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Analysis of Manpower System with Alert Human Resource Personnel

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Abstract

Manpower planning is concerned with matching the supply of people with the jobs available in any organization. Every year, during the months of appraisal, organizations record high rates of employee turnover. Due to various reasons, manpower employed leave the system periodically. Loss of manpower also occurs due to dismissal and death of employees. This loss of manpower has to be compensated by suitable recruitment. But, recruitment cannot be made frequently since it involves cost. Also recruitment of new employees and giving them training to suit the needs of the organization works out to be costlier than retaining the available employees. Hence the Human Resource Department has to be alert and avoid manpower loss due to resignations. There is a maximum amount of loss of man power that can be permitted in the organization which is called the threshold beyond which the manpower system of the organization reaches a point of break down. In this paper we introduce the concept of Human Resource Department alertness and find the joint Laplace Stieltjes transform of time to recruit (T) and recruitment time (R).

Mathematics Subject Classification: 90B05

Keywords: Manpower Planning, Manpower system, Shortage, Cumulative Shortage Process, Alertness, Human Resource Department

1 Introduction

Manpower system (MPS) of many organizations record high rates of employee turnover during the period of appraisal. After the appraisal, several employees walk out of their jobs, disappointed by the outcome. The reason for their unhappiness could be the new set of responsibilities, lack of opportunities, new policies, low pay or unfulfilled demands.

Whatever be the reason, businesses suffer losses due to the attritions, because it is usually the most valuable employees who walk out first. The cost of replacing these ambitious, self-motivated and talented employees is exhorbitant.

Hence, to retain top talent, the management should give them a reason to continue working with the organization. Even during induction and training programmes, the employees can be asked to choose the strengths they want to develop. Human Resource Personnel play a very crucial role in this area. If Human Resource Department (HRD) is alert to the retention of employees, the outflow from the MPS of the organization can come down to a great extent.

The total flow out of the MPS due to Resignations, Dismissals and Deaths is termed shortage. When MPS is exposed to this shortage continuously due to heavy attrition rate, at one point of time, it will break down. This point is called the threshold level. It can be interpreted as that point at which immediate recruitment is necessitated. If HRD is alert to the attritions in the MPS, the loss of manpower can be reduced.

The shortage of MPS depends on individual propensity to leave the organization due to factors such as current length of service, age, salary, place of residence in relation to work place, sex, marital status and the general employment situation. Such models were discussed by Grinold and Marshall [4].

V. Subramanian [6] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. Lesson [5] has given methods to compute shortages (Resignation, Dismissal and Death) and promotion intensities. Markovian models are designed for shortage and promotion in MPS by Vassiliou [3]. Esary et al. [2] have discussed cumulative Damage Process (CDP) related to the shock models in reliability theory. Gaver [1] has assumed two types A and B of shocks that can occur in a device causing major and minor damages. The expected time to recruit and the variance for MPS having threshold which follows Exponentiated Exponential distribution, have been obtained by S. Parthasarathy and R. Vinoth [7]. S. Mythili and R. Ramanarayanan [8] have found covariance between time to recruit and recruitment time.

In this paper, we introduce the concept of HRD alertness and find the relation between Time to recruit (T) and Recruitment time (R). In our model, two types A and B of man power losses can occur in the MPS of an organization. These can cause major and minor shortages respectively. Type A denotes top managerial level manpower loss. Type B denotes manpower loss other than top managerial level (subordinate group) Cumulative major shortages lead to the breakdown of the MPS of the organization. Minor losses cause problems of shortages in the MPS which do not have impact on the functional operation of the MPS and hence do not lead to threshold level.

We focus our attention on models in which HRD is able to avoid minor shortages whenever it is alert (but not major shortages). We allow the manpower losses of type A to accumulate. The breakdown of the MPS is indicated when the total loss exceeds the threshold level. We assume that the MPS can withstand minor shortages due to type B manpower loss. When the breakdown of the system occurs, we assume that recruitment begins immediately and all the shortages due to type A and B are compensated by recruitment one at a time.

In this paper, we present two models in which we study HRD alertness related to attritions that are avoided; in model 1 type A shortages occur in accordance with Poisson process and type B shortages occur in accordance with a general process. In model 2 type A shortages occur in accordance with a general process and type B shortages occur in accordance with Poisson process. We find the joint Laplace-Stieltjes transform of Time to recruit and Recruitment time.

2 Model Description

2.1 Assumptions

1. Let t_0 be the instant at which the MPS of the organization begins its function. Also, at t_0, the system has no shortage.

