Possibility of Primordial Black holes as the source of gravitational wave events in the advanced LIGO detector

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The analysis of Gravitational Waves (GW) data from the Advanced LIGO provides the mass of each companion of binary black holes as the source of GWs. The observations reveal that the mass of events corresponding to the binary black holes are much larger than the mass of astrophysical black holes. In this work, we suggest the Primordial Black Holes (PBHs) as the source of LIGO events. Assuming that 100% of the dark matter is made of PBHs, we estimate the rate at which these objects make binaries, merge and produce GWs as a function of redshift. The gravitational lensing of GWs by PBHs can also enhance the amplitude of the strain. We simulate GWs sourced by the binary PBHs, with the detection threshold of $S/N > 10$ for both Livingston and Handford detectors. For the log-normal mass function of PBHs, we generate the expected distribution of events and compare our results with the observed events and find the best value of the mass function parameters (i.e $M_c = 25M_\odot$ and $\sigma = 0.6$) in the lognormal mass function. Comparing the expected number of events with the number of observed ones reveals that even assuming all the dark matter is made of PBHs are not enough to produce the observed GW events and the astrophysical blackholes should have the main contribution in the observed GW events.

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The recent discovery of gravitational waves (GWs) by the Advanced LIGO, sourced from the binary black hole mergers opened a new window in astronomy. The analysis of strain signals with the theoretical templates of the GW can determine the mass of binary black holes [1]. Since the mass of black holes from GWs are larger than the astrophysical black holes discovered yet [2], one of the possibilities could be that the source of observed GWs is due to the Primordial Black Holes (PBHs). Ten GW candidates are detected as a result of the first and the second runs of the Advanced LIGO (i.e. O1 and O2) [1] and VIRGO [3] in 48.6 days and 117 days run, respectively [4, 5]. During the third observing run O_3 (i.e. April 2019-March 2020)[6] with improve sensitivity of detectors [7, 8], tens of candidate GW events have been identified [9] however 4 events have been confirmed [10–13].

While most constraints placed on the PBHs assume they are produced with a monochromatic mass function, PBH formation demands them to follow an extended mass function [14]. Moreover, taking the more realistic extended mass function for the PBHs increases the chance that they form 100% of the dark matter [15]. Being the formulation that fits a very large class of inflationary PBH models [14], in this work we adapt the lognormal mass function for the PBHs [15] as

$$\psi(M) = \frac{f_{PBH}}{\sqrt{2\pi}\sigma M}\exp\left(-\frac{log^2(M/M_c)}{2\sigma^2}\right), \qquad (1)$$

where M_c is characteristic mass, σ is the width of mass function and f_{PBH} is fraction of dark matter made of PBHs.

While the PBHs in the early universe were produced individually, we can argue that very small fraction of them can make binary systems in a cosmological timescale. When two individual PBHs pass close to each other, as a result of gravitational interaction, they can radiate GWs. For efficient interaction, the PBHs can make binary systems due to the dissipation process of GW emission surpassing their initial kinetic energy [16]. The rate of binary formation per halo is given by [16]:

$$R(M) = \int_0^{R_{\rm vir}} 2\pi r^2 \left(\frac{\rho_{\rm nfw}(r)}{M_{\rm pbh}}\right)^2 \langle\sigma_{GW} v_{\rm pbh}\rangle dr, \qquad (2)$$

where $\sigma = \pi(\frac{85\pi}{3})^{2/7} R_s^2 (\frac{v_{\rm pbh}}{c})^{18/7}$ is the cross section for binary formation[17, 18]. $\rho_{\rm nfw} = \rho_s[(r/R_s)(1+r/R_s)^2]^{-1}$ is the NFW density profile with characteristic radius and density, R_s and ρ_s respectively. $R_{\rm vir}$ is the virial radius of the galactic halo at which the density of NFW profile reaches 200 times the mean cosmic density. $v_{\rm pbh}$ is the dispersion velocity of PBHs and $M_{\rm pbh}$ is the mass of the PBH.

