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ON FIXED POINT THEOREMS IN FUZZY METRIC SPACES USING A CONTROL FUNCTION

C. T. AAGE[1*] AND J. N. SALUNKE[2]

Abstract. In this paper, we generalize Fuzzy Banach contraction theorem established by V. Gregori and A. Sapena [Fuzzy Sets and Systems 125 (2002) 245-252] using notion of altering distance which was initiated by Khan et al. [Bull. Austral. Math. Soc., 30(1984), 1-9] in metric spaces.

1. Introduction and preliminaries

Fuzzy metric space is closely generalization of generalized Menger space. Kramosil and Michalek [19] introduced fuzzy metric space, George and Veermani [11] modified the notion of fuzzy metric spaces with the help of continuous t-norms. George and Veeramani[11] imposed some stronger conditions on the fuzzy metric space in order to obtain a Hausdorff topology. In [8], V. Gregori, A. Sapena proved that the topology induced by a fuzzy metric space in George and Veeramani's sense is actually metrizable. The aim of this paper is to generalize the Banach fixed-point theorem to (fuzzy) contractive mappings on complete fuzzy metric spaces in George and Veeraman sense using concept of alternating distance.

Definition 1.1. (Schweizer and Sklar [26]. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $([0,1,*)$ is a topological monoid with unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $(a, b, c, d \in [0,1])$.

Definition 1.2. (Kramosil and Michalek [19]). The 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:
$(i) M(x,y,0) = 0$,
$(ii) M(x,y,t) = 1$ for all $t > 0$ iff $x = y$,
$(iii) M(x,y,t) = M(y,x,t)$,
$(iv) M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,
$(v) M(x,y,*) : [0,\infty) \to [0,1]$ is left-continuous, $x, y, z \in X$ and $t, s > 0$.

To obtain a Hausdorff topology on the fuzzy metric space, the authors gave the following definitions in [11].

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*: Corresponding author.

Definition 1.3. (George and Veeramani [11]). The 3- tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:
$(i) M(x, y, t) > 0$,
$(ii) M(x, y, t) = 1$ iff $x = y$,
$(iii) M(x, y, t) = M(y, x, t)$,
$(iv) M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(v) $M(x, y, *) : (0, \infty) \to [0, 1]$ is continuous, $x, y, z \in X$ and $t, s > 0$.

Definition 1.4. (George and Veeramani [11]). Let $(X, M, *)$ be a fuzzy metric space. The open ball $B(x, r, t)$ for $t > 0$ with centre $x \in X$ and radius $r, 0 < r < 1$, is defined as $B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$. The family $\{B(x, r, t) : x \in X; 0 < r < 1, t > 0\}$ is a neighborhood's system for a Hausdorff topology on X, that we call induced by the fuzzy metric M.

Definition 1.5. (George and Veeramani [11]). In a metric space (X, d) the 3-tuple $(X, M_d, *)$ where $M_d(x, y, t) = t/t + d(x, y)$ and $a * b = ab$, is a fuzzy metric space. This M_d is called the standard fuzzy metric induced by d. The topologies induced by d.

The topologies generated by the standard fuzzy metric and the corresponding metric are the same.

Lemma 1.6. *$M(x, y, *)$ is nondecreasing for all $x, y \in X$.*

Remark 1.7. In a fuzzy metric space $(X, M, *)$, for any $r \in (0, 1)$ we can find an $s \in (0, 1)$ such that $s * s > r$.

Definition 1.8. (George and Veeramani [11]). A sequence (x_n) in a fuzzy metric space $(X, M, *)$ is a Cauchy sequence iff for each $\epsilon \in (0, 1)$ and each $t > 0$ there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$, for all $n, m \in n_0$.

A fuzzy metric space in which every Cauchy sequence is convergent is called a complete fuzzy metric space.

Theorem 1.9. *(George and Veeramani [11]). A sequence (x_n) in a fuzzy metric space $(X, M, *)$ converges to x if and only if $M(x_n, x, t) \to 1$ as $n \to \infty$.*

Definition 1.10. G-Cauchy Sequence [11, 12]. A sequence (x_n) in a fuzzy metric space $(X, M, *)$ is called a G-Cauchy if $\lim_{n\to\infty} M(x_n, x_{n+m}, t) = 1$ for each $m \in N$ and $t > 0$.