2. Let T represent the time (from t_0) to recruit and R represent subsequent recruitment time respectively.

3. Let $x_1, x_2, x_3, \ldots$ and $t_1, t_2, t_3, \ldots$ be the instants at which type A and type B manpower losses occur respectively. Let the time differences $[x_n - x_{n-1}]$ and $[t_n - t_{n-1}]$, $(n = 1, 2, 3, \ldots; x_0 = t_0 = 0)$ be independent and identically distributed sequences of random variables independent of each other with distribution functions $V(x)$ and $F(x)$ respectively.

4. Let $X_1, X_2, \ldots$ be the major shortages caused by type A manpower losses and Y, the threshold of the system.

5. The sequence of shortages $X_1, X_2, \ldots$ and the threshold Y are independent.

6. $X_1, X_2, \ldots$ are independent observations on a prototype shortage variable X which is non-negative and non-degenerate at zero.

7. Let the threshold Y have an exponential distribution with parameter c.

8. MPS can withstand type B manpower losses. During recruitment time, type B shortages are also compensated.

2.2 Analysis

Note that $X_1, X_2, \ldots$ are major shortages caused by type A manpower losses and Y is the threshold of the system. The number of occurrences of shortages for the system to collapse is $N = min\{k : X_1 + X_2 + \cdots + X_k \geq Y\}$
i.e., $N > k$ if and only if $X_1 + X_2 + \cdots + X_k < Y$, $k = 1, 2, \ldots$.
$P(N > k) = P(Y > X_1 + X_2 + \cdots + X_k) = E(e^{-c(X_1+X_2+\cdots+X_k)}) = [E(e^{-cX})]^k$, $k = 0, 1, 2, \ldots$, where $P(Y > y) = e^{-cy}$, $y \geq 0$ is the survival function for Y.
$P(N > k) = a^k$, $k = 0, 1, 2, \ldots$ where $a = E(e^{-cX})$. N has a geometric distribution since X is non-negative and non-degenerate at zero and $0 < a < 1$.
$$P(N = r) = a^{r-1}(1 - a),\ r = 1, 2, 3, \ldots. \tag{A}$$
P(System collapse does not occur due to manpower losses of type A during $(0, t]) = P(N >$ number of occurrences of losses during $(0, t])$.
The c.d.f of T, the time to start recruitment, may be derived.

$P(T > t) = \sum_{k=0}^{\infty} a^k P_k(t)$, where $P_k(t)$ represents the probability for the occurrence of exactly k shortages due to manpower losses of type A during the period t. Considering that R is the time required to complete the recruitment process, in what follows, we introduce the concept of HRD alertness and discuss such (T, R) models.

3 HRD Alertness

We focus our attention on models in which HRD is able to avoid minor shortages whenever it is alert to the situation (but not major shortages).

From time instant t_0, HRD has initial alertness. Therefore due to alertness, HRD can avoid W_1 attritions of type B. Then the first minor shortage occurs at the instant t_{W_1+1}. The shortage at this instant creates an alertness sensation to HRD. Again W_2 (say) attritions of type B are avoided. The second minor shortage occurs at $t_{W_1+W_2+2}$. Therefore, shortages due to attritions of type B occur at $t_{W_1+1}, t_{W_1+W_2+2}, t_{W_1+W_2+W_3+3}, \ldots$. We study the problem when $W_1, W_2, W_3, \ldots$ are identically distributed independent random variables, independent of the instants of occurrences and independent of the major shortage process. The inter-occurrence times of the minor shortages are independent and identically distributed positive random variables with distribution function $\sum_{n=0}^{\infty} F_{n+1}(t) P(W = n) = E_W[F_{W+1}(t)]$ where $F_n(t)$ denotes the n-fold convolution of $F(t)$ with itself and E_W denotes expectation with respect to W. Let $G(t) = E_W[F_{W+1}(t)]$. As the organization starts recruitment at time T, the number of minor losses due to type B has a probability function $P[S(T) = n|T] = G_n(T) - G_{n+1}(T)$. $G_n(x)$ is the n-fold convolution of $G(x)$ with itself. $G_0(x) = 1$ if $x \geq 0$; $G_0(x) = 0$, otherwise.

3.1 Model 1

V is exponential, F is general, W is general.

