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Prediction of Tip-Leakage Losses in Axial Turbines

Existing methods for predicting the tip-leakage losses in turbomachinery are based on a variety of assumptions, many of which have not been fully verified experimentally. Recently, several detailed experimental studies in turbine cascades have helped to clarify the physics of the flow and provide data on the evolution of the losses. The paper examines the assumptions underlying the prediction methods in the light of these data. An improved model for the losses is developed, using one of the existing models as the starting point.

Introduction

Beginning with the work of Betz (1926), various correlations and models have been suggested for predicting the effects of tip leakage on the performance of axial turbomachinery. However, until recently detailed data for the tip leakage flow were not available and the physics of the flow was not fully understood. As a result, the prediction methods were often based on assumptions that had not been verified experimentally. Recently, several detailed studies in turbine cascades have provided a much better understanding of this complex flow. These studies included measurements inside the tip gap as well as downstream of the trailing edge, thus allowing the development of the losses to be traced in some detail. Not all aspects of the flow have been clarified. For example, the effect of the inlet boundary layer thickness needs further investigation. Nevertheless, a critical assessment is now possible of a number of major assumptions used in the prediction methods.

In most earlier experimental studies, the tip leakage losses were estimated from measurements made downstream of the trailing edge only. For example, the flow downstream of compressor rotors has been studied extensively by Inoue and his co-workers (e.g., Inoue and Kuroumaru, 1984, 1989; Inoue et al., 1986) and by Lakshminarayana and his co-workers (e.g., Lakshminarayana et al., 1987). Schmidt et al. (1987a, 1987b) investigated the effect of leakage on the spanwise loss distribution in an isolated compressor rotor and similar measurements were made by Patel (1980) for an axial-turbine rotor. Patel also examined the effects of blade tip treatment. More recently, Yamamoto (1988, 1989) has investigated the tip leakage flow downstream of a linear turbine cascade. While such studies provide useful data and insights into some of the effects that influence the tip-leakage losses, they give a somewhat incomplete picture.

The three recent studies in, turbine cascades by Bindon (1989), Dishart and Moore (1990), and Yaras and Sjolander (1989) provide a considerably more detailed picture. Bindon examined three clearances in a linear cascade of turbine blades, although detailed data were obtained only for a clearance of 2.5 percent of the blade chord. Measurements were made in the tip gap and at the trailing edge plane. Bindon concluded that of the tip leakage losses generated up to the trailing edge, about 40 percent occurred within the gap, due mainly to the separation bubbles formed on the blade tip. In addition, he indicates that the fluid of low total pressure, which is discharged into the passage from the separation bubbles, contributes significantly to the mixing losses after the flow leaves the gap. He thus attaches great importance to the separation bubbles on the blade tip. Dishart and Moore (1990) investigated the tip-leakage losses in a linear turbine cascade with a clearance of 2.1 percent of axial chord. They measured the flow at the gap outlet and at 40 percent axial chord downstream of the trailing edge. The authors also calculated the fully mixed-out losses. They found that the losses measured at the gap exit represented only about 17 percent of the total mixed-out losses. They also found that nearly 90 percent of the final losses had occurred by the downstream measurement plane. Thus Dishart and Moore's results imply that the losses inside the gap play a smaller role in the overall losses than was concluded by Bindon. The most detailed data currently available were presented recently by Yaras and Sjolander (1989). The authors examined four clearances from 1.5 to 5.5 percent of the blade chord. Very detailed results were obtained for clearances of 2.0 and 5.5 percent: inside the clearance gap, at the trailing-edge plane, and at one axial chord length downstream. The tip-leakage loss models are evaluated primarily on the basis of these data. Therefore, the main results and conclusions are summarized in a later section.

It thus appears that, based on the available experimental studies, it should now be possible to develop a tip-leakage loss model that is reasonably consistent with the physics of the flow. The present paper reviews the commonly used tip-leakage loss models and examines the validity of the physical assumptions that underlie them. The examination is confined to models that produce total pressure loss coefficients. A number of methods have also been developed for predicting the change in stage efficiency with tip clearance. Some of these are primarily correlations of available data, with some physical reasoning being applied to choose the correlating parameters (e.g., Hesselgreaves, 1969; Amann et al., 1963; Moyle, 1990). Others incorporate some degree of flow modeling (e.g., Senoo and

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Ishida, 1986; Farokhi, 1988). However, models that predict the total pressure losses seem preferable since they deal with flow quantities that are directly measurable. They are thus inherently more closely linked to the physics of the flow. They also have the advantage of direct compatibility with the form in which the other losses through the blade row, such as profile and secondary losses, are usually handled.

An improved model for the tip-leakage losses is developed using one of the existing models as the starting point.

