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OPTIMIZATION OF A-WEIGHTED DISCRETE NOISE USING UNEQUALLY SPACED BLADES

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ABSTRACT

A fan with equally spaced blades causes noticeable discrete noise. An efficient way to cut the discrete noise down is using the unequally spaced blades. An optimization method to minimize the A-weighted sound level was given in this paper by designing the circumferential angles of the blades. The compositive attenuation properties made by the A-weighted function and a S_c parameter, and the limited human audible range of frequency all contribute to the noise reducing. Simulation was given and the efficacy of the optimization model was proved.

INTRODUCTION

A noticeable discrete noise is generated in a centrifugal fan because the blades are equally spaced. In the early 1960s, Lowson had put forwards the idea use non-uniformly spaced blades to reduce the discrete noise [1]. In 1970, Mellin and Sovran used the unequally spaced blades in the circumference to reduce the tonal noise and the formula to calculate the fan's discrete noise was also put forward [2]. Later, Ewald used the technique to cut the tonal noise of an axial flow fan down by 8 dB [3]. Duncan adopted unequally spaced stationary blades to get the same effect [4]. Unequally spaced blades were used in a centrifugal fan (showed in Fig. 1) to cut down the total A-weight discrete noise down. The basic theory in this optimization model was expatiated.

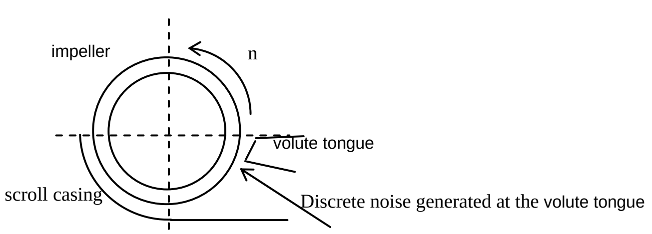

Figure 1. Cross section of a centrifugal fan

CALCULATION FORMULA OF A-WEIGHTED DISCRETE NOISE

CALCULATION FORMULA OF DISCRETE NOISE

In a centrifugal fan, the discrete noise is mainly caused by the fluctuating force at the tongue. The sound pressure at a distance r is [1]

$$p(x,t) = -\frac{\vec{F} \bullet \vec{r}}{4\pi cr} \bullet [\frac{\partial F}{\partial t}]. \qquad (1)$$

In equation (1) $\vec{F}$ represents the vector of fluctuating force, $\vec{r}$ represents the distance vector pointed from the source to the observer, $\boldsymbol{r}$ represents the absolute value of $\vec{r}$, $\boldsymbol{c}$ represents the of speed sound, "[]" represents the delayed propagation time.

We assume the n^{th}-order of the fluctuating force is $F(n\omega)e^{jnp\omega t}$ where $j=\sqrt{-1}$. Substitute it into equation (1), we get

$$p(x,n\omega) = -\frac{n\omega\vec{F} \bullet \vec{r}}{4\pi cr} \bullet F(n\omega)e^{jn\omega(t-\frac{r}{c})} \qquad (2)$$

where ω is the frequency.

The A-weighted total sound pressure is:

$$p_A(x) = \sqrt{\sum_{n=1}^{\infty}(p(x,n\omega) \bullet H_A(jn\omega))^2} \qquad (3)$$

where $H_A(n\omega)$ is the A-weighted function of sound pressure [6], it is

$$H_A(j\omega) = \frac{k_A(j\omega)^4}{(j\omega+\omega_1)^2(j\omega+\omega_4)^2(j\omega+\omega_2)(j\omega+\omega_3)}. \qquad (4)$$

In equation (4) $\boldsymbol{k_A}$, ω_1, ω_2, ω_3, ω_4 are constant numbers.

The discrete noise is calculated from equation (2) and (3). However, the fluctuating force is still unknown. Next, the fluctuating force will be calculated.

CALCULATION OF FLUCTUATING FORCE

Using Sear's theory, the fluctuating force caused by the turbulence flow at the volute tongue is [7]

$$F(n\omega)=\pi\cos(\beta)bL\rho Uu(n\omega)S_c\,. \qquad (5)$$

where β is the angle between the air flow and the normal of the plane of the volute tongue, $\boldsymbol{b}$ is the length of the plane of the volute tongue, $\boldsymbol{L}$ is the width of the outside plane of the volute tongue, $\boldsymbol{U}$ is the velocity of the main flow, $\boldsymbol{u}(\boldsymbol{n}\omega)$ is the pulsant part of the main flow, $\boldsymbol{S}_c$ is the constant number, which can be calculated by

$$S_c=\begin{cases}\dfrac{e^{-i\sigma\left[1-\frac{\pi^2}{2(1+2\pi\sigma)}\right]}}{\sqrt{1+2\pi\sigma}} & \dfrac{\omega b}{2c}<=1\\[2ex] \dfrac{e^{-i\sigma}}{\pi\sigma}\sqrt{\dfrac{i2}{M}}\displaystyle\int_0^{\sqrt{\frac{4\sigma M}{\pi(1+M)}}}e^{\frac{i\pi x^2}{2}}dx & \dfrac{\omega b}{2c}>1\end{cases} \qquad (6)$$

where $\sigma=\dfrac{\omega\bullet b}{2U}$ and $\boldsymbol{M}=\boldsymbol{U}/\boldsymbol{c}$.

