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1 Introduction

rate properties [1].

# A Probabilistic Lifetime Prediction Technique for Piping Under Creep Conditions

We outline here a simple approximate method of lifetime prediction for high-temperature, internally pressurized piping components that takes into account the very large scatter present in the creep data for commonly used piping materials. This scatter is so large as to make deterministic methods of life prediction quite problematic. The method presented here is based upon the well-known Monte Carlo technique, and makes use of the standard damage fraction as the basic measure of creep damage. The method yields predictions of the probability of failure after a fixed operating time, assumed to be large. [DOI: 10.1115/1.4001266]

# 2 Analysis of Experimental Data

One of the most vexing problems encountered in making lifetime predictions of high-temperature piping components is the scatter in the creep and creep rupture data under nominally identical testing conditions. This scatter can be quite substantial. In the case of creep rupture times, it can amount in the worst cases to an order of magnitude or more difference in failure times for specimens tested under nominally identical conditions. To a considerable extent, this is brought about by product form variations, heatto-heat differences, and random variations in test conditions, but it is present, even in data from tensile specimens taken from single heats of material and tested under carefully controlled conditions. Reference [1], for example, reports the results of a number of replicated creep failure tests on specimens of AISI type 316 stainless steel, taken from a single heat of material. Here, variations in the failure times of as much as a factor of 4.5 were noted. Tests on bicrystals loaded normal to the grain boundary have produced similar variations in the creep failure times [2]. Comparable amounts of scatter can be observed in the measurements of creep [6]

The existence of this scatter leads to the conclusion that the prediction of creep lifetime is best viewed from a probabilistic standpoint. In this context, then, one would not ask at what time a component would fail in creep under a given loading conditions, but would rather inquire as to the probability of failure after a certain period of time. We outline here a simple methodology for making probabilistic creep failure predictions of high-temperature piping under plane strain conditions that takes into account the scatter in the available creep and creep rupture data. The basis for this methodology is the well-known Monte Carlo method, which has seen extensive use in many diverse applications, e.g., creep crack growth in high-temperature pipe components [3]. Creep failure is predicted by the damage, or time, fraction, a widely used quantitative creep damage measure. We give an example of the application of this methodology to an internally pressurized pipe made of ASME SA-335 P11 (1.25 Cr-0.5 Mo) steel. Hereafter, the term "Grade 11" will be used to denote the material of this nominal composition in all product forms.

We begin by analyzing the available experimental creep and creep rupture data for Grade 11 steel, a material that is in common use in pressure vessel and piping systems. Reference [4] contains an extensive compendium of creep and creep rupture data for this material at various temperatures in the creep range. Similar compendia, although perhaps not quite so extensive, exist for other materials, e.g., A-106B steel [5]. From the available data [4], a total of four sets of creep data and ten sets of creep rupture data at various stress levels for Grade 11 steel at 1000°F (538°C) were selected for analysis. The criterion for selection was that each data set contain a minimum of four data points, so that the data fits to be discussed subsequently would be of reasonable quality. Each data set represents data taken using tensile specimens fabricated from a particular heat of the indicated steel and a particular product form (plate, bar, etc.).

The data were analyzed in the context of two widely used power-law relationships for modeling creep and creep failure data [6]

$$\dot{\varepsilon}^c = C \left(\frac{\sigma}{\sigma_o}\right)^n \tag{1}$$

and

$$t_f = A \left(\frac{\sigma}{\sigma_o}\right)^{-\nu} \tag{2}$$

or in logarithmic form

$$\ln \dot{\varepsilon}^c = \ln C + n \ln(\sigma/\sigma_o)$$

$$\ln t_f = \ln A - \nu \ln(\sigma/\sigma_o) \tag{3}$$

In Eq. (1),  $\dot{\varepsilon}^c$  is the axial (secondary) creep rate experienced under a constant uniaxial stress  $\sigma$ , while  $\sigma_o$  is an arbitrarily chosen reference stress, taken to be  $\sigma_o = 1000$  lbs/in.<sup>2</sup> (6.89 MPa). Likewise,  $t_f$  in Eq. (2) is the time to creep failure under a constant uniaxial stress  $\sigma$ . The quantities (C,n) and  $(A, \nu)$  are typically taken to be material constants, although, as Figs. 1 and 2 demonstrate, the experimental data, from which their values are derived, exhibit large amounts of scatter. These figures show, respectively, log-log plots of all of the selected data sets for Grade 11 steel for creep times-to-failure and creep rates versus stress. The scatter in the experimental data for the creep rate and the failure times at

