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The Validity of Mathematical Solutions for Determining Friction From the Ring Compression Test

The ring test has previously been experimentally calibrated for friction studies on the assumption of constant coefficient of friction under metal-working conditions. This investigation has demonstrated that the same experimental results may be used to calibrate the ring test on the assumption of constant interface friction factor. Use of available mathematical solutions, based on the concept of a constant interface friction factor, provides a possible means for the calibration of different initial ring geometries by computer solution. Excellent correlation has been shown between the shape of calculated curves and experimental ring test results on a wide variety of materials. However, the actual values of m obtained by using the theory to analyze experimentally determined shape changes appear to be somewhat in error to a degree depending on the initial specimen geometry and the general friction level under which it is deformed. This is due essentially to the assumption used in the theoretical treatment, that the surface frictional stresses are transmitted uniformly throughout the specimen thickness, not being generally valid in the practical situation except with very thin specimens.

I Introduction

In metal deformation processes, frictional forces are generated at the interface between the tools and deforming materials by virtue of the workpiece surface extension. These frictional forces have the following modifying effects:

(*a*) The total deformation loads are increased.
(*b*) The internal structure and surface characteristics (surface finish and surface defects) of the product are influenced.
(*c*) Wear is produced on the tooling material, thus reducing its useful life.
(*d*) Dimensional variations are produced in the processed material.

Because of these effects, friction is considered to be a major variable in metalworking operations and must be adequately controlled to optimize processing procedures for economically producing material with the desired geometry and internal structure. For effective friction control, quantitative data on the effects of lubrication and other processing variables, such as temperature, speed and pressure, are essential.

A number of studies have already been made in an attempt to obtain quantitative data on friction in metal processing by using the actual metalworking operation or by using simulative laboratory tests. Use of the particular metalworking process for these investigations has the disadvantage that it is difficult to separate the force necessary to overcome friction from the force necessary to give the required deformation, and more difficult to control the secondary process variables. Laboratory tests may furnish valuable measurements of frictional behavior under controlled conditions provided that these tests are capable of simulating process conditions such as temperature, deformation speed and deformation pressure. Of the many laboratory tests utilized for friction studies the ring test technique originated by Kunogi [1][1] and further developed by Male and Cockcroft [2] has, in the opinion of the present authors, the greatest capability for quantitatively measuring friction under normal processing conditions.

The ring test technique involves a simple forging operation performed on a flat ring-shaped specimen; the change in diameter produced by a given amount of compression in the thickness direction is related to the interfacial friction condition, as shown in Fig. 1. If friction were equal to zero, the ring would deform in the same way as a solid disk, with each element flowing radially outwards at a rate proportional to its distance from the center. With a small but finite interfacial friction force, outward flow takes place at a lower rate and, for the same degree of compression, the outside diameter is smaller than with zero friction. If the frictional force exceeds a critical value, it is energetically

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[1] Numbers in brackets designate References at end of paper.

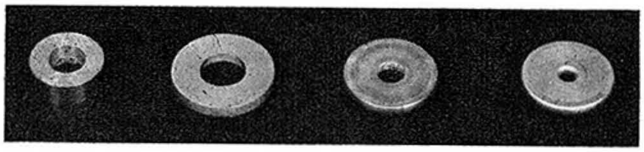

Fig. 1 Typical specimens deformed under different friction conditions: (a) undeformed; (b)–(d) deformed 50 percent: (b) low friction; (c) medium friction; (d) high friction

favorable for only part of the ring to flow outwards and for the remainder to flow inwards toward the center; thus the outside diameter after compression is still further reduced.

Measurement of the final internal diameter of compressed rings provides a particular sensitive means for studying interface friction since the internal diameter increases if the friction is small and decreases if the friction is large.

The ring test has an advantage when applied to the study of friction at elevated temperatures and/or high speeds: no direct measurement of force is required and no yield strength values of the deforming material are needed, hence the major difficulties of evaluation of compression tests under these conditions are eliminated. Correlation of changes in internal diameter with numerical values of friction can be obtained either by independent calibration or by the application of available theoretical analysis. Before a satisfactory mathematical solution for the compression of a ring was available, a pioneering independent calibration was made by experimentation [3]. Subsequent theoretical analyses [4, 5] have made possible more accurate and less laborious calibration of the ring test by mathematical computation.

The object of this report is to describe the mathematical calibration of the ring test and to compare this calibration with experimental results.

II Theory

The first satisfactory analysis of the compression of a flat ring was made by Avitzur [4] through an optimum upper bound mathematical solution and later verified by Hawkyard and Johnson [5] using a stress analysis approach. Both solutions are based on the assumptions that (*a*) there is no nonuniform distortion of cylindrical elements due to frictional constraint i.e., no barrelling; (*b*) the ring material obeys Mises' stress-strain rate laws, implying no strain hardening effect, no elastic deformation, and no volumetric change; and (*c*) a constant fricton factor, m, for a given die and material under constant surface and temperature conditions such that the interfacial shear stress, τ, is given by

$$\tau = m\,\frac{\sigma_0}{\sqrt{3}}$$

where σ_0 = basic yield stress of the ring material. The assumption of constant m and constant σ_0 automatically means that τ is constant. Certain investigations [6, 7, 8] on friction indicate that, for metalworking conditions, the assumption of a constant interfacial friction stress may be reasonably justified. When strain hardening of the ring material occurs [i.e., assumption (*b*) is violated] it is necessary that interpretation of the ring test results with the mathematical solution should be somewhat cautious.

