Beam Element Structural Dynamics Modification Using Experimental Modal Rotational Data

John A. Cafeo

Martin W. Trethewey

H. Joseph Sommer III

Department of Mechanical Engineering,
The Pennsylvania State University,
University Park, PA 16802

Structural dynamic modification (SDM) of a fixed-free (cantilever) beam to convert it into a fixed-fixed beam with experimental modal data is presented. The SDM focuses on incorporating experimental rotational degrees-of-freedom (DOF) measured with a novel laser measurement technique. A cantilever beam is tested to develop the experimental modal database including rotational degrees of freedom. A modal database from a finite-element model also is developed for comparison. A structural dynamic modification, with both databases, is performed using a Bernoulli-Euler beam to ground the free end of the cantilever beam. The hardware is then modified and a second experimental modal analysis of the resulting fixed-fixed beam performed. A finite-element model of the fixed-fixed beam also was created. Comparison of results from these four tests are used to assess the effectiveness of SDM using experimental modal rotational data. The evaluation shows that provided high quality experimental rotational modal data can be acquired, SDM work with beam elements can be effective in yielding accurate results.

1 Introduction

Real world structural modifications usually involve the addition of beam-type supports, braces, and plates (Ewins, 1975). These realistic modifications transfer both force and moments. Practical modification schemes with experimental modal analysis data are currently limited to point masses and springs or dampers connected between two points. The element restrictions are not inherent to the modification scheme rather they are due to the incomplete experimental database that does not contain rotational data (Elliott, 1985). The majority of transducers used in experimental modal analysis sense only translation and not rotation. Without experimental rotation information, it is impossible to add beam elements properly. Previous efforts to estimate the rotational information from the displacement based modal data have been utilized (Hermanski, Ostholt and Bielefeld, 1987; Avitabile, O'Callahan, Chou and Kalkunte, 1987; Yasuda, Riehle, Brown and Allemang, 1984). Unfortunately, these methods provide an indirect estimation of the rotational characteristics and can produce substantial errors (Cafeo, Trethewey and Sommer, 1992). In order for beam elements to be applied effectively to experimental modal data via SDM it is necessary to experimentally measure the structural rotational degrees of freedom directly.

The objective of this paper is to examine the utilization of beam elements in structural dynamics modification using an experimental modal data base containing rotational degrees of freedom. The modal database will be developed with a new measurement technique capable of measuring one translation and two rotational degrees-of-freedom (Trethewey, Sommer and Cafeo, 1993). Initially both a finite element and experimental modal analysis is performed on a cantilever beam to develop modal databases. A structural modification is then performed to predict the dynamic characteristics when the free end of the beam is grounded. The modification involves the attachment of a Bernoulli-Euler beam element (with fixed-fixed boundary conditions at one end) to the free end of the cantilever. For comparison, a second experimental evaluation is performed on a beam with two fixed-fixed boundary conditions. These results are examined to assess the application of beam elements with measured experimental modal data containing rotational degrees-of-freedom.

2 Beam Element Structural Dynamics Modification

Gathering experimental modal data has become an integral part of the modern design process in many industries such as automotive or aerospace. The data can be gathered on a prototype product to verify and fine tune the results of a finite-element analysis. The verified model can then be used to explore design changes. The data also can be gathered on a production product in order to solve an unforeseen vibration problem. Furthermore, structural dynamics modification techniques provide a means to evaluate the change in structural dynamics behavior for potential design utilizing only the ex-

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perimental modal database. Hence, the combination of experimental modal analysis and SDM is a powerful design tool.

There are several procedures for performing SDM. These have been summarized and discussed in (Snyder, 1986; Brandon, 1990). The basic premise is to take experimental modal data, and use it to estimate the modal parameters of a changed system by adding various modification elements. Most libraries of modification elements include concentrated masses at single points, springs between two points, and dampers between two points. Recently, several investigators have developed procedures to incorporate more realistic element such as beam and plate elements into the element libraries (Elliott, 1985; O'Callahan and Chou, 1984). However, these realistic elements are not in widespread use because of the lack of experimental rotational degrees-of-freedom in the modal database. These degrees-of-freedom are essential to add beam or plate elements.

The procedure developed by (Elliott, 1985) to perform SDM with beam elements is an extension of a modification procedure developed by (Luk and Mitchell, 1982) for simple springs, dampers, and lumped masses called Dual Modal Space Modification (DMSM). A brief overview of the method is presented as a basis for the work discussed herein.

