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Estimating the Demand Parameters for Single Period Problem, Markov-modulated Poisson Demand, Large Lot Size, and Unobserved Lost Sales

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Abstract. We consider a single-period single-item problem when the demand is a Markov-modulated Poisson process with hidden states, unknown intensities and continuous batch size distribution. The number of customers and lot size are assumed to be large enough. The estimators of demand mean and standard deviation for unobservable lost sales in the steady state are considered. The procedures are based on two censored samples: observed selling durations and the demands over the period. Numerical results are given.

Keywords: Stationary Markov-modulated Poisson Process, Newsvendor, Asymptotic Distribution, Selling Duration, Parameters Estimation, Unobserved Lost Sales.

1. INTRODUCTION AND PROBLEM STATEMENT

Consider a single-item single-period supply chain consisting of a supplier, a vendor, and customers. Let T be duration of the period. At the beginning of the selling season the vendor facing uncertain demand purchases a lot size Q, and replenishment during the period is impossible. This classic model is known as the newsvendor problem, see, for example, Arrow, Harris, and Marschak (1951); Silver, Pyke, and Peterson (1998).

Thus, two scenarios are possible during the selling season: the vendor runs short, i.e. there are lost sales, or there are leftovers at the end of the season and the vendor has to dispose of the remaining stock. To resolve this tradeoff between understocks and overstocks costs the vendor needs to know the demand distribution. However, usually the distribution is not known a priori and we need to relay on the historical sales data and use some statistical methods.

In general, there are different approaches to the handling of demand uncertainty in the framework of the newsvendor problem; see Khouja (2000) and more recently Rossia et al. (2014). Most of the papers have focused on cases where the demand is fully observed, for example, this is the case when the demand in excess of inventory on hand is backordered. But if this demand is unobservable, that is often the case; the information on demand gets censored by inventory availability. Early approaches to dealing with the censored demand induced by lost sales were presented by Hill (1992), Nahmias (1994), Agrawal and Smith (1996), and Lau and Lau (1996). Most recent papers concerning censored demand focus on the Bayesian approach; for references see, for example, Jain, Rudi, and Wang (2015). Among others we want to mention papers by Huh and Rusmevichientong (2009) and Huh et al. (2009, 2011) taking nonparametric approach.

In Kitaeva, Subbotina, and Stepanova (2015) we described the demand as a compound Poisson process with continuous batch size distribution. The model is rich enough to present the uncertainty of demand and seems to be suitable for many inventory control processes; see, for example, Kemp (1967) and Adelson (1966) for early papers; Monahan, Petruzzi, and Zhao (2002) and Babai, Jemai, and Dallery (2011) for more recent ones. The only drawback of the distribution is its complexity, which leads to expressions that are too complex to deal with analytically.

In that paper, we have solved the problem by considering the diffusion approximation of the demand process for fast moving items, i.e. we assumed that the number of customers and lot size are sufficiently large. We used the approximation to obtain the asymptotic distribution of time required for the sale of a fixed lot size. The distribution is strongly connected with asymptotic demand distribution, so we can use the timing of stock out to estimate the demand distribution under lost sales, i.e. when there are lost sales and the actual demand during the period is unobserved, we use observations of the moments at which the vendor runs short. The diffusion approximation of a compound Poisson process was used without a rigorous proof there.

In this paper we do not use the approximation and provide a proof of the result using Laplace transform method. We also model the demand as a two-state Markov-modulated Poisson process (MMPP), i.e. the intensity of a Poisson process of the customers arrivals is defined by the state of a Markov chain with two states: if at time t the Markov process has value i = 1, 2 then the customers are arriving according to a Poisson process with intensity $\lambda_i > 0$. The amounts required at each arrival are distributed according to probability density function (PDF) $p(\cdot)$ and are independent of everything else. If the two intensities are equal, this model becomes an ordinary compound Poisson process.

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MMPP processes submit a flexible way of modeling demand for inventory systems, see, for example, Song and Zipkin (1992, 1996), Abhyankar and Graves (2001). A collection of results about Markov-modulated Poisson processes is given in Fischer and Meier-Hellstern (1993). Here we are going to consider MMPP only in the steady state.