Let $R = R_1 + R_2 + \cdots + R_{S(T)+N}$ be the total recruitment time where $S(T)$ and N are the number of minor and major shortages that have occurred. Let R_n, $n = 1, 2, 3, \ldots$ be independent, identically distributed random variables independent of the shortage size. If $L(x)$ is the distribution function for recruitment time and $V(x) = 1 - e^{-hx}$,
$P(R \leq Y, S(T) = n|N, T) = L_{n+N}(y)[G_n(T) - G_{n+1}(T)]$

$$P(T \leq x), S(T) = n, R \leq y/N) = L_{n+N}(y) \int_0^x [G_n(z) - G_{n+1}(z)] dV_N(z) \tag{3.1}$$

$P(T \leq x, S(T) = n, R \leq y/N) = L_{n+N}(y) \int_0^x [G_n(z) - G_{n+1}(z)]e^{-hz}\frac{h(hz)^{N-1}}{(N-1)!}dz$

$$P(T \leq x, R \leq y/N) = \sum_{n=0}^{\infty} L_{n+N}(y) \int_0^x [G_n(z) - G_{n+1}(z)]e^{-hz}\frac{h(hz)^{N-1}}{(N-1)!}dz$$

From (A), $P(T \leq x, R \leq y)$

$$= \sum_{r=1}^{\infty}(1-a)a^{r-1}\sum_{n=0}^{\infty} L_{n+r}(y) \int_0^x [G_n(z) - G_{n+1}(z)]e^{-hz}\frac{h(hz)^{r-1}}{(r-1)!}dz. \quad (3.2)$$

Introducing Laplace-Stieltjes transforms

$$\begin{aligned}
&E(e^{-\epsilon T}e^{-\eta R}) \\
&= \sum_{r=1}^{\infty}(1-a)a^{r-1}\sum_{n=0}^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-\epsilon x}e^{-\eta y}[G_n(x) - G_{n+1}(x)]e^{-hx}\frac{h(hx)^{r-1}}{(r-1)!}dL_{n+r}(y)dx \\
&= hL^*(\eta)(1-a)\sum_{n=0}^{\infty}L^{*n}(\eta)\int_0^{\infty} e^{-x(\epsilon+h)}[G_n(x) - G_{n+1}(x)]\sum_{r=1}^{\infty}\frac{[hxaL^*(\eta)]^{r-1}}{(r-1)!}dx \\
&= hL^*(\eta)(1-a)\sum_{n=0}^{\infty}L^{*n}(\eta)\int_0^{\infty} e^{-x(\epsilon+h-haL^*(\eta))}[G_n(x) - G_{n+1}(x)]dx \\
&= \frac{hL^*(\eta)(1-a)}{(\epsilon+h-L^*(\eta)ha)}\sum_{n=0}^{\infty}L^{*n}(\eta)[G^{*n}(\epsilon+h-L^*(\eta)ha)] - [G^{*(n+1)}(\epsilon+h-L^*(\eta)ha)] \\
&= \frac{hL^*(\eta)(1-a)[1-G^*(\epsilon+h-L^*(\eta)ha)]}{(\epsilon+h-L^*(\eta)ha)[1-L^*(\eta)G^*(\epsilon+h-L^*(\eta)ha)]} \qquad (3.3)
\end{aligned}$$

where L^* and G^* denote the corresponding Laplace-Stieltjes transforms.

Now $G(t) = \sum_{n=0}^{\infty} F_{n+1}(t)P(W = n)$. This becomes $G^*(s) = F^*(s)\phi(F^*(s))$ where $\phi(r) = \sum_{r=0}^{\infty} p_n r^n$ is the generating function of the random variable W with $P(W = n) = r_n$, $n = 0, 1, 2, \ldots$. We obtain

$$E(T) = \frac{1}{h - ha} \quad (3.4)$$

$$E(R) = \frac{E(R_1)[1 - aF^*(h-ha)\phi(F^*(h-ha))]}{(1-a)[1 - F^*(h-ha)\phi(F^*(h-ha))]} \quad (3.5)$$

$$Cov(T, R) = \frac{-E(R_1)G^{*\prime}(h-ha)}{[1 - F^*(h-ha)\phi(F^*(h-ha))]^2} + \frac{aE(R_1)}{h(1-a)^2} > 0 \quad (3.6)$$

3.2 Model 2

In this model, we consider W to be geometric, F exponential and V general. Let $F(x) = 1 - e^{-\lambda x}$. We find,

$$G(t) = \sum_{n=0}^{\infty} F_{n+1}(t)P(W = n) = \sum_{r=0}^{\infty} Q^{-1}\left(\frac{P}{Q}\right)^r \int_0^t e^{-\lambda t}\frac{(\lambda t)^r}{r!}\, dt$$