The theoretical solutions of Avitzur and Hawkyard and Johnson both yield the following mathematical relationships for a ring specimen under compression (Fig. 2), where R_n is the radius of the metal flow divide within the ring (sometimes referred to as the neutral or "no-slip" radius):

1 When $R_n \leqslant R_i$

$$\left(\frac{R_n}{R_0}\right)^2 = \frac{\sqrt{3}}{2}\,\frac{\left[1+\left(\frac{R_i}{R_0}\right)^4\right]X^2}{\left[X(X-1)\left(1-\frac{R_i^4}{R_0^4}X\right)\right]^{1/2}} \qquad (1)$$

where

$$X = \left\{\frac{R_0}{R_i}\exp\left[-m\,\frac{R_0}{T}\left(1-\frac{R_i}{R_0}\right)\right]\right\}^2$$

2 When $R_i \leqslant R_n \leqslant R_0$

$$m\,\frac{R_0}{T} = \frac{1}{2\left(1+\frac{R_i}{R_0}-\frac{2Rn}{R_0}\right)} \times \ln\left[\left(\frac{R_0}{R_i}\right)^2 \frac{1+\sqrt{1+3\left(\frac{R_i}{R_0}\right)^4\left(\frac{R_0}{R_n}\right)^4}}{1+\sqrt{1+3\left(\frac{R_0}{R_n}\right)^4}}\right] \qquad (2)$$

Equation (1) is valid when R_n lies between R_i and 0, and

$$\frac{mR_0}{T} \leqslant \frac{1}{2\left(1-\frac{R_i}{R_0}\right)}\ln\left[\frac{3\left(\frac{R_0}{R_i}\right)^2}{1+\sqrt{1+3\left(\frac{R_0}{R_i}\right)^4}}\right] \qquad (3)$$

Equation (2) is valid when R_n lies between $\frac{R_0+R_i}{2}$ and R_i, and

$$\frac{mR_0}{T} \geqslant \frac{1}{2\left(1-\frac{R_i}{R_0}\right)}\ln\left[\frac{3\left(\frac{R_0}{R_i}\right)^2}{1+\sqrt{1+3\left(\frac{R_0}{R_i}\right)^4}}\right] \qquad (4)$$

Neither the basic yield stress of the material, σ_0, nor the interfacial shear stress, τ, appear in the final equations in terms of absolute values, only as a ratio, m. The basic assumption in the analysis is that this ratio remains constant for the material and deformation conditions. If the analysis is carried out for a small increment of deformation, σ_0 and τ can be assumed to be approximately constant for this increment and the solution is valid. Thus, if the shear factor m is constant for the whole operation, it would appear justifiable to continue the mathematical analysis in a series of small deformation increments using the final ring geometry from one increment as the initial geometry for the subsequent increment and so on. As long as the ratio of the interfacial shear stress, τ, and the material flow stress, σ_0, remained constant it would not be of consequence if the ring material strain hardened during deformation provided that the increase in work hardening in any one single deformation increment could be neglected. The progressive increase in interfacial shear stress accompanying strain hardening would also be of no consequence provided that it could be assumed to be constant over the entire die/ring interface during any one deformation increment. Thus, it is possible that the analysis could be justifiably applied to real materials even though it was initially assumed that the material would behave according to the Mises' stress-strain rate laws provided that the assumption of a constant interfacial shear factor, m, is correct.

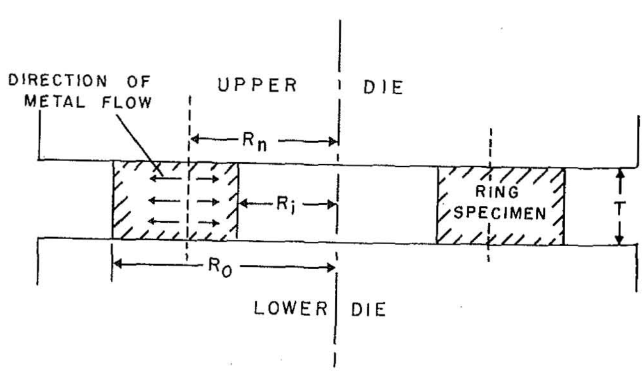

Fig. 2 Compression of flat ring-shaped specimen between flat dies

III Procedure

A Mathematical Calibration. Calibration curves relating the change in internal diameter of the ring, expressed as a percentage of the original internal diameter, to the percentage reduction in height on deformation for various values of the interfacial friction factor, m, were obtained for ring test specimens having the following starting geometries:

D_0	D_i	T
6	4.0	1
6	3.2	1
6	3.0	1
6	2.4	1
6	1.6	1

Mathematical solution of the relevant equations for this calibration was performed by writing a program in Fortran 4 and using an IBM Digital Computer 7044-7094 Direct Coupled System.

It is perhaps pertinent at this stage to mention that these solutions can be adjusted for application to rings of different heights, but of the same $D_0:D_1$ ratio, simply by multiplying the m value by the ratio of the heights to obtain the new m value. For example, with the standard ring ratio of 6:3:2, used by Male and Cockcroft, the m values for the 6:3:1 ring would have to be multiplied by 2; or with a ratio of 6:3:0.5, the m values would have to be multiplied by 0.5.

