Water Table Dynamics in Tile-Drained Fields

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ABSTRACT

We present a method for simulating water table dynamics in tile-drained fields that are subject to intermittent precipitation or irrigation. A stochastic state equation for the water table height midway between drain laterals is obtained by adding a random noise term to van Schilfgaarde's deterministic drainage model. The random term accounts for dynamics not modeled by van Schilfgaarde's equation, which is based on numerous simplifying assumptions. The continuous-discrete Kalman filter is used to obtain an estimate of the time variation of the water table height, as well as the variance of the estimate. The method is demonstrated using experimental data from the literature and is shown to have advantages over alternative approaches. Additional testing is necessary to fully assess the validity of the stochastic state equation and the utility of the filtering procedure.

In areas with shallow groundwater, it is common practice to use subsurface drainage systems to lower the water table. Drainage systems prevent waterlogging of surface soils and thereby enhance the potential for agricultural development. Researchers have studied extensively the problem of predicting water table shapes, depths, and dynamics for different drainage designs and soils; an overview can be found in the monograph edited by van Schilfgaarde (1974a). Because of the continuing desire to make low quality lands agriculturally productive, and because of present-day concerns about the effect of agricultural practices on water quality and supply, soil drainage systems remain a subject of practical and theoretical interest.

Mathematical modeling is frequently used to study water table dynamics in drained fields. Available models range from classic analytical drainage equations that are based on the Dupuit-Forchheimer theory of groundwater flow to numerical models that solve some form of the Richards' equation. Irrespective of their sophistication, mathematical drainage models will always be simplified representations of the actual flow and drainage processes that occur in the field. Consequently, modeling estimates and predictions of water table dynamics are approximate or uncertain.

As an alternative to modeling, water table dynamics can be studied by monitoring the water table in a drained soil. Direct measurements of the water table eliminate much uncertainty but are labor intensive and expensive, cannot be used to make forecasts, and are not easily extrapolated to other drainage designs and soils.

The objectives of our study were to develop a stochastic state equation that describes water table dynamics in tile-drained fields, and to use a filtering method to obtain estimates and predictions of the water table dynamics. Developing the state equation within a stochastic framework allows us to perform simulations while acknowledging and accounting for the approximate nature of the mathematical model. The filtering method combines modeling and field measurements to obtain estimates and predictions that are an improvement over those obtained using either measurements or modeling exclusively. Filtering methods are used widely in information and control sciences (e.g., Gelb, 1974) and have also been used in surface and subsurface hydrology (e.g., Bras and Rodríquez-Iturbe, 1985; Morkoc et al., 1985; Milly and Kabala, 1985; Graham and McLaughlin, 1989; Or and Hanks, 1992; Parlange et al., 1993; Katul et al., 1993; Nielsen et al., 1994).

This paper is organized as follows. First, van Schilfgaarde's (1965) deterministic model for the midway water table height is presented. A stochastic state model for the water table height is then obtained by adding a random noise term to van Schilfgaarde's equation. Next, we introduce a measurement model that accounts for spatial averaging of water table measurements and measurement errors. Finally, the Kalman filter is used to estimate and predict the midway water table dynamics, as well as the variance of the estimates and predictions. The filtering method is demonstrated using field data from the literature.

THEORY

Deterministic Model

Figure 1 shows a vertical cross section of an idealized tile-drained field. The drains are located a distance h above an impervious layer, with drain spacing L and effective drain diameter d. Midway between drain laterals, the water table height relative to the drain elevation is denoted m.

Van Schilfgaarde (1965, 1970, 1974b) describes the time variation of m in response to intermittent recharge as

$$\frac{dm}{dt} + \frac{m}{B} = u, \qquad B = fCLF/K \qquad [1]$$

where K is the isotropic saturated hydraulic conductivity, f is the drainable porosity, and $u(t)$ is the groundwater recharge rate. The dimensionless constant $F(h/L, d/L)$ is from Kirkham's (1958) solution for the steady-state water table height and is defined

$$F = \frac{1}{\pi}\left[\ln\frac{2L}{\pi d} + \sum_{n=1}^{\infty}\frac{1}{n}\left(\cos\frac{n\pi d}{2L} - \cos n\pi\right)\left(\coth\frac{2n\pi h}{L} - 1\right)\right] \qquad [2]$$

The dimensionless parameter C was introduced by Bouwer and van Schilfgaarde (1963) to correct for the fact that the rate of water table drop midway between drains is generally different from the average rate of drop between drains. The parameter C may be thought of as the ratio of the average

