

# Accounting and the Financial Accelerator

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## Abstract

We extend the general equilibrium economy of Holmström and Tirole (1997) to optimal reporting of productive assets and examine when the accounting process can contribute to financial acceleration. Given a small change in aggregate capital stock, the economy may respond with large readjustments in accounting policies, prices and investment activity. A neutral accounting system, defined as a policy that does not distort decision-making, is optimal when capital is abundant but, after a contraction in aggregate capital, the accounting system becomes initially liberal and then conservative. Surprisingly, accounting policies maximizing firm value, i.e., the net cash flows to shareholders, may lead to self-fulfilling equilibria with inefficient forced liquidations. The theory offers a stylized paradigm to evaluate accounting policies in the aggregate.

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The seminal contributions of Bernanke (1983), Bernanke and Gertler (1989) and Holmström and Tirole (1997) have examined economies with *multiple firms* facing financial constraints, demonstrating the existence of mechanisms that worsen the effects of adverse shocks, i.e., financial acceleration. Accountants have shown in *single-firm* models that accounting numbers play a key role in financing decisions and help alleviate financial constraints (Arya, Glover and Sivaramakrishnan 1997, Stocken and Verrecchia 2004, Göx and Wagenhofer 2009, Gao, Jiang and Zhang 2018).<sup>1</sup> But these models of standard-setting ignore how accounting choices across multiple firms aggregate to affect supply and demand of capital, in turn, affecting financial acceleration. Because models of financial acceleration do not incorporate accounting choices and models of standard-setting take interest rates (and, hence, financial acceleration) as a given, we know little of the interactions between accounting choices and the financial accelerator.

This study attempts to fill this void. We incorporate measurement into the general equilibrium model of Holmström and Tirole (1997) and use this framework to examine the joint determination of accounting policy, interest rates and investment in response to changes in aggregate fundamentals. By modelling the role of accounting in allocating capital, we ask whether accounting policies may magnify the effects of changes in fundamentals on economic activity.

Our analysis aims more broadly to explain why accounting matters in the aggregate (Jorgensen, Li and Sadka 2012, Crawley 2015, Breuer 2017). Procyclicality has been a recurrent, yet unresolved, problem in accounting measurement, starting with widespread concerns that accounting may contribute to business cycles (Plantin, Sapra and Shin 2008, Adrian and Shin 2010). In the simplest form of this theory, the accounting system reflects shocks to fundamentals by depleting equity, depressing the willingness of capital providers to finance new projects and then spiralling into lower aggregate economic ac-

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<sup>1</sup>This is only a small subset of an extensive literature in accounting that focuses on choosing information systems to solve an information asymmetry; see Stocken (2013) for a more complete treatment of models of communication in accounting.

tivity.

In models of aggregate shocks such as Bernanke and Gertler (1989) and Holmström and Tirole (1997), both the amount of aggregate capital and its market-clearing cost of capital will endogenously drive the response of the economy to a shock. However, these models focus on the response of investment to shocks but do not incorporate optimal reporting choices.<sup>2</sup> We develop a stylized general equilibrium model in which real capital is in limited supply, and demand for capital responds endogenously to accounting standards. Vice-versa, desirable accounting standards are affected by market-clearing prices in the capital market. Our objective is thus to build a theory in which the allocation of a scarce stock of capital and accounting standards are jointly determined.<sup>3</sup>

In the model, an owner-manager has a single productive asset that can be liquidated or operated subject to an agency problem. To solve the agency problem, lenders require the asset resale value to be sufficiently high, because they can resell it at the end of the period should the firm perform poorly. The asset value is unknown but may be revealed to lenders by an optimally-designed reporting system. Continuing the firm implies an opportunity cost equal to the interest that could have been earned on the proceeds from liquidation.

We examine first how the optimal reporting policy can contribute to financial acceleration under the assumption of a benevolent planner by considering how the economy and the reporting process responds to small changes in the fundamentals. Even a benevolent planner (maximizing end-of-period social surplus) would implement accounting policies

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<sup>2</sup>Our focus on optimal reporting systems is not new in accounting and draws from extensive research in this area, see, e.g., Arya et al. (1997), Stocken and Verrecchia (2004), Göx and Wagenhofer (2009), Heinle, Hofmann and Kunz (2012) or Gao et al. (2018). The main contribution of our study is to bring these insights into a general equilibrium model such that shocks to capital shock jointly determine the allocation of capital and accounting choices.

<sup>3</sup>A standard assumption, for example, is to assume that firms can use capital for an exogenous cost, as explained in various discussions on the productive effects of accounting, see, e.g., Kanodia and Sapra (2016). This approach assumes that firms can manufacture capital with a linear returns to scale technology and, for our research question, would imply that the endogenous manufacturing of capital can fully absorb aggregate shocks to capital stock.

that lead to large changes in activity for seemingly small shocks. Comparing investment responses to shocks in our model versus in an economy without agency problems, we show that the accounting process can initially dampen reductions in overall investment but, when aggregate capital is sufficiently depressed, contributes to financial acceleration.

We also characterize the form of accounting policies as a function of the aggregate capital. A neutral reporting system, defined as a policy that does not distort investments, is optimal when the level of aggregate capital is sufficiently high. As the aggregate capital stock decreases, increasing collateral initially requires a liberal reporting system, such that favorable events are measured less precisely than under the neutral system; as aggregate reporting declines further, the reporting system becomes conservative and follows stricter rules for continuation such that only precisely measured and favorable asset values can be continued, while all other imprecisely measured asset values are liquidated. The model thus predicts that more conservative reporting tends to be associated with negative aggregate shocks.

We consider next decentralized environments such that firms set their policy to maximize the ex-ante net payoff to shareholders, i.e., expected cash flows minus the competitive interest rate paid to lenders. This may (realistically) reflect the practical considerations of a regulator without the means to coordinate market expectations, that is, taking interest rates as a given. Without information asymmetries, decentralized and centralized regulation coincide and yield efficient allocations. By contrast, with collateral requirements, there exists an inefficient competitive equilibrium in which high interest rates trigger inefficient liquidations of low asset values, which, in turn, drains the amount of traded capital and sustains high market-clearing interest rates. This type of equilibrium exists for any aggregate capital because the combination of a policy misusing capital and high interest rates (caused by the scarcity of traded capital) is self-fulfilling, while more efficient competitive equilibria only exist with a sufficiently high capital stock. This result proves that, even in a competitive market, maximizing ex-ante value to sharehold-

ers need not implement efficient capital allocation because it ignores general equilibrium re-allocations.

We develop three intuitive extensions. First, in our baseline model, we simplify the analysis by using a reporting system that is a cutoff on asset values, which determines whether the firm reports high or low earnings. We recast the model through a mechanism where a regulator controls the distribution of reported signals and show that this cutoff structure is optimal in our setting. Second, while our baseline focuses on measurement of assets in the balance sheet prior to financing (i.e., held as collateral), we show that accounting signals that provide contractible information about outcomes unambiguously alleviate financial acceleration. Third, we consider a richer setting where firms need not always receive information and may strategically withhold their accounting signals (Dye 1985). In this setting, uncertainty about information endowment tends to increase inefficient continuations when capital is abundant and increase inefficient liquidations when capital is scarce.

**Literature Review.** Our analysis draws from an active research on the real effects of accounting measurement (Kanodia and Sapra 2016, Edmans, Heinle and Huang 2016). Within this area, our model intends to capture the consequences of credit market frictions on accounting, which is most closely related to a growing literature on banks, fragility in the lending system and accounting. Corona, Nan and Zhang (2014, 2018) emphasize how measurements can affect lending via the channel of bank competition. Lu, Sapra and Subramanian (2011) specifically focus on how measurements can affect the asset substitution problem in bank regulation.

Most close to ours are two studies that focus on stability and acceleration (or multiplier effect) caused by public disclosure. Zhang and Liang (2018) develop a model in which the nature of public disclosures affects coordination in economies with strategic complementarities. They show that an accounting system that is more objective, in reduc-

ing idiosyncratic noise, will reduce the frequency of inefficient investment runs. Zhang (2018) also examines an economy with strategic complementarities. Acceleration is driven by the interaction between coordination and competition. These studies are connected to an active recent literature which explains how accounting reports can interact with coordination, see, e.g., Gigler, Kanodia and Venugopalan (2013). Overall, these models can be interpreted as particular channels for financial acceleration because a small change in fundamentals can trigger runs in investment or trading decisions. But our concept of acceleration is quite different here, as we do not model strategic complementarities and, as in Holmström and Tirole (1997), focus on the acceleration caused by general equilibrium price readjustments.

Another set of studies provides us with an equilibrium determination of the cost of funds. Several prior studies focus on the interaction between information, risk premia and investment policy, see, e.g., Gao and Verrecchia (2012) or Cheynel (2013). However, these studies focus on the determination of the risk premium. In comparison, a key part of our analysis is that capital is scarce and must be allocated across firms; most models focusing on the risk premium formulate an economy with quasi-linear preferences where, by construction, there is a potentially large pool of capital. Following Holmström and Tirole (1997), we also put aside any effects due to aggregate risk-aversion in order to set aside risk-sharing effects already discussed in this literature.

# **1 The Model**

## **1.1 Overview**

We summarize the main ingredients of our approach and then develop the notations and general environment. The model is a stylized economy in which accounting reports provide a measurement of the external resale value of an asset (e.g., the fair value or “*price*

*that would be received to sell an asset or paid to transfer a liability in an orderly transaction,”* FAS 157). In particular, we shall focus here on one specific aspect of accounting information: “balance sheet” information about whether an asset has collateral value in a lending agreement. This is one important function of accounting, as measurements of assets (including impairments) can help lenders assess the ability of a firm to repay a loan.

Naturally, there are other critical roles of accounting and our main purpose here is not to present a complete model of all types of accounting information. In practice, accounting numbers provide information both from a balance sheet perspective, e.g., measuring assets at a particular point in time, and from an income statement perspective, measuring performance over a period with, say, revenue or cost of goods sold accounts. We focus on the former role here because it is information that is observed before financing and (in our model) implies more surprising and richer interactions with the financial accelerator. However, we show later on that a more precise contractible measure of income will also serve to unambiguously improve stewardship and alleviate financial acceleration. In other words, our model may be further re-interpreted as implying that information that increases the quality of the income statement benefits investment efficiency.

We depart from previous literature by assuming that the total amount of real capital in the economy (e.g., physical or intangible capital) is fixed, so that the rate of return must equate uses of capital with sources of capital. In a competitive equilibrium, the external capital raised by agents operating their asset (demand) equals the physical capital put for sale by agents selling assets (supply). In particular, while prior research focuses on the effect of accounting on investment levels, our model captures the allocative effects of accounting. Our study aims to clarify the joint determination of interest rate and reporting systems, and further characterizes how the *aggregate* economy adjusts to a contraction in resources.

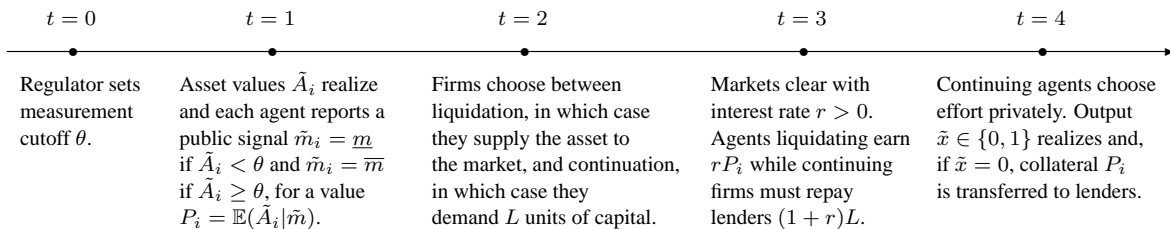


Figure 1: Model timeline

## 1.2 Information and Technology

The economy is populated by a continuum of agents  $i \in [0, 1]$  with mass normalized to one and a regulator setting accounting standards. Each agent owns a single productive asset with outside resale value  $\tilde{A}_i$ , drawn from an i.i.d. random variable with probability density function (p.d.f.)  $f$ , cumulative density function (c.d.f.)  $F$  and full support on  $\mathbb{R}^+$ . Normalizing the price per unit of traded capital to one, we interpret  $\tilde{A}_i$  as the amount of physical capital that an *external* party would obtain by buying the asset and using it in their own production technology (e.g., purchase  $\tilde{A}_i$  units of capital in the form of a real estate, patents, etc.). Hereafter, we omit the dependence on  $i$  when describing the problem of a generic agent.<sup>4</sup> A timeline of the model is presented in Figure 1 with details and notations below.

Similar to accounting for impairments or fair values, accounting reports provide noisy information about this external value prior to a sale. As in Dye (2002) and Laux and Stocken (2018), the accounting system maps the value of the asset into a binary report  $\tilde{m} \in \{\underline{m}, \overline{m}\}$  such that  $\tilde{m} = \overline{m}$  if and only if the asset value is above a reporting cutoff  $\tilde{A} \geq \theta$ . We may interpret a report as having high or low earnings, or a decision to write off an asset (Göx and Wagenhofer, 2009, 2010). We interpret the system as becoming more conservative as  $\theta$  increases (Kwon 2005), which is consistent with a more demanding

<sup>4</sup>This formulation describes uncertainty about the best use of an asset and, especially, as to whether the asset should be best used internally or sold to other users. In practice, the productivity of an asset may be correlated across multiple uses but, as long as the correlation is not perfect (which would remove any uncertainty), we should interpret  $\tilde{A}_i$  as the ratio of the economic benefits asset when used externally versus internally. For example, when  $\tilde{A}_i$  is high, the asset will be inefficiently used if held by agent  $i$ .



threshold to report higher values, such that higher realizations of  $\tilde{A}$  are measured more precisely (Gigler and Hemmer 2001, Gigler, Kanodia, Sapiro and Venugopalan 2009, Lu and Sapiro 2009, Li 2013, Caskey and Laux 2017). In Section 5.1, we show that this representation is without loss of optimality and can be derived as the solution to a general mechanism.<sup>5</sup>

Note that while we use the notation of asymmetry for expositional purposes, the function of the asymmetry in this model is to create an asset pool via the management of information. Specifically, a key aspect of this setting will be that continuing assets must be pooled together to source enough collateral (Göx and Wagenhofer, 2009, 2010). We may think about choices over different reporting methods (e.g., historical cost versus fair value) depending on the managers' intention to continue or liquidate (e.g., held-to-maturity versus trading securities). While we use the generic term of asymmetry (rather than particular transactions) for expositional purposes, we conjecture that many accounting rules used in practice may implement asymmetric pools.