B Correlation with Experimental Results. Both published [2, 9] and unpublished [3] experimental results obtained by Male on rings of various geometries were used for comparison with the calibration curves obtained from the mathematical computations.

In addition, compression tests were conducted on commercially pure aluminum at 600 deg C and on mild steel at 750 deg C. The specimen sizes used were 3.0 in. OD × 1.5 in. ID × 0.5 in. height and 3.0 OD in. × 1.5 in. ID × 0.25 in. height for both materials. Preheating was carried out in an electric resistance furnace without atmosphere control, and deformation was carried out on a high speed 500 ton hydraulic forge press using degreased tooling.

IV Results

A Mathematical Calibration. For each of the five different starting ring geometries the computer solution provided values for the internal, external, and neutral radii and the percentage change in internal diameter for each increment of deformation at set values of the interfacial friction factor, m. These values are available separately, in a U. S. Air Force Materials Laboratory Technical Report [10] and are therefore not presented here in detail, with the exception of the curves given in Fig. 3 for an initial ring geometry of 6:3:2. The shape of this family of curves is typical of the curves determined for other ring geometries. For these other geometries, only the relevant theoretical curves are given in particular figures for comparison with the experimental results.

B Correlation of Mathematical Calibration With Experimental Results.
(a) Maximum or Sticking Friction (m = 1.0) for Standard Ring (6:3:2). In the early experimental calibration studies [2, 3], conditions which gave sticking friction were identified by looking for the presence or absence of minute surface scratches [11] on the specimen surface which had been in contact with the die. Male and Cockcroft standard geometry ring specimens of aluminum deformed at 600 deg C were completely devoid of these scratches on either the original surface or the new surface created by the deformation, thus showing that the friction must have been sufficiently high so that at no stage was there any relative movement at the aluminum/die interface. Surface scratches were observed on all specimens deformed at temperatures below 600 deg C.

The experimental values obtained for aluminum rings deformed at 600 deg C for this sticking friction condition are compared with the mathematically derived curve for maximum friction ($m \doteq 1.0$) in Fig. 4. The following additional metals were compressed without lubricant at the elevated temperature indicated and the results correlate well with the results obtained on the aluminum: copper at 400 deg C, magnesium at 400 deg C titanium at 500 deg C, mild steel at 750 deg C, and zinc at 350 deg C. It can be seen that the experimentally derived curve for maximum friction is somewhat higher than the theoretical maximum curve.

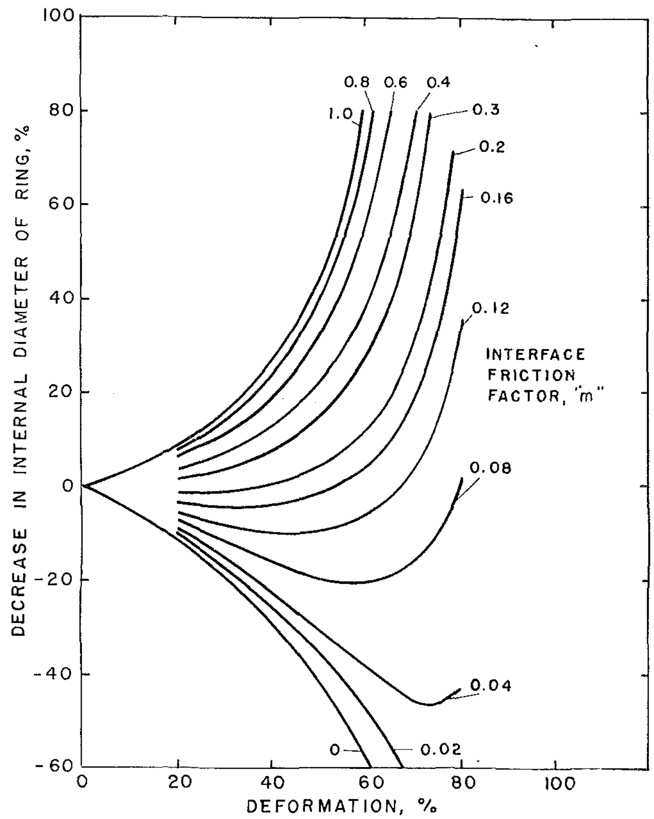

Fig. 3 Theoretical calibration curve for standard ring, 6:3:2

(b) Minimum or Zero Friction (m = 0) for Standard Ring (6:3:2). Experimentally, conditions approaching zero friction have been achieved in the following manner [2]: wax ring specimens of the standard geometry were warmed to within 2 deg C of the melting point and were compressed between polished steel dies which were a few degrees hotter.

Melting of an extremely thin surface layer of the wax specimens provided almost perfect interfacial lubrication. The calculated curve for zero friction (m = 0) and some of these experimental results are also given in Fig. 4. These results indicate that perfectly frictionless conditions were not achieved with this technique, but that variable friction conditions (m = 0.02 to m = 0.14) were experienced during testing. The theoretical curve tended to form a lower envelope for the experimental results.

(c) Other Friction Values for Standard Ring (6:3:2). The earlier experimental calibration of the ring test [2, 3] at intermediate friction levels involved the determination of the loads necessary to deform solid disk specimens (0.5 in. dia × 0.1 in.) by various amounts. These results were taken and values of the interfacial friction factor (m) were calculated using the analysis of Schroeder and Webster [12]. Yield stress values required for this analysis were obtained using the Polakowski technique [13] of compressing tall cylindrical specimens (height/diameter = 2), relubricating frequently during testing and remachining at intervals to remove any trace of barrelling. Values of m obtained in this way are shown in Fig. 5.