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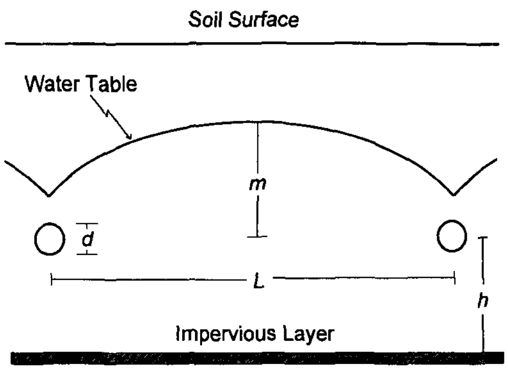

Fig. 1. Vertical cross section of an idealized tile-drained field.

downward water table flux between drains to the flux midway between drains, and usually has a value between 0.8 and 1.0 (Bouwer and van Schilfgaarde, 1963). The parameter B has units of time and combines the soil properties f and K, the system geometry parameters L and f, and the correction factor C into a single system-defining constant (van Schilfgaarde, 1974b). The key assumption underlying Eq. [1] is that the instantaneous drainage rate midway between drains is equal to the steady-state drainage rate that corresponds with the water table height m, where the steady-state drainage rate is given by Kirkham's (1958) solution to the steady flow problem.

Stochastic State Model

Equation [1] is an approximate model that relies on idealizations of soil properties, system geometry, and water flow processes. When an approximate model is used to describe a complex system, it may be desirable to quantify in some way the dynamics that are not accounted for in the model. The simplifications in Eq. [1] are too numerous and ill-defined to make an exact accounting. Rather than attempting to improve or refine Eq. [1], we assume that the unmodeled dynamics can be represented as an additive stochastic error term. With this assumption, water table dynamics are described by the stochastic state model

$$\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{x}{B} = u + w, \qquad [3]$$

where x is the state variable that corresponds with the midway water table height and w is a white noise, zero-mean random process with spectral density q. For simplicity, we take q to be constant in time.

Due to the randomness in Eq. [3], we can describe or predict the time evolution of x only in a statistical sense. The equation governing the time variation of the mean of x, $\langle x \rangle$, is found by taking the expectation of Eq. [3],

$$\frac{\mathrm{d}\langle x \rangle}{\mathrm{d}t} + \frac{\langle x \rangle}{B} = u \qquad [4]$$

Similarly, it can be shown that the time variation of the state variance, $P = \langle (x - \langle x \rangle)^2 \rangle$, is (e.g., Lewis, 1986)

$$\frac{\mathrm{d}P}{\mathrm{d}t} + \frac{2P}{B} = q \qquad [5]$$

The Kalman Filter

Given initial conditions $\langle x(t_0) \rangle = x_0$ and $P(t_0) = P_0$, Eq. [4] and [5] can be used to predict the time evolution of $\langle x \rangle$ and P. Suppose at time $t_k > t_0$ a measurement of x becomes available. We would like to use the information provided by the new measurement to update our predictions of $\langle x \rangle$ and P. The process of predicting and updating as measurements become available is known as filtering.

Let the state measurement be represented as

$$z_k = x(t_k) + v_k \qquad [6]$$

where z_k is the measured state at time t_k, $x(t_k)$ is the (unknown) true state at time t_k, and v_k is a zero-mean random measurement error with variance r_k. The error term v_k accounts for uncertainty arising from spatial averaging of point measurements, as well as any instrument or operator error.

The projection Eq. [4] and [5] and the measurement Eq. [6] form the basis of the filtering algorithm. Starting with estimates of x_0 and P_0, the projection equations are used to predict the evolution of $\langle x \rangle$ and P up until the time at which a state measurement becomes available. At that time, a linear combination of the projected and the measured state is taken as the updated state estimate, and the state variance estimate is updated accordingly.

