Robust Control for Uncertain Takagi–Sugeno Fuzzy Systems with Time-Varying Input Delay

Ho Jae Lee
e-mail: mylchi@control.yonsei.ac.kr

Jin Bae Park
e-mail: jbpark@control.yonsei.ac.kr

Department of Electrical and Electronic Engineering, Yonsei University, Seodaemun-gu, Seoul, 120-749, Korea

Young Hoon Joo
e-mail: yhjoo@kunsan.ac.kr
School of Electronic and Information Engineering, Kunsan National University, Kunsan, Chonbuk, 573-701, Korea

A control problem of Takagi–Sugeno fuzzy systems with a time-varying input delay and norm-bounded uncertainties is addressed. The input delay is well-known in making the closed-loop stabilization difficult. A sufficient condition for the robust fuzzy-model-based stabilization is derived based on the Lyapunov–Razumikhin stability theorem, without the assumption of the variation rate on the delay. A constructive design scheme is presented in the form of the iterative convex optimization problem. The effectiveness of the proposed method is demonstrated by a numerical simulation of a nonlinear mass-spring-damper system.
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1 Introduction

In the control engineering area, it is common to encounter some practical applications containing time delays in their model configurations. Examples include chemical processes [1], biological systems [2], and virtual laboratories [3]. It is widely believed that the time delay is one of the major sources of the instability of control systems [4–6].

To resolve the control problems associated with the time delay, two generalized approaches have attracted great attention: One is the Lyapunov–Krasovskii stability theorem and the other is the Lyapunov–Razumikhin approach [7]. Although the former has been widely adopted, it requires a supplementary property in terms of the time-derivative of the time delay: The upper bound of the time-derivative must be less than one. This requirement may not be satisfied in some specific applications. On the contrary, the latter can be successfully applied because any properties of the time-derivative of the given time delay are not necessary, although the obtained results may be conservative [4].

Turning our attention to another important technical issue, most plants in the industry have severe nonlinearities and uncertainties. They thus post additional difficulties to the stability analysis and controller design. Until now, various control techniques have been developed. Among them, the Takagi–Sugeno (*T--S*) fuzzy-model-based control is popular today since it is regarded as a powerful resolution to bridge the gap between the fruitful linear control theories and the fuzzy logic control targeting plants that are mathematically ill-defined, uncertain, and nonlinear. Plentiful works related can be found in [8–14] and references therein.

Despite the extensive studies in the fuzzy-model-based control literature to date, there are relatively few research results tackling the *T--S* fuzzy systems with time delay [13,14]. Moreover, although the input delay is a technically important issue of frequent occurrence, few related control strategies seem to be available. It remains yet to be a theoretically challenging issue, and thereby must be carefully handled.

Motivated by the above observations, this paper aims at solving the robust control problem for a continuous-time uncertain *T--S* fuzzy system with a time-varying input delay. The main contribution is to propose a constructive design tool in terms of matrix inequalities for *T--S* fuzzy system of interest. The stabilizing controller is designed so that the Lyapunov–Razumikhin stability is established. It should be noted that, under the stability condition obtained, any additional restriction on the time-derivative of the time delay is not necessary. Furthermore, the longer the time delay guaranteeing the stability of the closed-loop system is, the more desirable it is. Hence, an iterative convex optimization algorithm is presented to search the maximal bound of the admissible time delay.

The organization of this paper is as follows: Section 2 briefly reviews a continuous-time uncertain *T--S* fuzzy system with the input delay. The main results are presented in Sec. 3. In Sec. 4, an example is included to visualize the feasibility of the proposed method—control of a nonlinear mass-spring-damper system. Lastly, Sec. 5 concludes this paper.

2 Preliminaries and Problem Statement

Consider the time-varying input-delayed continuous-time uncertain *T--S* fuzzy system described by the following fuzzy rules:

$$R^i:\begin{cases}\text{IF } z_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is } \Gamma_n^i\\ \text{THEN } \dot{x}(t)=(A_i+\Delta A_i)x(t)+(B_i+\Delta B_i)u(t-d(t))\end{cases} \tag{1}$$

where R^i, $i\in\mathcal{I}_Q=\{1,2,\ldots,q\}$, denotes the ith fuzzy inference rule; $z_h(t)$, $h\in\mathcal{I}_N=\{1,2,\ldots,n\}$, the premise variable; Γ_h^i the fuzzy set of $z_h(t)$ in the ith rule; $x(t)\in\mathbb{R}^n$ the state; $u(t-d(t))\in\mathbb{R}^m$ the delayed control input, in which $d(t)$ is the time-varying delay represented by any admissibly bounded function satisfying $0\leqslant d(t)\leqslant\tau$; (A_i,B_i) the model of the ith rule; and $(\Delta A_i,\Delta B_i)$ real matrix functions representing uncertainties.

Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics of (1) of the retarded type is given by

$$\begin{aligned}\dot{x}(t)&=\sum_{i=1}^{q}\theta_i(z(t))((A_i+\Delta A_i)x(t)+(B_i+\Delta B_i)u(t-d(t)))\\ x(t)&=\phi(t),\quad \forall\, t\in[-\tau,0]\end{aligned} \tag{2}$$

where $\phi(t)$ is a smooth vector-valued function defined in Banach space $\mathbf{C}[-\tau,0]$, and $\omega_i(z(t))=\Pi_{h=1}^n\Gamma_h^i(z_h(t))$, $\theta_i(z(t))=\omega_i(z(t))/\Sigma_{i=1}^q\omega_i(z(t))$.

In this study, the following fuzzy rule for the fuzzy-model-based controller is employed:

$$R^i:\begin{cases}\text{IF } z_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is } \Gamma_n^i\\ \text{THEN } u(t)=K_ix(t)\end{cases}$$

where K_i is control gain matrix to be determined. Its defuzzified output is described by

$$u(t)=\sum_{i=1}^{q}\theta_i(z(t))K_ix(t). \tag{3}$$

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The closed-loop system with (2) and (3) is described by

$$\dot{x}(t)=\sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(t))\theta_j(z(t-d(t)))((A_i+\Delta A_i)x(t)+(B_i+\Delta B_i)K_jx(t-d(t))). \quad (4)$$

Since (4) has time-varying uncertain matrices, it is not easy to determine K_i. Hence, the uncertain matrix functions should be manageable under some reasonable assumptions.

Assumption 1. The uncertainties considered here are norm-bounded of the form:

$$[\Delta A_i \quad \Delta B_i]=D_iF_i(t)[E_{a_i} \quad E_{b_i}]$$

where $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F_i(t)^TF_i(t)\leqslant I$, in which D_i, E_{a_i}, and E_{b_i} are known real constant matrices of compatible dimensions.

Our goal is summarized as follows:

Problem 1. Find K_i for (3) such that (4) is robustly globally asymptotically stable in the sense of Lyapunov against the admissibly norm-bounded and structured uncertainties and any time-varying delay $d(t)$ less than or equal to the prescribed τ. Furthermore, if possible, find the maximal upper bound of τ within which the stability of the whole system is still preserved.

3 Main Results

Before proceeding, recall the following lemmas which will be used for the proofs of our results.

Lemma 1. For given vectors a, b, and any symmetric positive definite matrix P of appropriate dimensions, and any positive scalar α, the following inequality holds:

$$\pm 2a^Tb\leqslant \alpha a^TPa+\frac{1}{\alpha}b^TP^{-1}b.$$

Lemma 2. Given constant matrices D and E, and a symmetric constant matrix S of appropriate dimensions, the following inequality holds:

$$S+DFE+E^TF^TD^T<0$$

where F satisfies $F^TF\leqslant I$ if and only if for some $\epsilon>0$

$$S+[\epsilon D \quad \epsilon^{-1}E^T]\begin{bmatrix}\epsilon D^T\\ \epsilon^{-1}E\end{bmatrix}<0.$$

Remark 1. In case of $-2a^Tb<0$ in Lemma 1, its estimated upper bound may be not good and introduce conservatism. However, optimizing over α can rather reduce the introduced conservatism.

