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Characteristics of Laminated Sheet Metals in Tensile Deformation

Deformation behavior of laminated sheet metals is estimated by the upper bound method based on Hill's plasticity theory for anisotropic materials. The r-value and the stress-strain curve are theoretically derived for the laminated sheet metals whose component metals are combined at arbitrary angles to the rolling direction. Tension tests of the laminated sheet metals which are made by bonding in combinations of mild steel, brass, and aluminum sheet metals are carried out. The experimental results are compared with the theoretical ones. They are in good agreement.

1 Introduction

Recently, laminated sheet metals have been used in various fields, for example, as vibration isolation sheet, because of their special properties which cannot be obtained by conventional materials. However, a method for evaluating workability of the laminated sheet metals has not been established. Only a few investigations have been made.

Hawkins and Wright [1] investigated deep drawability of adhesive bonded and clad sheet metals of copper and mild steel. Hiraiwa and Kondo [2,3] also studied deep drawability and found that the limiting drawing ratio (LDR) does not always obey the composite law and that the LDR becomes larger because deformation of the sheet metal with a small LDR is affected by that with a large LDR. This trend is more remarkable when the difference in the workhardening exponents of the sheet metals is larger. Semiatin and Pieler [4] found that the stress-strain relationship of clad sheet metals of stainless steel and aluminum obtained in static tension test obeyed the composite law. They also studied local instability condition and localized necking condition. Ohsawa and Nishimura [5] made a theoretical consideration of the stress and strain in tensile deformation of three-layered sheet metals. However, this analysis is very complicated because nonlinear six-dimensional simultaneous equations must be solved.

This paper deals with an analysis of tensile deformation of the laminated sheet metals by using the upper bound method [6] based on Hill's plasticity theory for anisotropic materials [7]. This analysis gives the r-value (Lankford value) and the stress-strain relationship by taking account of the r-value, the workhardening coefficient K, and the workhardening exponent n of each component sheet which makes up the laminated sheet metal. The theoretical results are compared with experimental results for the laminated sheet metals which are produced by adhesive bonding of steel, aluminum, and brass sheet metals.

Contributed by the Production Engineering Division for publication in the JOURNAL OF ENGINEERING FOR INDUSTRY. Manuscript received May 1987.

2 Analysis

In the present analysis, the following assumptions were made for simplicity: (*1*) Anisotropic yield criterion proposed by Hill [7] is used, (*2*) The component sheets of the laminated sheet metal deform uniformly, (*3*) Bonding between the component sheets is perfect, therefore, separation or relative slip does not take place.

Let us take x and y axes on the sheet metal surface and z axis in the direction of thickness. The x axis is taken as the rolling direction of the sheet metal so that x and y directions coincide with the principal axes of anisotropy. Since the thickness of the sheet metal is small, the stress σ_z in z axis is neglected. Then the equivalent stress σ_{eq} is given by

$$\sigma_{eq}=\left[\left(\frac{3}{2}\right)\frac{F\sigma_y^2+G\sigma_x^2+H(\sigma_x-\sigma_y)^2+2N{\tau_{xy}}^2}{(F+G+H)}\right]^{1/2} \quad (1)$$

Equivalent strain increment $d\epsilon_{eq}$ is

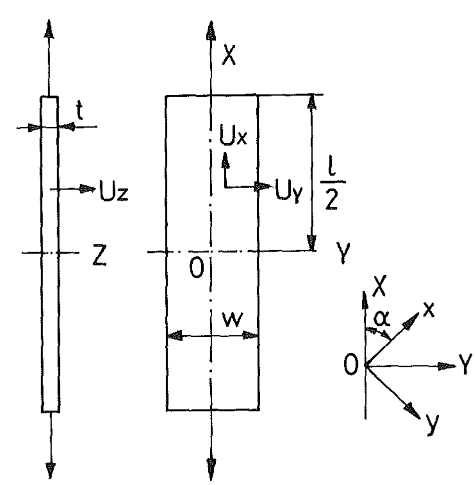


Fig. 1 Geometry of test specimen and admissible velocities

$$d\epsilon_{eq} = \left[\frac{2(F+G+H)}{3}\right]^{1/2} \left[F\left(\frac{Gd\epsilon_y - Hd\epsilon_z}{FG+GH+HF}\right)^2 + G\left(\frac{Hd\epsilon_z - Fd\epsilon_x}{FG+GH+HF}\right)^2 + H\left(\frac{Fd\epsilon_x - Gd\epsilon_y}{FG+GH+HF}\right)^2 + 2\,\frac{(d\epsilon_{xy})^2}{N}\right]^{1/2} \quad (2)$$

where F, G, H, and N are parameters of anisotropy.