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Fig. 1 Experimental creep failure data for Grade 11 (1.25 Cr– 0.5 Mo) steel [1]

constant values of stress is clearly evident. As an example, nine creep failure data points are available at a stress level of 35,000 lbs/in.<sup>2</sup> (241 MPa). The mean time-to-failure for these nine values is 490 h, with a large standard deviation of 250 h. The smallest time-to-failure at this stress level is 140 h, while the largest is 932 h, a difference of nearly a factor of 6.6. There are indications, moreover, that the scatter becomes greater with increasing failure times. For example, Ref. [4] gives a total of 14 data at a stress level of 20,000 lbs/in.<sup>2</sup> (138 MPa), most of which are not shown in Figure 1. For these points, the mean failure time is 3820 h, with a standard deviation of 2780 h and minimum and maximum failure times of 799 h and 10284 h, respectively. The straight line in Fig. 1 is a least-squares fits of Eq. (3) to the data. It can be seen that these relationships represent the mean trends in the data fairly well, but that the scatter in the data is substantial, with values of  $r^2 = 0.84$  for the creep failure data and  $r^2 = 0.77$  for the strain rate data, where r is the correlation coefficient for the least-squares fit.

We assume, for the purposes of analysis, that the scatter arises from two independent sources: the first, due to variations in chemical composition (heat-to-heat variation) and in product form, and the second, to variations present within a given heat and product form. To quantify the first of these, we conducted standard least-squares linear fits of Eq. (3), to each of the sets of creep and creep rupture data. This procedure yielded as many values of the parameters (ln C, n) and (ln  $A, \nu$ ) as there were data sets, i.e.,



Fig. 2 Experimental creep strain data for Grade 11 (1.25 Cr–0.5 Mo) steel [1]



Fig. 3 Values of ln(C) for four data sets on normal probability paper. The normal distribution corresponding to the calculated mean and variance is shown as a straight line.

four values of  $(\ln C, n)$  and ten values of  $(\ln A, \nu)$ . The means, variances, and covariances for the resulting  $(\ln C, n)$  and  $(\ln A, \nu)$  values were then calculated using standard statistical formulae. We note in passing that more sophisticated methods are available for this purpose, e.g., Ref. [7]. The mean values were  $\ln C = -28.30$ ,  $\bar{n} = 5.72$ ,  $\ln A = 21.98$ , and  $\bar{\nu} = 4.46$ , with the corresponding variance-covariance matrices being

$$G_{\ln C,n} = \begin{bmatrix} 43.43 & -16.12 \\ -16.12 & 6.10 \end{bmatrix}$$
$$G_{\ln A,\nu} = \begin{bmatrix} 15.84 & 4.75 \\ 4.75 & 1.44 \end{bmatrix}$$
(4)

We now assume that the pairs  $(\ln C, n)$  and  $(\ln A, \nu)$  are dependent random variables governed by a joint normal distribution, with  $(\ln C, n)$ ,  $(\ln A, \nu)$  independent of each other. That the pairs  $(\ln C, n)$  and  $(\ln A, \nu)$  are dependent variables is, of course, apparent from the rather large covariances. The assumption of a joint normal distribution implies that the marginal distribution functions for  $\ln C, n, \ln A$ , and  $\nu$  are normal. The assumption that these variables are, in fact, distributed in this fashion requires some verification. One common means of doing this, at least for the marginal distribution functions, is to plot the values of the variables on normal probability paper. Plotted in this way, a random quantity, dependent or not, that has a marginal normal distribution will plot as a straight line. Figures 3-6 show such plots for  $\ln C, n, \ln A$ , and  $\nu$ , respectively, along with the straight line that results from the assumed marginal normal distribution. In general, the data can be seen not to depart too far from linearity, providing reasonable justification that the pairs  $(\ln C, n)$  and  $(\ln A, \nu)$  are, in fact, distributed in the assumed fashion.