More specifically, let $\langle x(t_k) \rangle^-$ be the state projected by Eq. [4] at time t_k prior to the measurement update, and $\langle x(t_k) \rangle^+$ be the estimated state at time t_k after the update. Likewise, let $P(t_k)^-$ and $P(t_k)^+$ be, respectively, the projected and updated state variance at time t_k. Assuming w and v are uncorrelated with x_0 and each other, the Kalman filter equations for updating the state and variance estimates are (e.g., Lewis, 1986):

$$\langle x(t_k) \rangle^+ = \langle x(t_k) \rangle^- + K_k[z_k - \langle x(t_k) \rangle^-] \qquad [7]$$

$$P(t_k)^+ = (1 - K_k)P(t_k)^- \qquad [8]$$

$$K_k = P(t_k)^-/[P(t_k)^- + r_k] \qquad [9]$$

where K_k is referred to as the Kalman gain. Equations [7] through [9] are known as the continuous-discrete form of the Kalman filter because of the time-continuous projection Eq. [4] and [5] and the discrete measurement Eq. [6]. From [7] and [9], it is seen that $\langle x(t_k) \rangle^+$ is a linear combination of $\langle x(t_k) \rangle^-$ and z_k, with the weight given to the two terms being determined by the relative size of the projected state variance $P(t_k)^-$ and the measurement variance r_k. When $P(t_k)^-$ is large relative to r_k, more weight is given to the measurement than to the projection. Conversely, when $P(t_k)^-$ is small relative to r_k, more weight is given to the projection than to the measurement.

If v and w are Gaussian variates and the initial conditions and model parameters are known exactly, it can be shown that the Kalman filter provides an estimate of x that is optimal in the mean square error sense (e.g., Gelb, 1974). In the current problem we must estimate some model parameters and may not be able to verify the other conditions, meaning the filtered estimate may be suboptimal.

Parameter Value Identification

Parameters in the state and measurement models are L, h, d, f, K, C, q, r_k, x_0, and P_0. Most of the parameters can be estimated from field measurements, except for the model error spectral density q and the initial state variance P_0. It is therefore necessary to obtain values for these parameters by fitting the filter to a series of state measurements (e.g., Bras and Rodríquez-Iturbe, 1985).

Assume we have the series of state measurements z_k with variances r_k, $k = 1, \ldots, N$. Using the method of maximum likelihood, values for q and P_0 are found by maximizing the objective function (Bras and Rodríquez, 1985, p. 492–496)

$$\xi(q, P_0|z_k, r_k) = -N \ln(2\pi) - \sum_{k=1}^{N} \ln |r_k + P(t_k)^-| - \sum_{k=1}^{N} \frac{[z_k - \langle x(t_k)\rangle^-]^2}{r_k + P(t_k)^-}, \quad [10a]$$

subject to the constraint

$$q, P_0 \geq 0 \quad [10b]$$

The optimization was accomplished using a steepest-descent algorithm.

FIELD EXPERIMENT

Kirkham and de Zeeuw (1952) present field measurements that are designed to test soil drainage theory. The experiment site is located in the Netherlands and is a flat field that is reported to be "an area of high uniformity and low permeability" (Kirkham and de Zeeuw, 1952). The surface soil is a fine sand that contains some silt and clay. The low permeability is attributable to the clay and silt content being large enough to fill the space between sand particles, but not large enough to develop good soil structure (Kirkham and de Zeeuw, 1952). A relatively impermeable peat layer exists at 1.8 m below the surface. Tile drains are located 0.97 m below the surface and have an effective diameter of 0.09 m (Kirkham, 1958).

Approximately 75 saturated hydraulic conductivity measurements were made across the field at various depths using the auger hole method and the piezometer method (Kirkham and de Zeeuw, 1952). Above the impermeable layer at 1.8 m, the conductivity measurements vary from ≈50 to 150 mm d^{-1}, with a trend of decreasing conductivity with depth (see scatter plot of conductivity values in Fig. 4, Kirkham and de Zeeuw, 1952). Two measurements estimated the drainable porosity to be <6% in the soil layer from 0.01 to 0.19 m below the soil surface, and <2.5% in the layer from 0.3 to 0.6 m below the surface. A subsequent analysis by Kirkham (1964, p. 588) suggested that the effective drainable porosity was around 2%. The measured daily rainfall is shown in Fig. 2. Rainfall measured on Days 3, 10, and 17 are 2-d totals (measurements were not made on Days 2, 9, and 16). The field was cropped in clover (*Trifolium* spp.) during the experiment. Evapotranspiration rates were low throughout the study; open-pan evaporation averaged 0.4 mm d^{-1}.

Midway water table heights were measured daily for 21 d (24 November–14 December). Observation wells were located midway between drains and on a line perpendicular to the tile lines. The reported heights are the average of measurements made in six wells. The standard deviations of the daily measurements are also given (Kirkham and de Zeeuw, 1952). Results are reported for drain spacings of 8, 10, 12, and 16 m; we focus on the 8-m spacing data.

