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A MESH INDEPENDENT GTN DAMAGE MODEL AND ITS APPLICATION IN SIMULATION OF DUCTILE FRACTURE BEHAVIOUR

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ABSTRACT

Ductile fracture process involves the typical stages of nucleation, growth and coalescence of voids in the micro-scale. In order to take the effects of these voids on the stress carrying capability of a mechanical continuum during simulation, damage mechanics models, such as those of Rousselier and Gurson-Tvergaard-Needleman (GTN) are widely used. These have been highly successful in simulating the fracture resistance behaviour of different specimens and components made of a wide spectrum of engineering steels. However, apart from the material parameters, a characteristic length parameter has to be used as a measure of the size of the discretisation in the zone of crack propagation.

This inherent limitation of these local damage models prevents them from simulating the stress distribution near the sharp stress gradients satisfactorily, especially at transition temperature regime. There have been efforts in literature to overcome the effect of mesh-dependency by development of nonlocal and gradient damage models. A nonlocal measure (weighted average of a quantity over a characteristics volume) of damage is usually used in the material constitutive equation.

In this paper, the authors have extended the GTN model to its nonlocal form using damage parameter 'd' as a degree of freedom in the finite element (FE) formulation. The evolution of the nonlocal damage is related to the actual void volume faction 'f' in ductile fracture using a diffusion type equation. The coupled mechanical equilibrium and damage diffusion equations have been discretised using FE method. In order to demonstrate the mesh independent nature of the new formulation, a standard fracture mechanics specimen (i.e., 1T compact tension) has been analysed using different mesh sizes near the crack tip and the results have been compared with those of experiment. The results of the nonlocal model have also been compared with those of the local model. The effect of different GTN parameters on the fracture resistance behaviour of this specimen has been studied for the nonlocal model and these results have been compared with those of experiment.

Keywords: nonlocal model, damage mechanics, Gurson-Tvergaard-Neeedleman's model

1. INTRODUCTION

The ductile damage models such as Rousselier (1987) and Gurson-Tvergaard-Neeedleman's [Gurson 1977, Needleman and Tvergaard 1984, Tvergaard and Needleman 1984] models have been widely used for predicting load-deformation and fracture resistance behaviour of specimens and components in literature [Xia and Shih 1995; Ruggieri et al. 1996; Seidenfuss 1992; Seidenfuss and Roos 2004; Pavankumar et. al. 2005; Gullerud et al. 2000]. These so-called coupled models for ductile fracture of materials include the micro-mechanical effects of void nucleation, growth and coalescence of micro-voids in the constitutive equation used for description of the mechanical continuum. These models where the evolution of material damage at a point is a function of stress and strain field at the same point are called local models. The disadvantage of these local models lies in use of a constant finite element size in the fracture zone to simulate the crack initiation and crack growth process when using the finite element method for solving the boundary value problems.

There have been efforts by researchers to remove this problem by development of so-called "nonlocal" damage models [Svendsen 1999, Reusch et. el. 2003a,b]. In this work, the authors have developed a nonlocal form of the GTN damage model and its finite element (FE) formulation using material damage 'd' as a degree of freedom. The evolution of nonlocal damage 'd' is related to the local void volume fraction 'f' through a characteristic length 'C_{length}' and the equation governing this is written as [Reusch et. al. 2003a]

$$\dot{d} - \dot{f} - C_{length}\nabla^2\dot{d} = 0 \qquad (1)$$

When C_{length} is zero, the nonlocal formulation reduces to the local formulation. The derivation of Eq. (1) is briefly described here. The nonlocal variable in a material point $\vec{x}$, i.e. the nonlocal void volume fraction d , is mathematically defined as a weighted average of the local void volume fraction f in a domain Ω, i.e.,

$$d\left(\vec{x}\right)=\frac{1}{\Psi\left(\vec{x}\right)}\int_{\Omega}\Psi\left(\vec{y};\vec{x}\right)f\left(\vec{y}\right)d\Omega\left(\vec{y}\right) \qquad (2)$$

where $\vec{y}$ is the position vector of the infinitesimally small volume $d\Omega$ and $\Psi\left(\vec{y};\vec{x}\right)$ is the Gaussian weight function given by

$$\Psi\left(\vec{y};\vec{x}\right)=\frac{1}{8\pi^{3/2}l^{3}}\exp\left(-\frac{\left|\vec{x}-\vec{y}\right|^{2}}{4l^{2}}\right) \qquad (3)$$

