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COMPARISON OF WOOD, GAINES, PARABOLIC, HAYASHI, DHANNO AND POLYNOMIAL MODELS FOR LACTATION SEASON CURVE OF SIMMENTAL COWS

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ABSTRACT

The aim of this study was to determine a suitable nonlinear model explaining the lactation season curve of Simmental cows. Monthly milk yield records representing the test days milk yield of 777 Simmental cows were used to estimate lactation curve parameters by using Wood, Gaines, Parabolic, Hayashi, Dhanno and (second degree) polynomial models and to compare the shape of lactation season curves resulted from fitting all of these models. These models were given in explanation of the parameters and according to the lactation season, the parameters a, b and c for these models at all the periods were estimated. Secondly, the formula of some criteria, such as time to peak, peak production, turning point time and turning point production, are presented in mathematical procedure and then t the criteria values are calculated for all of the examined models. Furthermore, partial derivatives of the models according to the parameters were given in mathematical procedure. For the season, winter, spring, summer, fall and without season the lactation curve graphs for all the nonlinear models were drawn, respectively. Moreover, the best nonlinear model was used to determine the adjusted coefficient determination (R^2_{adj}), mean square prediction error (MSPE) and Bayesian Information Criteria (BIC). The runs test was used for determining whether data differ systematically according theoretical curves. Generally, polynomial model gave the best nonlinearmodel to the data compared to the other models.

Key words: Simmental cows, wood, gaines, parabolic, hayashi, dhanno and polynomial modelling, lactation curve.

INTRODUCTION

Milk yield, fertility, and health are the most substantial traits affecting performance in dairy production (Kucuk and Eyduran, 2010). Bulls and cows have been genetically evaluated on the basis of the 305-d-milk yield in selection programs (Yilmaz *et al.,* 2011). The best way to provide desirable genetic improvement of these animals in milk yield is to predict lactation shape parameters, used as selection criteria in breeding program and influenced by genetic and environmental factors..The monthly milk yield – lactation time data must be provided for estimating these shape parameters of several nonlinearmodels (Nelder, 1966; Wood, 1967; Rook *et al.,* 1993). Cagan and Ozyurt, (2008) mentioned significant genetic correlation between milk yield and persistency, which is known as a cow’s ability to maintain milk production after peak point in a lactation curve.

Although there were limited number of different complex modeling studies on the definition of lactation shape in dairy cattle were very few (Yılmaz *et al.,* 2011), there were a great number of studies on comparison of differerent nonlinearmodels with the aim of determining the best nonlinearmodel explaining relationship between milk yield-time (Landete-Castillejos and Gallego, 2000; Orhan and Kaygisiz, 2002; Cagan and Ozyurt, 2008; Ozyurt and Ozkan, 2009; Kucuk and Eyduran, 2010). Determination of the most suitable nonlinear model in a herd is very important for developing more effective selection strategies and regulating management practices. Hence, the aims of this study were to estimate lactation curve parameters of the nonlinearmodels, and to compare performances of the nonlinear models used to explain relationship between milk yield-time in Simmental cows.

MATERIALS AND METHODS

Data on monthly milk yield recordsfor the season representing the test days milk yield of 777 Simmental cows were collected from a herd maintained at the Farm Production Research Institute of Kazova, during the period from 1989 to 1996. Knowledge of the study material was provided by Kaygisiz, (1996).

For a nonlinear regression model;

$$y_i = f(t_i, p) + \varepsilon_i \qquad (1)$$

$i = 1, 2, ..., n$ where y is the found independent variable, t is the dependent variable, p is the vector of the unknown parameters and n is the number of observation. The estimators of the vector of the unknown parameters are found by minimizing the sum of squares error (SS_{err}) model as below,

$$SS_{err} = \sum_{i=1}^{n} (y_i - f(t_i, p))^2 \qquad (2)$$

Under the assumption that the ε_i is the normal and independent with mean zero and common variance σ^2. Since y_i and t_i are fixed observations, the sum of squares residual is a model of p. Least squares estimates of p are values which, when substituted into Eq. 2, will make the SS_{err} minimum and are found by first differentiating Eq. 2 with respect to each parameter and then setting the result to zero. This provides the normal equations. These normal equations take the form,

$$\sum_{i=1}^{n} (y_i - f(t_i, p)) \left[\frac{\partial f(t_i, p)}{\partial p_j} \right] = 0 \qquad (3)$$

for j = 1, 2,.., k where k is the the number of the parameter.

An iterative method must be employed to minimize the SS_{err}. Here the Levenberg-Marquards iterative method is an estimator method, which represent a compromise between the Gauss-Newton method and the steepest descent method. It is a method that combines the best features of both while avoiding their most serious limitations. Due this characteristic we decided to use the method described by Ismail *et al.,* (2003). Lactation curves were described by the following formulas in Table 1.

All of the examined models were fitted by the Levenberg-Marquards algorithm using NLIN of SAS (SAS, 2000). Starting grids were specified such that all solutions fell within the outer limits of the search grids. Time to peak, peak production, turning point time and turning point production over the whole period of lactation were calculated and compared with statistics obtained from the actual data.

The examined parameters were calculated by using the following formulas in Table 2. The partial derivatives of the models with respect to each parameter (a, b and c) are given in Table 3. The partial derivatives of the nonlinear models must be required for the Levenberg- Marquardt algorithm using NLIN procedure of SAS package.