When the tensile direction of the sheet metal is inclined at an angle α from the rolling direction, i.e., from x axis, let the tensile direction be X axis and the transverse direction be Y axis as shown in Fig. 1. Then, the strain increments $d\epsilon_x$, $d\epsilon_y$, $d\epsilon_z$, and $d\epsilon_{xy}$ are expressed by using $d\epsilon_X$ and $d\epsilon_Y$ as

$$\left.\begin{aligned} d\epsilon_X &= \frac{d\epsilon_X + d\epsilon_Y}{2} + \frac{d\epsilon_X - d\epsilon_Y}{2}\cos 2\alpha \\ d\epsilon_Y &= \frac{d\epsilon_X + d\epsilon_Y}{2} - \frac{d\epsilon_X - d\epsilon_Y}{2}\cos 2\alpha \\ d\epsilon_{XY} &= -\frac{d\epsilon_X - d\epsilon_Y}{2}\sin 2\alpha \\ d\epsilon_Z &= -(d\epsilon_X + d\epsilon_Y) \end{aligned}\right\} \quad (3)$$

The r-value in tension in X axis, r_α, is given [7] as a function of angle α by

$$r_\alpha = \frac{H(2N-F-G-4H)\sin^2\alpha\cos^2\alpha}{F\sin^2\alpha + G\cos^2\alpha} \quad (4)$$

The r-value for $\alpha = 0$ deg, r_0, is H/G then we have

$$G = \frac{H}{r_0} \quad (5)$$

Similarly, r_{90} and r_{45} for $\alpha = 90$ and 45 deg, respectively, are obtained and then F and N are expressed as

$$F = \frac{H}{r_{90}} \quad (6)$$

and

$$N = (F+G)\left(r_{45} + \frac{1}{2}\right) = \left(\frac{1}{r_0} + \frac{1}{r_{90}}\right)\left(r_{45} + \frac{1}{2}\right)H \quad (7)$$

Substitution of equations (5), (6), and (7) into equation (2) gives the equivalent strain increment $d\epsilon_{eq}$ as

$$d\epsilon_{eq} = \left[\frac{2}{3}\left(\frac{1}{r_{90}} + \frac{1}{r_0} + 1\right)\right]^{1/2} \left[\frac{1}{r_{90}}\left(\frac{\frac{d\epsilon_y}{r_0} - d\epsilon_z}{\frac{1}{r_{90}r_0} + \frac{1}{r_0} + \frac{1}{r_{90}}}\right)^2 + \frac{1}{r_0}\left(\frac{d\epsilon_z - \frac{d\epsilon_x}{r_{90}}}{\frac{1}{r_{90}r_0} + \frac{1}{r_0} + \frac{1}{r_{90}}}\right)^2 + \left(\frac{\frac{d\epsilon_x}{r_{90}} - \frac{d\epsilon_y}{r_0}}{\frac{1}{r_{90}r_0} + \frac{1}{r_0} + \frac{1}{r_{90}}}\right)^2 + \frac{2(d\epsilon_{xy})^2}{\left(\frac{1}{r_0} + \frac{1}{r_{90}}\right)\left(r_{45} + \frac{1}{2}\right)}\right]^{1/2} \quad (8)$$

Since it was assumed that the sheet metal deforms uniformly, admissible velocities U_X, U_Y, and U_Z in X, Y, and Z directions shown in Fig. 1 can be expressed as

$$U_X = aX, \quad U_Y = bY, \quad U_Z = cZ \quad (9)$$

where a, b, and c are constants. Defining the velocity U_X at $X = l/2$ as U, U is given by the velocity of tensile loading as $U = al/2$. The strain increments are obtained from equation (9) as

$$d\epsilon_X = a, \quad d\epsilon_Y = b, \quad d\epsilon_Z = c = -(a+b) \quad (10)$$

