

Predictive effect of economic and market variations on structural breaks in credit rating dynamics

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Abstract

Recent studies have shown that firms credit rating transition process is not stationary and may have structural breaks. To study the predictability of structural breaks, we develop a predictive model for latent structural breaks in firms rating transition dynamics, using historical records of (high-dimensional) economic and market fundamentals. As a large number of economic and market variables are sometimes involved in the study, we also introduce an inference procedure that select and estimate important economic factors at the same time from the high-dimensional factor space. Based on an empirical study using the U.S. firms' credit rating transition records and the history of economic and market variations from 1986 to 2013, we demonstrate that not all structural breaks are black-swan events and some of them can be estimated and predicted up to certain extent.

JEL classification: C13; C41; G12; G20

Keywords: Credit risk; Hidden Markov model; Stochastic structural break

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1. Introduction

During the last two decades, the U.S. credit market has experienced several structural breaks that were triggered by certain social and economic events, and then propagated through complicated economic mechanisms. For instance, the U.S. commercial paper markets experienced a severe disruption in the second half of 1998 due to a series of events including the Russian's default, Brazil's currency crisis, and the downturn of the LTCM (Long-Term Capital Management L.P.), while the financial crisis of 2007-2008 was resulted from a complex interaction of several financial and policy causes such as "high risk lending by U.S. financial institutions; regulatory failures; inflated credit rating; high risk, poor quality financial products" (Permanent Subcommittee on Investigations, 2011). Whereas these marketwide structural breaks could result in devastating economic, social and political consequences, it is important for the regulatory authority and financial practitioners to understand the question of to what extent such structural breaks can be predictable so that its devastating impact can be prevented or mitigated.

Based on U.S. firms' credit rating records and macroeconomic fundamentals, we develop a predictive model of structural breaks in credit market. In particular, the developed model extracts and aggregates the information on market structural breaks from individual firm's rating records, and then connects the market structural break information with the variations of economic and market conditions. The predictive effect of the model comes from the lagged macroeconomic covariates and the latent market structural break information in firms' rating records, which will be further discussed below. We show that: (i) market structural break information can be successfully extracted and aggregated from the level of individual firms, indicating the proposed model has an economic interpretation at microeconomic level; (ii) market structural breaks are not "highly improbable" or "black swan" events discussed by Taleb (2010) but, instead, predictable up to certain standard; (iii) the explanatory information for structural breaks is contained in a (high-dimensional) vector of macroeconomic factors, but need to be extracted carefully for the purpose of prediction. Our findings improve the understanding of predictive effect of macroeconomic covariates on market structural breaks, and throw light on developing an early-warning system that predicts or monitors structural breaks in credit market for the regulatory authority and commercial banks.

Our model has two components, the first is to extract and aggregate the market structural break information hidden in firms' credit rating transition records, and the second is to link the aggregated structural break information to macroeconomic fundamentals. The first component of the model is similar to the stochastic structural break model introduced by Xing et al. (2012) for firms' rating transition dynamics. In this component, firms are assumed to be homogeneous and structural breaks in credit market are represented as sharp changes of firms' credit rating dynamics (or rating generator matrices). Specifically, firms' credit rating transition generator matrices are assumed as piecewise homogeneous Markov chains with unobserved structural breaks. The economic rationale behind this assumption is as follows. Since credit rating measures a firm's ability to fulfill its future financial obligation under the current market conditions, a firm's rating transitions contain information on general market conditions and the sharp changes of firms' rating transition patterns reveal the variations of general market conditions. However, such kind of macro information hidden in firms' rating transition records is very weak, and need to be aggregated over the micro level data, that is, all firms' rating records. Another assumption we made is that structural breaks in firms' rating dynamics (i.e., rating generator matrices) follow a compounded Poisson process, in which the times of structural breaks follow a Poisson process with a time-varying rate and the entries of post-change generator matrices follow a Gamma distribution. These assumption corresponds to the facts that the number, time, and magnitude of structural breaks in firms' rating dynamics are not known in advance, and that the effect of market structural breaks can be either gradual or abrupt on firms' rating dynamics, and consequently, the magnitude of changes in generator matrices can be small or large.

The second component of our model is to connect the aggregated market structural break information with variations of macroeconomic fundamentals so that the probability of structural breaks can be predicted. Compared to the structural break model in Xing et al. (2012) which summarizes structural break through a fixed probability, our model allows the probability of structural breaks, or the rate of Poisson process mentioned above, depending on historical macroeconomic and financial market variations. This is important as it provides us a tool to investigate the causal effects of variations of macroeconomic fundamentals on market structural breaks. However, since market structural breaks can be resulted from different complex economic mechanisms and further research will undoubtedly investigate those mechanisms,

we do not claim that this paper provides a structural model on economic mechanism of structural breaks. Instead, we only provide a “reduced-form” description on the predictive effects on market structural breaks and hence demonstrates that market structural break events are not highly improbable and their relationship to economic and financial market variations can be understood econometrically.

In the effort to establish a connection between market structural breaks and economic and financial market variations, we have to face another challenge, that is, how many economic fundamentals or covariates should be included in the model so that a better prediction can be obtained. Since the mechanisms of market structural breaks are not considered (or too complicated to discuss) here, it is difficult to pre-conclude which economic covariates should be included in the model. Given the fact that each covariate may only have a weak effect in terms of market structural breaks, it is probably preferable to include economic covariates as many as we could find, especially for market practitioners or the regulatory authority. However, including many more economic covariates generate the curse of the dimensionality problem, and could diffuse the effective information on market structural breaks latent in variables on market variation and hence make the statistical inference inefficient. In order to overcome this difficulty, we use an expectation-maximization (EM) approach to estimate model parameters and effects of economic covariates. In particular, after the expectation step in the EM algorithm, we are able to represent the structural break events by estimated probabilities of structural breaks and then maximize the expected likelihood by solving a logistic regression problem if the number of economic covariates are not many. In the case that a large number of covariates are included in the model, we consider a penalized likelihood function in the maximization step. Such penalization regularizes the coefficient parameters of high-dimensional covariates, and allow us to select and estimate effective (or non-zero coefficients) covariates for structural breaks at the same time.

We use the proposed model and developed inference procedure to study the predictive effect of a set of economic and financial market variables on market structural breaks in credit rating dynamics of U.S. firms’ from January 1986 to March 2013. The study contains three parts. The first evaluates the predictive effects of a set of economic and financial market variables on market structural breaks and discuss two types of retrospective estimates of structural break probabilities from the model. The second conducts a prospective study that predict sequentially the probability of structural

breaks using the history of firms' rating transitions and economic covariates. The third part demonstrates how our model and inference procedure select and estimate important factors for structural breaks when high dimensional covariates are involved in the study.

The model developed here extracts and aggregates the information of market structural break from each firm's rating transition records and link the probability of structural breaks with macroeconomic fundamentals. The estimated probability of structural breaks helps us understand the risk of sudden shifts of general market conditions, and is related but distinct to the concept of systemic risk, which refers to the risk of collapse of the entire market caused by one or a few firms' risk events. From this perspective, the model serves a purpose for regulatory agencies on monitoring the stability of the market. Another potential application of the model is to help bank understand the risk of structural breaks from the "market" that consist of all her own counterparties and exposures. Under the guideline of Basel Accords, banks are allowed to build their internal rating system to assess the risk of all their counterparties and exposures. The proposed model can be extended there to estimate the probability of structural breaks, and hence allows the bank to take necessary actions to mitigate the loss caused by such instability.

The remainder of the paper is organized as follows. Section 2 provides the related literature on credit rating study in recent years. Section 3 develops a structural break model with economic covariates and its inference procedure using continuous credit rating history. In Section 4, we study the in-sample and out-of-sample performance of our model on rating transition records of U.S. firms and economic variables, and discuss the estimation results and their economic implication. Section 5 provides some concluding remarks.

2. Related literature

In order to investigate the connection between structural breaks in credit market and economic covariates, an intuitive way is to design a regression type model and include market structural information and economic covariates as response and explanatory variables, respectively. This requires finding a quantitative measurement on market structural breaks first. However, it is difficult to find such kind of measurement in the existing literature, since market structural break is presumably a phenomenon at macro level instead of an observable economic variable. Therefore, market structural breaks can not be represented directly by existing economic and financial

market fundamentals, but instead, need to be inferred from economic or financial records. On the other hand, since we want to use economic covariates as explanatory variables, we should not extract market structural information from macroeconomic fundamentals so that the issue of circular reasoning and data snooping can be avoided in the analysis. Furthermore, since market structural break presumably influence all firms' credit transitions, the pattern changes of firms' rating transitions must contain information on market structural breaks, and hence extracting such information from firms' rating transition records could provide us a full spectrum of structural break information over the market consisting of all firms. From this perspective, our model has a micro foundation for market structural break phenomenon.

We note that, as an information good provided by credit rating agencies (CRAs), a firm's credit ratings measure her ability to fulfill its future financial obligation under the current market conditions (Langohr and Langohr, 2008). It has been brought to the regulators' and public's attention that CRA's rating quality varies during different periods such as the East Asian financial crisis in 1997, the failure of Enron in 2001 and Worldcom in 2002, and the most recent 2007-09 financial crisis. Researchers have discussed this type of behavior from different perspectives. One perspective is how the conflict of interest between investors and information intermediaries affects the quality of the information disclosed to the market. Lizzeri (1999) discussed the role of monopoly or competitive information intermediaries in the revelation of the information of privately informed parties. Boot et al. (2006) considered credit rating as a coordination mechanism and explored the contractual relationship between a CRA and a firm through its credit watch procedures. Skreta and Veldkamp (2009) and Bolton et al. (2012) developed models to study ratings inflation due to the issuer shopping and other behaviors. Opp et al. (2013) develops a model to discuss the variation in credit rating standards over time and across asset classes. Another perspective is how a CRA's reputational concern affects its ratings quality. Mathis et al. (2009) discussed the issue whether reputation concerns are sufficient to discipline CRAs. Fulghieri et al. (2011) analyzed the incentives of CRAs to issue unsolicited ratings and the effects of this practice on CRA's rating strategies. Mariano (2012) examined the incentives of CRAs to reveal the information that they obtained about their client firms and explored the way competition and market power in the ratings industry affects such incentives. Bar-Isaac and Shapiro (2013) considered a model of ratings incorporating endogenous reputation and a time-varying market environment and concluded

that rating quality is countercyclical. Other perspectives of studying credit rating quality include the interaction between the business cycle and firms' incentives (Povel et al., 2007), feedback effects of credit ratings (Manso, 2013), and the role that regulation plays in enhancing the importance and market power of CRAs (White, 2010).