We call a fuzzy metric space $(X, M, *)$ G-complete if every G-Cauchy sequence in X is convergent. It follows immediately that a Cauchy sequence is a G-Cauchy sequence. The converse is not always true. This has been established by an example in [29].

The following concept of convergence was introduced in fuzzy metric spaces by Mihet[22].

Definition 1.11. Point Convergence or p-convergence[22]. Let $(X, M, *)$ be a fuzzy metric space. A sequence (x_n) in X is said to be point convergent or p-convergent to $x \in X$ if there exists $t > 0$ such that $\lim_{n\to\infty} M(x_n, x, t) = 1$. We write $x_n \to_p x$ and call x as the p-limit of (x_n).

The following lemma was proved in [22].

Lemma 1.12. [22] *In a fuzzy metric space* $(X, M, *)$ *with the condition* $M(x, y, t) \neq 1$ *for all* $t > 0$ *whenever* $x \neq y$*, p-limit of a point convergent sequence is unique.*

It has been established in [22] that there exist sequences which are p-convergent but not convergent .

V. Gregori and A. Sapena [10] established fixed point theorem for following types fuzzy contractive mappings.

Definition 1.13. Let $(X, M, *)$ be a fuzzy metric space. We will say the mapping $f : X \to X$ is fuzzy contractive if there exists $k \in (0, 1)$ such that

$$\frac{1}{M(f(x), f(y), t)} - 1 \leq k\big(\frac{1}{M(x, y, t)} - 1\big), \tag{1.1}$$

for each $x, y \in X$ and $t > 0$. (k is called the contractive constant of f.)

Definition 1.14. Altering distance function [18] An altering distance function is a function $\psi : [0, \infty) \to [0, \infty)$
(i) which is monotone increasing and continuous and
(ii) $\psi(t) = 0$ if and only if $t = 0$.

In 1984 Khan et al proved the following result.

Theorem 1.15. [18] *Let* (X, d) *be a complete metric space,* ψ *be an altering distance function and let* $f : X \to X$ *be a self mapping which satisfies the following inequality*

$$\psi(d(fx, fy)) \leq c\psi(d(x, y)$$

for all $x, y \in X$ *and for some* $0 < c < 1$*. Then* f *has a unique fixed point.*

Definition 1.16. A function $\phi : R \to R^+$ is said to satisfy the condition $*$ if it satisfies the following conditions
$(i)\phi(t) = 0$ if and only if $t = 0$,
$(ii)\phi(t)$ is increasing and $\phi(t) \to \infty$ as $t \to \infty$,
$(iii)\phi$ is left continuous in $(0, \infty)$,
$(iv)\phi$ is continuous at 0.

In this connection Binayak S. Choudhury et al have been studied the fixed point results in Menger Space, details see in [3, 4, 6, 7]. Recently C. T. Aage and B. S. Choudhury proved following result.

Theorem 1.17. *Let* (X, M, T) *be a fuzzy metric space in the sense of George and Veeramani and* $sup_{0 \leq a < 1} T(a, a) = 1$ *and the self mapping* $f : X \to X$ *satisfy*

$$M(fx, fy, \phi(t) \geq M(x, y, \big(\phi(\frac{t}{c})\big),$$

where $0 < c < 1$*,* $x, y \in X$ *and* $t > 0$ *and* ϕ *satisfies* $*$ *condition. Suppose that for some* $x_0 \in X$ *the sequence of* $\{f^n x_0\}$ *has a p-convergent subsequence. Then* f *has a unique fixed point.*

In this paper we generalize contractive condition (1.1) using alternating distance and establish fixed point theorem in G-complete fuzzy metric space in the sense of George and Veeramani.