The strain increments in x, y, and z directions are rewritten from equation (3) as

$$\left.\begin{aligned} d\epsilon_x &= \frac{a+b}{2} + \frac{a-b}{2}\cos 2\alpha \\ d\epsilon_y &= \frac{a+b}{2} - \frac{a-b}{2}\cos 2\alpha \\ d\epsilon_{xy} &= -\frac{a-b}{2}\sin 2\alpha \\ d\epsilon_z &= -(a+b) \end{aligned}\right\} \quad (11)$$

Substitution of equation (11) into equation (8) leads to the equivalent strain increment $d\epsilon_{eq}$ in terms of a and b.

The relationship between the equivalent stress σ_{eq} and the equivalent strain ϵ_{eq} is expressed as

$$\sigma_{eq} = K\,\epsilon_{eq}{}^n \quad (12)$$

where K and n are the workhardening coefficient and the workhardening exponent, respectively. The equivalent stress σ_{eq} can be derived in terms of a and b from equations (8), (11), and (12). The values of K and n can be obtained as follows. When the sheet metal is extended in the rolling direction, the relationship between the tensile stress σ_0 and the strain ϵ_0 is given [7] by

$$\sigma_0 = K\left[\frac{2(r_0 + r_{90} + r_0 r_{90})}{3(r_{90} + r_0 r_{90})}\right]^{\frac{n+1}{2}} \cdot \epsilon_0^n \quad (13)$$

Therefore, K and n can be obtained by a graphical representation of the stress-strain $(\sigma_0 - \epsilon_0)$ curve in tension test.

The energy dissipation increment dE due to plastic deformation of sheet metal is given by

$$dE = \sigma_{eq} d\epsilon_{eq} \cdot V = \sigma_{eq} d\epsilon_{eq} \cdot (lwt) \quad (14)$$

where V is the volume, l the length, w the width, and t the thickness of sheet metal. dE is expressed in terms of a and b because σ_{eq} and $d\epsilon_{eq}$ are so as mentioned above. The energy dissipation increment dE_L for the laminated sheet metal is the total of the energy dissipation increments for each component. For example, dE_L for two layer sheet metal is

$$dE_L = dE_{\mathrm{I}} + dE_{\mathrm{II}} = \sigma_{eq\mathrm{I}} d\epsilon_{eq\mathrm{I}} \cdot V_{\mathrm{I}} + \sigma_{eq\mathrm{II}} d\epsilon_{eq\mathrm{II}} \cdot V_{\mathrm{II}} \quad (15)$$

where subscripts I and II refer to the components I and II, respectively. Since the energy dissipation increment dE_L is expressed in terms of a and b, and a is given by $a = 2U/l$, dE_L can be regarded as a function of only b. Therefore, dE_L can be minimized by varying the value of b when the angels α_{I} and α_{II} are given. The r-value of the laminated sheet metal, r_L, is obtained from equation (10) as

$$r_L = \frac{d\epsilon_Y}{d\epsilon_Z} = \frac{b_c}{-(a+b_c)} \quad (16)$$

where b_c is the value of b which minimizes dE_L.

Table 1 Mechanical properties of component sheet metals

Metals	α (deg)	r-value	K	n
Steel	0	1.72	559	0.228
	45	1.50		
	90	1.97		
Brass	0	0.85	902	0.353
	45	1.43		
	90	0.87		
Aluminum	0	0.72	160	0.297
	45	1.18		
	90	0.87		

The external work done by the mean tensile stress σ_{XL} of the laminated sheet metal is equal to the energy dissipation increment dE_L and we have

$$2\sigma_{XL}(t_{\rm I}+t_{\rm II})wU=\sigma_{XL}(V_{\rm I}+V_{\rm II})a=dE_L \qquad (17)$$

where, $t_{\rm I}$ and $t_{\rm II}$ are the thicknesses and $V_{\rm I}$ and $V_{\rm II}$ the volumes of the component metals I and II, respectively. The stress σ_{XL} can be calculated from equation (17) in every incremental deformation, and then the stress-strain relationship can be obtained.

