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# ASSESSMENT OF MATHEMATICS TEACHERS' PROFESSIONAL COMPETENCE

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***Abstract***

Development of students' mathematical skills is associated with quality teaching, which means that mathematics teachers should be able to successfully solve mathematical, teaching, and professional problems. The article aims to describe the assessment system of mathematics school teachers' professional competence, which helps identify gaps in their training and design tailor-made retraining courses. 2,359 mathematics teachers from 13 regions of Russia participated in the research on 05–29 September 2017. Foremost, we conducted a survey and collected data about their teacher category and teacher expertise. Next, we provided a preliminary diagnostic test to enable the participants to self-assess their subject matter and teaching competencies. After that, they completed a three-part diagnostic test to assess their abilities to solve mathematical, teaching, and professional problems. Finally, the participants conducted video lessons. The three-part diagnostic test and video lessons allowed determining the professional competence level for every mathematics teacher. 24% participants showed level I of professional competence, 44% – level II, 9% – level III; 23% participants did not pass the basic level of professional competence. The results show that the mathematics teachers have difficulties in solving mathematical, teaching, or professional problems so tailor-made retraining courses are required. The developed assessment system underlies designing the courses.

**Keywords:** mathematics school teachers, professional competence level, teaching and professional problems, diagnostic test, video lessons

**Abstrak**

Pengembangan keterampilan matematika siswa dikaitkan dengan pengajaran yang berkualitas, yang berarti bahwa guru matematika harus mampu menyelesaikan dengan baik permasalah matematika, pengajaran, dan profesionalitas sebagai seorang pengajar. Artikel ini bertujuan untuk mendeskripsikan sistem penilaian kompetensi profesional guru matematika di sekolah, untuk membantu mengidentifikasi kesenjangan dalam pelatihan dan merancang kursus pelatihan ulang yang dibuat khusus untuk mereka. Sebanyak 2.359 guru matematika dari 13 wilayah di Rusia ikut berpartisipasi dalam penelitian ini yang dilakukan pada tanggal 5-29 September 2017. Kami melakukan survei dan mengumpulkan data tentang kategori guru dan keahlian guru mereka. Selanjutnya, kami menyediakan tes diagnostik awal untuk memungkinkan peserta menilai sendiri materi pelajaran dan kompetensi mengajar mereka. Setelah itu, mereka menyelesaikan tes diagnostik sebanyak tiga bagian untuk menilai kemampuan mereka dalam memecahkan masalah matematika, pengajaran, dan profesional. Terakhir, peserta membuat video pembelajaran. Tes diagnostik tiga bagian dan video pembelajaran memungkinkan penentuan tingkat kompetensi profesional untuk setiap guru matematika. Terdapat 24% peserta menunjukkan kompetensi profesional tingkat I, 44% di tingkat II, dan 9% di tingkat III; sebanyak 23% peserta tidak lulus kompetensi profesional tingkat dasar. Hasil penelitian menunjukkan bahwa guru matematika mengalami kesulitan dalam memecahkan masalah matematika, pengajaran, atau profesional sehingga diperlukan kursus pelatihan ulang yang dibuat khusus. Sistem penilaian yang dikembangkan ini mendasari perancangan kursus tersebut.

**Kata kunci:** guru matematika sekolah, tingkat kompetensi profesional, masalah pengajaran dan profesional, tes diagnostik, video pembelajaran

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Nowadays, it is important not only to teach the subject matter of mathematics, but also develop school students' mathematical skills. Mathematical skills development is one of teachers' main goals in mathematics school classes. To encourage the development, a highly qualified teacher should be able to effectively apply their knowledge, life experience and expertise, as well as personal and professional values. They all contribute to quality teaching.

Comparative studies of students' learning outcomes, which have been conducted since the 1980s, show that their improvement is directly connected to improving quality of teaching (Barber & Mourshed, 2008). Mathematics teachers' quality of teaching reflects their professional competence. Raven (2002) determines the latter as a multifunctional package of continuously updating knowledge, abilities (skills) and relationships that each person requires for a full and effective inclusion in society. Kozyrev et al. (2005) specify the concept and describe it as an integral characteristic that defines ability to solve professional problems in real-life situations of professional activity with the help of knowledge, professional and personal experience, as well as values. The pan-European Tuning project determines the main components of competence: knowledge and understanding, knowledge as an action, and knowledge as the basis of value relationships in professional activity (Kennedy, Hyland, & Ryan, 2009).

School teachers' beliefs about their quality of teaching are actually reflected in their teaching activity (Kardanova & Ponomareva, 2014). It is critical that mathematics teachers should respond to students' academic needs in learning basic mathematics at schools: from arithmetic to stereometry. With this aim in mind, the teachers should be ready to efficiently carry out their professional activity. So, assessment of their professional competence and competencies as its constituents is in focus. According to Shulman (1986), a set of competencies help solve problems arising in professional activity. The problems are tools of teachers' professional training (Kozyrev et al., 2005).

The problems can be mathematical, teaching, and professional. For mathematics teachers, abilities to solve teaching and professional problems characterise their quality of teaching. The solution to teaching problems means identifying school students' errors in solving mathematical problems and their causes, work organisation and assistance in searching for relevant solutions, encouraging students to think critically and self-assess their solution. The successful solution to professional problems allows creating favourable learning conditions. The problems relate to development of school students' communication skills, enhancement of social potential, ability to regulate conflict situations, and development of self-regulation.

The solution to the problems relies upon interrelated aspects of mathematics teachers' knowledge: subject matter, teaching, and practical (Ball, Thames, & Phelps, 2008; Silverman & Thompson, 2008). The subject matter knowledge, which covers the main content of mathematics, constitutes subject matter competency. The knowledge about general and private methods of teaching mathematics describes teaching knowledge as the basis of teaching competency. The use of practical knowledge, or theories-in-use, underlies practical competency (Cheng, 2019; Rollnick & Mavhunga, 2016). Each competency can be developed at basic and advanced levels if a teacher is able to apply their knowledge, expertise, and values.

There are a number of studies which focus on assessing the effective teaching of mathematics. The assessment of a mathematics teacher's knowledge is a complicated process, since it is multidimensional. Within the research project TEDS-M, Döhrmann, Kaiser, and Blömeke (2012) explored primary school teachers' beliefs about teaching and learning mathematics effectively. They designed a TEDS-M questionnaire, which examined beliefs about the subject matter of mathematics and teaching mathematics, rather than beliefs related directly to teaching activity. There was only an investigation into the knowledge component integrated into the subject matter and teaching techniques, which is insufficient for making judgements about development of mathematics teachers' professional competence.

The Teaching and Learning International Survey centred on continuous professional development, teacher appraisal, school leadership and management, school climate, teachers' instructional beliefs, teachers' pedagogical and professional practices (Strizek, Tourkin, Erberber, & Gonzales, 2014). However, the survey did not aim to assess mathematics teachers' professional competence. Moreover, the questions in the survey did not cover the subject matter of mathematics and mathematics teaching techniques, did not provide data on mathematics teachers' professional skills. In addition, the collected data presented a subjective assessment of the respondents.