In the DMSM method, modifications are implemented by changes in either the physical stiffness or mass matrices. Instead of this approach, Elliott's method is to describe the change in terms of a dynamic stiffness modification. The dynamic stiffness matrix for a beam element is based on the transfer matrix method. Because a continuum beam transfer matrix includes trigonometric and hyperbolic functions which have the mass and stiffness properties intertwined in their arguments, it is not possible to make simple changes to the mass or stiffness matrices as detailed further in the next section.

The modification procedure proceeds by specifying the dynamic stiffness change in physical space (Δd), then transforming this change into modal space I (ΔD_I) by pre- and post-multiplying by the modal matrix as shown in Eq. (1).

$$[\Delta \mathbf{D}_I(\omega)] = [\boldsymbol{\Phi}_I]^T [\Delta \mathbf{d}(\omega)][\boldsymbol{\Phi}_I] \tag{1}$$

The dynamic stiffness matrix for the modification is added to the original system dynamic stiffness matrix to complete the description of the change to the original system in modal space I, illustrated in Eq. (2).

$$([\mathbf{D}_I(\omega)] + [\Delta \mathbf{D}_I(\omega)])\{Q_I\} = \{0\} \tag{2}$$

This new system described by Eq. (2) must undergo another eigensolution to determine its modal characteristics. This process places the system into modal space II. Because the matrices of this equation are a function of ω, this is a transcendental eigenvalue problem. The eigenvalues of the new system occur at the values of ω for which the determinant of the left side of Eq. (2) becomes zero. In this study, a zero determinant search routine was used. Starting at zero, ω was increased by a fixed interval. When the sign of the determinant changed, an eigenvalue inside the interval is indicated. By decreasing the step size and repeating the iteration, the eigenvalue could be determined to a desired tolerance. After an eigenvalue was found, the step size was reset to its initial value and the search continued from the previous eigenvalue. This simplified approach is slow, and, if the initial step size is too large, closely spaced eigenvalues can be missed. However, for the intended application (i.e., the grounding of the free end of a cantilever beam) the natural frequencies are widely spaced and the solution method proved practical. For another structure/modification combination it may be necessary to use other eigenvalue search routines (Williams and Wittrick, 1970; Meirovitch, 1980).

Once the eigenvalues of the new system have been found, the following procedure discussed by (Craig, 1981) and used by (Elliott, 1985), it used to extract the eigenvectors. First, the dynamic stiffness matrix for the modified system is defined as shown in Eq. (3).

$$[\overline{\mathbf{D}}_I(\omega)] = [\mathbf{D}_I(\omega)] + [\Delta \mathbf{D}_I(\omega)] \tag{3}$$

Then, for a given mode, j, Eq. (4) yields $\mathbf{Y}^{(j)}$, the eigenvector.

$$[\overline{\mathbf{D}}_I(\omega_j)]\{\mathbf{Y}\}^{(j)} = \{0\} \tag{4}$$

Assuming the first element in each eigenvector is not a node in Modal Space II, it is set to unity (Craig, 1981). Equation (4) can then be written in partitioned form as shown in Eq. (5).

$$[\overline{\mathbf{D}}_I(\omega_j)]\{\mathbf{Y}\}^{(j)} = \begin{bmatrix} [\overline{\mathbf{D}_{AA}}(\omega_j)] & [\overline{\mathbf{D}_{AB}}(\omega_j)] \\ [\overline{\mathbf{D}_{BA}}(\omega_j)] & [\overline{\mathbf{D}_{BB}}(\omega_j)] \end{bmatrix} \begin{Bmatrix} 1 \\ \mathbf{Y}_s^{(j)} \} \end{Bmatrix} \tag{5}$$

Finally, solving for $\mathbf{Y}_s^{(j)}$ yields Eq. (6).

$$\{\mathbf{Y}_s^{(j)}\} = -[\overline{\mathbf{D}_{BB}}(\omega_j)]^{-1}[\overline{\mathbf{D}_{BA}}(\omega_j)] \tag{6}$$

Each of the eigenvectors have been arbitrarily scaled by assuming unity for the first element. These vectors must be normalized to unity length in order to preserve the scaling in the new system to unity modal mass. Once normalized, these vectors can then be used to assemble the modal matrix in modal space II as shown in Eq. (7).

$$[\boldsymbol{\Phi}_{II}] = [\{\hat{\mathbf{Y}}^{(1)}\}\{\hat{\mathbf{Y}}^{(2)}\}\cdots\{\hat{\mathbf{Y}}^{(m)}\}] \tag{7}$$

where m = number of modes.