Consequently, the r-value and the stress-strain curve can be theoretically derived for the laminated sheet metal whose components are combined at arbitrary angles $\alpha_{\rm I}$ and $\alpha_{\rm II}$ to the rolling direction. The above method gives the same results as that from the composite law when $r_{\rm I}=r_{\rm II}=1$; however, in other case a different value will be derived.

3 Experimental Procedures

3.1 Specimens. Materials used were cold rolled sheet, 60–40 brass sheet and aluminum sheet. The thickness of these sheet metals was 0.5 mm. Tension test specimens were produced by blanking the sheet metals so that the tensile direction was inclined at various angles to the rolling direction. The length of the parallel part of the specimen was 52 mm, the width 15 mm, the gage length 40 mm, the radius of shoulder 12.5 mm. Laminated specimens were made by bonding the above specimens with an adhesive of epoxy synthetic resin. Bond strength was measured by shear test and was about 28.5 to 31.0 MPa.

3.2 Tension Test. Tension tests were carried out by using an Instron universal testing machine at a cross head speed of 1 mm/min. The r-value was obtained at a tensile strain of 0.085 by measuring the width and the gage length with an optical shadowing projector or a micro comparator.

4 Results and Discussion

Table 1 shows the mechanical properties, the r-value, the workhardening coefficient K, and the workhardening exponent n, of the component sheet metals in tension to the directions of $\alpha=0$, 45 and 90 deg. By using these values the r-value and the stress-strain relationship of the laminated sheet metals can be theoretically obtained.

Figures 2(a), (b), and (c) show the calculated results of the r-value for laminated sheet metals with two component metals. In Fig. 2(a), the laminated sheet metal of Steel (0)-Brass (α), for example, means that the steel sheet is inclined at the angle α of 0 deg to the rolling direction and the brass sheet is inclined at various α from 0 to 90 deg. An advantage of the present method is that the r-value of the laminated