2. DISTRIBUTION OF THE SELLING TIME FOR COMPOUND POISSON DEMAND

To illustrate our approach, first consider the demand generated by customers arriving according to a Poisson process with unknown intensity λ and requiring amounts of varying size independent of the arrival process. The amounts required at each arrival (batch sizes) are i.i.d. continuous random variables with PDF $p(\cdot)$.

Let $\tau(Q)$ be the amount of time it takes to sell a lot Q, and $g(w,Q) = E\{e^{-w\tau(Q)}\}$ be the Laplace transform of its PDF.

Let us find asymptotic characteristics of $\tau(Q)$ as $Q \to \infty$.

Consider small time interval Δt. Denote ΔQ the purchases during the interval, then $\tau(Q) = \Delta t + \tau(Q - \Delta Q)$ and

$$g(w,Q) \approx e^{-w\Delta t} E_{\Delta Q}\{g(w, Q-\Delta Q)\} = \exp(-w\Delta t)\cdot$$

$$\cdot\left[(1-\lambda\Delta t) g(w,Q) + \lambda\Delta t \int_0^Q g(w,Q-x)\,p(x)dx + \lambda\Delta t\int_Q^\infty p(x)dx\right],$$

where $E_{\Delta Z}\{\cdot\}$ is the expectation with respect to ΔZ. Using $e^{-w\Delta t} = 1 - w\Delta t + o(\Delta t)$, we get the equation as $\Delta t \to 0$

$$(\lambda + w) g(w,Q) = \lambda\int_0^Q g(w,Q-x)\,p(x)dx + \lambda\int_Q^\infty p(x)dx \quad ,(1)$$

and initial moments $T_k(Q)$ of $\tau(Q)$ are defined by equation $T_k(Q) = (-1)^k g_w^{(k)}(0,Q)$, where $g_w^{(k)}(0,Q)$ is the k-th derivative of $g(w,Q)$ with respect to w at $w = 0$.

Taking into account that $g(0,Q) = 1$, we obtain from (1) that the mean of $\tau(Q)$ satisfies following equation

$$\lambda T_1(Q) = 1 + \lambda\int_0^Q T_1(Q-x)\,p(x)dx\,, \tag{2}$$

and $T_k(Q)$, $k \neq 1$, are defined by equations

$$\lambda T_k(Q) = kT_{k-1}(Q) + \lambda\int_0^Q T_k(Q-x)\,p(x)dx\,. \tag{3}$$

Denote $X(\cdot)$ and $F_k(\cdot)$ the Laplace transforms of functions $p(\cdot)$ and $T_k(\cdot)$, $k \geq 1$. From (2) and (3) we get

$$F_1(u) = \frac{1}{\lambda u\left(1 - X(u)\right)},$$

and $\lambda F_k(u) = kF_{k-1}(u) + \lambda F_k(u) X(u)$, $k \neq 1$. It follows

$$F_k(u) = \frac{k!}{\lambda^k u\left(1 - X(u)\right)^k},\ k = 1,2,\ldots,$$

and $T_k(Q) = \dfrac{k!}{2\pi j}\displaystyle\int_{c-j\infty}^{c+j\infty} \frac{\exp(uQ)}{\lambda^k u\left(1 - X(u)\right)^k}du$.

Let us consider the case $Q >> 1$. If $\mathrm{Re}\,u > 0$ then $|X(u)| < 1$ and $X(0) = 1$, therefore the zeros of the denominator lie on the imaginary axis and in the left half-plane. All the residues, except for the residue at $u = 0$, contain exponentially decreasing factor, so the residue at $u = 0$ gives the main contribution to $T_\kappa(Q)$ as $Q >> 1$.

