Table 2. For example, in the case of 100 observations, nominal 5% significance level, the rejection rate is approximately 23% in the case of $\phi_1 = 0.6$ and $\phi_1 = 0.3$, where as the rate is only 15% in the case when $\phi_1 = 0.3$ and $\phi_1 = 0.6$. Once again the difference could be caused by two factors, either ϕ_1 or ϕ_2, and this will be assessed shortly.

(ii) Assessing the Impact of the Volatility of the Variance Process, ϕ_1, and ϕ_2

Following Kim and Schmidt (1993), we considered how the degree of volatility of the variance process and its persistence affect the degree of over rejection. Kim and Schmidt state that "in the GARCH(1,1) model, it is roughly accurate to say that ϕ_1 determines the degree of volatility of the variance process, while $\phi_1+\phi_2$ determines its persistence." To analyse the effects, we consider the GARCH coefficients given by $\phi_1 = 0.1$ for $\phi_2 = 0.5, 0.8, 0.85, 0.9$ (Table 3) and then $\phi_1 = 0.3, 0.6, 0.65, 0.7$ where ϕ_2 is held constant at 0.3 (Table 4).

The results confirm those in Tables 1 and 2 in that as the degree of integratedness and degeneracy increases towards their limits the over rejection problem becomes increasingly significant.

Additionally, in the degenerate and integrated case, the size distortion once again increases in sample size, confirming that the asymptotic theory is inappropriate in these cases. Similarly, for some cases of non-degenerate and non-integrated processes it is not clear that the asymptotic theory is appropriate. From Table 3, we can see that if we allow the degree of integratedness/degeneracy to increase by varying ϕ_2 then, although the rejection rate increases, it does not do so significantly, except in the limit. Consequently, it appears that though ϕ_2 does have an impact on the rejection rate, it is not too significant.

If we now consider the cases where ϕ_0 and $\phi_1 + \phi_2$ are the same across Tables 1, 2, 3 and 4, for example $\phi_0 = 0.1$and $\phi_1 + \phi_2 = 0.9$, then it can be seen that as ϕ_1 decreases in value, the degree of over-rejection falls. In the case where $\phi_0 = 0.1$and $\phi_1 + \phi_2 = 0.9$, for a sample of 100 observations at a nominal 5% significance level, the rejection rate was approximately 23%, 15% and 7% for $\phi_1 = 0.6$, 0.3 and 0.1 respectively. In the limit when the GARCH process is both degenerate and integrated, the rejection rates are considerably different - being approximately 98%, 59% and 11% for $\phi_1 = 0.6$, 0.3 and 0.1 respectively.

Further, the increase in over-rejection as $\phi_1 + \phi_2$ increases falls as ϕ_1 falls. For example moving from $\phi_0 = 0.1$ and $\phi_1 + \phi_2 = 0.9$, through to $\phi_0 = 0$ and $\phi_1 + \phi_2 = 1$ causes the rejection rate to increase approximately 1.6 and 3.9 fold for $\phi_1 = 0.1$ and 0.3, respectively. Similarly, if we consider $\phi_0 = 0.1$ and $\phi_1 + \phi_2 = 0.9$, then we can see that the rejection rate for $\phi_1 = 0.3$ is approximately 2.7 times that when $\phi_1 = 0.1$. As we approach the integrated and degenerate case, these multiples increase such that when the limits are reached the rejection rate for $\phi_1 = 0.3$ is approximately 5.4 times that when $\phi_1 = 0.1$.

We can therefore conclude that, for a constant degree of degeneracy and integratedness, the magnitude of the volatility of variance coefficient determines the extent of over-rejection - the greater the value the greater the over-rejection. In addition, the relative rejection rates for different DGPs increases in: (1) the difference between the ϕ_1's; (2) the degree of degeneracy and (3) the degree of integratedness. It is interesting to note that even in the degenerate and integrated case, the Perron procedure appears to have reasonable size as the volatility parameter tends to zero. With respect to the ϕ_2 coefficient, we have seen that although the rejection rate for the test statistic increases as the coefficient increases, it does not appear to have as much of a significant impact as ϕ_1. Consequently, it appears that the volatility of variance coefficient has a considerable influence on the size of the test and not just the degree of integratedness and degeneracy. We now consider the individual impact of degeneracy and integratedness.