The fluctuating force is caused by the fluctuating flow on the face of the volute tongue; the fluctuating flow is caused mainly by the blade wake of the impeller. A reference frame $\xi_1 0\,\xi_2$ is set up at the volute tongue (showed in Fig.2). The wake of a single blade is

$$u(\xi_1,\xi_2)=u(\xi_1)e^{-(\xi_2/y_w)^2} \qquad (7)$$

where ξ_1 is the distance in the radius direction, ξ_2 is the distance in the circumference direction. $\boldsymbol{u}(\xi_1)$ is the maximum velocity losing caused by the wake, y_w is the character width of the blade wake.

The impeller is rotating at a speed of $\omega\boldsymbol{R}$ where $\boldsymbol{R}$ is the radius of the volute tongue. So the coordinate in the circumference direction of the $\boldsymbol{i}^{th}$ blade is $\xi_2=(\omega\boldsymbol{t}+\theta_i)\boldsymbol{R}$ where θ_i is the angle of the $\boldsymbol{i}^{th}$ blade. If the total $\boldsymbol{B}$ blades is uniformly spaced, θ_i is $2(\boldsymbol{i}-1)\pi/\boldsymbol{B}$. Deduced from equation (7) and $\xi_2=(\omega\boldsymbol{t}+\theta_i)\,\boldsymbol{R}$, the composed total pulsant flow of $\boldsymbol{B}$ blades is

$$u(t)=u(\xi_1)\sum_{i=1}^{B}e^{-\pi(R(\omega t+\theta_i)/y_w)^2}\,. \qquad (8)$$

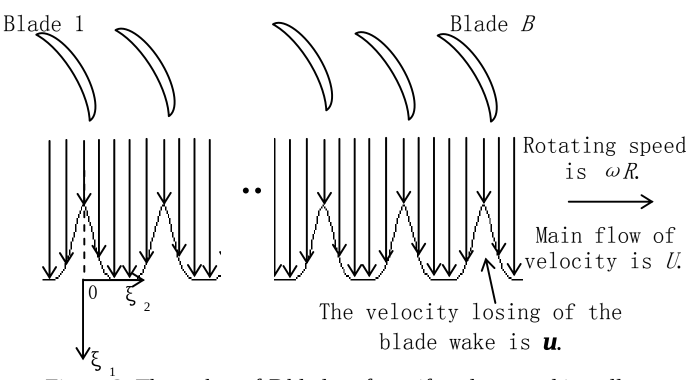

Figure 2. The wakes of ***B*** blades of a uniformly spaced impeller

By a Fourier analysis of equation (8), we get the series as

$$u(n\omega)=\frac{u(\xi_1)y_w}{2\pi R}e^{-\frac{1}{4\pi}\left(\frac{ny_w}{R}\right)^2}\sum_{i=1}^{B}e^{jn\theta_i}\,. \qquad (9)$$

In an uniformly spaced impeller (show in Fig.2), equation (9) is simplified as

$$u(n\omega)=\frac{u(\xi_1)By_w}{2\pi R}e^{-\frac{1}{4\pi}\left(\frac{ny_w}{R}\right)^2}\delta(n-mB) \qquad (10)$$

where $\delta(\boldsymbol{x})$ is the Dirac function and m is a positive integer. It is obvious that the fundamental frequency of the noise of a uniformly spaced impeller is $\boldsymbol{B}\omega$, but the fundamental frequency of the noise of a non-uniformly spaced impeller is ω. After all, the total A-weighted discrete noise of a non-uniformly spaced impeller can be calculated by the union of equation (2), (3), (4), (5), (6) and (9); that of a uniformly spaced impeller can be calculated by the union of equation (2), (3), (4), (5), (6) and (10).

OPTIMIZATION MODEL OF A-WEIGHTED DISCRETE FAN NOISE

DESIGN VARIABLE AND TARGET FUNCTION

From the analysis above, the spectrum of the discrete noise changes as the spacing fashion of the fan changes. Therefore, the angles of the other $\boldsymbol{B}$-1 blades, namely θ_2, θ_3, θ_4,… $\theta_{\boldsymbol{B}}$, are the design variables. (Because only relative angle is concerned, the angle of the first blade is zero and can be neglected in the design.) If group-distributing method is used in blade angle design, the angles blade of the first group, which are θ_2, θ_3, θ_4, … $\theta_{B/s}$, are chose as design variables, and s is the number of groups. The group-distributing method is a balancing method, which will be discussed later.