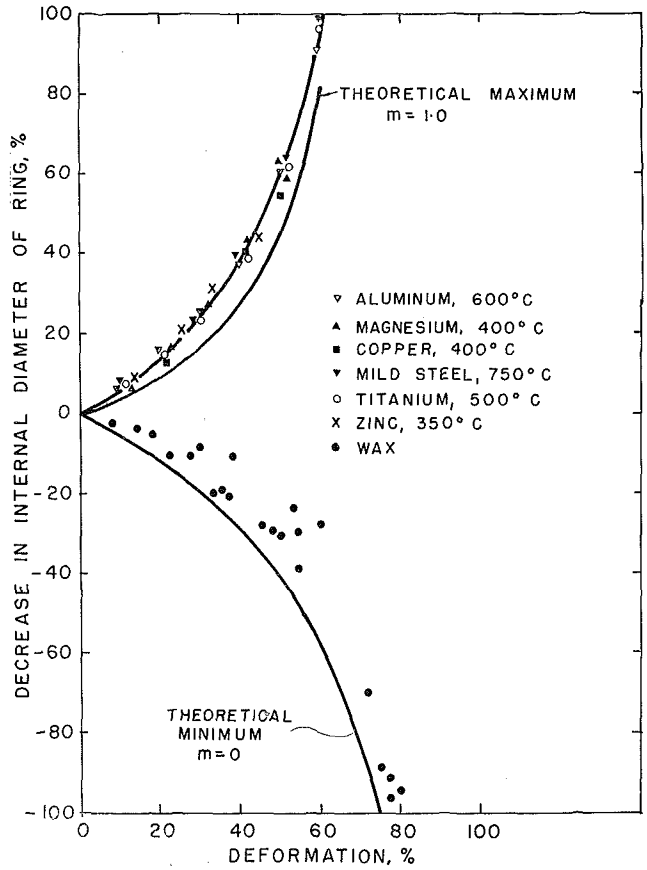

Fig. 4 Comparison of maximum and minimum theoretical and experimental curves

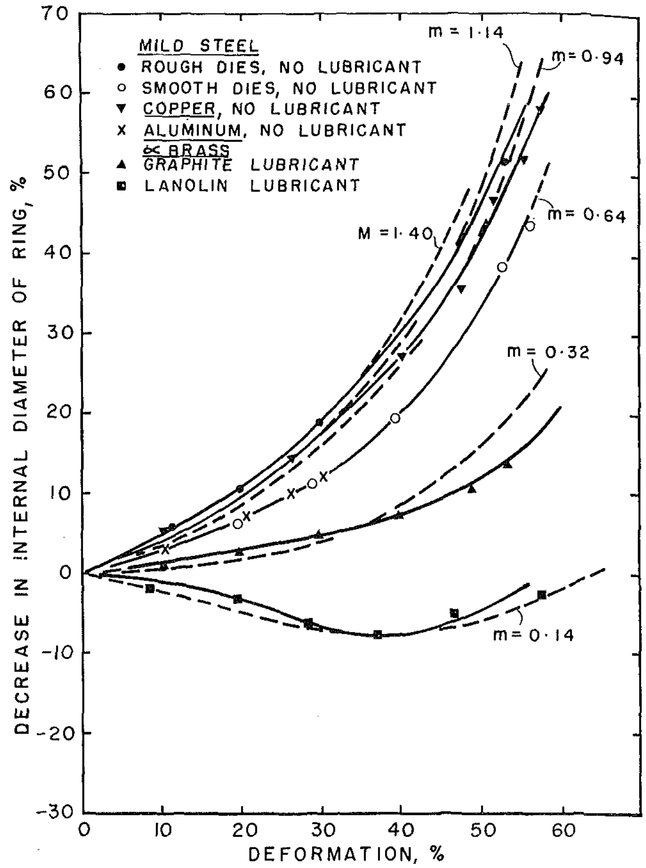

Fig. 6 Comparison of experimental and theoretical curves for materials and lubrication of Fig. 5

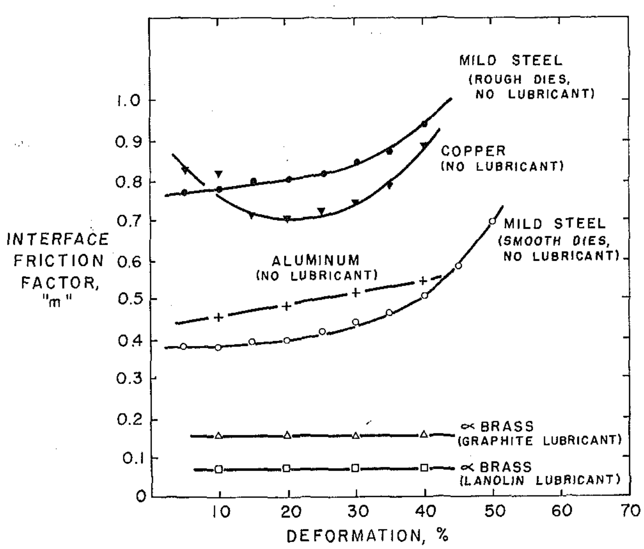

Fig. 5 Interface friction factor of several metals for various lubrication conditions at 20 deg C. Calculated from experimental data using the analysis of Schroeder and Webster

