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Journal of Biomechanics 35 (2002) 775–783

JOURNAL OF BIOMECHANICS

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Measurements of wave speed and reflected waves in elastic tubes and bifurcations

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Accepted 5 February 2002

Abstract

Wave intensity analysis is a time domain method for studying waves in elastic tubes. Testing the ability of the method to extract information from complex pressure and velocity waveforms such as those generated by a wave passing through a mismatched elastic bifurcation is the primary aim of this research. The analysis provides a means for separating forward and backward waves, but the separation requires knowledge of the wave speed. The PU-loop method is a technique for determining the wave speed from measurements of pressure and velocity, and investigating the relative accuracy of this method is another aim of this research.

We generated a single semi-sinusoidal wave in long elastic tubes and measured pressure and velocity at the inlet, and pressure at the exit of the tubes. In our experiments, the results of the PU-loop and the traditional foot-to-foot methods for determining the wave speed are comparable and the difference is on the order of $2.9 \pm 0.8\%$. A single semi-sinusoidal wave running through a mismatched elastic bifurcation generated complicated pressure and velocity waveforms. By using wave intensity analysis we have decomposed the complex waveforms into simple information of the times and magnitudes of waves passing by the observation site.

We conclude that wave intensity analysis and the PU-loop method combined, provide a convenient, time-based technique for analysing waves in elastic tubes.

Keywords: Wave speed; Wave reflection; Elastic bifurcations and tubes; Wave intensity; PU-loop

1. Introduction

The aim of this research is to test experimentally the relative accuracy of the PU-loop method for measuring wave speed in elastic tubes. Also, to test the ability of wave intensity analysis to determine the times and the magnitudes of waves passing by an observation site, in elastic tubes and bifurcations, from the complex pressure and velocity waveforms. The reflections coefficient of elastic bifurcations is also investigated.

While some researchers have chosen to investigate blood flow in arteries by constructing mock circulations with specifications that simulate in vivo conditions (Bowles et al., 1991), we have taken an alternative approach. The circulatory system is complicated and attempts to simulate it physically can sometimes generate more questions than useful answers. We therefore designed a simple but well characterised system that can be used for studying the times and magnitudes of waves in elastic tubes.

The wave speed, c, from the 1D wave theory (Skalak, 1972) is

$$c = \sqrt{\frac{1}{\rho D}}, \qquad (1)$$

where D is the distensibility of the conduit and ρ the density of the fluid. It has been observed that wave speeds in the elderly are higher than in younger people (Avolio, 1990). This phenomenon is thought to be a result of the aorta losing its elasticity, and the reduced distensibility leads to a higher wave speed. Accordingly, we studied the wave speed in a latex tube with a reinforced wall and compared the results with those obtained in the same sized tube with an ordinary elastic wall.

Moens (1878) and Kortweg (1878) arrived independently in the same year to the equation that is named after them, which is a special form of Eq. (1), and only valid for thin walled tubes with homogeneous elastic

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0021-9290/02/$ - see front matter
PII: S0021-9290(02)00025-8

properties

$$c = \sqrt{\frac{Eh}{\rho d}}, \quad (2)$$

where E is the Young's modulus, h is wall thickness and d is internal diameter. From Eq. (2) we expect different wave speeds for tubes with the same wall properties but different diameters. Therefore, we investigate the relative accuracy of the PU-loop method, by comparing its results to the results of the traditional foot-to-foot method in elastic tubes of different diameters but same wall thickness.

Several methods have been proposed to identify the reflection sites in vivo. Westerhof et al. (1973) used the input impedance to calculate the distance to the nearest reflection site and Van Den Bos et al. (1976) introduced a method based on the time delay between the forward and backward components of the pressure waveform. Murgo et al. (1980) suggested the temporal time from the initial pressure upstroke to the pressure inflection point is the time that takes the wave to run forward, be reflected and arrive back, and with knowledge of the wave speed the distance to reflection site can be determined. More recently Pythoud et al. (1996a) proposed the "reflection profile" method to determine the distances of the most important reflection sites by deconvoluting the backward pressure wave. In this paper, we examine the relative accuracy of wave intensity analysis to determine the distance to the reflection site of open or closed-ended elastic tubes. Also, because of the physiological significance of the arrival time of the reflected waves at the aortic root we investigate the ability of wave intensity analysis to determine the arrival times and the magnitudes of the waves reflected from a mismatched bifurcation and open and closed ended elastic tubes.

2. Methods

A schematic diagram of the experiment is shown in Fig. 1 and a description of the individual elements follows.

2.1. The pump

A positive displacement syringe pump was used to generate an approximately sinusoidal pulse wave. It consists of a cylinder with a 3 cm internal diameter, and a piston that is driven by a connecting rod attached to a wheel (Scotch yoke), which generates a sinusoidal pattern. An electric motor running at a constant speed of 20 rpm drives the wheel to produce a linear displacement of 4 cm and a displaced volume of 28 ml. In all of the experiments we generated and analysed a single half-sinusoidal pulse, in which the piston moved forward approximately from its bottom to its top dead centre position.