The main result is now presented in the following theorem:

Theorem 1. If there exist a symmetric positive definite matrix Q, matrices M_i, and positive scalars α_1, α_2, ϵ_{ij}, $\epsilon_{\alpha_{1_i}}$, and $\epsilon_{\alpha_{2_{ij}}}$ such that the following inequalities are satisfied:

$$\begin{bmatrix}\frac{1}{\tau}\Upsilon_{ij}+(\alpha_1+\alpha_2)Q+\epsilon_{ij}D_iD_i^T & (\cdot)^T & (\cdot)^T\\ M_i^TB_i^T & -\frac{1}{2}Q & (\cdot)^T\\ \frac{1}{\tau}(E_{a_i}Q+E_{b_i}M_j) & E_{b_i}M_j & -\epsilon_{ij}I\end{bmatrix}<0, \quad (i,j)\in\mathcal{I}_Q\times\mathcal{I}_Q \quad (5)$$

$$\begin{bmatrix}-\alpha_1Q+\epsilon_{\alpha_{1_i}}D_iD_i^T & (\cdot)^T & (\cdot)^T\\ QA_i^T & -Q & (\cdot)^T\\ 0 & E_{a_i}Q & -\epsilon_{\alpha_{1_i}}I\end{bmatrix}<0, \quad i\in\mathcal{I}_Q \quad (6)$$

$$\begin{bmatrix}-\alpha_2Q+\epsilon_{\alpha_{2_{ij}}}D_iD_i^T & (\cdot)^T & (\cdot)^T\\ M_j^TB_i^T & -Q & (\cdot)^T\\ 0 & E_{b_i}M_j & -\epsilon_{\alpha_{2_{ij}}}I\end{bmatrix}<0, \quad (i,j)\in\mathcal{I}_Q\times\mathcal{I}_Q \quad (7)$$

then (2) is robustly globally asymptotically stabilizable by (3) in the presence of the norm-bounded uncertainties and for all $d(t)\leqslant\tau$, where $\Upsilon_{ij}=QA_i^T+A_iQ+M_j^TB_i^T+B_iM_j$, $Q=P^{-1}$, $M_i=K_iP^{-1}$, and $(\cdot)^T$ denotes the transposed elements in the symmetric positions.

Proof. Choose a Lyapunov functional candidate as $V(x(t))=x(t)^TPx(t)$ where P is a symmetric positive definite. Clearly, $V(x(t))$ is positive definite and radially unbounded. The time derivative of $V(x(t))$ along any trajectory of (4) is given by

$$\dot{V}(x(t))=\sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(t))\theta_j(z(t-d(t)))(x(t)^T((A_i+\Delta A_i)^TP+P(A_i+\Delta A_i))x(t)+x(t-d(t))^TK_j^T(B_i+\Delta B_i)^TPx(t)+x(t)^TP(B_i+\Delta B_i)K_jx(t-d(t))). \quad (8)$$

Note that $x(t-d(t))=x(t)-\int_{t-d(t)}^{t}\dot{x}(\lambda)d\lambda$. Then, plugging it into (8) results in

$$\dot{V}(x(t))=\sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\sum_{l=1}^{q}\theta_i(z(t))\theta_j(z(t-d(t)))\Bigg(x(t)^T((A_i+\Delta A_i)^TP+P(A_i+\Delta A_i)+K_j^T(B_i+\Delta B_i)^TP+P(B_i+\Delta B_i)K_j)x(t)-2\int_{t-d(t)}^{t}x(t)^TP(B_i+\Delta B_i)K_j\theta_k(z(\lambda))\theta_l(z(\lambda-d(\lambda)))\times((A_k+\Delta A_k)x(\lambda)+(B_k+\Delta B_k)K_lx(\lambda-d(\lambda)))d\lambda\Bigg). \quad (9)$$

Applying Lemma 1 to (9) implies

$$\dot{V}(x(t)) \leqslant \sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\sum_{l=1}^{q} \theta_i(z(t))\theta_j(z(t-d(t)))\Bigg(x(t)^T((A_i+\Delta A_i)^TP + P(A_i+\Delta A_i) + K_j^T(B_i+\Delta B_i)^TP + P(B_i+\Delta B_i)K_j)x(t) + \int_{t-d(t)}^{t} \frac{1}{\alpha_1}x(t)^TP(B_i+\Delta B_i)K_j(A_k+\Delta A_k)P^{-1}(A_k + \Delta A_k)^TK_j^T(B_i+\Delta B_i)^TPx(t) + \frac{1}{\alpha_2}x(t)^TP(B_i+\Delta B_i)K_j(B_k + \Delta B_k)K_lP^{-1}K_l^T(B_k+\Delta B_k)^TK_j^T(B_i+\Delta B_i)^TPx(t) + \alpha_1 x(\lambda)^TPx(\lambda) + \alpha_2 x(\lambda-d(\lambda))^TPx(\lambda-d(\lambda))d\lambda\Bigg). \tag{10}$$

