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PREDICTIONS OF WEAR AND TRANSMISSION ERRORS OF CYLINDRICAL WORM GEARS

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ABSTRACT
Transmission error (TE) is an important transmission parameter for precision worm gears. Modern cutting methods in conjunction with modern software allow manufacturers to deliver worm gear products of high accuracy to the highly competitive market. However, the initial shape of a bronze wheel tooth changes dramatically due to bedding-in and wear when gears mesh under load, and hence transmission characteristics change.

A computer program is being developed to predict wear during bedding-in and constant wear rate stages for involute worm gears. A progressive wear over given number of tooth engagements is estimated using both the available experimental wear data and theoretical considerations. Being subtracted from an "as-cut" geometry, a new shape of worm wheel tooth surface can be predicted. The calculations can be executed iteratively for as many wear steps as necessary.

The model takes load sharing and contact stress distribution into account to estimate the lubrication oil film thickness and wear intensity. Contact patterns, TE, load cycles and meshing stiffness are also modeled.

A comparison between theoretical wear predictions and experimental wear data is made.

Predictions of wear and transmission errors are useful for optimization of existing worm gear design and for development of worm gears of new designs.

INTRODUCTION
Wormwheels' resistance to wear and pitting is mainly determined by their load capacity [1]. Worm gears are traditionally cut by trial and error in order to achieve a desirable shape of contact and a desirable tooth contact position to increase gear load capacity.

It is useful to predict worm gear transmission parameters (transmission errors (TE), marking patterns, load cycles) at the design stage in order to cut manufacturing cost and efforts. A model has been created which takes into account the complexity of worm gear geometry and manufacturing and assembly errors [2-4].

However, wear plays an important role in worm gears due to the presence of high sliding when in contact. Experimental and theoretical worm wear studies are going on world wide [4-8], but no methods have been developed to date to theoretically predict *progressive* wear changes in worm gears, and consequently, predict changes to TE and other transmission parameters due to wear. In order to do so, mathematical modeling of *progressive wear* for worm gears is required.

This paper suggests a novel method which can be used to theoretically calculate wear for a worm gear pair of given design as it progresses when gears are run under load. The method is based on ISO [1] which gives an outline for *mean wear* calculations. This ISO method has been extended to calculate *progressive wear*, using worm gear wear experimental data performed at Huddersfield ([9], Huddersfield University working papers), and various theoretical considerations. The method also takes into account the contact conditions and elastohydrodynamic lubrication (EHL) using recent developments at Cardiff University, UK (Sharif et al [10-11]).

Load cycle calculations are made using the compliance matrix method proposed by Conry and Seireg [12], using developments by Sudoh [13].

Changes in load cycle, TE, and marking patterns due to wear are calculated and predictions are compared with experiments.

NOMENCLATURE
b_H semi-axis of Hertzian contact
$[C]$ compliance matrix
$\vec{e}$ unit vector

$\vec{e}^T$ transposed unit vector
F total transmitted force
$h_{\min m}$ mean minimum oil film thickness over the meshing zone
i (subscript) line of contact number
J_w wear intensity
n number of contact lines
$\vec{P}$ column of load vectors
s_{gB} local sliding path
s_w worm gear wear
$\vec{v}_g$ worm and wheel sliding velocity
v_{2n} component of wheel velocity normal to the contact line
$\vec{\varepsilon}$ gap vector
1 (subscript) corresponds to worm
2 (subscript) corresponds to wheel

1. Suggested methodology for progressive wear calculations

The presented methodology for the calculation of progressive worm gear wear is based on considerations presented in ISO [1], as well as various theoretical considerations.

It is proposed to calculate wear of worm gears, s_w, as a function of the *local sliding path*, s_{gB}, and *wear intensity*, J_w, as follows:

$$s_w = J_w \times s_{gB} \qquad (1)$$

The *local sliding path*, s_{gB}, by definition, can be calculated directly from equation (2), [1]

$$s_{gB} = \frac{\left|\vec{v}_g\right| \times 2 \times b_H}{v_{2n}} \qquad (2)$$

Here:

$\vec{v}_g = \vec{v}_1 - \vec{v}_2$ - worm and wheel sliding velocity
v_{2n} - component of wheel velocity normal to the contact line
b_H - semi-axis of Hertzian contact

Wear intensity, J_w, can be calculated from an equation suggested by ISO [1] which, for mineral oils, can be re-written as:

$$J_w = 2.4 \times 10^{-11} \times h_{\min m}^{-3.1} \qquad (3)$$

Here, $h_{\min m}$ is *mean* minimum oil film thickness over the meshing zone [1]. Within the ISO procedure, $h_{\min}$ is calculated using Dowson and Higginson's line contact theory.