2.2. Tubes and bifurcations

In all of the experiments we used elastic tubes made of latex (3S Health Care, London, UK). Tubes were uniform along their lengths with a circular cross-sectional area and were obtained with a standard length of 110 cm. When we needed to use a longer tube, we attached two tubes by glueing them together with liquid latex. We constructed elastic bifurcations by similarly attaching the two daughter tubes to the mother tube and glueing them with liquid latex with a bifurcation angle of 30°. We used tubes with 1 and 0.5 in diameter for which we measured the wall thickness and found there was no difference between the two tubes. It was assumed that all tubes had the same Young's modulus.

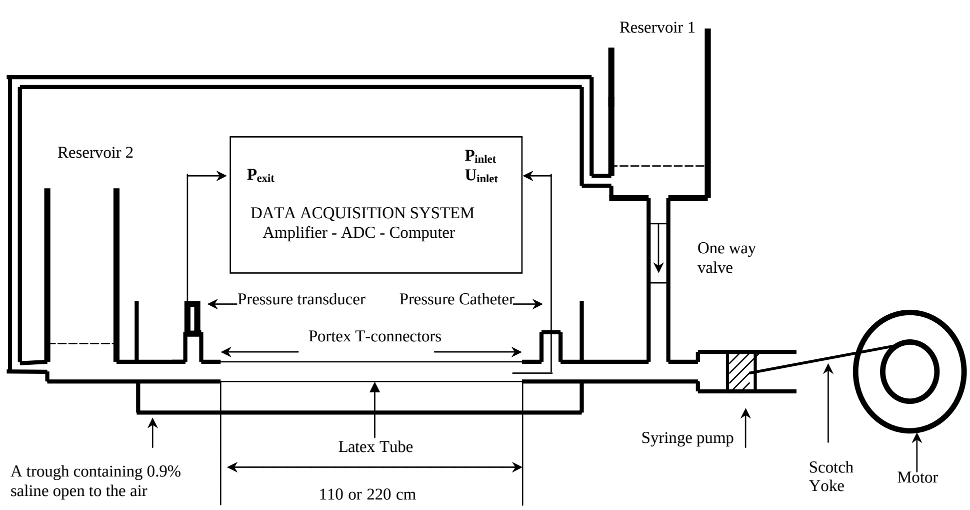


Fig. 1. A schematic diagram of the experiment. All elements of the experiment are on the same horizontal plane.

To eliminate gravitational and contact stresses between the elastic tubes and any hard surfaces during the experiments, the tube was immersed in an open trough containing 0.9% saline. The tubes were able to extend and distend freely and were fully immersed in the saline. We note that there will be a hydrostatic pressure difference between the top and bottom surfaces of the latex tube, however we expect its effect to be minimal and therefore ignored its effect.

To produce a stiffer wall than that of a normal latex tube, we used a similar method to that used by Papageorgio and Jones (1987a, b). A latex tube of 110 cm length and 0.5 in diameter was mounted on a well-polished wooden rod. A cotton thread was wound around the tube with a pitch of approximately 45° and was fixed to the tube by liquid latex, which was left to cure several hours.

The reflection sites were created by either applying a clamp causing a closed-end reflection, or by connecting the tube to a reservoir open to the atmosphere causing an open-end reflection.

2.3. Reservoirs

We used two reservoirs one upstream and one down stream, both were made of hard plastic, cylindrical and 10 cm in diameter. In order to minimise inertial effects, we made sure that the level of saline in the reservoirs was only high enough to prevent pump cavitations and air bubbles entering the system.

2.4. Measurements

Pressure and velocity at the inlet of the tube and pressure at the exit of the tube were recorded simultaneously. At the inlet of the tube, the pressure and velocity were measured using an 8F catheter (SVPC 684\A, Millar, Texas, USA) with a strain gauge pressure transducer and an electromagnetic flow sensor connected to a flow-meter (Cliniflow II, 710DE, Carolina Medical Electrical Inc., North Carolina, USA). At the exit of the tube, pressure was measured using a strain gauge pressure transducer connected to the tube through a side gate (Spectramed, California, USA). Data were sampled at a frequency of 125 Hz.