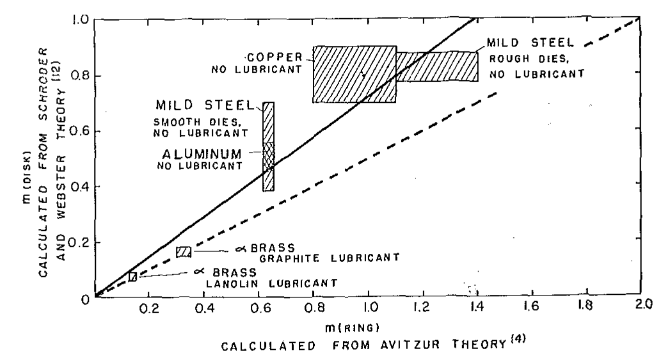

Fig. 7 Comparison of *m* values obtained from disk (Fig. 5) and ring (Fig. 6) compression tests carried out under identical friction conditions

Standard geometry ring test specimens were machined from the same material used for the solid disks and were deformed under identical conditions. Data for the percentage change in internal diameter as a function of amount of deformation in these tests are given in Fig. 6 together with the most appropriate theoretical curves. The m values given by these curves are compared with the m values obtained from the solid disk tests (Fig. 5) in Fig. 7. This comparison shows that the m values from the ring test theory are greater than those obtained from the disk tests by a factor of approximately 1.4. For low values of m, this factor appears to be as high as 2.0.

Superposition of the experimental values of the percentage change in internal diameter of rings as a function of amount of deformation for a number of materials [2, 3, 9] upon the theoretically derived curves clearly demonstrates the general applicability of these curves for the characterization of interfacial friction conditions, as shown in Fig. 8. It is, however, obvious from these results and those previously given in Fig. 4 that the Avitzur theoretical solution yields values of m in excess of unity.

(*d*) **Nonstandard Geometry, Ring Tests.** Ring specimens having the same height (0.250 in.) and external diameter (0.750 in.) but of different internal diameters were machined from a single bar of commercially pure aluminum and annealed in vacuum for 1 hr at 450 deg C, followed by furnace cooling. The specimen geometries used were as follows:

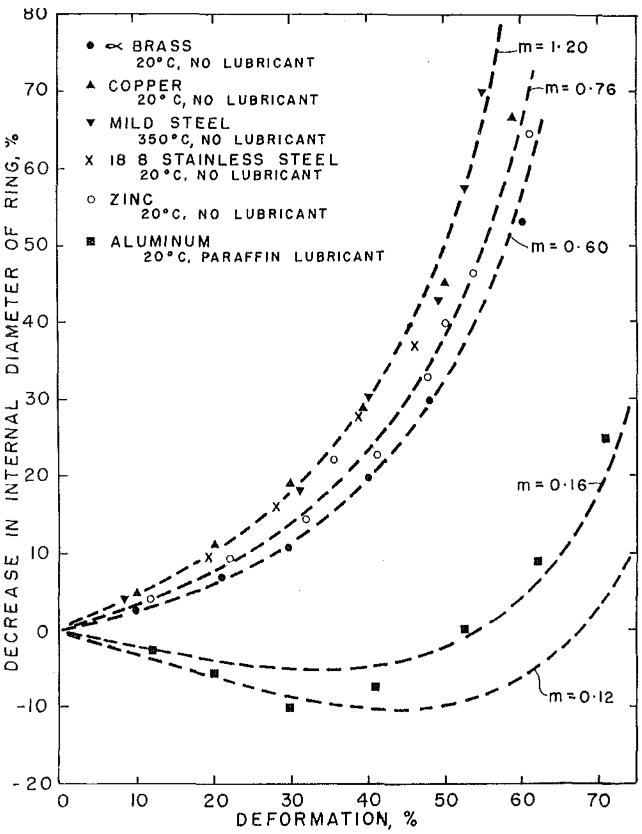

Fig. 8 Comparison of experimental and theoretical curves for interface friction factors for standard rings of various materials

Internal diameter (in.)	Geometry ratio (OD:ID:height)
0.500	6:4:2
0.400	6:3.2:2
0.300	6:2.4:2
0.200	6:1.6:2

A series of rings of each geometry were then deformed over a range of deformation between the same pair of flat dies at a press speed of 2 in./sec under the following frictional conditions: (*a*) 20 deg C, no lubricant; (*b*) 600 deg C, no lubricant; (*c*) 20 deg C, lubricated with lanolin.