In contrast to the above literature that focuses on extracting credit quality information from the rating records, our main purpose is to extract information on sharp variations of market conditions from firms' rating transitions and explore its connection with macroeconomic fundamentals. We believe that firms' rating transitions happen with certain rates in a stable economic environment, when the environment undergoes abrupt changes, such changes can be reflected via the sharp changes of patterns (or rates) of firms' rating transitions. Though such information on market structural change is very weak in individual firm's rating records, it can be amplified and extracted after aggregating all firms' rating records longitudinally. The intuition behind the paper is similar to the law of motion in a physical world. Consider a bus without windows that moves with constant velocity, passengers in the bus would hardly notice the movement of the bus, even they moves inside the bus, however, when the bus is suddenly accelerated or decelerated so that the velocity of the bus has a abrupt change, the passengers in the bus must show some movement as a result of the momentum change. In our study, although it is indeed a concern whether credit ratings provided by CRAs measure firms' credit quality truthfully or accurately, such concern is a little deviated from our concentration on extracting market structural break information. Actually, if firms' ratings provided by a CRA are systematically biased, the transition records may still contain enough information on market structural breaks.

Our model is also related to the credit risk literature that characterizes firms' credit rating dynamics. In credit risk management, it is convenient to assume that the firms' credit quality follows a time homogeneous Markov chain described by a rating transition matrix. The rating transition matrix can be estimated in a discrete-time setting, however, due to the well known advantage of using the continuous time Markov approach over the discrete one (Lando and Skødeberg, 2002; Christensen et al., 2004), a continuous time homogeneous Markov model is usually assumed and a rating transition generator matrix is estimated for the rating process (Küchler and Sørensen, 1997, Chapter 1). Recently, the assumptions of time homogeneity and Markovian behavior of the rating process have been challenged by the studies of

various non-Markov behaviors in credit ratings such as industry heterogeneity and ratings drift (Altman, 1998; Altman and Rijken, 2004; Liao et al., 2009). Furthermore, the stability of firms' ratings over credit cycles has also been discussed; see Cantor and Mann (2003a), Cantor and Mann (2003b), Frydman and Schuermann (2008), Bruche and González-Aguado (2010) and reference therein. Among these studies, Weissbach and Walter (2010) assumed a time-continuous discrete-state Markov process for rating transitions and derived a likelihood ratio test to examine the time-stationarity of rating transitions. They found that rating data reveals highly significant instationarity.

The closest paper in the literature of characterizing firms' rating transition dynamics is Xing et al. (2012), who modeled instationarity of ratings directly and developed a stochastic structural break model to characterize the rating transition dynamics in the presence of market structural break. In contrast to Xing et al. (2012) who assumes the instationarity or structural breaks happen with a fixed probability, we characterize the probability of structural break as the effect of variation of macroeconomic fundamentals. This allows us to investigate the causal effect of variation of macroeconomic fundamentals on structural breaks in credit rating dynamics or market general movements.

The current paper is also related to the literature that focuses on the dependence of firms' rating transitions and default on macroeconomic factors. Nickell et al. (2000) and Bangia et al. (2002) found that rating transitions probabilities varies over different economic regimes. Carling et al. (2007) used a duration model to incorporate the impact of macroeconomic conditions on credit defaults. Duffie et al. (2007) and Duffie et al. (2009) developed duration-based models that characterize the term structure of credit risk as a function of a small number of observed and/or unobserved firm-specific and macroeconomic factors, and found that average times-to-default decreases when economic activity decreases. Koopman et al. (2009) considered an intensity-based framework and studied the relation between macroeconomic fundamentals and cycles in rating activity. Figlewski et al. (2012) explored the impact of general economic conditions on rating changes by fitting reduced-form Cox intensity models with a broad range of macroeconomic and firm-specific variables. Comparing to these work, our study concentrates on the dependence of abrupt changes in firms' rating transitions on variations of macroeconomic factors. Intuitively, the time of abrupt changes in firms' rating transitions are same as the one when market has sudden movement.

Therefore, in order to explain the source of such sudden movement in the market, it is more appropriate to use the variations of economic fundamentals as predatory variables, instead of the fundamental themselves. This allows us to perform a logistic-type analysis in the inference procedure and provides a much better prediction of structural breaks using historical data. ¹

3. A predictive model for structural breaks

We assume that a rating transition process of an obligor follows a K -state non-homogeneous continuous time Markov process. This process is further characterized by a transition probability matrix $P(s, t)$ over the period (s, t) , in which the ij 'th element of $P(s, t)$ represents the probability that an obligor starting in state i at time s is in state j at time t . Since the purpose of our model is to incorporate abrupt changes into credit rating dynamics, we assume from now on that the non-homogeneous continuous time Markov process can be decomposed as piecewise homogeneous continuous time Markov processes with unobserved structural breaks.

3.1. Market structural breaks

We first assume that the structural breaks in firms' rating transition dynamics follows a Poisson process $\{N(t); t \geq 0\}$ with arrival rate $\eta(t)$, where $N(t)$ counts the number of structural breaks up to time t . That is, the number of structural breaks arrived in the period (s, t) follows a Poisson distribution with mean $\int_s^t \eta(u) du$, and assuming the k th structural break happens at time τ_k ($k \geq 1, \tau_0 = 0$), the interarrival time $\tau_k - \tau_{k-1}$ given τ_{k-1} follows an exponential distribution with rate $\int_s^t \eta(u) du$. To connect the rate function $\eta(t)$ of structural breaks with macroeconomic variables, we denote \mathcal{G}_t the information set consisting of all macroeconomic variables up to time t . Note that in the real world, econometricians can only observe a subset, rather than the whole set, of \mathcal{G}_t . We then assume that $X_1(t), \dots, X_q(t)$ are q observed economic variables at time t . Note that $X_i(t)$ ($i = 1, \dots, q$) are exogenous variables prior to or at time t . Let $\mathcal{F}_t = \sigma\{X(s) | 0 \leq s \leq t\}$ be the information set generated by $\{X(s)\}_{0 \leq s \leq t}$. Obviously, we have $\mathcal{F}_t \subset \mathcal{G}_t$. Since usually $\mathcal{F}_t \neq \mathcal{G}_t$, we assume that the dependence of future structural

¹We also use the model to study the dependence of structural breaks on economic fundamentals, and find such dependence is much weaker and actually negligible, comparing to the result of using variations of fundamentals.

breaks on current macroeconomic variables is through the observed set \mathcal{F}_{t-} , i.e.,

$$E\{dN(t)|\mathcal{G}_{t-}\} = E\{dN(t)|\mathcal{F}_{t-}\}, \quad (1)$$

in which $dN(t)$ is the increment $N\{(t+dt)-\} - N(t-)$ over the small interval $[t, t+dt)$. We further assume a multiplicative intensity model for the intensity of structural breaks (Andersen and Gill, 1982),

$$E\{dN(t)|\mathcal{F}_{t-}\} = \eta(t)dt, \quad (2)$$

and in which the function $\eta(t)$ takes the form

$$\eta(t) = \eta_0 \exp \left\{ \sum_{i=1}^q \theta_i X_i(t) \right\}, \quad (3)$$

where η_0 and $\theta_1, \dots, \theta_q$ are unknown parameters.

We also assume that there are n rating transitions observed during the sample period $(0, T)$. Since rating records refer to the rating at particular times, we consider an evenly spaced partition for the period $(0, T)$, $0 = t_0 < t_1 < \dots < t_L = T$, and assume that structural breaks can only happen at the times t_1, \dots, t_L . We define the variables $I_l = 1$ and $I_l = N(t_l-) - N(t_{l-1}-)$ for $l = 2, \dots, L$ to indicate if there is a structural break at t_{l-1} , then $\{I_l\}$ is a sequence of independent and identically distributed Bernoulli random variables with success probability $p_l = 1 - \exp(-\int_{t_{l-1}}^{t_l} \eta(t)dt)$. We note that

$$\begin{aligned} \frac{p_l}{1-p_l} &= \exp \left\{ \int_{t_{l-1}}^{t_l} \eta(t)dt \right\} - 1 \approx \int_{t_{l-1}}^{t_l} \exp \left\{ \sum_{i=1}^q \theta_i X_i(s) \right\} \eta_0 d(s) \\ &= (t_l - t_{l-1}) \eta_0 \exp \left\{ \sum_{i=1}^q \theta_i X_i(\xi) \right\} \\ &= \exp \left\{ \theta_0 + \sum_{i=1}^q \theta_i X_i(\xi) \right\} \end{aligned}$$

in which the approximation is based on the first-order Taylor expansion, and the second equality is based on the first mean value theorem for integration, $\xi \in [t_{l-1}, t_l]$ and $\theta_0 = \log(t_l - t_{l-1}) + \log \eta_0$. When the partition is fine enough so that variable $X_i(t)$ doesn't change much in the period (t_{l-1}, t_l) , the variable $X_i(\xi)$ in the last equality can be approximated by $X_i(t_{l-1})$. Note

that $X_i(\xi)$ can also be approximated by $X_i(t_l)$, but this does not provide us the predictability at time t_{l-1} . Then we obtain a logistic regression model for structural breaks,

$$\log \frac{p_l}{1 - p_l} = X'_{l-1} \theta, \quad (4)$$

where $X_{l-1} = (1, X_1(t_{l-1}), \dots, X_q(t_{l-1}))'$ and $\theta = (\theta_0, \theta_1, \dots, \theta_p)' \in \mathbb{R}^{q+1}$. Note that X_{l-1} includes exogenous variables prior to and at time t_{l-1} .