3. Analysis

The PU-loop method for determining wave speed is described in our previous work (Khir et al., 2001a), and essentially depends on the validity of the water hammer equation

$$\mathrm{d}P_{\pm} = \pm\rho c \mathrm{d}U_{\pm}, \tag{3}$$

where $\pm$ indicates forward and backward waves, $\mathrm{d}P$ & $\mathrm{d}U$ are the pressure and velocity differentials, respectively and c is the wave speed. Eq. (3) can be used for calculating wave speed if waves are unidirectional. During the earliest part of the cycle (half-sinusoid), it is likely that only forward waves are present at the inlet of the tube, since it would be too early for the arrival of reflected waves. If we plot the measured pressure against the measured velocity we obtain the PU-loop, whose slope during the very early part of the ejection period equals ρc.

To calculate the wave speed by the foot-to-foot method we used a technique introduced by Latham (1988) to identify the foot of the wave. The technique involves extrapolating the initial pressure upstroke and the pressure prior to the initiation of the pump. The intersection point of the two extrapolations is considered the foot of the pressure wave. The wave speed is calculated by measuring the time it takes for the foot of the wave to travel the known distance between the pressure transducers at the inlet and distal end of the tube.

We used wave intensity analysis for studying the waves in elastic tubes. The technique is a time domain analysis based on the method of characteristics (Parker and Jones, 1990). Wave intensity, $\mathrm{d}I$, is defined as the amount of energy carried by the wave per cross-sectional area of the vessel and can be calculated as

$$\mathrm{d}I = \mathrm{d}P\,\mathrm{d}U. \tag{4}$$

Wave intensity analysis has a number of distinct advantages; the method is a time domain analysis, which makes it easier to relate events directly to time, no assumption is made about linearity and this analysis can accommodate viscoelastic, convective and frictional effects. Originally, Parker and Jones suggested a linear method for the separation of waves into their forward and backward components. Pythoud et al. (1996b) proposed a nonlinear technique for the separation of waves, also based on the method of characteristics, but they found the difference between the linear and the nonlinear methods was on the order of 5–10%, which is of the same magnitude as the experimental error. Therefore, we used the linear method for wave separation. The forward and backward pressure and velocity differences are

$$\mathrm{d}P_{\pm} = \tfrac{1}{2}(\mathrm{d}P \pm \rho c \mathrm{d}U), \tag{5}$$

$$\mathrm{d}U_{\pm} = \frac{1}{2}\left(\mathrm{d}U \pm \frac{\mathrm{d}P}{\rho c}\right). \tag{6}$$

Therefore, we can calculate the wave intensity of the forward and backward waves as

$$\mathrm{d}I_{\pm} = \pm\frac{1}{4\rho c}(\mathrm{d}P \pm \rho c\,\mathrm{d}U)^2. \tag{7}$$

The separated backward wave intensity can be used to estimate the distance to a wave reflection site. In our experiments the reflection site was created by either clamping the tube, causing a closed-end reflection, or by connecting the tube to a reservoir open to the atmosphere, causing an open-end reflection. The onset of the separated backward wave intensity indicates the length of time that it took the wave to travel from the inlet of the tube, be reflected at the reflection site and arrive back to the inlet of the tube. If the wave speed is known, then the distance to the reflection site can be calculated, $L = \frac{1}{2} c\,\Delta t$, where L is the distance to the reflection site and Δt is time of onset of the separated backward wave intensity.

The analysis of reflections at mismatched bifurcations in 1D has been discussed by Caro et al. (1978). The pulse wave is partially reflected at the bifurcation and returned to the inlet of the mother tube and the transmitted waves continue to travel downstream in the daughter tubes. The transmitted waves are reflected when they meet the reservoir in one daughter tube and the clamp in the other. These two reflected waves will run backward towards the bifurcation and will be reflected again when they meet the bifurcation. The transmitted part of these reflected waves will continue running towards the inlet of the mother tube, and down the other daughter tube, while the re-reflected waves will travel forward in the same daughter tube. This process of generating new waves will continue until all waves diminish in magnitude because of the effect of dissipation.

The reflection coefficient of a bifurcation, R, is defined as the ratio of the magnitude of the reflected wave, $\mathrm{d}P_0$ compared to that of the incident wave, ΔP_0 and can be written as

$$R \equiv \frac{\mathrm{d}P_0}{\Delta P_0} = \frac{Y_0 - Y_1 - Y_2}{Y_0 + Y_1 + Y_2}, \tag{8}$$

where the admittance of vessel i is $Y_i = A_i/\rho c_i$, A is the cross-sectional area of the vessel and 0, 1 and 2 refer to the parent and daughter tubes, respectively.

If the product of Young's modulus and wall thickness is a constant in all of the vessels, as is the case in this study, then from Eq. (2) the relationship between the wave speed in the parent and daughter vessels varies as the inverse square root of the diameter

$$\frac{C_1}{C_0} = \left(\frac{A_0}{A_1}\right)^{1/4} \tag{9}$$

and the reflection coefficient for a symmetrical bifurcation reduces to

$$R = \frac{2^{1/4} - \alpha^{5/4}}{2^{1/4} + \alpha^{5/4}}, \tag{10}$$

where $\alpha = (A_1 + A_2)/A_0$.