Using a considerably more sophisticated statistical treatment, Evans and Ward [8] carried out similar characterizations of the minimum creep rate and the creep failure times in terms of generalized three-parameter gamma distributions, of which the lognormal distribution is a special case. If the pairs (C,n) and  $(A, \nu)$ are assumed to be governed by a joint log-normal/normal cumulative probability distribution function, then Eq. (3) imply that both  $\dot{\varepsilon}^c$  and  $t_f$  should be log-normally distributed. The distributions obtained by Evans and Ward for  $\dot{\varepsilon}^c$  and  $t_f$  do not differ too greatly from a log-normal distribution, providing additional support for the forms of the distributions adopted here.

Although this treatment adequately characterizes the scatter between mean trends in data sets, it does not take account of the scatter within a given data set, which may be significant. To deal

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Fig. 4 Values of n for four data sets on normal probability paper. The normal distribution corresponding to the calculated mean and variance is shown as a straight line.

with this source of scatter in the creep rupture data, we examine the variations  $\Delta \ln t_f$  between individual data points in a given creep rupture data set and the predicted value of  $\ln t_f$  obtained by



Fig. 5 Values of ln(A) for ten data sets on normal probability paper. The normal distribution corresponding to the calculated mean and variance is shown as a straight line.



Fig. 6 Values of  $\nu$  for ten data sets on normal probability paper. The normal distribution corresponding to the calculated mean and variance is shown as a straight line.

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Fig. 7 Deviation in individual specimen failure times  $\Delta t_t$  within a given data set from the data set mean trends

the least-squares linear fit to the data (see Fig. 7). The values of  $\Delta \ln t_f$  obtained from the ten creep rupture data sets are shown in Fig. 8, plotted on normal probability paper. Even though the resulting plot has a slight S-shaped variation, the assumption that the individual variations  $\Delta \ln t_f$  are normally distributed is not a bad one. The straight line in Fig. 8 shows a normal distribution fit to the data, based upon zero mean and a standard deviation of 0.2576.

#### **3** Monte Carlo Simulation and Lifetime Prediction

As outlined in Sec. 2, we assume that the variability in creep and creep failure data arises from two sources: variations between data sets due to variations in product form and chemical composition, and variations within a given data set due to random features within the creep and creep failure processes themselves. The first of these may be characterized by variations in the parameters  $(\ln C, n)$  and  $(\ln A, \nu)$  from data set to data set. The second source, at least with regard to the creep failure times, is characterized by individual specimen variations in the failure times  $\Delta t_f$  about the mean data set trends given by the second of Eq. (3). We assume that the parameters  $(\ln C, n)$  and  $(\ln A, \nu)$ , as well as the specimen variation  $\Delta t_f$ , are constant for a given Monte Carlo realization, but vary randomly from realization to realization, with the random variation being governed by the distribution functions described in Sec. 2. Given this, our goal is to make inferences about the statistical creep failure behavior of high-temperature piping.



Fig. 8 Individual specimen deviations  $\Delta t_f$  on normal probability paper

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The Monte Carlo simulation is a very attractive means of accomplishing this goal, primarily because it is able to handle fairly complex situations, and is at the same time relatively simple to implement into a computer code. The basic tool for the Monte Carlo simulation is a numerical pseudo random number generator that is capable of generating large quantities of random numbers that are uniformly distributed on the interval (0,1). Pseudo random number generators are now widely available that will carry out this task quickly and efficiently. In the present case, the random variables are the parameters in the creep rate and failure time equations (3),  $\ln C$ , n,  $\ln A$ , and  $\nu$ , and the quantity  $\Delta \ln t_f$ . Reference [9] gives a detailed description of how the output of a uniform random number generator may be used to generate random pairs of  $(\ln C, n)$  and  $(\ln A, \nu)$  that have a joint normal distribution with the desired means and variances, using the standard Rosenblatt transformation. Likewise, the Rosenblatt transformation may be used to generate uncorrelated pairs of  $\Delta \ln t_f$ , having zero mean and the specified variance.