3.2. Credit rating transitions with structural breaks

Before we specify a piecewise homogeneous continuous-time Markov process with structural breaks for firms' rating transitions, we shall note that a continuous time homogeneous Markov process is usually specified by the generator matrix of its probability transition matrix. For example, if a rating migration process of a firm during the period $(0, t)$ is a continuous time homogeneous Markov chain with transition matrix $P(0, t)$, the matrix $P(0, t)$ can be represented through its generator matrix Λ , that is, for any time $t > 0$,

$$P(0, t) = \exp \left(\int_0^t \Lambda ds \right) = \sum_{k=0}^{\infty} \frac{\Lambda^k t^k}{k!},$$

in which $\Lambda = (\lambda^{(i,j)})$ satisfies $\lambda^{(i,i)} = -\sum_{j \neq i} \lambda^{(i,j)}$ for $1 \leq i \leq K$, and $\lambda^{(i,j)} \geq 0$ for $1 \leq i \neq j \leq K$.

When we consider a piecewise homogeneous continuous-time Markov process in which the break points represent the structural break times in the rating transition dynamics, we note that the generator matrix $\Lambda(t)$ becomes a piecewise constant. In particular, if there is no structural break during the period (s, t) so that the generator $\Lambda(u)$ become a constant for $u \in (s, t)$, the Markov process associated with the transition matrix $P(s, t)$ is homogeneous and

$$P(s, t) = \exp \left(\int_s^t \Lambda(u) du \right) = \exp \left[(t - s) \Lambda(t-) \right].$$

If there are M structural breaks during the period (s, t) , the generator matrix $\Lambda(t)$ become piecewise constant. Assume these M breaks happen at $\tilde{\tau}_1, \dots, \tilde{\tau}_M$ during (s, t) , i.e., $s < \tilde{\tau}_1 < \dots < \tilde{\tau}_M < t$, the transition matrix

during (s, t) can be expressed as

$$\begin{aligned} P(s, t) &= \prod_{k=1}^{M+1} P(\tilde{\tau}_{k-1}, \tilde{\tau}_k) = \prod_{k=1}^{M+1} \exp\left(\int_{\tilde{\tau}_{k-1}}^{\tilde{\tau}_k} \Lambda(u) du\right) \\ &= \prod_{k=1}^{M+1} \exp\left[(\tilde{\tau}_k - \tilde{\tau}_{k-1})\Lambda(\tilde{\tau}_{k-})\right], \end{aligned}$$

in which $\tilde{\tau}_0 = s, \tilde{\tau}_{M+1} = t$. Note that the exponent in the above equation usually can not be simplified to $P(s, t) = \exp\left[\int_s^t \Lambda(u) du\right]$, because the components $P(s, \tilde{\tau}_1), \dots, P(\tilde{\tau}_M, t)$ may not commute (Lando, 2004, p.160).

Since we have specified a Poisson process $N(t)$ for structural breaks, the generator matrix at time t can be characterized by $\Lambda(t) = Q_{N(t)}$. We further assume that generator matrices Q_1, Q_2, \dots are independent and identically distributed random matrices such that the off-diagonal elements $\lambda^{(i,j)}$ follow independently a Gamma(α_{ij}, β_i) prior distribution with the density function

$$g(\lambda^{(i,j)}) = \frac{\beta_i^{\alpha_{ij}}}{\Gamma(\alpha_{ij})} [\lambda^{(i,j)}]^{\alpha_{ij}-1} \exp(-\lambda^{(i,j)}\beta_i), \quad (i, j) \in \mathcal{K}, \quad (5)$$

in which $\mathcal{K} = \{(i, j) | i \neq j, 1 \leq i \leq K-1, 1 \leq j \leq K\}$. Note that the elements of the last row in the generator matrix, representing the rating migrations from the default category to others, are usually assumed to be zero, so we don't need to model the dynamics of those elements.

Finally, we assume that firms' rating migration from state i to state j at the period (s, t) are conditional independent given the generator matrix at the period (s, t) .

3.3. Macroeconomic effect on structural breaks

To estimate the macroeconomic effect on structural breaks or the logistic regression parameter $\theta \in \mathbb{R}^q$ in (4), we also need to estimate prior parameters α_{ij} and β_j ($i, j \in \mathcal{K}$) for generator matrices. As there are too many parameters in the model, we consider an expectation-maximization approach that treats the generator matrices $\{\Lambda(t)\}$ as the missing data. Denote, for the period (s, t) , $K_{s,t}^{(i,j)}$ the number of transitions from category i to category j , $S_{s,t}^{(i)}$ the amount of time that firms spend in category i , $\lambda_{s,t}^{(i,j)}$ the ij th entry in the generator $\Lambda(t)$, and $\mathcal{Y}_{s,t}$ the observed rating transitions. The log likelihood

$l_c(\Phi)$ of the complete data $\{K_{t_{l-1},t_l}^{(i,j)}, S_{t_{l-1},t_l}^{(i)}, \lambda_{t_{l-1},t_l}^{(i,j)}; (i,j) \in \mathcal{K}, 1 \leq l \leq L\}$ can be expressed as

$$\ell_c = \ell_1 + \ell_2(\{\alpha_{ij}, \beta_i\}) + \ell_3(\theta), \quad (6)$$

where

$$\begin{aligned} \ell_1 &= \sum_{l=1}^L \sum_{i=1}^K \left\{ \sum_{j \neq i} K_{t_{l-1},t_l}^{(i,j)} \log \lambda_{t_{l-1},t_l}^{(i,j)} - \left(\sum_{j \neq i} \lambda_{t_{l-1},t_l}^{(i,j)} + 1 - K \right) S_{t_{l-1},t_l}^{(i)} \right\}, \\ \ell_2(\{\alpha_{ij}, \beta_i\}) &= \sum_{l=1}^L \sum_{i=1}^K \left\{ \sum_{j \neq i} (\alpha_{ij} - 1) \log \lambda_{t_{l-1},t_l}^{(i,j)} - \left(\sum_{j \neq i} \lambda_{t_{l-1},t_l}^{(i,j)} \right) \beta_i \right. \\ &\quad \left. + \sum_{j \neq i} (\alpha_{ij} \log \beta_i - \log \Gamma(\alpha_{ij})) \right\} \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}}, \end{aligned}$$

and

$$\begin{aligned} \ell_3(\theta) &= \sum_{l=1}^L \left\{ [\log(1 - p_l)] \mathbf{1}_{\{\Lambda(t_l) = \Lambda(t_{l-1})\}} + (\log p_l) \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}} \right\} \\ &= \sum_{l=1}^L \left\{ (X'_{l-1} \theta) \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}} - \log [1 + \exp(X'_{l-1} \theta)] \right\}. \end{aligned}$$

In the expectation step of an EM algorithm, we need to take expectation for ℓ_c given the observed rating transitions $\mathcal{Y}_{0,T}$ and macroeconomic history $\mathcal{F}_{t_{L-1}}$ so that the “missing data” $\{\lambda_{t_{l-1},t_l}^{(i,j)}\}$ can be integrated out. This requires to replace $\mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}}$, $\log \lambda_{t_{l-1},t_l}^{(i,j)}$, and $\lambda_{t_{l-1},t_l}^{(i,j)}$ by their expectations respectively. In particular, $\mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}}$ are replaced by the conditional probabilities of structural breaks given $\mathcal{Y}_{0,T}$ and \mathcal{F}_T , that is,

$$E(\ell_3(\theta) | \mathcal{Y}_{0,T}, \mathcal{F}_T) = \sum_{l=1}^L \left\{ (X'_{l-1} \theta) y_l - \log [1 + \exp(X'_{l-1} \theta)] \right\}, \quad (8)$$

in which

$$y_l = P(\Lambda(t_l) \neq \Lambda(t_{l-1}) | \mathcal{Y}_{0,T}, \mathcal{F}_{t_{L-1}}) = P(I_l = 1 | \mathcal{Y}_{0,T}, \mathcal{F}_{t_{L-1}}) \quad (9)$$

and can be computed explicitly (see Section 3.4). In the maximization step, we need to maximize the expected ℓ_c over the parameter space of θ , α_{ij} and

β_j . Since parameters θ and $\{\alpha_{ij}, \beta_j\}$ are separated in different functions, we can estimate θ by maximizing function (8) over the parameter space of θ . Note that q represents the number of macroeconomic factors included in the model. If q is small, we can use iteratively reweighted least squares procedure to maximize (8) and obtain an maximum likelihood estimate for θ ; see Lai and Xing (2008, Section 4.1).

When q is much larger than L or there are a large number of macroeconomic factors in the model, we run into the curse of dimensionality problem, and hence it is not realistic to maximize (8) over the whole parameter space of θ . However, under such circumstance, our focus is not to estimate the effect of each individual macroeconomic factor on structural breaks any more, because most of such effects caused by individual factors must be negligible, or numerically, most elements in vector θ would be zero. Furthermore, we are always interested in identifying a small number of important macroeconomic factors that generate significant effects on structural breaks from the high dimensional space of economic variables. This requires us to select a small subset from a large set of macroeconomic variables (i.e., nonzero θ_i), and in the mean while, give accurate estimates of those effects (i.e., the numerical values of nonzero θ_i 's).

To solve this sparsity problem, we borrow the idea of regularization for sparse high-dimensional problems from statistics and econometrics literature; for reviews on this topic, see Stock and Watson (2006), Bai and Ng (2008), and Fan et al. (2011). The regularization here means a process of introducing additional constraint on parameters in order to solve the ill-posed problem that the number of parameters is much larger than the sample size (i.e., the number of period L in our data). Since it is difficult to maximize the target function (8) directly, we consider a penalized target function

$$\tilde{\ell}_3(\theta) = -\frac{1}{L}E(\ell_3(\theta)|\mathcal{Y}_{0,T}, \mathcal{F}_{t_{L-1}}) + \gamma\Phi(\theta), \quad (10)$$

in which the penalty function $\Phi(\theta)$ is a weighted average of L_1 and squared L_2 norms of parameters

$$\Phi(\theta) = \phi \sum_{j=1}^q |\theta_j| + (1 - \phi) \sum_{j=1}^q \theta_j^2; \quad (11)$$

see Zou and Hastie (2005). The penalty $\Phi(\theta)$ is a combination of the L_1 penalty and squared L_2 penalty on degree-scaled differences of coefficients

between linked covariates. It includes both sparsity and smoothness with respect to the correlated structure of the regression coefficients of macroeconomic factors, and hence allows us to control the amount of regularization for sparsity and smoothness at the same time through the tuning parameters γ and ϕ , respectively. In particular, when $\phi = 0$, it is equivalent to expose some prior distribution on the high-dimensional vector θ , and when $\phi = 1$, the penalty reduces to the L_1 norm of θ and can shrink the estimates of most θ_i 's to 0; see Tibshirani (1996). As our purpose is to shrink the effect of unimportant factors to 0 so the dimension of the problem can be reduced, ϕ should be chosen to be close or equal to 1. Given the values of ϕ and γ , the minimization of (10) can be solved by the cyclical coordinate decent method proposed by Friedman et al. (2010).