In studying the waves running through the bifurcation we refer to the mother tube as 0 and the daughters as tubes 1 and 2. The waves travelling forward and backward will be denoted by '+' and '−' respectively. Therefore, a wave that runs in the path 0–0 means that it runs from the inlet of the mother tube, is reflected by the bifurcation then runs backward towards the inlet of the mother tube. The path 0 1–1–0 means that the wave runs from the inlet of the mother tube towards the bifurcation, is transmitted into the daughter tube 1, is reflected backwards to the bifurcation and finally runs backwards to the inlet of the mother tube.

4. Results

4.1. PU-loops

We tested the relative accuracy of the PU-loop for determining wave speed in our experiments by comparing its results to the results of the traditional foot-to-foot method. We tested the method in latex tubes with 1, 0.5 in diameter and a latex tube with hardened wall with 0.5 in diameter. Figs. 2a and b show the wave speed measured by both methods in a 1 in diameter latex tube. The results are given in Table 1 and the overall difference between both methods is in the order of $2.9 \pm 0.8\%$. We calculated the relative difference between the results of the PU-loop and foot-to-foot methods as the ratio of the difference between the results of both methods (foot-to-foot minus PU-loop) to their average (Bland and Altman, 1986).

4.2. Estimation of the distance of the reflection site

We clamped the tubes at different sites. From the measured pressure and velocity, we calculated the wave speed and the separated wave intensities using (7). We then calculated the distance to the clamp or to the open-ended reflection site in unclamped tubes and compared it with the physically measured distance. Fig. 3a shows the measured pressure and velocity in a 0.5 in latex tube that is clamped 30 cm away from the inlet of the tube. Fig. 3b shows the separated wave intensity, where the calculation of the occlusion site is 31 cm giving an error of 3.3%. Table 2 includes the results of all of the experiments, compared with the measured distances. The overall average error of wave intensity analysis in detecting the distance of the reflecting site is in the order of $1.7 \pm 4.6\%$. The error is calculated as the ratio of the difference between the calculated and the measured distances (measured minus calculated) to the measured distance.

4.3. Analysis of reflections in bifurcations

Fig. 4 shows typical pressure and velocity waveforms measured at the inlet of the mother tube which is 105 cm away from the centre of a bifurcation with $\alpha = 0.5$. This

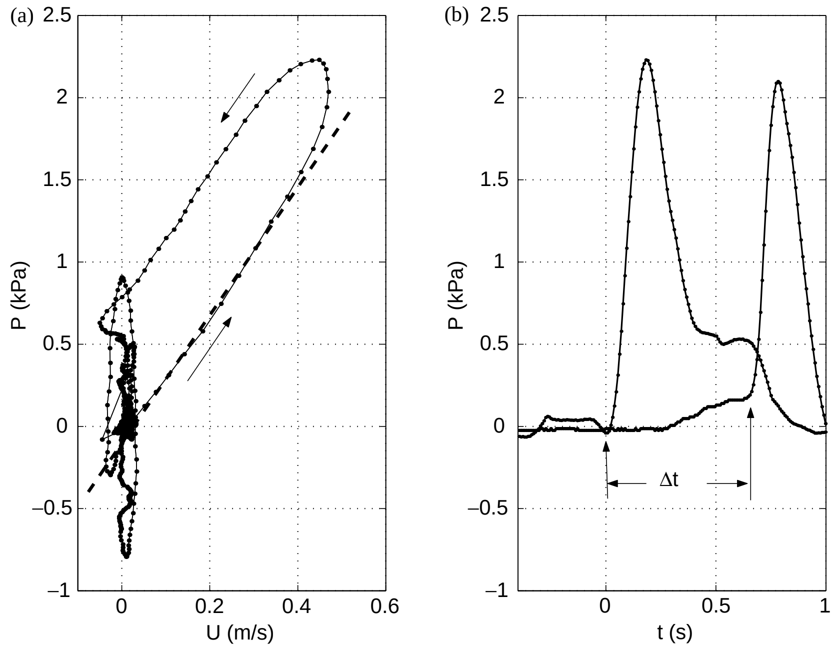


Fig. 2. Wave speed measured in a 1 in diameter latex tube by (a) PU-loop method. The slope of the initial part of the PU-loop indicates wave speed of 3.47 m/s and the arrows indicate the direction of the loop. (b) foot-to-foot method. The time that takes the wave to run from one site to the other, Δt, is 62 ms as indicated by the arrows which point at the foot of the wave at each site. The distance between the two sites is 220 cm and the calculated wave speed is 3.55 m/s.