Various types of teacher knowledge, which are necessary for teaching mathematics, are identified in Chapman's article (2013). Nevertheless, the knowledge component cannot give an idea of how mathematics teachers' professional competence is developed. In turn, Koponen, Asikainen, Viholainen, and Hirvonen (2017) explain the relationship between teachers' knowledge and their qualification. The researchers developed a 72-item survey to measure teacher educators' and graduated teachers' perceptions about what the latter had learned well or poorly during their studies. Again, this research considers subjectively only the knowledge component of preservice teachers' readiness for professional activity, which is clearly not enough to assess their professional competence.

As regards teachers' knowledge assessment, the Learning Mathematics for Teaching form ("Learning mathematics for teaching," 2007) is also of interest. Orrill and Cohen (2016) used this 73-item instrument to teach proportional reasoning. The instrument measured common content knowledge and specialised content knowledge, but it did not address the activity component of professional competence.

Apart from the subject matter knowledge, mathematics teachers should implement diverse teaching practices. A sample of 644 Greek-Cypriot preservice and inservice elementary school teachers and university students with strong mathematical background demonstrated knowledge with respect to four teaching practices. They include providing and evaluating explanations; selecting and using representations; analysing student errors, misconceptions, and non-conventional solutions; selecting tasks (Charalambous, 2016).

The described studies do not equally address subject matter, teaching, and practical competencies as the main components of mathematics teachers' professional competence. The researchers tend to focus more on the first component, while the second and especially the third one lag behind. In particular, they prioritise solutions to mathematical problems, but do not develop ability to solve teaching or professional problems.

It is important to consider the three competencies together in assessment and seek for relevant assessment tools. It is proposed to use teachers' assessment materials obtained as a result of analysing their performance review, surveys, as well as observing practical activities at competitive lessons. In her study, Smoleusova (2015) considers components of professional competence quite fully, but only according to one side of professional activity. Another approach is a formal assessment of students' learning outcomes so social aspects (for example, a student's family) strengthen or weaken development of professional competence (Huber & Skedsmo, 2016). Zee and Koomen (2016) highlight the relationship between teachers' self-assessment of their professional activity and their students' attitude towards the learning process. Kalyar, Ahmad, and Kalyar (2018) determine the relationship between teachers' motivation and their focus on mastery.

In other studies, there is an assessment with a specific tool. Temnyatkina and Tokmeninova (2018) propose the design of a teacher's performance assessment based on their students' performance. Other studies (Meuwissen & Choppin, 2015; Mohamadi & Malekshahi, 2018) create a teacher's formative assessment based on observations of their activities and identification of their performance feedback. So, testing or questioning, rather than solving problems mainly underlies the assessment. The tools reflect teachers' knowledge but do not contribute to development of their professional competence. Therefore, the research problem is: How can we assess mathematics teachers' professional competence, with subject matter, teaching, and practical competencies included? The article aims to describe the assessment system of mathematics school teachers' professional competence.

## METHOD

2,253 mathematics teachers working in school grades 5–11 participated in the research, which was conducted in 13 regions of Russia on 05–29 September 2017. We aimed to determine the professional competence level (Table 1) for every mathematics teacher on the basis of a diagnostic test and video lessons.

**Table 1.** Characteristics of the professional competence levels

| Level | Characteristics of the level |
|---|---|
| Level I | the ability to solve problems of professional activity with a predetermined condition (without considering the variability of real-life contexts) |
| Level II | the ability to solve problems of professional activity in a changing situation, which reflects various real-life contexts, to propose and choose various tools to solve them |

| Level | Characteristics of the level |
|---|---|
| Level III | the ability to solve problems of professional activity in a situation of uncertainty, which involves developing increasingly complex real-life contexts and variability of tools, as well as using new resources to solve them |

**At the first stage of the research** held on 05–14 September 2017, we conducted a survey of research participants. We found out that they were mathematics teachers from 1,460 schools (Table 2). Their median age was 45 years old, with a different percentage of teachers in each age category: 15.60% (20–30 years old), 22.09% (31–40), 44.84% (41–55), 17.47% (56–older). 36.14% had a higher teacher category, 35.18% were awarded the first teacher category, no teacher category was characteristic of 28.68% participants. Their teacher expertise was different: 12.55% (under 5 years old), 12.15% (5–10), 20.62% (11–20), and 54.66% (over 20).

**Table 2.** Locations of research participants

| Russian regions | Number of research participants | Number of schools involved |
|---|---|---|
| Volgograd region | 186 | 13 |
| Kabardino-Balkar Republic | 196 | 152 |
| Kurgan region | 186 | 149 |
| Leningrad region | 199 | 155 |
| Moscow region | 191 | 158 |
| Republic of Adygeya | 97 | 68 |
| Republic of Ingushetia | 148 | 91 |
| Republic of Tatarstan | 204 | 188 |
| Ryazan region | 190 | 114 |
| Tomsk region | 184 | 91 |
| Khabarovsk territory | 182 | 114 |
| Chechen Republic | 216 | 95 |
| Yaroslavl region | 116 | 72 |

**The second stage of the research** provided a preliminary diagnostic test, with PDF tasks to allow the participants to self-assess their subject matter and teaching competencies. The mathematics teachers could freely access the test on the teachers' level website of Herzen State Pedagogical University of Russia (Figure 1). At that, they could solve a variety of mathematical and teaching problems by themselves and/or study the provided solutions.

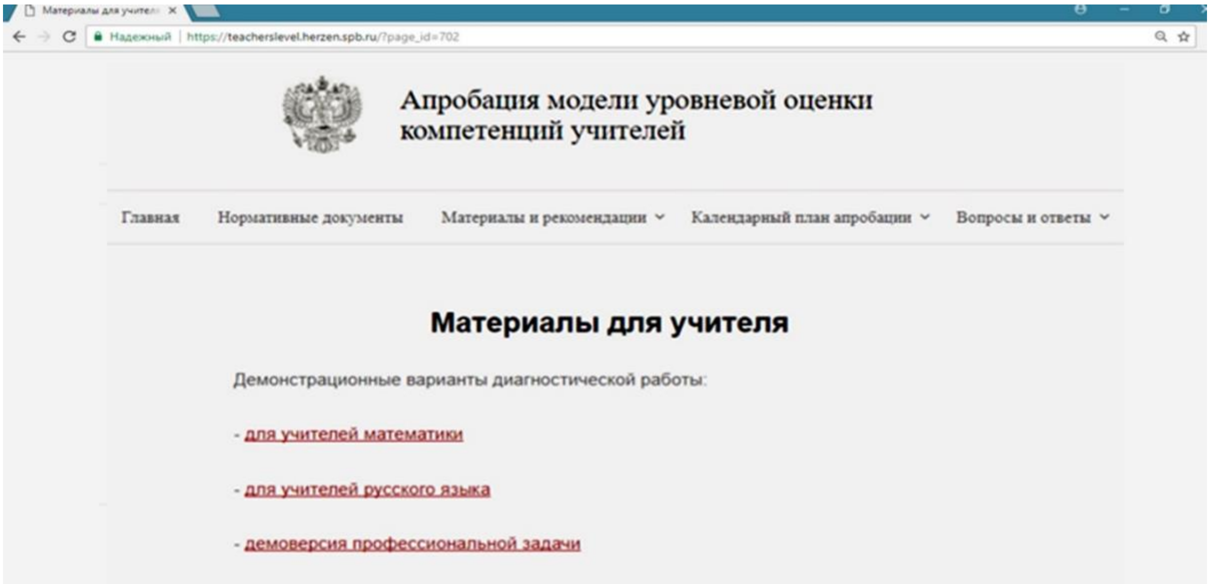


**Figure 1.** A webpage with a preliminary diagnostic test

Part 1 of the preliminary diagnostic test includes six basic mathematical problems, which target the subject matter knowledge about arithmetic, algebra, planimetry, and stereometry. The mathematics teachers are to solve the problems and provide a number as a single answer. There is an example of such a problem below, with the correct solution $x = 7$.