To complete the inference procedure on θ , we also need to estimate $\{\alpha_{ij}, \beta_i\}$. This requires us to maximize the expected values of ℓ_2 , i.e., setting the first derivatives of ℓ_2 with respect to α_{ij} 's and β_i 's to 0 and solving the equation. Such procedure provides us explicit formulas for the estimates of α_{ij} and β_i ; see Appendix A for technical details.

3.4. Probabilities of structural breaks

As a structural break is considered as the sharp change of economic and financial market environment, the probabilities of structural breaks show how sharp the change of economic conditions is. If the variations of economic and market conditions are gradual and very smooth, we expect this probability to be small; otherwise, it should be significantly different from 0. Our model provides three types of descriptions on probabilities of structural breaks. The first is based on the estimated regression coefficient from the previous section. Specifically, provided the estimated coefficients $\hat{\theta}$, we obtain the following partial estimate of p_l via (4)

$$\tilde{p}_l = \frac{\exp(X'_{l-1}\hat{\theta})}{1 + \exp(X'_{l-1}\hat{\theta})}, \quad l = 1, \dots, L. \quad (12)$$

We call this a partial estimate because, given the estimated coefficients, it makes only use of the macroeconomic information via (4) and ignores the structural break information hidden in firms' rating transition records.

The second estimate of probabilities of structural breaks is the posterior probabilities of structural breaks given firms' rating transition records $\mathcal{Y}_{0,T}$

and the history of macroeconomic factors $\mathcal{F}_{t_{L-1}}$, which is the y_l given by (9). As y_l can not be calculated directly, we consider the event

$$\begin{aligned} C_{ml} &= \{I_m = 1, I_{m+1} = \dots = I_l = 0, I_{l+1} = 1\} \\ &= \{\Lambda(t_{m-1}-) \neq \Lambda(t_m-) = \dots = \Lambda(t_l-) \neq \Lambda(t_{l+1}-)\}, \end{aligned}$$

where $m < l$. C_{ml} indicates that two structural breaks happen at times t_{m-1} and t_l , respectively, but there are no any changes during (t_{m-1}, t_l) . Then the event of having a structural break at time t_l can be written as the union of C_{ml} 's, that is,

$$\{I_{l+1} = 1\} = \cup_{m=1}^l C_{ml}.$$

Given the fact that $\{C_{ml}; m = 1, \dots, l\}$ are disjoint events, the posterior probabilities of structural breaks at t_l can be expressed as

$$\hat{p}_l = y_l = \sum_{m=1}^l P(C_{ml} | \mathcal{Y}_{(0,T)}, \mathcal{F}_{t_{L-1}}), \quad l = 1, \dots, L. \quad (13)$$

We then note that $P(C_{ml} | \mathcal{Y}_{(0,T)}, \mathcal{F}_{t_{L-1}})$ in (13) can be computed explicitly; see its derivation in Appendix B for details. Appendix B also provides explicit formulas for the posterior mean of generator matrix and transition probability matrix in the period (t_{l-1}, t_l) , given firms' rating transition records $\mathcal{Y}_{0,T}$ and the history of macroeconomic variables.

The probabilities (13) can be used to conclude whether a structural break occurs at time t_l . In fact, if there does exist a structural break at t_l and the length $t_l - t_{l-1}$ of each period is fixed, one can show that the probability \hat{p}_l converges to 1 as the number of observed transitions at t_l goes to infinity. The key of the proof for this property is to consider likelihood ratio tests of structural break at each time t_l , then \hat{p}_l becomes a function of different likelihood ratios which are dominated by the one corresponds to some C_{ml} . We skip the proof in the paper as it is tedious but technically not difficult.

The third estimate of structural break probability is prospective instead of retrospective. As the discussion above provides us an estimate of θ based on rating transition records $\mathcal{Y}_{0,T}$ and macroeconomic covariates $\mathcal{F}_{t_{L-1}}$, we could make use of the estimated coefficients $\hat{\theta}$ and observed macroeconomic factors at $t_L = T$ to construct a predictor for the structural break probability in the next period (t_L, t_{L+1}) (or $(T, (1 + \frac{1}{L})T)$)

$$\hat{p}_{L+1} = \frac{\exp(X'_L \hat{\theta})}{1 + \exp(X'_L \hat{\theta})}. \quad (14)$$

Since the structural break probability measures essentially the sharpness of the change of economic and market conditions, \widehat{p}_{L+1} in (14) provides us an objective and forward-looking description on how fast the economic and market condition is going to change in the next period, given the most current information of firms' rating transitions and economic conditions.

4. Data analysis

4.1. Data description

The data in our analysis consists of Standard & Poor's monthly credit ratings of firms and 18 time series on the U.S. economy over 28 years starting January 1985 and ending March 2013. They are obtained from COMPUS-TAT and downloaded from the Federal Reserve Bank of St. Louis, respectively.²

In the monthly credit rating dataset, there are a total of 2,584,350 rating records and 23,464 firms whose ratings were recorded at the end of each month. The data contain ten rating categories, *AAA*, *AA*, *A*, *BBB*, *BB*, *B*, *CCC*, *CC*, *C* and *D* (default), and 25 rating subcategories. Subcategories are obtained by possibly adding “+” or “-” to the letter grade of categories, which shows relative standing within the major rating categories. We then clean the rating data as follows. We first group *C* and *CC* into *CCC* as the records in the former two rating categories are relatively few, and then remove rating records of two invalid ratings “N.M.” and “Suspended”. After the above data-cleaning steps, we extract the initial rating and transition information from the rating records. Then we obtain 5,538 initial rating and 7,313 transition records covering 5,539 firms, and eight rating categories, *AAA*, *AA*, *A*, *BBB*, *BB*, *B*, *CCC*, and *D*. Note that the first rating transition of a firm happens after it migrates away from its initial rating, and in 1985, there are 1,286 initial ratings and only one rating transition. Hence our analysis will be based on the data starting from January 1986.

Since structural breaks in rating transition dynamics reflect the sharp changes of general economic and market environments, we consider macroe-

²The Standard & Poor rating record starts from 1981. However, the data we obtained from COMPUSTAT contain very few ratings records during 1981 and 1985. Furthermore, some of macroeconomic time series downloaded from the Federal Reserve Bank have no records prior to 1985.

economic variables that represent directions of general economic and financial market conditions. Specifically, we consider the following macroeconomic variables in the study.

- (1) *Real GDP growth.* The U.S. real GDP in total is only available quarterly, so we constructed monthly series of real GDP by linear interpolation. We then compute the growth rate of monthly real GDP for the model.
- (2) *Growth rate of industrial production.* As real GDP consists of economic activities from government, corporate and non-corporate business, and other sections that may not be directly related to credit market, we include the growth rate of industrial production to strengthen the effect of activities from the corporate sector.
- (3) *Change rates of unemployment rate and mean duration.* The unemployment rate and mean duration indicates the overall health of the economy. High unemployment rate and long unemployment mean duration should decrease (or increase) the hazard of upgrade (or downgrade) transitions.
- (4) *Inflation measured through CPI, PPI and oil prices.* Inflation is widely considered as an important economic variable and commonly measured by percentage changes of the seasonally adjusted Consumer Price Index (CPI). As this may not reflect the inflation faced by domestic producer, we also compute the monthly percentage changes in the seasonally adjusted Producer Price Index (PPI) and West Texas intermediate spot oil prices. Hence we have three inflation series that measure different aspects of prices in the economy.
- (5) *Growth rates of the government and consumer debts.* We consider the total public debt of the federal government and the total outstanding credit of consumer owned and securitized. The government debt is quarterly, so we turn it into monthly by the method used for the real GDP. The government and consumer debts are widely believed to be related to the most recent financial crisis in 2007-2009.
- (6) *Change rates of consumer sentiment.* Research has shown consumer sentiment can have a significant impact on consumers' and investor's behavior (Baker and Wurgler, 2006; Lemmon and Portniaguina, 2006). We use data from the University of Michigan Survey of Consumer Sentiment as the measure for consumer sentiment. We then use the change rate in consumer sentiment to indicate how economic agents' subjective

beliefs and expectations about the economy vary over time.

- (7) *Chicago Fed National Activity Index (CFNAI)*. This composite series captures the overall economic conditions and summarizes the behavior of 85 economic series in the categories of production and income, employment, unemployment and hours, personal consumption and housing, and sales, orders and inventories. The CFNAI is published monthly in the form of a 3-month moving average by the Federal Reserve Bank of Chicago.
- (8) *Short- and long-term interest rates*. The interest rate series consist of the monthly rates of 3-month Treasury Bill and 10-year Treasury Constant Maturity. High interest rates may increase difficulty in raising funds to make debt service payments and cause general tightness in the economy, and as a simple proxy of the interest rate term structure, the difference of the long- and short-term interest rates reflects the forward-looking expectation of the tightness of the money market. Furthermore, the 3-month interest rates can be considered as a proxy of the instruments of U.S. money policy.
- (9) *Stock market performance measured by S&P 500 returns and volatility*. These two series show the stock market performance and give an indication of the general healthiness of the stock market. In particular, volatility shows the extent of stock market instability, and is estimated month by month as the annualized standard deviation of S&P500 daily returns within the month.
- (10) *Growth rate of money supply*. This is measured by the growth rate of the M2 series. As one of the monetary policy instruments, it can be affected by private demand for credit and liquidity.
- (11) *Growth rate of all U.S. commercial banks' net assets*. Assets and liabilities of commercial banks show the borrowing and lending activities in the economy and are important variables on the general healthiness of an economy.
- (12) *Corporate bond Aaa and Baa credit spreads*. Research has shown that credit spreads have significant predictive value for real output (Stock and Watson, 1989; Friedman and Kuttner, 1992). We measure the spread here as the difference between Moody's Aaa and Baa corporate bond yields and 10-year Treasury bonds, respectively, so that the effects of firms with different credit quality on structural breaks can be distinguished.

