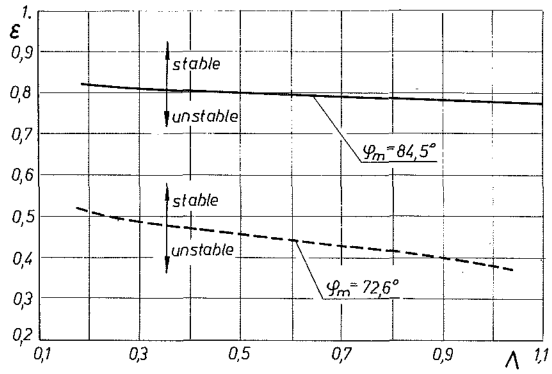

Fig. 6 Stability threshold lines for different pivot angle φ_m. For $\varphi_m = 59.4$ deg the line lies below $\epsilon = 0.2$.

load characteristics are satisfactory if $\varphi_m \approx 0.66\alpha_p$, but these pads do not have tendencies to loose their stability.

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Empirical Formulas to Evaluate the Friction Coefficient in Non-Newtonian Lubrication

J. M. Francois[1]

I Introduction

Modern lubricants often exhibit shear thinning due to the presence of high molecular weight polymers as additives. For some fluids, the viscosity can change by a factor of 10 to 100, owing to the presence of macromolecules. Therefore, the influence of this non-Newtonian property on the performances of lubricating systems must be predicted. To account for the fact that the viscosity of a fluid changes with the shear rate, the Reiner "generalized Newtonian fluid" has been induced. As quoted by Metzner [2], Bird [3] or Pearson [4], this model is very useful for applications. The corresponding fluid film flow is governed by a nonlinear partial differential equation, which generalizes the classical Reynolds equation [5]. Various approaches have been proposed to extend the classical Reynolds equation, in order to incorporate shear-thinning effects; see for instance Carlson and Winer [6] or more recently Dien and Erold [7]. All these studies require numerical techniques which are time consuming because of the intrinsic nonlinear character of the problem. The corresponding codes are therefore difficult to implement into microcomputers. Within that framework, a rapid evaluation of the performances of a lubricating system (slider bearing, journal bearing . . .) would be of interest.

Such an approach, based on the use of empirical formulas, had been presented [1] in order to evaluate the load capacity of a finite width slider bearing lubricated by the so-called Rabinowitsch fluid. The concept of an equivalent Newtonian viscosity, including both rheological, geometrical and kinematical parameters had been worked out. In this paper, the same analysis is extended to the computation of the friction coefficient in the same conditions.

II Flow Configuration

The flow is described with reference to a fixed rectangular coordinate system (O, x, y, z). The lubricating system considered is a linear slider bearing. The flow is bounded between the plane $y = 0$, translating at velocity U in the x-direction, and the fixed surface (S), defined by the equation:

$$y = h(x) = h_e + (h_s - h_e)x \tag{1}$$

where h_e and h_s are, respectively, the inlet and outlet film thickness. The two surfaces are separated by a small gap (thin film assumption). Besides, the upper surface is of finite width, which involves the additional condition:

$$-l/2 \le z \le l/2 \tag{2}$$

where l is the bearing width.

III Fluid Rheological Model

Wada and Hayashi [8] have shown that the Rabinowitsch model [9] would fit reasonably well the viscosity behavior of oils added with polyisobutylene, for shear rates ranging from 0 to $10^5\ s^{-1}$, which is representative of the actual working conditions of lubricants. In 2-D flows, the following shear stresses-shear rates relationships hold:

$$\frac{\partial u}{\partial y} = \frac{\tau_{xy}}{\mu_0} + \alpha(\tau_{xy}^2 + \tau_{yz}^2)\tau_{xy} \tag{3a}$$

$$\frac{\partial w}{\partial y} = \frac{\tau_{yz}}{\mu_0} + \alpha(\tau_{xy}^2 + \tau_{yz}^2)\tau_{yz} \tag{3b}$$

where τ_{xy} and τ_{yz} are, respectively, the shear stresses in the x-direction and in the z-direction, u and w are, respectively, the components of the velocity in the x-direction and in the z-direction. μ_0 is the zero shear rate viscosity and α is a parameter accounting for non-Newtonian effects.