The target is to minimize the total of the A-weighted discrete noise. The target function is

$$f_{\min}([\theta])=\sqrt{\sum_{n=n1}^{n2}(p(x,n\omega)\bullet H_A(jn\omega))^2} \qquad (11)$$

where $\boldsymbol{n}_1$ is a minimum integer that is greater than $20\times2\pi/\omega$, $\boldsymbol{n}_2$ is the maximal integer that is less than $20000\times2\pi/\omega$. Equation (11) is a little different from equation (3) in the summation which takes into account the fact that the audibility frequency range of human is between 20Hz and 20000Hz.

CONSTRAINT CONDITION

In the design of the angles of blades, the balancing problem of centrifugal force caused by the unequal spaced blade and the aerodynamic performance should be considered.

BALANCING OF THE FAN

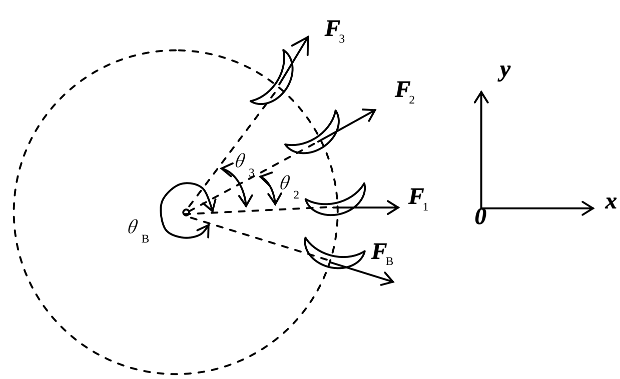

Figure 3. Centrifugal force of each blade

The balancing centrifugal force is an important problem in centrifugal fan design. The centrifugal force is composed of the acceleration acentric force and the aero-dynamical force. It is assumed there is little changing of the aerodynamic force of each blade after reconfiguring of the blade. Therefore, $\boldsymbol{F}_1=\boldsymbol{F}_2=\ldots=\boldsymbol{F}_B$. The balance equation deduced from Fig. 3 is

$$\begin{cases} 1+\sum_{i=2}^{B}\cos\theta_i = 0 \\ \sum_{i=2}^{B}\sin\theta_i = 0 \end{cases} . \tag{12}$$

Therefore, equation (12) is automatically satisfied. Another way to realize equilibrium is the group-distributing method. In this method the total set of blades is grouped into several groups. Each group has the same number of blades. The configuring fashion of each group is same. Therefore, equation (12) is auto satisfied. In reference [5], the function $\theta'_i=\theta_i+A\sin(s\bullet\theta_i)$ is used to determine the angle of unequally spaced blade. It is also a group-distributing method where s is the number of groups; $\boldsymbol{A}$ is the modulate scope of angle; θ_i is the angle of equally spaced blade.

AERODYNAMIC PERFORMANCE REQUIREMENT

To keep the aerodynamic performance after reconfiguring the angles of blades, the maximum allowable changing of angle is confined. Its range is between $2\pi/\boldsymbol{B}-\gamma$ and $2\pi/\boldsymbol{B}+\gamma$ where γ is a value and γ=25%×$2\pi/\boldsymbol{B}$ is used [5]. Therefore, the constraint condition is

$$\begin{cases} \max\left(\left|\partial_i-\partial_{i-1}\right|,\ \partial_1, 2\pi/s-\partial_{\frac{B}{s}-1}\right) < \dfrac{2\pi}{B}+\gamma \\ \min\left(\left|\partial_i-\partial_{i-1}\right|,\ \partial_1, 2\pi/s-\partial_{\frac{B}{s}-1}\right) > \dfrac{2\pi}{B}-\gamma \end{cases} \quad i=2\ldots\frac{B}{s}-1 \tag{13}$$

where $\boldsymbol{s}$ is the number of groups. If $\boldsymbol{s}$=1 it changes to the direct balancing method.

Therefore, if direct balancing method is used the constraint condition is given by equation (12) and equation (13); if group-distributing method is used the constraint condition is given by equation (13).

CALCULATION EXAMPLE

A calculation example is given for a centrifugal fan, which has a total of 48 blades. The rotation speed is 860r/min. The radius at the volute tongue is 0.21m. The width of the blade wake is 0.014m. Other parameters are $\boldsymbol{U}$=5m/s, β=5°, $\boldsymbol{L}$=1m and $\boldsymbol{b}$=0.1m.

Group-distributing method is used in the configuring of angles. A total two groups is used. The optimal angles of the first group are calculated to be 9.4°, 15°, 20.6°, 30°, 39.4°, 48.8°, 58.1°, 67.5°, 73.1°, 78.8°, 84.4°, 90°, 95.6°, 101.3°, 106.9°, 112.5°, 121.9°, 131.3°, 140.6°, 150°, 159.4°, 165°, 170.6° and 180°.