From the Razumikhin stability theorem [7], and assuming that for any real number $\delta > 1$, we have $V(x(\lambda)) < \delta V(x(t))$, $\forall \lambda \in [t-2\tau, t]$. Suppose that

$$(A_k+\Delta A_k)P^{-1}(A_k+\Delta A_k)^T \leqslant \alpha_1 P^{-1}, \quad k \in \mathcal{I}_Q \tag{11}$$

$$(B_k+\Delta B_k)K_lP^{-1}K_l^T(B_k+\Delta B_k)^T \leqslant \alpha_2 P^{-1}, \quad (k,l) \in \mathcal{I}_Q \times \mathcal{I}_Q \tag{12}$$

then, it is not difficult to understand the right-hand side of (10) is less than

$$\sum_{i=1}^{q}\sum_{j=1}^{q} \theta_i(z(t))\theta_j(z(t-d(t)))(x(t)^T((A_i+\Delta A_i)^TP + P(A_i+\Delta A_i) + K_j^T(B_i+\Delta B_i)^TP + P(B_i+\Delta B_i)K_j)x(t) + 2d(t)x(t)^TP(B_i + \Delta B_i)K_jP^{-1}K_j^T(B_i+\Delta B_i)^TPx(t) + d(t)\delta(\alpha_1+\alpha_2)x(t)^TPx(t)). \tag{13}$$

From the observation that (13) is monotonically increasing with respect to $d(t)$, if the following holds,

$$(A_i+\Delta A_i)^TP + P(A_i+\Delta A_i) + K_j^T(B_i+\Delta B_i)^TP + P(B_i+\Delta B_i)K_j + 2\tau P(B_i+\Delta B_i)K_jP^{-1}K_j^T(B_i+\Delta B_i)^TP + \tau\delta(\alpha_1+\alpha_2)P < 0, \quad i,j \in \mathcal{I}_Q \tag{14}$$

then (4) is robustly globally asymptotically stable against all time-varying input delays not larger than τ and the structured uncertainties. Moreover, from the continuity of the eigenvalues of (14) with respect to δ, there exists $\delta > 1$ sufficiently small such that (14) with $\delta = 1$ still holds.

With some efforts, we can show that (5)–(7) guarantee the negative definiteness of (14) whenever $x(t)$ is not zero. First, we show that (6) and (7) are directly derived from (11) and (12). Inequality (11) can be represented as follows:

$$[(A_i+\Delta A_i)P^{-1}][P][P^{-1}(A_i+\Delta A_i)^T] - \alpha_1 P^{-1} < 0. \tag{15}$$

Applying the Schur complement and Assumption 1 to (15) gives

$$\begin{bmatrix} -\alpha_1 P^{-1} & (\cdot)^T \\ P^{-1}A_i^T & -P^{-1}\end{bmatrix} + \begin{bmatrix} D_i \\ 0 \end{bmatrix} F_i(t)[0 \quad E_{a_i}P^{-1}] + \begin{bmatrix} 0 \\ P^{-1}E_{a_i}^T \end{bmatrix} F_i(t)^T[D_i^T \quad 0] < 0. \tag{16}$$

According to Lemma 2, (16) holds if and only if there exists a constant $\epsilon_{\alpha_{1_i}}^{1/2} > 0$ such that

$$\begin{bmatrix} -\alpha_1 P^{-1} & (\cdot)^T \\ P^{-1}A_i^T & -P^{-1}\end{bmatrix} + \begin{bmatrix} D_i & 0 \\ 0 & P^{-1}E_{a_i}^T \end{bmatrix}\begin{bmatrix} \epsilon_{\alpha_{1_i}} I & 0 \\ 0 & \epsilon_{\alpha_{1_i}}^{-1} I\end{bmatrix}\begin{bmatrix} D_i^T & 0 \\ 0 & E_{a_i}P^{-1}\end{bmatrix} < 0.$$