Finally, the eigenvectors for the modified system in physical space are defined by Eq. (8).

$$[\boldsymbol{\Phi}] = [\boldsymbol{\Phi}_I][\boldsymbol{\Phi}_{II}] \tag{8}$$

Close examination of Eq. (8) reveals the eigenvectors for the modified system are a linearly weighted sum of the eigenvectors of the original system.

This scheme experiences potential computational problems if the first eigenvector element, which was arbitrarily scaled to unity, is small. If a small numerical value exists during normalization it can be readily rectified by selecting another eigenvector component to be used as the unity reference. Furthermore, if there is a suspicion that this problem exists, the process may be repeated several times using different components as the basis for normalization.

2.1 Beam Modification Dynamic Stiffness Matrix. Because beams are fourth-order elements, they require four boundary conditions; two translations and two rotations. This is illustrated by the transfer matrix for a Bernoulli-Euler beam with mass as given by (Pilkey and Chang, 1978) and shown in Eq. (9), where ρ is the mass per unit length, E is the modulus of elasticity, and I is the area moment of inertia.

$$\begin{Bmatrix} v \\ \theta \\ M \\ V \end{Bmatrix}_{i+1} = \begin{bmatrix} e_1 & -e_2 & -e_3/EI & -e_4/EI \\ \alpha e_4 & e_1 & e_2/EI & e_3/EI \\ \alpha EIe_3 & EIe_0 & e_1 & e_2 \\ \alpha EIe_3 & -\alpha EIe_3 & -\alpha e_4 & e_1 \end{bmatrix} \begin{Bmatrix} v \\ \theta \\ M \\ V \end{Bmatrix}_i \tag{9}$$

$$e_0 = \frac{\eta}{2}[\sinh(\eta l) - \sin(\eta l)]$$

$$e_1 = \frac{1}{2}[\cosh(\eta l) + \cos(\eta l)]$$

$$e_2 = \frac{1}{2\eta}[\sinh(\eta l) + \sin(\eta l)]$$

$$e_3 = \frac{1}{2\eta^2}[\cosh(\eta l) - \cos(\eta l)]$$

$$e_4 = \frac{1}{2\eta^3}[\sinh(\eta l) - \sin(\eta l)]$$

$$\eta^4 = -\alpha = \frac{\rho\,\omega^2}{EI}$$

This equation relates the state vector (v is vertical translation, θ is the bending rotation, M is the bending moment, and V is the shear force) at one end of the beam to the state vector at the other end. Essentially, all the dynamic information of the beam is concentrated at its endpoints. Partitioning Eq. (9) leads to Eq. (10)

$$\begin{Bmatrix} \{\mathbf{q}_{i+1}\} \\ \{\mathbf{f}_{i+1}\} \end{Bmatrix} = \begin{bmatrix} [L_1] & [L_2] \\ [L_3] & [L_4] \end{bmatrix} \begin{Bmatrix} \{\mathbf{q}_i\} \\ \{\mathbf{f}_i\} \end{Bmatrix} \tag{10}$$

If Eq. (10) is solved for the force vectors, $\mathbf{f}_{i+1}$ and $\mathbf{f}_i$, it can be rearranged and put into dynamic stiffness form as shown in Eq. (11)

$$\begin{Bmatrix} \{\mathbf{f}_i\} \\ \{\mathbf{f}_{i+1}\} \end{Bmatrix} = \begin{bmatrix} -[L_2^{-1}][L_1] & [L_2^{-1}] \\ [L_3]-[L_4][L_2^{-1}][L_1] & [L_4][L_2^{-1}] \end{bmatrix} \begin{Bmatrix} \{\mathbf{q}_i\} \\ \{\mathbf{q}_{i+1}\} \end{Bmatrix} \tag{11}$$

or

$$\{f\} = [d_s]\{q\}$$

In order to switch from transfer matrix sign convention to stiffness matrix sign convention, two transformation matrices are defined as shown in Eqs. (12) and (13).

$$\{f\} = [C_p]\{p\}$$

where

$$[\mathbf{C}_p] = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \tag{12}$$

$$\{\mathbf{q}\} = [C_u]\{u\}$$

where

$$[\mathbf{C}_u] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

Finally, the dynamic stiffness matrix in local coordinates, $\mathbf{d}$, can be defined by Eq. (14)

$$\{p\} = [\mathbf{C}_p]^T[\mathbf{d}_s][\mathbf{C}_u]\{u\} = [\mathbf{d}]\{u\} \tag{14}$$