Review of Existing Tip-Leakage Loss Models

Experimental Background. The loss models will be evaluated primarily by comparison with the present authors' own data for the tip-leakage flow field in a planar cascade of turbine blades (Yaras and Sjolander, 1989, 1990; Yaras et al., 1989). This flow is idealized in a number of respects. The results were obtained at realistic Reynolds numbers but under essentially incompressible conditions and for a free stream with a low level of turbulence intensity. There was also no relative motion between the tip wall and the tip of the blade. Nevertheless, the cascade flow has many of the essential features of the tip leakage flow in the actual machine. In any case, there is a long-standing and successful tradition of using turbomachinery performance correlations and models which are based at least partly on cascade results. For reference, the main results obtained from the Yaras–Sjolander measurements are summarized here.

It is conventional to divide the losses in turbomachinery blade rows into several components. It is usually assumed that these can be evaluated independently and then combined linearly to obtain the total loss. Thus, for a blade row with clearance the overall loss coefficient might be written:

$$Y = Y_p + Y_{\text{sec}} + Y_{\text{tip}}, \tag{1}$$

where Y_p is the blade profile loss, Y_{sec} is the loss due the secondary or endwall boundary layer flow, and Y_{tip} is the tip-leakage loss. The secondary loss is obtained at zero clearance and the value is generally assumed to remain unchanged as the clearance gap is opened. The tip clearance loss is then taken as the loss that must be added to the secondary loss to obtain the total observed loss in the blade end region. This simple, pragmatic decomposition is widely recognized as rather unsatisfactory physically (e.g., Vavra, 1960; Dunham and Came, 1970).

For the Yaras–Sjolander cascade, the blade wake was identifiable over the entire span. Thus, it was relatively easy to distinguish and separate the profile losses from the other components of the loss. On the other hand, it was clear that there was a strong interaction between the tip-leakage and endwall boundary layer aspects of the flow. It was evident that a substantial fraction of the original endwall boundary layer fluid passed through the tip gap and became an indistinguishable part of the tip-leakage flow. The remaining endwall boundary layer fluid was swept across the passage and rolled up into a passage vortex in the usual way. However, the vortex was far smaller than the secondary vortex that formed at zero clearance. Likewise, the losses associated with this identifiable "secondary flow" were much smaller.

The alternative breakdown of the losses that emerges from the Yaras–Sjolander study is shown schematically in Fig. 1. The pattern corresponds roughly to the fully mixed-out losses. It was found that the losses increased substantially downstream of the trailing edge but that most of the additional loss production occurred over about the first axial chord length.

The losses inside the gap itself were found to be a relatively small fraction of the total losses. The secondary loss is now taken as the loss that can be clearly assigned to a secondary flow structure, namely the passage vortex. As shown in Fig. 1, this component of the loss fell off very quickly with clearance; even at 2 percent clearance, it accounted for an insignificant fraction compared with the loss in the tip-leakage flow. However, this may not be a general result. The Yaras–Sjolander cascade had a thin endwall boundary layer at the inlet and a relatively low turning of about 45 deg. The secondary flow is therefore comparatively weak. More data are needed on the effects of inlet boundary thickness. Rotation may also have some influence since, in a turbine blade row, the scraping effect would tend to enhance the passage vortex.

The remaining loss, which is identifiably associated with the tip-leakage flow, was referred to as "end loss" by Yaras and Sjolander (1989b). This was to avoid confusion with the common usage of the term "tip-leakage loss." However, as indicated in Fig. 1, the latter term is being readopted since it is more descriptive. The end loss is then taken as the total loss that occurs in the endwall region, apart from the profile loss. In the Yaras–Sjolander experiment, the non-gap end loss (that is, the loss in the endwall region apart from the loss inside the gap itself) was made up almost entirely of tip-leakage loss.

Nomenclature

c = blade chord length
C = empirical constant relating local gap loss coefficient to clearance size (see Eq. (19))
C_D = discharge coefficient for tip gap
C_L = lift coefficient
C_p = $(P-P_1)/_{1/2}\rho V_1^2$ = static pressure coefficient
C''_{Po} = mass-averaged total pressure loss coefficient based on inlet dynamic pressure
C''_{q_s} = mass-averaged coefficient of secondary kinetic energy for downstream flow
C''_{q_n} = mass-averaged coefficient of gap kinetic energy normal to chord line
d = diameter of tip leakage vortex
E = kinetic energy
h = blade span
k_s = fraction of zero-clearance secondary loss still present with clearance (Eq. (16))
K = retained lift coefficient (Eq. (4))
K_E, K_G = constants related to the blade loading distribution (Eqs. (12), (18))
$\dot{m}_g$ = mass flow rate through tip gap
$\dot{m}_p$ = mass flow rate for the blade passage
P = static pressure
P_{PS} = blade pressure-side pressure
P_{SS} = blade suction-side pressure
S = blade spacing
t_{MAX} = blade maximum thickness
V = velocity
V_N = component of gap velocity normal to chord line
x' = coordinate in chordwise direction
Y = mass-averaged total pressure loss coefficient based on outlet dynamic pressure
Y' = loss coefficient averaged over local mass flow rate (e.g., in the gap)
α = flow angle, relative to axial direction
δ = boundary layer thickness
ρ = density
σ = blade row solidity = c/S
τ = tip gap height

