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A metastable wet steam turbine stage model

Wageeh Sidrak Bassel *, Arivaldo Vicente Gomes

Instituto de Pesquisas Energeticas e Nucleares, Travessa 'R' 400-Cidade Universitaria, CEP 05508-900 São Paulo, SP, Brazil

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Abstract

A model for the prediction of the efficiency of axial flow steam turbine stage is described, where the flow through turbine cascade is considered non-homogeneous and metastable. At the exit an oblique shock brings it to equilibrium. The losses in the cascade are expressed according to Dunham and Came (Trans. ASME (1970)) and Kacker and Okapuu (J. Eng. Power (1982)) which is imprivemente of Ainley and Mathieson (1951) method. Two phase flow frictional multiplier is used as a correction factor for pressure coefficient. The model is compared with data of performance evaluation of large steam turbines of PWR power plants and results shows a good agreement.

1. Introduction

The performance of axial flow turbine was determined by means of loss model first deduced by Ainely and Metheison (1951). The work has been revised by Dunham and Came (1970) and later by Kacker and Okapuu (1982). The loss model is widely used in gas turbine and the efficiency can be determined within accuracy of 1.5%. The application of this method in steam turbine results of higher errors. Craig and Cox (1971) proposed 1% less of efficiency for each 1% of mean stage wetness.

The solution of three dimensions Navier–Stokes equation—by numerical method—for compressible single phase was developed for turbine cascade flow. According to Cofer (1996), the application of similar method for two phase flow of wet steam turbine not yet established up to now.

The present work is simple and can be used for wet steam turbine stage with adequate accuracy. The model is based upon system of non-linear algebraic conservation equations of mass, momentum in axial and tangential directions, energy and loss model. In these equations it is considered slip between vapor and liquid phases and the loss model is corrected by two phase flow frictional multiplier. Since the residence time of steam when passing through blade row in the order of 0.1 ms, it is assumed that there is no heat transfer between the two phases and there is no sufficient time for formation of new liquid drops so both the vapor and liquid phases are in metastable state. The metastable state is transformed in stable equilibrium state at the entrance of new subsequent row by means of oblique equilibrium shock.

* Corresponding author.

E-mail addresses: wsbassel@net.ipen.br (W.S. Bassel), avgomes@net.ipen.br (A.V. Gomes).

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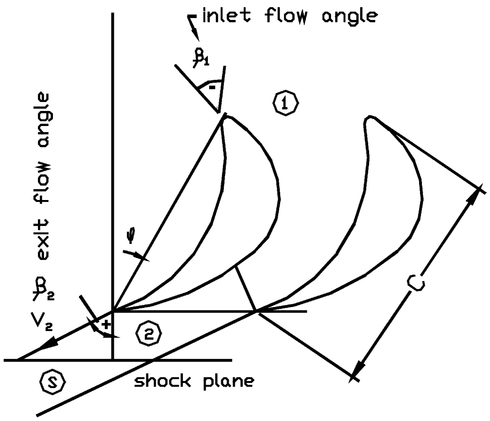

Fig. 1.

2. Governing physical equations

Fig. 1 shows steam turbine blade terminology used in this work. The governing physical equations which describes the flow for fixed and moving blades are the conservation equations of mass, momentum and energy, pressure loss coefficient and equation of state. Equation of conservation of mass:

$$v_{g2}sh_2\left(\alpha_2\rho_{g2}+\frac{(1-\alpha_2)\rho_{l2}}{S_2}\right)=v_{g1}sh_1\left(\alpha_1\rho_{g1}+\frac{(1-\alpha_1)\rho_{l1}}{S_1}\right). \quad (1)$$

Equation of conservation of momentum in axial direction:

$$p_2sh_2+mv_{g2}\left(x_2+\frac{1-x_2}{S_2}\right)+(C_L\sin\beta_m+C_D\cos\beta_m)ch_m\rho_{g2}\frac{V_{g2}^2}{2}=p_1sh_1+mv_{g1}\left(x_1+\frac{1-x_1}{S_1}\right), \quad (2)$$

where $h_m=(h_1+h_2)/2$.