(iii) The Individual Impact of the Degree of Degeneracy on the Size of the Perron Procedure

To assess the impact that the degree of degeneracy has on the size properties of the Perron procedure we consider a series of simulations that hold the degree of integratedness constant and vary the degree of degeneracy. The first case that we consider is the non-integrated one where ϕ_1 = 0.3 and ϕ_1 = 0.65. We allow the degree of degeneracy to vary from ϕ_0 = 0 to ϕ_0 = 100, the results are summarised in Table 5. It can be seen that the rejection rate appears to be very insensitive to the degree of degeneracy except in the limit and consequently we can state that the Perron procedure is robust to degeneracy except in the limit. Some additional experiments were conducted where ϕ_0 = 0.0001 and 0.000001 and it was found that, although these were slightly different to those reported in the table, the difference was not significant and as a whole they confirmed the insensitivity of the test to the degree of degeneracy.

Similar experiments were carried out on (ϕ_1, ϕ_2) = (0.3, 0.7), (0.1, 0.9), (0.1, 0.85), (0.1, 0.5), (0.5, 0.1), (0.5, 0.45), (0.5, 0.5) for which Table 6 summarises the results at the 5% nominal significance level for 100 observations. It can be seen that the rejection rate is insensitive to the degree of degeneracy. It is worth noting that when the series is near degenerate (ϕ_0 = 0.000001), then, as long as the volatility of variance coefficient is small, the tests have good size, 0.051 and 0.07 for (ϕ_1, ϕ_1) = (0.1, 0.5) and (0.1, 0.85) respectively. As can be seen, the increase in the rejection rate is not too large as we approach integratedness, and even in the limit when (ϕ_1, ϕ_1) = 1, the rejection rate is still only 0.073 at the nominal 5% significance level, if $\phi_1 = 0.1$ and $\phi_2 = 0.9$, compared to 0.258 when (ϕ_1, ϕ_2) = (0.5, 0.5). In the limit, when the GARCH process is degenerate, the rejection rate is too high but is still highly dependent on the volatility of the variance.

Having considered varying the degree of near-degeneracy, it is now necessary to consider the impact of holding the degree of near-degeneracy constant whilst allowing the degree of near-integratedness to vary.

(iv) The Individual Impact of the Degree of Integratedness on the Size of the Perron Procedure

In assessing the impact that the degree of integratedness has on the size of the Perron test, we hold the degree of degeneracy and the volatility of the variance constant and then allow ϕ_2 to vary such that we go from an a non-integrated case to the limit of an integrated GARCH process. We then consider a different value of ϕ_1 and then repeat the exercise to assess the impact of the volatility of the variance and the degree of integratedness.

Table 7 reports the results for the simulations where $\phi_0 = 0.01$ and $\phi_1 = 0.3$, where ϕ_2 is allowed to take the values 0.6, 0.65 and 0.7. It can be seen that as we increase the degree of integratedness, the rejection rate increases and this is more marked as the sample size increases. For example, the rejection rate for 100 observations at the 5% nominal significance rate increases from 0.150 to 0.195 as ϕ_2 increases from 0.6 to 0.7, an increase of approximately 30%, whilst for a sample of 1000 observations, the rejection rate rises from 0.171 to 0.436, an increase of over 150%.

As would be expected the variance of the volatility also plays a significant role in determining the rejection rate, and this can be seen by considering the results in Table 8 where $\phi_1 = 1$ and the degree of integratedness is increased. The variance of the volatility parameter is lower than that used in Table 7, and as would be expected, there has been a corresponding decrease in the rejection rate. For example if we consider the nominal 5% significance level when $\phi_1 + \phi_2 = 0.9$, the rejection rates are 0.150 and 0.055 for a sample of 100 observations for $\phi_1 = 0.3$ and 0.1 respectively.

The above analysis has considered various experiments to pinpoint the main factors in determining the over-rejection problem. However, from an empirical point of view it is of interest to consider GARCH specifications that are likely to be encountered in applied work. Specifically, though we have shown that the Perron test exhibits size problems when the error specification is allowed to be a GARCH(1,1), are we likely to experience problems in practice?

(v) The Impact of Continuous Time Approximation - GARCH on the Perron Procedure

Although we have analysed what the impact of the volatility of variance, degeneracy and integratedness has on the size of the test, we need to also ensure that we consider cases that are likely to be empirically relevant. To analyse this, we consider various parameterisations as defined by Nelson (1990, 1992) and Nelson and Foster (1991). Employing the coefficient values given in Kim and Schmidt, we have $\omega = 0.01$, $\alpha = 0.3$ for $\gamma = 1$, 0.25, 0.09, and 0.01. The resulting values of the GARCH specification are $(\phi_1, \phi_2, \phi_3) = (0.01, 0.3, 0.7)$, (0.0025, 0.15, 0.85), (0.0009, 0.09, 0.91) and (0.0001, 0.03, 0.97) respectively. Table 9 summarises these results.

