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The teaching of fractions

A comparative study of a Swedish
and a Hong Kong classroom

ULLA RUNESSON & IDA AH CHEE MOK

The aim of this paper is to illustrate how the topic of fractions can be taught differently by making a comparison between two cultures. We have studied mathematics teaching in classrooms in Hong Kong and Sweden. One of our basic assumptions is that the way in which the content is taught in a classroom has an important implication for what students may possibly learn. With reference to the framework of Variation Theory, two different spaces of learning are delineated. The Hong Kong lesson demonstrated a pattern of many juxtaposed variations, whereas the Swedish lessons presented a pattern of sequential and wide spreading character.

“The fish is always the last one to understand water”

Being constantly in an environment, one usually takes things for granted and may fail to see the characteristics of the environment as special or different. This argument is relevant to comparative studies and explains why comparison within a culture and across different cultures may bring about a better understanding of mathematics teaching itself. Comparison can be made at different levels and with different foci. For instance, large-scale studies such as Third International Mathematics and Science Study (TIMSS) (Hiebert et al., 2003) and Programme for International Student Assessment (PISA) (OECD, 2004) inform about students' achievement, curriculum and systems. They present national images, which are often eye-catching and sometimes get quick responses from policy-makers (Ainley, 2004). However, educators, researchers and teachers working closely with students at the frontline may prefer to receive reports containing more tangible features about pedagogical instruction,

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which are more likely to provide insights for improvement inside the classrooms.

Our study attempts to meet this challenge by comparing mathematics lessons in Hong Kong and Sweden. Our aim was to capture features which may not be easily observed within one culture, but which might become more visible in the contrast, in order to get a better understanding of the teaching practice *per se*.

The topic of fraction is well recognised in the literature as difficult to learn (See e.g. Behr et al., 1992; Carpenter, Fennema, and Romberg, 1993; Freudenthal, 1983).

Theoretical framework: a theory of awareness and variation.

This study is premised on a conceptual framework based on Variation Theory (Marton, Runesson, and Tsui, 2004). There are two fundamentals in Variation Theory; learning always has an object, and this object of learning is experienced differently (Marton, 1981; Marton and Booth, 1997). The difference between different ways of seeing the same thing has to do with those aspects of the object of learning that are discerned and held in the individual's focal awareness at the same time. Aspects can be discerned differently; some come into the focal awareness, and thus are in the forefront of the attention, while others remain in the background (Gurwitsch, 1964). The meaning or the way something is seen is constituted as a pattern of discerned aspects, of parts and wholes, and of foreground and background. Consequently, the way in which critical aspects of the object of learning are brought out in a learning situation is significant for what is possible to discern and, thus, to learn (Marton and Morris, 2002; Marton and Tsui, 2004).

Discernment requires an experienced variation. An aspect cannot be discerned without being experienced as a dimension of variation. So, to make learning possible, it is necessary to open up variation of those aspects we want the learner to notice. What is varied and what is kept constant create both constraints and opportunities for learning. Those aspects that are varied (e.g. by making contrasts) are likely to be discerned and those which are varied at the same time, learners are likely to discern *at the same time*.

Drawing on these theoretical assumptions, we have studied two different classrooms – one in Sweden the other in Hong Kong – by looking at the pattern of simultaneous variation and invariance that was brought out in the interaction between teachers and learners.

Contextualizing the study in two cultures

Although Swedish students perform relatively well in international comparisons such as PISA, they display a lower level of interest in mathematics than their counterparts in other OECD countries. Swedish students consider mathematics to be difficult and boring. The Swedish National Curriculum gives prominence to problem solving, every-day mathematics and communication. Nevertheless arithmetic dominates in Swedish mathematics classrooms at the expense of conceptual understanding, for instance. Swedish mathematics classrooms are very heterogeneous, and individual practice is common. Students very often work with tasks in the textbooks in their own pace (Liljestrand and Runesson, 2004). Consequently whole class discussions about the same topic do not occur very often.

Since the mid-90's, efforts have been made in Hong Kong to look for means to develop classroom environments with characteristics of pedagogy widely recommended in the West. Consequently, the scenario of a uniform teacher-centred pattern has changed (Mok and Morris, 2001). The Hong Kong lesson discussed in this paper is a product of these reform activities. On the other hand, the high position of Hong Kong students' performances in the league table suggests that it is worthwhile studying the pedagogy at classroom level in detail.