After the binary formation stage, the binary system can further emit GWs and lose the orbital energy and finally inspiral to merge. This merging happens on a timescale which is much smaller than the Hubble time at $z = 0$ and hence at the current time, we can safely assume that once a binary is formed it will certainly also merge to produce GWs [16]. In order to estimate how far back we can go in time to have the first merging, we compare the merging timescale [19] with the age of the Universe (i.e. $4\pi G M_{tot}/v_{pbh}^3 \sim 1/H(z)$). Numerical estimation from this equation results in $z \approx 6$.

In what follows we estimate the number of binary PBHs that can be formed from single PBHs. We follow the work of Bird et al. [16] where the calculation is done for the local universe. The total merger rate per unit volume is as follows,

$$\Gamma = \int \frac{dn}{dM} R(M) dM, \qquad (3)$$

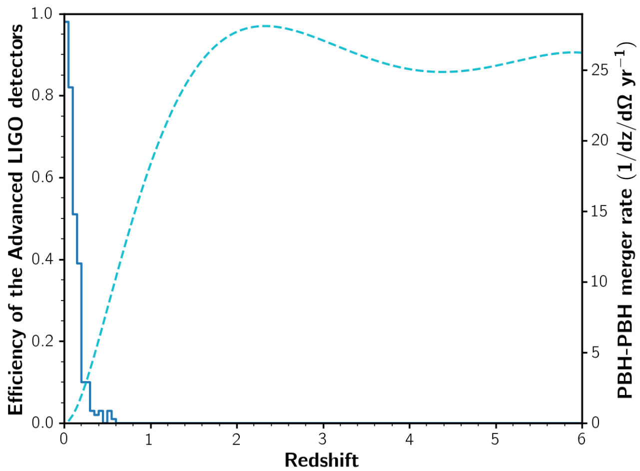

FIG. 1: The dashed-line represents the total PBH merger rate per steradian per redshift per year from equation (4). Here we assume that 100% of dark matter is made of PBHs. The solid-line histogram shows the redshift dependence of efficiency of Advanced LIGO with $SNR > 10$ detection threshold.

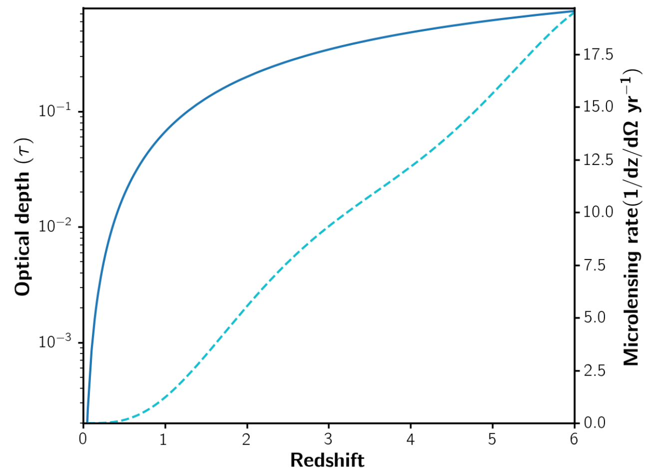

FIG. 2: This plot assumes that the whole of dark matter is composed of PBHs(light blue).The multiplication of the optical depth, $\tau(z)$ (dark blue curve) and the events rate, $N(z)$, as a function of redshift. This curve represents the rate of microlensed GWs, sourced by the PBHs

where dn/dM is the halo mass function that we adopt the Press-Schechter formalism [20], $R(M)$ from equation (2), is the rate of mergers in a given halo of mass M.

Considering that halos are formed at the higher redshifts [21], we take into account the redshift dependence in equation (3) and write the rate of events in comoving volume as

$$\frac{dN(z)}{dzd\Omega} = c\frac{\chi^2(z)}{H(z)}\int_0^{M_{vir}(z)}\frac{dn(z)}{dM}R(M,z)dM, \quad (4)$$

where c is the speed of light, $\chi(z)$ is the comoving distance, $H(z)$ is the Hubble expansion rate and $dn(z)/dM$ is the halo mass function of halos at the redshift, z. It should be noted that the upper limit of the integral decreases with increasing the redshift where at the higher redshifts the larger mass halos have not been virialized yet [22].