Denote $Y(u) = (1 - X(u))/u$ and the initial moments of purchase $a_k = \displaystyle\int_0^\infty s^k p(s)ds$, $k = 1,2\ldots$, then

$$T_k(Q) = \frac{1}{\lambda^k}\lim_{u\to 0}\frac{d^k}{du^k}\left(\frac{\exp(uQ)}{Y(u)^k}\right)$$

and as $Q >> 1$

$$T_1(Q) = \frac{Q}{\lambda Y(0)} - \frac{Y'(0)}{\lambda Y(0)^2} = \frac{Q}{\lambda a_1} + \frac{a_2}{2\lambda a_1^2}. \tag{4}$$

Note that $T_1(0) \neq 0$ because here $\tau(Q)$ is the time from the beginning of the sale period until the arrival of the first buyer who cannot make a purchase due to the zero inventories.

Analogously we receive as $Q >> 1$

$$T_2(Q) = \frac{1}{\lambda^2}\left[\frac{Q^2}{Y(0)^2} - \frac{4QY'(0)}{\lambda Y(0)^3} - \frac{2Y''(0)}{Y(0)^3} + \frac{6Y'(0)^2}{Y(0)^4}\right] \approx$$

$$\approx \frac{1}{\lambda^2}\left[\frac{Q^2}{a_1^2} + \frac{2Qa_2}{a_1^3}\right],$$

and the variance of $\tau(Q)$

$$Var\{\tau(Q)\} = D(Q) \approx \frac{Qa_2}{\lambda^2 a_1^3}. \tag{5}$$

Let us denote $\Phi(w,u)=\int_0^\infty g(w,s)\exp(-us)ds$. From (1), we get $\Phi(w,u)=\lambda\frac{1-X(u)}{u\left(w+\lambda-\lambda X(u)\right)}$, and

$$g(w,Q)=\frac{\lambda}{2\pi j}\int_{c-j\infty}^{c+j\infty}\frac{1-X(u)}{u\left(w+\lambda-\lambda X(u)\right)}\exp\left(uQ\right)du .$$

By the convolution theorem

$$g(w,Q)=\int_0^Q\phi\left(w,Q-x\right)\int_x^\infty p(y)dydx ,$$

where $\phi\left(w,Q\right)==\frac{\lambda}{2\pi j}\int_{c-j\infty}^{c+j\infty}\frac{\exp\left(uQ\right)}{w+\lambda-\lambda X(u)}du$.

Consider the case $Q>>1$ and $w<<1$. Zeros of the denominator, $\mathrm{Re}\,w>0$, lie on the imaginary axis and in the left half-plane because for $\mathrm{Re}\,u>0$ function $\frac{\exp(uQ)}{w+\lambda-\lambda X(u)}$ has to be analytical. As $Q>>1$, the residue at zero gives the main contribution to $\phi(w,Q)$.

Note that if $w=0$ then $u=0$, and $X(0)-1=0$, $X'(0)\neq 0$. It follows that the zeros for $w\neq 0$ can be sought as the Bürmann-Lagrange expansion $u_0=\sum_{n=1}^\infty d_n w^n$, where

$$d_n=\frac{1}{n!}\lim_{z\to 0}\frac{d^{n-1}}{dz^{n-1}}\left[\frac{z^n}{\left(\lambda\left(X(z)-1\right)\right)^n}\right].$$

According to the expansion we get as $w<<1$

$$u_0=-\frac{1}{\lambda a_1}w+\frac{a_2}{2\lambda^2 a_1^3}w^2+o(w^2) .$$

By the residue theorem for $Q>>1$ and $w<<1$ $\phi(w,Q)\sim-\frac{\exp\left(u_0 Q\right)}{X'\left(u_0\right)}$, and

$$g(w,Q)\sim-\frac{1}{X'\left(u_0\right)}\int_0^Q\exp\left(u_0\left(Q-x\right)\right)\int_x^\infty p(y)dydx .$$

Consider random variable $V=\left(\tau(Q)-q\right)/\left(\sqrt{q}\delta\right)$, where $q=\frac{Q}{\lambda a_1}$, $\delta=\sqrt{\frac{a_2}{\lambda a_1^2}}$. It follows from (4) and (5) that $E\{V\}=0$ and $Var\{V\}=1$ as $Q>>1$. As Q tends to infinity the moment generating function of V

$$g_V(w,Q)=\exp\left(\frac{w\sqrt{q}}{\delta}\right)g\left(\frac{w}{\sqrt{q}\delta},Q\right)\sim\exp\left(\frac{w^2}{2}\right),$$

i.e. V convergences in distribution to the standard normal random variable.