Implementing the study

The Swedish data used in this paper are drawn from a larger data set consisting of 25 lessons in grade six and seven created in the study by Runesson (Runesson, 1999) which aimed at finding differences between teachers with respect to how a topic was handled. The selection of data in the study presented here was made for making a deeper analysis of a particular sub-construct of the topic. The lessons were audio taped and transcribed verbatim. The five teachers taught the same topic: fractions.

Research co-operation between Sweden and Hong Kong provided us with the opportunity to revisit the Swedish data by matching with a lesson in Hong Kong. Our study aimed at finding similarities and differences in how the same topic can be taught by contrasting mathematics teaching in two different cultures. At the beginning, we noticed a great difference between the curricula of the two places. Therefore, we chose one topic that was common to both – comparison of fractions with different denominators. The Hong Kong lesson was with a primary four (age 10, grade 4) class. Once the teacher was identified, we asked her to teach a lesson about comparing fractions with different denominators in the way she usually did.

Despite all the efforts to match the content of the lessons, the topic was taught in Grades 6 and 7 (age 12 and 13 respectively) in Sweden and in Grade 4 in Hong Kong (age 10); and some differences inevitably occurred. In the Hong Kong lesson the students worked with finding the common denominators of two fractions. In the Swedish lessons, the task was to find another fraction with the same value (e.g. $2/6=1/3$). Unlike the Hong Kong data, which are drawn from one single lesson, the Swedish data derive from several lessons by the same teacher.

The data were analysed in parallel and in shifts. Once the Hong Kong data had been collected, a preliminary comparison between the Swedish and the Hong Kong lessons was done and the lessons from the two places appeared to be very different. The next step in the analysis was to describe these discrepancies at a more detailed level. This involved identifying patterns of variation/invariance in the lessons. For instance, if the teacher illustrated the same fraction differently (e.g. showing one quarter of a pizza and writing $1/4$) we characterised this pattern of variation as varying the representation while keeping the number constant.

Results – The Hong Kong lesson

The Hong Kong lesson appeared to have only one objective, comparing fractions with different denominators. Nevertheless, this objective was visited and revisited via several tasks that were either in the form of questions in the worksheets or examples on the board. As a result, the Hong Kong lesson shows a pattern of variation consisting of many dimensions of variation. Each of these dimensions tells the learners a bit more about comparing fractions and these dimensions are related to each other. To summarize, they are:

1. Alternative representations of the method of amplification (producing an equivalent fraction with a larger denominator).
2. The denominators can be seen as a critical aspect of the method of amplification.
3. Generalization of the application of the method of amplification.
4. Variation in the fractional parts of different wholes ($5/8 < 1/2$?).
5. The contrast between the methods of comparison, namely, diagrams, the method of amplification and comparing with $1/2$.

Alternative representations of the method of amplification

Referring to Worksheet 1 (figure 1), the teacher elaborated that she wanted them to think of a method either by adding lines to the diagrams or other methods. After the class carried out the work, the teacher asked two students to report their group discussion results.

Worksheet 1 - The method of amplification

There are four pairs of fractions ($\frac{3}{4}$ & $\frac{5}{8}$, $\frac{1}{2}$ & $\frac{2}{3}$, $\frac{3}{4}$ & $\frac{5}{6}$, $\frac{2}{5}$ and $\frac{3}{10}$) represented both in numeral representations and diagrams. The students are asked to compare each pair of fractions to complete a sentence “__ is larger than __ by __” and they can use the diagrams to help them answer the questions. At the end of the worksheet, the students are asked to write down their method.

Figure 1. *Worksheet 1 in the Hong Kong lesson*

The task in Worksheet 1 is to compare the fractions. However, the focus is not on the answers but on the method of comparing fractions. Embedded in the design of the task there is a variation in the representation of the method of amplification. The students can add lines to the diagram; find the numerical difference between the two fractions or find equal denominators (also known as the method of amplification). The task becomes a means of encouraging students to suggest different methods to create a *variation*.

The denominators – a critical aspect of the method of amplification

After finishing Worksheet 1, the teacher explained the method of amplification directly with the example of comparing $\frac{3}{10}$ and $\frac{2}{15}$ on the board. She asked the students to apply the method of amplification to get the same denominators directly. His instruction solicited several answers from the students: “ten divided by two times three”, “ten times three, fifteen times two”, “thirty”, “sixty”, “ninety” and “one hundred and twenty”. Then the teacher asked a question about which value would be the most convenient. She followed this by probing for the reasons and then written work on the board. Despite the directive nature of the interaction, the teacher’s questions invited multiple answers from the students; these form a dimension of variation and draw the students’ attention to the denominators. At this point, the students see the need to choose the most convenient denominator to simplify the fractions. This instance demonstrates how variation can be used to draw the learners’

attention to a critical aspect (denominators) of the object of learning (the amplification). This variation was not stand-alone but followed directly from what they had experienced in worksheet 1. Therefore, it produced a zooming effect: from the method of amplification to choosing the appropriate denominators.