Subscripts

1, 2 = inlet, outlet
m = mean value through blade row
p = profile or passage

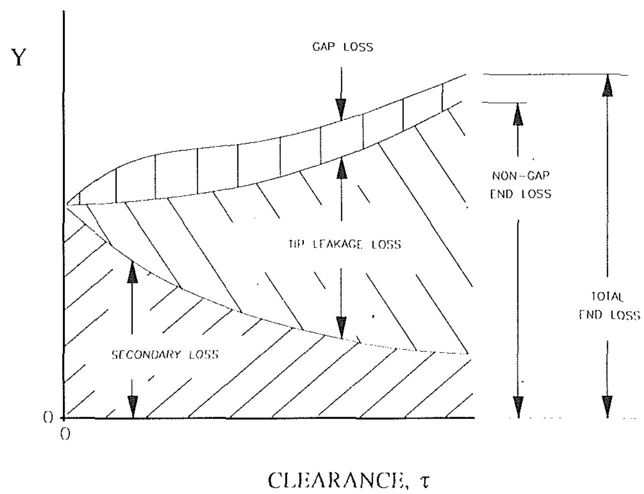

Fig. 1 Schematic breakdown of the losses in the end region (excluding profile losses)

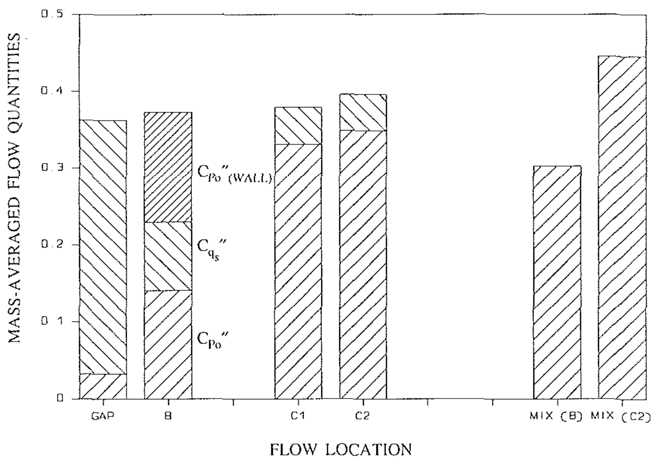

Fig. 2 Downstream development of losses and secondary kinetic energies for τ/c = 0.055 (from Yaras and Sjolander, 1989)

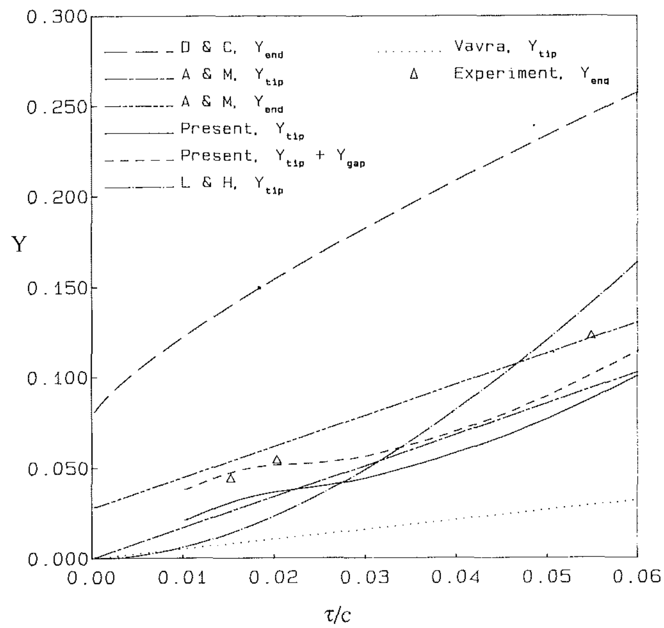

Fig. 3 Comparison between tip-leakage loss models and measurements (D & C: Dunham and Came, 1970; A&M: Ainley and Mathieson, 1951; L & H: Lakshminarayana and Horlock, 1965)

The loss processes contributing to the tip-leakage loss can be summarized in terms of Fig. 2 (taken from Yaras and Sjolander, 1989b). The stacked bars show the total pressure losses and secondary kinetic energies measured at several locations for a clearance of 5.5 percent of the blade chord. Plane B is at the trailing edge and planes C1 and C2 are two closely spaced planes one axial chord length downstream. The fully mixed-out losses calculated from the data at planes B and C2 are also shown. For the gap flow, the "secondary" kinetic energy is in fact that corresponding to the component of gap velocity normal to the blade chord. For the downstream locations it is that associated with the components of velocity in the plane normal to the downstream mean velocity. All quantities are averaged over the mass flow rate through one passage and over half the blade span.