Thus, as $Q>>1$ we can consider the selling time $\tau(Q)$ as the normal random variable with parameters $T_1(Q)=Q/(\lambda a_1)$ and $D(Q)=Qa_2/\left(\lambda^2 a_1^3\right)$. The same asymptotic result has been obtained in Kitaeva, Subbotina, and Stepanova (2015), where the diffusion approximation of the demand process has been considered.

For exponential batch size distribution the results of comparing the exact and approximate probability density functions of selling time are presented in Kitaeva, Subbotina, and Zhukovskiy (2015).

3. DISTRIBUTION OF THE SELLING TIME FOR MMPP DEMAND

Denote $P=\begin{pmatrix}-p_{11} & p_{11}\\ p_{22} & -p_{22}\end{pmatrix}$ an infinitesimal generator of a two-state continuous-time Markov chain determining the customers' arrival rate in a Poisson process, i.e. the arrival rate is $\lambda_i>0$ when the Markov chain is in state *i*. Let $\pi=\left(\pi_1,\pi_2\right)$, $\pi_1=\frac{p_{22}}{p_{11}+p_{22}}$, $\pi_2=\frac{p_{11}}{p_{11}+p_{22}}$ be the steady state vector of the Markov chain and $\tau_i(\mathrm{Q})$ be an amount of time it takes to sell a lot Q if at $t=0$ the chain is in state *i*. All other assumptions and notations are the same.

Using the same technique as previously we receive the system of equations for the Laplace transforms $g_i(\omega,Q)$ of PDF of $\tau_i(\mathrm{Q})$ in the steady state

$$\left(\lambda_i+\omega\right)g_i\left(\omega,Q\right)=\sum_{j=1}^n p_{ij}g_j\left(\omega,Q\right)+$$
$$+\lambda_i\int_0^Q g_i\left(\omega,Q-x\right)p(x)dx+\lambda_i\int_Q^\infty p(x)dx.$$

Conditional means $T_i(Q)$ and second initial moments $V_i(Q)$ are defined by equations

$$\lambda_i T_i(\mathrm{Q})=1-\sum_{j=1}^2 p_{ij}T_j(\mathrm{Q})+\lambda_i\int_0^Q T_j\left(Q-x\right)p(x)dx, \qquad (6)$$

$$\lambda_i V_i(\mathrm{Q})=2T_i(\mathrm{Q})-\sum_{j=1}^2 p_{ij}V_j(\mathrm{Q})+\lambda_i\int_0^Q V_j\left(Q-x\right)p(x)dx.$$

Using the Laplace transform we get from (6)

$$W_i(\omega) = \frac{p_{22} + p_{11} + \lambda_{\bar{i}}\omega Y(\omega)}{\omega^2 Y(\omega)\left[\lambda_1\lambda_2\omega Y(\omega) + \lambda_0\left(p_{11} + p_{22}\right)\right]},$$

where $X(\omega) = \int_0^\infty \exp(-\omega x) p(x) dx$, $Y(\omega) = \frac{1 - X(\omega)}{\omega}$,

$W_i(\omega) = \int_0^\infty T_i(x)\exp(-\omega x)dx$, $\lambda_0 = \pi_1\lambda_1 + \pi_2\lambda_2$; and $\bar{i} = 2$, if $i = 1$, $\bar{i} = 1$, if $i = 2$.