We note that the above 20 variables can be grouped into two categories,

the first 12 variables (constructed from the first seven items) represent the improving or worsening of general economic conditions, and the rest provides a description of current situations of financial market. We do not want to claim that all important macroeconomic factors on structural breaks have been included in our analysis, as further research will undoubtedly discover more significant factors on structural breaks. Our main purpose here is to explore the predictability of structural breaks using firms' rating records and macroeconomic factors. As structural breaks essentially describe how fast the change of general economic and financial market conditions is, we further compute the lag-1 difference of these 20 series as explanatory variables in the model. Table B.2 defines these 20 differenced variables as Y_1, \dots, Y_{20} , respectively, and adds a vector of 1's as covariate Y_0 , corresponding to the intercept.

The 20 macroeconomic factors constructed above are contemporaneous variables. Since we are interested in their predictive effects on structural breaks, one should use lagged values of these variables. Intuitively, these macro factors are fundamentals of overall healthiness of economic and financial markets conditions, the effects of these variables on structural breaks can not be instantaneous. Actually, one should expect some time for these factors to take effect. However, it is not clear how many lags one should go back and include for these factors. Hence we consider two situations here, one is to impose an exponentially weighted lag structure on the factors (Sections 4.2 and 4.3), and the other is to perform select important variables from these factors with up to 20 lags (Section 4.4).

4.2. Effects of aggregated macroeconomic factors

As $\{Y_{i,t}; t = 1, \dots, L, i = 1, \dots, 20\}$ are the monthly differenced series constructed in the previous section, we construct the following aggregated macroeconomic factors using the lagged data,

$$X_{i,t} = \frac{\sum_{k=1}^H \delta^{k-1} Y_{i,t-k}}{\sum_{k=1}^H \delta^{k-1}},$$

where δ is the decay factor and H is the length of the lag window. As there are no rules to choose parameters H and δ , we explored decay factors $\delta = 0.8, 0.9, 1.0$ and lag windows of $H = 12, 24$ months. The result doesn't show any sensitivity to these choices, so we choose to report the findings

based on $\delta = 0.9$ and $H = 18$. Table B.1 shows the correlations of these aggregated macroeconomic factors. Note that most correlations are small and moderate. One exception is the high correlation (0.823) between the differenced CFNAI and the growth rate difference of industrial production, due to the fact that the CFNAI summarizes of production and other economic activities. Moderate correlations include those among interest rates and bond credit spreads (-0.555, -0.665, -0.503 and -0.521), the changes of the 3-month T-Bill and unemployment rates (-0.618), the changes of Moody's Baa corporate bond yield spread and the CFNAI (0.581), the changes of unemployment rate and mean duration (0.529), and the changes of unemployment rate and growth rate difference of the total outstanding consumer credit (-0.505).

Insert Table B.1 about here.

We then begin our analysis by estimating the developed model at the level of $K = 8$ rating categories from January 1986 to March 2013. In particular, we denote the beginning of 1986 as the time 0 and the end of March 2013 as the time T . Since the rating records are monthly, we partition the sample period from January 1986 to March 2013 to $L = 327$ intervals and each interval corresponds to a calendar month. Then we use the algorithm in Section 3.3 to estimate the parameters $\{\beta_i, \alpha_{ij}\}$, and θ in the model. In particular, the estimated $\{\beta_i, \alpha_{ij}\}$ are given by

$$(\widehat{\beta}_i) = (853.00 \quad 917.20 \quad 1163.83 \quad 1087.56 \quad 777.07 \quad 660.73 \quad 378.82 \quad \cdot),$$

and

$$(\widehat{\alpha}_{ij}) = \begin{pmatrix} \cdot & 0.932 & 0.932 & 0.932 & 0.932 & 0.932 & 0.932 & 0.932 \\ 0.936 & \cdot & 0.931 & 0.936 & 0.936 & 0.936 & 0.936 & 0.936 \\ 0.937 & 0.937 & \cdot & 0.937 & 0.937 & 0.937 & 0.937 & 0.937 \\ 0.936 & 0.936 & 0.936 & \cdot & 0.936 & 0.932 & 0.936 & 0.936 \\ 0.934 & 0.934 & 0.934 & 0.934 & \cdot & 0.9341 & 0.934 & 0.934 \\ 0.932 & 0.932 & 0.932 & 0.932 & 0.932 & \cdot & 0.931 & 0.932 \\ 0.928 & 0.928 & 0.928 & 0.928 & 0.928 & 0.927 & \cdot & 0.907 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

Table B.2 shows the estimated regression coefficients and their standard errors. We didn't report the p-values in the table as they all are very small (less than 0.1%), suggesting all the factors are significant. Among the impact

of variables X_1, \dots, X_{20} on the occurrence of structural breaks, we notice that the growth rate changes of the U.S. real GDP, the public debt of the U.S. Federal government and the industrial production have the most significant effects on structural breaks. Specifically, the coefficient for the growth rate changes of the U.S. federal government debt is 446.28 with standard error 4.50, suggesting that if the growth rate of the government debt increases by one percent, the odds ratio of structural break $p_l/(1 - p_l)$ at month l is increased by a factor of $e^{4.46} \approx 86.49$. The coefficients of growth rate changes of the US real GDP and industrial production have similar absolute values (453.62 and 400.27) but opposite signs, indicating that the growth rate changes of non-industrial production have a positive impact on the occurrence of structural breaks. The coefficients of monthly changes of unemployment rate, the monthly changes of CFNAI, and the monthly growth rate changes of consumer debt (given by 153.10, 151.79, and 153.09, respectively) are at the same level, and hence have similar effects in terms of increasing the odds ratio of structural breaks. Comparatively, the effect of the change of unemployment duration change (-33.94) is opposite to and less significant than that of unemployment rate change. The coefficients of changes of Aaa and Baa corporate bond credit spreads (given by 103.37 and -189.71, respectively) have different signs, meaning that they have opposite effect on the occurrence of structural breaks, but the effect of Baa spread changes is a little more significant than that of Aaa spread changes.

Insert Table B.2 about here.

We find that the effects of the 10-year Treasury rate changes and the growth rate change of M2 money stock are similar (with coefficients -73.88 and -89.97, respectively), while efficient of the 3-month T-Bill rate change is relatively small (-7.73), suggesting that the variations of term structure of interest rates and money supply, as tools of monetary policy instruments, have significant impact on structural breaks in the market. We also notice that the changes of inflation rates measured by CPI, PPI and oil prices have very different effects on structural breaks (the coefficients are 99.21, -56.61, and 1.80). In particular, the effect of variations of oil price have the smallest effect, although it seems significant statistically. The coefficients of growth rate changes of consumer sentiment and commercial bank loans are similar (25.78 and 22.58), and hence have similar effects on structural breaks. Compare to these two factors, the growth rate change of bank's net

asset has the same magnitude but opposite sign of effects (-25.31). Finally, we notice that the coefficients for variations of S&P500 returns and volatilities are only 1.18 and 8.10, respectively, indicating the variation of stock market performance have very small impact on structural breaks.

4.3. Retrospective analysis of structural breaks

Provided the regression coefficients and priors of rating transition matrices estimated above, we estimate the structural break probabilities at each month by equations (12) and (13). The top and middle panels of Figure B.1 show the posterior probabilities of structural breaks \hat{p}_l and the partial estimate of structural break probabilities \tilde{p}_l , respectively, for $l =$ January 1986, ..., March 2013. We also mark the three periods of economic recessions announced by NBER as shaded areas in Figure B.1 so that we can see that the periods of structural breaks are not necessarily coincident with beginning or ending periods of economic recessions. We note that the top panel in Figure B.1 shows that most structural break probabilities \hat{p}_l are almost zero except at a few periods. Specifically, there are seven months at which \hat{p}_l are much larger than zero, and they are June 1986 (0.963), Mar 1991 (0.710), May 1992 (0.622), October 1998 (0.765), May 2003 (0.996), September 2008 (0.967), and June 2009 (0.999). To interpret these probabilities, we make an effort to link them to some major economic events during those periods. Specifically, the structural break at July 1986 might relate to the shift of the US economy in 1986 from a rapid recovery to a slower expansion. The break at April 1991 signifies the economic recovery from the 1990-1991 economic recession. May 1992 is the ending month of an unstable period for the rating transition dynamics of U.S. firms, as argued by (Xing et al., 2012, Section 3.2), and it is also the period that the U.S. economy began to expand rapidly after the recovery. October 1998 indicates the beginning of a period of credit turmoil caused by a series of devastating events in the second half year of 1998 such as Russia's default, Brazil's currency crisis and the severe disruption of the LTCM's crisis to the US commercial paper markets. The structural break at May 2003 is the month when the Fed announced the lowest federal funds rate over the past 40 years and also the point that the U.S. economy turns from recovery to expansion. The structural change in September 2008 corresponds to the period that the 2007-2009 financial crisis hits its peak, and the structural break in June 2009 is the month that NBER announced that the most recent economic recession is ended.

Insert Figure B.1 about here.