From equation (4), we plot $dN(z)/dzd\Omega$ in Figure (1) (in dashed line), as the merging rate per redshift per steradian per years, assuming that 100% of dark matter is made of PHBs. It is noteworthy that the peak at $z \approx 2$ comes from the peaking comoving volume at this redshift. Integrating the curve in Figure (1) over redshift, the result is about 1724 of events per year for $z < 6$. However, as we will determine the detection efficiency of LIGO, only the close-by events can be detected. GW signals for distant sources can also be magnified by gravitational lensing. Assuming the whole dark matter is made of PBHs, they can also play the role of lensing.

In what following we simulate the GWs from PBHs binary merging in the Universe and measure the detectability of events by LIGO detector. For simulating an ensemble of GW-sources and lenses, we assume: (a) a mass function for PBHs is lognormal [15] and (b) the redshift distribution of the PBHs follows the comoving volume. Then we calculate the antenna pattern function for Hanford and Livingston and include the background noise to determine GW strain in the detector frame [1, 23].

In the next step, we calculate the microlensing optical depth as a function of redshift to calculate the number of GWs being lensed. The optical depth as the probability of lensing for a homogeneous distribution of PBHs is give by [24]

$$\tau = \frac{3H_0\Omega_{\rm PBH}}{2D_{\rm s}}\int_0^{z_{\rm s}}\frac{(1+z)^2D_{\rm ls}D_{\rm l}{\rm d}z}{c\sqrt{\Omega_{\rm M}(1+z)^3+\Omega_\Lambda}}, \quad (5)$$

where H_0 is the current value of the Hubble parameter, $\Omega_{\rm M}$ is the dark matter density parameter and Ω_Λ is the dark energy density parameter. $D_{\rm s}$, $D_{\rm ls}$ and $D_{\rm l}$ are comoving observer-source distance, lens-source distance and the lens distance, respectively. Here we assume PBHs composed the whole dark matter (i.e. $\Omega_{\rm M} = \Omega_{\rm PBH}$). Figure (2-solid curve) shows the optical depth as a function of redshift. The multiplication of the optical depth, $\tau(z)$ to the events rate, $N(z)$ would result in the rate of the microlensed-GWs event. This rate is shown in Figure (2- dashed curve).

In the gravitational lensing of gravitational waves since the mass of binary merging system is in the order of lens, then the wavelength of GW, λ is in the order of the Schwarzchild radius of the lens, R_{sch}. In this case we need to deal problem with the wave optics approach. The propagation of gravitational wave with a perturbation due to a lens, behave similar to the electromagnetic equation (i.e. $\Box h^{\mu\nu} = 0$) [25, 26] where the metric is

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The GWs can be magnified similar to the electromagnetic radiation during the microlensing, however since the time scale of GW is very short, we can take statics configuration for the relative position of the source, lens and the observer.

From the generic solution of the wave equation, we can calculate the magnification factor for the strain of the GW during the lensing as [27]

$$\mu_{GW}(\beta; k) = \sqrt{\pi f} J_0(f\beta), \tag{6}$$

where $f = 2kR_{\rm sch}$, k is the wavenumber, β is the impact parameter of the source on the lens plane normalized to the Einstein angle and J_0 is the Bessel function of the first kind. We note that the magnification for light is square of relation (6) as for the electromagnetic waves we measure intensity of light (i.e. $I \propto E^2$) while for the GW we measure the strain (i.e. $h_{\mu\nu}$). In the limit of $k \to \infty$, we can recover the geometric optics relation for the magnification. In our simulation, we limit the minimum impact parameter to be in the range of $0 < \beta < 1$.

In what follow, we consider lognormal mass functions for PBHs with different parameters of M_c and σ of the equation (1). We perform a Monte-Carlo simulation and general binary PBHs over the redshift range of $(0 - 6)$. We also take into account the microlensing of the GWs with the magnification given by equation (6).