Using the same as in the previous part asymptotic considerations as $Q >> 1$, we get

$$T_i(Q) = \frac{Q}{a_1\lambda_0} + \frac{a_2}{2a_1^2\lambda_0} - \frac{\lambda_{\bar{i}}(\lambda_1 - \lambda_2)\pi_2}{\lambda_0^2(p_{11} + p_{22})},$$

and the total expectation

$$E\{\tau\} = T(Q) = \pi_1 T_1(Q) + \pi_2 T_2(Q) =$$

$$= \frac{Q}{a_1\lambda_0} + \frac{a_2}{2a_1^2\lambda_0} - \frac{(\lambda_1 - \lambda_2)^2\pi_1\pi_2}{\lambda_0^2(p_{11} + p_{22})}.$$

Similarly the total variance

$$Var\{\tau\} = D(Q) = \frac{Qa_2}{a_1^3\lambda_0^2} + 2Q\frac{(\lambda_1 - \lambda_2)^2\pi_1\pi_2}{a_1\lambda_0^3(p_{11} + p_{22})}.$$

Consider the asymptotic behaviour of the moment generating function $g(\omega, Q) = \pi_1 g_1(\omega, Q) + \pi_2 g_2(\omega, Q)$. Denote $\Phi(\omega, u) = \int_0^\infty g(\omega, Q)e^{-uQ}dQ$ its Laplace transform. Then

$$\Phi(\omega, u) = -\frac{\lambda_0(p_{11} + p_{22}) + \lambda_1\lambda_2 uY(u) + \lambda_0\omega}{d(\omega, u)}uY(u),$$

where

$$d(\omega, u) = \omega^2 + \omega\left((\lambda_1 + \lambda_2)uY(u) + p_{11} + p_{22}\right) +$$

$$+\lambda_0(p_{11} + p_{22})uY(u) + \lambda_1\lambda_2 u^2 Y(u)^2.$$

Taking into account that $d(\omega, u_0) = 0$, where

$$u_0(\omega) = -\frac{1}{\lambda_0 a_1}\omega + \left(\frac{a_2}{2\lambda_0^2 a_1^3} + \frac{(\lambda_1 - \lambda_2)^2\pi_1\pi_2}{a_1\lambda_0^3(p_{11} + p_{22})}\right)\omega^2 + o(\omega^2),$$

the moment generating function of $z = \frac{\tau(Q) - T(Q)}{\sqrt{D(Q)}}$ tends to the moment generating function of the standard normal distribution as $Q \to \infty$.

4. PARAMETERS ESTIMATION AND SIMULATION

In Kitaeva et al. (2016) it is shown that for stationary demand when the intensity of the arrival process is high and the duration of the period is sufficiently large we can consider demand X as a normal random variable with the mean $m_X = a_1 m_T$ and variance $\sigma_X^2 = m_T(a_2 - a_1^2) + \sigma_T^2 a_1^2$, where m_T and σ_T^2 are the mean and variance of the number of arrivals during T. From, for example, Yoshihara, Kasahara, and Takahashi (2001) it follows that for a time-stationary MMPP

$$\mu_X = a_1\lambda_0 T, \ \sigma_X^2 = T\left(a_2\lambda_0 + 2a_1^2\frac{\pi_1\pi_2(\lambda_1 - \lambda_2)^2}{p_{11} + p_{22}}\right).$$

Suppose we have observed n periods, and in m cases we have had leftovers and in $n - m$ cases we have had lost sales.

We consider three types of parameters m_X and σ_X estimators: estimators using only leftovers periods $\hat{\mu}_x$ and $\hat{\sigma}_x$; estimators using only lost sales periods $\hat{\mu}_x^{(t)}$ and $\hat{\sigma}_x^{(t)}$; and weighted estimators

$$\hat{\mu} = \frac{m}{n}\hat{\mu}_x + \frac{n-m}{n}\hat{\mu}_x^{(t)}, \ \hat{\sigma} = \frac{m}{n}\hat{\sigma}_x + \frac{n-m}{n}\hat{\sigma}_x^{(t)}.$$

We estimate the parameters given two samples: the selling durations, i.e. the moments of time at which the vendor runs short, $t_1, t_2, \ldots, t_{n-m}$, $\forall i = 1, \ldots, n-m \ \ t_i \le T$, if there are lost sales; and the sizes of sales during T $x_1, x_2, \ldots, x_m$, $\forall i = 1, \ldots, m \ \ x_i \le Q$, if there are leftovers at the end of the period. In Jain, Rudi, and Wang (2015) it is shown that for Poisson and normal demand distributions in the case of stock-out all the information contained in the timing of sales occurrences is captured by the timing of stock-out.