Statistical Evaluation: There are different statistical tests for ranking and evaluating models. Sometimes results from these different tests seem contradictory, so an overall assessment is needed in this situation. The number of runs of sign of residuals, MSPE, R^2_{adj}, BIC and plot of residuals against predicted values and lack-of-fit test are "goodness of fit" criteria used to determine the best nonlinearmodel.

In assessing goodness of fit, it is essential to first examine a graph of a curve superimposed on the data points. Many potential problems are easiest to spot graphically. It is inappropriate to use the results of a nonlinear regression program without first examining a graph of the data together with the fit curve. In addition to viewing the graph, several statistical methods can be used for quantitating goodness of fit.

The runs test is a simple and robust method, is used to determine whether data differ systematically from a theoretical curve. A run is a series of consecutive points with a residual of the same sign (positive or negative). The runs test statistic is calculated from the number of runs associated with a particular fit of the data (Motulsky and Ransnas, 1987).

The parameters of the nonlinear lactation season curves were estimated using the NLIN procedure of SAS. An assessment of the error of predicted relative to observed values was made by calculation of the mean square prediction error (MSPE):

$$MSPE = \sum_{i=1}^{n} (O_i - P_i)^2 / n$$

where i = 1, 2, . . , n

n is the number of experimental observations, and O_i and P_i are the observed and predicted values, respectively (Bibby and Toutenburg, 1977).

The adjusted coefficient determination (R^2_{adj}) is a rescaling of R^2 by the degrees of fredom so that it involves a ratio of mean squares rather than sums of squares. Similar to R^2, it should be computed from the residual mean squares:

$$R^2_{adj} = 1 - \frac{MSPE}{MS(\text{corrected total})}$$

The adjusted coefficient determination is more comparable than R^2 for models that involve different numbers of parameters. A model with large R^2_{adj} is more favourable. The numerical value of R^2_{adj} the closer to one is indicates a better fit when the models are compared (Sit and Melanie, 1994).

Comparison of models was based on Bayesian Information Criteria (BIC), which are model order selection criteria based on parsimony and impose a penalty on more complicated models for inclusion of additional parameters. For that reason, BIC criteria is better than R^2_{adj}. BIC combines maximum likelihood (data fitting) and choice of model by penalizing the (log) maximum likelihood with a term related to model complexity as follows:

$$BIC = nLn\left(\frac{RSS}{n}\right) + KLn(n)$$

where RSS is the residual sum of squares, K is the number of free parameters in the model and n is the sample size. A smaller numerical value of either BIC and MSPE indicates a better fit when comparing models (Schwarz, 1978; AlZahal *et al.,* 2007).

The BIC and R^2_{adj} are useful for comparing models with different numbers of parameters. They have therefore more advantageous than MSPE.

RESULTS AND DISCUSSION

To identify the lactation curves, the models which have many different structures have been reported in the literature. Six different models were evaluated using data collected in different seasons of Simmental dairy cows. Comparison of their predictive ability allows identification of a mathematical model capable of describing and providing a better perspective on the shape of the lactation curve of Simmental dairy cows.

The Figures 1-10 illustrates the estimated lactation season, winter, spring, summer and fall, curve for all of the modelling, respectively. All of the nonlinearmodels gave fit well for each season. However, the only Wood and Dhanno modelling had the peak points. Similar result was also reported by Aziz *et al.,* (2006). But the only Wood and Dhanno modelling have the peak points. Several studies have shown differences in the general shape of the lactation curve (Ferris *et al.,* 1985; Perochon *et al.,* 1996; Ramirez-Valverde *et al.,* 1998; Orman and Ertuğrul, 1999; Landete-Castillejos and Gallego, 2000; Orhan and Kaygisiz, 2002), the most common shape being a rapid increase after calving to a peak a few weeks later followed by a gradual decline until the cow is dried off. The models that shows the best this kind of curve are Wood and Dhanno models.

On the other hand, the residual plots show how far each point is from the curve. The residuals of all models except fall season are randomly above and below zero. The residuals from the Hayashi and Gaines models, however, show a systematic pattern, with negative residual for the first and last point and positive residuals for the middle points. Such systematic deviations indicate that the data are not well-described by those equations. Similarly, the researchers such as Motulsky and Ransnas (1987) reported that these systematic deviations for the data set were not appropriate.

The runs test statistic is calculated from the number of runs associated with a particular fit of the data. The number of runs of sign of the residuals and significance values are given in table 4.

According to these results, the residuals of all seasons and overall indicates that the curves do not deviate systematically from the points (P>0.05) in Table 4. In a similar study, Şahin and Efe, (2010) used Durbin Watson test instead of runs test in their studies. Also autocorelation was not observed in this study.

The critical points and their values (Time to Peak, Maximum Yield, Turning Point Time and Turning Point Production) of all of the modelling for each season are given in Table 5. For all season, by examining Table 5, Wood and Dhanno models were explained many important parameters in terms of dairy cattle. In addition, the results of Wood and Dhanno models were close to each other. In addition, the results of Wood and Dhanno models were close to each other. Gaines model did not have any critical point and moreover this model was simplistic and did not provide a physiological basis for the lactation curve (Val-Areeola *et al.,* 2004). For Hayashi and polynomial models, the critical points could not be accounted because of the structures of the curves. Parabolic model has only turning point and its production for all season except summer.