Equation of conservation of momentum in tangential direction:

$$mu_{g2}\left(x_2+\frac{1-x_2}{S_2}\right)-(C_L\cos\beta_m-C_D\sin\beta_m)ch_m\rho_{g2}\frac{V_{g2}^2}{2}=mu_{g1}\left(x_1+\frac{1-x_1}{S_1}\right). \quad (3)$$

Equation of conservation of energy:

$$\frac{V_{g2}^2}{2}\left(x_2-\frac{1-x_2}{S_2^2}\right)+H_2=\frac{V_{g1}^2}{2}\left(x_1-\frac{1-x_1}{S_1^2}\right)+H_1, \quad (4)$$

$$H_2=x_2H_{g2}+(1-x_2)H_{l2}, \quad (5)$$

$$V_{g2}^2=v_{g2}^2+u_{g2}^2, \quad (6)$$

mean flow angle β_m:

$$\tan\beta_m=\frac{1}{2}\left(\frac{u_{g1}}{v_{g1}}+\frac{u_{g2}}{v_{g2}}\right). \quad (7)$$

The exit flow angle β_2:

$$\tan\beta_2=\frac{u_{g2}}{v_{g2}}. \quad (7a)$$

The inlet flow angle β_1:

$$\tan\beta_1=\frac{u_{g1}}{v_{g1}}. \quad (7b)$$

The pressure loss coefficient $Y_{bifasic}$ is defined by,

$$Y_{two\ phase}=\frac{p_1+(1/2)\rho_{g1}V_1^2-(p_2+(1/2)\rho_{g2}V_2^2)}{(1/2)\rho_{g2}V_2^2}, \quad (8)$$

where

$$Y_{two\ phase}=\phi Y, \quad (8a)$$

where ϕ is the two phase frictional multiplier, and

$$Y=Y_p+Y_{sec}+Y_{tet\,+}\,Y_{tc}. \quad (8\text{-b})$$

Y_p is the profile loss coefficient, Y_{sec} secondary loss coefficient, Y_{tet} trailing edge coefficient and Y_{tc} tip clearance coefficient. These coefficients are determined according to Dunham and Came (1970), Kacker and Okapuu (1982).

The two phase frictional multiplier ϕ is the ratio of the two phase pressure drop estimated by Friedel correlation to vapor phase pressure drop estimated at blade inlet.

The equations of state, enthalpy and entropy in metastable state are estimated by curve fitness from data of Keenen and Keys steam table as follows.Equation of state:

$$\frac{p_2}{\rho_{g2} R T_2} = 1 + \rho_{g2} B(T) + \rho_{g2}^2 C(T), \tag{9}$$

$$B(T) = -0.076898 + 0.074717 \times 10^3/T_2 - 0.020749 \times 10^6/T_2^2, \tag{9a}$$

$$C(T) = 0.001731 - 0.000834 \times 10^3/T_2, \tag{9b}$$

where p is the pressure in Pa; ρ, density in kg m^{-3}; T, temperature in K; R, gas constant for steam, equal 461.5 J kg^{-1} K.

The enthalpy in kJ kg^{-1} is:

$$H_{g2} = H_0 + \frac{RT_2}{1000}\left\{\rho_{g2}(B(T) - T_2)\frac{dB(T)}{dT} + \rho_{g2}^2\left(C(T) - \tfrac{1}{2}T_2\right)\frac{dC(T)}{dT}\right\}, \tag{10}$$

$$H_0 = 2369.131 - 1.231567\ T_2 + 0.008379\ T_2^2 - 0.000007455\ T_2^3. \tag{10a}$$