We assume that the customer satisfy the demand as far as possible when the amount required is more than the remainder; and the lot sizes are the same for each period.

The first equation for the estimators $\hat{\mu}_x$ and $\hat{\sigma}_x$ $\hat{\mu}_x + \hat{\sigma}_x\Psi(h) = Q$ we derive from equation $h = \Phi\left(\frac{Q - \mu_X}{\sigma_X}\right)$, where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(\frac{-t^2}{2}\right)dt$, $\Psi(\cdot) = \Phi^{-1}(\cdot)$, $h = m/n$.

By equating the theoretical and empirical conditional means, we obtain the second equation

$$\hat{\mu}_x - \hat{\sigma}_x \frac{\exp\left(-\frac{1}{2}\left[(Q-\hat{\mu}_x)/\hat{\sigma}_x\right]^2\right)}{\sqrt{2\pi}\Phi\left[(Q-\hat{\mu}_x)/\hat{\sigma}_x\right]} = \bar{x},$$

where $\bar{x} = \frac{1}{m}\sum_{i=1}^{m} x_i$.

So we get following system

$$\hat{\mu}_x - \hat{\sigma}_x F(h) = \bar{x}, \quad \hat{\mu}_x + \hat{\sigma}_x \Psi(h) = Q,$$

where $F(h) = \frac{\exp\left(-\Psi^2(h)/2\right)}{\sqrt{2\pi h}}$.

Thus, we receive the estimators

$$\hat{\sigma}_x = \frac{Q-\bar{x}}{\Psi(h)+F(h)}, \quad \hat{\mu}_x = \frac{\bar{x}\Psi(h)+QF(h)}{\Psi(h)+F(h)}.$$

Similarly we obtain the system of equations for $\hat{\nu}$ and $\hat{s}$

$$\hat{\nu} + \hat{s}\Psi(1-h) = 1, \quad \hat{\nu} - \hat{s}F(1-h) = \tilde{\tau},$$

where the dimensionless quantities $\nu = \frac{E\{\tau\}}{T}$, $s = \frac{\sqrt{Var\{\tau\}}}{T}$, and $\tilde{\tau} = \bar{t}/T$; $\bar{t} = \frac{1}{n-m}\sum_{i=1}^{n-m} t_i$; $\hat{\nu}$ and $\hat{s}$ are the corresponding estimators.

Thus, taking into account the connection between the parameters μ_X, σ_X^2 and $E\{\tau\}$, $Var\{\tau\}$; see Kitaeva et al. (2016), we get

$$\hat{\mu}_x^{(t)} = Q\frac{\Psi(1-h)+F(1-h)}{\tilde{\tau}\Psi(1-h)+F(1-h)},$$

$$\hat{\sigma}_x^{(t)} = Q(1-\tilde{\tau})\sqrt{\frac{\Psi(1-h)+F(1-h)}{\left(\tilde{\tau}\Psi(1-h)+F(1-h)\right)^3}}.$$

The numerical study is designed in the following manner.

First, we consider uniform at [2, 8] batch size distribution. It follows that a_1 = 5 and a_2 = 28. Intensities $\lambda_1 = 1$, $\lambda_2 = 2$, $p_{11} = 0.5$, $p_{22} = 2$ and period $T = 200$. Second, batch size is assumed to be exponentially distributed with a_1 = 4. It follows that a_2 = 32. Intensities $\lambda_1 = 2$, $\lambda_2 = 3$, $p_{11} = 0.3$, $p_{22} = 0.7$, and period $T = 250$.

We replicate the procedures ten times for n = 100 and n = 300. Tables 1 and 2 report the mean absolute percent errors (MAPE) of corresponding estimates in percentages, for example, $MAPE(\hat{\mu}_x) = \frac{1}{10}\sum_{i=1}^{10}\frac{\left|\hat{\mu}_x^{(i)} - \mu_X\right|}{\mu_X}100\%$.