The length parameter l determines the size of the volume, which effectively contributes to the nonlocal quantity and is related to the scale of the microstructure. The above integral nonlocal kernel holds the property that the local continuum is retrieved if $l \rightarrow 0$. By expanding $f\left(\vec{y}\right)$ in Taylor's series and substituting in Eq. (2) and doing some algebra, we obtain Eq. (1) [Reusch et. al. 2003a]. The above new formulation has been used to predict the load-deformation and fracture resistance behaviour of a standard 1T CT (compact tension) specimen using different mesh sizes near the crack tip. The results of nonlocal FE damage analysis have been compared with the corresponding results of the local model. The results have also been compared with the experimental data.

2. DEVELOPMENT OF A NONLOCAL GTN DAMAGE MODEL

The yield potential of the local Gurson-Tvergaard-Neeedleman's (GTN) damage model is written as [Gurson 1977, Needleman and Tvergaard 1984, Tvergaard and Needleman 1984]

$$\phi=\frac{\sigma_{eq}^{2}}{\sigma_{m}^{2}}+2.q_{1}f^{*}\cosh\left(q_{2}\frac{3}{2}\frac{\sigma_{hydro}}{\sigma_{m}}\right) -1-q_{3}f^{*2}=0 \qquad (4)$$

where σ_{eq} is the von-Mises equivalent stress, f^{*} is the modified ductile void volume fraction, σ_{hydro} is the mean hydrostatic stress and σ_{m} is the true stress in the matrix material which is a function of equivalent plastic strain ε_{eq} in the matrix material (i.e., the material stress-strain curve), q_1, q_2 and q_3 are the constants introduced by Tvergaard and Needleman (1984) in order to simulate the observed experimental fracture behaviour in many different materials more accurately. The function f^{*} was introduced by Tvergaard and Needleman (1984) and the modified void volume fraction f^{*} is related to the actual void volume fraction f through the relationship

$$f^{*}=\begin{cases} f & \text{if} \quad f \le f_{c} \\ f_{c}+\dfrac{f_{u}^{*}-f_{c}}{f_{f}-f_{c}}\left(f-f_{c}\right) & \text{if} \quad f > f_{c} \end{cases} \qquad (5)$$

where f_c is the critical void volume fraction (signifying void coalescence) and f_f is the actual void volume fraction at final fracture and f_u^{*} is the modified void volume fraction at fracture (usually $f_{u}^{*}=\frac{1}{q_{1}}$).

The void growth rate is obtained using the plastic incompressibility condition of the matrix material as

$$\dot{f}_{growth}=\left(1-f\right)\dot{\varepsilon}_{kk}^{p} \qquad (6)$$

The voids quantified by f are either initially present or nucleated by the deformation process. In the latter case, some void nucleation law has to be specified.

Chu and Needleman (1980) proposed the following law for calculating increment of void volume fraction due to nucleation, which tells that the increase in void volume fraction f is due to void growth and void nucleation processes. Hence, the increment of void volume fraction includes the growth law, i.e., Eq. (6) and a strain controlled void nucleation mechanism (also a stress-controlled mechanism if it is favourable in the material) which is described as

$$\begin{aligned}\dot{f} &= \dot{f}_{nucleation}+\dot{f}_{growth} \\ &= A\dot{\varepsilon}_{eq}^{p}+B\left(\dot{\sigma}_{eq}+\dot{\sigma}_{hydro}\right)+\left(1-f\right)\dot{\varepsilon}_{kk}^{p}\end{aligned} \qquad (7)$$

where the constants A and B for strain-controlled nucleation are written as

$$A=\frac{f_{N}^{\varepsilon}}{\sqrt{2\pi}s_{n}^{\varepsilon}}\exp\left(-\frac{1}{2}\left(\frac{\varepsilon_{eq}^{p}-\varepsilon_{n}}{s_{n}^{\varepsilon}}\right)^{2}\right),\ B=0 \qquad \text{for } \dot{\varepsilon}_{eq}^{p}>0 \qquad (8)$$

and for stress-controlled nucleation

$$A=0,\ \mathrm{B}=\frac{f_{n}^{\sigma}}{\sqrt{2\pi}s_{n}^{\sigma}}\exp\left(-\frac{1}{2}\left(\frac{\sigma_{eq}+\sigma_{hydro}-\sigma_{n}}{s_{n}^{\sigma}}\right)^{2}\right) \qquad \text{for } \dot{\sigma}_{eq}+\dot{\sigma}_{hydro}>0 \qquad (9)$$

where f_n^{ε} and f_n^{σ} are the values of void volume fractions of void nucleating particles at mean nucleation strain and stresses ε_n and σ_n respectively. s_n^{ε} and s_n^{σ} are the standard deviations of equivalent plastic strain and sum of equivalent and hydrostatic stress respectively, which are responsible for void nucleation (assuming a Gaussian distribution for the void nucleation process). The superscripts ε and σ denote the strain- and stress-controlled nucleation respectively.