RESULTS

To apply Eq. [4] it is necessary to specify the relationship between rainfall and the groundwater recharge rate u. As a first approximation, we assume the rainfall rate is constant between rainfall measurements and take the recharge rate to be equal to the rainfall rate divided by the drainable porosity (no time lag),

$$u(t) = \frac{1}{f}\left(\frac{R_k}{t_{k+1} - t_k}\right) \qquad t_{k+1} > t > t_k \quad [11]$$

where R_k is the measured rainfall between times t_k and t_{k+1}. Evapotranspiration is assumed to be negligible.

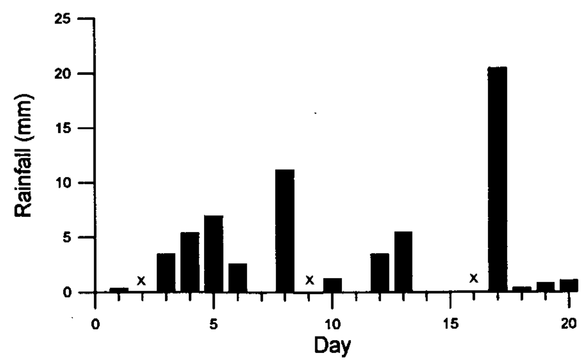

Fig. 2. Rainfall measured by (Kirkham and de Zeeuw, 1952). Measurements were not made on Days 2, 9, and 16; measurements on Days 3, 10, and 17 are 2-d totals.

This simple representation of recharge may be a reasonable approximation given the shallow water table and low drainable porosity.

Based on the field measurements discussed above, we arrive at the following values for the model parameters: L = 8 m, d = 0.09 m, h = 0.86 m, f = 0.02, and K = 0.1 m d^{-1}. The value K = 0.1 m d^{-1} is near the middle of the range of conductivities measured by Kirkham and de Zeeuw, 1952) and is the value used by Kirkham (1958) in his steady-state analysis of the same data set. The drainable porosity value of 2% is based on Kirkham's (1964) calculated value of 1.96% and the field measurement that indicated a drainable porosity of <2.5% at depths where the water table is expected to be found most often. The choice of parameter values for drainable porosity and hydraulic conductivity is further discussed below. Because the data do not provide a basis for choosing otherwise, the flux correction parameter is assumed to be unity (C = 1). We take the first day of measurements to be Day Zero, and set the initial state x_0 equal to the first water table measurement z_0 with r_0 = 0. The measurement variance for the next 20 d, r_k, k = 1, . . . , 20, is equal to the sample variance of the six spatially averaged daily water table measurements; we thus assume any instrument or operator error is negligibly small.

As noted earlier, we obtain values for q and P_0 by fitting the filter to a series of state measurements. We use the first 10 water table measurements, z_k, k = 1, . . . , 10, and fit the two parameters by maximizing Eq. [10]. Ordinarily it would be advisable to use all available data when fitting unknown parameters, but here we use only the first 10 measurements so that the filter can be evaluated on data for which there was no filter or model calibration. The result of the fitting is q = 0.0026 m^2 d^{-1} and P_0 = 0 m^2.

With all parameters quantified, the Kalman filter is now used to estimate the midway water table height from time t = 0 through time t = 20 d. The filtered water table estimate is shown in Fig. 3. The solid line is $\langle x \rangle$ and the dashed lines are $\langle x \rangle \pm 2(P)^{0.5}$. Also shown for reference are the measured water table heights z_k (solid squares) and measurement error standard deviations [error bars equal to $z_k \pm 2(r_k)^{0.5}$]. The filtered water table provides a good representation of the water table

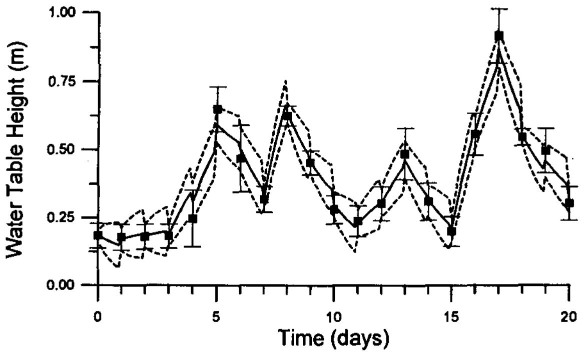

Fig. 3. Water table dynamics modeled using the Kalman filter. The water table height is relative to the drain elevation. The solid line is the state estimate $\langle x \rangle$ and the dashed lines are $\langle x \rangle \pm 2(P)^{0.5}$. Also shown are the measured water table heights z_k (solid squares) and measurement standard deviations [error bars equal to $z_k \pm 2(r_k)^{0.5}$].

dynamics. At the time of the measurement updates, the state variance P is, in general, slightly smaller than the measurement error variance r_k. As expected from filtering theory, the state variance (uncertainty) grows as the water table is projected between measurement updates.