Table 1. Uniform distribution, $\mu_x = 1200$, $\sigma_x \approx 85.79$

MAPE(·)% / Q	$\hat{\mu}_x$	$\hat{\mu}_x^{(t)}$	$\hat{\mu}$	$\hat{\sigma}_x$	$\hat{\sigma}_x^{(t)}$	$\hat{\sigma}$
n = 300						
1100	0.98	0.48	0.54	13.57	5.15	6.22
1200	0.41	0.43	0.42	5.11	6.20	5.56
1300	0.46	1.75	0.59	3.79	10.33	4.54
n = 100						
1100	1.50	0.72	0.82	14.97	9.10	9.73
1200	0.62	0.56	0.58	9.32	7.12	8.17
1300	0.59	2.33	0.74	7.23	14.68	8.16

Table 2. Exponential distribution, $\mu_x = 2300$, $\sigma_x \approx 141.7$

MAPE(·)% / Q	$\hat{\mu}_x$	$\hat{\mu}_x^{(t)}$	$\hat{\mu}$	$\hat{\sigma}_x$	$\hat{\sigma}_x^{(t)}$	$\hat{\sigma}$
n = 300						
2200	0.62	0.27	0.35	8.84	3.89	5.11
2300	0.29	0.29	0.29	4.31	5.43	4.79
2400	0.28	0.70	0.37	3.55	8.97	4.80
n = 100						
2200	0.83	0.50	0.58	10.96	9.50	9.77
2300	0.66	0.77	0.70	8.29	8.91	8.59
2400	0.55	0.82	0.61	8.51	9.92	8.62

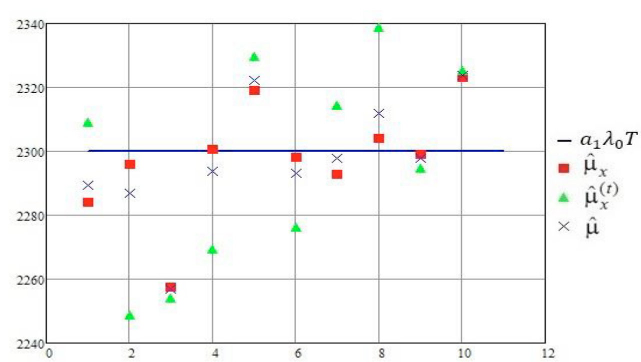

Fig. 1. Exponential distribution, Q = 2400, n = 100.

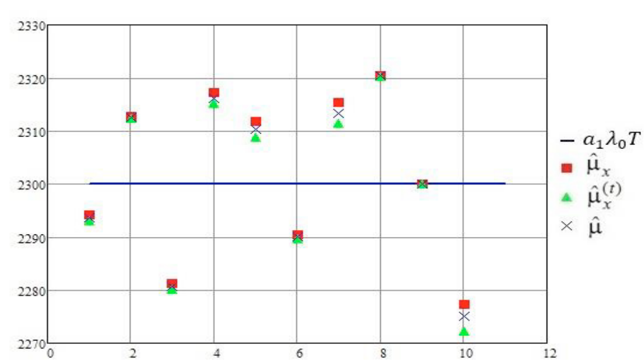

Fig. 2. Exponential distribution, Q = 2300, n = 100.

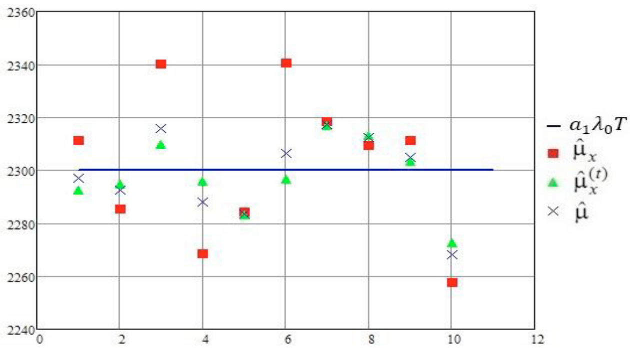

Fig. 3. Exponential distribution, Q = 2200, n = 100.

In Figures 1−3 the numerical results for exponential batch size distribution are represented for n = 100, $Q = 2400 > \mu_X$, $Q = 2300 = \mu_X$, and $Q = 2200 < \mu_X$.

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