The clearance flow was discharged from the gap with a large amount of kinetic energy normal to the gap exit. Much of this gap kinetic energy was found to have been recovered by the time the flow reached the trailing edge. Evidently, the gap discharge did not act as a simple sudden expansion. Instead, the relatively orderly roll-up of the tip leakage vortex apparently allows a significant recovery of static pressure. The losses then increased substantially over the first axial chord downstream of the trailing edge. At that point the loss generation seemed to be largely complete. Interestingly, it was found that the end loss observed at the downstream plane agreed well with the sum of the losses within the gap itself together with a loss equal to the kinetic energy in the flow at the gap outlet. This was true at both the clearances for which detailed data were obtained. The gap energy, which was initially recovered, was thus ultimately lost. The loss appeared to occur through two main mechanisms. The energy was partly recovered as secondary kinetic energy of the tip-leakage vortex. This energy was eventually lost as the vortex mixed with the surrounding free-stream fluid. Secondly, the presence of the tip-leakage vortex seemed to lead to higher shear stresses at the endwall and therefore to higher entropy production there. It was tentatively concluded that the loss production at the endwall, over the axial chord length of downstream distance, was in fact substantial. This conclusion was based on the indirect evidence of the very different values of mixed-out loss obtained at planes B and C, as shown at the right of the bar chart. The most obvious loss mechanism not taken into account by the mixing calculations is the viscous loss production on the endwall. It is seen that when the difference in the mixed-out losses (marked $C''_{Po(\mathrm{WALL})}$ on the figure) is added to the loss observed at B, the trailing edge plane, it agrees well with the final loss obtained at plane C, one chord length downstream.

Finally, the nature of the tip-leakage vorticity field needs to be mentioned. A number of loss models calculate the tip leakage losses in terms of the "induced drag" of the blade. They therefore make assumptions about the strength of the trailing vortex system. The vorticity aspects of the present flow were examined by Yaras and Sjolander (1990). It was found that, unlike the case of a finite wing, the circulation of the tip leakage vortex is considerably less than the bound circulation of the blade, and varies with clearance. The physics of this odd result is not fully understood. However, a number of other researchers have come to the same conclusion: for example, Lakshminarayana and Horlock (1965), Lewis and Yeung (1977) and Inoue et al., (1986). The data of Inoue et al. show the value of the shed circulation also depends on the relative wall speed.

The tip-leakage loss models are then examined in the light of these experimental observations. The losses predicted by most of the models are compared with the results of the Yaras–Sjolander experiment in Fig. 3.

The models can be divided into two broad categories: those that arrive at the losses indirectly as a result of a momentum balance, and those that consider energy directly.

Models Based on Momentum Considerations. This group of models determines the effect of the tip-leakage flow initially in the form of a drag force on the blade. The drag force is generally seen as the equivalent of the induced drag of a finite wing. To convert the resulting momentum deficiency into a total pressure loss, it is necessary to assume that the loss is distributed uniformly over the mass flow. Thus, these models implicitly predict the fully mixed-out value of the losses. This is something of a practical disadvantage. The measurements show that fully mixed-out conditions are approached about one axial chord length or further downstream. The momentum-based models will therefore overestimate the losses at the inlet to a downstream blade row that is closely spaced.

Betz (1926) appears to have been the first to model some aspects of the tip-leakage flow. The author calculated the induced drag due to the vortices shed at the trailing edge of the blade. The strength of the shed vorticity was obtained from the gradient of the bound circulation distribution, with the assumption that the bound circulation went to zero at the blade tip. However, as noted earlier, there is considerable experimental evidence that the circulation in the tip-leakage vortex is less than the bound circulation and varies with both clearance and rotation.