As γ decreases, the process becomes increasingly degenerate and integrated and ϕ_1 falls. It can be seen that the over-rejection problem decreases and the values appear to approach their asymptotic values as outlined in Perron (1997). In particular, given these results and those previously reported, the asymptotic values appear to be approached as the volatility coefficient falls, indicating that the majority of the over-rejection outlined in the previous experiments is indeed caused by this parameter. In reality, as the process becomes increasingly degenerate and integrated, the volatility parameter becomes smaller and the over-rejection problem is reduced to the extent that Perron's asymptotes are approached. The effects of the integratedness and degeneracy together appear to counteract each other to some extent, so that the degree of over-rejection is reduced in such cases.

We also conduct experiments for $\omega = 0.1$ and 0 for which the related GARCH specifications are (ϕ_1, ϕ_2, ϕ_3) = (0.1, 0.3, 0.7), (0.025, 0.15, 0.85), (0.009, 0.09, 0.91), (0.001, 0.03, 0.97) and (ϕ_1, ϕ_2, ϕ_3) = (0.0, 0.3, 0.7), (0.0, 0.15, 0.85), (0.0, 0.09, 0.91) and (0, 0.03, 0.97) respectively. These results are reported in Tables 10 and 11. The results in these tables make it apparent that the main determinant of the over-rejection problem is in fact the volatility of the variance parameter. This can be seen since even in a degenerate case, the over-rejection decreases as the volatility parameter falls.

(vi) The Impact of Continuous Time Approximation - GARCH on the Perron Procedure as the Magnitude of the Break Increases

To complete the Monte Carlo analysis, we considered allowing the magnitude of the break to increase such that δ in the DGP takes a value of 0, 1, 2, 5 or 10 when the GARCH process is given by $\phi_1 = 0.03$ and $\phi_2 = 0.97$ and ϕ_0 is allowed to vary, taking the values $\phi_0 = 0.001$, 0.0001 and 0. Once again, we set $\alpha = 1$ in the DGP. Perron (1997) reports that in the presence of normal errors, with constant conditional variance, the test has reasonable size characteristics when δ is less than 5 standard deviations and this conclusion is shown to be applicable when we allow the errors of the DGP to be a GARCH process as shown in Table 12[7]. For example, when $\delta = 2$ then the rejection rate in the presence of normally distributed errors is 0.048 compared to 0.045, 0.047 and 0.060 when $\phi_1 = 0.03$, $\phi_2 = 0.97$ and $\phi_0 = 0.001$, 0.0001 and 0 respectively. If we consider the case when $\delta = 5$, we can see that, although there is an impact even in the case of normally distributed errors, the over-sizing increases as the process becomes more degenerate, from a rejection rate of 0.108 to 0.164 when $\phi_1 = 0.03$, $\phi_2 = 0.97$ and $\phi_0 = 0.001$ and 0 respectively. Consequently, we can conclude that the presence of an increasingly degenerate GARCH process does not have a significant impact on the size of the test except when the break in the mean of the series is in excess of 5 standard deviations.

(5) Conclusions

[7] We only report the rejection rates for a sample size of 100 at the 5% nominal level due to space constraints.

The aim of this paper was to investigate the impact of GARCH on the Perron procedure for a unit root in the presence of a structural break. Kim and Schmidt (1993) and Haldrup (1994) both report severe over-sizing with respect to the Dickey-Fuller and Perron Z-test test, rendering the test potentially problematic in the presence of GARCH processes. A similar conclusion can be made with respect to the impact of GARCH on the Perron procedure, which allows for a structural break. Our analysis shows that the main factor that influences the rejection rate for the Perron procedure is the volatility of the variance of the GARCH process. If this value is large, there is a serious over-sizing of the test; however it is unlikely that the value of this will be too large in empirical work and consequently the test should have appropriate actual size. This conclusion remains valid even in the degenerate and integrated case. We show that the degree of degeneracy does not have an impact on the size of the test except in the limit, whereas the over-sizing increases as the degree of integratedness increases, both are however dependent on the volatility of the variance. The results for the continuous time approximation GARCH confirmed that, when the GARCH process is similar to that which we would typically find in empirical work, the test has reasonable size characteristics. When we allow for different magnitudes of the break coefficient in the presence of GARCH, the resulting size characteristics are consistent with those reported by Perron (1997) only if the magnitude of the break is less than 5 standard deviations. If the break is of a greater magnitude than 5 standard deviations, then over sizing becomes progressively worse as the degree of degeneracy increases. Consequently, we can conclude that the Perron procedure, though severely affected in certain cases, appears to be robust to the existence of GARCH when it approximates that which we typically find in practice as long as the magnitude of the break is small.