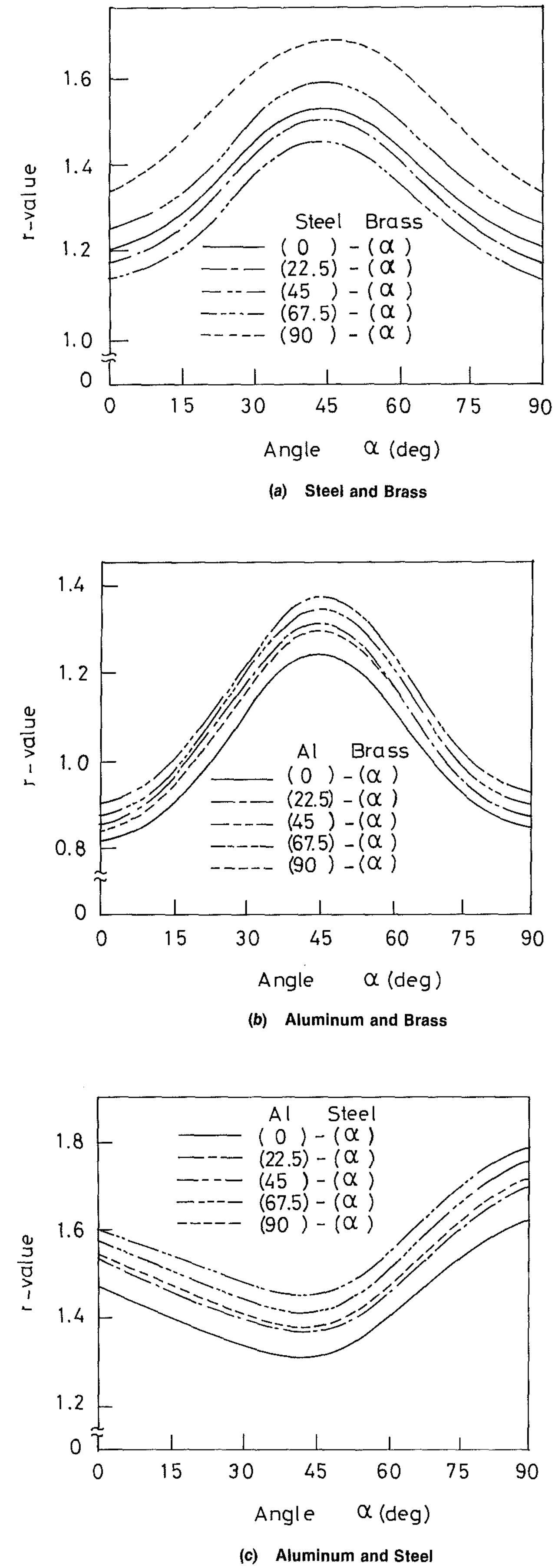


Fig. 2 Calculated results of r-value for laminated sheet metals whose component metals are combined at various angles α

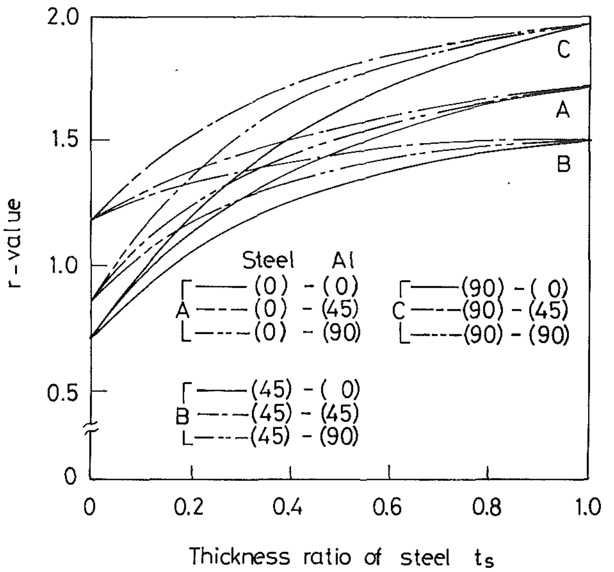


Fig. 3 Relationship between r-value and thickness ratio of steel t_s for laminated sheet metal with steel and aluminum

sheet metal whose component metals are combined at arbitrary angles α can be theoretically obtained by giving r_0, r_{45}, and r_{90} of the component metals. On the other hand, to determine the r-value of the laminated sheet metal from the composite law, the r-values of the component metals at every angle must be known. This means that the r-value cannot be predicted from the composite law in practice.

Figure 3 shows the effect of thicknesses of the component metals on the r-value for the laminated sheet metal with the steel and the aluminum. t_s is thickness ratio of steel. According to the composite law, the r-value of the laminated sheet metal is proportional to the r-value and the thickness of each component sheet metal, that is, relationship between the r-value and t_s is linear. However, the present analysis gives nonlinear relationship and slightly higher values than those from the composite law.

Figures 4(a), (b), and (c) show the tensile stress σ_{XL} at a strain of 0.1 in the stress-strain curve. The tensile stress σ_{XL} of the laminated sheet metal whose component metals are combined at arbitrary angles can be theoretically derived.

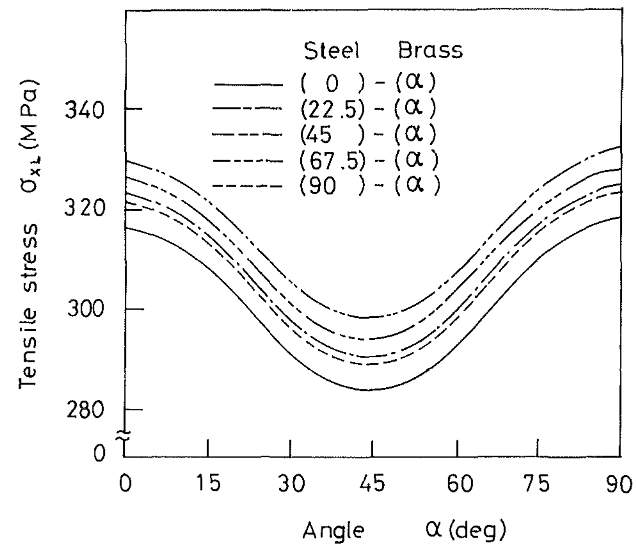
Figure 5 and Fig. 6 show comparison between the experimental and calculated results of the r-value and the tensile stress σ_{XL}, respectively, for the laminated sheet metal of Steel (0)-Brass (α). The calculated values are rewritten from Fig. 2(a) and Fig. 4(a). It is found from these figures that the calculated results are in good agreement with the experimental ones.