Table 1. Models used to describe the lactation curve of dairy cows

Models	Functional form
Wood	$y(t) = at^b e^{-ct}$ (Wood, 1967)
Gaines	$y(t) = ae^{-bt}$ (Thornley and France, 2005)
Hayashi	$y(t) = b(e^{-c/t} - e^{-t/ac})$ (*Hayashi et al.,* 1986)
Polynomial	$y(t) = a + bt + ct^2$ (Dave, 1971)
Dhanoa	$y(t) = at^{bc} e^{-ct}$ (Dhanoa and Le Du, 1982)
Parabolic	$y(t) = ae^{(bt - ct^2)}$ (Sikka, 1950)

where y is the milk yield (kg/day), t is time of lactation (month), e is the base of natural logarithm, a, b and c are the parameters which characterize the shape of the curve.

The least squares estimates of the parameters and the goodness of fit of the nonlinear models for daily mean milk production-month of lactation relationship are given in Table 6. All models were seamlessly fit to the data set. According to the meanings, the parameter values of the models were in aggreement with each other and did not indicate an extreme deviation. These results were in agreement with results reported by the

Table 2. Critical points and their formulas for all of the models

Models	Critical Points and Their Formulas			
	Time to Peak (Month)	Maximum Yield (Kg)	Turning Point Time (Month)	Turning Point Production (Kg)
1. Wood	b/c	$a(b/c)^b e^{-b}$	$\frac{b+\sqrt{b}}{c}$, $\frac{b-\sqrt{b}}{c}$	$a(\frac{b+\sqrt{b}}{c})^b e^{(-b-\sqrt{b})}$, $a(\frac{b-\sqrt{b}}{c})^b e^{(-b+\sqrt{b})}$
2. Gaines	-	-	-	-
3. Hayashi	$\frac{ln(a)ac}{a-1}$	$b(a^{(\frac{a}{1-a})} - a^{(\frac{1}{1-a})})$	$\frac{2\,ln(a)ac}{a-1}$	$b(a^{(\frac{2a}{1-a})} - a^{(\frac{2}{1-a})})$
4. Polynomial	$-b/2c$	$a-\frac{b^2}{4c}$	-	-
5. Dhanoa	b	$ab^{bc}e^{-bc}$	$\frac{bc+\sqrt{bc}}{c}$, $\frac{bc-\sqrt{bc}}{c}$	$a(\frac{bc+\sqrt{bc}}{c})^{bc} e^{(-bc-\sqrt{bc})}$, $a(\frac{bc-\sqrt{bc}}{c})^{bc} e^{(-bc+\sqrt{bc})}$
6. Parabolic	$b/2c$	$ae^{(\frac{b^2}{4c})}$	$\frac{b+\sqrt{2c}}{2c}$, $\frac{b-\sqrt{2c}}{2c}$	$ae^{(\frac{(b+\sqrt{2c})(b-\sqrt{2c})}{4c})}$

-:does not have any critical points.

Table 3. Partial derivatives of all of the models

Used models	Partial derivatives of the models according to the parameters	Used models	Partial derivatives of the models according to the parameters
1. Wood	$\frac{\partial y}{\partial a} = t^b e^{-ct}$	4. Polynomial	$\frac{\partial y}{\partial a} = 1$
	$\frac{\partial y}{\partial b} = at^b \ln(t) e^{-ct}$		$\frac{\partial y}{\partial b} = t$
	$\frac{\partial y}{\partial c} = -at^b te^{-ct}$		$\frac{\partial y}{\partial c} = t^2$
2. Gaines	$\frac{\partial y}{\partial a} = e^{-bt}$	5. Dhanoa	$\frac{\partial y}{\partial a} = t^{bc} e^{-ct}$
	$\frac{\partial y}{\partial b} = -ate^{-bt}$		$\frac{\partial y}{\partial b} = at^{bc} c \ln(t) e^{-ct}$
3. Hayashi	$\frac{\partial y}{\partial a} = -\frac{bte^{(\frac{-t}{ac})}}{a^2 c}$		$\frac{\partial y}{\partial c} = at^{bc} b \ln(t) e^{-ct} - at^{bc} te^{-ct}$
	$\frac{\partial y}{\partial b} = e^{(\frac{-t}{c})} - e^{(\frac{-t}{ac})}$	6. Parabolic	$\frac{\partial y}{\partial a} = e^{(bt-ct^2)}$
	$\frac{\partial y}{\partial c} = b(\frac{te^{(\frac{-t}{c})}}{c^2} - \frac{te^{(\frac{-t}{ac})}}{ac^2})$		$\frac{\partial y}{\partial b} = ate^{(bt-ct^2)}$
			$\frac{\partial y}{\partial c} = -at^2 e^{(bt-ct^2)}$

Table 4. The Runs Test for each season and overall

Models	Winter		Spring		Summer		Fall		Overall season	
	Runs	Sig.	Runs	Sig.	Runs	Sig.	Runs	Sig.	Runs	Sig.
1. Wood	4	0.314	4	0.361	4	0.314	4	0.314	7	0.737
2. Gaines	3	0.094	5	0.737	4	0.314	3	0.094	7	0.737
3. Hayashi	3	0.094	5	0.737	4	0.314	3	0.094	7	0.737
4. Polynomial	4	0.314	4	0.314	4	0.314	4	0.314	7	0.737
5. Dhanoa	4	0.314	4	0.314	3	0.094	4	0.314	4	0.314
6. Parabolic	4	0.314	4	0.314	4	0.314	4	0.314	7	0.737

Table 5. For all of the models the critical points and their values for each season