*Solve the equation*: $\sqrt{63 - 2x} = x$. *If there is more than one solution, give the least variable* $x$.

There are also six basic teaching problems to solve, which test teaching knowledge. While solving the problems, the mathematics teachers are involved in ordering typical of one problem, matching – one problem, and choosing single answers – four problems. There is an example of a teaching problem; its solution requires the single answer *b*.

*Find the natural numbers x for which* $2\frac{5}{9} < \frac{x}{9} < 3\frac{7}{9}$. *A student's solution x=6 is not correct.*

*The reason for the error is that he/she: a) ignored the ordered set of natural numbers; b) compared only fractions in mixed numbers; c) was unable to convert the mixed number into an improper fraction; d) had misconceptions about positioning fractions on the number line; e) was unable to compare ordinary fractions.*

The correct solutions to the basic mathematical problems, which are assessed 1 point maximum, allow identifying the basic level of subject matter and teaching competencies.

Part 2 comprises three advanced teaching problems to solve, which test teaching knowledge. The mathematics teachers are to provide their own scenarios of solving each teaching problem by following a set of instructions. The instructions for teaching problem 1 allow specifying school students' knowledge and skills to solve a mathematical problem, 2 – presenting difficulties in students solving a mathematical problem, 3 – providing an assessment of students' solution. At that, the teachers have to solve such mathematical problems by themselves. Here is an example of teaching problem 3.

*In the equilateral triangle ABC, the altitude is 12 and the base is 32 (Figure 2). Determine the radius of the circumference.*

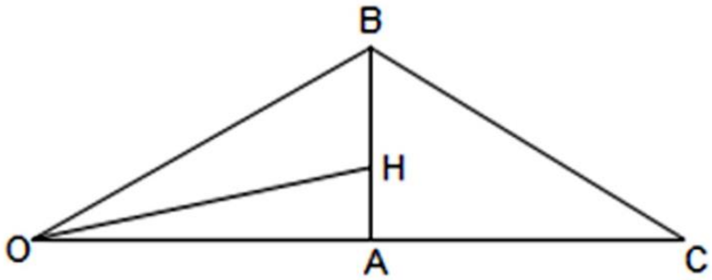


**Figure 2.** The equilateral triangle for problem 3 in the preliminary diagnostic test

*The student solved this problem as follows:*

$$0 = 256 + 144 - 24R, 24R = 400, R = \frac{400}{24} = 16\frac{2}{3}.$$

*Solution:* $R = 16\frac{2}{3}$.

*Instructions:*

*1. Comment on this solution. Specify errors and flaws if there are any.*

*2. Give a quantitative and qualitative assessment of the solution.*

*3. Offer a relevant teaching technique to avoid errors in solving such problems.*

The correct solutions to the advanced teaching problems help determine levels I, II, III of professional competence. The assessment of the teaching problems is based on a 5-point scale.

**At the third stage of the research**, the participants were to complete a diagnostic test, which we had made to identify gaps in the mathematics teachers' professional competence. The test assessed their abilities to solve mathematical, teaching, and professional problems revealed in Parts 1, 2, 3 of the tests (Table 3).

**Table 3.** Correlation of professional competence with the structure of the diagnostic test

| Parts | Blocks | Professional competencies | Levels of professional competence |
|---|---|---|---|
| 1 | 6 basic mathematical problems | subject matter competency | basic |
| | 6 basic teaching problems | teaching competency | basic |
| 2 | 3 advanced teaching problems | subject matter competency teaching competency | I, II, III |

| Parts | Blocks | Professional competencies | Levels of professional competence |
|---|---|---|---|
| 3 | 1 professional problem | subject matter competency<br>teaching competency<br>practical competency | I, II, III |

We were elaborating all the problems covered in arithmetic, algebra, planimetry, and stereometrics from 25 May 2017 to 05 July 2017. For that, we used past tests from the standardized state exam in mathematics qualifying for the General Certificate of Secondary Education in Russia.

The diagnostic test was provided on 25–27 September 2017. The mathematics teachers had 240 minutes to complete Parts 1 and 2 of the diagnostic test, which fully correspond to Parts 1 and 2 of the preliminary diagnostic test. The teachers were not allowed to use any supplementary material to do the diagnostic test. The mathematics experts assessed these parts from 25 September to 1 October 2017.

Part 1 embraces 12 basic mathematical and teaching problems typical of those proposed in the preliminary diagnostic test. The participants can receive 1 point for each problem, which they correctly solve online on the university website. The minimum score to acquire the basic level is 10 points and above. We calculated and analysed how the participants solved each problem. The results could demonstrate if they managed to acquire the basic level of subject matter and teaching competencies. We also considered dependence of the diagnostic test results (Part 1) on the participants' teacher category and teacher expertise.

To test teaching knowledge, Part 2 contains three advanced teaching problems, with 5 points maximum for each. We assessed the participants' solution to three teaching problems and analysed the correct problem solution in accordance with some subject matter and teaching criteria, which explain the best scores (Table 4). We also compared the correct problem solution in Parts 1 and 2 of the diagnostic tests.

**Table 4.** Criteria for assessing advanced teaching problems

| Teaching problem | Subject matter criteria | Teaching criteria |
|---|---|---|
| 1 | A well-argued answer (4 points) | A good ordering of questions and tasks (1 point) |
| 2 | The correct solution to the problem (1 point) | The correct list of errors and their causes, and revision to avoid errors (4 points) |
| 3 | The correct list of errors (2 points) | The well-grounded assessment and its correctness (3 points) |

In Part 3, which was assessed 40 points maximum, the teachers presented solutions to one professional problem. Each professional problem contains a description of its conditions: a situation of a teacher's professional activity, the context that reveals the degree of uncertainty of the situation, and the problem statement. In addition to the condition's description, the professional problem is supplied with a set of instructions, which made the teachers demonstrate the problem solution. The set aligns with evaluation criteria of mathematics teachers' professional activity.