For convenience, the non-Newtonian parameter α will be used in a nondimensional form as follows:

$$\tilde{\alpha} = \left(\frac{\mu_0 U}{h_s}\right)^2 \alpha \tag{4}$$

Wada and Hayashi's experiments [8] have shown that, in standard working conditions of mineral oils added with polyisobutylene, the values of parameter $\tilde{\alpha}$ were ranging from 0 to 1.13.

IV Approximate Method of Resolution

IV.I. Bearing of Infinite Width Lubricated by a Non-Newtonian Fluid. In the asymptotic case $l/L \rightarrow \infty$, the friction coefficient $(C_{fNN})_\infty$ is computed for several values of the dimensionless non-Newtonian parameter $\tilde{\alpha}$ and of the thickness parameter $a (a = he/hs)$. The friction coefficient

[1]Graduate Student, Université Claude Bernard, 69622 Villeurbanne Cedex, France

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$(C_{fNN})_\infty$ is normalized by the friction coefficient $(C_{fN})_\infty$ obtained in the same conditions with a Newtonian fluid of viscosity μ_0.

It has been plotted in Fig. 1 the ratio $(C_{fNN})_\infty/(C_{fN})_\infty$ versus parameter $\tilde{\alpha}$ for several values of the thickness parameter a ranging from 1.2 to 3.4 by steps of 0.2.

The curves of this figure show that the friction coefficient is increasing with the non-Newtonian parameter $\tilde{\alpha}$. This result is in agreement with the analytic works of Dubois et al. [10] or Tanner [11]. Oliver and Shahidullah's experiments [12] have shown that, for some polymer thickened oils, the friction coefficient may decrease. This contradiction is due to the fact that the Rabinowitsch model defined by equations (3*a*) and (3*b*) is a purely viscous model which does not take into account elastic effects existing for some non-Newtonian lubricants. Bourgin [13] showed that the presence of elastic effects could induce a reduction of the friction coefficient.

The general shape of the curves of the Fig. 1 suggest that an Equivalent Newtonian Viscosity (E.N.V.) μ_{eq} be introduced as follows:

$$\frac{(C_{fNN})_\infty}{(C_{fN})_\infty}=\frac{\mu_{eq}}{\mu_0} \qquad (5)$$

where coefficient μ_{eq} is a function of both parameters $\tilde{\alpha}$ and a. The non-Newtonian friction coefficient $(C_{fNN})_\infty$ can then be considered as the friction coefficient of the same linear slider-bearing lubricated by a Newtonian fluid of equivalent viscosity μ_{eq} and will be denoted $(C_{fNeq})_\infty$. Therefore, equation (5) turns out to be:

$$(C_{fNeq})_\infty=\frac{\mu_{eq}}{\mu_0}(C_{fN})_\infty \qquad (6)$$

The equivalent Newtonian viscosity μ_{eq} is determined in such a way that the curves of Fig. 1 should be adequately fitted. The following form is proposed:

$$\mu_{eq}(\tilde{\alpha},a)=\mu_0\times\sqrt{1+\left(\frac{1}{2}+\frac{1}{k_1a+k_2a^2+1}\right)\tilde{\alpha}+\left(\frac{1}{k_3a+k_4}\right)\tilde{\alpha}^2} \qquad (7)$$

where coefficients k_1, k_2, k_3, k_4 are equal to:

$$k_1=-1.3146 \qquad (8)$$

$$k_2=0.73099 \qquad (9)$$

$$k_3=-1.2780 \qquad (10)$$

$$k_4=0.7093 \qquad (11)$$

IV.2. Bearing of Finite Width Lubricated by a Newtonian Fluid. The friction coefficient C_{fN} has been computed for different values of the aspect ratio l/L and of the thickness parameter a. This friction coefficient has been normalized by the corresponding value $(C_{fN})_\infty$ obtained in the same conditions for the case of an infinite width bearing. We define a side leakage correcting factor (f) depending both on the aspect ratio and the thickness parameter:

$$C_{fN}=f(l/L,a)\times(C_{fN})_\infty \qquad (12)$$

where function f is assumed to be of the form:

$$f(l/L,a)=exp\frac{1}{(c_1a+c_2)(l/L)+(c_3a+c_4)(l/L)^2} \qquad (13)$$

where the coefficients C_1, C_2, C_3, and C_4 have the following values:

$$C_1=-0.1328 \qquad (14)$$

$$C_2=-0.9125 \qquad (15)$$

$$C_3=-0.0410 \qquad (16)$$

$$C_4=-0.0548 \qquad (17)$$

Equation (13) holds for a real Newtonian fluid but can be extended to an equivalent Newtonian fluid. Thus, one has:

$$C_{fNeq}=f(l/L,a)\times(C_{fNeq})_\infty \qquad (18)$$

IV.3. Additional Correcting Factor and General Formula. It is necessary to account for the fact that those effects may depend on the fluid non-Newtonian behavior. For that purpose, an ultimate correcting factor, say Ψ, depending on parameter $\tilde{\alpha}$ only is introduced in order to adjust C_{fNeq} to the actual non-Newtonian coefficient C_{fNN}. The following expression is retained for Ψ:

$$\Psi(\tilde{\alpha})=\frac{C_{fNN}}{C_{fNeq}}=n_1\tilde{\alpha}^2+n_2\tilde{\alpha}+n_3 \qquad (19)$$

where coefficients n_1, n_2, and n_3 are, respectively, equal to:

$$n_1=0.04125 \qquad (20)$$

$$n_2=-0.06475 \qquad (21)$$

$$n_3=0.9837 \qquad (22)$$

In addition, we have noted that, for l/L greater than 2.5, it was preferable to set $\Psi=1$.

A composite formula can be obtained to compute the non-Newtonian friction coefficient for the finite witdth configuration C_{fNN} knowing the Newtonian friction coefficient in the infinite width case $(C_{fN})_\infty$:

$$C_{fNN}=\Psi(\tilde{\alpha})\times f(l/L,a)\times\frac{\mu_{eq}(\tilde{\alpha},a)}{\mu_0}(C_{fN})_\infty \qquad (23)$$

The accuracy of the formula proposed here has been tested on several examples (see Fig. 2). Clearly, the agreement is excellent.

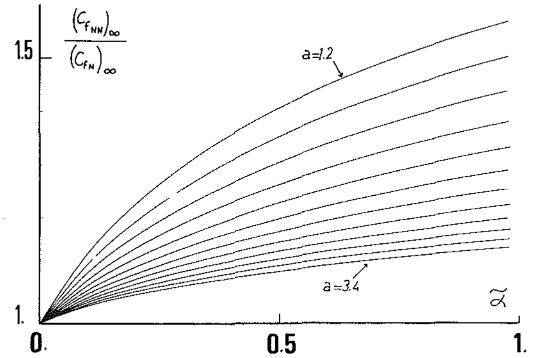

Fig. 1 Normalized non-Newtonian friction coefficient (numerically computed) versus parameter $\tilde{\alpha}$, for $1/L=\infty$ and $a=1.2, 1.4 \ldots, 3.4$

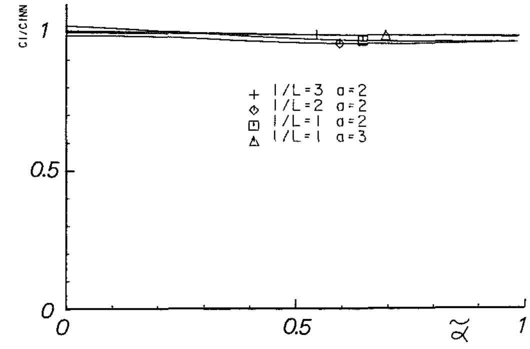

Fig. 2 Friction coefficient ratios (C_f: numerical; C_{fNN}: empirical) versus parameter $\tilde{\alpha}$ for $1/L=3$, $a=2$, $1/L=2$, $a=2$, $1/L=2$, $1/L=1$, $a=2$ and $1/L=1$, $a=3$

V Conclusion

An empirical formula has been proposed to compute the friction coefficient of a slider-bearing lubricated by a non-Newtonian (characterized by the Rabinowitsch model).

The analysis is based on the concept of an "equivalent Newtonian viscosity" depending on the fluid rheological parameters and on geometrical and kinematical characteristics of the flow.