Figure 7 shows a comparison between the experimental and calculated results of the r-value for various combinations of the component metals. The calculated and experimental results are in good agreement. Therefore it can be said that the present analysis is effective for estimating the tensile deformation of the laminated sheet metals.

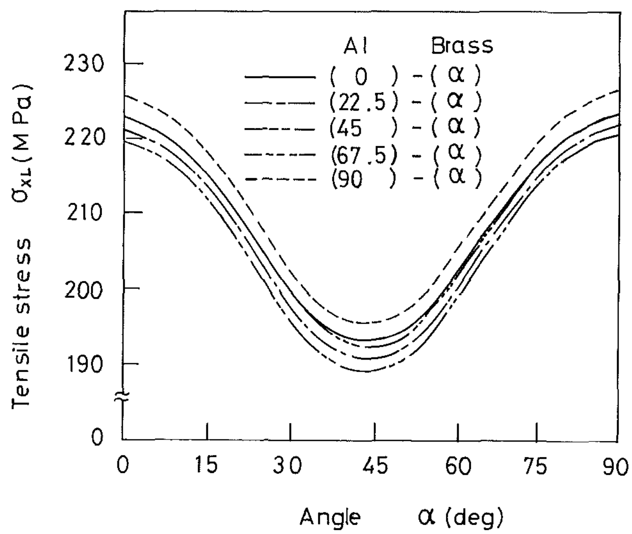
It is well known that the limiting drawing ratio (LDR) increases with increasing r-value. Therefore, the LDR of a sheet metal having a small r-value can be improved by laminating with other sheet metal having a large r-value. Deformation of the laminated sheet metal is dependent on the combination of the component metals.

5 Conclusions

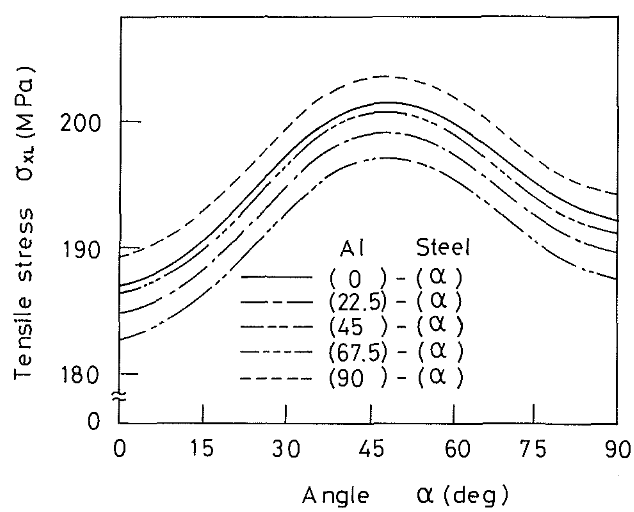
Tensile deformation of the laminated sheet metal was analyzed by the upper bound method based on Hill's plasticity theory for anisotropic materials. Tension tests of the adhesive-



(a) Steel and Brass



(b) Aluminum and Brass



(c) Aluminum and Steel

Fig. 4 Calculated results of tensile stress σ_{XL} at strain of 0.1 for laminated sheet metals whose component metals are combined at various angles α