The suggested alternative approach differs in two ways.

Firstly, it takes *local* information on minimum oil film thickness $h_{\min}$ into account, as it changes from contact point to contact point, for every wear step. This information is received (from research project collaborators) from results on contact analysis and oil film thickness by Sharif's et al [8-9] (using EHL).

Secondly, results of wear tests at Huddersfield are used to adjust the coefficient in equation (3).

2. Summary of methodology used for calculation of load cycle

The method for load cycle calculations is based on an approach suggested by Conry and Seireg [12] with compliance matrix for a particular worm gear case provided by Sudoh [13]. This section presents the summary of the method.

If there were no gap between the worm thread and wheel tooth surfaces, the TE, α, due to their deflections for a certain meshing position, could be calculated from equation (4) ([12], [13]):

$$\begin{bmatrix} [C_1]_1 & & & \\ & [C_1]_2 & & \\ & & \ldots & \\ & & & [C_1]_n \end{bmatrix} \begin{bmatrix} \vec{P}_1 \\ \vec{P}_2 \\ \ldots \\ \vec{P}_n \end{bmatrix} + \begin{bmatrix} [C_2]_1 & & & \\ & [C_2]_2 & & \\ & & \ldots & \\ & & & [C_2]_n \end{bmatrix} \begin{bmatrix} \vec{P}_1 \\ \vec{P}_2 \\ \ldots \\ \vec{P}_n \end{bmatrix} = \alpha \vec{e} \qquad (4)$$

Here:

n - number of contact lines,

$$[C_1] = \begin{bmatrix} [C_1]_1 & & & \\ & [C_1]_2 & & \\ & & \ldots & \\ & & & [C_1]_n \end{bmatrix}$$
- worm compliance matrix,

$$[C_2] = \begin{bmatrix} [C_2]_1 & & & \\ & [C_2]_2 & & \\ & & \ldots & \\ & & & [C_2]_n \end{bmatrix}$$
- wheel compliance matrix,

$\vec{P}_i$, $i = 1 \ldots n$ - column of load vectors,

$\vec{e}$ - unit vector.

In order to take a gap between the worm thread and wheel tooth surfaces into account, equation (4) is rewritten as follows:

$$[C]\left[\vec{P}\right] + \vec{\varepsilon} \geq \alpha\, \vec{e} \tag{5}$$

where,

$[C] = [C_1] + [C_2]$ - combined worm and wheel compliance matrix,

$\vec{\varepsilon}$ - gap vector.

An equality sign in equation (5) means that load is transmitted at the contact point (i.e. the gap is smaller than deformation), and an inequality sign means that there is no contact.

The inequality (5) can be re-written as an equality (6):

$$[C]\vec{P} + \vec{\varepsilon} + \vec{Y} = \alpha\, \vec{e} \tag{6}$$

where

$$\vec{Y} = \begin{cases} = 0, & \text{gap} \leq \text{deformation} \\ > 0, & \text{gap} > \text{deformation} \end{cases} \tag{7}$$

The total transmitted force, F, is:

$$F = \vec{e}^{\,T} \bullet \vec{P} \tag{8}$$

where $\vec{e}^{\,T}$ is a transposed unit vector.

Equations (6)-(8) have to be solved simultaneously. The linear programming iterative method developed by Conry and Seireg [12] can be used to do so.

3. An example of progressive wear calculations

This section shows a result of the use of described methodology for a particular worm gear pair. Changes in transmission parameters due to wear over several hours of work under load are presented. Interim wear results for several points shown in Figure 1 are also provided.

Parameters of the worm pair are given in Table 1.