The entropy in kJ kg^{-1} K is:

$$i_{g2} = i_o + \frac{R}{1000}\left\{\ln\frac{p_2}{10^5} + T_2\rho_{g2}\frac{dB(T)}{dT} + 2\rho_{g2}B(T)^2 - 2\rho_{g2}C(T) + 2\rho_{g2}T_2\frac{dC(T)}{dT}\right\}, \tag{11}$$

$$i_o = 4.927682 + 0.007063\ T_2 + 0.000001462\ T_2^2 - 7.284401 \times 10^{-9}\ T_2^3, \tag{11a}$$

$$i_2 = x_2\, i_{g2} + (1 - x_2) i_{l2}, \tag{12}$$

the slip ratio S_2 is given as

$$S_2 = (\rho_{l2}/\rho_{g2})^{0.33}, \tag{13}$$

and void fraction α_2 is given as:

$$\alpha_2 = \frac{1}{1 + ((1 - x_2)/x_2)(\rho_{g2}/\rho_{l2} S_2)}. \tag{14}$$

In the above system of equations the inlet flow velocity and all the thermodynamic properties are known. Considering that there is no heat transfer between as the two phases and no formation of new liquid drops it can be assumed:

$$x_2 = x_1,$$

$$H_{l2} = H_{l1},$$
$$\rho_{l2} = \rho_{l1},$$

$$i_{l2} \cong i_{l1}.$$

The solution of non-linear system of algebraic equations from Eq. (1) to Eq. (14) gives the numerical values of the following variables:

(v_{g2}, u_{g2}, V_{g2}, p_2, T_2, ρ_{g2}, C_L, C_D, H_{g2}, H_2, i_{g2}, i_2, S_2, α_2).

The flow, after exit plane of the blade, suffers from obstacle represented by the subsequent blade row, due to this it is assumed that an oblique shock is established in plane parallel to exit blade plane transforming the flow to stable equilibrium conditions.

The governing physical equations for the equilibrium shock are: the tangential velocities, relative to the shock plane, before and after the shock are equal.

$$u_{gs} = u_{g2}, \tag{15}$$

conservation of mass:

$$m_{gs} + m_{ls} = m, \tag{16}$$

$$m_{ls} = v_{gs}(1 - \alpha_s) s h_2 \rho_{ls}/S_s, \tag{17}$$

$$m_{gs} = v_{gs}\alpha_s s h_2 \rho_{gs}. \tag{18}$$

Equation of conservation of momentum:

$$p_s s h_2 + m_{gs} v_{gs} + m_{ls} v_{gs}/S_s = p_2 s h_2 + m_{g2} v_{g2} + m_{l2} v_{g2}/S_2. \tag{19}$$

Equation of conservation of energy

$$m_{gs}\left(H_{gs} + \frac{v_{gs}^2}{2}\right) + m_{ls}\left(H_{ls} + \frac{v_{gs}^2}{2S_s^2}\right) = m_{g2}\left(H_{g2} + \frac{v_{g2}^2}{2}\right) + m_{l2}\left(H_{l2} + \frac{v_{g2}^2}{2S_2^2}\right). \tag{20}$$

The equations from Eq. (16) to Eq. (20) joint with the seven equations of pressure as a function of temperature, and density, enthalpy and entropy of liquid and vapor phase as a function of temperature too, in thermodynamic equilibrium state,

Table 1
Characteristics of turbine stage—illustrative example

Description	Fixed blade	Moving blade
Spacing (s, mm)	29.6	49.4
Cord (c, mm)	51	90
Inlet blade height (h_1, mm)	182	208
Exit blade height (h_2, mm)	195	235
Main radius (mm)	1405	1419
Exit flow angle (°)	73	73.3
Number of blades	298	180
Velocity at mean radius (m s^{-1})		264.82
Type		Unshrouded

forms a system of non-linear algebraic equations, where the solution gives

$(m_{gs}, m_{ls}, \alpha_s, v_{gs}, p_s, T_s, H_{gs}, H_{ls}, \rho_{gs}, \rho_{ls}, i_{gs}, i_{ls})$.