The well-known and widely used Ainley and Mathieson (1951) model takes much the same approach. Their tip-leakage model was in fact adapted from an earlier theory for secondary flows developed by Carter (1948). Carter had determined the incidence induced by the secondary flow based on conventional theory for the downwash at a finite wing. Both Carter and Ainley and Mathieson again assumed that the full bound circulation would appear in the trailing vortex system. The resultant expression given by Ainley and Mathieson is as follows:

$$Y_{\mathrm{tip}}=\frac{\cos^2(\alpha_2)}{\cos^3(\alpha_m)}\,\frac{1}{s/c}\,0.5\left(\frac{\tau}{s}\right)\frac{c_L^2}{h/c}. \tag{2}$$

Figure 3 shows the loss variation predicted by this model for the Yaras–Sjolander cascade geometry. The figure also shows that when Ainley and Mathieson's secondary loss is added to their tip-leakage loss, the agreement with the measured end loss is quite reasonable. However, this agreement is probably fortuitous since the simple decomposition of the losses, which is assumed by the Ainley and Mathieson approach, is not supported by the Yaras–Sjolander measurements.

Dunham and Came (1970) modified Ainley and Mathieson's model in the light of some later cascade data, which suggested a nonlinear variation of the tip-leakage losses with clearance size. The resultant expression is as follows:

$$Y_{\mathrm{tip}}=\frac{\cos^2(\alpha_2)}{\cos^3(\alpha_m)}\,\frac{1}{(s/c)^2}\,0.47\left(\frac{\tau}{c}\right)^{0.78}\frac{c_L^2}{(h/c)}. \tag{3}$$

However, as seen from Fig. 3, this model substantially overestimates the losses. This behavior was also noted by Kacker and Okapuu (1982).

In their scheme for predicting the tip-leakage losses in compressors, Lakshminarayana and Horlock (1965) followed a procedure somewhat similar to that of Ainley and Mathieson. They simulated the tip leakage vortex with a Rankine vortex and determined the drag induced on the blade by this vortex. The vortex was modeled in considerable detail, taking into account its location and alignment relative to the blade, its initiation point, and the estimated radius of the core. Furthermore, the authors had noted the lower value of the circulation for the tip-leakage vortex compared with the bound circulation and therefore took this into account. The expression for the tip-leakage loss is then:

$$Y_{\mathrm{tip}}=\frac{\cos^2(\alpha_2)}{\cos^3(\alpha_m)}\,\frac{1}{(s/c)}\,\frac{c_L^2(1-K)}{8\pi(h/c)}\,(A-B), \tag{4}$$

where

$$A=\ln\left(\frac{e^{2\pi h/s}-1}{\left(1+\coth\left(\frac{2\pi\tau}{s}\right)\right)(e^{2\pi h/s}-1)+2}\right),$$

$$B=\ln\left(\frac{e^{\pi d/s}-1}{\left(1+\coth\left(\frac{2\pi\tau}{s}\right)\right)(e^{\pi d/s}-1)+2}\right),$$

and $(1-K)$ is the fraction of the bound circulation shed in the tip-leakage vortex. The model was found to be quite sensitive to the assumed diameter, d, of the vortex. Figure 3 shows that the trend in the losses predicted by the model is not in good agreement with the turbine cascade measurements. Quantitative comparison requires that a secondary loss component be added to the prediction. If, for example, the Ainley and Mathieson secondary loss were used, the predicted end loss would clearly be much higher than the measured. In a modified version of the model (Lakshminarayana, 1970), the losses associated with the spanwise flow in the blade surface boundary layer were taken into account; the change in the predicted loss was quite small.

Models that predict the losses based on an induced drag are essentially inviscid models. The viscous losses in the blade boundary layers are of course accounted for by the profile loss component, although as noted by Lakshminarayana (1970) the profile losses near the tip may be increased somewhat by the tip-leakage vortex. It could also be argued that the viscous losses on the endwall will be taken into account by the separate secondary loss model. However, the Yaras–Sjolander experiment suggested that there was enhanced viscous loss production on the endwall, downstream of the trailing edge, which was directly attributable to the leakage flow. This is thought to be due to the higher wall shear stresses that occur beneath the vigorous tip-leakage vortex. This indicates that an additional loss ought to be added to the purely inviscid loss component, which is derived from the induced drag analysis. However, Fig. 3 shows that most of the models already tend to overpredict the end losses. Therefore, addition of a viscous loss component, as suggested by the physics, would only make the agreement poorer.

Models Based on Energy Considerations. The alternative to the momentum-based models is to consider the mechanical energy changes directly.