After the report  $\tilde{m}$  is publicly revealed, the agent chooses over two mutually exclusive actions. First, the agent can liquidate the firm, and then sells the asset for a price  $P = \mathbb{E}(\tilde{A}|\tilde{m} = m)$  and the proceeds are re-invested at the competitive interest rate  $r$ , earning a payoff  $\pi_0(P) \equiv (1+r)P$ . Second, the agent can retain the asset and operate the firm. This requires additional external funds in the form of  $L > 0$  units of capital that must be raised from competitive investors, subject to an agency problem along the lines of Holmström and Tirole (1997) described below.

The firm's operations yield contractible cash flows  $\tilde{x} \in \{0, 1\}$ , where the probability of  $\tilde{x} = 1$  is equal to  $q \in (0, 1)$  if the manager exerts effort and reduced by  $\Delta q \in (0, q)$  if the manager shirks.<sup>6</sup> Effort is unobservable and involves a personal cost  $c > 0$  to the

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<sup>5</sup>This representation is also intuitive in our setting, because it only provides information that has non-zero social value. There are other implementations of the optimal reporting mechanism, but they would all feature extra information that does not affect actions. As long as there is an infinitesimal cost of reporting the information, the solution would correspond to our baseline specification. This is similar to the reduction to signal structures where posterior expectations match with actions in Kamenica and Gentzkow (2011).

<sup>6</sup>As noted earlier, we could also model a direct performance evaluation value of accounting by assuming

agent. Further,  $q - \Delta q - L < 0$  so that inducing low effort is never desirable. The asset can be used as collateral to raise  $L$ , which we operationalize by assuming that the asset can be sold at date 3 for its external value  $P$  if the agent needs liquidity to repay investors.<sup>7</sup>

As is well-known in this type of model, the agent can achieve  $\pi_1(P) \equiv q - c - (1 + r)L + P$  if operating the firm with high effort can be elicited, equal to the payoff of the project  $q$  net of the effort cost  $c$ , hereafter, the net operating cash flow, minus the expected payment to price-protected investors  $(1 + r)L$  plus the value of the collateral. However, to elicit high effort, the value of the collateral must be large enough:<sup>8</sup>

$$P \geq W^-(r) \equiv q \frac{c}{\Delta q} - q + (1 + r)L. \quad (1)$$

Even when (1) is satisfied, the agent will be better-off selling the asset when the asset has sufficiently high external value  $\pi_0(P) \geq \pi_1(P)$ , that is:

$$P \geq W^+(r) \equiv \frac{q - (1 + r)L - c}{r}. \quad (2)$$

In short, the firm is operated when  $P \in \mathcal{W}(r) \equiv [W^-(r), W^+(r)]$ . To avoid pathological equilibria, we assume that the agent operates the firm when indifferent and restrict the attention to  $r \in (0, r_{\max})$  where  $r_{\max}$  is given by  $W^-(r_{\max}) = W^+(r_{\max})$ . This guarantees that  $W^-(r) < W^+(r)$ , so that there is always a region of asset values in which the

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that the accounting system also generates a signal, which can be used to compensate the agent. Adding this element to the model would imply a more efficient resolution of the agency frictions and, as such, a more precise signal would reduce the need for collateral and help sustain competitive equilibria with lower interest rates and neutral measurements of assets.

<sup>7</sup>For expositional purposes, we assume that the asset does not depreciate over the course of its operations. The analysis is almost identical if, realistically, we reduce the end-of-period value of the asset to  $\delta P$  with  $\delta \in [0, 1]$ . The results are also unchanged if the agent observes  $\tilde{A}$  but does not have the means to make a verifiable disclosure, since investors would only pay the equilibrium price  $P$ .

<sup>8</sup>As in Holmström and Tirole (1997), the agent must offer a security with repayment  $s_x$  conditional on  $\tilde{x} = x$  such that (i) investors break even, i.e.,  $qs_1 + (1 - q)s_0 = (1 + r)L$ , (ii) the agent chooses high effort, i.e.,  $\Delta q(1 - s_1(P) + s_0(P)) \geq c$ , and (iii) the agent can repay at most the project payoff and the collateral  $s_x \leq x + P$ . To derive (1), set  $s_0 = P$  in (iii) and, then, solve for  $s_1 = ((1 + r)L - (1 - q)P)/q$  from (i); the bound  $W^-(r)$  then follows from substituting  $(s_0, s_1)$  into (ii). Intuitively, when  $P < W^-(r)$ , eliciting high effort would require an upside payment that is so large that investors would never be able to break even and recover their expected rate of return  $(1 + r)L$ .

firm can be operated.

### 1.3 Equilibrium Concept

We are now equipped to write the supply and demand equations that determine the equilibrium interest rate  $r$ . Each asset value  $A$  maps to an accounting report  $m \in \{\underline{m}, \bar{m}\}$  and an associated price  $P = \mathbb{E}(\tilde{A}|m)$ . Hence, we can denote  $\mathcal{D}(r)$  as the subset of realizations of  $\tilde{A}$  such that  $\mathbb{E}(\tilde{A}|m) \in \mathcal{W}(r)$  induces continuation and demands  $L$  units of capital. Vice-versa, we denote  $\mathcal{S}(r) = \mathbb{R}^+ \setminus \mathcal{W}(r)$  as the set of firm values such that the agent liquidates and supplies the asset to the capital market. Assuming that the economy is initially endowed with  $K$  units of capital, for markets to clear, the equilibrium interest rate  $r$  must satisfy<sup>9</sup>

$$\underbrace{L \int_{\mathcal{D}(r)} f(A) dA}_{\text{inverse demand curve}} = K + \underbrace{\int_{\mathcal{S}(r)} Af(A) dA}_{\text{inverse supply curve}}.$$

Adding  $\int_{\mathcal{D}(r)} Af(A) dA$  on both sides, this equation can be rewritten as

$$\int_{\mathcal{D}(r)} (A + L) f(A) dA = K + \mathbb{E}(\tilde{A}), \quad (3)$$

such that the right-hand side is the (fixed) total amount of useable capital in the economy and the left-hand side is the total used capital  $A + L$  for firms choosing to operate. To rule out corners, we further assume that

$$\int_0^{W^+(r_{\max})} (A + L) f(A) dA < K + \mathbb{E}(\tilde{A}) < \int_{\mathcal{D}(0)} (A + L) f(A) dA,$$

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<sup>9</sup>Implicitly, we assume that once capital is sold for price  $P$  and redeployed to a new firm, this new firm learns the true value of the capital and can readjust its capital by either buying extra capital ( $L - A$ ) or selling excess capital ( $P - A$ ).

guaranteeing that there is always excess supply (demand) of capital if the interest rate is small (large).

Initially, we shall assume that the regulator is benevolent and maximizes the ex-ante surplus of all agents (firms and investors), considering the consequences of the choices of accounting standard on the equilibrium interest rate. In our economy, all end-of-period payoffs must be from firms' operations and their collateral, so that each firm generates a surplus  $p - c - (1 + r)L + A$  for the agent running the asset and  $(1 + r)L$  for its investors. The regulator maximizes the total expected surplus (in utils) as given by

$$\Sigma(\theta, r) \equiv \int_{\mathcal{D}(r)} (q - c + A)f(A)dA$$

subject to the market clearing constraint (3).

Note that this objective function makes the strong assumption that the regulator is able to both understand and control expectations about interest rates. Later in the analysis, we show how policies that maximize the net payoffs of firms but cannot control the interest rate – possibly, a more realistic description of the current accounting policies – may lead to competitive equilibria with lower total surplus.

## 2 Competitive Equilibrium

By way of a benchmark, we derive the first-best equilibrium of the model in the absence of any informational friction: effort is contractible and  $\tilde{A}$  is publicly observed. The agent can design a forcing contract, which pays only when high effort is chosen, and the firm operates the asset when  $P \leq W^+(r)$ , where the bound  $W^+(r)$  is defined in (2). That is, the firm is better-off selling the asset when the outside resale value is sufficiently high. An increase in the interest rate  $r$  shrinks the set of asset values with continuation  $[0, W^+(r)]$  and expands the set of asset values with asset sale  $(W^+(r), \infty)$ .

Supply and demand have regular comparative statics in this environment. For a higher interest rate, the inverse demand  $L \int_0^{W^+(r)} f(A)dA$  slopes downwards as fewer firms demand capital, while the inverse supply  $K + \int_{W^+(r)}^\infty Af(A)dA$  slopes upwards as more firms put their asset for sale. The combination of these two effects yields a market-clearing interest rate  $r_{fb} \in (0, r_{\max})$  that satisfies

$$\int_0^{W^+(r_{fb})} (L + A)f(A)dA = K + \mathbb{E}(\tilde{A}). \quad (4)$$

**Proposition 1** *In the first-best equilibrium, firms sell their assets if and only if  $A \geq W^+(r_{fb})$ , where the interest rate  $r_{fb} > 0$  is uniquely given by (4). The interest rate  $r_{fb}$  is increasing in the firm's net operating cash flow  $q - c$  and decreasing in  $K$ .*

Proposition 1 characterizes the behavior of the first-best interest rate  $r_{fb}$ . If the exogenous supply of capital  $K$  decreases, the shift in the supply curve causes an increase in the equilibrium interest rate. Similarly, an increase in the net operating cash flow  $q - c$  increases demand for capital and the interest rate.

Even if  $\tilde{A}$  is not initially known, a benevolent planner can implement the first-best allocations by setting a reporting cutoff at  $\theta_{fb} \equiv W^+(r_{fb})$  such that all assets with  $\tilde{A} \leq \theta_{fb}$  correspond to low earnings  $\underline{m}$  and all assets values with  $\tilde{A} > \theta_{fb}$  correspond to high earnings  $\overline{m}$ . Hereafter, we refer to the information system that implements the first-best allocations as *neutral*, where neutrality is simply meant as an anchor point against which we define liberal or conservative accounting, thus, interpreting distortions away from neutrality as a consequence of informational friction. We say that the system  $\theta$  is conservative (liberal) if it features more (less) precise high asset values  $\theta > \theta_{fb}$  ( $\theta < \theta_{fb}$ ) relative to the level that would maximize first-best production efficiency.<sup>10</sup>

Next, consider the economy subject to informational frictions, where an agent must

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<sup>10</sup>Naturally, this definition is not essential for our study. The reader may define conservatism in a relative sense, noting that the system becomes more conservative as the cutoff to recognize high asset values increases, or choosing any arbitrary anchor point  $\theta_0$  (say, as the mode of the c.d.f.) for neutrality, adjusting the thresholds on aggregate capital for each system to be optimal accordingly.

satisfy the collateral constraint  $P \geq W^-(r)$ . This constraint can be incompatible with first-best allocations. As the stock of capital  $K$  decreases, the first-best competitive interest rate  $r_{fb}$  increases because there is more intense competition for scarce capital. This, in turn, reduces  $W^-(r_{fb})$ , which makes obtaining financing more difficult. From the collateral constraint (1), implementing first-best allocations in the presences of informational frictions requires the following condition on collateral:

$$\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta_{fb}) \geq W^-(r_{fb}). \quad (5)$$

When this condition fails, the allocation at  $\theta_{fb}$  is infeasible and so is any other allocation prescribing continuation for any  $\tilde{A} \leq \theta$ . To see why, recall that  $\theta_{fb}$  is determined at the point at which demand equates supply. Any other choice  $\theta < \theta_{fb}$  would prescribe fewer liquidations and implying excess supply of capital; a cutoff  $\theta > \theta_{fb}$  would symmetrically imply excess demand of capital. As a result, no cutoff  $\theta \neq \theta_{fb}$  can sustain allocations with continuations when  $\tilde{A} \leq \theta$ .

**Proposition 2** *A benevolent planner implements the first-best cutoff  $\theta_{fb}$  if  $K \geq K_{fb}$  where  $K_{fb}$  is (uniquely) defined by*

$$K_{fb} + \mathbb{E}(\tilde{A}) = F(\theta_{fb})(W^-(r_{fb}) + L). \quad (6)$$

*Otherwise, there is no feasible reporting cutoff  $\theta$  such that firms with  $\tilde{A} \leq \theta$  continue.*

Although inducing the first-best capital allocation is infeasible when  $K < K_{fb}$ , the planner can, alternatively, use a reporting system with the form prescribed by Göx and Wagenhofer (2009) and which raises collateral by inducing continuation for firms with  $\tilde{A} \geq \theta_b$ . This is a second-best allocation because those firms with collateral  $\tilde{A} \leq \theta_b$  are forced to liquidate assets that generate almost no capital for external users. Plugging this

allocation into the market-clearing condition (3), the cutoff  $\theta_b$  is uniquely given by<sup>11</sup>

$$\int_{\theta_b}^{\infty} (A + L)f(A)dA - K - \mathbb{E}(\tilde{A}) = 0. \quad (7)$$

**Proposition 3** *If  $K < K_{fb}$ , a benevolent planner implements a cutoff  $\theta_b$  given by the unique solution to (7). The reporting system  $\theta_b$  is liberal, i.e.,  $\theta_b < \theta_{fb}$ , when  $K + \mathbb{E}(\tilde{A}) > L - K$ , and conservative, i.e.,  $\theta_b > \theta_{fb}$ , otherwise.*

We conclude this section by illustrating how an economy would transition between accounting policies and how the induced aggregate investment adjusts to different levels of aggregate capital. In Figure 2, when the economy has enough aggregate capital, the benevolent planner will choose neutral accounting and implement the first-best allocation of capital.