The work done is given by the equation:

$$W = U\,\Delta F, \quad (21)$$

where U is mean radius speed, ΔF is the change of the tangencial momentum of the moving blade, based on absolute velocity, that is given by the equation:

Table 2
Inlet flow conditions

Mass flow rate (m, kg s^{-1})	161.05
Static pressure (p_1, bar)	2.393
Steam quality (x_1)	0.9766
Vapor phase enthalpy (H_{g1}, kg kg^{-1})	2714.93
Liquid phase enthalpy (H_{l1}, kJ kg^{-1})	529.13
Mixture enthalpy (kJ kg^{-1})	2663.78
Vapor phase entropy (i_{g1}, kJ kg^{-1} K)	7.0678
Liquid phase entropy (i_{l1}, kJ kg^{-1} K)	1.5918
Mixture entropy (i_1, kJ kg^{-1} K)	6.9395
Vapor phase density (kg m^{-3})	1.3347
Liquid phase density (kg m^{-3})	937.9
Mixture density (m^3 kg^{-1})	1.3666
Inlet axial velocity (v_1, m s^{-1})	73.37
Inlet tangential velocity (u_1, m s^{-1})	−9
Slip ratio (S_1)	8.69
Void fraction (α_1)	0.9997

Table 3
Exit flow conditions of fixed blades

Exit conditions before shock plane	
Frictional two phase flow multiplication factor (φ)	1.4
Two phase flow pressure drop coefficient ($Y_{\text{two phase}}$)	0.1228
Exit axial velocity (v_2, m s^{-1})	80.9
Exit tangential velocity (u_2, m s^{-1})	261.92
Exit velocity (V_2, m s^{-1})	274.13
Exit static pressure (p_2, bar)	1.9492
Static temperature (T_2, °C)	107.55
Vapor phase density (ρ_{g2}, kg m^{-3})	1.141
Lift coefficient (C_L)	0.6786
Drag coefficient (C_D)	0.0395
Vapor phase enthalpy (H_{g2}, kJ kg^{-1})	2680.15
Mixture enthalpy (H_2, kJ kg^{-1})	2629.75
Vapor phase entropy (i_{g2}, kJ kg^{-1} °C)	7.07014
Mixture entropy (i_2, kJ kg^{-1} °C)	6.94178
Slip ratio (S_2)	9.1548
Void fraction (α_2)	0.99973
Conditions after shock plane	
Vapor phase mass rate of flow (m_{gs}, kg s^{-1})	0.521873
Liquid phase mass rate of flow (m_{ls}, kg s^{-1})	0.01856
Steam quality (x_s)	0.96565
Void fraction (α_s)	0.999615
Slip ratio (S_s)	9.27
Vapor phase axial velocity (v_{gs}, m s^{-1})	82.029
Static pressure (p_s, bar)	1.9498
Static temperature (T_s, °C)	119.38
Vapor phase enthalpy (H_{gs}, kJ kg^{-1})	2705.43
Liquid phase enthalpy (H_{ls}, kJ kg^{-1})	501.024
Vapor phase density (ρ_{gs}, kg m^{-3})	1.1017
Liquid phase density (ρ_{ls}, kg m^{-3})	943.3
Vapor phase entropy (i_{gs}, kJ kg^{-1} °C)	7.13657
Liquid phase entropy (i_{ls}, kJ kg^{-1} °C)	1.52089
Mixture entropy (i_s, kJ kg^{-1} °C)	6.94368

$$\Delta F = \left[x_2(u_{g2} - U) + (1 - x_2)\left(\frac{u_{g2}}{S_2} - U\right) \right]_{\text{moving}} + \left[x_s\, u_{gs} + (1 - x_s)\,\frac{u_{gs}}{S_s} \right]_{\text{fixed}}. \quad (22)$$

The dynamic enthalpy drop is given by:

$$\Delta H_{\text{dynamic}} = \Delta H_{\text{static}} + KE_{\text{inlet}} - KE_{\text{exit}}, \quad (23)$$

where $\Delta H_{\text{dynamic}}$ is the dynamic enthalpy drop, ΔH_{static} is the static enthalpy drop, KE_{inlet} is the kinetic energy of inlet flow based upon absolute velocity, KE_{exit} is the kinetic energy of exit flow based upon absolute velocity.

$$\eta = W/\Delta H_{dynamic}, \quad (24)$$

where η is the stage efficiency.