The subsequent changes in shape on deformation were measured and curves obtained for the percentage changes in internal diameter as a function of amount of deformation. When the appropriate theoretical curves for the various initial specimen geometries were superimposed on these results, they were very similar. For this reason, complete data for only two ring geometries are shown in Figs. 9 and 10. Summarizing the data for the same test material and conditions, with different initial specimen geometry, the following interfacial friction factors were obtained:

	Interface Friction Factor (m)		
Ring geometry	20 deg C No lubricant	600 deg C No lubricant	20 deg C Lanolin lubricant
6:4.0:2	1.0–2.0	1.0–2.0	0.10–0.14
6:3.2:2	1.0–1.5	2.0	0.08–0.10
6:2.4:2	0.9–1.4	2.0	0.08
6:1.6:2	0.8–1.0	1.5–2.0	0.06–0.12

In general terms, good correlation is shown for the four initial ring geometries and the three interfacial friction conditions, thus indicating that the interfacial friction factor (m) is a reasonable

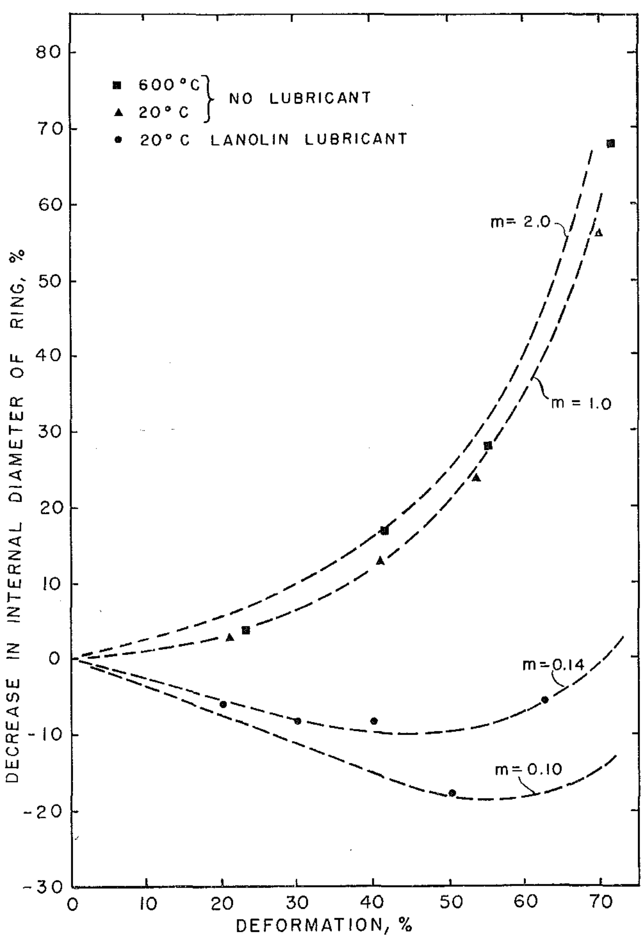

Fig. 9 Comparison of experimental and theoretical curves for aluminum rings (6:4:2)

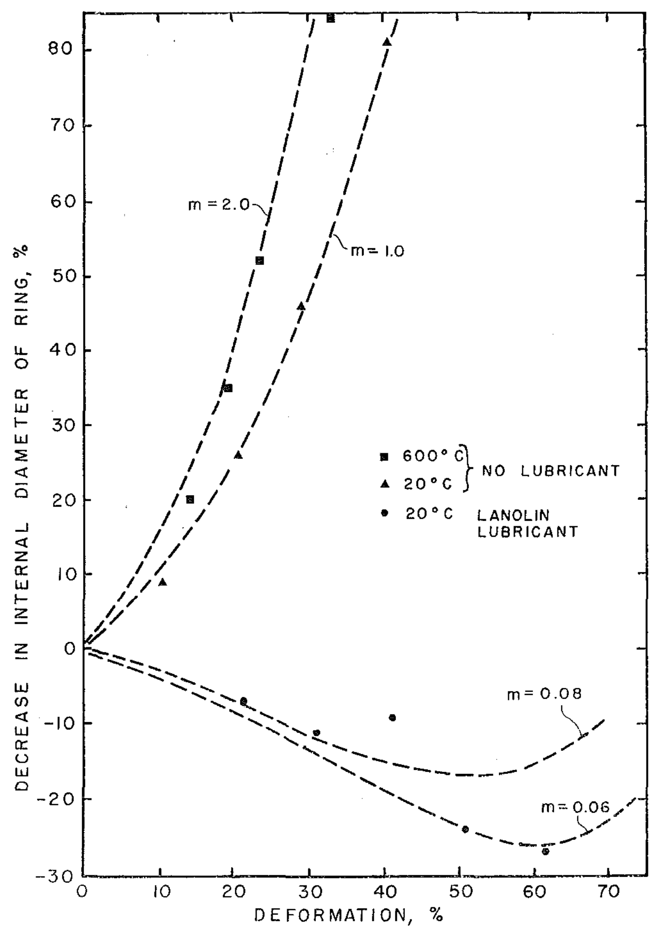

Fig. 10 Comparison of experimental and theoretical curves for aluminum rings (6:1.6:2)