In the nonlocal model, the local void volume fraction 'f' at any material point is replaced with the nonlocal damage parameter 'd' in order to take the effect of surrounding material points (over a region characterized by the characteristics length) and hence the modified nonlocal yield function of the GTN model can be written as

$$\phi = \frac{\sigma_{eq}^2}{\sigma_m^2} + 2.q_1 d \cosh\left(q_2 \frac{3}{2} \frac{\sigma_{hydro}}{\sigma_m} \right) - 1 - q_3 d^2 = 0 \quad (10)$$

The weak form of the governing mechanical equilibrium equation (i.e., $\nabla.\sigma + f_b = 0$; f_b being body force per unit volume; σ being Cauchy's stress tensor) and the damage equilibrium equation (1) can be defined for the mechanical continuum Ω as [Samal et. al. 2007]

$$\int_\Omega \delta u \left(\nabla.\sigma + f_b \right) d\Omega = 0$$
$$\int_\Omega \delta \dot{d} \left(\dot{d} - \dot{f} - C_{length} \nabla^2 \dot{d} \right) d\Omega = 0 \quad (11)$$

Using the standard displacement based finite element (FE) method of discretisation of the mechanical continuum and using the standard shape functions for an element, we can derive the FE equations of a single element as

$$\begin{bmatrix} K_{uu} & K_{ud} \\ K_{du} & K_{dd} \end{bmatrix} \begin{Bmatrix} \Delta \hat{u} \\ \Delta \hat{d} \end{Bmatrix} = \begin{Bmatrix} R_m^{ext} - R_m^{int} \\ -R_d^{int} \end{Bmatrix} \quad (12)$$

The element level equations derived so, can be assembled and solved for the global degrees of freedom when we specify the required boundary and loading conditions. It may be noted that the stiffness terms K_{ud}, K_{du} and K_{dd} in the element stiffness matrix are contributions of the nonlocal model. R_m^{ext}, R_m^{int} and R_d^{int} are the externally applied mechanical force vector, internal mechanical force vector and internal equivalent damage force vectors respectively. The details of the procedure for derivation of the finite element equations are given in [Samal et. al. 2007].

The void volume fraction evolves from the initial void volume fraction f_0 in the material (volume fraction of relevant inclusions or void nucleation sites) and with straining; it increases to the critical void volume fraction for coalescence f_c. After f_c, the increase in void volume fraction gets accelerated and final fracture of the material point occurs at final void volume fraction f_f. In the continuum damage mechanics models, these parameters are determined from combined numerical simulation and metallurgical observations of microscopic voids as described in Seidenfuss (1992).

Usually, a fracture mechanics specimen such as compact tension specimen is numerically simulated and by comparing the experimentally obtained J-integral value at crack initiation (J_{IC}) with the numerically simulated value, the value of f_c is fixed. Many times, the notched tensile specimens are also used. The critical void volume fraction for coalescence f_c is determined by comparing the true fracture strain from experiment with that of numerical simulation. It may be noted that the void volume fractions such as f_0 and f_c etc. are material properties and hence they can be used for simulation in all types of specimens and components of the same material. In nonlocal models, the characteristics length parameter C_{length} is determined by comparing the experimentally measured J-resistance curve of a standard fracture mechanics specimen with the numerical simulation curve. Analysis is done with various values of C_{length} and the result of analysis, which describes the experiment more closely, is selected. It was also shown by Reusch et al. (2003a) that $C_{length} \approx \frac{l_c^2}{4}$. The characteristics length of the material l_c represents the mean distance between relevant inclusions of the material (Seidenfuss 1992).

3. RESULTS AND DISCUSSION

To demonstrate that the finite element results converge as the mesh size is reduced in case of the nonlocal formulation, a standard fracture mechanics (i.e., CT) specimen has been analysed. The CT specimen has the following dimensions, i.e., 25mm thickness (B), 50mm width (W), 60mm height (H). The specimen has a 20% side-groove in order to ensure plane strain conditions during the test.