Before proceeding further, we need to say something about the piping stresses. We assume that plane strain conditions hold, and that a constant internal pressure p is applied to the pipe. The initial stresses are then given by the well-known elastic solution, but as time progresses, creep effects modify the stress distribution, especially the  $\sigma_{\theta\theta}$  component. If the creep strains are described by Eq. (1), then the pipe stresses approach a limiting distribution. At the inner radius r=a, where the stresses can be expected to be highest and where creep failure typically initiates, this limiting distribution is given by [6]

$$\sigma_{rr}(r=a,t\to\infty) = -p$$

$$\sigma_{\theta\theta}(r=a,t\to\infty) = \frac{p}{\left(\frac{b}{a}\right)^{2/n} - 1} \left(1 + \frac{2-n}{n} \left(\frac{b}{a}\right)^{2/n}\right)$$

$$\sigma_{zz}(r=a,t\to\infty) = \frac{p}{\left(\frac{b}{a}\right)^{2/n} - 1} \left(1 + \frac{1-n}{n} \left(\frac{b}{a}\right)^{2/n}\right) \tag{5}$$

where b is the outer radius and n is the creep exponent appearing in Eq. (1). A basic assumption of the methodology described here is that the piping system has been in service for a sufficient length of time so that the inner radius stresses over the bulk of the service period are well-approximated by Eq. (5). This is quite a reasonable assumption for piping systems that have been in service for hundreds of thousands of hours.

There are several possibilities for the multiaxial generalization of the uniaxial stress  $\sigma$  that appears in Eq. (1), which we denote by  $\sigma_m$ . Among the most commonly used are the von Mises effective stress  $\bar{\sigma}$  defined by

$$\overline{\sigma} = \sqrt{\frac{1}{2}} [(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{rr} - \sigma_{zz})^2 + (\sigma_{\theta\theta} - \sigma_{zz})^2]$$
(6)

and the maximum principal tensile stress, which in this case is  $\sigma_{\theta\theta}$ . Depending upon which of these is chosen, we then set  $\sigma_m = \overline{\sigma}$  or  $\sigma_m = \sigma_{\theta\theta}$ . In either case, the multiaxial generalization of Eq. (2) becomes

$$t_f = A \left(\frac{\sigma_m}{\sigma_o}\right)^{-\nu} \tag{7}$$

We are now in a position to describe the basic Monte Carlo simulation procedure. On each Monte Carlo trial, a pseudo random number generator was used to generate four independent random numbers that are uniformly distributed on the interval (0,1). These were then transformed into two pairs of random numbers  $(\ln C, n)$  and  $(\ln A, \nu)$  that have joint normal distributions with the desired means, standard deviations, and correlation coefficients. The value of  $\ln C$  was discarded, as it does not appear in either Eq. (5) and (7), and hence, was not required. Likewise, a random value for the quantity  $\Delta t_f$  was generated.

A consideration of the means and variances for the parameters n and  $\nu$  indicates that, in approximately 1% of the trials, this procedure will generate negative values of n and  $\nu$ , which are not allowed by the physics of the situation. To deal with this situation, only those Monte Carlo trials that generated positive values of n and  $\nu$  were accepted, and trials resulting in negative values of these parameters were discarded. This resulted in a slight skewing of the Monte Carlo values, but one that can be expected to have a negligible effect on the subsequent results. A more accurate analysis would have involved a joint normal/truncated normal distribution for  $(\ln C, n)$  and  $(\ln A, \nu)$ , but would have been much more difficult to implement.

For fixed values of internal pressure p, ratio of outer to inner radius b/a, and a value of creep exponent n that changes on each Monte Carlo trial, explicit values for the stresses, and hence, the value of  $\sigma_m$ , were calculated from Eq. (5). Then a time to creep failure  $t_f$  for each Monte Carlo trial was obtained from

$$\ln t_f = \ln A - \nu \ln \left(\frac{\sigma_m}{\sigma_o}\right) + \Delta \ln t_f \tag{8}$$

The value of  $t_f$  computed here, of course, changed randomly from trail to trial. The actual time-to-failure of the component  $T_f$ , on the other hand, is both unknown and unknowable, else there would be no need for the Monte Carlo estimation procedures described here.

We now describe how the lifetime prediction was done, using a procedure that follows the prescriptions laid down by the ASME Boiler and Pressure Vessel Code [10]. In the simplest case, all conditions are constant throughout the lifetime of the component, and the remnant lifetime was estimated from the well-known damage fraction f given by

$$f = \frac{t_c}{t_f} \tag{9}$$

where  $t_c$  is the total component operating time. By definition, a value of  $f \ge 1$  signals creep failure, with values of f > 1 denoting a failed component. For f < 1, the quantity 1-f is the fraction of the remnant lifetime. On each Monte Carlo trial, a random value of  $t_f$  was computed from Eq. (8) and the resulting Monte Carlo time fraction f calculated from Eq. (9). We use lower case f for the damage fraction here to denote the values calculated from the Monte Carlo trials, which we wish to distinguish from the actual damage fraction for the component, denoted by F. As is the case with the actual component failure time, F is unknown and unknowable.