Generalization and variation of the 'whole'

The second worksheet (see figure 2) created two very different dimensions of variation.

Worksheet 2

Applying the method of amplification and $5/8 < 1/2$?

Worksheet 2 consists of 3 questions:

1. Compare with the method of amplification $1/6$ and $2/9$. ___>___.
2. Compare with the method of amplification $5/8$, $1/2$ and $3/4$. ___>___>___.
3. Is it possible for $5/8$ to be smaller than $1/2$? Discuss with classmates and explain the results with words and diagrams.

Figure 2. *Worksheet 2 in the Hong Kong lesson*

Questions 1 and 2 both involve variation in applying the method of amplification to similar but different situations, to 'generalize' what they had learned in a slightly different case (Marton, Runesson, and Tsui, 2004). Question 3 leads students to see another meaning of the fraction. Seeing $5/8$ and $1/2$ as two numerical values, $5/8$ is greater than $1/2$. In order to say the opposite, one has to consider the fractions as fractional parts of different wholes. By guiding students to consider pizzas of different sizes, they are then able to see the other meaning of fraction, a fractional part of a whole. This is not an easy question since the students need to draw upon the part-whole concept. With the teacher as a facilitator in the group and whole-class discussions, they paid attention to the various aspects of the comparison of fractions. Excerpt 1.HK shows how the teacher guided the students to consider the possibilities.

Excerpt 1. HK – The fractional parts of two different pizzas [transl.]

T: Stop for a while, go back to [your] seats. I heard that many groups said [that] it's impossible during the discussion (One student at his seat said: It's possible). Now I tell you a story and see whether you can think of the answer. Okay. A father went to a pizza restaurant with his two daughters. Then he ordered two pizzas. When the pizzas were ready [on the table], the father then said, "Little sister, this one eighth of pizza is yours". Then, he [divided another pizza] for his elder daughter and said, "Elder sister, this one eighth of pizza is yours". The little sister looked at [the pieces of the pizza in their dishes], she didn't agree on [her father's division method], she asked why the piece of pizza that elder sister got was bigger although they both got one eighth [of pizza]. Okay, [after listening to this story], now I give [some time] for you to think about, is it possible that five eighth is smaller than one half?

S: It's possible...

T: Good, starts [your] discussion again, now start the discussion again.

During the discussion, the teacher drew the attention of the class to a comparison between $\frac{1}{4}$ of two triangles of different sizes. Some groups discussed the queries of equal parts. By the end of the discussion, the class had some idea how $\frac{5}{8}$ could possibly be smaller than $\frac{1}{2}$.

Variation of methods – Comparing with $\frac{1}{2}$

The next part of the lesson brought the method of comparison to the foreground again. The students suggested different methods: by looking at the diagrams, by dividing the diagrams into smaller triangles (an iconic version of the method of amplification) (student S32) and comparing with $\frac{1}{2}$ (student S33). The contrast makes the method of comparison a dimension of variation. (See excerpt 2.HK).

Excerpt 2. HK – Comparing with $\frac{1}{2}$ [transl.]

S32: Because they... (T: what?) Because the diagrams are the same. (T: The diagrams are the same.) The five eighth occupies five triangles. So, it's eight equal parts. (T: Okay) Then it is one quarter [which] occupies two triangles of the left diagram, one half then occupies four triangles of the left diagram.

T: Hmm, you are actually doing amplification, how about you? I said [there is] no need to do amplification.

S33: If [we] observe [them] precisely, you will find [that] one half ... five eighth will point out a little, the coloured parts will point out a little. (T: Ah!) And five eighth is more than a half, one half is equal to a half, one quarter is not as much as a half.

Results – The Swedish lessons

When analysing how 'comparison of fractions with different denominators' and 'different fractions with the same value' were taught in the Swedish lessons, we found that these sub-constructs were not presented in the same lesson, but were extended over time. One task was given for comparison and another one for finding different fractions with the same value, and these were given to the students in different lessons. The following analysis illustrates this sequential character.