After generating GW events in our simulation, we add the corresponding noise to the data and use the specification of LIGO for calculating the detection efficiency of LIGO detector. For analyzing simulated data, we use PyCBC inference [28] package with dynamic nested sampling MCMC algorithm, dynesty [29]. It is based on sampling the likelihood function for a hypothesis that gives a measure of the existence of a signal in the data. The sampler performs the full Bayesian parameter estimation for each injection. Then we can obtain maximum SNR recovered from injection. We assume a threshold of $S/N > 10$ for criteria of significant detection. For each injection, we used the inbuilt injection creation of PyCBC package [1] using IMRPhenomPv2 waveform. We chose two detector network (Hanford, Livingston) for this analyse. We run the pipeline for $16304 \times 4 \times 6$ (accounting for $M_c = 20, 25, 30$ and $35 M_\odot$ and $\sigma = 0.1, 0.2, 0.3, 0.4, 0.5$ and 0.6 of the model (1)) injections using detector sensitivity during the O2 run. It is known that LIGO noise vary over long periods of time [30], in order to model the detector sensitivity (accounting for non-gaussian and/or non-stationary background noise) we made $32^{\rm Hanford} \times 32^{\rm Livingston} = 1024$ number of Power Spectral Densities (PSD) of 4096 sec random data and perform the PyCBC pipeline to analyse the injections for these random times. We used a fake Gaussian noise (via the fake-strain option) that is colored by a given PSD.

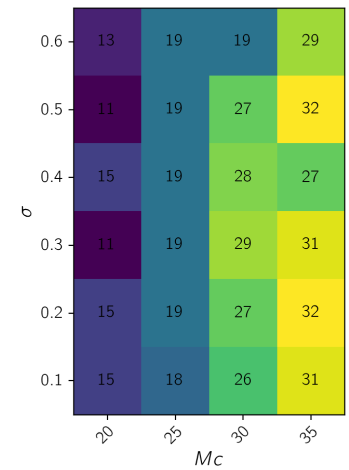

FIG. 3: The number of detected events from the simulated events for source located at $z < 1$, for various log-normal parameters of σ and M_c PBH mass function, assuming $f = 1$.

These injections are created with zero spin BBH components[2]. The orientation (inclination and polarization angles), and location (right-ascension and declination) of the GW-sources are distributed uniformly in the polarization sphere and in the sky, respectively [1, 31].

In the following, we calculate the redshift-dependent detection efficiency function of Advanced LIGO observatory as

$$\epsilon(z) = \frac{\Delta N_{detected}(z, z + \Delta z)}{\Delta N_{theory}(z, z + \Delta z)}, \tag{7}$$

where $\Delta N_{detected}$ is the number of events with $SNR > 10$ between $(z, z + \Delta z)$ and ΔN_{theory} is the number of theoretical events we generate between $(z, z + \Delta z)$. In the Figure (1), the histogram with solid line represents the efficiency function of LIGO in terms of redshift. This function declines to zero for redshifts less than one. Multiplying the efficiency function to the normalized distribution of events based on the theoretical assumptions (i.e. $\Delta N_{expected} = \Delta N_{theory} \times \epsilon(z)$) results in the expected number of events.

In order to find the best parameters for the mass function of PBHs, we compare the mass distribution of observed GWs sources with that of our simulated events [1]. Here we use the mean and the width of the detected distribution of mass of events (derived from the analysis of the binary blackhole) and compare them with the

[1] PyCBC inference documentation >> Simulated BBH example >> 1. Create the injection

[2] Although it would be more physical to set a non-zero distribution for each component spin, we assume that it doesn't affect the signal recovery of injections.

simulation. The lognormal mass function with characteristic mass of $M_c = 25M_\odot$ and $\sigma = 0.6$ in equation (1) shown in Figure (3) has the best compatibility between the theory and the observation.

Now, we compare the number of observed events with that of our expect from the theory. Integrating $\Delta N_{expected}/dz$ over z, we calculate the overall expected number of events where according to the efficiency function, we are able to detect events up to distance of $z \sim 1$. We note that in our simulation 38% of GW events with high redshifts have been microlensed and however for the lower redshifts (i.e. $z < 1$) the number of microlensed events are negligible. The comparison of the observed GW events (7 event from 117 days observation by O2) with the expected number of event assuming the 100% of dark matter is made of PBHs (0.5 event for 117 days), reveals that $N_{observed} \gg N_{expected}$ which means that $f > 1$. In order to explain this result within the context of the theory has been used in this work, we propose the following two explanation. (i) the number of binary blackholes in Figure (1) is underestimated. If we assume a part of PBHs population has been born in the binary systems then we would have a larger number of binary PBHs in this calculation. (ii) The second possibility is that astrophysical blackholes are responsible for the GW events. The problem with the second solution is that the astrophysical objects are not able to produce such a massive blackholes.

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