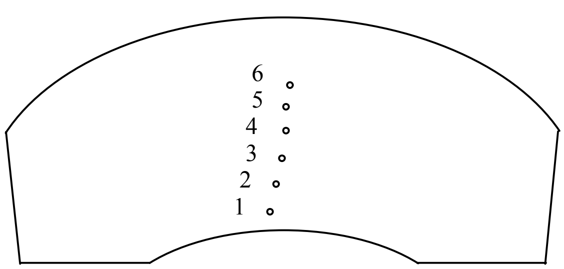

Figure 1. Contact positions of worm and wheel

Table 1 Parameters of worm gear pair

Parameter	Symbol	Value	Units
Number of worm starts	z_1	1	
Number of teeth	z_2	50	
Pressure angle	α_n	20	deg
Module	m	4.997	mm
Center distance	a	152.4	mm
Face width of wheel	b	35.58	mm
Worm reference diameter	d_1	54.903	mm
Wheel reference diameter	d_2	249.897	mm
Worm material	Case Harden Steel		
Wheel material	Phosphor Bronze		

Gear were run under 2000Nm load at 150 rpm for a wear step of 6 hours. Mineral oil Vitrea 220 was used.

Results by Sharif et al and Sudoh et al applied for our particular pair for every contact position in Figure 1 are summed up in table 2.

Wear tests were used to modify the equation (3) as follows:

$$J_w = 4 \times 10^{-13} \times h_{\min}^{-3.1} \tag{9}$$

to "match" the gear pair under consideration and oil used in the tests. The same tests were used to adjust Sudoh's compliance matrix to the gear pair of given design.

Using the above data, local wear increments were calculated for a *wear step* of 6 hours and the results are presented in sections 4.1-4.4. A comparison with the experimental results is given.

3.1 Wear increments

Wear increments for the points in Figure 1 over 6 hours calculated by the methods discussed above are shown in Figure 2.

It can be seen that generally wear increases towards the root of a tooth, as one would expect (because contact lines are more dense near the root of a tooth than those at the tip). Heavier wear close to the pitch line of wheel is due to load increase there (load sharing is discussed in section 4.2 in more detail).

Table 2 Interim parameters for wear calculations for given worm gear pair at contact positions per in Figure 1

Position	Load (N)	Semi-axis b_H (mm)	Film thickness $h_{\min}$ (μm)	Slip-roll Ratio $\frac{v_{12}}{v_{2n}}$
1	5959	0.696	0.012	26.3
2	8513	0.748	0.011	28.2
3	16175	0.881	0.009	32.0
4	11237	0.776	0.010	36.6
5	11067	0.736	0.010	39.6
6	11067	0.661	0.010	45.8

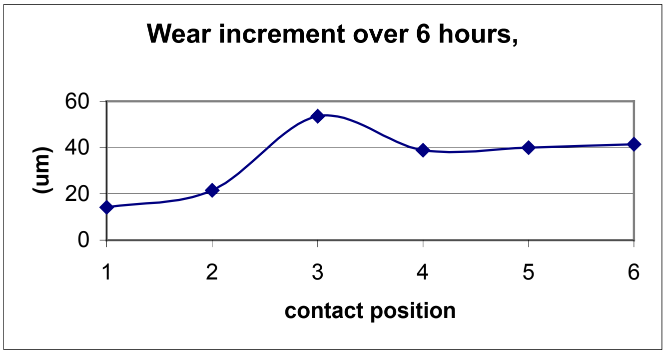

Figure 2. Wear increment under 2000Nm load for 6 hours

3.2 Load cycle

Using equation (6)-(8) and data from Table 1, load cycles for one wheel tooth with new and worn geometries are calculated and shown in Figure 3. It can be seen that the maximum load for this particular design is located at about the middle of the tooth, for both new and worn geometries. For the worn wheel geometry, the load is more symmetrically distributed than that of a new wheel.

3.3. Transmission Errors

Transmission errors for the new and worn wheels are modeled and given in Figures 4a and 4b respectively. The corresponding measured transmission errors are shown in Figures 4c and 4d.

Comparison of the theoretical and experimental results for a new wheel show that at zero loads the TE is small. At higher loads, the TE is bigger, and of a similar magnitude and shape.