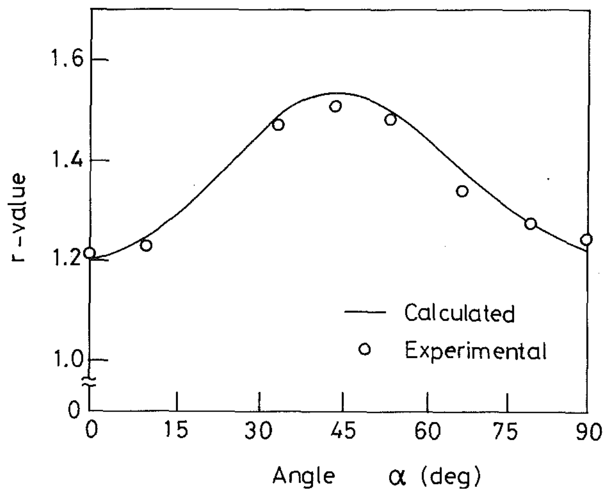


Fig. 5 Comparison between experimental and calculated results of r-value for laminated sheet metals with steel of $\alpha=0$ deg and brass of various α

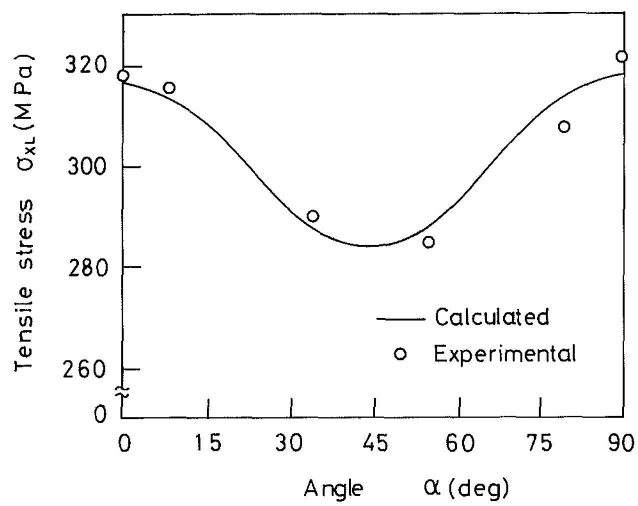


Fig. 6 Comparison between experimental and calculated results of tensile stress σ_{XL} at strain of 0.1 for laminated sheet metals with steel of $\alpha=0$ deg and brass of various α

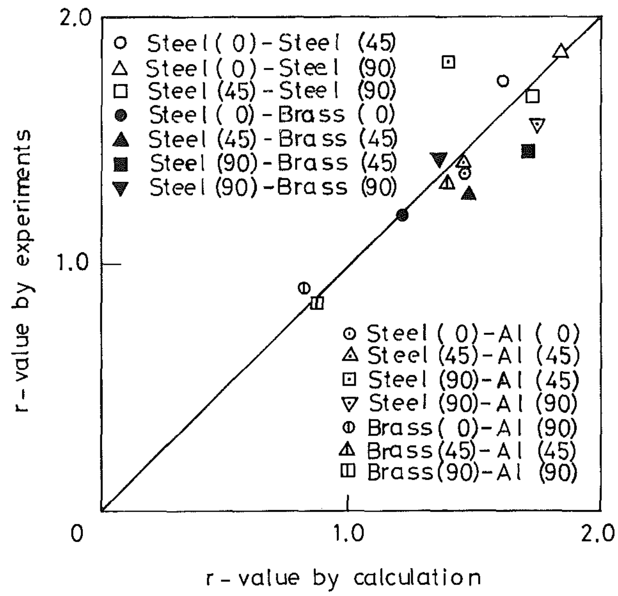


Fig. 7 Comparison between experimental and calculated results of r-value

bonded sheet metal were carried out and the experimental results are compared with the calculated ones.

The r-value of the laminated sheet metal whose component metals are combined at arbitrary angles α to the rolling direction was theoretically derived by giving the r-values r_0, r_{45}, and r_{90} of the component metals in tension to the direction of $\alpha=0$, 45 and 90 deg, respectively. The stress-strain curve of the laminated sheet metal was also derived by giving the workhardening coefficient and the workhardening exponent of the component metals in tension to the direction of $\alpha=0$ deg. The calculated results agreed well with the experimental ones. Therefore, the present method was found to be effective for estimating the tensile deformation of the laminated sheet metals.

The authors wish to express their appreciation to Professor K. Yamaguchi of Kyoto Institute of Technology for his valuable comments.

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