In lesson one, different fractions with the same value were addressed for the first time. The teacher, Ms I, showed a 'chocolate bar' (squared 3x4) and explained that three twelfths equals one quarter. This idea was brought up again in lesson two. However, this time a new teaching aid was used for presenting a diagrammatic method. In this lesson the task was to mark 1/4 of a circle partitioned into eighths in order to realize that 1/4=2/8.

In another lesson (SW lesson 3) two fractions with different denominators were also compared, but this time the students were expected to use a new teaching aid for finding the correct answer (figure 3).

1							
1/2				1/2			
1/4		1/4		1/4		1/4	
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

Figure 3. A teaching aid for comparing fractions

Excerpt 3. SW lesson 3– Finding the bigger fraction [transl.]

(The teacher wrote 3/5 and 2/3 on the board and instructed the students how to use the aid.)

T: --- so, you'll have to consider this a bit. What do you think? Joan.

S1: They are just as big, no ...

T: What do you think?

S2: It is fifteen, fifteen ... no I don't know.

T: No. Does anyone have an opinion? Dan?

S3: Two thirds.

T: You think this is bigger (pointing to $2/3$) ... anyone who thinks this (pointing to $3/5$) is bigger?

S4: I think they are as big.

T: You think they are as big. If you can't see, because it's not so easy, there are methods, so we have a very good picture in the book on page 8 and I want you to take a look at that, because you will get there, if you are not already. And this time there are no cakes, but there is a, we can call it a 'ribbon' or something. And if you imagine that at the top there is one whole on this band ... and then there are two halves beneath, and then three thirds and so on. If you take a look at this picture now, you can rather easily see. Which is the bigger three fifths or two thirds? Or are they perhaps as big?... What do you think, Mary?

S5: Ah ... two thirds is the bigger ...

T: ... is the bigger, because then you get more to the right. It is like a measuring tape. This is a bigger part (pointing to two thirds of the whole) than three fifths. And you can see that if you go up vertically and down in this. You are allowed to use this picture when you solve this problem, because it is not so easy to see otherwise.

Using the aid for comparing the fractions was the only method presented to the students. This method does not require any special understanding of the magnitude of the fractions. The students can find the correct answer by reading the relative position of the numbers on the teaching aid. One student, S2 (line 4), said "fifteen, fifteen", but then changed her mind "no I don't know", this could be interpreted as if she was trying to make a link to the common denominator (fifteenths). However, the teacher did not follow up the student's suggestion, and missed the opportunity of making a connection between the concepts in the first lesson with the method of comparison in the second lesson and also the chance of seeing one method as a variation of the other.

We have other instances showing how similar situations were presented as disjoint instances and without any connection or reference to previous lessons. For instance, the teacher did not make any references to the students' previous experience of this when she started lesson six by giving each student a cuboid built from 1 cm cubes. Regardless of the

total of number of cubes one half of them were always white. When the students had answered what part of the total numbers were white (e.g. $4/8$, $6/12$, $3/6$, $5/10$), the teacher wrote the fractions " $4/8$, $6/12$, $3/6$, $5/10$ " on the board, and asked the students to find similarities. One student cried out: "I know! Everyone is one half". The teacher continued:

Excerpt 4. SW lesson 6 – Everything is one half [transl.]

T: You will soon discover, it's one half. And that means that all these numbers are just as much. One half of something is the same as three sixths of something or four eighths. However, your cuboids were not the same size, 'cause it depends on whether it was one half of this size [showing a cuboid] shh! ... or one half of this size. Of course the result will be different, yeah, but the ratio in the fraction is the same.

In this task the relative and the absolute size of the white part were compared. The relative size was kept constant, while the absolute size varied. So a variation of relative and absolute size was opened. In this way the students' attention was drawn to 'size' as an aspect of fractions. And, the difference of relative and absolute size of a fraction could visually be experienced.

The next task was to find other names for one third. The teacher did not give any instructions on how to do this. She just asked for other names, and the students were supposed to find those. Several suggestions were given; $4/12$, $5/15$, $3/9$. And the teacher asked: "Can anyone tell me – how can I see they are the same as one third?"

Excerpt 5. SW lesson 6 – Comparing numerators and denominators [transl.]

T: Yes. So this one is three times as big as that one [pointing to the denominators in $3/9$ and $1/3$ respectively, i.e. comparing 9 and 3, and 3 and 1 respectively]. Mm, check with this one (comparing fractions equal to $1/2$ written on the board). That's correct too. One half is equal to all these. Can you see that? You can see that immediately. Can't you? Yes, Jenny?

S1: The one on the top is twice as much as the one on the bottom ...