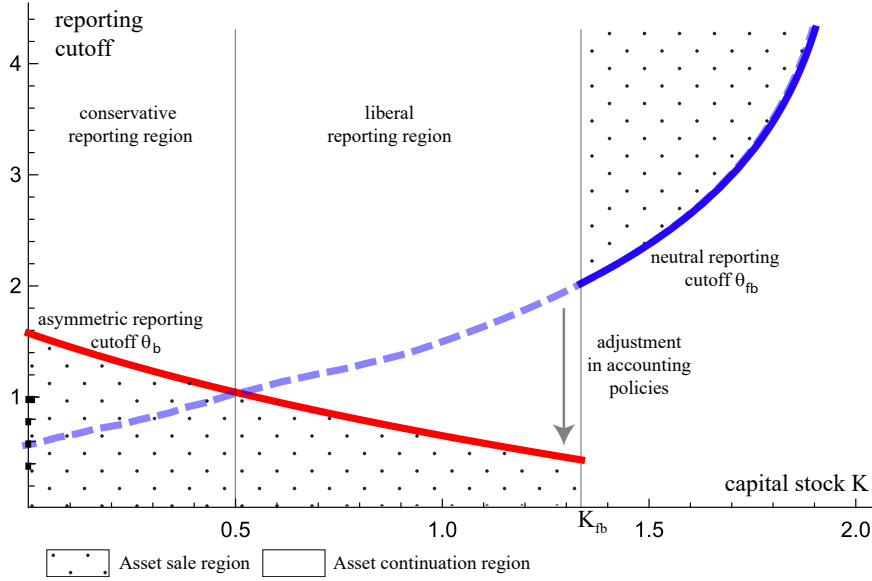


Figure 2: Optimal cutoff as a function of aggregate capital

As capital stock decreases below  $K_{fb}$ , the first-best allocation is no longer feasible

<sup>11</sup>The market-clearing interest rate that is a solution in the benevolent planner's problem is typically not unique: in Proposition 2, any interest rate such that  $\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta_b) < W^-(r)$  and  $\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta_b) \geq W^+(r)$  will clear markets because it induces to sell the asset conditional on low earnings and continue conditional on high earnings.

and the reporting choice must change to a cutoff  $\theta_b$  such that collateral values below  $\theta_b$  are liquidated. Note that this implies a discontinuous change to aggregate activity, because the nature of the reporting system changes from continuing conditional on low collateral to continuing conditional on high collateral. Initially, this is achieved via liberal accounting  $\theta_b < \theta_{fb}$  because only the lowest collateral need to be liquidated and, therefore, the reporting system imprecisely measures favorable events. As the aggregate capital stock becomes lower, competition for capital increases interest rate to a point where only the highest collateral values can be financed: then, high collateral values must be measured precisely with a conservative reporting system.

### 3 Policies maximizing firm value

Our next steps will be to derive the competitive equilibrium when the policy is chosen to maximize firm value. We define firm value as the expected final cash flow net of payments made to (competitive) capital providers. Formally, given that a firm realizes  $q - c - (1 + r)L + P$  if it operates and  $(1 + r)P$  if it does not operate, the net value from continuing is<sup>12</sup>

$$\sigma(\theta; r) \equiv \int_{\mathcal{D}(r)} (q - c - (1 + r)L - rP_A) f(A) dA, \quad (8)$$

where  $P_A$  is the price achieved by a firm with assets  $\tilde{A} = A$  and is equal to  $\mathbb{E}(\tilde{A}|\bar{m})$  if  $A \geq \theta$  or  $\mathbb{E}(\tilde{A}|m)$  otherwise.

**Definition 1** *A competitive equilibrium  $(r, \theta)$  is defined as an interest rate  $r$  and a reporting system with a cutoff  $\theta$  such that (i) markets clear, i.e., (3) is satisfied, and (ii) the policymaker maximizes net value, i.e.,  $\theta \in \operatorname{argmax}_{\theta'} \sigma(\theta'; r)$ .*

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<sup>12</sup>Writing the objective as a net benefit from continuation simplifies the expression but, naturally, the objective function is equivalent to maximizing the shareholder value  $g(A)$  where  $g(A) = q - c - (1 + r)L + P_A$  if  $A \in \mathcal{D}(r)$  and  $g(A) = (1 + r)P_A$  otherwise.



Note that this objective is no longer a normative statement about what policymakers *should* do (to maximize welfare) but can already be interpreted in terms of various plausible perspectives on how accounting policies might emerge in practice. First, accounting regulators may not be able to coordinate market expectations on new interest rates and, for practical purposes, may focus on maximizing the value to shareholders given the current prevailing market conditions. Second, we may also think about accounting policies as being, at least in part, implemented *ex ante* by individual firms to maximize shareholder value, in which case, firms would choose the policy that maximize firm value rather than end-of-period total surplus.

Accordingly, we refer to this setting as a *decentralized* policymaker because policymakers, whether individual firms or standard-setters, do not control or apprehend the entire equilibrium. Naturally, this objective is also implicit in existing models of accounting choice where the cost of investment or flows to competitive capital providers are netted out from the surplus. But does maximizing firm value necessarily entail any loss in investment efficiency? Interestingly, without information asymmetries, the policy-maker would implement first-best allocations when maximizing firm values.

**Proposition 4** *If effort is contractible, there exists a unique competitive equilibrium and it is such that the interest rate is  $r_{fb}$  and the cutoff  $\theta_{fb}$  implements the first-best allocation.*

Next, consider competitive equilibria when effort is not contractible. To do so, we proceed to two steps: first, analyzing the policymaker's choice of reporting system and the resulting continuation decisions for any given prevailing interest rate  $r$  and, second, writing the resulting supply and demand to recover the equilibrium interest rate  $r$  that clears markets in (3).

The first step of the analysis draws from Göx and Wagenhofer (2009) and Bertomeu and Cheynel (2015) adapted to the general equilibrium environment and is a key preliminary to our analysis. Specifically, holding the interest rate  $r$  as a given, we state the

optimal cutoff chosen by the policymaker below.

**Lemma 1** *Let  $\bar{r}$  be defined by  $\mathbb{E}(\tilde{A}) = W^-(\bar{r})$  such that, in the aggregate, total available collateral could potentially meet collateral constraints for all firms. For any given interest rate  $r$ , the reporting cutoff  $\theta^*(r)$  that maximizes  $\sigma(\theta; r)$  is given by:*

- (a) *if  $r \leq \bar{r}$ , the reporting cutoff  $\theta^*(r) = \min(W^+(r), \theta_a(r))$  where  $\theta_a(r)$  is an increasing function defined by  $\mathbb{E}(\tilde{A} | \tilde{A} \leq \theta_a(r)) = W^-(r)$  and such that firms operate the asset when  $\tilde{A} < \theta^*(r)$ ;*
- (b) *otherwise,  $\theta^*(r) = \theta_b(r)$  where  $\theta_b(r)$  is an increasing function defined by  $\mathbb{E}(\tilde{A} | \tilde{A} \geq \theta_b(r)) = W^-(r)$  and such that firms operate the asset when  $\tilde{A} \geq \theta^*(r)$ .*

*Hereafter, define  $\underline{r} < \bar{r}$  such that  $\theta^*(r) = W^+(r)$  when  $r \leq \underline{r}$ .*

In Lemma 1, the policymaker adjusts to the prevailing interest rate demanded by capital providers to minimize the distortions due to the collateral constraint  $W^-(r)$ . If  $r < \underline{r}$ , the interest rate is sufficiently low so that the policymaker can implement the ideal cutoff  $\theta^*(r) = W^+(r)$  and the collateral constraints do not bind. For a moderate interest rate  $r \in (\underline{r}, \bar{r}]$ , the policymaker must tolerate some excessive continuations on  $[W^+(r), \theta_a(r)]$  as a manner to subsidize the collateral of firms with low assets. As  $r$  further increases beyond  $\bar{r}$ , the policymaker must keep increasing investment distortions to increase collateral, now requiring inefficient liquidations for some firms with low collateral  $\tilde{A} < \theta_b(a)$ .

We obtain the operating decisions from Lemma 1 and inject them into the (inverse) excess demand

$$\Delta(r) \equiv \int_{\mathcal{D}(r)} (A + L)f(A)dA - K - \mathbb{E}(\tilde{A}).$$

As in Holmström and Tirole (1997), we plot this function in Figure 3, varying the interest rate  $r$  until excess demand equals zero, i.e.,  $\Delta(r) = 0$ . For interest rates up to  $r = \underline{r}$ , the policymaker implements the cutoff  $W^+(r)$ , increasing in  $r$ , and the excess demand features conventional comparative statics, decreasing as outside capital becomes

more costly. In contrast, for moderate interest rates  $r \in [\underline{r}, \bar{r}]$ , the collateral constraint binds and an increase in the interest rate requires an increase in the expected asset value for operating firms to meet the minimal collateral  $W^-(r)$ . This is achieved by expanding the continuation region and inducing more firms to continue and demand capital. Hence, excess demand *increases* in response to a higher interest rate in this region.

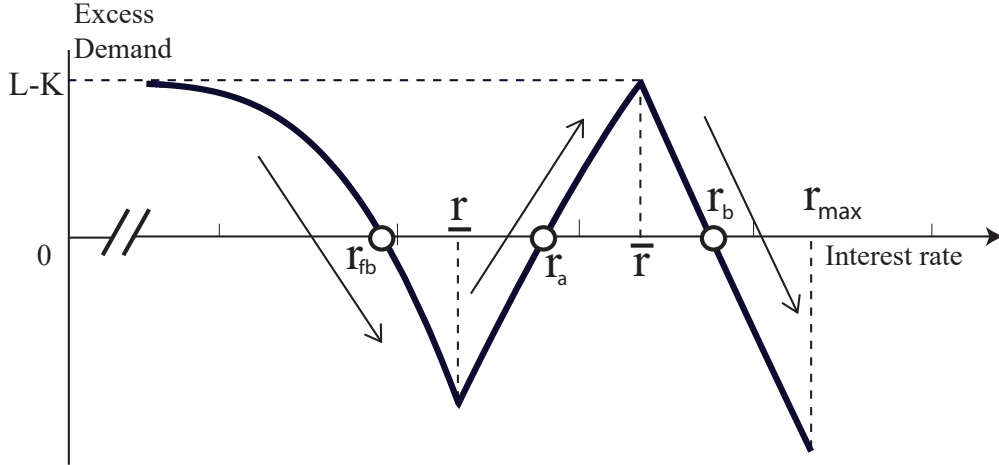


Figure 3: Excess Demand as a function of the interest rate

The resulting excess demand is U-shaped on  $[0, \bar{r}]$ , decreasing from  $L - K$  when the interest rate is zero, since all firms would be better-off retaining their asset without any positive returns, to a local minimum at  $\Delta(\underline{r})$  at  $r = \underline{r}$ , and then increasing back to  $L - K$  because, then, the policymaker needs to pool all firms to generate sufficient collateral  $\mathbb{E}(\tilde{A}) = W^-(\bar{r})$ .

If  $\Delta(\underline{r}) < 0$ , corresponding to the environment illustrated in Figure 3, there are two competitive equilibria  $(r_i^*, \theta_i^*)_{i=1,2}$  and the reporting cutoff  $\theta_i^*$  is such that firms with assets  $\tilde{A} \geq \theta_i^*$  continue. But both cutoffs must also satisfy the market-clearing constraint  $\Delta(r_i^*) = 0$ : this requires that the two cutoffs be equal to  $\theta_1^* = \theta_2^* = W^+(r_{fb})$  because, as the first-best cutoff  $W^+(r_{fb})$  allocates the aggregate stock of capital  $\mathbb{E}(\tilde{A}) + K$ , any other cutoff different from it would use a different amount of capital and could not clear markets. Put differently, the two equilibria feature neutral accounting, allocating capi-

tal efficiently: as a result, agents achieve their highest possible ex ante utility despite the existence of collateral constraints.

Closer inspection of the excess demand also shows that the equilibria  $(r_i^*, \theta_i^*)_{i=1,2}$  exist as a pair. The excess demand shifts upwards as aggregate capital  $K + \mathbb{E}(\tilde{A})$  decreases so this condition for existence can be equivalently written as a lower bound on aggregate capital. Intuitively, resolving collateral constraints requires capital to be sufficiently abundant. Note also that in the region  $[0, \underline{r}]$ , the reporting system chosen by the policymaker coincides with the first-best policy  $W^+(r)$ , so the first equilibrium interest rate must be the first-best interest rate  $r_1^* = r_{fb}$ . In contrast, in the region  $[\underline{r}, \bar{r}]$ , the reporting cutoff  $W^+(r_{fb})$  set by the policymaker satisfies  $\mathbb{E}(\tilde{A} | \tilde{A} \leq W^+(r_{fb})) = W^-(r_2^*)$ , implying a higher interest rate  $r_2^* = r_a > r_{fb}$  where

$$\mathbb{E}(\tilde{A} | \tilde{A} \leq W^+(r_{fb})) = W^-(r_a). \quad (9)$$

This equilibrium has distributional consequences and is not equivalent to the first-best equilibrium  $(r_{fb}, W^+(r_{fb}))$ . The higher interest rate  $r_a$  (which can only exist with informational frictions) makes lenders with  $\tilde{A} > W^+(r_{fb})$  and other capital providers supplying  $K$ , e.g., financial institutions and other passive investors, better-off because they earn a higher return. However, it hurts the agents with  $\tilde{A} \leq W^+(r_{fb})$  operating the asset.