It is important to note that the equivalent Newtonian viscosity which had been previously defined in order to evaluate the loading capacity [1] is not the same that the one defined for the friction coefficient determination. The same remark can be done concerning the side leakage effects. Thus, the concept of an equivalent Newtonian viscosity must be carefully defined, according to the characteristics which are to be evaluated.

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Comparison of Algorithms for In Situ Estimation of Squeeze-Film Bearing Characteristics

C. R. Burrows,[1] M. N. Sahinkaya,[1] N. C. Kucuk,[1] and M. L. Tong[1]

Nomenclature

$A_{11}, \ldots$ etc. = system coefficient
c = radial clearance
$c_{xx}, \ldots$ etc. = dimensional damping coefficients
$C_{xx}, \ldots$ etc. = nondimensional damping coefficients
$C_{11}, \ldots$ etc. = noise characteristic coefficients
${}^0\mathbf{C}, {}^1\mathbf{C}, \ldots$ etc. = optimum and estimated damping coefficient vector
d = mass unbalance eccentricity
$\mathbf{e}$ = error vector
F_x, F_y = oil-film forces in x and y directions, respectively
$\mathbf{G}, \eta, \psi$ = matrices defined in Appendix 1
$\mathbf{K}$ = Kalman filter gain matrix
k_s = bearing support stiffness
l = bearing land length
m = bearing mass
$\mathbf{P}$ = variance-covariance matrix in Kalman filter
Q = bearing parameter, $\pi\mu R(l/c)^3$
R = bearing radius
t = time
T = sampling interval
U_x, U_y = modelling errors in x and y direction, respectively
x, y = orthogonal displacements
$\mathbf{Y}$ = observation matrix
ϵ_0 = static eccentricity ratio
ϵ_1, ϵ_2 = error terms
$\boldsymbol{\theta}$ = parameter matrix
$\boldsymbol{\Lambda}$ = error variance matrix
μ = lubricant viscosity
ϕ_0 = static attitude angle
ω = rotational speed
ω_n = natural frequency ($\sqrt{k/m}$)
$(\ )^T$ = transpose
$(\dot{\ })$ = time derivative
$(\hat{\ })$ = estimate
$(\ )^0$ = initial value

[1]University of Strathclyde, Division of Dynamics and Control, Glasgow, Scotland.

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Introduction

The present paper is concerned with the limited objective of examining the relative performance of three algorithms to estimate squeeze-film bearing characteristics from the synchronous unbalance response. This is done under controlled conditions by using a computer solution of Reynolds' equation from which optimum coefficient values are determined from the oil-film forces [1]. The data are contaminated with different levels of noise and the three algorithms are assessed by their ability to recover the optimum parameter values.

Generation of Data

A dynamic nonlinear simulation program (NLSP in Fig. 1) was written to represent a rigid rotor symmetrically supported by two roller-bearing elements each of which operates within a squeeze-film damper. The simulated system parameters were: $m=33.43$ kg, $l=22.7$ mm, $c=0.1$ mm, $R=64.8$ mm, $\mu=2.66\times10^{-3}$ Nsm^{-2}, $k_s=21.54\times10^6$ Nm^{-1}, $d=12.5\times10^{-6}$ m.

The oil-film forces F_x and F_y were obtained by solving Reynolds' equation for a 2π film. The oil-film forces can be linearised for small pertubations of the journal to give:

$$\begin{aligned} F_x(t) &= -c_{xx}\dot{x}(t) - c_{xy}\dot{y}(t) + U_x(t) \\ F_y(t) &= -c_{yx}\dot{x}(t) - c_{yy}\dot{y}(t) + U_y(t) \end{aligned} \quad (1)$$

where $c_{xx} \ldots$ etc. are the damping coefficients and U_x and U_y are the modelling errors due to nonlinearities etc. An uncavitated squeeze-film bearing exhibits no stiffness effects, and the use of Reynolds' equation precludes the inclusion of inertia coefficients. These restrictions are useful at this stage where the object is to evaluate the algorithms. The estimation of inertia coefficients is considered elsewhere [2].

The computer generated data for F_x, F_y, $\dot{x}$, and $\dot{y}$ were used