WINTER	used models					
	Wood	Gaines	Hayashi	Polynomial	Dhanoa	Parabolic
Time to Peak (Month)	1.216	-	*	*	1.165	*
Maximum Yield (Kg)	14.910	-	*	*	14.948	*
Turning Point Time (Month)	4.668	-	*	*	4.578	8.400
Turning Point Production (Kg)	12.388	-	*	*	12.462	9.347
SPRING						
Time to Peak (Month)	0.864	-	*	*	1.208	*
Maximum Yield (Kg)	14.437	-	*	*	14.231	*
Turning Point Time (Month)	3.996	-	*	*	4.684	7.410
Turning Point Production (Kg)	12.312	-	*	*	11.841	9.929
SUMMER						
Time to Peak (Month)	*	-	*	*	1.275	*
Maximum Yield (Kg)	*	-	*	*	14.648	*
Turning Point Time (Month)	*	-	*	*	4.846	*
Turning Point Production (Kg)	*	-	*	*	12.151	*
FALL						
Time to Peak (Month)	1.216	-	*	*	1.529	*
Maximum Yield (Kg)	14.910	-	*	*	15.631	*
Turning Point Time (Month)	4.668	-	*	*	5.439	8.430
Turning Point Production (Kg)	12.388	-	*	*	12.836	10.150
OVERALL						
Time to Peak (Month)	0.747	-	*	*	1.318	*
Maximum Yield (Kg)	14.930	-	*	*	14.525	*
Turning Point Time (Month)	3.747	-	*	*	4.948	6.743
Turning Point Production (Kg)	12.863	-	*	*	12.027	10.520

*: unavailable values because of the curve
-:does not have any critical points.

researchers such as Yılmaz and Kaygisiz, (1999); Orhan and Kaygisiz, (2002). In addition, the standard errors of parameter values of the models were quite small. Occuring the small errors increases the confidence of the model used in the study. Similar result was also reported by Olori *et al.,* (1999), Cilek and Keskin, (2008), Silvestre *et al.,* (2009), Gantner (2010). The results obtained by examining in terms of goodness of fit is quite satisfactory. The highest R^2_{adj} value, the lowest MSPE and BIC value represents the best consistency (Motulsky and Ransnas, 1987). Firstly, The adjusted coefficient determination changed in the range of 0.8980-0.9945 for all seasons and all models. The value of the adjusted coefficient determination estimated in this study were higher than the reported values by Kitpipit *et al.,* (2008), Orman and Okan, (1999).

The results of the other criteria of goodness of fit are in harmony with the adjusted coefficient determination. In this context, by examining adjusted coefficient determination and the values of MSPE and BIC, the polynomial model compared to the other models shown the best consistency except in the summer season. In the summer seson, wood model shown the best consistency. According to the result of criteria of goodness of fit, Hayashi model shown the lowest consistency except in the summer season. In this study, the values of MSPE and BIC changed in the range of 0.031-0.580 and -31,214 - -0,764, respectively. These values were quite small according to the reports of the researchers such as Aziz *et al.,* (2006). A similar study was done by Val-Areeola *et al.,* (2004). The results of Wood and Gaines models were consistent in our study. However, the value of MSPE was small in our study. As a result in this study, it has been mentioned that a few lactation models was to be fit the data of Simmental cows and some of the criteria to be followed.

Table 6. Parameter estimates with standart error and Goodness of fit measurements for all lactation season curve models. Standard errors are given in parentheses

WINTER	Parameters			Goodness of fit		
	A	B	C	R^2_{adj}	MSPE	BIC
1. Wood	16.474 (0.467)	0.124 (0.062)	0.102 (0.017)	0.9575	0.280	-9.424
2. Gaines	16.738 (0.526)	0.070 (0.006)	-	0.9475	0.389	-4.759
3. Hayashi	0.130 -	-16.667 (0.526)	0.009 -	0.9407	0.391	-4.727
4. Polynomial	15.378 (0.399)	-0.424 (0.167)	-0.038 (0.015)	0.9824	0.116	-15.935
5. Dhanoa	(16.499 1.087)	1.165 (0.780)	0.100 -	0.9628	0.245	-7.096
6. Parabolic	15.215 (0.541)	-0.016 (0.017)	0.005 (0.002)	0.9745	0.168	-12.203
SPRING						
1. Wood	15.752 (0.377)	0.076 (0.050)	0.088 (0.014)	0.9672	0.184	-13.549
2. Gaines	15.904 (0.386)	0.069 (0.005)	-	0.9624	0.211	-13.156
3. Hayashi	0.130 -	-15.904 (0.388)	0.009 -	0.9621	0.213	-10.812
4. Polynomial	15.005 (0.386)	-0.593 (0.161)	-0.017 (0.014)	0.9809	0.107	-18.968
5. Dhanoa	15.696 (0.937)	1.208 (0.676)	0.100 -	0.9681	0.179	-12.520
6. Parabolic	14.950 (0.494)	-0.033 (0.016)	0.003 (0.002)	0.9760	0.135	-16.705
SUMMER						
1. Wood	16.196 (0.026)	-0.051 (0.035)	0.051 (0.009)	0.9821	0.089	-20.837
2. Gaines	16.090 (0.264)	0.064 (0.003)	-	0.9795	0.102	-20.442
3. Hayashi	0.130 -	-16.486 (0.317)	0.009 -	0.9714	0.142	-14.817
4. Polynomial	16.112 (0.411	-1.021 (0.172)	0.027 (0.015)	0.9755	0.122	-17.673
5. Dhanoa	16.132 (1.403)	1.275 (1.021)	0.100 -	0.9211	0.392	-4.692
6. Parabolic	16.331 (0.464)	-0.072 (0.013)	0.001 (0.001)	0.9779	0.110	-18.719
FALL						
1. Wood	16.474 (0.467)	0.124 (0.062)	0.102 (0.017)	0.9575	0.279	-9,424
2. Gaines	17.189 (0.355)	0.061 (0.004)	-	0.9671	0.187	-14.413
3. Hayashi	0.130 -	-17.363 (0.641)	0.009 -	0.8980	0.580	-0.764
4. Polynomial	16.368 (0.209)	-0.500 (0.087)	-0.026 (0.008)	0.9945	0.031	-31.214
5. Dhanoa	17.068 (0.894)	1.529 (0.672)	0.100 -	0.9747	0.144	-14.690
6. Parabolic	16.236 (0.295)	-0.022 (0.008)	0.004 (0.000)	0.9912	0.050	-26.531
OVERALL SEASON						
1. Wood	16.175 (0.330)	0.062 (0.045)	0.083 (0.012)	0.9743	0.144	-16.068
2. Gaines	16.305 (0.335)	0.067 (0.004)	-	0.9713	0.161	-15,875
3. Hayashi	0.130 (0.000)	-16.300 (0.367)	0.009 -	0.9661	0.190	-11,913
4. Polynomial	15.498 (0.329)	-0.639 (0.138)	-0.013 (0.012)	0.9860	0.078	-22,116
5. Dhanoa	15.979 (0.931)	1.318 (0.681)	0.100 (0.000)	0.9697	0.170	-13,045
6. Parabolic	15.475 (0.425)	-0.037 (0.425)	0.003 (0.001)	0.9823	0.100	-19,734