In this work, a 2-D plane strain finite element analysis (with 8-noded iso-parametric quadrilateral elements) has been used to determine the load-CMOD (crack mouth opening displacement) and fracture resistance behaviour of the specimen. The geometry of the specimen is shown in Fig. 1. The initial crack to width ratio (a_0/W) of the CT specimen is 0.522. Due to symmetry, only half of the specimen has been analysed with imposition of symmetric boundary conditions. Experiments on the CT specimen have been carried out at MPA, University of Stuttgart, Germany. The material of the CT specimen is a low alloy German pressure vessel steel. The material stress-strain curve used in the analysis is shown in Fig. 2. The initial and critical void volume fractions used in the analysis are: $f_0 = 0.0003$, $f_c = 0.05$. The initial void volume fraction has been obtained from quantification (measurement of area fraction) of relevant inclusions in the initial microstructure of the material. For this purpose, initial microstructure of the material (along different planes or orientations) has been studied under SEM and the initial void volume fraction determined. The critical void volume fraction has chosen from literature as the typical value of void volume fraction at coalescence (used in the continuum damage mechanics analysis) for these types of materials. The characteristic length parameter value has been used as 0.01 for $C_{length}\left(\approx l_c^2/4\right)$ where $l_c = 0.2$ for this material (l_c represents mean distance between relevant inclusions distributed in the material matrix those are responsible for nucleation and growth of voids). The other material properties used in the analysis are: $f_f = 0.3$, $f_n = 0$, $\varepsilon_n = 0.3$, $s_n = 0.1$, $q_1 = 1.5$, $q_2 = 1.1$, $q_3 = 2.25$. The parameters f_n, ε_n and s_n are known as void nucleation parameters. Strain-controlled void nucleation mechanism has been assumed for this material. In the subsequent analyses, the effect of variation of material or GTN parameters on the results of the nonlocal model has been studied. The parameters q_2 and f_n have been varied for the above purpose.