Table 1
Average wave speed calculated by the PU-loop and foot-to-foot methods in latex tubes

Tube size	PU-loop (m/s)	Foot-to-foot (m/s)	Difference %
1 in	3.4±0.8	3.5±0.8	2.2
0.5 in	4.5±1.2	4.6±1.1	2.9
0.5 in "hard wall"	5.3±1.1	5.5±1.2	3.7
		Average difference	2.9±0.8

bifurcation has a mother tube of 1 in diameter and the two daughter tubes of 0.5 in diameter each. One of the daughter tubes is clamped 70 cm away from the centre of the bifurcation and the other daughter is connected to open to the air reservoir at a distance of 105 cm from the centre of the bifurcation. The wave intensity is calculated and the reflected waves are shown as individual peaks in Fig. 5. Each individual peak represents one or more reflected waves, depending on their arrival time to the measurement site. Table 3 contains a description of the possible waves with their paths and a comparison between the theoretical and the calculated arrival times of reflected waves to the measurement site at the inlet of the mother tube.

The measured reflection coefficient in this experiment calculated as the ratio of the magnitude of the second pressure peak minus the pressure at the onset of the second peak to the magnitude of the first pressure peak, is $R = 0.48$. From Eq. (10) the bifurcation of this experiment is expected to produce a reflection coefficient of $R = 0.45$. Experimental result is in good agreement with theoretical results and the difference is on the order of 7%.

5. Discussion and conclusions

As seen in Fig. 1, the path of the waves included more than the latex tube; portex connectors were used to give access to the measurement catheter and polyurethane tubing to connect the pump and the downstream reservoir. However, since they were made of a much more rigid material than the latex tubing, their wave speeds are much larger and the time taken to traverse these parts of the wave path will be an order of magnitude smaller. For example, the wave speed in the 20 cm long polyurethane tube connecting the latex tube to the downstream reservoir, using Eq. (2), is approximately 43 m/s ($E = 10$ Mpa, $h = 0.3$ cm, $d = 1.5$ cm and $\rho = 1080\,\mathrm{kg/m^3}$). The wave travel time in that tube is <5 ms, which is less than one sampling period. We have therefore neglected those rigid parts of the path of the wave travel in our wave timing calculations, since they would affect the results negligibly.

We assumed that reflections caused at an open-ended tubes have no time lag. The reservoir is more rigid than the polyurethane connecting tubes and the latex tubes, hence the wave speed in the reservoir will be so high that it will affect the results negligibly. Also, the amount of saline injected by the pump is approximately 28 ml,

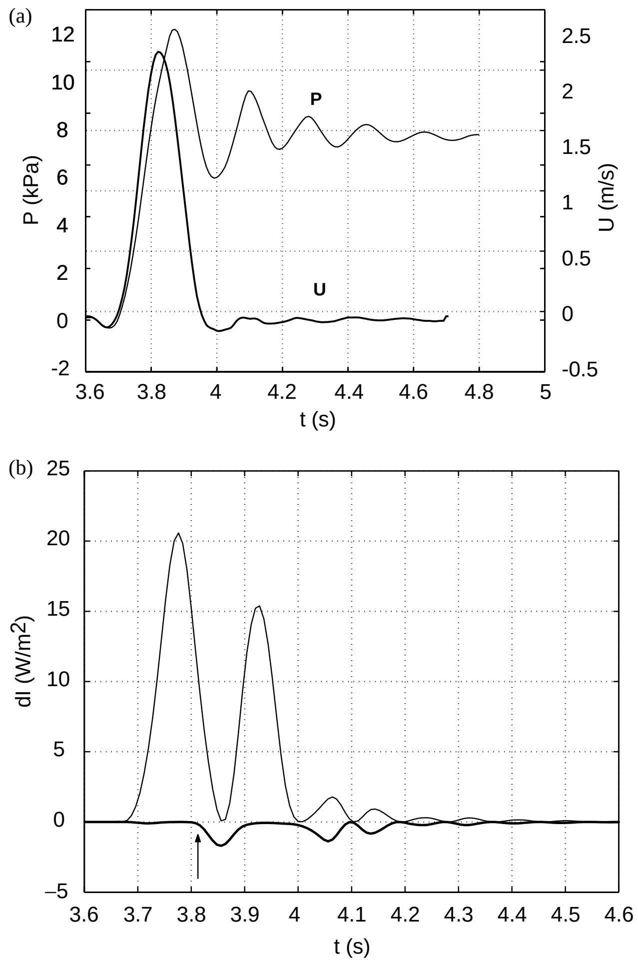


Fig. 3. (a) Pressure and velocity measured at the inlet of a latex tube of a 0.5 in diameter that is clamped at 30 cm away from its inlet. (b) Forward and backward wave intensities calculated from the measured data. Wave speed is 4.5 m/s and the onset of the reflected wave is 0.14 s as indicated by the arrow. Calculated distance is 31 cm giving an error of 3.3%.

which will increase the level of saline in the reservoir only 3.6 mm. This gives us confidence that the effect of oscillation will be minimal and the downstream reservoir would behave as an open-end. We isolated the upstream reservoir during ejection by including a one-way valve, Fig. 1.