The professional problem can be the following: *When working with your students, you are faced with an urgent problem of modern teenagers: a low level of communication skills. You would like to incorporate games into your lesson plan. What school and classroom supplies can be used to enhance development of students' communication skills at the mathematics lesson?*

From 14 to 23 September 2017, the participants were to study the professional problem, solve it by following a set of instructions, and present their answer. The teachers were to show how they solved the problem on 5 pages maximum (a 12-point font, single-spaced text) and explained the answer on 10 pages maximum. They could use any relevant supplementary material at that.

On 16–29 September 2017, the mathematics experts assessed the participants' solution to the professional problem and analysed it. The assessment covered such aspects as stage-by-stage description of the problem solution, teaching material, and linguistic accuracy (Table 5), which equaled 16, 22, and 2 points correspondingly.

**Table 5.** The assessment criteria of the professional problem in the diagnostic test (Part 3)

| Aspects of criteria | Criteria |
|---|---|
| Stage-by-stage description of the problem solution | The task to solve is well-presented.<br>The task is relevant to the situation of a teacher's professional activity.<br>The context, in which the situation is presented, is thoroughly described.<br>The questions to answer are formulated well, specific steps to find the answer are identified.<br>There is a comprehensive list of questions and relevant steps to find a solution to the professional problem in the given context.<br>There is information, sources, and techniques adequate to solving the professional problem. |

| **Aspects of criteria** | **Criteria** |
|---|---|
| | The information, sources, and techniques presented in a table are sufficient. |
| | The solution to the professional problem is presented in stages. |
| Teaching material | There is teaching material to illustrate the solution. |
| | The teaching material is adequate to the solution. |
| | The teaching material is age- and gender-specific. |
| | The teaching material meets requirements of the secondary education standard. |
| | The teaching material is accurate. |
| | The solution to the professional problem is well-grounded. |
| | The solution prevents students from being alienated from school. |
| | The solution assessment is accurate. |
| | There are described other situations of professional activity, where the solution can be employed. |
| | There are specific steps to prevent from violating the ethical code. |
| | There are short-term consequences of the solution. |
| Linguistic accuracy | The text is linguistically accurate and is logically arranged. |

The teachers solved advanced teaching and professional problems in writing and uploaded their anonymous papers on the university website. They learnt about the results of each part in the diagnostic test two weeks after uploading their papers.

**The fourth stage of the research**, which was held on 16–29 September 2017, involved the participants conducting video lessons, with 25 students in each class.

The teachers could receive up to 38 points. Each criterion (Table 6) could be assessed 2 points maximum. The assessment of video lessons was held on 1–3 October 2017.

**Table 6.** The assessment criteria of a video lesson

| **Aspects of criteria** | **Criteria** |
|---|---|
| Lesson topic and lesson objective | Choice of the lesson topic/objective<br>Students' awareness of the lesson topic/objective |
| Subject matter | Subject matter knowledge<br>Correlation of the subject matter with the lesson topic/objective<br>Selection of relevant learning materials<br>Interdisciplinary relationships<br>Age-specific subject matter<br>Enriching subject matter |
| Interaction in class | Relevance of teaching techniques<br>Stimulation of students' positive motivation<br>Stimulation of students' cognitive activity<br>Applying a student-centred approach<br>Age-specific communication with students |
| Development of students' self-assessment | Explanation of self-assessment<br>Selection of assessment techniques<br>Techniques to develop students' self-assessment |
| Lesson maintenance | Educational tools<br>Speed of a lesson<br>Friendly environment in class |

Table 7 presents the matrix of points that the teachers could score after solving advanced teaching and professional problems in the diagnostic test, and conducting a video lesson. The scored points from Parts 2 and 3 of the test as well as the video lesson correspond to the teachers' professional competence level (I, II, III). The results of Part 1 are not included since they refer only to the basic level of subject matter and teaching competencies.

**Table 7.** The correlation of the total score with the professional competence level

| **Diagnostic test / Video lesson** | | **Professional competence** | | |
|---|---|---|---|---|
| **Parts** | **Score** | **Level I** | **Level II** | **Level III** |
| Part 2 | 5 (teaching problem 1) | 5 | 5 | 5 |
| | 5 (teaching problem 2) | ≥ 3 | 5 | 5 |
| | 5 (teaching problem 3) | ≥ 3 | ≥ 4 | 5 |
| Part 3 | 40 | ≥ 21 | 22–31 | 32–40 |
| Lesson | 36 | 18–23 | 24–31 | 32–38 |
| Total | 91 | 50–59 | 60–76 | 79–91 |

## RESULTS AND DISCUSSION

The diagnostic test results are displayed in accordance with its each part. Figure 3, which illustrates a percentage of participants who correctly solved basic mathematical and teaching problems, shows a normal distribution of the points scored in Part 1. Mathematical problem 6 proved to be the most difficult to solve as only 20.10% participants managed to use methods in number theory correctly. However, mathematical problem 3 was the easiest one, with 91.30% involved to identify and employ mathematical facts in order to answer the provided question.

The smallest number of participants (28.99%) succeeded in solving teaching problem 7, which assessed basics of teaching mathematics. Only 54.37% coped with the least difficult teaching problem 12, which tested the mathematical subject matter knowledge as well as skills to select, interpret, and employ mathematical facts relevant to teaching activity.

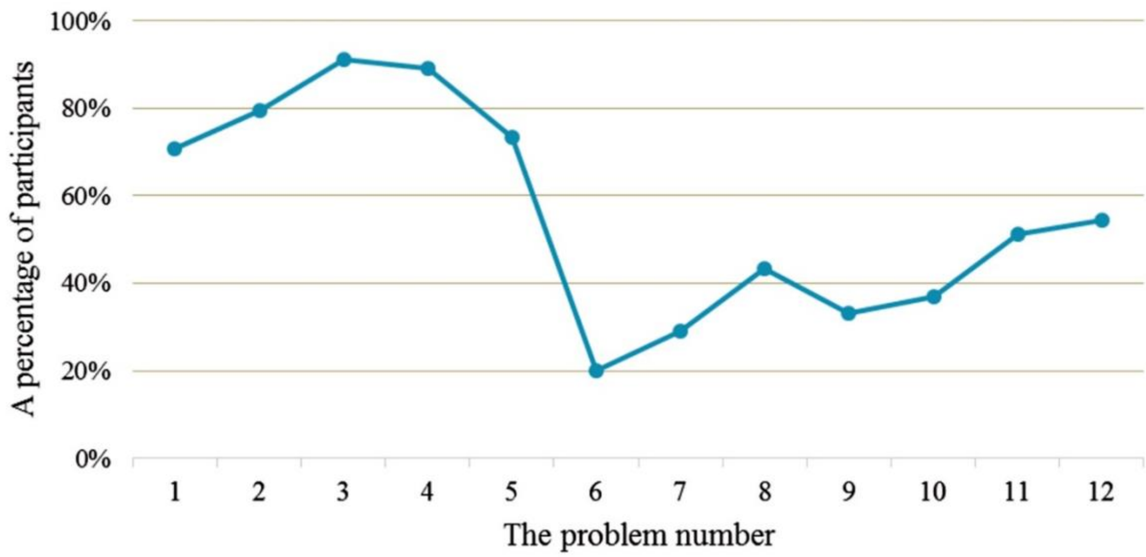


**Figure 3**. The results of the correct problem solution in the diagnostic test (Part 1)

Figure 4 correlates a percentage of participants with the number of total points gained in Part 1. It demonstrates a lower basic level of teaching competency compared to subject matter competency. 19.20% participants gained 8 points, 17.94% – 7, 15.99% – 6, while the highest score (12 points) was found only among 0.39% participants, the lowest one (0 points) – 0.44%. The median score was 7.