The structural break probabilities \hat{p}_l are posterior estimates that make use of firms' rating transition records and the history of economic variations, and hence have an advantage over models that only use information at micro or macro level. As shown in the middle panel of Figure B.1, the partial estimates of structural break probabilities \tilde{p}_l are generally less than \hat{p}_l . Specifically, for the seven structural break periods $l =$ June 1986, Mar 1991, May 1992, October 1998, May 2003, September 2008, and June 2009, the partial estimate \tilde{p}_l are 0.226, 0.143, 0.007, 0.565, 0.501, 0.502, and 0.345, respectively. These values are much smaller than the corresponding posterior estimates \hat{p}_l , indicating that firms' rating transition records do contain information on structural breaks. To see the contribution of macroeconomic variables on extracting structural break information, we compare our result with Xing et al. (2012)'s that only used Standard and Poor's monthly credit ratings of firms during January 1985 - September 2009. They obtained five estimated structural breaks with probabilities 0.992 (July 1986), 0.527 (April 1991), 0.868 (January 1999), 0.969 (June 2003), and 0.572 (October 2008). Among these values, the structural break probabilities at July 1986, April 1991, June 2003, and October 2008 are much smaller than those of corresponding periods in our result. Xing et al. (2012) also argued that May 1992 is the ending month of a credit turmoil period that starts at April 1991, but their estimated structural break probability is less than 0.15, which is much smaller than our estimate 0.622. Furthermore, For the highly unstable period of credit market during the second half of 1998, the result in Xing et al. (2012) showed the structural break time during that period is January 1999, while it is October 1998 in our analysis. Note that the latter is more close to the beginning month of the turmoil period of credit market in 1998. These evidence show that incorporating macroeconomic variations does provide more accurate estimates on structural breaks.

4.4. Prospective analysis of structural breaks

As the above analyzes the probabilities of structural breaks retrospectively, we now look into the prospective aspect of our model on structural breaks by sequentially forecasting the structural break probabilities using historical data. Specifically, at the end of each month, we use firms' rating transition and macroeconomic records $(\mathcal{Y}_{(0,t_L)}, \mathcal{F}_{t_{L-1}})$ as the training sample to estimate the model parameters by the method in Section 3.3, and then

compute the one-month ahead prediction of the structural break probability at the next period (t_L, t_{L+1}) by equation (14). We apply the above procedure for $t_L =$ January 1996, February 1996, \dots , March 2013. The bottom panel of Figure B.1 shows these predicted structural break probabilities. Most predicted probabilities are close to zero except the ones at January 1999, October 2001, November 2002, November 2006, February 2008, October 2008, January 2009, and May 2010 are larger than 0.1. As the variations of economic and market conditions are smooth in most periods (that is, the corresponding values of \hat{p}_{L+1} are close to 0), our discussion concentrates on the periods with nonzero structural break probabilities that are listed above.

We first consider the period January 1999 in which the predicted structural break probability is only 0.143. This indicates that, compared to the second half of 1998 during which the market environment is very unstable, the economic and market variations in January 1999 is expected to shift to a relatively calm period. For October 2001, the predicted structural break probability is 0.511, suggesting the economic and market conditions was about to have a big shift. Actually, only after one month, the NBER announced the end of 2001 economic recession. For November 2002, although the predicted probability \hat{p}_L is 0.404, the retrospective estimates of structural break probabilities around the period are very small, we are not clear that this prediction of structural break is simply a false alarm, or as an example of Lucas' critique on macroeconomic policymaking (Lucas, 1976), the potential structural break was eliminated by some economic policy interventions in November 2002. Similar discussions can be applied to November 2006, as the corresponding prediction of structural break has the value of 0.351. Related to the most recent 2007-2008 financial crisis are the next three periods, February 2008, October 2008, January 2009. The NBER announced the start of the crisis in December 2007, which is two months ahead of the period of February 2008 in which the predicted structural break probability is 0.571. Note that the retrospective estimates of structural break probability around February 2008 do not catch up the beginning of the financial crisis at all. In October 2008 which is the period that the financial crisis hits its peak, the prediction $\hat{p}_{L+1} = 0.999$ indicates that the economic environment would have a sharp change. Actually, in late November 2008, the U.S. Federal Reserve started using quantitative easing, as part of the US monetary policy, to alleviate the financial crisis. Our model also predicts the structural break probability at January 2009 is 0.999, indicating the economic environment had a sharp change again. This might correspond to the fact

the US economic growth reached the lowest point since 2008. In May 2010, the predicted structural change probability is 0.642, this can be explained by the fact the U.S. economy had begun to improve after the first round of quantitative easing policy used by the U.S. Fed.

4.5. Selection of macroeconomic factors in a high-dimensional space

The analysis in preceding sections is based on the 20 aggregated covariates that summarize the lag effects of macroeconomic variables, in this section, we consider the problem of selecting variables from a set of macroeconomic factors. We consider two sets of covariates, one is the 20 aggregated covariates $\{X_{i,t}; i = 1, \dots, 20\}$ used in Sections 4.2-4.4, and the other treats economic factors with lags up to 24 months as different variables, that is,

$$\tilde{X}_{24(i-1)+h,t} = Y_{i,t-h}, \quad h = 1, \dots, 24, i = 1, \dots, 20.$$

Note that the second set of covariates contain 480 factors, which is much larger than the number of samples $L = 327$ in terms of the probabilities of structural breaks. For both sets, we apply the penalized inference procedure in Section 3.3 to estimate the regression coefficient θ . Specifically, we estimate θ by maximizing the penalized expected log-likelihood (10) with the penalty function (11). As the procedure involves two tuning parameters, we choose the parameter ϕ in (11) to be 0.9 and 1 so that the effect of unimportant macroeconomic factors can be shrunk to 0.³ We also let γ vary within certain range so that the variations of covariate effects on structural break can be observed, specifically, $\gamma = e^{-1/2}, e^{-1}, \dots, e^{-10}$. Figure B.2 plots the estimated coefficients in the regularized regression with covariates $\{X_{i,t}\}$ and $\{\tilde{X}_{24(i-1)+h,t}\}$ and weight parameter $\phi = 1$ and 0.9, respectively, versus the tuning parameters $\log \gamma = -10, -9, \dots, -1$. Note that all coefficients in the four panels are shrunk to zero when the tuning parameter γ is larger than e^{-3} .

We now focus on the details of Figure B.2. The top two panels show the estimated regression coefficients of $\{X_{i,t}\}$ for $\phi = 1$ and 0.9, respectively. Although the shrinkage for $\phi = 1$ is smoother than that for $\phi = 0.9$, both cases select almost the same subsets of $\{X_{i,t}\}$ for tuning parameter γ . For example, when $\gamma = e^{-10}$, the estimated θ in both cases are similar to the

³For the values of ϕ larger than 0.8, the selected variables are similar, so we only present the results of $\phi = 0.9$ and 1.

result in Section 4.2. When $\gamma = e^{-5}$, both cases select the covariates $X_4, X_5, X_7, X_{13}, X_{16}$ and X_{17} , and shrink the coefficients of other factors to 0. The selected six variables are the variations of 10-year Treasury rates, Moody’s Aaa corporate bond credit spread, change of monthly unemployment rate, growth rate of the US real GDP, growth rate of consumer sentiment, and growth rate of bank’s total net asset. Note that this provides a compact description on an economy consisting of labors (or consumers), firms, financial intermediary, and output, and matches our intuition obtained from the complete analysis in Section 4.2. The bottom panels of Figure B.2 show estimated regression coefficients of $\tilde{X}_{24(i-1)+h,t}$ for $\phi = 1$ and 0.9, respectively. Note that these two studies select not only economic variables but their lagged values as well. When $\gamma = e^{-10}$, all the 400 variables are selected in both studies. When $\gamma = e^{-5}$ and $\phi = 1$, 13 lagged variables are selected. In particular, they are $Y_{6,t-8}, Y_{7,t-4}, Y_{9,t-5}, Y_{12,t-2}, Y_{13,t-4}, Y_{13,t-13}, Y_{14,t-2}, Y_{14,t-18}, Y_{17,t-10}, Y_{17,t-12}, Y_{17,t-23}, Y_{19,t-7}, Y_{19,t-9}$, and indicates which lagged covariates are more important than others and how far the effect of the selected economic variables on structural breaks can be traced back. When $\gamma = e^{-5}$ and $\phi = 0.9$, the result is similar except that more variables are selected as the shrinkage in this case is much slower than that in $\phi = 1$.

Insert Figure B.2 about here.

5. Concluding remarks

We have developed a predictive model for structural breaks in credit rating dynamics. The model extracts and aggregates the information on market structural breaks from individual firm’s rating records, and then connects the market structural break information with the variations of economic and market conditions. The model and the empirical analysis here also show the market structural break phenomena are not exactly “highly improbable” or “black swan” events, instead, they can be predicted up to certain extent. Furthermore, the explanatory information for structural breaks that is hidden in the space of a high-dimensional vector of macroeconomic factors can be sparse, however, such sparsity can be overcome by a state-of-the-art approach developed recently by statisticians and econometricians.

We find through the retrospective analysis that the identifiability of structural breaks can be improved when the variations of economic and market

conditions are incorporated. The prospective analysis of structural breaks shows further that our model is indeed able to predict market structural breaks to certain extent, although there are not easy econometric tools to check the predicted structural breaks are real or not. For the sparsity problem of finding important explanatory variables from a high-dimensional space of economic variables, the variable selection approach is shown to be effective and able to choose an interpretable subset of macroeconomic factors that covers important aspects of an economy.

The model in the paper assumes that firms are homogeneous and the credit ratings provided by CRAs are unbiased and accurate. Note that in the real world, firms are heterogeneous and the credit ratings assigned by CRAs are sometimes biased and inaccurate, an interesting and challenging question is whether the two assumptions in our model can be removed without sacrificing the predictability of structural breaks via firms' rating records and histories of economic variables. Intuitively, firms' heterogeneity can be introduced by incorporating firm-specific covariates into firms' rating transition dynamics, and the quality of credit ratings provided by CRAs can be characterized via the models discussed in the literature. However, such relaxation introduces much noisy information into the study and creates much difficulty in econometric theory, which should be investigated in further studies.

Acknowledgement

Xing's research is supported by the National Science Foundation under grant DMS-1206321.