3. Illustrative example

The above model was applied in the analysis of steam turbine stage of a low pressure turbine of a nuclear power plant. Table 1 shows the stage specification, Table 2 shows inlet flow conditions, Table 3 shows the result of calculation of the fixed blades. Table 4 shows the result of calculations of the moving blades.

The calculated results are as follow:

$W = 68.2$ kJ/kg (specific work done),

$\eta = 88.55\%$ (efficiency).

The estimated stage efficiency, obtained during performance evaluation of steam turbine, by measuring exit power and exit enthalpy indirectly, was 86.3%; the difference is due to some other types of losses, like leakage in the stator blades and losses due to expansion of steam from exit fixed blade to inlet moving blades, that was not considered in this calculation.

Table 4
Exit flow conditions of moving blades

Exit conditions before shock plane	
Frictional two phase flow multiplication factor (φ)	1.47
Two phase flow pressure drop coefficient ($Y_{two\ phase}$)	0.25463
Exit axial velocity (v_2, m s^{-1})	82.3
Exit tangential velocity (u_2, m s^{-1})	274.39
Exit velocity (V_2, m s^{-1})	286.46
Exit static pressure (p_2, bar)	1.5162
Static temperature (T_2, °C)	98.96
Vapor phase density (ρ_{g2}, kg m^{-3})	1.1055
Lift coefficient (C_L)	0.6313
Drag coefficient (C_D)	0.0637
Vapor phase enthalpy (H_{g2}, kJ kg^{-1})	2667.31
Mixture enthalpy (H_2, kJ kg^{-1})	2592.9
Vapor phase entropy (i_{g2}, kJ kg^{-1} °C)	7.148967
Mixture entropy (i_2, kJ kg^{-1} °C)	6.95565
Slip ratio (S_2)	9.8864
Void fraction (α_2)	0.99966
Conditions after shock plane	
Vapor phase mass rate of flow (m_{gs}, kg s^{-1})	0.85409
Liquid phase mass rate of flow (m_{ls}, kg s^{-1})	0.040625
Steam quality (x_s)	0.95459
Void fraction (α_s)	0.999563
Slip ratio (S_s)	10.02
Vapor phase axial velocity (v_{gs}, m s^{-1})	84.38
Static pressure (p_s, bar)	1.5173
Static temperature (T_s, °C)	111.68
Vapor phase enthalpy (H_{gs}, kJ kg^{-1})	2693.94
Liquid phase enthalpy (H_{ls}, kJ kg^{-1})	468.37
Vapor phase density (ρ_{gs}, kg m^{-3})	0.8724
Liquid phase density (ρ_{ls}, kg m^{-3})	949.4
Vapor phase entropy (i_{gs}, kJ kg^{-1} °C)	7.2202
Liquid phase entropy (i_{ls}, kJ kg^{-1} °C)	1.437
Mixture entropy (i_s, kJ kg^{-1} °C)	6.9576

Table 5
Comparison between metastable and homogeneous equilibrium models (case 1)

Parameter	Metastable model	Homogeneous equilibrium model
Inlet pressure (bar)	16.143	16.143
Inlet steam quality (%)	90.2	90.2
Two phase frictional multiplier for stator	1.47	Not applicable
Pressure loss coefficient Y for stator	0.1164	0.0792
Two phase frictional multiplier for rotor	1.47	Not applicable
Pressure loss coefficient Y for rotor	0.1557	0.1059
Exit stage pressure (bar)	13.75	13.85
Exit stage steam quality (%)	89.28	89.27
Stage efficiency	0.865	0.941
Stage efficiency after correction	Not applicable	83.8
Estimated stage efficiency from heat balance	85	