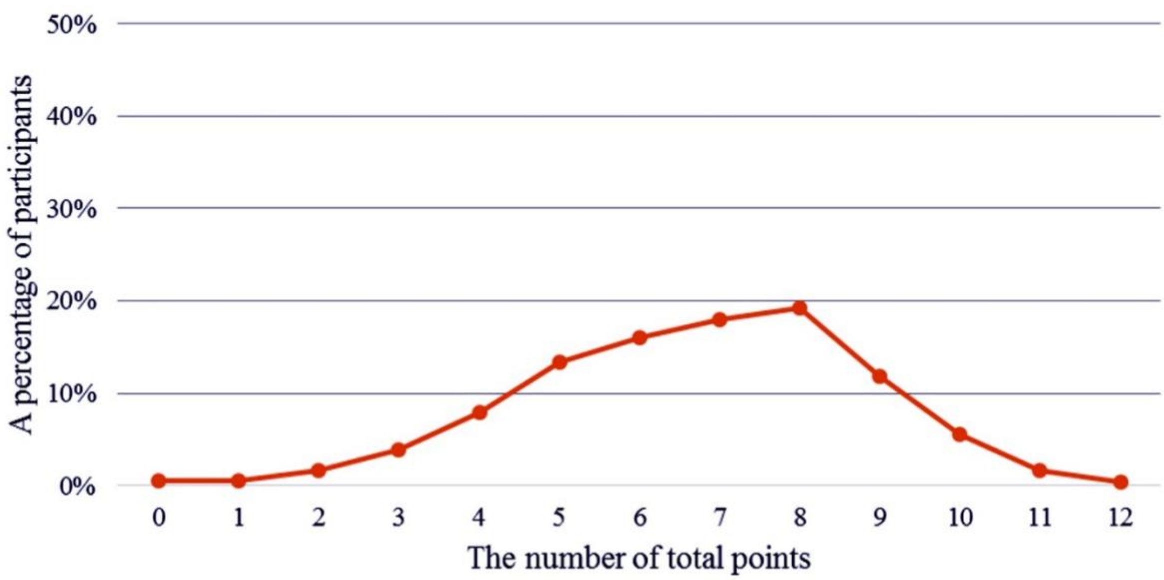


**Figure 4.** The total results of the diagnostic test (Part 1)

We compared the correct problem solution among the participants with different teacher categories (Figure 5) and concluded that a higher teacher category allowed them to receive higher results. 26.10% correctly solved the most difficult basic mathematical problem, 96.90% – the easiest one. Correspondingly, there were 17.57% and 91.41% with the first teacher category, 15.63% as well as 84.05% – with no teacher category. As regards the most difficult and easiest basic teaching problems, the results were as follows: 53.10% and 31.40% among those with a higher teacher category, 55.09% and 32.23% – the first teacher category, 55.10% and 21.88% – no teacher category.

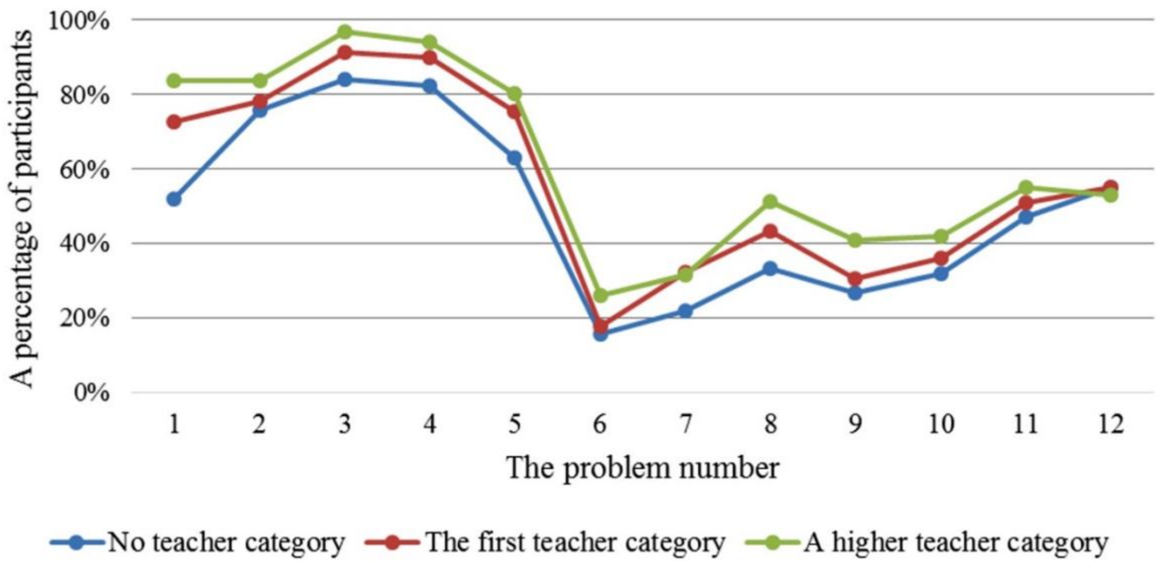


**Figure 5.** The results of the diagnostic test (Part 1) related to the participants' teacher category

The participants' teacher expertise influenced the correct problem solution but not so significantly (Figure 6), which is evident in solving the most difficult and easiest problems. On average 22.05% participants with longer teacher expertise (over 10 years) managed to solve the most difficult basic mathematical problem, unlike 15.33% of those with shorter teacher expertise (under 10 years). The average percentage of teachers who succeeded in solving the easiest mathematical problem was 92.98% and 84.18% correspondingly. There are no striking differences in solving basic teaching problems since the success did not largely depend on the teacher expertise. The percentage of teachers (53.63% and 31.43%), who had been working for over 10 years, was similar to 54.45% and 22.54% with teacher expertise of under 10 years, as they relate to solving the most difficult and easiest teaching problems.