An interesting direction for future research would be an analysis of the impact of different error structures on the ability of the Perron test to accurately “date” the break. The reason that this is of relevance is that, although we have shown that the test for a unit root and structural break against an alternative of a stationary process with a structural break appears to be reasonably robust in the presence of GARCH of the form that we are likely to encounter in empirical work, this study has not assessed the effect of GARCH and /or structural breaks on the test's ability to date the break. It would also be of interest to assess which of the various unit root testing procedures available is most robust to various empirically relevant stylised regularities, including the impact of structural breaks, GARCH and unconditional fat tails. In doing so, one could potentially highlight which tests are best in certain circumstances and which ones are most problematic. This would prove to be beneficial as researchers would have a better understanding of the tools that they employ, and which of a set of contradictory results to favour in a given setting.

Table 1 - The impact on the Perron Procedure as $\phi_1 + \phi_2 \rightarrow 1$ for $\phi_0 = 1 - \phi_1 - \phi_2$ and $\phi_1 = 0.3$

	(0.1, 0.3, 0.6)			(0.05, 0.3, 0.65)			(0, 0.3, 0.7)		
	100	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**
1%	0.060	0.098	0.070	0.073	0.125	0.125	0.451	0.959	0.996
2.5%	0.103	0.163	0.127	0.130	0.190	0.196	0.534	0.974	0.996
5%	0.150	0.219	0.179	0.168	0.243	0.255	0.588	0.978	0.996
10%	0.214	0.285	0.238	0.235	0.328	0.331	0.653	0.984	0.998

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$

Table 2 - The impact on the Perron Procedure as $\phi_1 + \phi_2 \rightarrow 1$ for $\phi_0 = 1 - \phi_1 - \phi_2$ and $\phi_2 = 0.3$

	(0.1, 0.6, 0.3)			(0.05, 0.65, 0.3)			(0, 0.7, 0.3)		
	100	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**
1%	0.135	0.167	0.152	0.169	0.244	0.220	0.964	1.000	1.000
2.5%	0.188	0.245	0.218	0.211	0.330	0.291	0.976	1.000	1.000
5%	0.231	0.289	0.272	0.267	0.388	0.351	0.982	1.000	1.000
10%	0.303	0.345	0.357	0.336	0.447	0.436	0.986	1.000	1.000

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$

Table 3 - Assessing the Impact of ϕ_2, for $\phi_1 = 0.1$ as $\phi_1 + \phi_2 \rightarrow 1$ for $\phi_0 = 1 - \phi_1 - \phi_2$

	(0.4, 0.1, 0.5)			(0.1, 0.1, 0.8)			(0.05, 0.1, 0.85)			(0, 0.1, 0.9)		
	100	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**
1%	0.013	0.013	0.015	0.018	0.026	0.018	0.018	0.024	0.028	0.035	0.309	0.613
2.5%	0.032	0.036	0.035	0.039	0.049	0.043	0.044	0.076	0.073	0.061	0.414	0.710
5%	0056	0.071	0.062	0.069	0.095	0.075	0.077	0.118	0.119	0.108	0.503	0.753
10%	0.106	0.121	0.114	0.124	0.160	0.129	0.133	0.184	0.188	0.162	0.599	0.802

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$

Table 4 - Assessing the Impact of ϕ_1, for $\phi_2 = 0.3$ as $\phi_1 + \phi_2 \rightarrow 1$ for $\phi_0 = 1 - \phi_1 - \phi_2$

	(0.4, 0.3, 0.3)			(0.1, 0.6, 0.3)			(0.05, 0.65, 0.3)			(0, 0.7, 0.3)		
	100	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**
1%	0.046	0.024	0.018	0.135	0.167	0.152	0.169	0.244	0.220	0.964	1.000	1.000
2.5%	0.069	0.061	0.046	0.188	0.245	0.218	0.211	0.330	0.291	0.976	1.000	1.000
5%	0.101	0.099	0.081	0.231	0.289	0.272	0.267	0.388	0.351	0.982	1.000	1.000
10%	0.150	0.153	0.148	0.303	0.345	0.357	0.336	0.447	0.436	0.986	1.000	1.000