Both methods for determining wave speed compared in this study, PU-loop and foot-to-foot exhibited some level of sensitivity. While the PU-loop method is sensitive to the choice of the portion of the curve used to determine the slope, the foot-to-foot is sensitive to determining the foot of the wave. However, the results of the two methods agree well and the small differences between the results of the two methods could not be attributed to any specific reason.

We compared the experimental results of measuring the wave speed using the PU-loop with the theoretical results that can be obtained by using Eq. (2). We measured the wall thickness of latex tube sizes in the range of 0.5–1 in diameter and found them all to have the same wall thickness, 0.25 mm. From Eq. (9), the ratio of the wave speed of a 1 in diameter tube to that of a 0.5 in diameter tube should be $\sqrt{0.5}$. The ratio of wave speeds measured using the PU-loop was 0.733, which is in good agreement with the theoretical prediction.

The in vitro set up used in this study is similar to that used by Stergiopulos (1998), who estimated pressure-dependant wave speeds taking into consideration the nonlinear elastic properties of the wall. His analysis yielded errors on the order of 20% in calculating the wave speed and a similar error in estimating the distances to the reflection sties. In our experiments the PU-loop and wave intensity analysis yielded differences in calculating wave speed and distance of reflection approximately 3% and 2%, respectively.

From Table 3 and Fig. 4 we can see that producing a single half sinusoidal wave in a rather simple system with one bifurcation can generate surprisingly complex pressure and velocity waveforms, which are difficult to interpret. However wave intensity analysis made it possible to determine the timing of the waves passing the observation site at the inlet of the mother tube with a reasonable accuracy.

There is a similarity between wave intensity curves in elastic tubes, Fig. 3b, and those seen in the aortic root in human studies. Fig. 6 shows the wave intensity curve calculated from pressure and velocity measurements in the ascending aorta of man in control conditions during peripheral vascular surgery using strain gauge pressure transducer and Doppler ultrasound (Khir et al., 2001b). In elastic tubes there is an initial forward compression wave ($dP > 0$) that is due to the piston of the pump moving forward and increasing the pressure in the tube. This wave is followed by a backward wave due to the arrival of the reflected wave from the clamp. The backward wave is followed by a forward expansion wave ($dP < 0$) that is due to the piston slowing down until it finally stops in the second half of the semi sinusoidal pulse wave. Similarly, in the human aortic root we observe a forward compression wave due to the contraction of the heart, followed by a backward compression wave due to the reflections arriving from different sites and finally the forward expansion wave due to the slowing down of contraction decelerating the blood in the aorta (Parker et al., 1988). Although the forward compression and expansion waves in this study should be approximately equal because each of them is produced by the approximately half semi-sinusoidal action of the piston, the forward compression and expansion waves seen in the aortic root of human are not usually equal, suggesting that the cardiac contraction phase is not symmetrical. Whilst we are not drawing similarity between the simple pump used in

Table 2
The results of detection of the reflection sites calculated by using wave intensity analysis. The calculated distance is compared to the measured distance of the reflection sites in closed and open-end reflection sites. Closed-end reflection sites were created at various distances by cross clamping the tubes. The overall average of the error produced by wave intensity analysis in all experiments is 1.7±4.6%.

Reflection site measured distance (cm)	Tube					
	1 in latex ($c = 3.4$ m/s)		0.5 in latex ($c = 4.5$ m/s)		0.5 latex with hardened wall ($c = 5.3$ m/s)	
	Calculated distance (cm)	Error %	Calculated distance (cm)	Error %	Calculated distance (cm)	Error %
30 (closed end)	29	−3.3	31.5	5.0	28	−6.6
40 (closed end)	—	—	—	—	38	5.0
60 (closed end)	61	1.6	62	3.3	—	—
80 (closed end)	—	—	—	—	74	−7.5
90 (closed end)	86	−4.4	92	2.2	94	4.4
120 (closed end)	118	−1.6	—	—	—	—
150 (closed end)	154	2.6	—	—	—	—
180 (closed end)	176	−2.2	—	—	—	—
200 (closed end)	195	−2.5	—	—	—	—
220 (open end)	204	−7.2	—	—	—	—
200 (open end)	—	—	189	−5.5	—	—
110 (open end)	—	—	—	—	106	−4.0
Average error		2.12±3.13		1.25±4.54		1.74±6.02