The specific modification discussed in this paper is to convert a fixed-free (cantilever) beam database by fixing the free end and converting it into a fixed-fixed beam. To accomplish this, the dynamic stiffness matrix defined by Eq. (14) must be mapped into Δd which is in global coordinates. This is performed in Eq. (15), where n is the number of nodes in the cantilever beam model.

$$[\Delta\mathbf{d}(\omega)] = \begin{bmatrix} 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & d_{11} & d_{12} & d_{13} & d_{14} \\ 0 & \cdots & \cdots & d_{21} & d_{22} & d_{23} & d_{24} \\ 0 & \cdots & \cdots & d_{31} & d_{32} & d_{33} & d_{34} \\ 0 & \cdots & \cdots & d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \begin{pmatrix} x_1 \\ \theta_1 \\ \vdots \\ \vdots \\ x_{n+1} \\ \theta_{n+1} \end{pmatrix} \tag{15}$$

It is common practice in finite-element analysis to eliminate the rows and columns of those degrees of freedom that are zero (Chandrupatla and Belegundu, 1991), thus reducing the size of the problem. Using this approach, Eq. (15) can be reduced by eliminating the last two rows and columns as shown in Eq. (16)

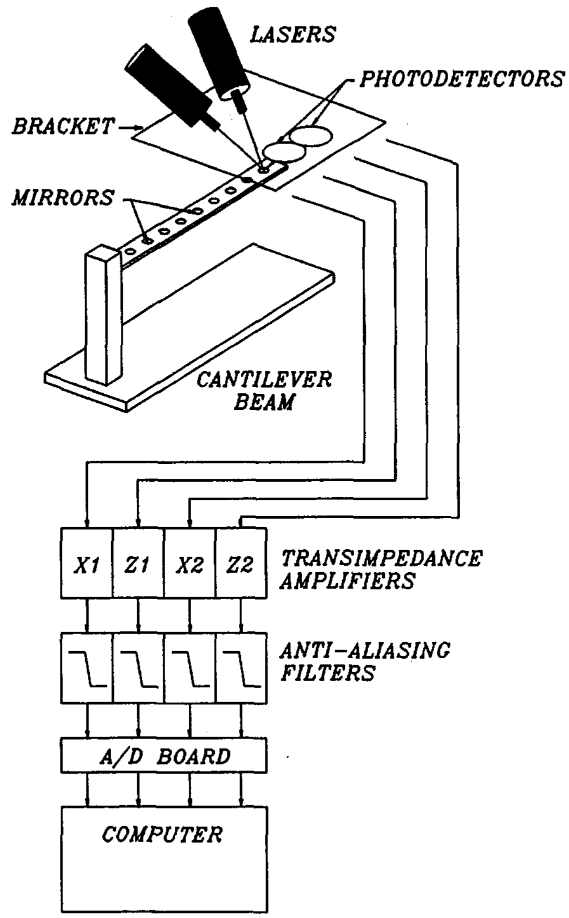

Fig. 1 Schematic of three degree-of-freedom laser vibrometer and cantilever beam test showing mirrors

$$[\Delta\mathbf{d}(\omega)] = \begin{bmatrix} 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & d_{11} & d_{12} \\ 0 & \cdots & \cdots & d_{21} & d_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ \theta_1 \\ \vdots \\ x_n \\ \theta_n \end{pmatrix} \tag{16}$$

Equation (16) is the matrix that is used in Eq. (1) to produce the modification matrix in modal space I.

3 Experimental Modal Analysis of a Cantilever and Fixed-Fixed Beam

A steel cantilever beam, 773 mm long with a rectangular cross section 6.35 mm by 24.13 mm, was chosen as the original structure. Sixteen flat circular front surface mirrors (18.0 mm dia. and 0.2 mm thickness) were mounted equidistantly along the beam with double-sided tape to provide reflective targets for the 3 degree-of-freedom laser vibrometer as depicted in Fig. 1. An instrumented modal hammer was used as the excitation source. The centerline of the beam at mirror site 3 was chosen as the driving point. The transducer was moved to each mirror site along the beam and five ensemble sets of time input and response data were acquired at each site. The time histories were processed with an FFT algorithm for frequency domain analysis. FFT results at each test site were averaged over the five ensemble data sets at each site to estimate the respective auto and cross spectral quantities. Finally, the frequency response functions (FRF) and ordinary coherence functions for each of the test sites along the beam were formed.