a, *b* and *c* are parameters that define the scale and shape of the curve.

R^2_{adj} = The adjusted coefficient determination

MSPE= Mean Square Prediction Error

BIC = Bayesian information criteria.

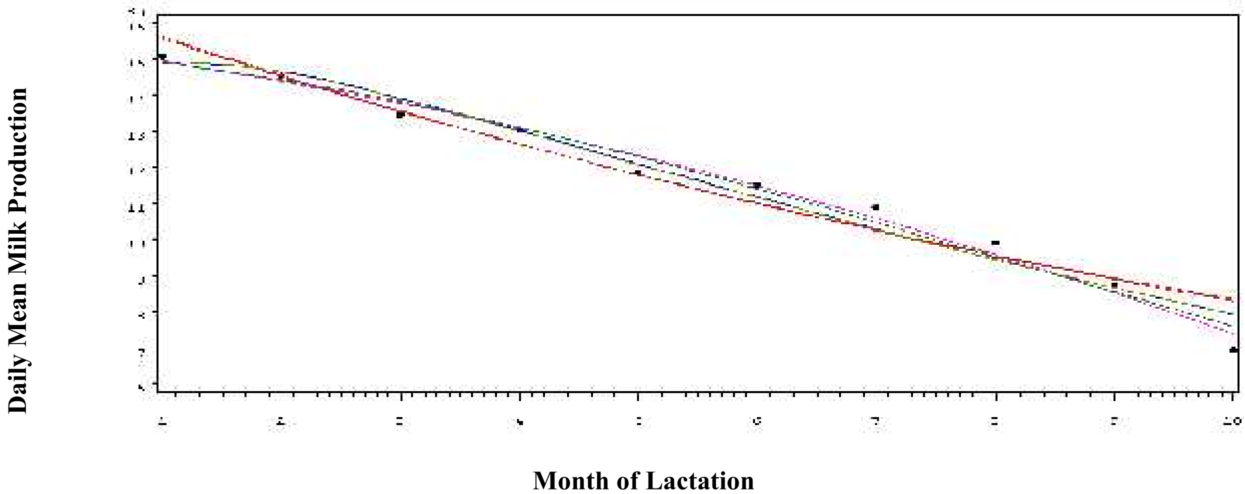


Figure 1: Estimated Lactation Curve for Winter

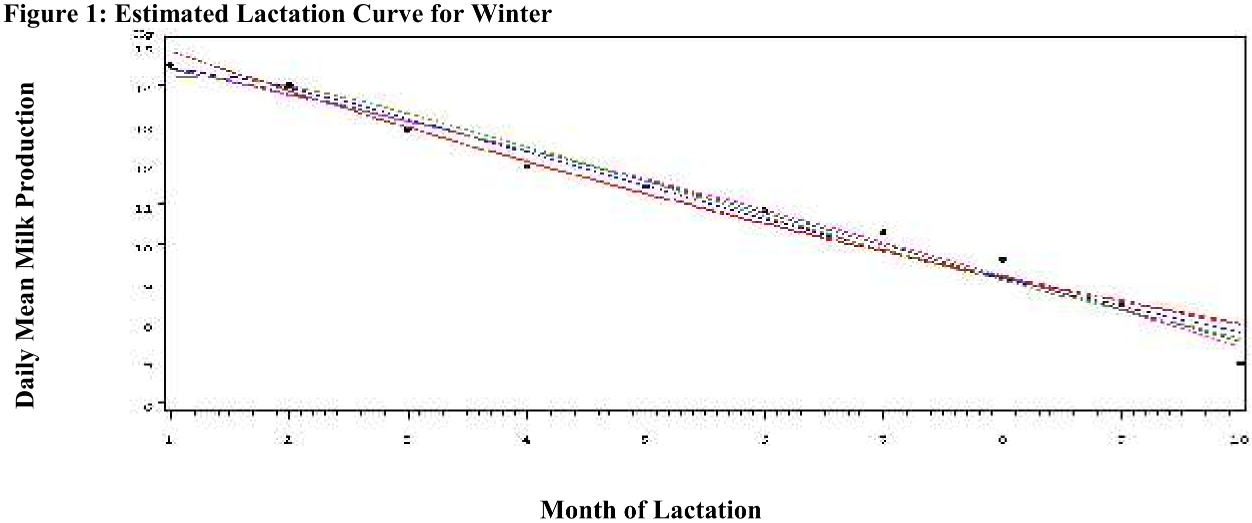