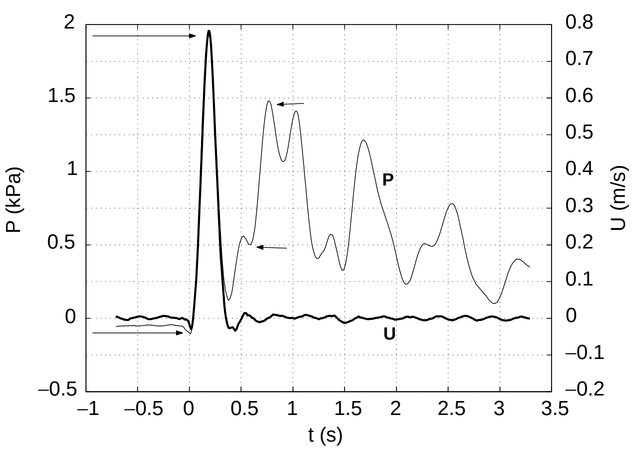


Fig. 4. Pressure and velocity measured at the inlet of the mother tube of a bifurcation with $\alpha = 0.5$. Mother tube is 105 cm long and 1 in diameter. Both daughter tubes are 0.5 in diameter each. One daughter is connected to a reservoir that is open to the air, 105 cm away from the centre of the bifurcation and the other daughter tube is clamped at 70 cm away from the centre of the bifurcation. The ratio of the pressure difference between the two small arrows, dP_0, to the pressure difference between the long arrows, ΔP_0, is the reflection coefficient, $R = 0.48$, which is in good agreement with the theoretical value $R = 0.45$.

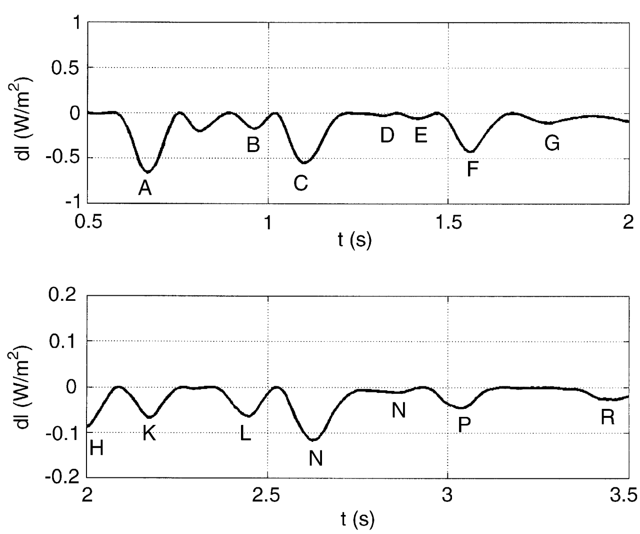


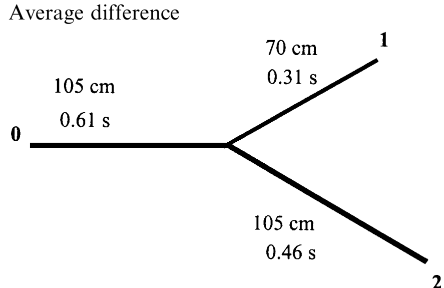
Fig. 5. Reflected waves arriving back to the inlet of the mother tube in a bifurcation with $\alpha = 0.5$. The mother tube is 105 cm long and 1 in diameter. Both daughter tubes are 0.5 in diameter. One of the daughter tubes is connected to a reservoir that is open to the atmosphere, 105 cm away from the centre of the bifurcation, and the other daughter is clamped 70 cm away from the centre of the bifurcation. Peaks of the reflected waves represent individual waves whose time is marked by the onset of each reflected wave. Time is continuous but the abscissa for $t > 2$ has been expanded in order to show smaller waves at later times.

this study and the heart, we believe that the underlying mechanics of generating the compression and expansion waves are similar.

The waves reflected from the end of the daughter tubes are re-reflected as they pass the bifurcation on their way back towards the inlet of the mother tube. Some of the reflected and re-reflected waves may arrive back to the inlet of the mother tube simultaneously, and be detected by the analysis as a single summation wave, as is the case for wave K in Fig. 5 and Table 3. Other reflected and re-reflected waves may arrive back to the

Table 3
The waves paths with the theoretical and the calculated arrival time to the inlet of the mother tube. (–) denotes a backward wave, (0) the mother tube, 105 cm long and 1 in diameter, (1) the daughter tube that is occluded at a distance to 70 cm from the centre of the bifurcation and (2) the daughter tube that is connected to the open-to-the air reservoir, 105 cm away from the centre of the bifurcation and. Both daughters are 0.5 in diameter. The sketch includes the length and time it takes the wave to run through and back each arm of the bifurcation. Wave speed in the mother tube is 3.4 m/s and for both daughter tubes is 4.5 m/s.