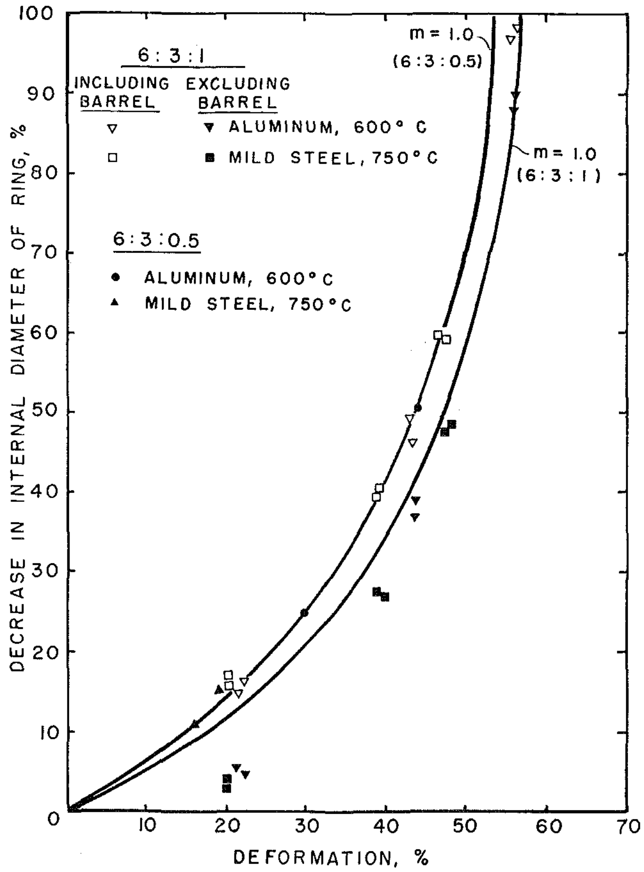

Fig. 11 Comparison of experimental and theoretical curves for maximum friction with rings of initial geometry 6:3:1 and 6:3:0.5

process parameter which can be used for identifying interfacial friction conditions.

In addition, ring specimens of dimensions 3.0 in. OD × 1.5 in. ID × 0.5 in. height (6:3:1) and also 0.25 in. height (6:3:0.5) were machined from commercially pure aluminum and mild steel and were deformed without lubricating at 600 deg C and 750 deg C, respectively, after heating in air. These conditions were those which had previously [2] been determined as giving full sticking. The results are given in Fig. 11 together with the relevant theoretical curves calculated using the Avitzur analysis. It can be seen that, with the initial ring geometry 6:3:1, values of m in excess of unity are obtained, but that with the ratio 6:3:0.5 the experimental points obtained agree very well with the theoretical curve for $m = 1.0$.

V Discussion of Results

The ring test has previously been experimentally calibrated [2, 3] on the assumption of constant coefficient of friction under metalworking conditions using an indirect technique. Only late in the preparation of this manuscript was the theoretical treatment of Burgdorf [14] found for the compression of a flat ring based on the concept of a constant coefficient of friction. For this reason, no evaluation of Burgdorf's analysis has been included here.

Mathematical analysis of the compression of a ring carried out by Avitzur and by Hawkyard and Johnson, and based on the assumption of a constant interfacial friction factor, m, and a constant material flow stress, σ_0, provides a possible means for the calibration of different initial ring geometries for friction evaluation. As outlined earlier, this theoretical treatment should be able to be applied to real materials when strain hardening occurs provided that the mathematical computations are carried out for a successive series of small increments of deformation. Excellent correlation has been shown between the shape of the theoretical curves and experimental ring test results on a wide variety of materials and a number of different initial ring geometries. However, it is noted that a number of experimental curves, when analyzed theoretically, yielded values of m considerably in excess of unity.

The theoretical analyses assume that there is no nonuniform distortion of cylindrical elements due to the frictional constraints, i.e., no barrelling. Or, more exactly, the effect of the frictional retardation stresses at the die/specimen interfaces is assumed to be transmitted uniformly throughout the specimen thickness. This is obviously a simplification of actual conditions and would be expected to give increasing error with increasing friction stress and increasing specimen height. Considerable barrelling was, in fact, observed on specimens of most geometries deformed with high interface frictions, therefore it is not surprising that there is a general lack of correlation between the theoretical and experimental curves under these conditions.

Concurrent work [15] being carried out to investigate the possibility of using the compression of flat ring-shaped specimens for the determination of material flow stress values under metalworking conditions has revealed a similar lack of correlation at low frictional values when using an initial ring geometry of 6:3:2. The general effect was to overestimate the effect of friction when using the Avitzur theory to analyze experimentally determined shape changes. On reducing the specimen height to give a geometry of 6:3:1 the theory appeared to work well under these relatively low friction conditions. However, even with this new geometry, the lack of correlation was still evident under conditions of high friction as shown in Fig. 11. Further reducing the specimen height to give an initial ring geometry of 6:3:0.5 gave experimental results under conditions of maximum friction which agreed well with the curve for maximum friction derived theoretically. This data is also shown in Fig. 11.

Under conditions of maximum friction, the most probable reason for obtaining agreement between theory and practice when using an initial ring geometry of 6:3:0.5 is that the theoretical assumption of the surface frictional retardation stress being transmitted uniformly throughout the specimen thickness is now approximately true for the practical situation. The correctness of this assumption manifests itself in the general lack of barrelling observed in the deformed rings of initial geometry 6:3:0.5 by comparison with that formed in deformed rings of initial geometry 6:3:1.

It thus appears that the theoretical assumptions made in the Avitzur analysis are not justified experimentally over all conditions of interfacial friction until the initial specimen geometry approaches 6:3:0.5. Under conditions of low interface friction, the theoretical assumptions are met when using an initial geometry of 6:3:1. Experimentally, neither of these geometries are particularly satisfactory with which to work especially at high temperatures, principally because of the large ratio of surface area contacting the dies to the workpiece thickness giving a quenching effect, and also the relatively high loads necessary to effect a particular increment of deformation.