**Proposition 5** *Suppose that aggregate capital is large enough, i.e.,  $K \geq K_{fb}$ , where  $K_{fb}$  is defined in (6), then there exist two equilibria  $(r_i^*, \theta_i^*)_{i=1,2}$  where  $r_1^* = r_{fb} < r_2^* = r_a$  defined by (9) and such that both equilibria exhibit a neutral reporting system  $\theta_1^* = \theta_2^* = W^+(r_{fb})$ . If  $K < K_{fb}$ , there is no competitive equilibrium with  $r \leq \bar{r}$ .*

As the interest rate increases beyond  $\bar{r}$ , there is no longer sufficient collateral to operate all firms with low asset values and the policymaker implements a continuation region

$\mathcal{D}(r) = [\theta_b, \infty]$  in which only firms with high earnings  $\bar{m}$  can meet the collateral constraint and operate. The excess demand then returns to conventional comparative statics: as the interest rate increases  $r$ , more excessive liquidations are required ( $\theta_b(r) \uparrow$ ), and the continuation region shrinks with a corresponding decrease in excess demand. This implies a third possible equilibrium such that the reporting cutoff  $\theta_b$  satisfies the market-clearing condition

$$\int_{\theta_b}^{\infty} (A + L)f(A)dA - K - \mathbb{E}(\tilde{A}) = 0. \quad (10)$$

**Proposition 6** *There exists a unique competitive equilibrium  $(r_b, \theta_b)$  with  $r \in [\bar{r}, r_{\max}]$ , where  $\theta_b$  is given by (10) and the interest rate  $r_b$  satisfies  $\mathbb{E}(\tilde{A}|\tilde{A} \geq \theta_b) = W^-(r_b)$ . The reporting system  $\theta_b$  is liberal when  $K + \mathbb{E}(\tilde{A}) > L - K$ , and conservative when  $K + \mathbb{E}(\tilde{A}) < L - K$ .*

Proposition 6 demonstrates an important implication of the equilibrium model. Even when the economy has sufficient capital available  $K + \mathbb{E}(\tilde{A})$  to, potentially, operate all firms in the first-best capital allocation, there is always a competitive equilibrium with a higher interest rate  $r_b > r_{fb}$  which implies distorted operating decisions because some firms with  $\tilde{A} \leq \theta_b$  fail to meet their collateral constraints and (inefficiently) liquidate.

From an investment perspective, the equilibrium is sustained because the policymaker facing a high interest rate  $r_b$  responds by setting a reporting system such that firms with high asset values continue in order to increase collateral above  $W^-(r_b)$ . But this type of reporting system drains the supply of capital by ensuring that high-value assets are not sold. In the competitive equilibrium, the capital market responds to lower supply with a higher interest rate, whereby high interest rates become self-fulfilling.

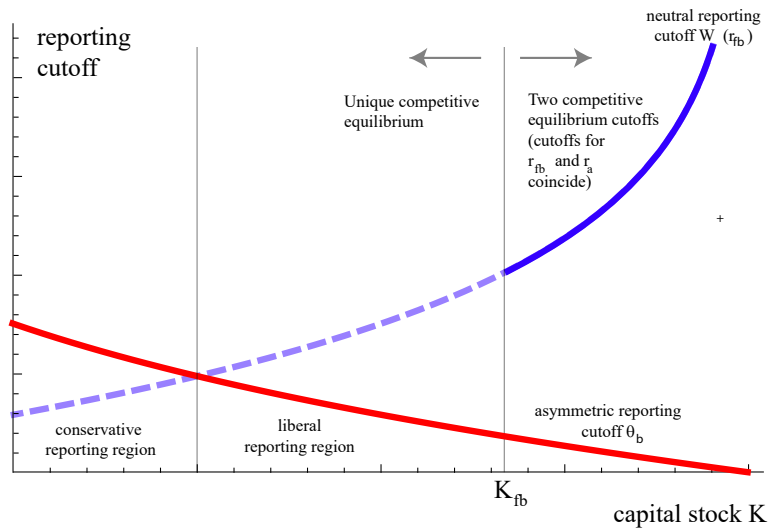


Figure 4: Cutoff as a function of aggregate capital

In Figure 4, we contrast the competitive equilibrium with a decentralized policymaker to Figure 2, when a benevolent regulator can set the preferred equilibrium. The competitive equilibrium is unique and corresponds to the equilibrium with a benevolent regulator when  $K < K_{fb}$ . However, in the region with  $K \geq K_{fb}$ , the inefficient equilibrium (in red) coexists with the competitive equilibrium that the benevolent regulator would choose.

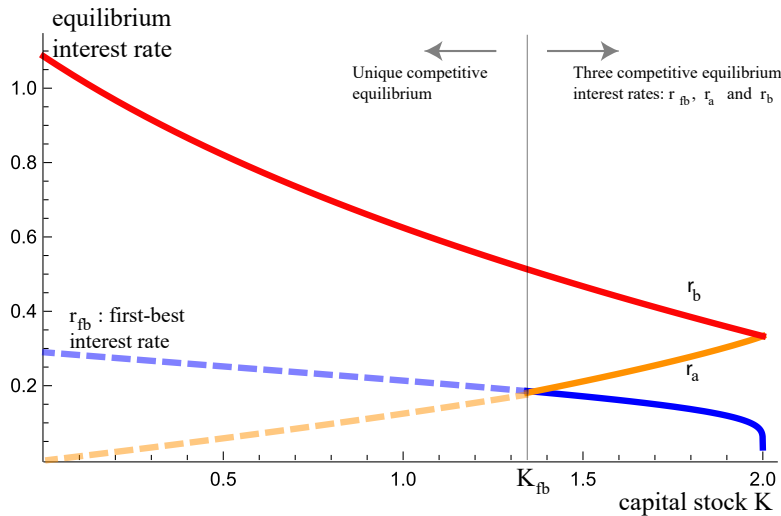


Figure 5: Interest as a function of aggregate capital

Figure 5 displays the equilibrium interest rates as a function of aggregate capital  $K$  and contrasts them with the first-best interest rate. One may conjecture that only exceptional circumstances would trap an economy on the inefficient competitive equilibrium with high interest rates  $r_b$ . However, there are economic forces that may plausibly lead the economy to implement the high interest rate equilibrium even when equilibria with more efficient allocations exist. First, the equilibrium with higher interest rate is preferred by external capital providers supplying  $K$  and, therefore, a focus on maximizing value to capital providers relative to economic efficiency would naturally gravitate toward the equilibrium  $(r_b, \theta_b)$ .<sup>13</sup> Second, even if the economy were to set a competitive equilibrium  $r^* \in \{r_{fb}, r_b\}$ , it would have to readjust toward  $r_a^*$  for a sufficiently large decline in capital. Once the economy is coordinating on  $(r_b, \theta_b)$ , it would require a discontinuous coordinated adjustment in expectations to restore  $(r_a, \theta_a)$  or  $(r_{fb}, \theta^*)$ .<sup>14</sup>

## 4 Accounting Acceleration

Until this point, we considered acceleration intuitively as a mechanism in which a change in capital stock and informational frictions can cause large readjustments to the competitive equilibrium. Below, we explore a more formal definition of acceleration by examining how investment adjusts in response to contractions in capital. Note that the equilibrium approach helps examine the response of the economy to contractions. If we had (as is usual) taken the interest rate as a given, firms would simply be able to draw more capital at the fixed interest rate and the model would not capture implied scarcity in

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<sup>13</sup>We use the policymaker in this model as a placeholder for all institutions involved in setting standards, and our purpose is not to blame accounting standard-setters for any possible inefficiency. In the United States, for example, the FASB has noted that it agrees with the principles of measuring economic efficiency but it is, currently, not endowed with sufficient resources to make this assessment (see further discussions in Concepts Statements No.1) and are bound by the SEC to a limited mandate focusing on capital providers.

<sup>14</sup>In addition, the equilibrium  $(r_a, \theta_a)$  is unstable, that is, for any small perturbation  $r > r_a$  ( $r < r_a$ ), a Walrasian auctioneer would observe higher (lower) excess demand and increase (decrease) the interest rate toward  $r_b$  ( $r_{fb}$ ). As to the multiplicity of equilibria caused by informational frictions, our approach is closer to the recent study of Corona, Nan and Zhang (2014) who show that banks' optimal reporting choices will affect their supply of loans and thus may cause regions with multiple equilibria.

aggregate capital.

To address this question, we analyze, as in Holmström and Tirole (1997), the response of interest rates and investment to financial shocks by considering two types of shocks. First, the economy features a contraction in the capital stock, or in short *capital crunch*, if the overall available capital  $K$  is reduced. To be concrete, a capital crunch could occur if foreign investors experience a domestic shock and pull out some of their assets from the country; alternatively, the economy could experience a negative shock to the productivity of all-purpose capital because some capital can no longer be profitably used or deployed.

Second, the economy features a contraction in asset values if the external resale value of a firm's productive assets is reduced. We model it as a proportional decrease by writing  $\tilde{A} = (1 - \epsilon)\tilde{a}$  and increasing  $\epsilon > 0$  while holding  $\tilde{a}$  fixed. In our model, such a situation will occur, for example, if assets become more specialized or firm-specific.<sup>15</sup> Figure 6 presents an intuition for the effects of a capital crunch or asset value contraction in each competitive equilibrium.

**Corollary 1** *Conditional on a capital crunch, the interest rates  $r_{fb}$  and  $r_b$  increase, and the interest rate  $r_a$  decreases and the reporting cutoff  $\theta_{fb}$  decreases while the reporting cutoff  $\theta_b$  increases.*

Figure 6a plots a capital crunch and the shifts of the excess demand curve. In all types of equilibria, a capital crunch must lead to fewer firms operating in order for markets to clear. In the equilibria  $(r_{fb}, \theta_{fb})$  and  $(r_b, \theta_b)$ , this leads to an increase in the interest rate so that more liquidations can be induced. In comparison, in the middle equilibrium with interest rate  $\theta_a$ , the policymaker responds to a higher interest rate by increasing pooling for continuing firms and, therefore, a capital crunch must lead to a lower interest rate in order to coincide with lower continuations.

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<sup>15</sup>If a shock affects an entire industry, for example, the asset when sold may no longer be used in a manner related to how it was used by the firm and instead may need to be fully reconverted. Consider the following example. If the asset is an inventory of gold jewelry, its asset value might be the value of the jewelry when sold by a different seller. If, however, an economic shock affects the entire industry, the gold might have to be melted to be used in the semiconductor industry, thus losing value as an asset.



**Corollary 2** *Conditional on a contraction in asset values, the interest rate  $r_{fb}$  increases, while the interest rates  $r_a$  and  $r_b$  decrease, and the reporting cutoff  $\theta_{fb}$  decreases, while the reporting cutoff  $\theta_b$  increases.*

A contraction in asset values is plotted in Figure 6b. The effects are similar to those of a capital crunch along some dimensions. In all equilibria, the contraction causes more liquidations (again as a result of market-clearing). However, the effect on the interest rate can be different from a capital crunch. The first-best interest rate  $r_{fb}$  adjusts intuitively, increasing when there is less aggregate capital  $\mathbb{E}(\tilde{A})$ . The interest rates  $r_a$  and  $r_b$ , however, decrease in response to a contraction in asset values. In the case of  $r_a$ , the policymaker responds to higher interest rates by increasing continuations (of higher asset values) to meet the average collateral constraint; hence, a lower interest rate is required to meet the market-clearing constraint. In the case of  $r_b$ , lower asset values would tend to induce liquidations for a greater proportion of firms, which will have a large effect on demand, implying that the equilibrium interest rate must decrease.

Next, we define (relative) acceleration in terms of the effect of the financial shock on the difference between total investment in the competitive equilibrium with the agency problem relative to the total investment with contractible effort. Formally, the accelerator  $\alpha$  is defined as

$$\alpha(X) = \frac{\partial\{\mu_F(\mathcal{D}(r^*)) - \mu_F(\mathcal{D}(r_{fb}))\}}{\partial X},$$

where  $\mu_F(Y) \equiv \text{Prob}(\tilde{A} \in Y)$ . A positive value for  $\alpha(X)$  means that for an increase in a variable  $X$ , the investment in the economy with frictions decreases more than it would without frictions.

From Proposition 5, the accelerator is zero when the competitive equilibrium is at  $r \in \{r_{fb}, r_a\}$ , since these two equilibria do not involve any investment distortion relative to contractible effort. We shall thus focus next on the accelerator for the competitive equilibrium with  $r = r_b$ .

**Corollary 3** *In a competitive equilibrium with  $r = r_b$ , there is always a financial acceleration conditional on a capital crunch, i.e.,  $\alpha'(K) < 0$ . Further, there is a financial acceleration conditional on a contraction in asset values, i.e.,  $\alpha'(\epsilon) < 0$ , if and only if  $\mu_F(\mathcal{D}(r_b)) > \mu_F(\mathcal{D}(r_{fb}))$ .*

A shock to the capital stock affects the economy with frictions more strongly than it affects the investment with contractible effort. Recall that a decrease in  $K$  must reduce aggregate investment since less capital is available. The model implies a decrease in investment even with contractible effort. The decrease in investment occurs as a result of asset sales when assets are least valuable, which then further depletes the amount of liquidated capital that may be sold to other firms. This effect will cause a financial acceleration in that it worsens the use of capital in response to the capital crunch.

In comparison, the existence of a financial acceleration given a contraction in the value of assets is conditional on the magnitude of the shock. Recall some firms  $\tilde{A} \geq W^+(r_b)$  are inefficiently operated relative to first-best. After the contraction, this region may shrink as the reporting cutoff increases, which implies that more capital can be used by other firms. In the aggregate, this may offset the decrease in aggregate investment. Resolving this trade-off, we show that the financial accelerator is present in economies whose investment levels are depressed relative to an economy without agency problems.