As noted above, filtering combines measurement and modeling approaches. For comparison purposes, it is worthwhile to consider what can be obtained using either measurement or modeling exclusively. In the case of using only measurements, no information exists about the evolution of the water table or the uncertainty between measurements, so we can only assume that the measured water table and its uncertainty remains the same until a new measurement becomes available. This type of measurement model has been referred to as the *persistence* model (Milly and Kabala, 1985). Figure 4 shows the persistence model for the midway water table height. The measurements are shown as solid squares. The error bars arise from spatially averaging the six measurements and are $\pm 2(r_k)^{0.5}$.

The water table prediction resulting when only modeling is used is shown in Fig. 5. This result is obtained by solving Eq. [1] subject to the initial condition $m(t_0) = m_0$, where m_0 is the water table measurement made at time zero. The water table height used as the initial condition is shown as a solid square. The remaining measurements are not used in the simulation and are shown as open squares. The model prediction is shown as a solid line and is a reasonable representation of the measured data. However, this approach does not

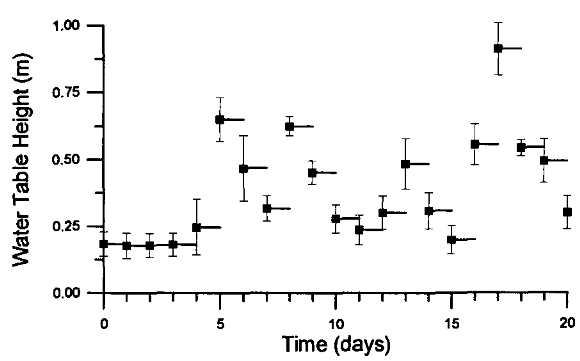

Fig. 4. Water table dynamics as represented by the persistence model.

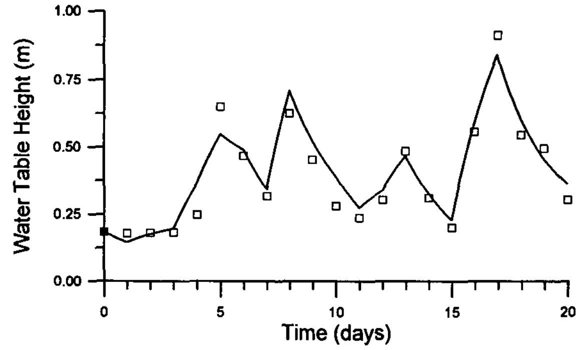

Fig. 5. Water table dynamics modeled using the deterministic model, Eq. [1]. Measurements shown as open squares are not used and are included only for comparison.

provide any information about the uncertainty of the prediction.

When there is an abundance of data available for constructing the persistence model, it may be argued that for many practical purposes the persistence model is a sufficient representation of the water table and related uncertainties, particularly if some sort of interpolation scheme is assumed. However, we emphasize that once sufficient data is obtained to calibrate the filter, it is possible to predict the water table with much less data than are required by the persistence model. Suppose, for example, we made water table measurements for 10 d and fitted the filter parameters q and P_0 as before. Then suppose we make only one more water table measurement, at Day 15. The resulting filtered water table is shown in Fig. 6, and the persistence model in Fig. 7. Figure 6 shows that the state variance grows initially following the measurement at Day 10 and then reaches a maximum value, which is determined by q, about 1 d later (Day 11). The variance is reduced following the measurement at Day 15 and then grows again as the prediction gets further away from the update. In this case, the filtering method is clearly superior to the persistence method in terms of modeling water table dynamics.