$= \lambda Q^{-1} \int_0^t e^{-\lambda Q^{-1} t} dt \qquad$ (where $P(W=n) = Q^{-1}\left(\frac{P}{Q}\right)^n, Q-P=1$)

The cdf of the inter-occurrence times of shortages due to type B attritions follow the exponential distribution $G(t) = 1 - e^{-\lambda Q^{-1} t}$.

$$P(T \leq x, S(T) = n, R \leq y|N) = L_{n+N}(y) \int_0^x e^{-\lambda Q^{-1} z} \frac{(\lambda Q^{-1} z)^n}{n!} dV_n(z) \quad (3.7)$$

$$P(T \leq x, R \leq y) = \sum_{r=1}^{\infty} P(N=r) \sum_{n=0}^{\infty} L_{n+N}(y) \int_0^x e^{-\lambda Q^{-1} z} \frac{(\lambda Q^{-1} z)^n}{n!} dV_n(z) \quad (3.8)$$

$$E(e^{-\epsilon T} e^{-\eta R}) = \frac{(1-a)L^*(\eta)[\epsilon + \lambda Q^{-1} - \lambda Q^{-1} L^*(\eta)]}{\{1 - aL^*(\eta)V^*[\epsilon + \lambda Q^{-1} - \lambda Q^{-1} L^*(\eta)]} \quad (3.9)$$

4 Numerical Illustration

$G(t) = \sum_{n=0}^{\infty} P(W=n) F_{n+1}(t) = E_W(F_{W+1}(t))$

$G^*(s) = \frac{\lambda}{\lambda+s} \phi\left(\frac{\lambda}{\lambda+s}\right)$ when $F(x) = 1 - e^{-\lambda x}$.

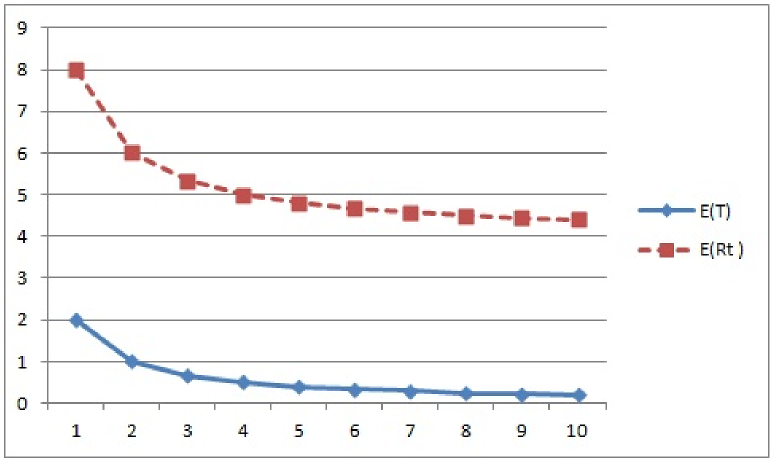
By taking $F(x) = 1 - e^{-2x}$, we draw the graph of $E(T)$, $E(R)$ and $Cov(T,R)$ using equations (3.4), (3.5) and (3.6) respectively for two values of $P(W=0)$.

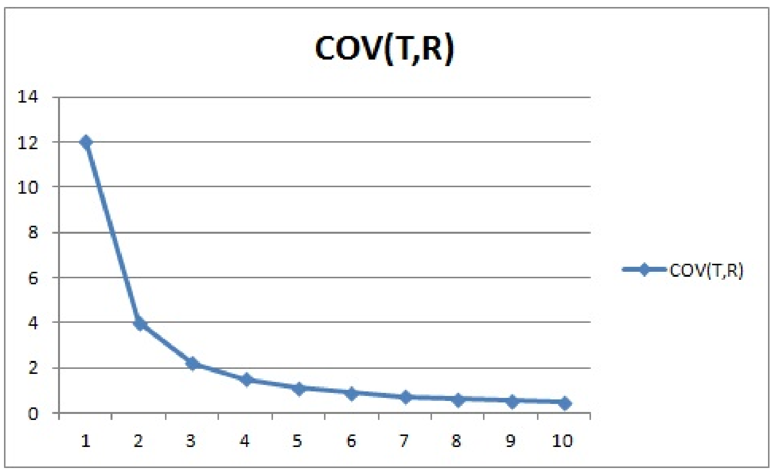
4.1 Numerical Illustration 1

Taking $P(W=0) = \frac{1}{2}$, we have $G^*(s) = \frac{\lambda}{\lambda+2s}$ and $G^{*\prime}(s) = \frac{-2}{(\lambda+2s)^2}$ where $\lambda = 2$. Allowing h to vary from 1 to 10, we draw graph of $E(T)$, $E(R)$ and graph of $Cov(T,R)$.