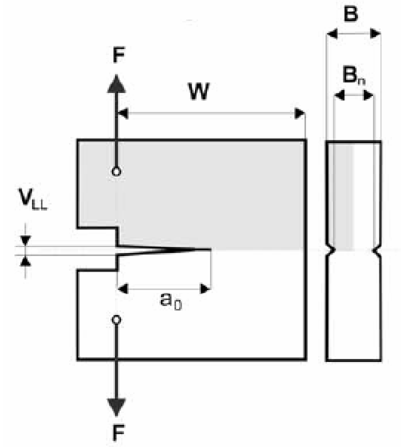


Fig. 1: Standard 1T CT specimen

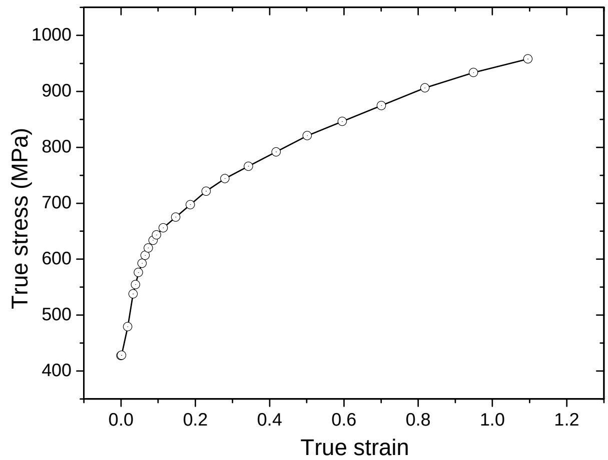


Fig. 2:True stress-strain curve used in the analysis of CT specimen

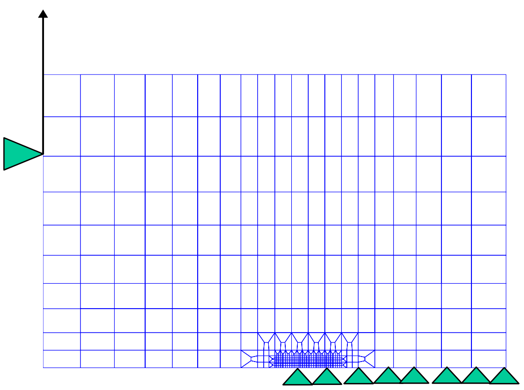


Fig. 3(a): Finite Element mesh for the CT specimen

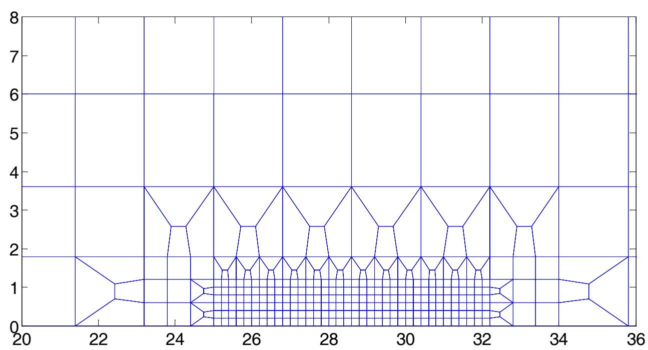
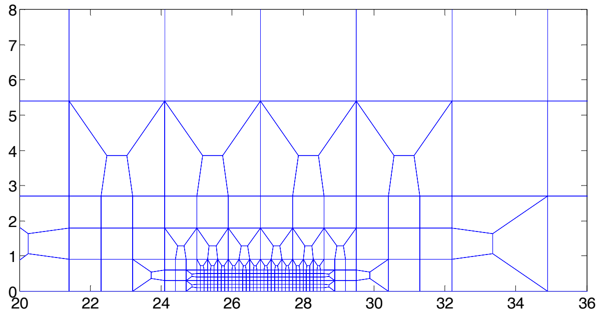


Fig. 3(b): Enlarged view of FE mesh of the CT specimen (near the crack tip, 0.2mm mesh)



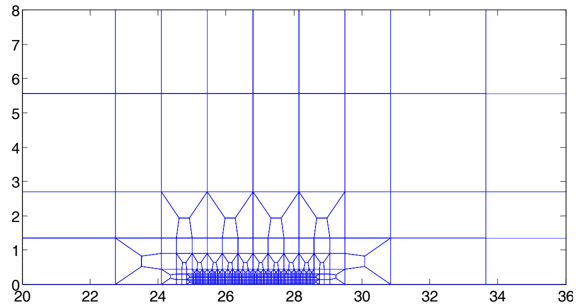
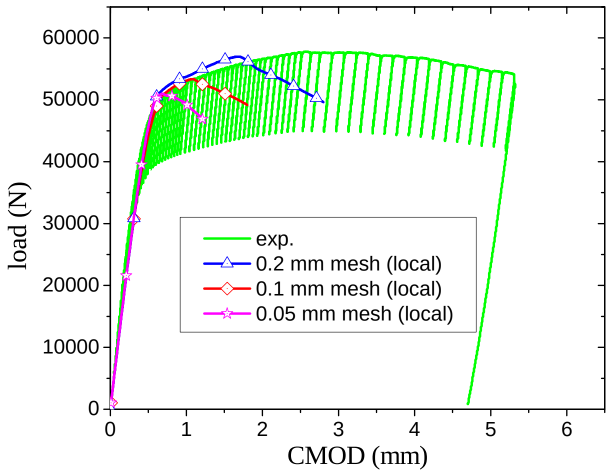


Fig. 3(c): Enlarged view of FE mesh of the CT specimen (near the crack tip, 0.1mm mesh)

Fig. 3(d): Enlarged view of FE mesh of the CT specimen (near the crack tip, 0.05mm mesh)



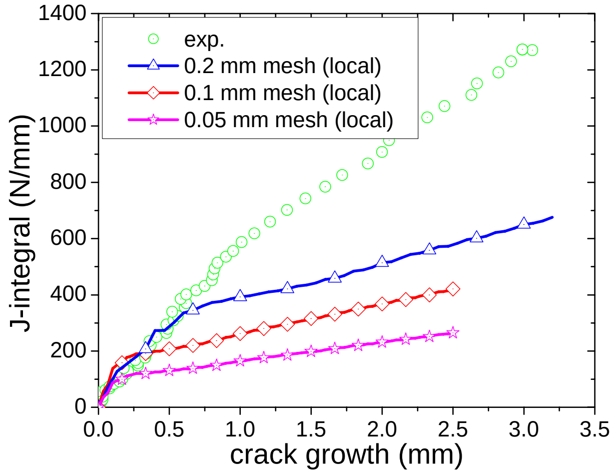


Fig. 4: Load-CMOD response of CT specimen (local model vs. experiment)