A complex exponential algorithm was used to estimate the

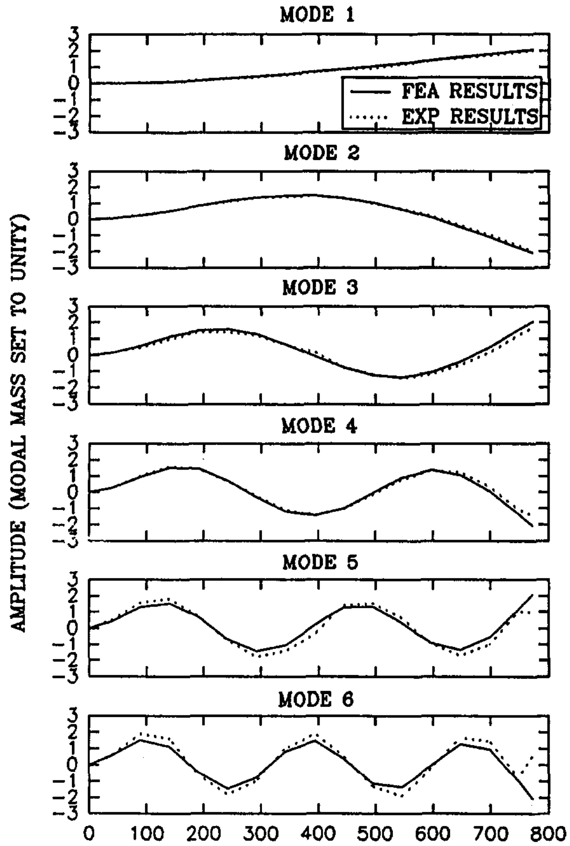

Fig. 2 Translation mode shapes (modes 1-6) for the cantilever beam, both experimental and finite-element model

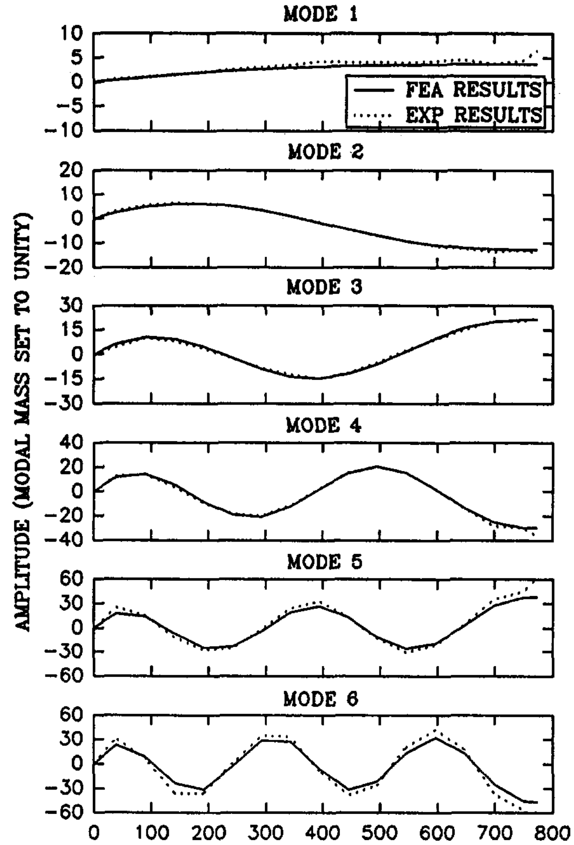

Fig. 3 Rotation mode shapes (modes 1-6) for the cantilever beam, both experimental and finite-element model

Table 1 Theoretical, FEA, and experimental cantilever beam natural frequencies

Theoretical (Hz)	FEA (Hz)	Experimental (Hz)
8.84	8.84	8.51
55.23	55.31	53.42
154.90	154.88	150.89
303.77	303.55	294.32
502.10	501.94	485.96
749.52	750.24	727.44

modal parameters and residues from the family of frequency response functions (Brown et al., 1979). The modal residues extracted with the complex exponential algorithm for each mode represent the eigenvectors. In order to use these eigenvectors in the modification routine, they must be scaled to unity modal mass as shown in Eq. (17) (Formentti, 1977).

$$\{\Phi_{\exp}^{(j)}\} = \sqrt{\frac{\omega_j}{r_{\text{driving point}}^{(j)}}}\,\{r^{(j)}\} \tag{17}$$

The unscaled residue vector, available from the modal extraction procedure, is divided by the square root of the driving point residue for that mode. If the driving point residue is negative, the residue vector is first multiplied by -1. The resulting vector is then multiplied by the square root of the natural frequency for that mode. The eigenvector is now scaled to unity modal mass.