2. Main Results

Theorem 2.1. *Let $(X, F, *)$ be a G-complete fuzzy metric space and $f : M \to M$ be a self mapping satisfying the following inequality*

$$\frac{1}{M\big(fx, fy, \phi(ct)\big)} - 1 \leq \psi\Big(\frac{1}{M(x, y, \phi(t))} - 1\Big) \tag{2.1}$$

where $x, y \in M$, $0 < c < 1$, ϕ is a function which satisfies Definition (1.16) and $\psi : [0, 1) \to [0, 1)$ is such that ψ is continuous, $\psi(0) = 0$ and $\psi^n(a_n) \to 0$, whenever an $a_n \to 0$ as $n \to \infty$ and $t > 0$ is such that $M(x, y, \phi(t)) > 0$. Then f has a unique fixed point.

Proof. Let $x_0 \in X$. Define the sequence $\{x_n\}$ as follows $fx_n = x_{n+1}$.

We assume that $x_{n+1} \neq x_n$ for all $n \in N$, otherwise f has a fixed point. By virtue of the properties of ϕ, we can find $t > 0$ such that $M(x_0, x_1, \phi(t)) > 0$. Then by an application of (2.1) we have

$$\frac{1}{M(x_1, x_2, \phi(ct))} - 1 = \frac{1}{M(fx_0, fx_1, \phi(ct))} - 1 \leq \psi\Big(\frac{1}{M(x_0, x_1, \phi(t))} - 1\Big) \tag{2.2}$$

Again $M(x_0, x_1, \phi(t)) > 0$ implies $M(x_0, x_1, \phi(\frac{t}{c})) > 0$. Then again by an application of (2.1) we have

$$\frac{1}{M(x_1, x_2, \phi(t))} - 1 = \frac{1}{M(fx_0, fx_1, \phi(t))} - 1 \leq \psi\Big(\frac{1}{M(x_0, x_1, \phi(\frac{t}{c}))} - 1\Big). \tag{2.3}$$

Repeating the above procedure successively n times we obtain

$$\frac{1}{M(x_n, x_{n+1}, \phi(t))} - 1 \leq \psi^{n-1}\Big(\frac{1}{M(x_0, x_1, \phi(\frac{t}{c^n}))} - 1\Big). \tag{2.4}$$

Again (2.2) implies that $M(x_1, x_2, \phi(ct)) > 0$.

Then following the above procedure we have

$$\frac{1}{M(x_n, x_{n+1}, \phi(ct))} - 1 \leq \psi^{n-1}\Big(\frac{1}{M(x_1, x_2, \phi(\frac{ct}{c^{n-1}}))} - 1\Big). \tag{2.5}$$

Repeating the above step r times, in general we have for $n > r$,

$$\frac{1}{M(x_n, x_{n+1}, \phi(c^r t))} - 1 \leq \psi^{n-r}\Big(\frac{1}{M(x_r, x_{r+1}, \phi(\frac{(c^r t)}{c^{n-r}}))} - 1\Big). \tag{2.6}$$

Since $\psi^n(a_n) \to 0$ whenever $a_n \to 0$, we have from (2.6), for all $r > 0$

$$M(x_n, x_{n+1}, \phi(c^r t)) \to \infty \text{ as } n \to \infty \tag{2.7}$$

Let $\epsilon > 0$ be given, then by virtue of the properties of ϕ we can find $r > 0$ such that $\phi(c^r t) < \epsilon$. It then follows from (2.7) that

$$M(x_n, x_{n+1}, \epsilon) \to 1 \text{ as } n \to \infty. \tag{2.8}$$

Again

$$M(x_n, x_{n+p}, \phi(\epsilon)) \geq \underbrace{M(x_n, x_{n+1}, \frac{\epsilon}{p}) * M(x_{n+1}, x_{n+2}, \frac{\epsilon}{p}) * \cdots * M(x_{n+p-1}, x_{n+p}, \frac{\epsilon}{p})}_{p-times}.$$

Taking $n \to \infty$ and using (2.8) we have for any integer p, $M(x_n, x_{n+p}, \epsilon) \to \infty$ as $n \to \infty$. Hence $\{x_n\}$ is a G-Cauchy sequence. As $(X, M, *)$ is G-complete, $\{x_n\}$ is convergent and hence $x_n \to z$ as $n \to \infty$ for some $z \in X$. Again

$$M(fz, z, \epsilon) \geq M(fz, x_{n+1}, \frac{\epsilon}{2}) * M(x_{n+1}, z, \frac{\epsilon}{2}). \tag{2.9}$$