In his influential study on the tip-leakage problem, Rains (1954) adopted this approach. Rains suggested that the tip-leakage fluid rolls up into a vortex whose energy cannot be recovered. In other words, the kinetic energy in the gap flow normal to the blade chord is assumed to be lost eventually and that this loss accounts for the whole of the tip-leakage loss. As discussed earlier, this simple idea is in fact supported by the Yaras–Sjolander cascade experiment. The key to the application of this idea is then a prediction of the mass flow rate through the gap and of the magnitude of the velocity component normal to the gap. Rains himself developed a somewhat involved analysis for the resulting efficiency drop in the stage. Subsequently, Vavra (1960) used the idea with some simplifications to arrive at the following loss coefficient:

$$Y_{\mathrm{tip}}=\frac{\cos^2(\alpha_1)}{\cos^3(\alpha_m)}\,\frac{1}{s/c}\,\frac{4(2)^{0.5}}{5}\,k\omega^3\,\frac{\tau}{h}\,c_L^{1.5}, \tag{5}$$

where ω accounts for the flow resistance in the gap and k is the contraction factor for the flow through the gap. It will be noticed that the energy approach leads to a weaker dependence on the blade loading than the momentum approach (Eqs. (2)–(4)). Hesselgreaves (1969) and Lewis and Yeung (1977) followed Vavra's approach but set ω and k to 1.0. As shown in Fig. 3, Vavra's model would predict a much lower loss than was measured in the turbine cascade even if a plausible value of secondary loss were added to it. This would be the case particularly at higher clearances.

Since the underlying idea is physically sound, the problem with Vavra's model must lie in the prediction of the gap kinetic energy. In an earlier paper (Yaras et al., 1989), the present authors examined the gap flow in detail and devised a simple model that appears to predict the flow, including its kinetic energy, with good accuracy. These results are combined with some of the ideas from Vavra's model to arrive at an improved tip-leakage loss model.

Improved Tip-Leakage Loss Model

The kinetic energy associated with the component of the gap velocity normal to the blade chord can be written:

$$\Delta E = \int_{\dot{m}_g} 0.5\ V_N^2 d\dot{m}_g. \tag{6}$$

Yaras et al. (1989) showed that the leakage flow passed through the gap with little change in chordwise momentum. Therefore, the driving pressure difference experienced by the fluid went entirely to accelerate it normal to the chord line. Furthermore, most of the fluid experienced no loss as it passed through the gap. Thus, Bernoulli's equation can be applied to the acceleration process:

$$V_N = [2(P_{PS} - P_{SS})/\rho]^{(1/2)} \tag{7}$$

It is well known that the blade loading near the tip is highly distorted by the presence of the leakage flow (e.g., see Sjolander and Amrud, 1987). However, Yaras et al. found that this distorted pressure field did not extend very far into the tip gap. Toward the endwall, where most of the leakage mass flow occurs, the fluid experiences essentially the undistorted blade pressure difference; that is, the pressure difference that would occur on the blade tip profile in the absence of clearance. Thus, the chordwise distribution of pressure difference used in Eq. (7) is the blade loading at the tip, neglecting the effects of the clearance. This is comparatively easy to calculate. In Vavra's model the velocity given by Eq. (7) is multiplied by a flow resistance factor. This implies a loss of kinetic energy inside the gap, which is not supported by the measurements.

The mass flow rate through the gap can be expressed as

$$d\dot{m}_g = C_D \rho V_N \tau dx', \tag{8}$$

where C_D is a discharge coefficient. Vavra used a constant value of 0.5 for the discharge coefficient. The measurements of Yaras et al. (1989) gave somewhat larger values and indicated that C_D varied with the aspect ratio of the gap. The measured variation is shown in Fig. 4. The values shown on the figure are somewhat larger than those quoted by Yaras et al. The discharge coefficients in the earlier paper apply to mass flow rates calculated from the component of velocity normal to the blade mean line. The C_D was therefore recalculated to be consistent with the velocity component normal to the chord line, as used in the model. The variation in C_D with clearance shown in Fig. 4 reflects the observed size of the separation bubble, relative to the gap height, on the tip of the blade. The trend may be somewhat different for a different blade design. Therefore, it is probably more reasonable to use a mean value, of about 0.7 to 0.8, for the discharge coefficient. Dishart and Moore (1989) found an average value of about 0.7 for their experiment.

The expression for the gap kinetic energy then becomes

$$\Delta E = \left(\frac{2}{\rho}\right)^{0.5} C_D \tau \int_0^c (P_{PS} - P_{SS})^{1.5} dx'. \tag{9}$$

For a design that is well advanced, so that the blade loading is known, Eq. (9) could be used to investigate the effect on the tip-leakage losses of changes in the loading, such as at off-design conditions.