Figure 2: Estimated Lactation Curve for Spring

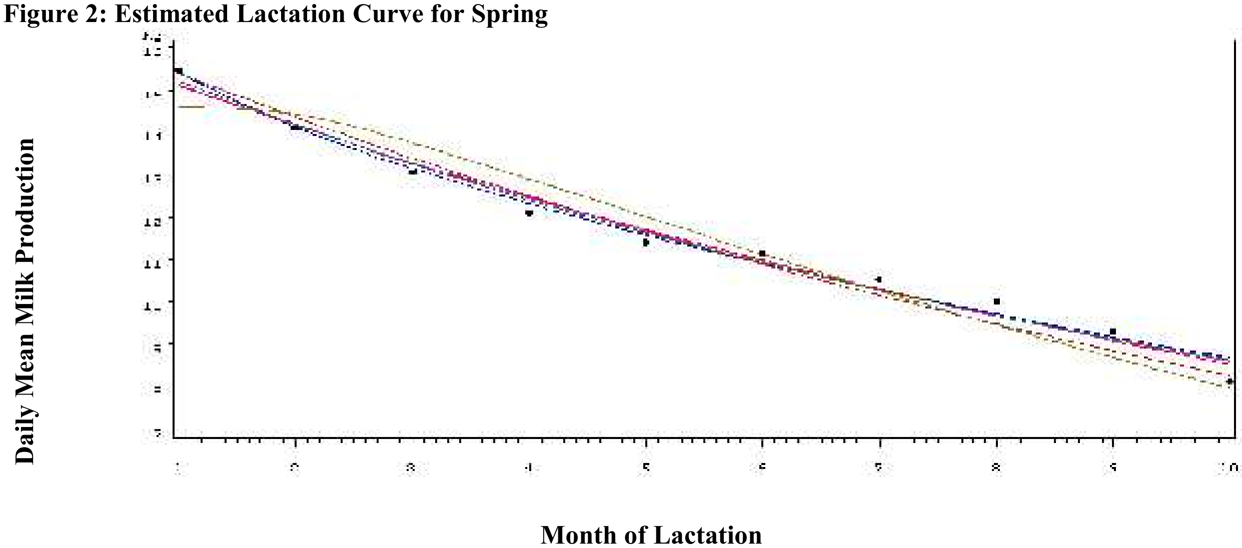


Figure 3: Estimated Lactation Curve for Summer

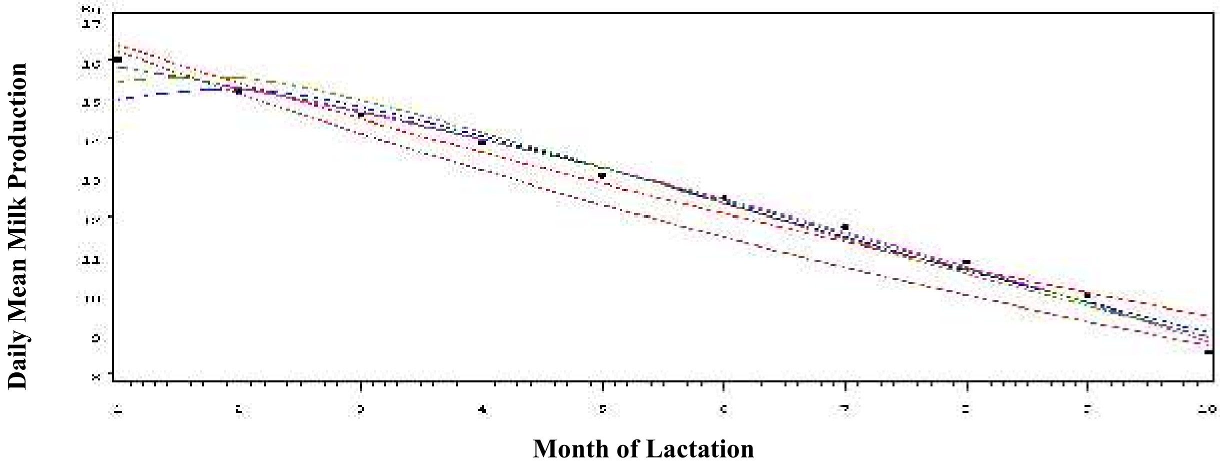


Figure 4: Estimated Lactation Curve for Fall

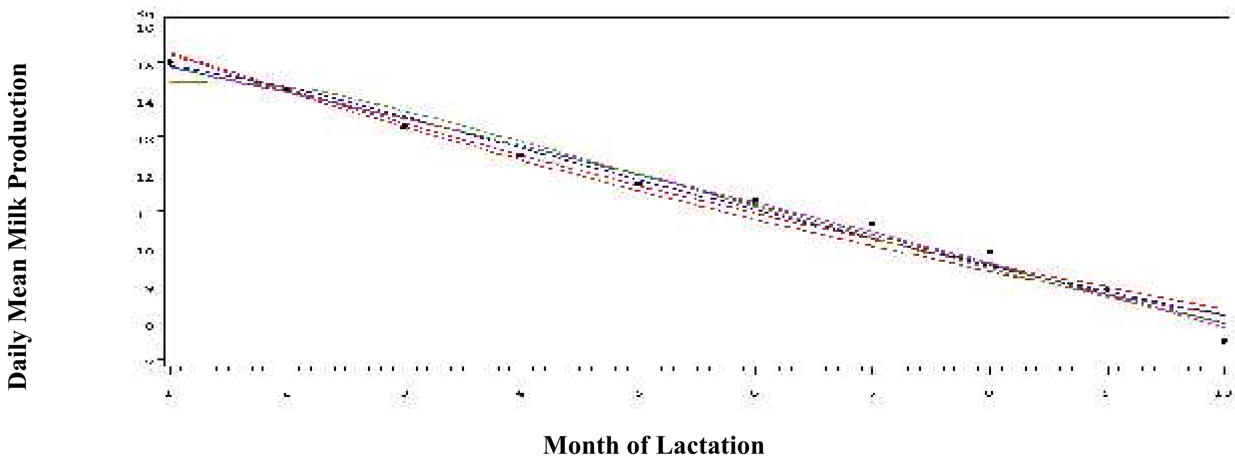
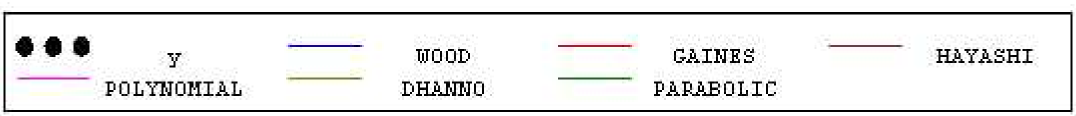