Fig. 5: J-resistance curve of the CT specimen (local model vs. experiment)

The finite element (FE) mesh of the CT specimen is shown in Fig. 3(a) along with the enlarged view near the crack tip for 0.2 mm mesh (Fig. 3b), 0.1 mm mesh (Fig. 3c) and 0.05 mm mesh (Fig. 3d) respectively. The above three different mesh sizes (i.e., 0.2, 0.1 and 0.05 mm) have been used near the crack tip to study the mesh dependent behaviour of both local and nonlocal models. Fig. 4 shows the load-CMOD results of local model for three different mesh sizes. When the mesh size is refined, the local model predicts faster crack growth and hence lower load-CMOD response.

The results of local GTN model are also compared with those of experiment and it may be observed from Fig. 4 that the local model is unable to predict the load-CMOD behaviour of the CT specimen even for the mesh size of 0.2mm which is usually used as the characteristics mesh size for these types of materials in the local damage mechanics analysis. The reason lies in the value of initial void volume fraction that has been used in this analysis (i.e., 0.0003). This type of problem is usually faced in the analysis with GTN model and is widely reported in literature. In order to avoid such problem, many researchers use an artificial low value of initial void volume fraction (i.e., 0 or of the order 1e-6) and use a non-zero value of the void nucleation parameter f_n (of the order 0.005). However, such an approach is many times difficult to explain from the physical point of view. The same trend is also observed in the J-resistance (J-integral vs. crack growth) behaviour (Fig. 5), i.e., the crack growth is faster when the mesh size is decreased. The J-integral has been evaluated using the area under the load-displacement diagram with suitable η and γ factors for the CT specimen as used in the experiment according to relevant ASTM standard.

It may be noted from Fig. 5 that the local model is also not able to predict the experimental J-resistance behaviour even for the FE mesh with 0.2mm mesh near the crack tip. However, in case of the nonlocal GTN model, the load-CMOD and the J-resistance curves compare very well with those of experiment for the three different mesh sizes (Figs. 6 and 7) and there is also very little difference among the results of different mesh sizes. Hence, the mesh independent nature of the nonlocal model is demonstrated.

The effect of variation of GTN parameters on the results of the nonlocal GTN model is considered next. Here, the two parameters (i.e.,

q_2 and f_n) has been varied. For the FE analysis, the mesh with square elements of 0.1mm side near the crack tip has been used. Two values of q_2 (i.e., 1 and 1.1) and two values of f_n (0 and 0.005) have been used in the FE analysis. The other parameters are same as those used in the previous analyses. The fracture resistance behaviours of the CT specimen as predicted by the nonlocal GTN model for q_2 equal to 1 and 1.1 are shown in Fig. 8 along with the comparison with the experimental result. According to the yield criterion in Eq. (10), the effect of q_2 is to accelerate shrinking of the yield surface when the stress triaxility is present and hence, the growth of void volume fraction become faster for increased values of q_2. The same trend is observed in Fig. 8 and it can be noted that the predicted result compares very well with that of experiment when q_2 is chosen as 1.1. The effect of void nucleation parameter f_n on the predictions of the nonlocal model is shown in Fig. 9. It can be observed that there is very little difference between the two results of the nonlocal model (i.e., with $f_n = 0$ and $f_n = 0.005$). This is because of the fact that the results become insensitive to variation of f_n (within small ranges which are typical for these types of materials) when f_0 values are dominant. The nonlocal model could successfully predict the experimentally observed behaviour with a value of initial void volume fraction which was obtained from direct physical measurements and hence is more appealing. This is in contrast to the behaviour of the local GTN damage model.

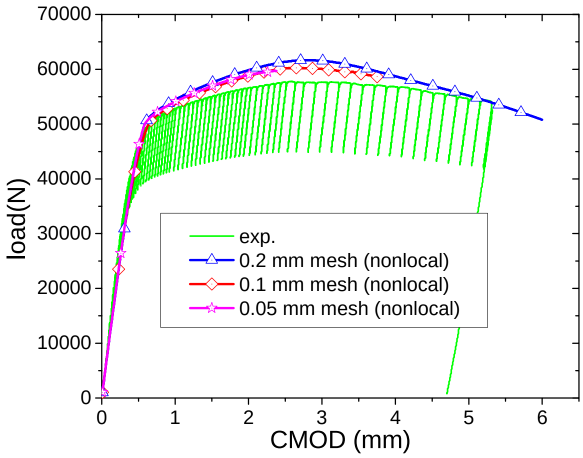


Fig. 6: Load-CMOD response of CT specimen (nonlocal model vs. experiment)