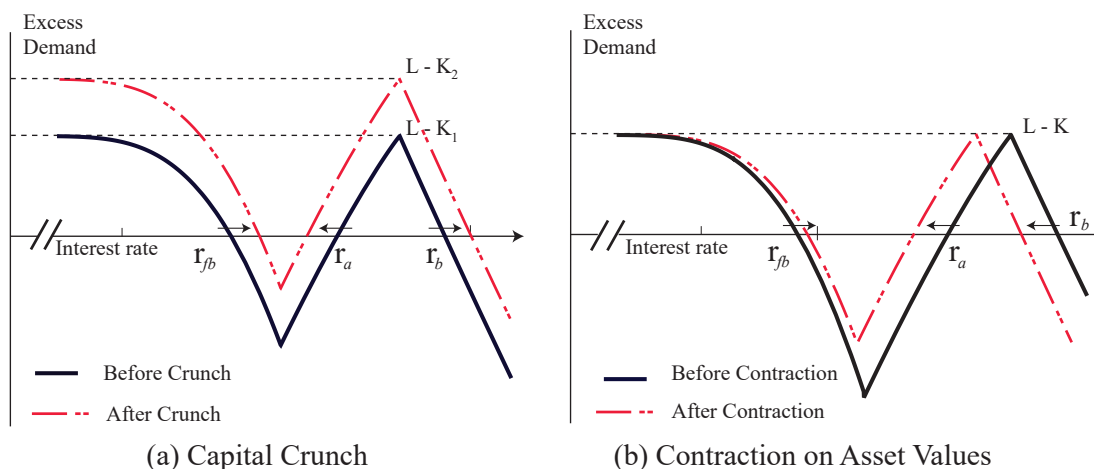


Figure 6: Financial Shocks

## 5 Further Discussions

### 5.1 Persuasion Mechanism

We briefly show here that it is optimal for the policymaker to use a cutoff  $\theta$  with a binary signal. In fact, among all optimal signal structures, the cutoff used in the baseline model implements the optimal allocations with the least amount of information and, therefore, would be strictly preferred if there existed a small cost of producing the information.

We define a reporting system as a mapping from  $A \in \mathbb{R}^+$  to a discrete distribution<sup>16</sup> over posterior expectations  $\mu_A$  and, for any  $P$ ,  $\mu_A(P)$  denotes the probability of a posterior expectation about  $\tilde{A}$  equal to  $P$ .<sup>17</sup>

<sup>16</sup>We skip technicalities by restricting the attention to discrete distribution over posteriors; however, in our setting, this is without loss of generality given that only two actions (operating the firm or selling the asset) are induced.

<sup>17</sup>Ex ante persuasion is implicit in many earlier accounting studies, see, e.g., Penno (1984), Wagenhofer (1990) or Arya et al. (1997), in the sense that the consequences of various commitments to an information system on real decisions are considered. One benefit of the formulation in Kamenica and Gentzkow (2011) is that it allows us to optimize over any feasible information system without a-priori specifying a family of possible signal structures. Recent work in accounting on Bayesian persuasion has studied how to design measurements to increase efficiency, see, e.g., Michaeli (2017) or Jiang and Yang (2016) or focuses on the interplay between persuasion and voluntary disclosures, see, e.g., Friedman, Hughes and Michaeli (2015) or Bertomeu, Cheynel and Cianciaruso (2018).

Let  $\mu = \int \mu_A dF(A)$  and the random variable  $\tilde{P}$  denote the posterior expectation. Kamenica and Gentzkow (2011) show that there exists a feasible signal structure if and only if the reporting system  $(\mu_A(P))_{A \in \mathbb{R}^+}$  is Bayes plausible, that is, for any  $P$  with  $\mu(P) > 0$ ,

$$P = \frac{\int A \mu_A(P) dF(A)}{\int \mu_A(P) dF(A)}.$$

A Bayes plausible reporting system is such that a posterior expectation is derived from Bayes rule by taking a conditional expectation over all values  $\tilde{A}$  that may have led to this expectation. We adapt the baseline model by defining an optimal reporting mechanism as a probability measure  $\mu_A^*(r) \in \max_{\mu_A} \sigma(\mu_A; r)$  and maximize across all possible Bayes plausible reporting systems.

**Proposition 7** *There exists an optimal Bayes plausible reporting system with a cutoff  $\theta^*(r)$  such that  $\tilde{P} = \mathbb{E}(\tilde{A} | \tilde{A} \leq \theta^*(r))$  if  $A \leq \theta^*(r)$  and  $\tilde{P} = \mathbb{E}(\tilde{A} | \tilde{A} > \theta^*(r))$ .*

The proof of the Proposition is cumbersome but the result is intuitive, and we lay out a simple heuristic for the result in the following section. Since only two actions are induced by the policy-maker, namely, whether to operate the firm or sell the asset, the policymaker can use without loss of generality a binary signal structure such that the signal matches the action. Operating a firm has two effects: an opportunity cost effect and a collateral effect. The opportunity cost effect entails a loss  $rA$  since the capital could have been reinvested on the market, while the collateral effect increases expected collateral for continuing firms by a factor proportional to  $A - W^-(r)$ . Both effects are per unit of asset value and, hence, the policymaker optimally decides whether to put low or high asset values, leading to the threshold strategy used in the baseline.

## 5.2 Equilibrium with more precise performance measurement

While our focus in the baseline model is on the measurements of assets used in collateralized lending, another critical function of accounting reports is to inform stakeholders

about the performance (output) of agents. In this section, we review how the functions are not separate but closely interrelated (Hemmer and Labro 2008). Our objective is to provide an illustration of this general principle in our current environment.

Suppose that the final cash flow is not readily observable but, instead, the firm provides a (noisy) interim accounting report  $r$  which, for simplicity, we set equal to  $r = h$  with probability  $v_x$  conditional on  $\tilde{x} = x$ , and  $r = l$  otherwise, such that  $v_1 > v_0$ .<sup>18</sup> Similar to the baseline model, we may implement a solution (when eliciting effort is feasible) with a contract such that the firm is transferred to investors if  $r = l$ , since this is the outcome with the lowest likelihood ratio and most consistent with low effort. To keep things as simple as possible (although this is not central to the argument), we assume that the collateral can only be seized at the interim stage.

The (equilibrium) value of the firm to investors can be derived by Bayesian updating as

$$s_l \equiv P_A + \frac{qv_1}{qv_1 + (1-q)v_0} \quad (11)$$

and, otherwise, the agent repays an amount  $s_h$ . The optimal contract is such investor break even:

$$(qv_1 + (1-q)v_0)s_h + (1 - qv_1 - (1-q)v_0)s_l = (1+r)L. \quad (12)$$

Effort is feasible if and only if

$$\Delta q(v_1(P_A + 1 - s_h) - v_0P_A) \geq c.$$

and solving for  $s_h$  in (11)-(12) and reinjecting in the above incentive-compatibility con-

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<sup>18</sup>This specification is chosen purely for expositional purposes so that it easily nests the baseline model as a special case (with  $v_1 = 1$  and  $v_0 = 0$ ). But one could write the model with no loss to the argument such that the interim signal is realized first and then  $\tilde{x}$  randomly realizes as a random variable correlated to the interim signal.

dition set at equality, the firm can be financed when

$$P_A \geq W^-(r) + k(v_1, v_0),$$

where  $k(\cdot)$  is a positive term that represents the effect of the quality of the report and is equal to zero when  $v_1 = 1$  and  $v_0 = 0$  since this corresponds to the baseline model.<sup>19</sup> The upper liquidation bounds  $W^+(r)$  is unchanged because it depends only on the net value of the firm.

As intuitive, a more precise signal on managerial output will loosen the incentive constraint and allow for more firms to be financed, shifting the range of capital stock such that the first-best allocations  $\theta_{fb}$  can be sustained. In other words, better reporting over outcomes suggest greater use of neutral accounting, more aggregate activity and less financial acceleration. On the other hand, when  $\theta_{fb}$  is no longer feasible, the prevailing equilibrium will be  $\theta_b$  which is uniquely determined by market-clearing (7) and does not depend on the quality of the reporting system. In summary, the quality of the information system can affect which reporting systems are feasible in the aggregate economy.

### 5.3 Voluntary disclosure

Assume that the firm does not receive the message from the reporting system with an exogenous probability  $q \in (0, 1)$  (Dye 1985); for example, one interpretation is that some assets may have inherent characteristics that are difficult to measure. With probability  $1 - q$ , the firm receives the intended message from the reporting system and may either report truthfully or withhold.<sup>20</sup>

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<sup>19</sup>With some cumbersome algebra, this term can be derived as

$$k(v_1, v_0) = \frac{\Delta q v_1 (L(1+r) - q - (1-q)v_0) + c(v_0(1-q) + qv_1)}{\Delta q (v_1(1-qv_0) - (1-q)v_0^2)},$$

and has the expected comparative statics in  $v_1$  and  $v_0$ .

<sup>20</sup>Note that our objective here is modest because we attempt to illustrate additional intuitions as to how strategic withholding may directionally affect the analysis in the baseline model. In a more general theory,

We build on existing results from the baseline model. Define the reporting system in terms of induced public beliefs after the voluntary disclosure stage. We label  $P_{nd}$  as the belief conditional on non-disclosure, and  $\bar{P} > P_{nd}$  as the belief conditional on a truthful disclosure. Naturally, in this model, firms with  $\tilde{A} \leq \theta$  will always withhold so that

$$P_{nd} = \frac{q\mathbb{E}(\tilde{A}) + (1 - q)F(\theta)\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta)}{q + (1 - q)F(\theta)}.$$

and  $\bar{P} = \mathbb{E}(\tilde{A}|\tilde{A} \geq \theta)$ .

We adapt Lemma 1. The pricing function needs to be adjusted by the pricing function  $P_{nd}$ . For  $r \leq \bar{r}$ , where  $\bar{r}$  is the threshold in the baseline, the reporting cutoff is  $\theta^*(r) = \max(W^+(r), \theta'_a(r))$  with  $\theta'_a(r)$  defined by

$$W^-(r) = \frac{q\mathbb{E}(\tilde{A}) + (1 - q)F(\theta'_a(r))\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta'_a(r))}{q + (1 - q)F(\theta'_a(r))}.$$

If  $r > \bar{r}$ , the collateral constraint is unchanged because only firms that disclose can be financed, implying that the policy coincides with the baseline at  $\theta^*(r) = \theta_b(r)$ .

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the characteristics of the reporting system would determine the likelihood of receiving information – however, a complete theory of Bayesian persuasion and ex-post (voluntary) disclosure is far beyond our scope. A limitation of nearly all voluntary disclosure models is that disclosure frictions – whether a disclosure cost à la Verrecchia (1983) and the information endowment à la Dye (1985) are modelled as exogenous to any other comparative static (in particular, the uncertainty prior to the disclosure decision) within the model. If, say, an increase in the variance of the manager’s expected cash flow signal (i.e., the manager has more precise information) made the finer information more difficult to measure and came with a decrease in the probability of information endowment, then the classic result that the probability of disclosure increases would no longer hold. One workaround is to model information acquisition, endogenizing  $q$ . However, there is no known framework to specify the cost function as a function of any possible reporting system.

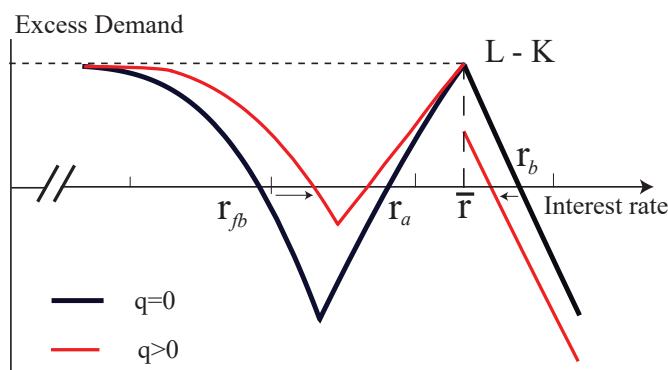


Figure 7: Competitive equilibrium with uncertain information endowment

We plot the excess demand and the effect of a change in  $q$  in Figure 7, which illustrates how the friction may change excess demands. At the first-best equilibrium  $r = r_{fb}$ , non-disclosing firms operate and, hence, a higher probability of being uninformed increases the excess demand and leads to a higher market-clearing interest rate. The effect on  $r = r_a$  can be more ambiguous in general because, while non-disclosing firms also operate, the higher  $q$  leads to a lower reporting cutoff  $\theta'_a(r)$ . When the first effect dominates because  $r_a$  is close enough to  $r_{fb}$ , a lower probability of information endowment reduces the interest rate. Finally, the equilibrium  $r = r_b$  is such that non-disclosing firms sell their asset, which implies that a higher amount of capital is put for sale on the market, leading to (jointly) a decrease in the interest rate and fewer firms operating when  $q$  increases.

## 6 Concluding Remarks: Empirical and Policy Implications

We conclude with several new domains where future work would be of interest to researchers and policymakers.

*Feedback between Accounting and the Economy.* While the effect of economic conditions on accounting is implicit in standard-setting theories, we show that accounting



can accelerate the effect of macroeconomic fluctuations. In terms of motivating economic facts, Figure 8 reveals that the existence of adverse write-downs and impairments has a very clear cyclical components. There is an extensive empirical literature debating the possible connection between accounting and financial crises (Ryan 2008, Laux and Leuz 2010, Khan 2010) but, to our knowledge, it has been centered primarily on the banking sector (“Wall Street”) . Our study suggests to incrementally consider the collateral channel in firms (“Main Street”).