Using the properties of ϕ-function, we can find a $t_2 > 0$, such that $\phi(t_2) < \frac{\epsilon}{2}$. Again $x_n \to z$ as $n \to \infty$. Hence there exists $N \in \mathbb{N}$ such that for all $n > N$,

$$M(x_n, z, \phi(t_2)) > 0.$$

Then we have for $n > N$,

$$\begin{aligned}\frac{1}{M(fz, x_{n+1}, \frac{\epsilon}{2})} - 1 &\leq \frac{1}{M(fz, fx_n, \phi(t_2))} - 1 \\ &\leq \psi\Big(\frac{1}{M(z, x_n, \phi(\frac{t_2}{c}))} - 1\Big).\end{aligned}$$

Letting $n \to \infty$, utilizing $\phi(0) = 0$ and continuity of ψ , we obtain

$$M(fz, x_{n+1}, \frac{\epsilon}{2}) \to \quad \text{as } n \to \infty. \tag{2.10}$$

Making $n \to \infty$ in (2.9), using (2.10), by continuity of ψ and the fact that $x_n \to z$ as $n \to \infty$ we have,

$$M(fz, z, \epsilon) = 1 \text{ for every } \epsilon > 0.$$

Hence $z = fz$. Next we establish the uniqueness of the fixed point. Let x and y be two fixed points of f. By the properties of ϕ there exists $s > 0$ such that$M(x, y, \phi(s)) > 0$. Then by an application of (2.1) we have

$$\frac{1}{M(x, y, \phi(cs))} - 1 = \frac{1}{M(fx, fy, \phi(cs))} - 1 \leq \psi\Big(\frac{1}{M(x, y, \phi(s))} - 1\Big). \tag{2.11}$$

Again $M(x, y, \phi(s)) > 0$ implies $M(x, y, \phi(\frac{s}{c})) > 0$. Then replacing s by $\frac{s}{c}$ in (12) we obtain

$$\frac{1}{M(x, y, \phi(s))} - 1 \leq \psi\Big(\frac{1}{M(x, y, \phi(\frac{s}{c}))} - 1\Big).$$

Repeating the above procedure n times we have

$$\frac{1}{M(x, y, \phi(s))} - 1 \leq \psi^n\Big(\frac{1}{M(x, y, \phi(\frac{s}{c^n}))} - 1\Big) \to 0 \text{ as } n \to \infty \text{ (by the properties of } \psi \text{).}$$

This shows that $M(x, y, \phi(s)) = 1$ for all $s > 0$.

Again from (2.11) it follows that $M(x, y, \phi(cs)) > 0$. Repeating the same argument with s replaced by cs we have $M(x, y, \phi(cs)) = 1$ and in general we have, $M(x, y, \phi(c^n s)) = 1$ for all $n \in N \cup \{0\}$. By the properties of ϕ for any given $\epsilon > 0$ there exists $r \in N \cup \{0\}$ such that $\phi(c^r s) < \epsilon$, so that from the above we have $M(x, y, \epsilon) = 1$ for all $\epsilon > 0$, that is x = y. This establishes the uniqueness of the fixed point. □

Theorem 2.2. *Let $(X, M, *)$ be a fuzzy metric space with the condition $M(x, y, t) \neq 1$ for all $t > 0$ whenever $x \neq y$, and $f : X \to X$ be a self mapping which satisfies the inequality (2.1) in the statement of Theorem 2.1. If for some $x_0 \in X$, the sequence $\{x_n\}$ given by $x_{n+1} = fx_n, n \in N \cup \{0\}$ has a p-convergent subsequence then f has a unique fixed point.*

Proof. Let $\{x_{n_k}\}$ be a subsequence of $\{x_n\}$ which is p-convergent to $x \in X$. Consequently there exists $s > 0$ such that

$$lim_{k\to\infty} M(x_{n_k}, x, s) = 1. \tag{2.12}$$

Further, following (2.8) we have $\lim_{i\to\infty} M(x_{n_i}, x_{n_{i+1}}, s) = 1$. Therefore given $\delta > 0$ there exist $k_1, k_2 \in N \cup \{0\}$ such that for all $k' > k_1$ and $k'' > k_2$ we have,

$$M(x_{n_{k'}}, x, s) > 1 - \delta$$
$$\text{and } M(x_{n_{k''}}, x_{n_{k''}+1}, s) > 1 - \delta.$$