In terms of general sensitivity of measurement and ease of experimentation, the ring geometry 6:3:2 has much to commend it. This was the initial geometry used by Male and Cockcroft [2] and since seems to have been adopted as an unofficial standard geometry. Experimental curves obtained in the early calibration work in terms of constant coefficient of friction, μ, exhibit a close correlation with the theoretical curves calculated using the concept of a constant interfacial shear factor, m, thus suggesting that there may possibly be an empirical relationship between μ and m. A direct comparison of data from the Male and Cockcroft calibration and from the theoretical solution shown in Fig. 3 is given in Fig. 12. This comparison shows that the relationship between μ and m varies with the particular deformation considered, although a number of experimental results obtained with rings tend to concentrate around the comparison line for 50 percent deformation, thus suggesting that a very approximate relationship of

$$\mu = \frac{m(\text{ring})}{2\sqrt{3}}$$

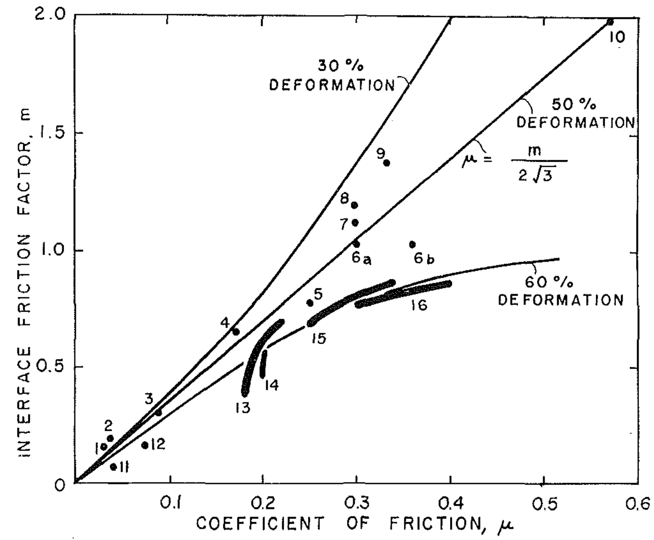

Fig. 12 Relationship between coefficient of friction, μ, and interface friction factor, m

Ring tests:

1. **α Brass, 20 deg C, lanolin lubricant**
2. **Aluminum, 20 deg C, paraffin lubricant**
3. **α Brass, 20 deg C, graphite lubricant**
4. **Mild steel, 20 deg C, smooth dies, no lubricant**
5. **Zinc, 20 deg C, no lubricant**
6. **Data of Jain and Bramley for mild steel, 1120 deg C, no lubricant (Ref. 16)**
   - **(a) Using Male and Cockcroft calibration**
   - **(b) Using Avitzur analysis**
7. **Copper, 20 deg C, no lubricant**
8. **Mild steel, 350 deg C, no lubricant**
9. **Mild steel, 20 deg C, rough dies, no lubricant**
10. **Data from a variety of metals exhibiting sticking friction at elevated temperatures**

Disk tests:

11. **α Brass, 20 deg C, lanolin lubricant**
12. **α Brass, 20 deg C, graphite lubricant**
13. **Mild steel, 20 deg C, smooth dies, no lubricant**
14. **Aluminum, 20 deg C, no lubricant**
15. **Copper, 20 deg C, no lubricant**
16. **Mild steel, 20 deg C, rough dies, no lubricant**

may be contemplated, and which would be more exactly true at low friction levels.

It is interesting to note that the curve for 60 percent deformation agrees well with data obtained for the compression of flat disks and correlating the values obtained in the early calibration work [2] with the m values (Fig. 5) presently calculated from the original results. Because of the specimen geometry used in the disk tests, there is little doubt about the true validity of the m values from this source. The inference is therefore that, when using an initial ring geometry 6:3:2, by the time the specimen has undergone 60 percent reduction in height, the resulting geometry renders the practical situation with regard to the distribution of surface frictional effects uniformly throughout the thickness in accord with the theoretical assumption.

Further supporting evidence for this is obtained from the results given in Fig. 11 where the value for the final hole diameter was taken by measuring from the inside corner of the faces in contact with the dies rather than the minimum diameter including the barrelling, as is normally the case. These results show that as the deformation increases, the value of the hole diameter measured from the inside corner of the faces approximates more closely to the diameter measured from the inside of the barrel, i.e., the barrel shape decreases considerably.

VI Conclusion

This work has shown that the concept of a constant interfacial friction factor, m, can be usefully used to characterize interfacial friction conditions during a flat forging operation. The mathematical solution for the compression of a flat ring made by Avitzur can be applied successfully to obtain realistic values of m by the analysis of experimentally determined shape changes, provided that the initial ring is small in thickness when compared with the internal and external diameters. An initial geometry (external dia:internal dia:thickness) of 6:3:0.5 has been found to be adequate for this purpose.

When using the Male and Cockcroft standard ring geometry of 6:3:2, the Avitzur solution gives inaccurate values of m which appear to be just double the true values. The reason for this is most probably because considerable barrelling takes place during deformation, thus deviating from the theoretical assumption.

The close correlation between the Avitzur analysis and practical results when using rings of initial geometry 6:3:0.5 now gives the distinct possibility of using this technique for the determination of accurate values of a materials flow stress under metalworking conditions provided that measurements are made of the deformation load. Measurement of the resulting shape change of the ring indicates the degree of frictional resistance which then, through the theoretical analysis, directly relates the material flow stress to the average applied pressure.

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