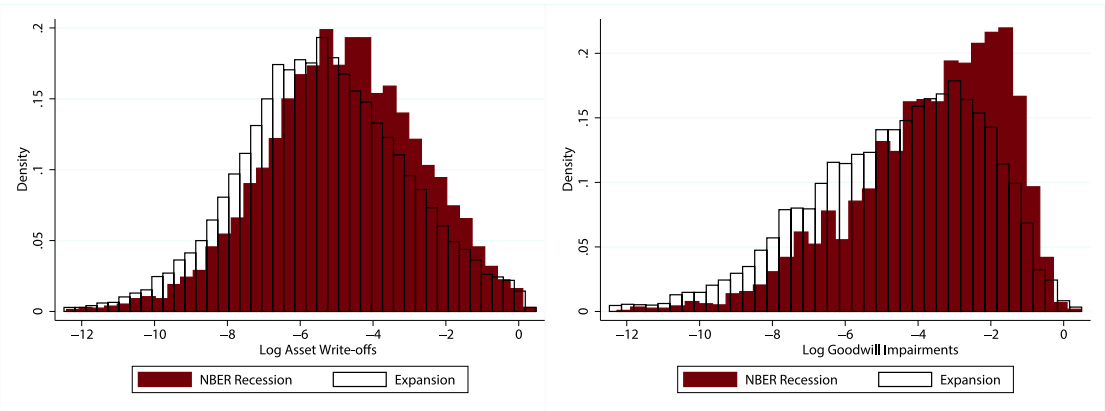


Figure 8: Tangible asset and goodwill impairments

First, the economy always features multiple competitive equilibria conditional on favorable conditions, especially, allocatively efficient equilibria with low interest rates in which firms efficiently sell high-value assets and a high-interest rate equilibrium where firms with low collateral are (inefficiently) forced out the credit market and to liquidate their assets at low prices. Given a sufficiently adverse shock to capital or collateral (presumably, any shock that reduces the ability of firms from using their capital to sell goods for a profit) low-interest equilibria cease to exist and the economy may discontinuously move to an equilibrium with accounting systems such that firms with low collateral are identified. This feature of the model explains how seemingly small changes in fundamentals can cause large changes in economic outcomes as the economy transitions

between equilibria.<sup>21</sup>

Second, we show that within the high-interest equilibrium, the accounting system may magnify the effect of the shock by increasing the effect of shocks to aggregate capital stock or collateral to the economy. For various shocks, we show that a more conservative accounting system (defined here as a system requiring better outcomes to issue high earnings) accompanies an economy with more inefficient liquidations and lower economic activity. This suggests that inefficient liquidations caused by accounting signals increase as a recession deepens.

*Asymmetric reporting (conservatism) and the Economy.* An open question in accounting research is whether there is an optimal level of asymmetry in reporting as a function of overall economic conditions. In other words, should the reporting system become more conservative during a recession? We show that a neutral system (defined as an accounting system that creates no frictions to capital allocations) can be maintained when there is abundant capital. However, as the economy moves toward a recession, the accounting system becomes more asymmetric. It is first liberal as, initially, most firms can meet their collateral constraints and, hence, favorable news need only be measured imprecisely. However, as the economy deteriorates and capital shrinks, the system becomes more demanding to select firms that meet their collateral constraints. Hence, we predict an association between average level of conservatism and the economic cycle: accounting is either neutral or liberal in relatively more favorable conditions, and becomes more conservative during periods of recession.

With the notable exception of Jenkins, Kane and Velury (2009), who show that, consistent with our main prediction, conservatism increases during recessions, this prediction has not yet been the object of systematic testing and, in most empirical studies, the busi-

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<sup>21</sup>Like Holmström and Tirole (1997) model, ours is stated in “real” terms, so we define the rate of return demanded by investors (interest rate) as the real interest rate. Also, by policy, we do not mean only the actions of standard-setters. In practice, what a firm views as an accounting policy is the interaction between the accounting policies of the firm and the actions of policymakers.

ness cycle is controlled with time-fixed effects. There is, however, a growing empirical literature in accounting studying how accounting practices (taken as a given) can influence the construction of macro indicators and forecast of future performance (Konchitchki and Patatoukas 2014, Konchitchki 2015, Crawley 2015, Nallareddy and Ogneva 2017). Our theory suggests to examine how measures of conservatism may change as a function of the economic cycle, either over time-series of a given country or across countries exposed to different economic shocks.

*Policy in General Equilibrium.* The current mandate of accounting standard-setters is to provide decision-useful information to capital providers which, in the context of this model, would prescribe full-information. In practice, however, maximizing welfare is a more complex problem, that features a trade-off between information and the potential costs of releasing information that would hurt the firm (possibly in the form of inefficient capital allocations). Traditionally, this is viewed as a concern for achieving higher value in the market and/or increasing investment (growth). We provide here an alternative interpretation. A standard-setter interested in maximizing value will tend to take expected rate of returns as a given and may implement an asymmetric accounting policy that is optimal for the given interest rate, even though there exists a more desirable competitive equilibrium with neutral accounting and lower interest rate. Addressing this issue requires full consideration of the equilibrium consequence of accounting on interest rates and some ability to coordinate markets toward a better equilibrium. Given that more efficient equilibria cannot be sustained during periods of (deep) recession, the policymaker should readjust the “accounting playbook” as a function of the cycle.

*Asset divisibilities.* Our main results apply to discrete investments such that a firm may or may not receive financing required to run a project requiring an indivisible amount of sunk costs. Many projects, requiring a sunk investment in R&D or significant equipments,

expansion into a foreign location, investment in a location or network externalities may have such fundamental indivisibilities. On the other hand, other types of investment can be continuously scaled in the form of supporting new locations in a franchise business or expanding a production line. If the firm can continuously expand subject to *linear* returns to scale, there will be less value in pooling assets because all firms receive financing in proportion to their collateral. We thus conjecture that the effects documented here are likely to be more prevalent for industries that need large incompressible investments.

There is, nevertheless, considerably more work to be done to fully understand the behavior of accounting in dynamic competitive equilibria. We have, in this model, followed the setting in Holmström and Tirole (1997) by taking a snapshot of the economy with a fixed amount of aggregate capital and where, by construction, accounting can change how this capital is used but not the overall amount of investment. This choice was made, of course, to focus a stylized model on shocks to the economy, which would occur even in a more complex environment. Over longer periods of time, however, the economy constantly adjusts its capital and, therefore, the overall stock of capital will evolve over time. We conjecture that such a perspective may be necessary to answer a key unanswered question in accounting: how does accounting policy affects the relationship between investment in capital and aggregate growth?

## Appendix

**Proof of Proposition 1:** Let  $\Gamma(\cdot)$  be defined by

$$\Gamma(r) = \int_0^{W^+(r)} (L + A)f(A)dA - K - \mathbb{E}(\tilde{A}), \quad (\text{A-1})$$

such that the first-best interest rate  $r_{fb}$  must satisfy (4), i.e.,  $\Gamma(r_{fb}) = 0$ .  $\Gamma(r)$  is continuous in  $r$  and such that  $\Gamma(r_{\max}) = \int_0^{W^+(r_{\max})} (L + A)f(A)dA - K - \mathbb{E}(\tilde{A}) > 0$  and  $\Gamma(0) = \int_0^{W^+(0)} (L + A)f(A)dA - K - \mathbb{E}(\tilde{A}) < 0$ . Further, in (2),  $W^+(r)$  is decreasing in  $r$ , so that  $\Gamma(r)$  is also decreasing in  $r$ . Hence,  $\Gamma(r_{fb}) = 0$  has a unique solution  $r_{fb} \in (0, r_{\max})$ .

Applying the implicit function theorem,

$$\begin{aligned} \frac{\partial r_{fb}}{\partial K} &= \frac{1}{\frac{\partial W^+(r)}{\partial r}|_{r=r_{fb}} f(W^+(r_{fb}))(L + W^+(r_{fb}))} \propto \frac{\partial W^+(r)}{\partial r}|_{r=r_{fb}} < 0, \\ \frac{\partial r_{fb}}{\partial X} &= -\frac{\frac{\partial W^+(r_{fb})}{\partial X} f(W^+(r_{fb}))(L + W^+(r_{fb}))}{\frac{\partial W^+(r)}{\partial r}|_{r=r_{fb}} f(W^+(r_{fb}))(L + W^+(r_{fb}))} \propto \frac{\partial W^+(r)}{\partial X}|_{r=r_{fb}}, \end{aligned} \quad (\text{A-2})$$

with  $X = p, c$  or  $\Delta q$ , implying  $\frac{\partial r_{fb}}{\partial p} > 0$ ,  $\frac{\partial r_{fb}}{\partial c} < 0$  and  $\frac{\partial r_{fb}}{\partial \Delta q} = 0$  in (A-2).  $\square$

**Proof of Proposition 2:** We know from Proposition 1 that  $r_{fb}$  is decreasing in  $K$  and the market-clearing (4) implies that  $\theta_{fb}$  is increasing in  $K$ . It then follows that (5) is equivalent to a cutoff  $K \geq K_{fb}$  where, subject to the assumed conditions (in-text) to avoid corners,  $K_{fb}$  satisfies (5) at equality.

Next, we rewrite (5) in simpler form: recall from Proposition 1 that the market clearing

$$\underbrace{\int_0^\theta (L + A)f(A)dA}_{=F(\theta)L + F(\theta)\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta)} = K + \mathbb{E}(\tilde{A}) \quad (\text{A-3})$$

has a unique solution at  $\theta = \theta_{fb} = W^+(r_{fb})$ . Substituting out  $\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta_{fb})$  in (5) from (A-3) readily yields (6). If this condition is not satisfied, then any other allocation  $\theta$  such that firms with  $\tilde{A} \leq \theta$  would have to satisfy (A-3), contradicting the uniqueness of  $\theta_{fb}$ .  $\square$

**Proof of Proposition 3:** As we have ruled out market-clearing in a corner such that all firms continue or all firms sell their asset, and we know from Proposition 2, that a cutoff such that firms with  $\tilde{A} \leq \theta$  continue is infeasible, the only remaining candidate solution is a cutoff such that firms with  $\tilde{A} \geq \theta$  continue. The market-clearing condition

$$\int_{\theta}^{\infty} (A + L)f(A)dA - K - \mathbb{E}(\tilde{A}) = 0$$

has a unique solution which we denote  $\theta_b$ , so it remains to check whether this solution is feasible. By construction, this cutoff satisfies market-clearing. In addition, consider an interest rate  $r_b$  defined by  $\mathbb{E}(\tilde{A}|\tilde{A} \geq \theta_b) = W^-(r_b)$ . This guarantees that firms with  $\tilde{A} \geq \theta_b$  continue. Further,  $\mathbb{E}(\tilde{A}|\tilde{A} < \theta_b) < \mathbb{E}(\tilde{A}|\tilde{A} \geq \theta_b) = W^-(r_b)$ , so that firms with  $\tilde{A} \leq \theta_b$  liquidate, implying that  $(r_b, \theta_b)$  is an equilibrium.<sup>22</sup>

We are left to show when the reporting cutoff  $\theta_b$  is liberal or conservative. We know that both the competitive cutoff  $\theta_b$  and, from Proposition 1, the neutral accounting  $\theta_{fb}$  must clear the market:

$$\int_{\theta_b}^{\infty} (A + L)f(A)dA - K - \mathbb{E}(\tilde{A}) = 0 \quad (\text{A-4})$$

$$\int_0^{\theta_{fb}} (A + L)f(A)dA - K - \mathbb{E}(\tilde{A}) = 0. \quad (\text{A-5})$$

Adding together (A-4) and (A-5),  $\theta_b < \theta_{fb}$  if and only if

$$\underbrace{\int_0^{\infty} (A + L)f(A)dA - 2K - 2\mathbb{E}(\tilde{A})}_{=\mathbb{E}(\tilde{A})+L-2K-2\mathbb{E}(\tilde{A})=L-2K-\mathbb{E}(\tilde{A})} > 0.$$

The reporting system is liberal when  $K + \mathbb{E}(\tilde{A}) > L - K$  and conservative when  $K + \mathbb{E}(\tilde{A}) < L - K$ .  $\square$

**Proof of Proposition 4:** It is readily verified that  $q - c - (1 + r)L - rP_A \geq 0$  if and only

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<sup>22</sup>The interest rate  $r_b$  guarantees that the cutoff  $\theta_b$  can be sustained as part of an equilibrium but other interest rates may have been used. For example, a lower interest rate  $r'$  such that  $W^-(r') \in (\mathbb{E}(\tilde{A}|\tilde{A} < \theta_b), \mathbb{E}(\tilde{A}|\tilde{A} \geq \theta_b)]$ .

if  $P_A \leq W^+(r)$ , which implies that the policy-maker solving (8) implements a reporting cutoff  $\theta = W^+(r)$  such that firms continue if and only if  $A \leq W^+$ . Market-clearing in (3) then requires that

$$\int_0^{W^+(r)} (A + L)f(A)dA = K + \mathbb{E}(\tilde{A}),$$

which we have shown in Proposition 1 has a unique solution  $W^+(r) = W^+(r_{fb}) = \theta_{fb}$ . Hence, the competitive equilibrium must coincide with the first-best interest rate and allocations.  $\square$

**Proof of Lemma 1:** The policy-maker may choose one of the following cutoffs: (a) set  $\theta$  such that a firm continues if and only if  $\tilde{A} \leq \theta$ , or vice-versa, (b) set  $\theta$  such that a firm continues if and only if  $\tilde{A} > \theta$ . Case (a) implies an optimal choice  $\theta \geq W^+(r)$  that reduces the region of inefficient continuations  $[W^+(r), \theta]$  subject to the collateral constraint  $\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta) \geq W^-(r)$ . Therefore,  $\theta = \max(W^+(r), \theta_a)$  where  $\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta_a) = W^-(r)$ . Case (b) implies an optimal choice  $\theta \leq W^+(r)$  that reduces the region of inefficient liquidations  $[0, \theta]$  subject to the collateral constraint  $\mathbb{E}(\tilde{A}|\tilde{A} \geq \theta) \geq W^-(r)$ . Therefore,  $\theta = \theta_b$  where  $\mathbb{E}(\tilde{A}|\tilde{A} \geq \theta_b) = W^-(r)$ .