T: Yes, you can see that at once, that's one half. Can you see something else? Or it is like a rule? Yeah?

S2: Two times five is 10.

- T: Two times five is ten, she says. Yes, if you compare the numerators and the denominators, for instance ... yeah?
- S3: the one on the top is (inaudible) five times bigger than ...
- T: ... Right, can you see that? This one is five times as much as this one (pointing to the numerators) and this one is five times as much as this one (pointing to the denominators). Yes of course. Let's look at another one (comparing $6/12$ and $1/2$). Yes Kate?
- S4: Except; now it is six times as much.
- T: Yes, but if this one is six times as much as that one (comparing the numerators), this one is evidently six times as much as that one (comparing the denominators) So, if I say something with seven (Writes $7/$ on the board). What must I write here, below, to get one half? Linda?
- S5: 14.
- T: Yes, $7/14$ is exactly one half. It is rather good if ... you don't have to know this by heart, but it's very good if you have a sense of this being equal to one half.

The above excerpt shows a scenario that appeared very often in the data. That is, the teacher let the students discover a way of solving the problem without explicitly showing any methods. Instead the teacher pointed out the ratio between the numerators and denominators in the pair of fractions.

Summary and Discussion

The object of research in this study was possibilities to learn, not in a general sense, but possibilities to learn *the same thing*. We studied the object of learning, and more specifically, how the object of learning was enacted as the learners and the teacher interacted in the classroom. The most prominent difference between the two classroom cultures with respect to how the object of learning was handled was that the construal of fractions was different.

The Hong Kong lesson is very condensed, in the sense that it shows a pattern of many juxtaposed variations related to different aspects of comparing fractions. In contrast, the Swedish students experienced a different pattern of variation with a sequential, widespread character. That is, there was little connection between the concepts and skills between lessons. Therefore, it is unlikely for students to discern how these are

related. In the Hong Kong class, however, the students may experience the intertwined relationship between these dimensions of variation that forms a special arrangement or simultaneity of variation. Such experiences give the students a chance to consider several aspects of the object of learning simultaneously and to see a connection between the critical aspects of the mathematical object (i.e., equivalence and comparison of fraction in the examples in this paper), whereas such opportunity is less likely to happen in the Swedish case. Therefore, the possibility of making connection makes a significant difference.

What does the difference in age of the pupils entail? It is most likely that the age of the pupils affects how they deal with the mathematics content. One might expect a tendency towards simplification by teachers of younger pupils. For instance, it is reasonable to assume that a teacher holds the idea that learning would be simplified if aspects of what is learned were brought out one at a time; that bringing in too many aspects simultaneously would confuse the students, and hence constrain learning. From this point of view, it is worth noticing that the opposite was found in this study. In the Hong Kong classroom the pupils were about three years younger than their counterparts in Sweden and yet a space of learning consisting of many simultaneous dimensions of variation was afforded to the learners, whereas for the older Swedish pupils dimensions of variation were brought out in sequence.

A small-scale comparative study like this necessarily raises the question of how representative it is. However, it has never been our aim to claim that it is representative at a national level. What we have described are two different ways of handling the same topic, or two different patterns of variation and simultaneity when teaching the same topic within two different school cultures. Our analysis shows that the same topic (fractions) was taught so differently in the two cultures that it is likely that very different understanding would be created by the students. Our aim was not to come up with suggestions as to how a topic (fractions with different denominators) should be taught in classrooms *in either country*, nor to explain differences in learning outcomes between the two countries. Instead we wanted this study to open our eyes and make visible what is not easily seen within our own culture, without saying anything about the typical Swedish or the typical Hong Kong classroom. Comparisons of this kind can shed light on the traditions within mathematics classrooms, and the results of a study like this appeal to teachers' professional knowledge and could be inspiration and help for reflection on practice. Seeing what *could be* the case sheds light on what *is* done and *what is the case in our own practice*.

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Sammanfattning

Syftet med denna artikel är att visa hur jämförelse av oliknämninga bråk kan hanteras på olika sätt i undervisningen. Detta görs genom att jämföra undervisning i Sverige och Hong Kong. Data har analyserats utifrån ett variationsteoretiskt perspektiv och utgångspunkten är ett antagande om att hur innehållet behandlas har betydelse för vad som är möjligt för eleverna att lära. Vi fann att i det svenska klassrummet behandlades innehållet så att det fick en sekventiell karaktär, medan det i Hong Kong gavs en mer komplex karaktär genom att flera aspekter behandlades samtidigt.