This was done repetitively for a total of *N* trials, where *N* is a large number, taken to be  $N=10^5$  here. The *N* values of *f*, thus obtained, were then ordered from lowest to highest, say  $f_i$ , i = 1, ..., N, and a probability estimator  $Pr_i$  (in percent) assigned to each  $f_i$ . A commonly used estimator, the one that we employed here, is

$$\Pr_{i} = \frac{100i}{N+1}; \quad i = 1, \dots, N$$
(10)

The significance of the probability estimator  $Pr_i$  corresponding to  $f_i$  is that it represents the Monte Carlo estimate that the actual (but unknown) life fraction F for the component is less than or equal to  $f_i$ , i.e.,  $Pr_i=Pr\{F \le f_i\}$ . Of particular interest is the value of Pr corresponding to f=1. This is exactly the estimated survival probability  $Pr_s$ , in other words, the probability that the true damage fraction  $F \le 1$  after an operating period  $t_c$ , and hence, the probability that the component will survive for a period of time  $t_c$ . Conversely,  $Pr_f=1-Pr_s$  is the estimated failure probability, the probability that F > 1, prior to  $t_c$ .

If conditions, e.g., temperature and internal pressure, are not constant throughout the lifetime of the component, but may be assumed to be piecewise constant over a period of time  $t_{c_i}=n_jt_c$ ,

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Fig. 9 Temperature histogram

 $n_j < 1$ , then we may make use of a relationship known as the Robinson's rule [6], similar to Miner's law in fatigue, to calculate the damage fraction f

$$f = \sum_{j} \frac{t_{c_j}}{t_{f_j}} = \sum_{j} \frac{n_j}{t_{f_j}} t_c$$
(11)

where  $t_{f_i}$  is the creep failure time under condition *j*.

As an example, we outline briefly here how piecewise constant temperature histories might be handled. This is done through the use of the Larson–Miller parameter  $P_{LM}(\sigma)$  defined by [6]

$$P_{\rm LM}(\sigma) = T(20 + \log_{10} t_f)$$
(12)

where *T* is the temperature in degrees Rankine and  $t_f$  is the failure time in hours. The Larson–Miller parameter is the most widely used of a number of methods for estimating the creep failure time at a given temperature, given the failure time at another temperature at the same stress level. For example, suppose that the Larson–Miller parameter has been determined for a fixed stress level at a given temperature and corresponding creep failure time. Then the creep failure time at a different temperature, say  $T_1$ , and the same stress value may be estimated from Eq. (12) as

$$t_{f_1} = 10^{P_{\rm LM}(\sigma)/T_1 - 20} \tag{13}$$

Piecewise constant pressure histories may be handled in a similar, albeit slightly more complicated, fashion.

## 4 Example

We now give a simple example of the application of the technique described in Sec. 3. We consider a pipe made of Grade 11 steel under a constant internal pressure of 650 lbs/in.<sup>2</sup> (4.48 MPa), with a ratio of outer to inner radius of b/a=1.5. The temperature history is considered to be piecewise constant about 1000°F (538°C), and is shown in histogram form in Fig. 9, in terms of the fraction of the total operating time spent at a particular temperature. A total of  $N=10^5$  Monte Carlo trials were carried out for a component operating time of  $t_c=500,000$  h (approximately 57.1 years), assuming that the creep properties of the material were statistically distributed, as described previously. The creep stresses were computed from Eq. (5), and two different multiaxial stress generalizations,  $\sigma_m = \bar{\sigma}$  and  $\sigma_m = \sigma_{\theta\theta}$ , were used. Figure 10 shows the results, in terms of the Monte Carlo probability estimator  $p=\Pr\{F \le f\}$  versus damage fraction f.