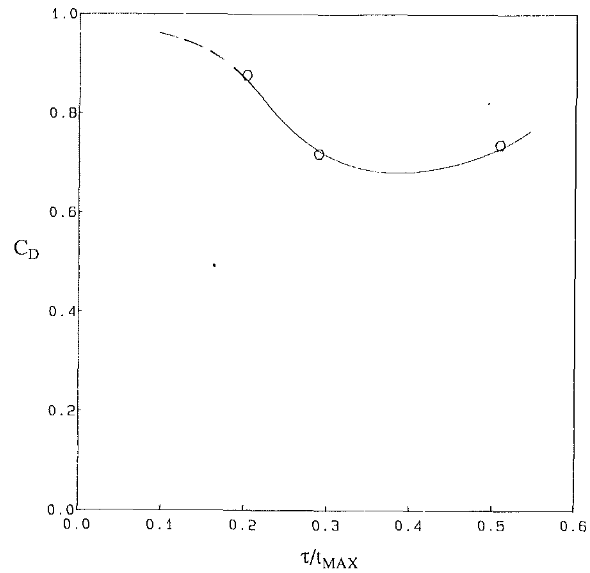

Fig. 4 Variation of gap discharge coefficient with clearance for Yaras-Sjolander Cascade

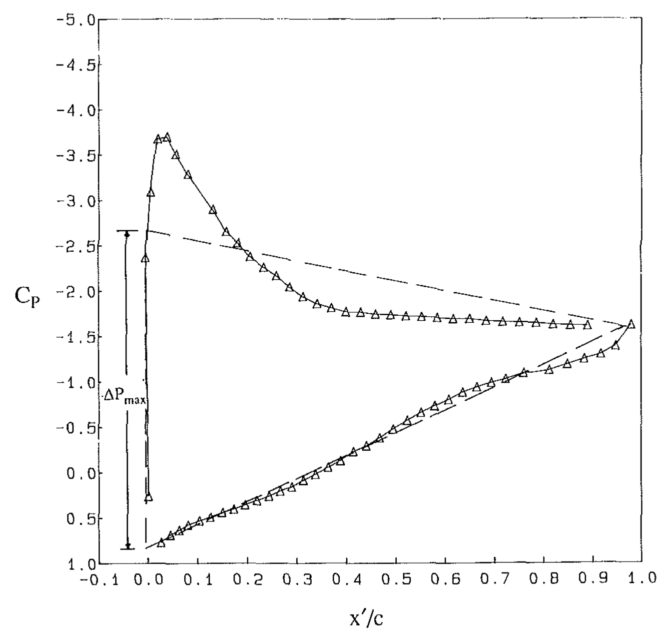

Fig. 5 Measured blade loading distribution and triangular approximation (Yaras-Sjolander cascade)

For use in the early stages of a design, an approximate loading distribution can be assumed. This was also done by Hesselgreaves (1969) in his version of the Vavra model. The simplest choice is a linear variation. The assumed variation can then reflect in broad terms the intended design philosophy for the blade; that is, whether it is expected to be front-, mid-, or aft-loaded. As shown in Fig. 5, a front-loaded, triangular distribution is a reasonable approximation for the Yaras-Sjolander blade:

$$P_{PS} - P_{SS} = \Delta P_{\max}\left(1 - \frac{x'}{c}\right). \tag{10}$$

The integral in Eq. (9) can then be evaluated for the assumed loading distribution. If it is also assumed that the lift force is roughly normal to the chord line, then

$$C_L = \frac{\int_0^c (P_{PS} - P_{SS}) dx'}{\frac{1}{2} \rho V_m^2 c}, \tag{11}$$

where V_m is the mean velocity through the blade row. Evaluating Eq. (11) with the assumed loading, the expression for the kinetic energy loss becomes

$$\Delta E = K_E \rho c C_D \tau V_m^3 C_L^{1.5}, \tag{12}$$

where $K_E = 0.566$ for a front- or aft-loaded blade,
$= 0.5$ for a mid-loaded blade.

The loss is seen to be relatively insensitive to the loading distribution.

The loss coefficient Y, for a cascade flow where absolute and relative frame of references are the same, is defined as follows:

$$Y = \frac{\Delta E}{E} = \frac{\Delta E}{\dot{m}_p V_2^2/2}. \tag{13}$$

The passage kinetic energy, E, can also be written in terms of the mean velocity:

$$E = \frac{\rho hs}{2} \frac{V_m^3 \cos^3(\alpha_m)}{\cos^2(\alpha_2)}. \tag{14}$$

The tip-clearance loss coefficient then becomes,

$$Y_{\text{tip}} = 2K_E \, \sigma \frac{\tau}{h} C_D \frac{\cos^2(\alpha_2)}{\cos^3(\alpha_m)} C_L^{1.5}, \tag{15}$$

where σ ($= c/S$) is the solidity of the blade row. Since for given flow turning C_L varies inversely with σ, Eq. (15) indicates that the tip-leakage loss will vary as $\sigma^{-0.5}$.