The TE change with load in case of a worn wheel shows a similar pattern for the model and experimental results.

The presence of high frequencies in measured TE, especially for zero load, can be observed. This is due to manufacturing errors resulting in high spots on contacting surfaces which are removed when gears are subjected to a load and with wear. It is not possible to model this process.

In general, the similarity between the measured and theoretically predicted TE is encouraging.

3.4. Marking Patterns

Similarly, marking patterns of the new and worn wheels, theoretical and measured, are given in Figures 5. They demonstrate the effect of wear in that the contact spreads wider across the tooth, and its shape changes. The model predicts pretty accurately the changes towards the root of a tooth.

Bearing in mind the difficulties in controlling the thickness of the blue powder used when marking patters are taken, one can conclude that the similarity of the model as compared with the experimental results is very good.

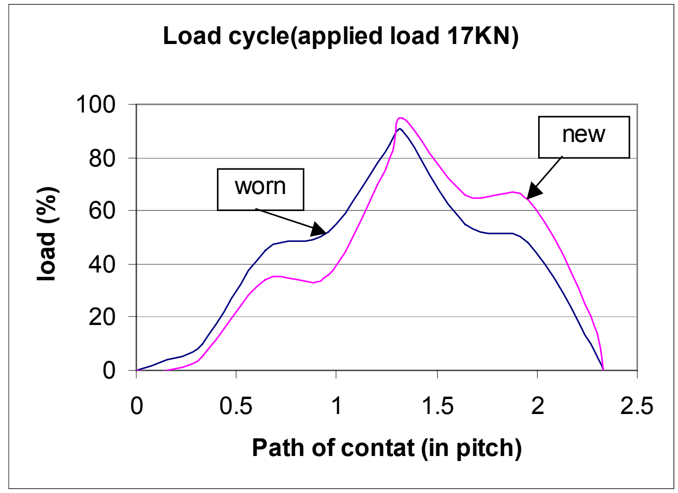

Figure 3. Load cycle of one wheel tooth

4. Conclusions

The paper describes a novel method for theoretical calculations of a *progressive* wear in involute worm gears and demonstrates its application to a pair of a particular design. As a result, the model can be used to theoretically predict the changes in important transmission parameters such as TE and marking patterns due to wear.

Theoretical results are compared with tests and correlation looks encouraging.

Experiments were made to provide corrections to the standard wear equations given by ISO in order to take into account geometry of a particular gear pair and parameters of the particular oil used.

The method can be extended so that it can be used for a wide range of gears and oils. However, additional tests are required in this case to provide empirical data. Once this has been completed, the model will lay a solid foundation for the optimization design of worm gears.