Appendix A. Estimation formulas of α_{ij} and β_i in Section 3.3

To estimate α_{ij} and β_i by the EM approach, one should take expectations for the complete log-likelihood (6) conditional on firms' rating transition history \mathcal{Y}_{0,t_L} and the record of economic variations \mathcal{F}_{t_L-1} , and then maximize the expected log-likelihood. Taking derivatives on the expected log-likelihood with respect to α_{ij} and β_i and setting them to 0 yield a series of equations on α_{ij} and β_i . Solving these equations provide us the following updating

formulas for α_{ij} and β_i ,

$$\widehat{\beta}_{i,\text{new}} = \frac{\sum_{l=1}^L y_l \sum_{j \neq i} \widehat{\alpha}_{ij,\text{old}}}{\sum_{l=1}^L E\left[\left(\sum_{j \neq i} \lambda_{t_{l-1},t_l}^{(i,j)} \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}}\right) \middle| \mathcal{Y}_{(0,T)}\right]},$$

$$\frac{\Gamma'(\widehat{\alpha}_{ij,\text{new}})}{\Gamma(\widehat{\alpha}_{ij,\text{new}})} = \Psi(\widehat{\alpha}_{ij,\text{new}}) = \frac{\sum_{l=1}^L E(\log \lambda_{t_{l-1},t_l}^{(i,j)} \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}} \middle| \mathcal{Y}_{(0,T)})}{\sum_{l=1}^L y_l + \log(\widehat{\beta}_{i,\text{old}})},$$

in which $\Psi(\cdot)$ is the Digamma function, and the equation $\Psi(\cdot) = a$ can be solved numerically by grid search. The above formulas involve conditional expectations of $\lambda_{t_{l-1},t_l}^{(i,j)} \cdot \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}}$, $\log \lambda_{t_{l-1},t_l}^{(i,j)} \cdot \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}}$, and $\mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}}$. Using the argument in Appendix B, we can show that

$$E(\lambda_{t_{l-1},t_l}^{(i,j)} \cdot \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}} \middle| \mathcal{Y}_{(0,T)}) = \sum_{l \leq k \leq L} \pi_{lkl} \frac{K_{t_l,t_k}^{(i,j)} + \alpha_{ij}}{S_{t_l,t_k}^{(i)} + \beta_i}.$$

Similar decomposition can be applied to the conditional expectation of $\log \lambda_{t_{l-1},t_l}^{(i,j)} \cdot \mathbf{1}_{\{\Lambda(t_l) \neq \Lambda(t_{l-1})\}}$, so that each conditional expectation in the decomposition can be computed numerically.

Appendix B. Posterior probabilities of C_{mk}

To compute the posterior probabilities of C_{mk} , we first derive the posterior distribution of $\Lambda(t_l) = (\lambda_{t_{l-1},t_l}^{(i,j)})$ given \mathcal{Y}_{0,t_L} and $\mathcal{F}_{t_{l-1}}$, using the method similar to that in Xing et al. (2012). Denote, for the period (t_{m-1}, t_l) , $K_{t_{m-1},t_l}^{(i,j)}$ the number of transitions from category i to category j , $S_{t_{m-1},t_l}^{(i)}$ the amount of time spent in category i , $\lambda_{t_{m-1},t_l}^{(i,j)}$ the ij th entry in the generator $\Lambda(t)$, and $(\mathcal{Y}_{t_{m-1},t_l}, \mathcal{F}_{t_{l-1}})$ the observed rating transitions and macroeconomic variables over the period (t_{m-1}, t_l) . Let $f(\cdot | \mathcal{Y}_{(t_l,t_L)}, \mathcal{F}_{t_{l-1}})$ be the density function of $\lambda_{t_{l-1},t_l}^{(i,j)}$ given $(\mathcal{Y}_{(t_l,t_L)}, \mathcal{F}_{t_{l-1}})$, and similar definition applies to $f(\cdot | \mathcal{Y}_{(0,t_l)}, \mathcal{F}_{t_{l-1}})$ and $f(\cdot | \mathcal{Y}_{(0,t_L)}, \mathcal{F}_{t_{l-1}})$.

Note that the $\lambda_{t_{l-1}, t_l}^{(i,j)}$ is a reversible Markov chain and its stationary distribution g is same as the prior Gamma distribution with parameters α_{ij} and β_i . Applying the Bayes' theorem and using yields the assumption that rating migrations of firms are conditionally independent in the period (t_{l-1}, t_l) given the generator matrix $\Lambda(t_l)$, we obtain the following

$$f(\lambda_{t_{l-1}, t_l}^{(i,j)} | \mathcal{Y}_{(0,T)}, \mathcal{F}_{t_{L-1}}) \propto f(\lambda_{t_{l-1}, t_l}^{(i,j)} | \mathcal{Y}_{(0,t_l)}, \mathcal{F}_{t_{l-1}}) f(\lambda_{t_{l-1}, t_l}^{(i,j)} | \mathcal{Y}_{(t_l, t_L)}, \mathcal{F}_{t_{L-1}}) / g(\lambda_{t_{l-1}, t_l}^{(i,j)}). \quad (\text{B.1})$$

We let $R_l = \max\{t_{m-1} | I_m = 1, m \leq l\}$, i.e., R_l is the most recent structural breaks up to time t_{l-1} , and $p_{m,l} = P(R_l = t_{m-1} | \mathcal{Y}_{t_{m-1}, t_l}, \mathcal{F}_{t_{l-1}})$. Note that the conditional distribution of $\lambda_{t_{m-1}, t_l}^{(i,j)}$ given $R_l = t_{m-1}$ and $(\mathcal{Y}_{t_{m-1}, t_l}, \mathcal{F}_{t_{l-1}})$ is $\text{Gamma}(K_{t_{m-1}, t_l}^{(i,j)} + \alpha_{ij}, S_{t_{m-1}, t_l}^{(i)} + \beta_i)$. Then we can show by brute force calculation that the posterior distribution of $\lambda_{t_{l-1}, t_l}^{(i,j)}$ given $(\mathcal{Y}_{(0,t_l)}, \mathcal{F}_{t_{l-1}})$ is

$$\sum_{m=1}^l p_{m,l} \text{Gamma}(K_{t_{m-1}, t_l}^{(i,j)} + \alpha_{ij}, S_{t_{m-1}, t_l}^{(i)} + \beta_i), \quad (\text{B.2})$$

in which the mixture weights $p_{m,l}$ can be calculated recursively by $p_{m,l} = p_{m,l}^* / \sum_{m=1}^l p_{m,l}^*$, and

$$p_{m,l}^* = \begin{cases} p_l f_{l,l} / f_{0,0} & m = l, \\ (1 - p_l) p_{m,l-1} f_{m,l} / f_{m,l-1} & m < l. \end{cases}$$

The terms $f_{m,l}$ and $f_{0,0}$ in the above equations are defined as follows,

$$f_{m,l} = \prod_{i,j \in \mathcal{K}} \Gamma(K_{t_{m-1}, t_l}^{(i,j)} + \alpha_{ij}) / (S_{t_{m-1}, t_l}^{(i)} + \beta_i)^{(K_{t_{m-1}, t_l}^{(i,j)} + \alpha_{ij})}, \quad f_{0,0} = \prod_{i,j \in \mathcal{K}} \Gamma(\alpha_{ij}) / \beta_i^{\alpha_{i,j}}.$$

Since $\{\lambda_{t_{l-1}, t_l}^{(i,j)}\}$ is a reversible Markov chain and its stationary distribution is given by $\text{Gamma}(\alpha_{ij}, \beta_i)$, as an analog of (B.2), the posterior distribution of $\lambda_{t_{l-1}, t_l}^{(i,j)}$ given $(\mathcal{Y}_{(t_l, T)}, \mathcal{F}_{t_{L-1}})$ can be derived as

$$p_l \text{Gamma}(\alpha_{ij}, \beta_i) + (1 - p_l) \sum_{k=l+1}^L \tilde{p}_{k,l+1} \text{Gamma}(K_{t_l, t_k}^{(i,j)} + \alpha_{ij}, S_{t_l, t_k}^{(i)} + \beta_i), \quad (\text{B.3})$$

in which the mixture weight $\tilde{p}_{k,l+1} = \tilde{p}_{k,l+1}^* / \sum_{k=l+1}^L \tilde{p}_{k,l+1}^*$ and

$$\tilde{p}_{k,l+1}^* = \begin{cases} p_l f_{l+1, l+1} / f_{00} & k = l + 1, \\ (1 - p_l) q_{l+2, k} f_{l+1, k} / f_{l+2, k} & k > l + 1. \end{cases}$$

Combining (B.2) and (B.3) via (B.1) yields the posterior distribution of $\lambda_{t_{l-1}, t_l}^{(i,j)}$ given $(\mathcal{Y}_{(0,T)}, \mathcal{F}_{t_{L-1}})$ is expressed as ($1 \leq l < L$)

$$\sum_{1 \leq m \leq l \leq k \leq L} \pi_{mlk} \text{Gamma}(K_{t_{m-1}, t_k}^{(i,j)} + \alpha_{ij}, S_{t_{m-1}, t_k}^{(i)} + \beta_i), \quad (\text{B.4})$$

in which $\pi_{mlk} = \pi_{mlk}^* / \sum_{1 \leq m \leq l \leq k \leq L} \pi_{mlk}^*$ and

$$\pi_{mlk}^* = \begin{cases} p_l p_{m,l} & m \leq l = k, \\ (1 - p_l) p_{m,l} \tilde{p}_{k,l+1} f_{m,k} f_{00} / (f_{m,l} f_{l+1,k}) & m \leq l < k. \end{cases}$$

Note that for $m \leq l \leq k$, the mixture element $\text{Gamma}(K_{t_{m-1}, t_k}^{(i,j)} + \alpha_{ij}, S_{t_{m-1}, t_k}^{(i)} + \beta_i)$ corresponds to the distribution of $\lambda_{t_{l-1}, t_l}^{(i,j)}$ conditional on the event C_{mk} . This shows that the probability of the event C_{mk} conditional on $(\mathcal{Y}_{(0,T)}, \mathcal{F}_{t_{L-1}})$ is given by

$$P(C_{mk} | \mathcal{Y}_{(0,T)}, \mathcal{F}_{t_{L-1}}) = \pi_{mlk}.$$

That is, $\{\pi_{mlk}; 1 \leq m < l \leq k \leq L\}$ represents the conditional distribution of the two structural breaks surrounding time t_l .