Table 6
Comparison between metastable and homogeneous equilibrium models (case 2)

Parameter	Metastable model	Homogeneous equilibrium model
Inlet pressure (bar)	2.393	2.393
Inlet steam quality (%)	97.65	97.65
Two phase frictional multiplier for stator	1.4	Not applicable
Pressure loss coefficient *Y* for stator	0.12228	0.08709
Two phase frictional multiplier for rotor	1.47	Not applicable
Pressure loss coefficient *Y* for rotor	0.25463	0.1732
Exit stage pressure (bar)	1.5173	1.64
Exit stage steam quality (%)	95.46	95.66
Stage efficiency (%)	88.5%	Near 100%
Stage efficiency after correction	Not applicable	97.34
Estimated stage efficiency from heat balance	86.3	

Table 7
Comparison between metastable and homogeneous equilibrium models (case 3)

Parameter	Metastable model	Homogeneous equilibrium model
Inlet pressure (bar)	0.2017	0.2017
Inlet steam quality (%)	0.922	0.922
Two phase frictional multiplier for stator	1.24	Not applicable
Pressure loss coefficient *Y* for stator	0.281	0.227
Two phase frictional multiplier for rotor	1.24	Not applicable
Pressure loss coefficient *Y* for rotor	0.285	0.255
Exit stage pressure (bar)	0.077	Note 2
Exit stage steam quality (%)	88.5	Note 2
Stage efficiency (%)	0.778 note 1	Note 2
Stage efficiency after correction	Not applicable	Note 2
Estimated stage efficiency from heat balance	0.71	

The above calculations was repeated using concept of thermodynamic equilibrium between phases, vapor and liquid, during flow through the blade rows. Correlations of enthalpy, entropy, specific volume of saturated vapor and saturated liquid, also the pressure temperature relationship of saturation state was developed. These correlations substituted the Eqs. (9)–(11) of metastable state. The pressure loss coefficient Y is used without multiplication of two phase frictional multiplier. Thus represents the treatment of the problem by the same method used in gas turbine, yielding homogeneous equilibrium solution of wet steam turbine stage. Carig and Cox suggested reduction of 1% of efficiency for each 1% mean humidity. Three real cases were analyzed with both methods, the metastable and homogeneous equilibrium for comparison, the results are shown in Tables 5–7.

Note 1

Case 3 represents the final stage of low pressure steam turbine, the exit kinetic energy is completely lost, so the efficiency in this case is expressed by the work done divided by the isentropic static enthalpy drop. The rotor blade is highly tapered by reducing the blade cord and thickness from hub to tip, in order to improve the mechanical vibration aspect. Due to blade taper the estimated efficiency at midline estimated by the present work is 9% greater than the real mean blade efficiency.

Note 2

The calculated exit conditions of the stator of equilibrium homogeneous model had no physical meaning—even the mathematical solution converged—thus there were negative change in entropy, negative absolute pressure and temperature. The expected exact solution is flow with stationary or moving shock within the blade row. The treatment of such phenomena is out of scope of this work.

Appendix A. Nomenclature

c	cord (m)
C_D	drag coefficient
C_L	lift coefficient
h	blade height (m)
H	enthalpy (kJ kg^{-1})
m	mass flow rate through one blade spacing (kg s^{-1})
p	static pressure (N m^{-2})
s	spacing (m)
S	slip ratio
x	steam quality
V	relative flow velocity (m s^{-1})
v	component of V in axial direction (m s^{-1})
u	component of V in tangential direction (m s^{-1})
α	void fraction
β	flow angle (with axial direction)
ρ	density (kg m^{-3})

Subscript

1	blade inlet
blade exit	before shock plane
s	blade exit, after shock plane
g1	vapor phase at blade inlet
l1	liquid phase at blade inlet
g2	vapor phase at blade exit
l2	liquid phase at blade exit
gs	vapor phase after shock plane
ls	liquid phase after shock plane

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