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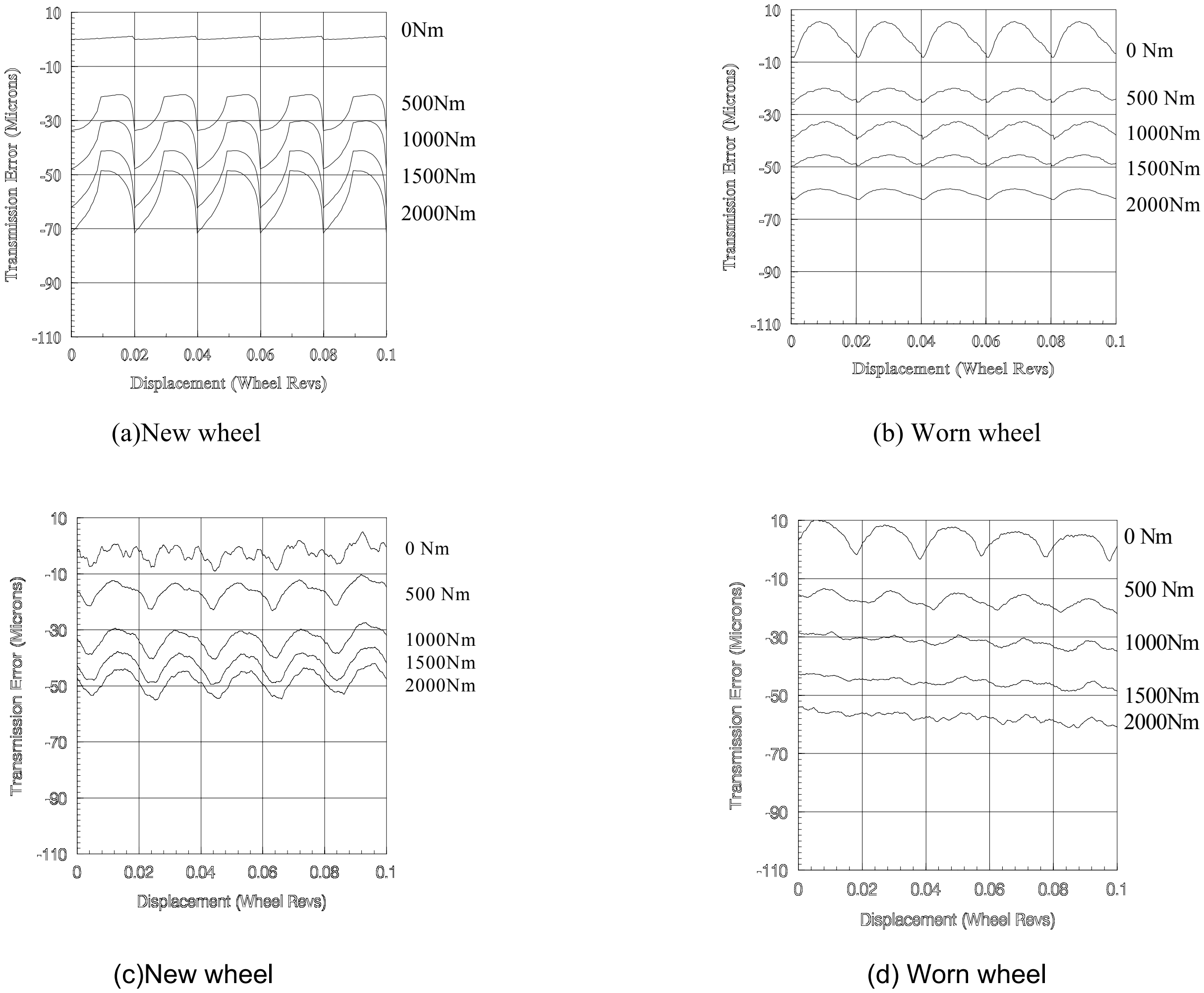


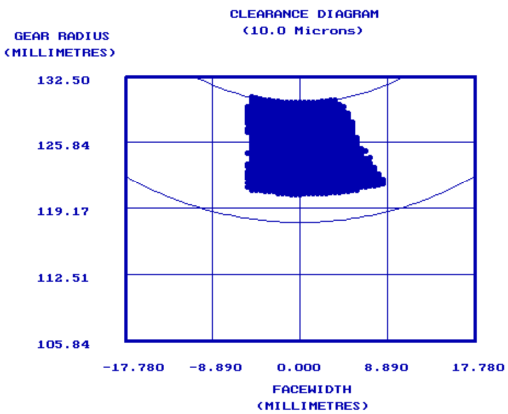
(a)New wheel

(b) Worn wheel

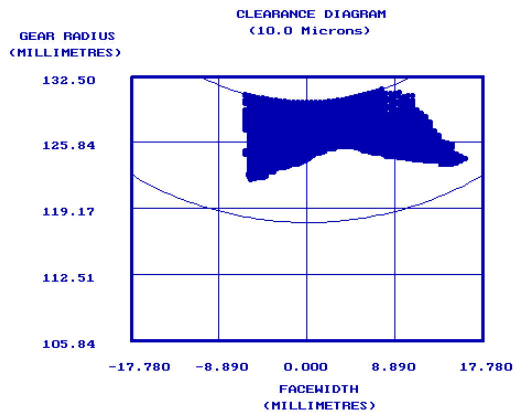
(c)New wheel

(d) Worn wheel

Figure 4. Transmission errors
a and b – modeled
c and d - measured



(a)New wheel



(b) Worn wheel

(c)New wheel

(d) Worn wheel

Figure 5. Marking patterns
a and b – modeled
c and d – measured

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