To conclude the proof, note that case (a) requires  $\mathbb{E}(\tilde{A}) \geq W^-(r)$  and case (b) requires  $\mathbb{E}(\tilde{A}) \leq W^-(r)$ . The two cases are identical when  $\mathbb{E}(\tilde{A}) = W^-(r)$ .  $\square$

**Proof of Proposition 5:** We have shown in text that for the competitive equilibrium  $(r_1^*, \theta_1^*) = (r_{fb}, W^+(r_{fb}))$  to exist, it must hold that  $\Delta(r) \leq 0$ . The rest of the statement follows from the fact that, from Proposition 2, an equilibrium with continuation when  $A \leq \theta$  is feasible if and only if  $K \geq K_{fb}$ .  $\square$

**Proof of Proposition 6:** The proof immediately follows from Proposition 3.  $\square$

**Proof of Corollary 1:** We know from Proposition 1 that  $\frac{\partial r_{fb}}{\partial K} < 0$ .

$$\frac{d\theta_{fb}}{dK} = \frac{dW^+(r_{fb})}{dK} = \frac{1}{f(W^+(r_{fb}))(L + W^+(r_{fb}))} > 0.$$

*Capital crunch on  $r_a^*$ .*

We write the equilibrium equations  $\Phi_u$  and  $\Gamma_u$  as follows:

$$\begin{aligned} \Phi_u(\theta, r, K) &= F(\theta)\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta) - F(\theta)W^-(r, K), \\ \Gamma_u(\theta, r, K) &= LF(\theta) - (1 - F(\theta))\mathbb{E}(\tilde{A}|\tilde{A} \geq \theta) - K. \end{aligned}$$

We define the Jacobian  $J_u(\theta, r, K)$  as follows:

$$J_u(\theta, r, K) = \begin{bmatrix} \partial\Phi_u(\theta, r, K)/\partial\theta & \partial\Phi_u(\theta, r, K)/\partial r \\ \partial\Gamma_u(\theta, r, K)/\partial\theta & \partial\Gamma_u(\theta, r, K)/\partial r \end{bmatrix} = \begin{bmatrix} (\theta - W^-(r))f(\theta) & -\frac{\partial W^-(r)}{\partial r}F(\theta) \\ (\theta + L)f(\theta) & 0 \end{bmatrix},$$

where the determinant of  $J_u(\theta, r, K)$  is non-zero. Applying the implicit function theorem,

$$\begin{bmatrix} \frac{\partial\theta_{fb}}{\partial K} \\ \frac{\partial r_a}{\partial K} \end{bmatrix} = -J_u^{-1}(\theta_{fb}, r_a, K) \begin{bmatrix} \frac{\partial\Phi_u(\theta_{fb}, r_a, K)}{\partial K} \\ \frac{\partial\Gamma_u(\theta_{fb}, r_a, K)}{\partial K} \end{bmatrix} = \begin{bmatrix} \frac{1}{(\theta_{fb}+L)f(\theta_{fb})} > 0 \\ \frac{\theta_{fb}-W^-(r_a)}{\frac{\partial W^-(r_a)}{\partial r_a}(\theta_{fb}+L)\int_0^{\theta_{fb}} f(A)dA} > 0 \end{bmatrix}.$$

*Capital crunch on  $r_b$  and  $\theta_b$ .*

We write the equilibrium equations  $\Phi_l$  and  $\Gamma_l$  as follows:

$$\begin{aligned} \Phi_l(\theta, r, K) &= (1 - F(\theta))\mathbb{E}(\tilde{A}|\tilde{A} \geq \theta) - (1 - F(\theta))W^-(r, K), \\ \Gamma_l(\theta, r, K) &= L(1 - F(\theta)) - F(\theta)\mathbb{E}(\tilde{A}|\tilde{A} \leq \theta) - K. \end{aligned}$$

Define the Jacobian  $J_l(\theta, r, K)$  as follows:

$$J_l(\theta, r, K) = \begin{bmatrix} \partial\Phi_l(\theta, r, K)/\partial\theta & \partial\Phi_l(\theta, r, K)/\partial r \\ \partial\Gamma_l(\theta, r, K)/\partial\theta & \partial\Gamma_l(\theta, r, K)/\partial r \end{bmatrix} = \begin{bmatrix} (W^-(r) - \theta)f(\theta) & -\frac{\partial W^-(r, K)}{\partial r} \int_{\theta}^{+\infty} f(A)dA \\ -(\theta + L)f(\theta) & 0 \end{bmatrix},$$

where the determinant of  $J_l$  is non zero. Applying the implicit function theorem,

$$\begin{bmatrix} \frac{\partial\theta_b}{\partial K} \\ \frac{\partial r_b}{\partial K} \end{bmatrix} = -J_l^{-1}(\theta_b, r_b, K) \begin{bmatrix} \frac{\partial\Phi_l(\theta_b, r_b, K)}{\partial K} \\ \frac{\partial\Gamma_l(\theta_b, r_b, K)}{\partial K} \end{bmatrix} = \begin{bmatrix} -\frac{1}{(\theta_b+L)f(\theta_b)} < 0 \\ \frac{\theta_b-W^-(r_b)}{\frac{\partial W^-(r)}{\partial r}|_{r=r_{II}^*}(\theta_b+L)\int_{\theta_b}^{\infty} f(A)dA} < 0 \end{bmatrix}. \square$$

**Proof of Corollary 2:** Conditional on a contraction in asset values,  $\tilde{A} = (1 - \varepsilon)\tilde{a}$ , where  $\tilde{a}$  is a random variable with probability density probability  $f_a(\cdot)$  and cumulative distribution probability  $F_a(\cdot)$ .



We make the dependence on the contraction in asset values explicit in  $\Gamma(\cdot)$  defined in (A-1):

$$\begin{aligned}\Gamma(r, \varepsilon) &= \int_0^{\frac{W^+(r)}{1-\varepsilon}} (L + (1-\varepsilon)a) f_a(a) da - K - (1-\varepsilon)\mathbb{E}(a). \\ \frac{\partial r_{fb}}{\partial \varepsilon} &= -\frac{\frac{\partial \Gamma(r, \varepsilon)}{\partial \varepsilon}}{\frac{\partial \Gamma(r, \varepsilon)}{\partial r} \Big|_{r=r_{fb}}} = -\frac{\frac{W^+(r_{fb}) f_a(\frac{W^+(r_{fb})}{1-\varepsilon})}{(1-\varepsilon)^2} (L + W^+(r_{fb})) + \int_{\frac{W^+(r_{fb})}{1-\varepsilon}}^{+\infty} a f_a(a) da}{\frac{1}{1-\varepsilon} \frac{\partial W^+(r)}{\partial r} \Big|_{r=r_{fb}} (L + W^+(r_{fb})) f_a(\frac{W^+(r_{fb})}{1-\varepsilon})} > 0 \\ \text{and } \frac{dW^+(r_{fb})}{d\varepsilon} &= \frac{\partial r_{fb}}{\partial q} \frac{\partial W^+(r)}{\partial r} \Big|_{r=r_{fb}} < 0.\end{aligned}$$

Given  $\tilde{A} = (1-\varepsilon)\tilde{a}$  and  $\theta = (1-\varepsilon)\theta'$ , we define  $\Phi_u$  and  $\Gamma_u$  as follows:

$$\begin{aligned}\Phi_u(\theta', r, \varepsilon) &= (1-\varepsilon)F_a(\theta')\mathbb{E}(\tilde{a}|\tilde{a} \leq \theta') - F_a(\theta')W^-(r), \\ \Gamma_u(\theta', r, \varepsilon) &= LF(\theta') - (1-F_a(\theta'))(1-\varepsilon)\mathbb{E}(\tilde{a}|\tilde{a} \geq \theta') - K.\end{aligned}$$

The Jacobian is:

$$J_u(\theta', r, \varepsilon) = \begin{bmatrix} (\theta'(1-\varepsilon) - W^-(r))f_a(\theta') & -\frac{\partial W^-(r)}{\partial r} F_a(\theta') \\ (\theta'(1-\varepsilon) + L)f_a(\theta') & 0 \end{bmatrix}.$$

Applying the implicit function theorem,

$$\begin{aligned}\begin{bmatrix} \frac{\partial \theta'_{fb}}{\partial \varepsilon} \\ \frac{\partial r_a^*}{\partial \varepsilon} \end{bmatrix} &= -J_u^{-1}(\theta'_{fb}, r_a, \varepsilon) \begin{bmatrix} \frac{\partial \Phi_u(\theta'_{fb}, r_a, \varepsilon)}{\partial \varepsilon} \\ \frac{\partial \Gamma_u(\theta'_{fb}, r_a, \varepsilon)}{\partial \varepsilon} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{(1-F_a(\theta'_{fb}))\mathbb{E}(\tilde{a}|\tilde{a} \geq \theta'_{fb})}{f_a(\theta'_{fb})(\theta'_{fb}(1-\varepsilon)+L)} < 0 \\ \frac{(\theta'_{fb}(1-\varepsilon)+L) \int_0^{\theta'_{fb}} a f_a(a) da + (\theta'_{fb}(1-\varepsilon) - W^-(r_a)) \int_{\theta'_{fb}}^{+\infty} a f_a(a) da}{-\frac{\partial W^-(r)}{\partial r} \Big|_{r=r_{Ib}^*} (\theta'_{fb}(1-\varepsilon)+L) \int_0^{\theta'_{fb}} f_a(a) da} < 0 \end{bmatrix}.\end{aligned}$$

Similarly, given  $\tilde{A} = (1-\varepsilon)\tilde{a}$  and  $\theta = (1-\varepsilon)\theta'$ , we define  $\Phi_l$  and  $\Gamma_l$  as follows:

$$\begin{aligned}\Phi_l(\theta', r, \varepsilon) &= (1-F_a(\theta'))(1-\varepsilon)\mathbb{E}(\tilde{a}|\tilde{a} \geq \theta') - (1-F_a(\theta'))W^-(r), \\ \Gamma_l(\theta', r, \varepsilon) &= L(1-F_a(\theta')) - F_a(\theta')(1-\varepsilon)\mathbb{E}(\tilde{a}|\tilde{a} \leq \theta') - K.\end{aligned}$$

The Jacobian  $J_l(\theta', r, \varepsilon)$  is given by

$$J_l(\theta', r, \varepsilon) = \begin{bmatrix} (W^-(r) - \theta'(1 - \varepsilon))f_a(\theta') & -\frac{\partial W^-(r)}{\partial r} \int_{\theta'}^{+\infty} f_a(a)da \\ -(\theta'(1 - \varepsilon) + L)f_a(\theta') & 0 \end{bmatrix}.$$

Applying the implicit function theorem,

$$\begin{bmatrix} \frac{\partial \theta'_b}{\partial \varepsilon} \\ \frac{\partial r_b}{\partial \varepsilon} \end{bmatrix} = -J_l^{-1}(\theta'_b, r_b, \varepsilon) \begin{bmatrix} \frac{\partial \Phi_l(\theta'_b, r_b, \varepsilon)}{\partial \varepsilon} \\ \frac{\partial \Gamma_l(\theta'_b, r_b, \varepsilon)}{\partial \varepsilon} \end{bmatrix}.$$

$$= \begin{bmatrix} \frac{\int_0^{\theta'_b} a f_a(a) da}{f_a(\theta'_b)(\theta'_b(1-\varepsilon)+L)} > 0 \\ -\frac{-W^-(r_b) \int_0^{\theta'_b} a f_a(a) da + L \int_{\theta'_b}^{+\infty} a f_a(a) da + (1-\varepsilon)\mathbb{E}(\tilde{a})\theta'_b}{\frac{\partial W^-(r)}{\partial r}|_{r=r_b}(\theta'_b(1-\varepsilon)+L) \int_{\theta'_b}^{+\infty} f_a(a) da} < 0 \end{bmatrix}$$

$$\begin{aligned} \text{because in equilibrium, } H(\theta'_b, r_b) &= -W^-(r_b) \int_0^{\theta'_b} a f_a(a) da + L \frac{W^-(r_b)}{(1-\varepsilon)} \int_{\theta'_b}^{+\infty} f_a(a) da \\ &= \frac{W^-(r_b)}{(1-\varepsilon)} \left( -(1-\varepsilon) \int_0^{\theta'_b} a f_a(a) da + L \int_{\theta'_b}^{+\infty} f_a(a) da \right) \\ &= \frac{W^-(r_b)}{(1-\varepsilon)} K. \square \end{aligned}$$

**Proof of Corollary 3:**

$$\begin{aligned} \frac{\partial F(W^+(r_{fb}))}{\partial K} &= \frac{\partial W^+(r_{fb})}{\partial K} f(W^+(r_{fb})) = \frac{1}{L + W^+(r_{fb})}, \\ \frac{\partial (1 - F(\theta_b))}{\partial K} &= -\frac{\partial (\theta_b)}{\partial K} f(\theta_b) = \frac{1}{L + \theta_b}. \end{aligned}$$

Hence, comparing the change in investment in first-best versus second-best boils down to ordering

$W^+(r_{fb})$  versus  $\theta_b$ . Because  $W^+(r_{fb}) > \theta_b$ , there is always a financial accelerator.

We derive below the financial accelerator that follows a contraction on asset values.

$$\begin{aligned}\frac{\partial F(W^+(r_{fb}))}{\partial \varepsilon} &= \frac{\partial W^+(r_{fb})}{\partial \varepsilon} f(W^+(r_{fb})) = -\frac{1 - F(W^+(r_{fb}))}{L + (1 - \varepsilon)W^+(r_{fb})} \\ \frac{\partial(1 - F(\theta_b))}{\partial \varepsilon} &= -\frac{\partial \theta_b}{\partial \varepsilon} f(\theta_b) = -\frac{F(\theta_b)}{L + (1 - \varepsilon)\theta_b}.\end{aligned}$$

We know that  $W^+(r_{fb}) > \theta_b$ , so that if  $1 - F(W^+(r_{fb})) < F(\theta_b)$ , there is a financial accelerator.  $\square$

**Proof of Proposition 7:** We rewrite the payoff of the policy-maker as  $\sigma(\mu_A; r) = \mathbb{E}(\nu(\tilde{P}; r))$ , where:

$$\begin{aligned}\nu(\tilde{P}; r) &= \mathbf{1}(\tilde{P} \in \mathcal{W}(r))(p - c - (1 + r)L - r\tilde{P}), \\ &\leq \hat{\nu}(P; r) \equiv \mathbf{1}(\tilde{P} \leq W^+(r))(p - c - (1 + r)L - r\tilde{P}).\end{aligned}\tag{A-6}$$

where  $\mathbf{1}(\omega)$  is a indicator random variable equal to one conditional on the event  $\omega$ . This payoff is bounded by  $\hat{\nu}(P; r)$ , the payoff of the policy-maker with contractible effort.