Table 1: $P(w=0) = 1/2, E(R_1) = 2, a = 0.5, h = 1$ to 10

h	1	2	3	4	5	6	7	8	9	10
$E(T)$	2	1	0.67	0.5	0.4	0.33	0.29	0.25	0.22	0.2
$E(R)$	8	6	5.33	5	4.8	4.67	4.57	4.5	4.44	4.4
$COV(T,R)$	12	4	2.22	1.5	1.12	0.89	0.73	0.63	0.54	0.48



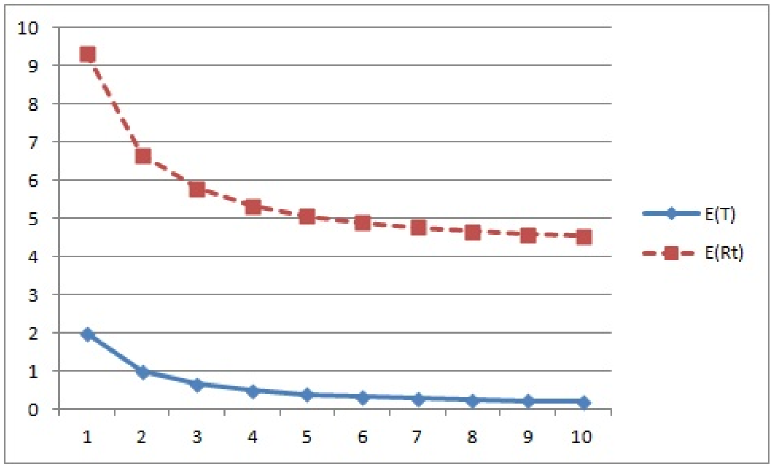


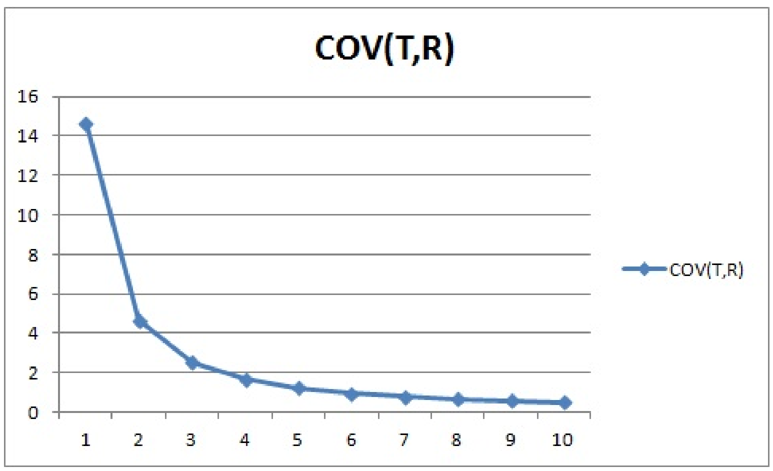
4.2 Numerical Illustration 2

Taking $P(W = 0) = \frac{1}{3}$, we have $G^*(s) = \frac{2\lambda}{2\lambda+3s}$ and $G^{*\prime}(s) = \frac{-6\lambda}{(2\lambda+3s)^2}$ where $\lambda = 2$. Allowing h to vary from 1 to 10, we draw the graph of $E(T)$, $E(R)$ and graph of $Cov(T, R)$.

Table 2: $P(w = 0) = 1/2, E(R_1) = 2, a = 0.5, h = 1$ to 10

h	1	2	3	4	5	6	7	8	9	10
$E(T)$	2	1	0.67	0.5	0.4	0.33	0.29	0.25	0.22	0.2
$E(R)$	9.33	6.67	5.78	5.33	5.07	4.89	4.76	4.67	4.59	4.53
$COV(T, R)$	14.67	4.67	2.52	1.67	1.23	0.96	0.79	0.67	0.58	0.51





As h increases, $E(T)$, $E(R)$ and $Cov(T, R)$ decrease. As $P(W = 0)$ decreases, $E(T)$ remains the same. $E(R)$ and $Cov(T, R)$ increase.

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