A finite-element dynamic analysis of the cantilever beam was performed for comparison purposes using FEA with 16 Bernoulli-Euler beam elements (Chandrupatla and Belegundu, 1991). The natural frequencies for the first six modes of the experiment are compared to the finite-element mode results as well as to a theoretical solution (Young, 1989) in Table 1.

All three sets of data compare very favorably. Both the theoretical and finite-element solutions are slightly higher than the experimental values. This is probably due to the inability to create a perfectly clamped boundary condition in the experiments, which will result in lower frequencies of resonance. The translation mode shapes for the experiment and finite-element model are compared in Fig. 2 while the rotation mode shapes are compared in Fig. 3. In general, good agreement is seen between the experimental mode shapes and the finite-element model mode shapes. However, the data from the free end in the experimental database show some variation in both translation and rotation modes five and six. The mirror for this site was mounted such that the center of the mirror was at the tip of the beam and half of the mirror was not in contact with the beam. This was done to measure translation and pitch as close to the attachment site for the subsequent modification at the tip of the beam as possible. Because data at the other measurement sites compare very well to the finite-element results for all modes, the variations at the tip measurement site is attributed to the overhung mirror mounting.

An experimental modal analysis of a steel fixed-fixed beam, 923 mm long and made from the same material stock as the cantilever beam, was then conducted. This beam effectively represents the original cantilever beam with a 150 mm long free-fixed modification attached to the tip of the cantilever. The same testing procedure and modal parameter extraction process was used. A finite-element model of the fixed-fixed beam also was analyzed to verify the experimental results. Data from this experiment were compared to SDM results to assess the quality of the SDM process.

The natural frequencies for the first five experimental modes are compared to the finite-element model results as well as a theoretical solution (Young, 1989) in Table 2. Again, excellent agreement is observed between all three sets of data. Both the

Table 2 Theoretical, FEA, and experimental fixed-fixed beam natural frequencies

Theoretical (Hz)	FEA (Hz)	Experimental (Hz)
39.47	39.43	37.44
108.73	108.69	103.61
213.24	213.07	204.88
352.45	352.27	340.99
526.92	526.37	505.59

Table 3 Natural frequencies of the fixed-fixed beam based on fixed-fixed experimental results, SDM predictions using experimental cantilever database, and SDM predictions using FEA cantilever database

Fixed-Fixed Experimental Results (Hz)	SDM Predicted using Experimental Cantilever Database (Hz)	Percent Error	SDM Predicted using FEA Cantilever Database (Hz)	Percent Error
37.44	37.19	0.6	39.53	5.6
103.61	112.89	9.0	110.13	6.3
204.88	209.96	2.5	224.00	9.3
340.99	345.68	1.4	378.55	11.0
505.59	548.21	8.4	552.93	9.4

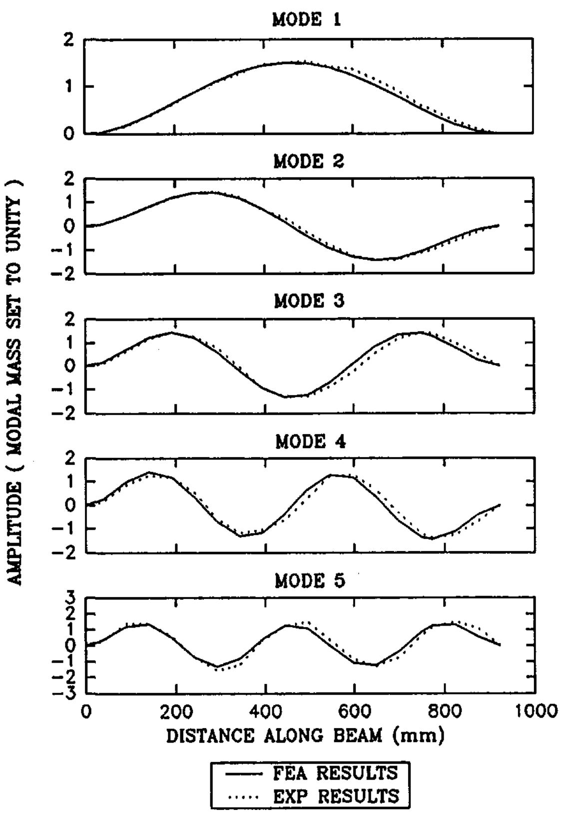

Fig. 4 Translation mode shapes (modes 1-5) for the fixed-fixed beam, both experimental and finite-element model