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$

Table 5 - The Impact of the Degree of Degeneracy on the Perron Procedure, $\phi_1 = 0.3$ and $\phi_2 =$

0.65 for ϕ_0 = 0 and 0.01

	(0, 0.3, 0.65)			(0.01, 0.3, 0.65)		
	100	**500**	**1000**	**100**	**500**	**1000**
1%	0.762	1.000	1.000	0.073	0.140	0.145
2.5%	0.815	1.000	1.000	0.130	0.217	0.217
5%	0.848	1.000	1.000	0.168	0.279	0.272
10%	0.887	1.000	1.000	0.235	0.347	0.335

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Simulations were also conducted for ϕ_0= 1.0 and 100, but the results were identical to the corresponding results for ϕ_0 = 0.01 and hence these are not presented. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$

Table 6 - The Impact of the Degree of Degeneracy on the Perron Procedure, where (ϕ_1, ϕ_2) = (0.3, 0.7), (0.1, 0.9), (0.1, 0.85), (0.1, 0.5), (0.5, 0.1), (0.5, 0.45), (0.5, 0.5) and ϕ_0 = 0.000001, 0.0001, 0.01, 1.0 and 100

	Degree of Degeneracy	
	0.000001	**0.0001**
(ϕ_1, ϕ_2)		
(0.1, 0.9)	0.073	0.072
(0.3, 0.7)	0.196	0.195
(0.5, 0.5)	0.258	0.257
(0.1, 0.85)	0.077	0.076
(0.3, 0.65)	0.158	0.158
(0.5, 0.45)	0.225	0.225
(0.1, 0.5)	0.051	0.051
(0.5, 0.1)	0.123	0.123

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Simulations were also conducted for ϕ_0= 0.01, 1.0 and 100, but the results were identical to the corresponding results for ϕ_0 = 0.0001 and hence these are not presented. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$

Table 7 – The Impact of the Degree of Integratedness on the Perron Procedure ϕ_0 = 0.01, ϕ_1 = 0.3 and ϕ_2 = 0.6, 0.65, 0.7

	(0.01, 0.3, 0.6)			(0.01, 0.3, 0.65)			(0.01, 0.3, 0.7)		
	100	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**
1%	0.059	0.089	0.080	0.073	0.140	0.145	0.090	0.252	0.292
2.5%	0.103	0.138	0.135	0.130	0.217	0.217	0.152	0.344	0.374
5%	0.150	0.189	0.171	0.168	0.279	0.272	0.195	0.408	0.436
10%	0.217	0.274	0.245	0.235	0.347	0.335	0.269	0.472	0.515

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$

Table 8 – The Impact of the Degree of Integratedness on the Perron Procedure
ϕ_0 = 0.01, ϕ_1 = 0.1 and ϕ_2 = 0.5, 0.8, 0.85 and 0.9

	(0.01, 0.1, 0.5)			(0.01, 0.1, 0.8)			(0.01, 0.1, 0.85)			(0.01, 0.1, 0.9)		
	100	500	1000	100	500	1000	100	500	1000	100	500	1000
1%	0.013	0.010	0.013	0.018	0.010	0.025	0.018	0.030	0.029	0.020	0.070	0.107
2.5%	0.032	0.042	0.039	0.039	0.051	0.057	0.043	0.067	0.066	0.036	0.134	0.181
5%	0.055	0.075	0.064	0.069	0.084	0.104	0.076	0.105	0.100	0.078	0.180	0.240
10%	0.106	0.133	0.107	0.124	0.139	0.154	0.132	0.163	0.164	0.127	0.262	0.316

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$

Table 9 - The Impact of Continuous Time Approximation - GARCH on the Perron Procedure,
ω = 0.01

	γ = 1 (0.01, 0.3, 0.7)			γ = 0.25 (0.0025, 0.15, 0.85)			γ = 0.09 (0.0009, 0.09, 0.91)			γ = 0.01 (0.0001, 0.03, 0.97)		
	100	500	1000	100	500	1000	100	500	1000	100	500	1000
1%	0.090	0.252	0.292	0.042	0.166	0.192	0.020	0.087	0.125	0.013	0.015	0.020
2.5%	0.152	0.344	0.374	0.072	0.246	0.281	0.040	0.165	0.219	0.027	0.062	0.080
5%	0.195	0.408	0.436	0.106	0.311	0.343	0.078	0.227	0.286	0.044	0.105	0.120
10%	0.269	0.472	0.515	0.171	0.375	0.412	0.124	0.305	0.365	0.092	0.159	0.190

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$. The values for the GARCH coefficients are derived from the Nelson (1992) formulation given by $\phi_0 = \omega\gamma$, $\phi_1 = \alpha\gamma^{(1/2)}$, $\phi_2 = 1 - \phi_1$, where ω = 0.01, γ = 1, 0.25, 0.09, and 0.01.