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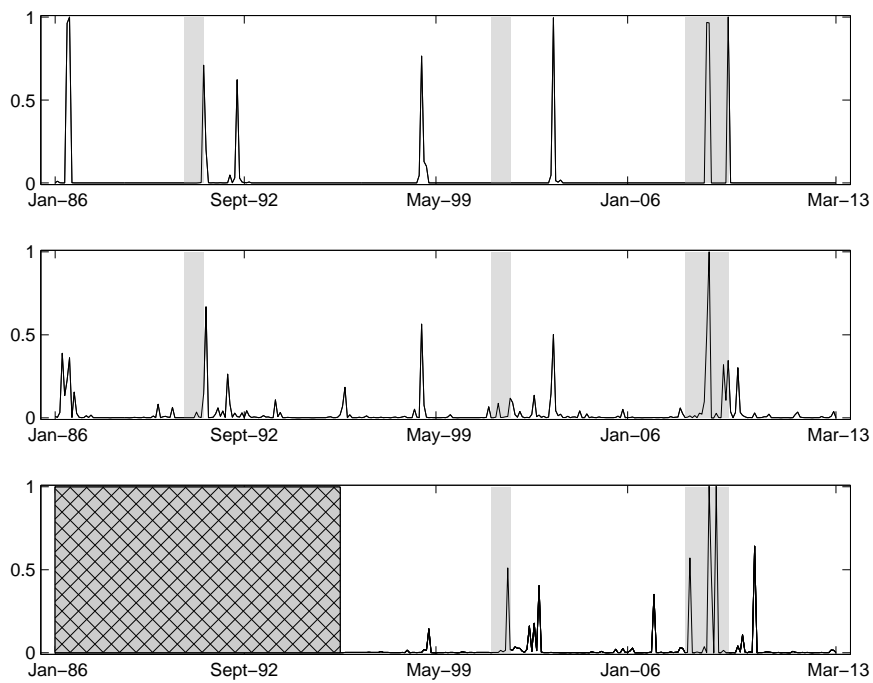


Figure B.1: Probabilities of structural breaks (Top: Posterior probabilities \hat{p}_l ; Middle: Partial estimates \tilde{p}_l ; Bottom: Predicted probabilities \hat{p}_{L+1}).

Table B.1: Correlation among macroeconomic variables

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}
X_1	1	-.380	.006	-.097	-.017	.002	.087	.020	-.082	-.035	-.021	-.062	.119	-.137	-.041	.193	.033	-.119	-.069	-.025
X_2	-.380	1	-.179	-.082	.421	.507	-.021	-.147	-.251	.045	-.202	-.283	-.219	-.105	.314	-.193	.010	.229	.078	-.138
X_3	.006	-.179	1	.545	-.555	-.503	-.618	-.340	.142	.072	.047	.071	-.054	.013	-.027	-.042	.021	-.180	.332	.210
X_4	-.097	-.082	.545	1	-.665	-.521	-.189	-.046	.153	.045	.122	.159	.012	.135	-.225	-.016	-.039	-.033	.141	.161
X_5	-.017	.421	-.555	-.665	1	.872	.240	-.083	-.224	.015	-.199	-.457	-.267	-.298	.307	-.089	.047	.208	-.161	-.118
X_6	.002	.507	-.503	-.521	.872	1	.253	-.183	-.300	.001	-.214	-.581	-.329	-.358	.284	-.005	.027	.242	-.200	-.170
X_7	.087	-.021	-.618	-.189	.240	.253	1	.529	-.090	-.135	.023	-.018	.192	.075	-.097	.088	.004	.187	-.505	-.218
X_8	.020	-.147	-.340	-.046	-.083	-.183	.529	1	.016	-.037	.033	.352	.286	.219	-.042	.004	-.001	-.051	.000	.053
X_9	-.082	-.251	.142	.153	-.224	-.300	-.090	.016	1	.063	.461	.140	.075	.059	-.102	-.193	-.036	-.047	.069	.100
X_{10}	-.035	.045	.072	.045	.015	.001	-.135	-.037	.063	1	-.025	-.050	-.165	-.048	.057	-.096	-.061	.068	.139	.076
X_{11}	-.021	-.202	.047	.122	-.199	-.214	.023	.033	.461	-.025	1	.155	.147	.145	-.197	-.093	-.071	-.072	-.019	.101
X_{12}	-.062	-.283	.071	.159	-.457	-.581	-.018	.352	.140	-.050	.155	1	.447	.823	-.267	-.078	-.019	-.206	.077	.117
X_{13}	.119	-.219	-.054	.012	-.267	-.329	.192	.286	.075	-.165	.147	.447	1	.375	-.073	.065	-.012	-.314	-.127	-.012
X_{14}	-.137	-.105	.013	.135	-.298	-.358	.075	.219	.059	-.048	.145	.823	.375	1	-.220	-.129	-.045	-.143	-.001	.072
X_{15}	-.041	.314	-.027	-.225	.307	.284	-.097	-.042	-.102	.057	-.197	-.267	-.073	-.220	1	-.104	.006	-.012	.179	-.005
X_{16}	.193	-.193	-.042	-.016	-.089	-.005	.088	.004	-.193	-.096	-.093	-.078	.065	-.129	-.104	1	.069	-.030	-.157	.009
X_{17}	.033	.010	.021	-.039	.047	.027	.004	-.001	-.036	-.061	-.071	-.019	-.012	-.045	.006	.069	1	.035	.031	.063
X_{18}	-.119	.229	-.180	-.033	.208	.242	.187	-.051	-.047	.068	-.072	-.206	-.314	-.143	-.012	-.030	.035	1	-.139	-.114
X_{19}	-.069	.078	.332	.141	-.161	-.200	-.505	.000	.069	.139	-.019	.077	-.127	-.001	.179	-.157	.031	-.139	1	.148
X_{20}	-.025	-.138	.210	.161	-.118	-.170	-.218	.053	.100	.076	.101	.117	-.012	.072	-.005	.009	.063	-.114	.148	1

Table B.2: Macroeconomic covariates and estimates of their coefficients ($\Delta\{\cdot\}$ represents the lag-1 difference of the variable)

Variable	Definition	Coefficient	Std
Y_0	intercept	-6.728	0.011
Y_1	$\Delta\{\text{S\&P500 monthly return}\}$	1.176	0.018
Y_2	$\Delta\{\text{S\&P500 monthly realized volatility}\}$	8.103	0.116
Y_3	$\Delta\{\text{3-month T-Bill rate}\}$	-7.730	0.413
Y_4	$\Delta\{\text{10-year Treasury rate}\}$	-73.875	0.706
Y_5	$\Delta\{\text{Moody's Aaa corporate bond yield}\}$	103.366	3.328
Y_6	$\Delta\{\text{Moody's Baa corporate bond yield}\}$	-189.709	2.282
Y_7	$\Delta\{\text{monthly unemployment rate}\}$	153.098	1.498
Y_8	$\Delta\{\text{mean duration of unemployment}\}$	-33.935	0.481
Y_9	$\Delta\{\text{the inflation rate measured by CPI}\}$	99.209	3.021
Y_{10}	$\Delta\{\text{the inflation rate measured by PPI}\}$	-56.613	3.315
Y_{11}	$\Delta\{\text{the inflation rate measured oil price}\}$	1.797	0.097
Y_{12}	$\Delta\{\text{CFNAI}\}$	151.788	3.140
Y_{13}	$\Delta\{\text{growth rate of US real GDP}\}$	453.619	6.906
Y_{14}	$\Delta\{\text{growth rate of industrial production}\}$	-400.270	3.306
Y_{15}	$\Delta\{\text{growth rate of M2 money stock}\}$	-89.965	3.782
Y_{16}	$\Delta\{\text{growth rate of consumer sentiment}\}$	25.780	0.251
Y_{17}	$\Delta\{\text{growth rate of bank's total net asset}\}$	-25.311	0.674
Y_{18}	$\Delta\{\text{growth rate of public debt of the U.S. Fed. Gov.}\}$	446.281	4.496
Y_{19}	$\Delta\{\text{growth rate of the total outstanding consumer credit}\}$	153.086	2.542
Y_{20}	$\Delta\{\text{growth rate of loans at all commercial banks}\}$	22.583	3.899

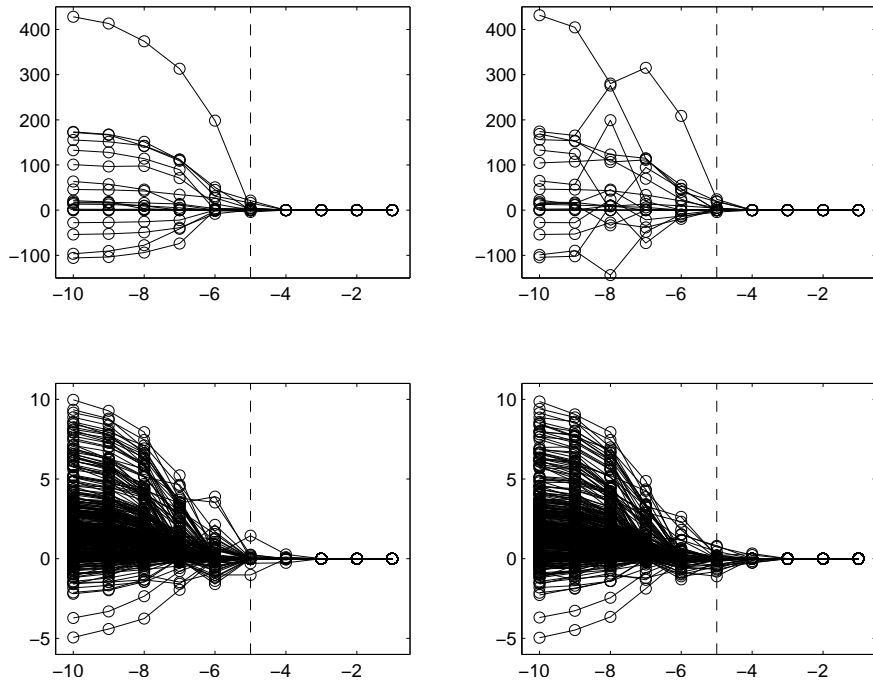


Figure B.2: Regularized model parameter paths of $\{X_{i,t}; i = 1, \dots, 20\}$ (top) and $\{X_{24(i-1)+h,t}; h = 1, \dots, 24, i = 1, \dots, 20\}$ (bottom) versus $\log \gamma$. The left and right panels corresponds to the cases of $\phi = 1$ and 0.9 , respectively. A vertical line is drawn at $\log \gamma = -5$.