Alternatively, we could suppose that we stop collecting water table data entirely after fitting q and P_0. In this case, no updates are made after Day 10 and the filter is used only for prediction. The resulting filtered water table prediction is shown in Fig. 8 and is similar

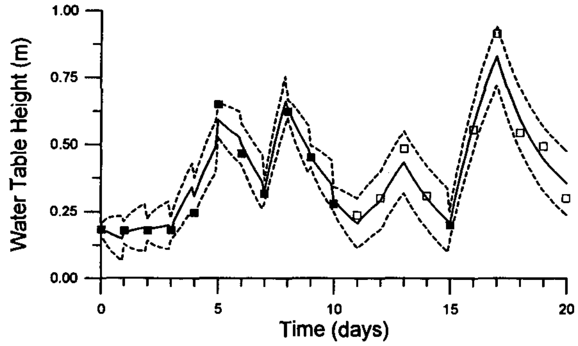

Fig. 6. Filtered water table dynamics. Measurements shown as open squares are included for comparison and are not used in the filter calculations.

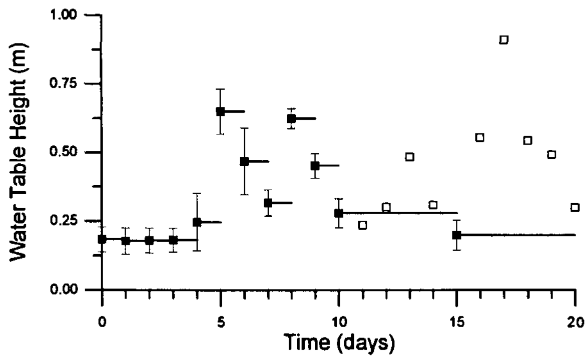

Fig. 7. The persistence model for the case of limited water table data.

to that seen in Fig. 6, except of course for a lack of reduction in the state variance at Day 15.

Fitting K and f

An interesting issue is the appropriateness of fitting certain model parameters rather than relying on physical measurements. There is, after all, considerable ambiguity as to how point measurements of spatially variable soil properties can be translated into the effective, lumped parameters that are used in the state model. For example, we arbitrarily took the (approximate) median value of the conductivity measurements to be the effective conductivity, although it could be argued that another value may be equally or more appropriate.

Values for K and f were obtained independent of the Kalman filter equations by fitting the deterministic model, Eq. [1], to the water table data. The fitting is accomplished by minimizing

$$J(f, K|z_k) = \sum_{k=1}^{N} [z_k - m(t_k)]^2 \qquad [12a]$$

subject to the constraints

$$K \geq 0 \qquad [12b]$$

$$0 \leq f \leq 1 \qquad [12c]$$

where $m(t_k)$ is the water table height at $t = t_k$ computed by Eq. [1]. We again use the first $N = 10$ measurements and the resulting parameters are $f = 0.018$ and $K = 0.12$ m d^{-1}, which are similar to the field measurements used above ($f = 0.02$ and $K = 0.10$ m d^{-1}). In this

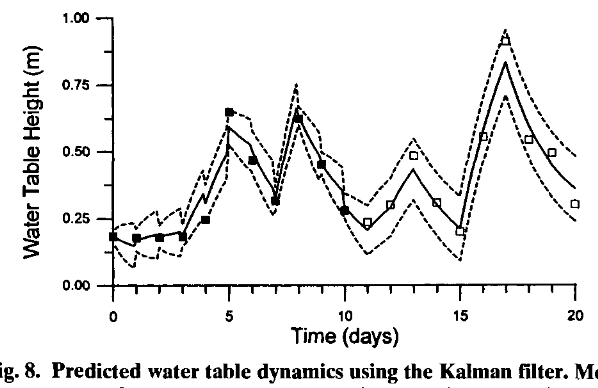

Fig. 8. Predicted water table dynamics using the Kalman filter. Measurements shown as open squares are included for comparison and are not used in making the prediction.

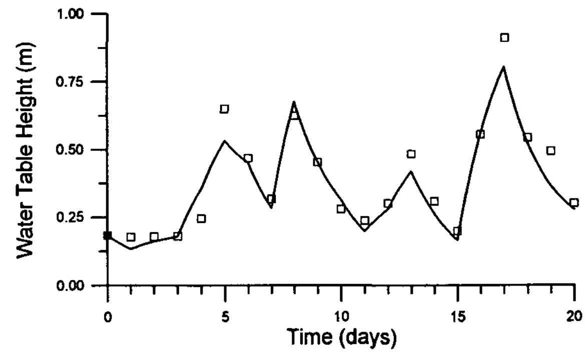

Fig. 9. Deterministic model prediction of water table dynamics when f and K are fit.