Figure 5: Estimated Lactation Curve for all lactation Overall



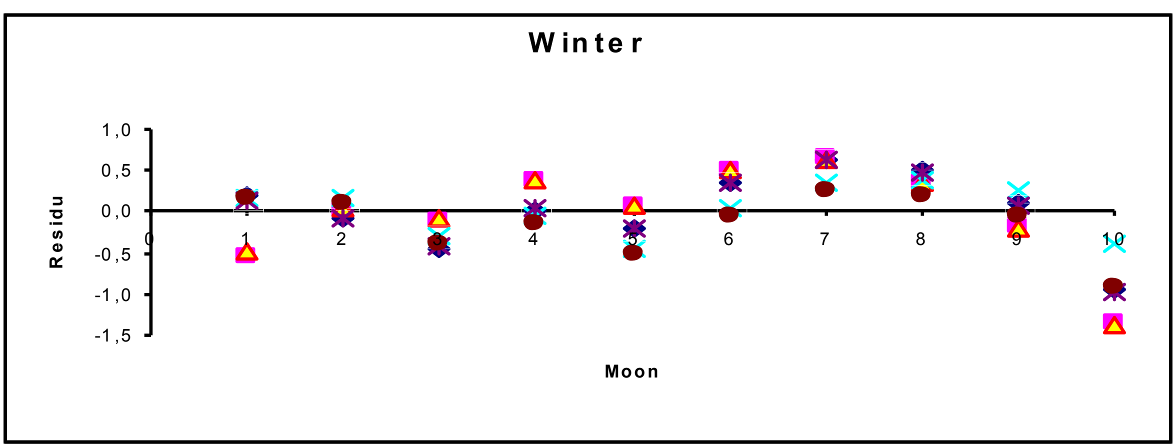


Figure 6: The Residual Plots for Winter

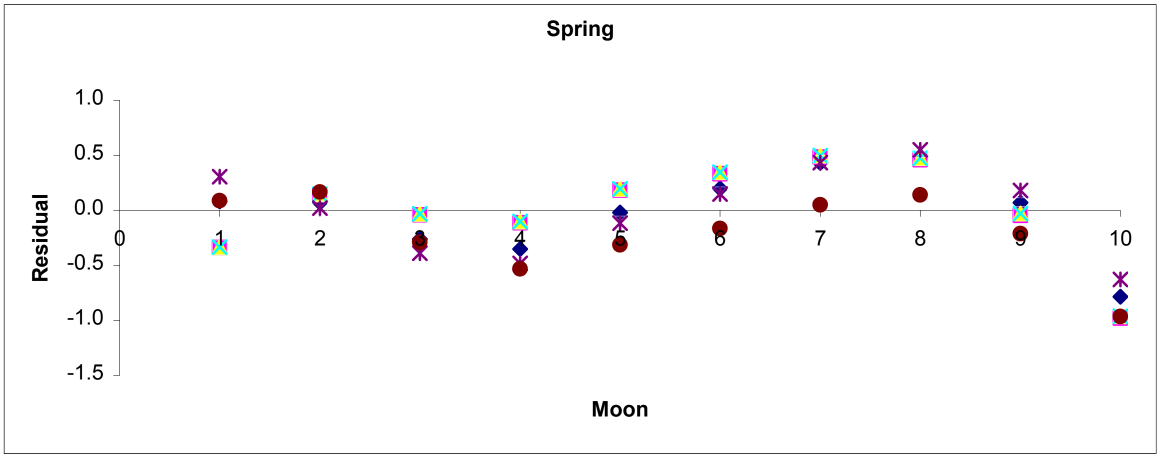


Figure 7: The Residual Plots for Spring

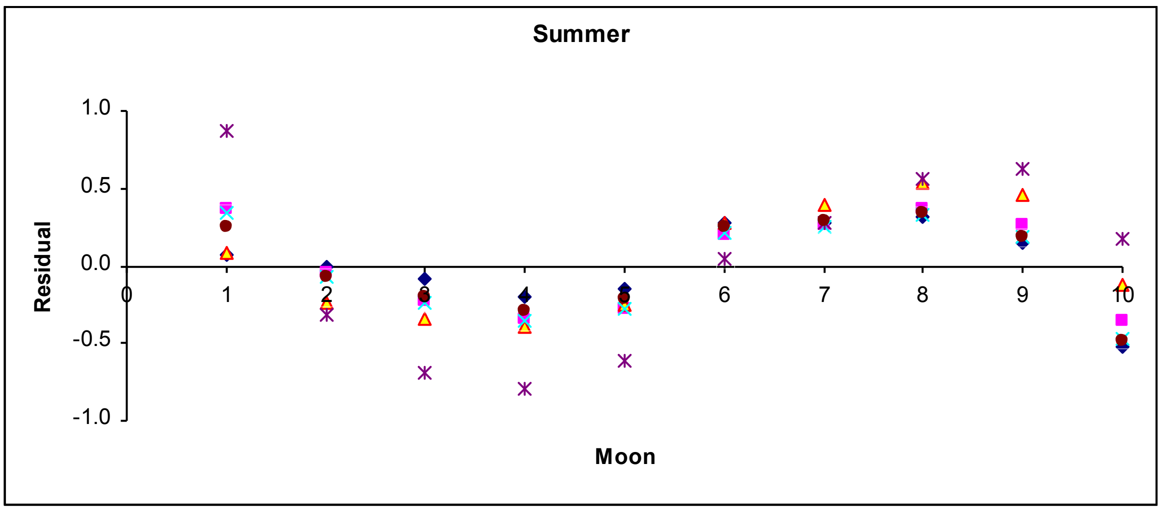