Table 10 - The Impact of Continuous Time Approximation - GARCH on the Perron Procedure,
ω = 0.1

	1 (0.1, 0.3, 0.7)			0.25 (0.025, 0.15, 0.85)			0.09 (0.009, 0.09, 0.91)			0.01 (0.001, 0.03, 0.97)		
	100	500	1000	100	500	1000	100	500	1000	100	500	1000
1%	0.090	0.226	0.307	0.037	0.148	0.197	0.020	0.060	0.111	0.011	0.014	0.031
2.5%	0.152	0.312	0.400	0.069	0.226	0.283	0.035	0.109	0.188	0.026	0.045	0.068
5%	0.195	0.383	0.455	0.099	0.292	0.346	0.067	0.152	0.247	0.043	0.071	0.111
10%	0.270	0.459	0.518	0.164	0.363	0.426	0.128	0.217	0.331	0.091	0.127	0.192

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$. The values for the GARCH coefficients are derived from the Nelson (1992) formulation given by $\phi_0 = \omega\gamma$, $\phi_1 = \alpha\gamma^{(1/2)}$, $\phi_2 = 1 - \phi_1$, where ω = 0.1, γ = 1, 0.25, 0.09, and 0.01.

Table 11 - The Impact of Continuous Time Approximation - GARCH on the Perron Procedure,
ω = 0.0

	1 (0.0, 0.3, 0.7)			0.25 (0.0, 0.15, 0.85)			0.09 (0.0, 0.09, 0.91)			0.01 (0, 0.03, 0.97)		
	100	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**
1%	0.451	0.959	0.996	0.088	0.666	0.874	0.028	0.258	0.555	0.012	0.014	0.056
2.5%	0.534	0.974	0.996	0.140	0.752	0.919	0.050	0.380	0.662	0.027	0.050	0.112
5%	0.588	0.978	0.996	0.193	0.788	0.935	0.095	0.459	0.713	0.047	0.082	0.158
10%	0.653	0.984	0.998	0.268	0.825	0.956	0.147	0.522	0.763	0.092	0.145	0.233

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$. The values for the GARCH coefficients are derived from the Nelson (1992) formulation given by $\phi_0 = \omega\gamma$, $\phi_1 = \alpha\gamma^{(1/2)}$, $\phi_2 = 1 - \phi_1$, where $\omega = 0.0$, $\gamma = 1$, 0.25, 0.09, and 0.01.

Table 12 - The Impact of Continuous Time Approximation - GARCH on the Perron Procedure, where the coefficient on the structural break variables is allowed to take the values 0, 1, 2, 5 and 10

		(ϕ_0, 0.03, 0.97)		
Magnitude of Break	**Normal**	ϕ_0 **= 0.001**	ϕ_0 **= 0.0001**	ϕ_0 **= 0**
0	0.045	0.043	0.044	0.047
1	0.047	0.043	0.045	0.052
2	0.048	0.045	0.047	0.06
5	0.087	0.108	0.149	0.164
10	0.451	0.460	0.546	0.590

The above table reports the rejection rates when the nominal 5% critical values, as outlined in Perron (1997), are employed to test for the presence of a unit root and structural break. Sample sizes of 100, 500 and 1000 are considered and the GARCH coefficients are expressed as (ϕ_0, ϕ_1, ϕ_2) where ϕ_0, ϕ_1 and ϕ_2 refer to the value of the coefficients in $h_t = \phi_0 + \phi_1 u_{t-1}^2 + \phi_2 h_{t-1}$. The values for the GARCH coefficients are derived from the Nelson (1992) formulation given by $\phi_0 = \omega\gamma$, $\phi_1 = \alpha\gamma^{(1/2)}$, $\phi_2 = 1 - \phi_1$, where $\omega = 0.1$, 0.01 and 0.0 and $\gamma = 0.01$.

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