instance, the fitted values are so close to the measured values that we expect that model predictions will be affected only minimally. The water table predicted by the deterministic model, Eq. [1], using the fitted parameter values is shown in Fig. 9. As expected, there is only a slight change from the prediction seen in Fig. 5. The closeness of the measured and fitted parameter values provides support for the validity of Eq. [1], although it should be remembered that the "measured" drainable porosity value of 2% is based on a single field measurement that indicated a drainable porosity of <2.5%, and on Kirkham's (1964) analysis that calculated a drainable porosity of 1.96%. Thus, claiming 2% as an independently measured value may be questionable. In any event, when model parameter data are scarce or uncertain, fitting the parameters provides a viable alternative.

When the fitted values of f and K are used, the subsequent optimization of Eq. [10] with $N = 10$ yields $q = 0.0013$ m^2 d^{-1} and $P_0 = 0.00097$ m^2. Because K and f have been fitted, Eq. [1] better matches the data and the optimal model error spectral density q is found to be smaller than before ($q = 0.0026$ m^2 d^{-1}). The resulting filtered water table is shown in Fig. 10. The smaller model error results in a state variance that is generally smaller than that shown in Fig. 3.

SUMMARY AND CONCLUSIONS

We have presented a method for estimating and predicting water table dynamics in tile-drained fields. A stochastic state model was used to model the water table height midway between drain laterals, and the Kalman filter was used to estimate the time evolution of the

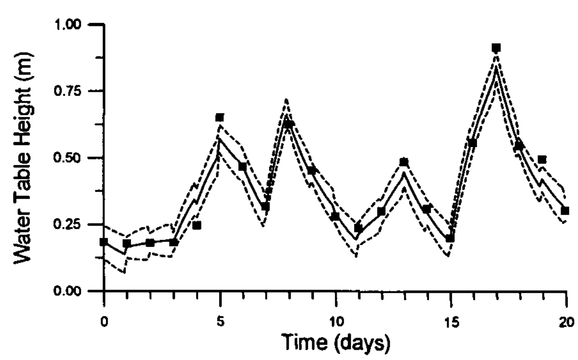

Fig. 10. Filtered water table dynamics when f and K are fit.

state mean and variance. The method was demonstrated using experimental data and was shown to have advantages over methods that rely exclusively on either modeling or field measurements.

With the widespread availability of high-speed computers, the trend in simulating drainage and other subsurface processes has been to develop more and more complex numerical models based on partial differential equations with spatially distributed parameters. The justification for the more complex models is that older analytical models with lumped parameters invoke too many simplifying assumptions and are not sufficiently accurate. However, difficulties in validating numerical models, the large data requirements of numerical models, and the high computational costs of numerical models may cast some doubt on the utility of the numerical approach.

In a general sense, the approach presented here represents an effort to overcome some of the limitations of an analytical water table model by recasting it as a stochastic state model that accounts for simplifying assumptions through the introduction of an error term. The resulting model equations are easy to solve and have relatively minor data requirements. Although the results presented here look promising, the success of this approach can be assessed only after testing it on a wide range of soil and drainage conditions.

With regard to the particular model used in this work, some modifications may be necessary for more general usage. First, in determining the recharge rate u, it will be necessary to provide a more detailed account of the surface and vadose zone water fluxes. In fields with high evapotranspiration rates and thick unsaturated zones, the simple rainfall–recharge relationship used here will be inadequate. Second, the soil surface elevation does not enter into Eq. [3], and consequently there is no provision for surface ponding. For a sufficiently high recharge, the model predicts that the water table rises to a height that is above the soil surface. This occurred, for example, when the filter was used to estimate water table dynamics in the Kirkham and de Zeeuw (1952) field with drains spaced 16 m apart. In such cases, it will be necessary to switch to a ponding–runoff model when the water table reaches the soil surface. Similarly, there is no accounting for the effects of regional flow on the water table elevation. Lastly, in some soils, the heterogeneity of hydraulic properties may result in water table dynamics that are significantly different than that described by Eq. [1] and the assumption that unmodeled dynamics can be accounted for with an additive stochastic error term may be inappropriate. For the case of layered heterogeneity, it is possible to write an equation for the time variation of m that is in the same form as Eq. [1] (see van Schilfgaarde, 1965, 1970, 1974b). The use of this equation should alleviate some difficulties that may be encountered in the analysis of layered soils.

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