We use equation $\Delta_{dB}=20\log_{10} p_A/f_{\min}$ to calculate the reducing in A-weighted sound level where $\boldsymbol{p}_A$ is the total sound level calculated by equation (3) and $\boldsymbol{f}_{\min}$ is the optimum target. 1.4 dBA of noise reducing is found.

The spectrum contents of the pulsant velocity before and after reconfiguring of the angles of blades are showed respectively in Fig. 4.1 and Fig. 4.2. The amplitude of spectrum lines is expressed in ratios to the amplitude of fundamental frequency. In Fig. 4.1 the fundamental frequency is $\boldsymbol{B}\omega/(2\pi)$=688Hz. The spectrum line with the biggest amplitude is the fundamental frequency. In Fig. 4.2 the fundamental frequency is $\omega/(2\pi\boldsymbol{s})$. The spectrum line with the biggest amplitude is 57Hz, which is 1/12 of the fundamental frequency of the uniformly spaced impeller. The spectrum lines are dispersed into a wider band and the scope of the fundamental frequency is greatly reduced.

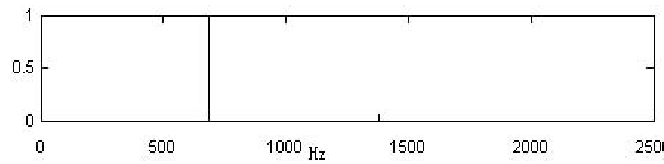

Figure 4.1. Uniformly spaced impeller

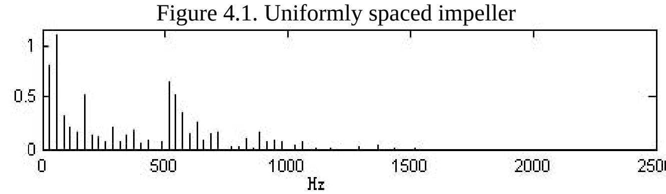

Figure 4.2. Non-uniformly spaced impeller

Figure 4. Spectrum contents of the pulsant velocity before and after reconfiguring of angles

Deduced from equation (2), (3) and (5), the attenuation multiplication, which is $\boldsymbol{S}_c\bullet\boldsymbol{H}_A(j\omega)$, is used in the calculating of the A-weight sound power. The curve of $20\log_{10}(S_c\bullet H_A(j\omega))$ is a function of the parameters of the volute tongue and the velocity $\boldsymbol{U}$, which is not changed because those parameters are not changed after reconfiguring of the angles. As showed in Fig. 5 there is a great attenuation at the low and high frequency range. If we could make most of the spectrum lines of the discrete noise fall into those range, a great deal of reducing of the A-weighted sound level will be achieved. This action is automatically done in the optimization.

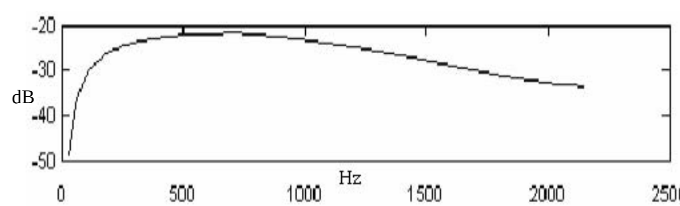

Figure 5. The total attenuation curve

Another group-distributing method is used in which the number of groups is 24. After calculation the optimal relative angles are 9.4° and 5.6°. There is also 1.4 dBA reducing of total sound level. Its configuration of blades is showed in Fig. 6.

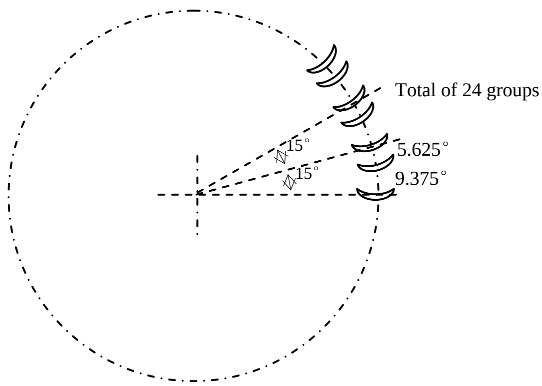

Figure 6. The configuring of blades when s=24

CONCLUSION

This paper gave an optimization model to minimize the total A-weighted sound level by reconfiguring of the angle of the blades. In the optimization model the balancing and performance constriction were considered. Simulation was done. The low and high frequency attenuation properties of synthesis attenuation curve and the human audible sound frequency range is between 20Hz and 20000Hz are the main reasons for reduction of A-weighted discrete noise.

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