Taking $k_0 = \max\{k', k''\}$, we obtain that for all $j > k_0$,

$$M(x_{n_j}, x, s) > 1 - \delta \tag{2.13}$$

and

$$M(x_{n_j}, x_{n_{j+1}}, s) > 1 - \delta. \tag{2.14}$$

So we obtain

$$\begin{aligned} M(x_{n_{j+1}}, x, 2s) &\geq M(x_{n_{j+1}}, x_{n_j}, s) * M(x_{n_j}, x, s) \\ &\geq (1-\delta) * (1-\delta) \quad [\text{ by (2.13) and (2.14)}]. \end{aligned}$$

Let $\epsilon > 0$ be arbitrary. As $(1 * 1) = 1$ and $*$ is a continuous t-norm, we can find $\delta > 0$ such that $(1-\delta) * (1-\delta) > 1 - \epsilon$. It follows from (2.13) and (2.14) that for given $\epsilon > 0$ it is possible to find a positive integer k_0 such that for all $j > k_0$, $M(x_{n_{j+1}}, x, 2s) > 1 - \epsilon$. Hence $\lim_{j\to\infty} M(x_{n_j+1}, x, 2s) = 1$, that is

$$x_{n_j+1} \to_p x. \tag{2.15}$$

Again, following the properties of ϕ-function we can find $t > 0$ such that

$$\phi(t) \leq 2s < \phi(\frac{t}{c}).$$

Also from (2.15) it is possible to find a positive integer N_1 such that for all $i > N_1$

$$M(x_{n_i+1}, x, 2s) > 0.$$

Consequently for all $i > N_1$,

$$\begin{aligned} \frac{1}{M(x_{n_i+1}, fx, 2s)} - 1 &\leq \frac{1}{M(fx, fx_{n_i}, \phi(t))} - 1 \\ &\leq \psi\Big(\frac{1}{M(x, x_{n_i}, \phi(\frac{t}{c}))} - 1\Big) \leq \psi\Big(\frac{1}{M(x, x_{n_i}, 2s)} - 1\Big). \end{aligned}$$

Taking $i \to \infty$ in the above inequality, and using (2.12) and the continuity of ψ we obtain $M(x_{n_i+1}, fx, 2s) \to 1$ as $i \to \infty$, that is,

$$x_{n_i+1} \to_p fx \text{ as } i \to \infty. \tag{2.16}$$

Using (2.15), (2.16) we have $fx = x$ which proves the existence of the fixed point. The uniqueness of the fixed point follows as in the proof of Theorem 2.1.

Example 2.3. Let $(X, M, *)$ be a complete fuzzy metric space where $X = \{x_1, x_2, x_3\}$, $a * b = \min\{a, b\}$ and $M(x, y, t)$ be defined as

$$M(x_1, x_2, t) = M(x_2, x_1, t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 0.9, & \text{if } 0 < t \leq 3, \\ 1, & t > 3. \end{cases}$$

$$M(x_1, x_3, t) = M(x_3, x_1, t) = M(x_2, x_3, t) = M(x_3, x_2, t)$$
$$= \begin{cases} 0, & \text{if } t \leq 0, \\ 0.7, & \text{if } 0 < t < 6, \\ 1, & \text{if } t \geq 6. \end{cases}$$

$f : X \to X$ is given by $fx_1 = fx_2 = x_2$ and $fx_3 = x_1$. If we take $\phi(t) = t_2, \psi(t) = 2t^3$ and $c = 0.8$, then it may be seen that f satisfies the inequality (2.1) and x_2 is the unique fixed point of f.

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[1] School of Mathematical Sciences, North Maharashtra University, Jalgaon. P.O. 425001, Fax-02572257406, India.

E-mail address: caage17@gmail.com

[2] School of Mathematical Sciences, North Maharashtra University, Jalgaon. P.O. 425001, Fax-02572257406, India.

E-mail address: drjnsalunke@gmail.com