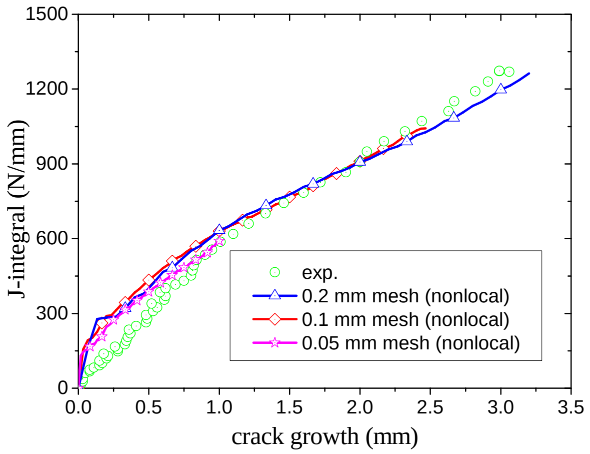


Fig. 7: J-resistance curve of the CT specimen (nonlocal model vs. experiment)

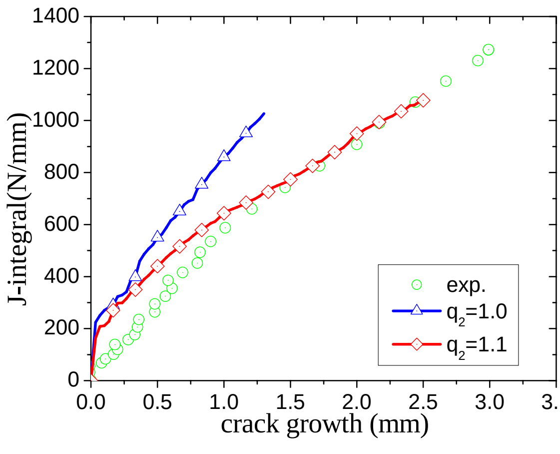


Fig. 8: J-resistance behaviour of the CT specimen with 0.1mm mesh using nonlocal model (effect of GTN parameter q_2)

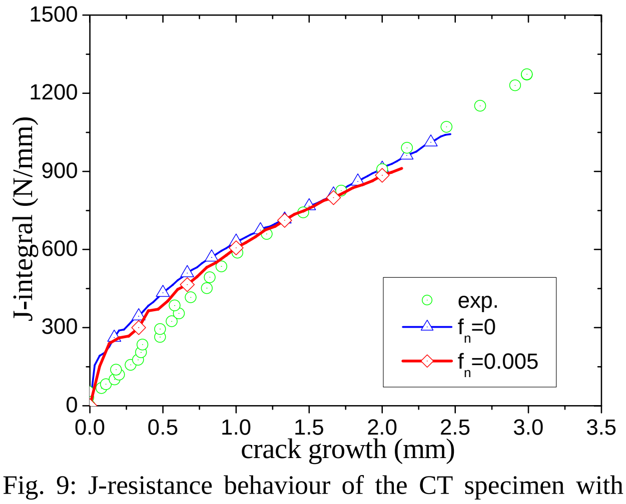


Fig. 9: J-resistance behaviour of the CT specimen with 0.1mm mesh using nonlocal model (effect of GTN parameter f_n)

4. CONCLUSIONS

In this work, a nonlocal form of the Gurson-Tvergaard-Neeedleman's damage mechanics model have been developed and is implemented in a finite element framework to solve the boundary value problems. Analysis has been carried out for a standard 1T CT specimen using both local and nonlocal damage theories. It was observed that the results of nonlocal model are mesh-independent and hence they can be used to predict ductile fracture behaviour of components without using a FE mesh of particular size. This important property is very useful to simulate fracture behaviour of components having large stress gradients, which otherwise is more difficult to realize with local damage models. The other important aspect of the nonlocal GTN model is its ability to use an initial void volume fraction which can be obtained from a direct physical measurement in the metallographic microstructure of the material. In case of the local GTN models, one has to resort to artificially low values of initial void volume fraction and use suitable values of the void nucleation parameters in order to obtain a satisfactory prediction of experimental data.

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