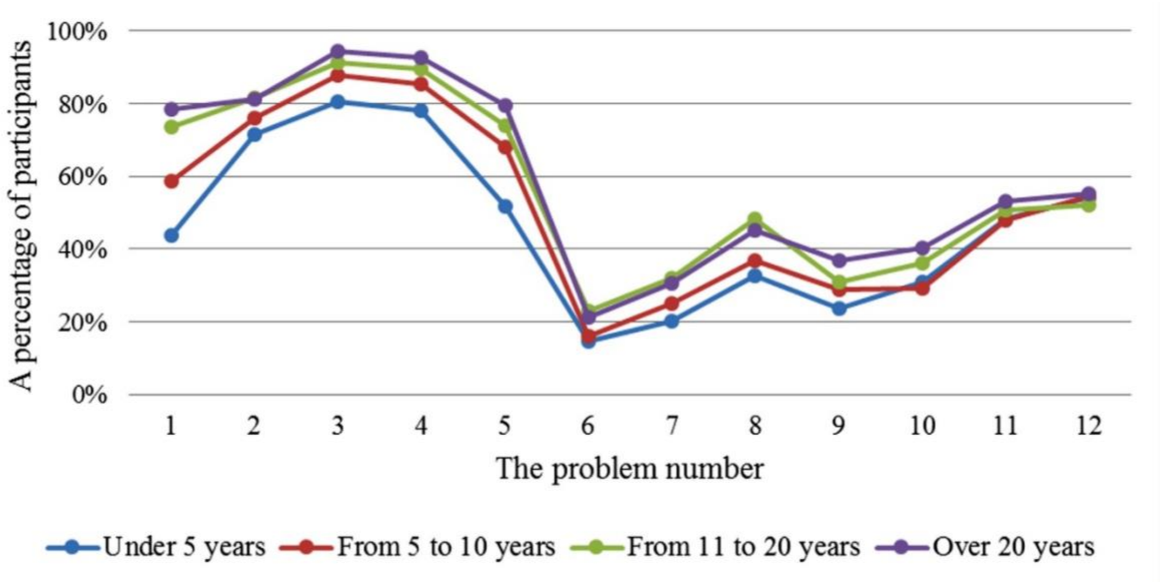


**Figure 6.** The results of the diagnostic test (Part 1) related to the participants' teacher expertise

Figure 7 demonstrates how a percentage of participants, who correctly solved three advanced teaching problems provided in Part 2, correlates with the total score gained. The median score was 8 points out of 15.

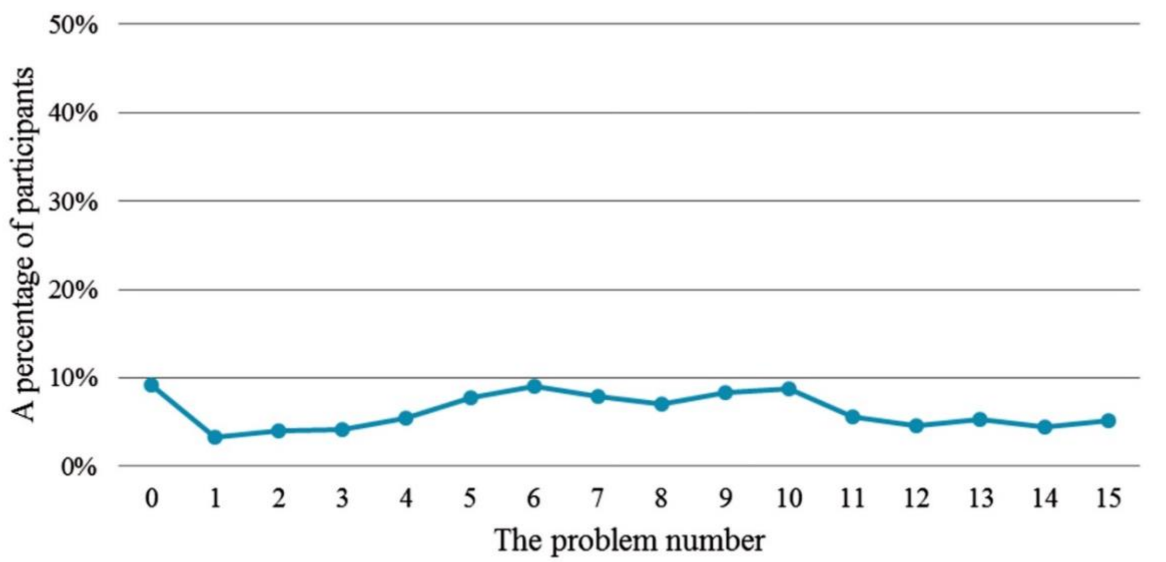


**Figure 7.** The results of the correct problem solution in the diagnostic test (Part 2)

Figure 8 reveals how many points the participants received for solving each advanced teaching problem in accordance with their percentage.

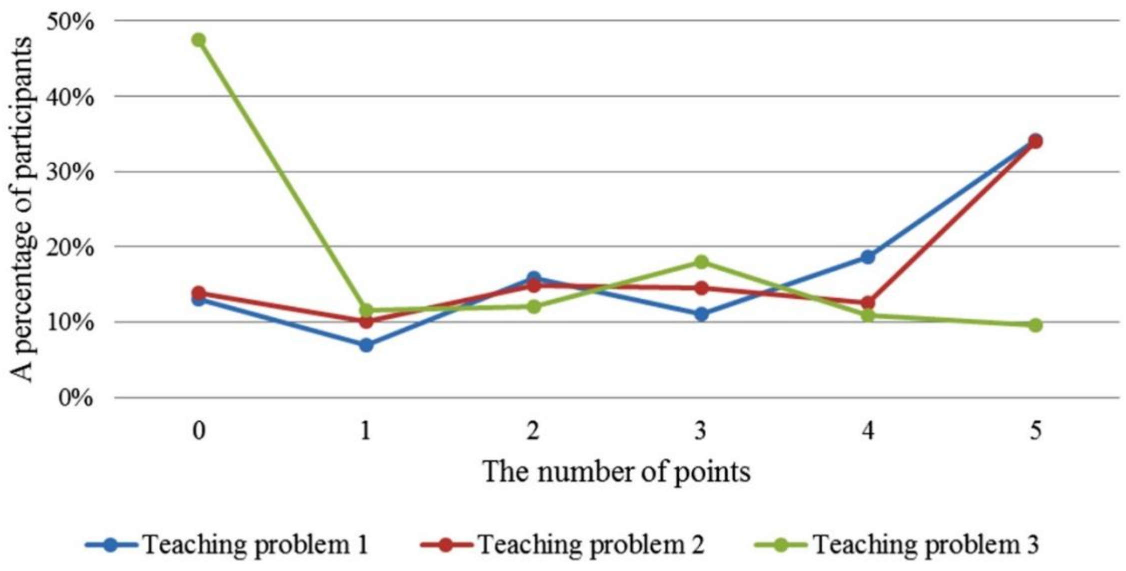


**Figure 8.** The total results of the diagnostic test (Part 2)

The results of solving teaching problem 1 correctly highlight some difficulties. Firstly, a half of the participants (49%) failed to correctly solve the advanced problem. Secondly, less than a half (45%) were able to compose a sequence of questions and tasks for the students, who sought to find a solution to this problem. The results of solving teaching problem 2 showed that 14% participants were able to identify all the errors and their causes, but could not explain how to organise their students' work to eliminate them. 44.7% of them were able to partially or fully propose a teaching technique to eliminate them. The obtained results relevant to teaching problem 3 demonstrated a low percentage of participants (19.6%) who successfully solved it. They could not correctly evaluate their students' work on solving a mathematical problem.