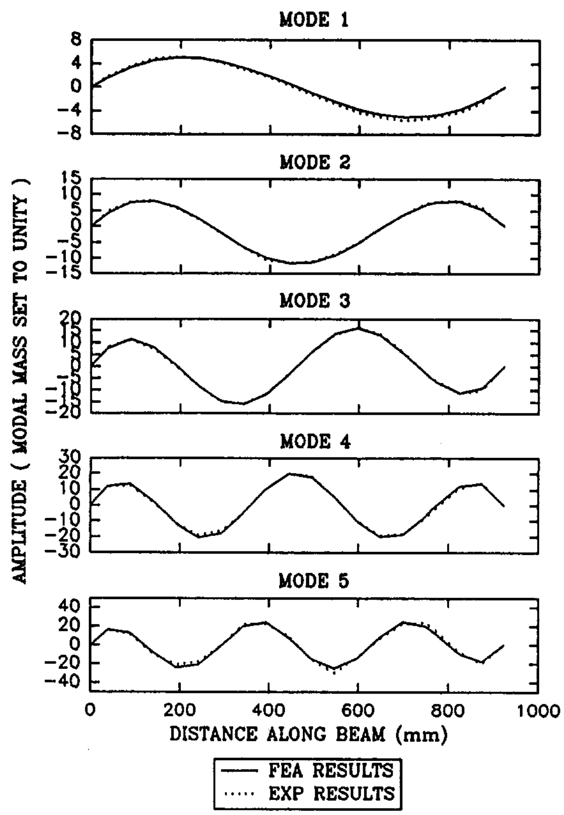

Fig. 5 Rotation mode shapes (modes 1-5) for the fixed-fixed beam, both experimental and finite-element model

theoretical and finite-element solutions are slightly higher than experimental values because of the difference in the theoretical and experimental boundary conditions. The translation mode shapes for the experiment and finite-element model of the fixed-fixed beam are compared in Fig. 4 and the rotation mode shapes in Fig. 5. Excellent agreement is seen for all modes.

4 Modification of Cantilever Beam into a Fixed-Fixed Beam

Using the procedure outlined in Section 2, the experimental database from the cantilever beam was modified and the first five modes of the resulting fixed-fixed beam were extracted. For comparison purposes, the finite-element model database for the cantilever beam also was modified and the first five natural frequencies were extracted. A frequency comparison between actual fixed-fixed beam experimental results, SDM predictions using the experimental cantilever database, and SDM predictions using the FEA cantilever database are shown in Table 3.

These results show that, in general, the frequencies predicted using the experimental database are better than those predicted using the FEA database. It should be remembered that the initial FEA database was also slightly more stiff, so it is not surprising that SDM based on the FEA database tends to predict higher frequencies than using experimental data. This fact, however, is not unique to this experiment, and will in general be the case when comparing experimental modal analyses to corresponding FEA results. Another observation is that error does not increase with increasing mode number as might have been expected. This is similar to the observation in (Elliott, 1985) who observed that the accuracy of the SDM natural frequencies and mode shapes are very dependent on which modes, and not necessarily the number of modes that are included in the original database because the SDM modes are linearly weighted sums of the original modes as shown in Eq. (16).

The number of modes in the original database is an important indicator of possible errors. In a discretized model of a physical structure, the number of degrees of freedom and, therefore, modes of vibration is some number *N*. Typically, only a few of these are available for the modification procedure. This reduction in the number of modes is called modal truncation and the errors due to it have been studied by (Braun and Ram, 1991; Avitabile and O'Callahan, 1991). Their results indicated that the accuracy of the results of SDM is heavily

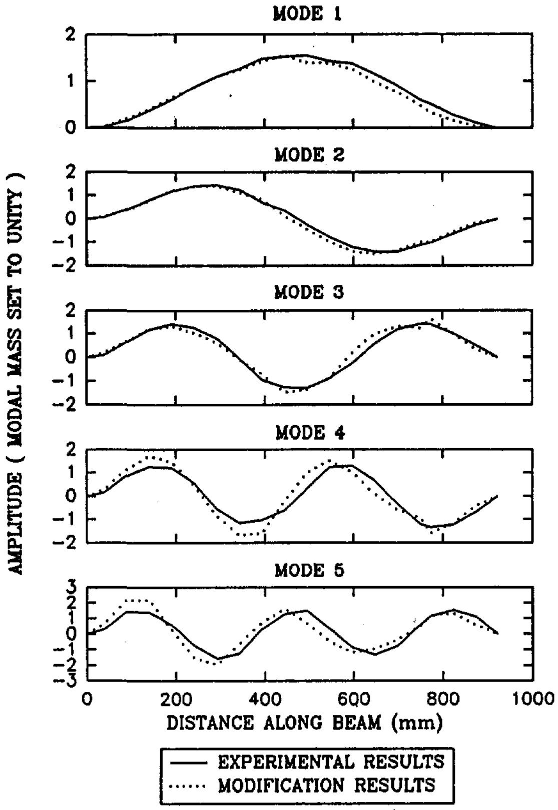

Fig. 6 Translation experimental and SDM predicted mode shapes using the experimental cantilever beam database for the fixed-fixed beam modification