Figure 8: The Residual Plots for Summer

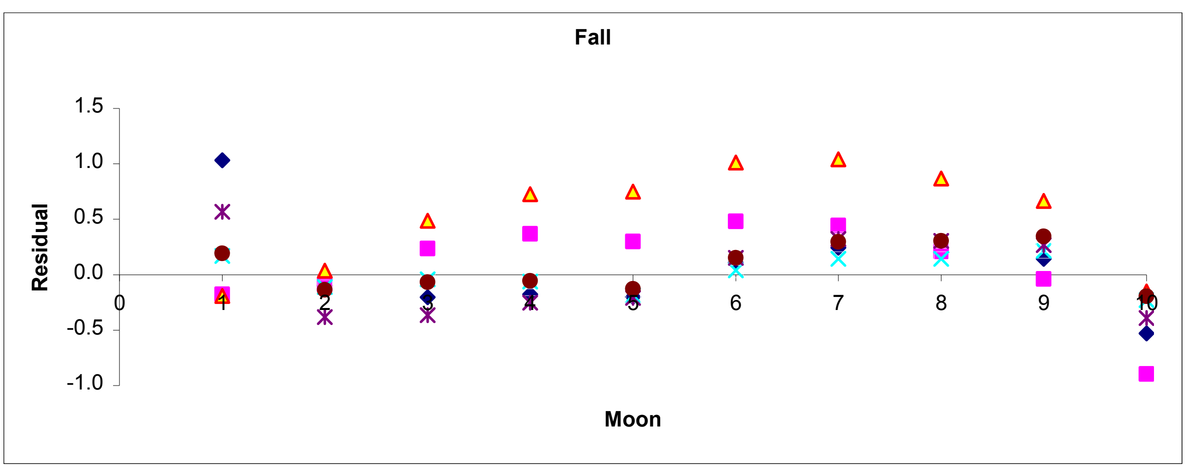


Figure 9: The Residual Plots for Fall

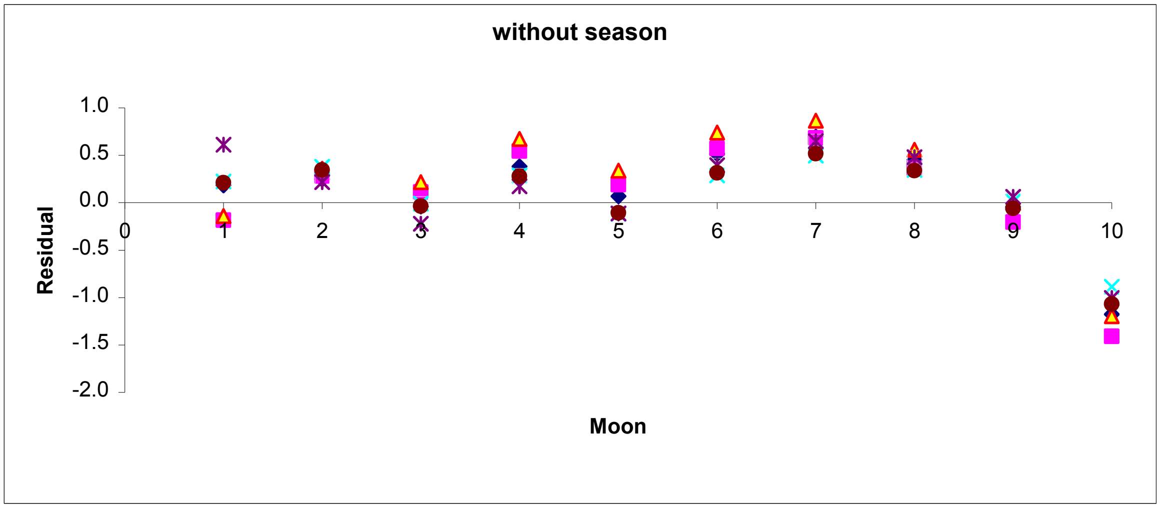


Figure 10: The Residual Plots for Overall

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