In Figure 9 we can see that that the participants, who scored higher points for solving the teaching problems in Part 2, showed higher results in Part 1 of the diagnostic test. Still, these participants (about 6%) scored less than 6 points, and only about 16% acquired the basic level.

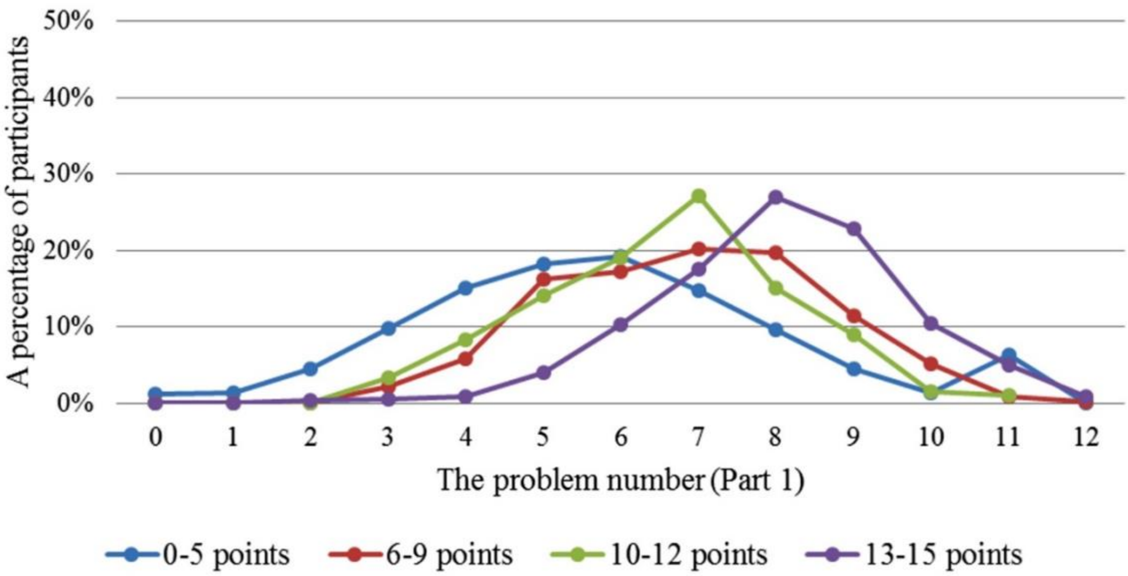


**Figure 9.** The total results of the diagnostic test (Parts 1 and 2)

In Figure 10, there is a correlation between the median and maximum points gained in accordance with the assessment criteria of the professional problem in the diagnostic test (Part 3).

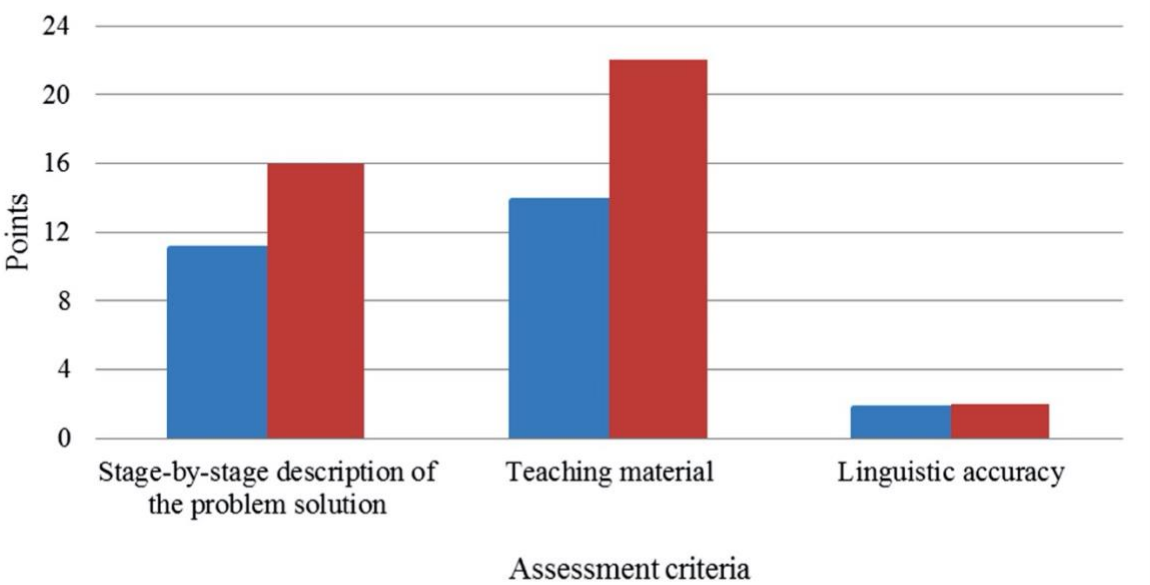


**Figure 10.** The median results in the diagnostic test (Part 3)

Evidently, the participants had difficulties in describing the solution to the professional problem stage by stage. Only 14% mathematics teachers managed to gain the maximum total score of 40 points, which indicates their success in completing Part 3. 19.6% teachers failed to receive the minimum total score of 21.

Figure 11 correlates the median with maximum points that the mathematics teachers gained when conducting their video lessons. The median score proved to be 25.9, out of 38 maximum points.

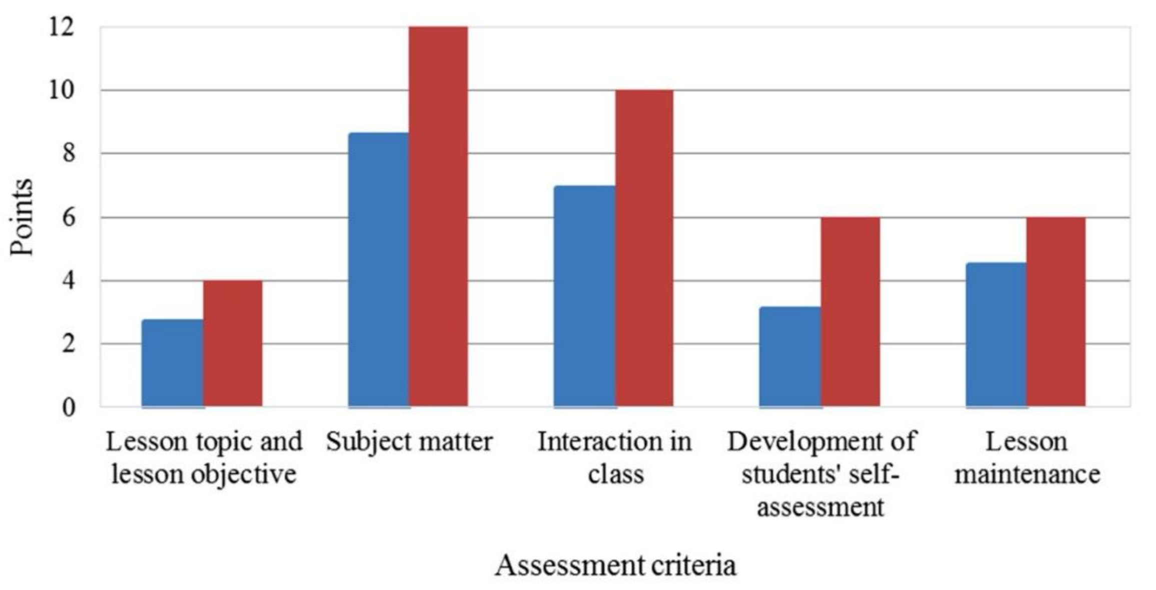


**Figure 11.** The median results of conducting video lessons

The total results of the diagnostic test and conducting video lessons indicated that 23% mathematics teachers did not pass the basic level of professional competence. 24% participants showed level I of professional competence, 44% – level II, 9% – level III.