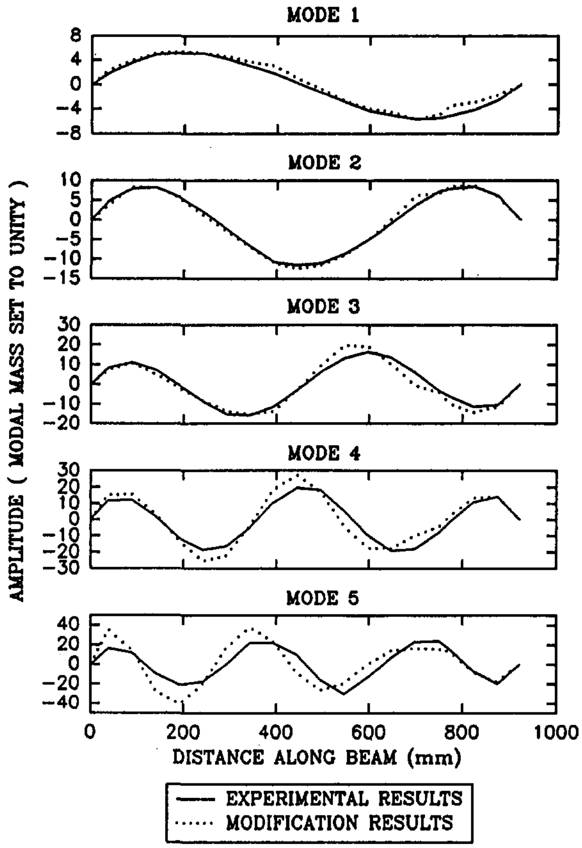

Fig. 7 Rotation experimental and SDM predicted mode shapes using the experimental cantilever beam database for the fixed-fixed beam modification

dependent on the collection of modes used in the original database. They show also that the resulting modes of the modified system are not all equally affected by the truncation of modes in the unmodified set of modes. Modal truncation errors are most apparent in this study by comparing the FEA results. Differences in frequencies between those in Table 3, column 4, and Table 2, column 2 are significant. Because there is no measurement error associated with these results, any deviations can be attributed to modal truncation errors.

Experimental translation mode shapes are compared to SDM mode shape predictions from the experimental cantilever database in Fig. 6, while experimental rotation mode shapes are compared to SDM predictions from the experimental cantilever database in Fig. 7. Excellent agreement for modes 1 through 3 are seen, while modes 4 and 5 capture general trends.

Excellent agreement in both natural frequencies and mode shapes between experimentally measurements and SDM predictions answers one of the major questions posed by (Elliott, 1985). That is, "How well will this modification procedure work when using an experimental database which will have some errors associated with it?". The data provided by the three degree-of-freedom laser vibrometer provides a database sufficiently accurate for this method to work well.

5 Summary

Data from a three degree-of-freedom laser vibrometer was used to provide a modal database for SDM beam addition to predict fixed-fixed beam performance based on experimental cantilever beam measurements. Frequency errors were 0.6, 9.0, 2.5, 1.4, and 8.4 percent for the first five modes of the predicted fixed-fixed beam compared to the experimental frequencies measured for the fixed-fixed beam. Mode shape agreement for modes 1 through 3 were excellent while modes 4 and 5 captured the general trends. This is the first documented use of experimental rotational data in the SDM process.

Further, because the experimental modal databases are better models of actual structures than corresponding FEA models, modal properties predicted by the SDM procedure using the experimental database are correspondingly better than those predicted using a FEA database. Frequency errors were 5.6, 6.3, 9.3, 11.0, and 9.4 percent for the first five modes of the predicted fixed-fixed beam using the FEA database compared to the experimental frequencies measured for the fixed-fixed beam.

This successful utilization of higher-order modification elements with experimental modal data opens the possibility for the incorporation of other structural members. Potentially three-dimensional beam or plate elements can be added to the library of available modification members. The incorporation of these elements would allow more realistic design modifications to be evaluated with experimental modal data.

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