Wave	Wave path	Theoretical arrival time (s)	Calculated arrival time (s)	Difference %
A	0–0	0.61	0.6	2.0
B	0 1–1 0	0.92	0.9	2.0
C	0 2–2–0	1.06	1.04	2.0
D	0–00–0	1.22	1.28	−5.0
	0 1–1 1–1–0	1.23	1.28	
E	0 1–1 2–2–0	1.38	1.35	2.0
	0 2–2 1–1–0			
F	0 2–2 2–2–0	1.53	1.48	3.0
	0–00 1–1–0			
G	0–0 0 2–2–0	1.68	1.68	0.00
H	0–0 0–0 0–0	1.83	1.88	−3.0
	0 1–1 1–1–0 0–0	1.84		
K	0–0 0 1–1–0 0–0	2.14	2.08	3.0
	0 2–2 2–2–0 0–0			
	0–0 0–0 0 1–1–0			
L	0–0 0–0 0–0 0–0	2.44	2.37	3.0
	0–0 0 1–1–0 0 1–1–0	2.45		
M	0–0 0 1–1–0 0 2–2–0	2.6	2.58	2.0
	0 2–2 2–2–0 0 2–2–0			
N	0–0 0 1–1–0 0–0 0–0	2.75	2.73	1.0
	0–0 0–0 0 1–1–0 0–0			
P	0–0 0–0 0–0 0–0 0–0	3.05	2.95	3.0
R	0–0 0 2–2–0 0 1–1 1–1–0	3.36	3.38	−1.0
	0–0 0–0 0–0 0–0 0 1–1 2–2–0	3.37		
		Average difference		1.0±2.5



inlet of the mother tube at different times such as in waves D, H, L and R but still detected by the analysis as one wave. The reason is due to the very short interval between the arrival of these waves. The complexity of waves in this simple system with one bifurcation gives us an idea about how much more complicated is the arterial tree with waves reflecting, re-reflecting and interacting in a huge network of bifurcations. It is worth noting that reflected waves in the human arterial system arriving back to the aortic root are detected as one backward wave in the wave intensity analysis, Fig. 6. This is due to the shorter segments of vessels, and multiple reflections and re-reflections.

The PU-loop and wave intensity analysis have been used to study arterial waves in humans (Khir et al., 2001b). Apart from the possible difficulty in obtaining simultaneous pressure and velocity measurements in humans, the main limitation in using these methods is the possibility of a time delay between the measurements of the pressure and velocity. If the velocity lags the pressure, the initial slope of the PU-loop will yield an incorrect, higher wave speed and if the pressure lags the velocity, the PU-loop will yield a lower wave speed. Also, any lag between the pressure and velocity may introduce artificial waves or exaggerate existing ones in the wave intensity analysis (Khir, 1999). Therefore, it is important to account for any time delay between the pressure and the velocity prior to carry out the analysis.

We conclude that the accuracy of the PU-loop method is at least as good as the traditional foot-to-foot method for measuring wave speed. The method has an advantage that may be clinically relevant; the method uses one measurement site, which makes it less invasive than other methods where two sites are required. Also,

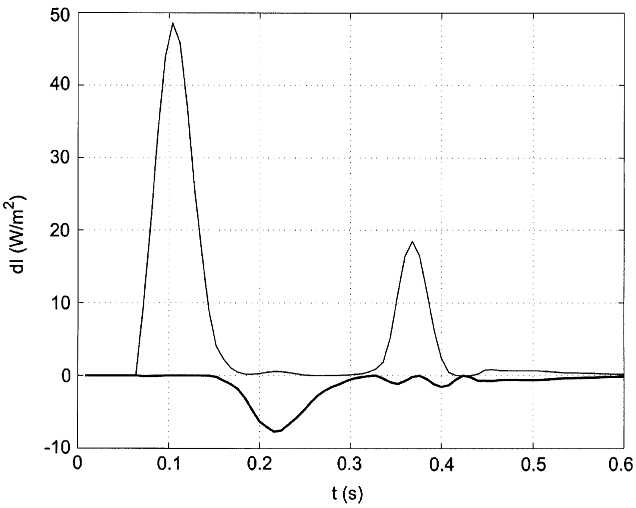


Fig. 6. Wave intensity curve in human calculated from measurements of pressure and velocity in the aortic root. This curve is similar to wave intensity curve calculated in latex tubes when a semi-sinusoidal wave is produced at its inlet. The three main characteristics of the curve is a forward compression wave followed by a backward expansion wave and a forward expansion wave.

the method is easy to implement requiring only the determination of the slope of the initial linear part of the PU-loop.

Wave intensity analysis is a convenient time domain method for studying the propagation of waves in elastic tubes and the arterial system. Wave intensity analysis and the PU-loop method provide a way for separating the forward and backward components of a wave, hence determining the predominant direction of waves from measurements of pressure and velocity. The results have shown that the method is able to decompose a complex waveform into simple results providing a reasonably accurate information on the times and magnitudes of waves passing by the observation site.

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