The results of the diagnostic test confirm that the mathematics teachers' basic level of subject matter competency grows with the teacher category and teacher expertise. However, the correct solution to basic teaching problems, which aimed to assess the basic level of teaching competency, remains insufficiently high even among the mathematics teachers with a higher teacher category and longer teacher expertise. In addition, the success in solving some problems does not always depend on the teacher category and teacher expertise.

Some difficulties in solving advanced teaching problems suggest that not enough attention is paid to organisation of school students' work on mathematical problems. In fact, this work is an essential part in recognising students' knowledge and skills to solve a mathematical problem, which are known as students' mathematical literacy skills (Prabawati, Herman, & Turmudi, 2019). In this regard, pedagogic efficacy can contribute to students' achievement in performing mathematical operations (Fung et al., 2017). Additionally, not every teacher was able to explain how to successfully organise students' work to eliminate errors in solving mathematical problems. Such work may involve tasks to help students understand mathematical concepts in depth, rather than memorise formulae (Halimah, Subanji, & Septi Nur Afifah, 2019). Another drawback was the teachers' inabilities to effectively assess students' solutions. For instance, they can find wrong solutions to be correct or may not be aware of errors in solutions (Mokh, Othman, & Shahbari, 2019). In this case, comparative judgement may be advantageous (Jones & Inglis, 2015).

The analysis of the presented solutions to the professional problem enabled to make some inferences. In particular, the majority of mathematics teachers exhibited a fairly high willingness to work with the subject matter. It can become the basis of implementing motivation as the main condition to improve the quality of teaching mathematics (Timerbaeva, Fazleeva, & Shakirova, 2019). Moreover, the majority of teachers pinpointed significance of the professional problem, rather than referred to the context where the situation of professional activity could be found.

Nevertheless, some teachers misunderstood the real-life professional context, which affected acquisition of practical competency. The participants' failures in solving professional problems are associated with their formal approach to the problem solution, without utilising their expertise. At times, the teachers could not fully grasp the content of the professional problem so their answer lacked a well-elaborated description of the problem solution. In addition, the teachers failed to follow the instructions to solve the professional problem consistently and accurately, or they demonstrated solutions to several problems instead of one. Moreover, the use of Internet sources to provide the problem solution or presumably copying other teachers' answers resulted in similar descriptions of such solutions. These are the reasons why the mathematics' teachers were not good at implementing their professional competence.

The results of conducting video lessons demonstrate an absence of providing the lesson's specific objectives as a common problem that the teachers faced. There are some striking differences between the median points, which are significantly lower especially for developing students' self-assessment, and maximum ones. The differences illustrate the teachers dominating in class, rather than creating favourable conditions to encourage their students' independent work.

The total results pinpointed some professional difficulties arising in solving teaching and professional problems. They resulted from the teachers' inabilities to solve practical tasks of constructing the educational process. In particular, they found it difficult to organise their students' work on mathematical problems, search for their solutions, and evaluate their students' activities. In classes the teachers were not capable enough of implementing a study programme in mathematics since some of them neglected real-life conditions of professional activity to support student progress.

It is important to eliminate barriers to teaching competency acquisition so formative assessment can assist. Alongside with professional problems, performance measures can help assess practical competency (Howell, Stone, & Kane, 2019). The measures are associated with students' abilities to perform in a variety of assessment tasks (Poh, Muthoosamy, Lai, & Hoe, 2015).

The current research makes a significant contribution to the training of mathematics teachers due to the horizontal and vertical structure of professional competence. The interrelated structural components (subject matter, teaching, and practical competencies) reveal the horizontal structure, whereas levels I, II, III refer to the vertical one. Therefore, we developed tools to assess each component both vertically and horizontally. In particular, we elaborated teaching and professional problems that allow identify in gaps in the professional training of mathematics teachers. They enable to design tailor-made retraining courses aimed at developing mathematics teachers' professional competence.

## CONCLUSION

The article describes the assessment system to identify gaps in mathematics school teachers' professional competence and reveals the results of its implementation, which enable to design tailor-made retraining courses. The research findings show that the mathematics teachers with a higher teacher category and longer teacher expertise succeeded in conducting a video lesson as well as passed the basic level of the diagnostic test and obtained higher scores. The findings prove that the provided assessment system corresponds to the aim of the research.

We assessed the mathematics teachers' level of professional competence after they had completed a three-part diagnostic test and conducted video lessons. These activities proved to be adequate to real-life situations of professional activity since mathematics teachers regularly deal with mathematical, teaching, and professional problems in mathematics school classes.

The teachers did not find mathematical problems difficult to solve, unlike basic and advanced teaching problems. Advanced teaching problems required knowledge about which teaching techniques to use in order to help students solve mathematical problems and eliminate any possible errors they could make. Some mathematical teachers failed to use the techniques properly and could not cope with the effective assessment of students' solutions to mathematical problems. On the other hand, professional problems were the most difficult to solve due to the fact that most teachers did not consider real-life conditions of professional activity to support students' progress in solving mathematical problems.

The total scores of the diagnostic test and conducting video lessons were attributed to different levels of professional competence: 24% participants acquired level I, 44% – level II, 9% – level III. 23% did not pass the professional level assessment. These results explain the need to offer a tailor-made professional retraining programme for each mathematics teacher, which meets their needs. To master solving mathematical, teaching, or professional problems in tailor-made retraining courses is likely to improve the professional competence level.

A minor focus on video lessons serves as a limitation. It is planned to describe in detail peculiarities of the lesson topic and lesson objective, subject matter, interaction in class, development of students' self-assessment, and lesson maintenance as they relate to different teacher categories and teacher expertise. In addition, the analysis of video lessons may show how conducting lessons contributes to acquisition of the mathematics teachers' professional competence.

When analyzing the results, we considered only the teacher category and teacher expertise, but ignored other important aspects of the mathematics teachers' professional activity. This major limitation can be mitigated if we analyze some other aspects. They are the teachers' place of residence, the number of students at school, experience of teaching mathematics outside the school, experience in organizing teaching, the volume of the teaching load, experience in preparing students for the All-Russian Olympiads, methods used to monitor learning outcomes, participation in extracurricular activities, applying information communication technologies, use of teaching technologies.

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