The simulations conducted using the von Mises multiaxial generalization indicate a survival probability of approximately 96% over an operating period of 500,000 h. Otherwise stated, the probability of failure during this period is 4%. The maximum principal



Fig. 10 Damage fraction probability after 500,000 h operating time

tensile stress multiaxial generalization shows a higher survival probability of approximately 97% over the same period, with a corresponding failure probability of 3%.

The shape of the curves shown in Fig. 10, rising steeply for small damage fraction values and then flattening for larger values, deserves a bit of discussion. The steep early rise of the curve is a reflection of the fact that a large number of the individual Monte Carlo trials resulted in quite low values of damage fraction after 500,000 h. To be more precise, approximately 90% of the trials yielded damage fractions of less than 0.06, a relatively small value, after 500,000 h. On a more fundamental level, this behavior is a function of the fact that the extreme values of the input  $(A, \nu)$  distributions play a major role, as the value of *f* becomes larger.

# 5 Discussion

We have presented here a probabilistic lifetime assessment methodology for high-temperature piping system components that takes into account the very wide scatter present in creep and creep failure data. This scatter is sufficiently large as to render existing deterministic techniques for lifetime prediction highly problematic. In the face of this scatter, a common practice is to make use of the minimum expected properties, a highly conservative approach. However, this practice neglects the totality of the material data base, and hence, is quite likely to lead to overly conservative predictions of component lifetime. We feel that a more rational component lifetime prediction scheme is one that takes account of the scatter in the data in a systematic fashion. This immediately implies that probabilistic, and not deterministic, life prediction techniques should be used. There are several techniques available for carrying out such calculations, but the Monte Carlo technique is especially attractive for this application, primarily due to its ease of use. The implementation described here requires only very modest computational resources. A drawback of the Monte Carlo technique is that it may require a rather large number of trials to accurately estimate small failure probabilities, say less than 0.5%. However, failure probabilities that are this small are far from critical, and hence, this should not constitute a drawback in the present case. A generalization of the time fraction rule, known as Robinson's rule, Eq. (11), is used to account for situations, in which conditions such as temperature or pressure vary in a piecewise constant fashion over the lifetime of the simulation.

The methodology outlined here makes use of the widely used damage fraction rule given by Eq. (9) for the calculation of creep damage. We depart from the standard deterministic implementation, however, in that the creep time-to failure  $t_f$  is taken to be a random variable, through Eq. (8). The random time-to failure  $t_f$  itself is dependent upon four random parameters, A,  $\nu$ , n, and

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 $\Delta \ln t_f$ , whose statistical properties are estimated from experimental creep and creep failure data. The first three of these are obtained from the variations in the values of these quantities obtained from fits of Eq. (3) to creep and creep rupture data sets, and hence, reflect variations in mean data set trends. The last,  $\Delta \ln t_f$ , is obtained from the variations in the fitted equations (3) to the individual values of  $\ln t_f$  within a data set. Thus, the first three parameters take account of the interdata set variations due to factors such as product form variations and variations in chemical composition between heats. The quantity  $\Delta \ln t_f$ , on the other hand, arises from variations within individual data sets, and thus models the effects of the intradata set variations upon the failure time. Intradata set variations on the creep rate  $\dot{\varepsilon}^c$  have, however, been neglected in the present formulation. The creep rate is not a direct contributor to the time-to-failure, and enters only indirectly through the creep exponent n, which affects the piping stresses through Eq. (5).

We have assumed in calculating the failure times that the stresses at the inner radius, the most likely point of creep failure, are given by the steady-state creep stresses, Eq. (5), for a cylinder under plane strain conditions. This is, of course, an approximation to the actual, more complicated time-dependent state stress. However, given the extremely long lifetimes associated with many piping components, it is quite a reasonable approximation. A more accurate description of the stresses would likely require a detailed finite element analysis on each Monte Carlo trial, greatly increasing the numerical effort involved in the implementation. Along these lines, it should be noted that the technique we have outlined here is applicable in principle to high-temperature components of arbitrary complexity. However, it is likely that in such cases, a detailed numerical analysis would be required for a determination of the component stresses, even for a determination of the steadystate stresses.

To sum up, we have presented a simple failure prediction methodology for piping systems that yields predictions of component survival probability, or equivalently of failure probability, after a fixed period of time. We believe that this methodology provides a much more rational basis for lifetime prediction than deterministic lifetime assessments that either ignore or treat in a very simplistic fashion, the very large scatter in the material creep and creep failure data.

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