It is envisaged that Eq. (15) would be used in a loss system in which the total loss through the blade row is calculated from

$$Y = Y_p + (Y_{\text{sec}})_{\text{HUB}} + (Y_{\text{end}})_{\text{TIP}}, \tag{16}$$

where $(Y_{\text{end}})_{\text{TIP}} = Y_{\text{gap}} + k_s Y_{s,0} + Y_{\text{tip}}$ and $Y_{s,0}$ is the secondary loss coefficient at the tip for zero clearance. The factor k_s is the fraction of the secondary loss that remains identifiably associated with a secondary flow structure, principally the passage vortex. In loss systems such as that of Ainley and Mathieson, k_s is implicitly taken to be 1.0. However, as described earlier, k_s appeared to be much less than 1.0 for the Yaras–Sjolander cascade. In general, k_s might be expected to be a function of the inlet boundary layer thickness, perhaps as δ/τ, and also of the rotation. With the enhancement of the passage vortex by the scraping effect, it is conceivable that k_s could become larger than 1.0. More data are needed on these aspects.

The tip-leakage losses calculated from Eq. (15), using the measured values of C_D, are compared with the measured $(Y_{\text{end}})_{\text{TIP}}$ in Fig. 3. They are somewhat lower than the measured values, as would be expected since Y_{gap} and $Y_{s,0}$ are not included in the predictions.

As noted earlier, the gap loss contributes a relatively small amount to the overall end loss. Dishart and Moore (1990) came to the same conclusion. Nevertheless, it is not negligible and it is desirable to include it in the model for completeness. For the Yaras–Sjolander experiment, Y_{gap} was reasonably constant for clearances of 2.0 to 5.5 percent, the range for which gap measurements were made. The loss coefficient averaged over the gap mass flow rate, Y'_{gap}, decreased roughly in proportion to the increase in flow rate as the gap size increased. The relationship between the two loss coefficients is:

$$Y_{\text{gap}} = Y'_{\text{gap}} \frac{\dot{m}_g}{\dot{m}_p}. \tag{17}$$

Calculating the gap flow rate from Eq. (8) with the assumed loading distributions and writing the passage mass flow rate as $\rho V_m \cos\alpha_m hS$, Eq. (17) becomes,

$$Y_{\text{gap}} = Y'_{\text{gap}} K_G \frac{\sigma C_D C_L^{1/2}}{\cos\alpha_m} \left(\frac{\tau}{h}\right), \tag{18}$$

where $K_G = 0.943$ for front- or aft-loaded blades,
$= 1.0$ for mid-loaded blades.

The gap loss is seen to be even less sensitive than the tip-leakage loss to the distribution of the blade loading.

To a good approximation, the local gap loss coefficient varied as $C/(\tau/c)$, with $C \cong 0.007$. Equation (17) can then be written:

$$Y_{\text{gap}} = CK_G \frac{\sigma C_D C_L^{1/2}}{\cos\alpha_m} \left(\frac{c}{h}\right). \tag{19}$$

It should be noted that there is considerable uncertainty in the local gap loss coefficient, Y'_{gap}, since some of the loss detected in the gap originated in the upstream boundary layer. It was assumed that about one quarter of the inlet boundary layer passed through the gap and the corresponding loss was subtracted from the measured gap loss. However, the assumed fraction could not be verified experimentally. In view of this, the value of C should be treated as very approximate. Figure 3 shows the sum of the losses calculated from Eqs. (15) and (19). It is seen that inclusion of the gap loss improves the agreement with the measurements.

Conclusions

Existing methods for predicting the tip-leakage loss coefficient were reviewed in the light of several detailed studies conducted recently in turbine cascades. It was found that methods that assume the loss is primarily due to an induced drag, resulting from the trailing vortex system, did not give very satisfactory results. These methods neglect the additional viscous loss production on the endwall that results from the presence of the tip-leakage vortex. Some of the recent data suggest that this loss production may be significant. However, if the momentum-based methods were modified to include the viscous loss production, they would produce even poorer results.

Recent experiments suggest that the kinetic energy carried by the normal component of the gap velocity is ultimately lost. Furthermore, this appears to account for essentially all the tip-leakage loss downstream of the gap. This confirms a hypothesis originally advanced by Rains and subsequently used by Vavra as a basis for a tip-leakage loss model. However, Vavra made several other assumptions that are not supported by the recent data and that led to poor predictions for the gap kinetic energy.

Vavra's model was used as the starting point to derive an improved tip-leakage loss model that is consistent with the recent experimental observations. The model gave very satisfactory agreement with the limited available data. More data are needed to clarify the interaction between the tip-leakage and secondary flows and the influence of the scraping effect that occurs with relative wall motion.

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