We prove the optimality of a cutoff reporting system assumed in Lemma 1 in several steps. First, we show that a cutoff is optimal when  $r \leq \underline{r}$  (steps 1-3), then we simplify the policy-maker's problem to a binary reporting choice subject to a binding collateral constraint (steps 4-7), and, finally, we show that the resulting program has a solution that features a cutoff as in the baseline.

*Step 1.* For any  $r$  and any Bayes-plausible mechanism,

$$\mathbb{E}(\hat{\nu}(\tilde{P}; r)) \leq \bar{\nu}(r) \equiv \mathbb{E}(\mathbf{1}(\tilde{A} \leq W^+(r))(p - c - (1 + r)L - r\tilde{A})).$$

*Proof.* From the definition of  $\hat{\nu}$  in equation (A-6),

$$\begin{aligned}\mathbb{E}(\hat{\nu}(\tilde{P}; r)) &= \mathbb{E}(\hat{\nu}(\mathbb{E}(\tilde{A}|\tilde{P}); r)) && (\mu \text{ is Bayes-plausible}) \\ &\leq \mathbb{E}(\mathbb{E}(\hat{\nu}(\tilde{A}; r)|\tilde{P})) && (\text{Jensen's inequality}) \\ &\leq \mathbb{E}(\hat{\nu}(\tilde{A}; r)) = \bar{\nu}(r).\end{aligned}\tag{A-7}$$

Since  $\bar{\nu}(r)$  does not depend on  $\mu_A$ , it is an upper bound on the profit of the payoff of the policy-

maker.  $\square$

*Step 2.* A reporting system attains  $\bar{\nu}(r)$  if and only if the event “ $\mathbf{1}(\tilde{P} \leq W^+(r)) = \mathbf{1}(\tilde{A} \leq W^+(r))$ ” has probability one.

*Proof.* The “if” part is immediate. To prove “only if”, assume that “ $\mathbf{1}(\tilde{P} \leq W^+(r)) \neq \mathbf{1}(\tilde{A} \leq W^+(r))$ ” has non-zero probability.

$$\begin{aligned} \mathbb{E}(\hat{\nu}(\tilde{P}; r)) &= \int \int \mathbf{1}(A \leq W^+(r))(q - c - (1+r)L - rP) d\mu_A(P) dF(A) \\ &\quad + \int \int \mathbf{1}(P \leq W^+(r), A > W^+(r))(q - c - (1+r)L - rP) d\mu_A(P) dF(A) \\ &\quad - \int \int \mathbf{1}(P > W^+(r), A \leq W^+(r))(q - c - (1+r)L - rP) d\mu_A(P) dF(A) \\ &= \bar{\nu}(r) + \int \phi(A) dF(A), \end{aligned}$$

where

$$\begin{aligned} \phi(A) &= \mathbf{1}(A > W^+(r))(q - c - (1+r)L - rA) \\ &\quad - \int \mathbf{1}(P > W^+(r))(q - c - (1+r)L - rP) d\mu_A(P). \end{aligned}$$

It follows from Jensen’s inequality that  $\mathbb{E}(\phi(\tilde{A})) < 0$ , implying  $\mathbb{E}(\hat{\nu}(\tilde{P}; r)) < \bar{\nu}(r)$ .  $\square$

*Step 3.* If  $W^-(r) \leq \mathbb{E}(\tilde{A} | \tilde{A} \leq W^+(r))$ , then  $\mu_A(\mathcal{W}(r)) = \mathbf{1}(A \leq W^+(r))$ .

*Proof.* By contradiction, if this claim is not true, we know from claim 2 that the policy-maker achieves strictly less the upper bound  $\bar{\nu}(r)$  in (A-7). Define next a Bayes-plausible improvement  $\mu'$  such that, for any  $A$ ,

(i) if  $A \leq W^+(r)$ ,  $\mu'_A(P_0) = 1$  and  $\mu'_A(P) = 0$  for any  $P \neq P_0$ ,

(ii) otherwise,  $\mu'_A(P_1) = 1$  and  $\mu'_A(P) = 0$  for any  $P \neq P_1$ ,

where  $P_0 = \mathbb{E}(\tilde{A} | \tilde{A} \leq W^+(r)) \in (W^-(r), W^+(r))$  and  $P_1 = \mathbb{E}(\tilde{A} | \tilde{A} > W^+(r)) > W^+(r)$ . It is readily seen that  $\nu'_A$  achieves  $\bar{\nu}(r)$ , a contradiction to  $\nu_A$  optimal.  $\square$

For all of the steps that follow, assume that  $W^-(r) > \mathbb{E}(\tilde{A} | \tilde{A} \leq W^+(r))$ . This is equivalent to  $r > \underline{r}$  where recall (in text) that  $\underline{r}$  is uniquely defined by  $W^-(\underline{r}) = \mathbb{E}(\tilde{A} | \tilde{A} \leq W^+(\underline{r}))$ . Recall that  $\mathcal{W}(r) = [W^-(r), W^+(r)]$  is the region of prices such that the firm operates the asset.

*Step 4.*  $\mu$  does not attain  $\bar{\nu}(r)$ .

*Proof.* We know from an argument identical to step 2 that  $\mu$  attains  $\bar{\nu}(r)$  if and only if the event “ $\mathbf{1}(\tilde{P} \in \mathcal{W}(r)) = \mathbf{1}(\tilde{A} \leq W^+(r))$ ” has probability one. Then, by contradiction, if  $\mu$  attains  $\bar{\nu}(r)$ ,  $\mathbb{E}(\tilde{P}|\tilde{P} \in \mathcal{W}(r)) = \mathbb{E}(\tilde{A}|\tilde{A} \leq W^+(r))$ . However, this contradicts the assumption that  $r > \underline{r}$ .  $\square$

*Step 5.* The following holds: (a)  $Prob(\tilde{P} \in \mathcal{W}(r)) > 0$  and (b)  $Prob(\tilde{P} \notin \mathcal{W}(r)) > 0$  if and only if  $\mathbb{E}(\tilde{A}) \neq W^-(r)$ .

*Proof.* (a) By contradiction, construct a Bayes-plausible improvement  $\mu'$  such that, for any  $A$ ,

(i) if  $A \in \mathcal{W}(r)$ ,  $\mu'_A(P_0) = 1$  and  $\mu'_A(P) = 0$  for any  $P \neq P_0$ ,

(ii) otherwise,  $\mu'_A(P_1) = 1$  and  $\mu'_A(P) = 0$  for any  $P \neq P_1$ ,

where  $P_0 = \mathbb{E}(\tilde{A}|\tilde{A} \in \mathcal{W}(r))$  and  $P_1 = \mathbb{E}(\tilde{A}|\tilde{A} \notin \mathcal{W}(r))$ .  $\mu'$  is preferred to  $\mu$  because

$$\int \nu(P; r) d\mu'(P) = Prob(\tilde{A} \in \mathcal{W}(r))\nu(P_0; r) > 0 = \int \nu(P; r) d\mu(P).$$

(b) For the ‘if’ part, note that  $Prob(\tilde{P} \in \mathcal{W}(r)) = 1$  implies, from Bayes plausibility, that  $\mathbb{E}(\tilde{P}|\tilde{P} \in \mathcal{W}(r)) = \mathbb{E}(\tilde{A})$ , which contradicts that  $\mathbb{E}(\tilde{P}|\tilde{P} \in \mathcal{W}(r)) \in \mathcal{W}(r)$ .

For the ‘only if’ part, assume that  $\mathbb{E}(\tilde{A}) = W^-(r)$  and consider the reporting system such that  $\mu_A(W^-(r)) = 1$  for any  $A$ , which yields  $\mathbb{E}(\nu(\tilde{P}, r)) = \nu(W^-(r), r) > \nu(P, r)$  for any  $P \neq W^-(r)$ . Hence, this reporting system is preferred to any other reporting system in which  $Prob(\tilde{P} = W^-(r)) < 1$ .  $\square$

*Step 6.* There exists a “straightforward” reporting system  $\mu$  that solves the policy-maker’s problem such that  $Supp(\mu) \subseteq \{P_0, P_1\}$  where  $P_0 \in \mathcal{W}(r)$  and  $P_1 \notin \mathcal{W}(r)$ .

*Proof.* This was shown in step 5 if  $\mathbb{E}(\tilde{A}) = W^-(r)$  so let us assume that  $\mathbb{E}(\tilde{A}) \neq W^-(r)$ . Let  $P_0$  and  $P_1$  be defined by

$$P_0 = \frac{\int \mathbf{1}(P \in \mathcal{W}(r)) P d\mu(P)}{\int \mathbf{1}(P \in \mathcal{W}(r)) d\mu(P)}, \quad P_1 = \frac{\int \mathbf{1}(P \notin \mathcal{W}(r)) P d\mu(P)}{\int \mathbf{1}(P \notin \mathcal{W}(r)) d\mu(P)}. \quad (\text{A-8})$$

Note that step 3 ensures that the denominator in each of these terms is non-zero. Then, define a

Bayes-plausible improvement  $\mu'$  as in step 3 for  $P_0$  and  $P_1$  defined in (A-8):

$$\begin{aligned}
\int \nu(\tilde{P}, r) d\mu' &= \mu'(P_0)[q - c - (1 + r)L - rP_0] \\
&= \int \mathbf{1}(P \in \mathcal{W}(r)) d\mu(P)[q - c - (1 + r)L - r \int \mathbf{1}(P \in \mathcal{W}(r)) P d\mu(P)] \\
&= \int \mathbf{1}(P \in \mathcal{W}(r))(q - c - (1 + r)L - rP) d\mu(P) = \int \nu(P, r) d\mu(P). \square
\end{aligned}$$

From step 6, we know that there is a straightforward reporting system  $\mu$  such that, for any  $A$ ,  $\tilde{P} = P_0 \in \mathcal{W}(r)$  with probability  $\mu_A(P_0)$  and  $\tilde{P} = P_1 \notin \mathcal{W}(r)$  with probability  $1 - \mu_A(P_0)$  solving

$$(Q) \max_{\mu'} \mathbb{E}_{\mu'}(\nu(\tilde{P}; r)) = \int \mu'_A(P_0) \nu(P_0; r) dF(A)$$

s.t.

$$P_0 = \frac{\int \mathbf{1}(P \in \mathcal{W}(r)) P d\mu'(P)}{\int \mathbf{1}(P \in \mathcal{W}(r)) d\mu'(P)} \in [W^-(r), W^+(r)] \quad (\text{A-9})$$

$$P_1 = \frac{\int \mathbf{1}(P \notin \mathcal{W}(r)) P d\mu'(P)}{\int \mathbf{1}(P \notin \mathcal{W}(r)) d\mu'(P)} \notin [W^-(r), W^+(r)]. \quad (\text{A-10})$$

We already showed the main result for the case  $\mathbb{E}(\tilde{A}) = W^-(r)$  in step 5 (in which case all firms operate the asset) so we assume for the remaining cases below that  $r > \underline{r}$  and  $\mathbb{E}(\tilde{A}) \neq W^-(r)$ .

*Step 7.* A solution to (Q) satisfies the relaxed problem, replacing constraints (A-9)-(A-10) by a single binding constraint:

$$P_0 = \frac{\int \mathbf{1}(P \in \mathcal{W}(r)) P d\mu'(P)}{\int \mathbf{1}(P \in \mathcal{W}(r)) d\mu'(P)} = W^-(r). \quad (\text{A-11})$$

*Proof.* Consider the problem  $(Q')$  without constraint (A-10). Then, at least one of the bounds  $P_0 \in [W^-(r), W^+(r)]$  must bind, since otherwise, from step 2, the reporting system with “ $\mathbf{1}(\tilde{P} \leq W^+(r)) = \mathbf{1}(\tilde{A} \leq W^+(r))$ ” would be optimal which, in turn, contradicts step 4. Further, if  $P_0 = W^+(r)$ ,  $\nu(P_0; r) = 0$  which, as shown in the proof of step 5(a), is suboptimal. It then follows that  $P_0 = W^-(r)$ .

We are left to show that constraint (A-10) is lax. If solving  $(Q')$  were to involve  $P_1 \in [W^-(r), W^+(r)]$ , this would imply that the firm is always continued, which, given that  $\mathbb{E}(\tilde{A}) \neq W^-(r)$ , contradicts step 5.  $\square$

We can conclude the proof by showing that the optimal reporting system for  $r > \underline{r}$  is a cutoff rule such that  $\mu_A(P_0)$  is a monotone step function. From step 7, we can simplify the constraints to a single constraint which can be rewritten

$$W^-(r) \int \mu'_A(W^-(r)) dF(A) = \int \mu'_A(W^-(r)) A dF(A) \quad (\text{A-12})$$

Differentiating the Lagrangian of this linear program,

$$\frac{1}{f(A)} \frac{\partial \mathcal{L}}{\partial \mu'_A(W^-(r))} = 1 - \lambda(W^-(r) - A),$$

where the multiplier  $\lambda$  must be non-zero or else the optimal policy would be to set  $\tilde{P} = W^-(r)$  and always continue which, by step 5, is suboptimal. Therefore, the problem has a bang-bang optimal policy equivalent to a cutoff  $\theta^*(r)$  defined by  $1 - \lambda(W^-(r) - \theta^*(r)) = 0$ .  $\square$

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