Using the Schur complement twice and denoting $P^{-1} = Q$ results in (6). We can again establish a similar argument to that above with (12) to obtain (7). Next, (14) can be rewritten as follows:

$$\frac{1}{\tau}((A_i+\Delta A_i)^TP + P(A_i+\Delta A_i) + K_j^T(B_i+\Delta B_i)^TP + P(B_i+\Delta B_i)K_j) + (\alpha_1+\alpha_2)P + [P(B_i+\Delta B_i)K_jP^{-1}][2P][P^{-1}K_j^T(B_i+\Delta B_i)^TP] < 0. \tag{17}$$

By applying the Schur complement, Assumption 1, and Lemma 2, (17) is equivalent to

$$\Psi_{ij} + \begin{bmatrix} \frac{1}{\tau}(E_{a_i}+E_{b_i}K_j)^T & PD_i \\ P^{-1}K_j^TE_{b_i}^T & 0 \end{bmatrix}\begin{bmatrix} \epsilon_{ij}^{-1}I & 0 \\ 0 & \epsilon_{ij} I\end{bmatrix} \times \begin{bmatrix} \frac{1}{\tau}(E_{a_i}+E_{b_i}K_j) & E_{b_i}K_jP^{-1} \\ D_i^TP & 0\end{bmatrix} < 0 \tag{18}$$

where

$$\Psi_{ij} = \begin{bmatrix} \frac{1}{\tau}(A_i^TP + PA_i + K_j^TB_i^TP + PB_iK_j) + (\alpha_1+\alpha_2)P & (\cdot)^T \\ P^{-1}K_j^TB_i^TP & -\frac{1}{2}P^{-1}\end{bmatrix}$$

if and only if there exists a constant $\epsilon_{ij}^{-1/2} > 0$. Sequential applying the Schur complement twice and a congruence transformation with diag$\{P^{-1}, I, I, I\}$ to (18), denoting $Q = P^{-1}$ and $M_i = K_iP^{-1}$ yields (5), which completes the proof of the theorem. ■

Remark 2. In order to diminish the conservatism introduced by the overestimated upper bound in (6) and (7), proper values of α_1 and α_2 should be chosen such that τ is maximized. However, the terms $\alpha_1 Q$ and $\alpha_2 Q$ make (5)–(7) be nonlinear, which are difficult to solve. Thus, an iterative convex optimization algorithm based on the linear matrix inequality (LMI) technique is utilized.

In order to find the maximal τ, without loss of generality, we replace ϵ_{ij} with τ in (5). Carrying out simple algebraic manipulation, the following LMI is equivalent to (5):

$$\begin{bmatrix} \frac{1}{\tau}(\Upsilon_{ij} + D_i^TD_i) + (\alpha_1+\alpha_2)Q & (\cdot)^T & (\cdot)^T \\ \frac{1}{\tau}(E_{a_i}Q + E_{b_i}M_j) & -\frac{1}{\tau}I & (\cdot)^T \\ M_j^TB_i^T & M_j^TE_{b_i}^T & -\frac{1}{2}Q \end{bmatrix} < 0. \tag{19}$$

Now, the following convex optimization algorithm is proposed.

Step 1: Find Q, M_i, and ϵ_{ij} such that the following LMI constraints are satisfied:

$$\begin{bmatrix} QA_i^T + A_iQ + M_j^TB_i^T + B_iM_j + \epsilon_{ij}D_i^TD_i & (\cdot)^T \\ E_{a_i}Q + E_{b_i}M_j & -\epsilon_{ij}I\end{bmatrix} < 0, \quad (i,j) \in \mathcal{I}_Q \times \mathcal{I}_Q.$$

Step 2: For Q given in the previous step, find α_1, α_2, $\epsilon_{\alpha_{1_i}}$, $\epsilon_{\alpha_{2_{ij}}}$, and M_i such that the following generalized eigenvalue problem (GEVP) has solutions

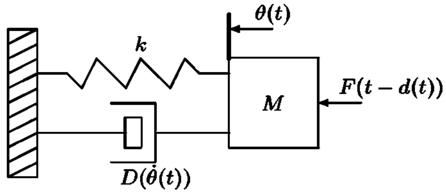

Fig. 1 Nonlinear mass-spring-damper system with an input delay

$$\underset{M_i,\alpha_1,\alpha_2,\epsilon_{\alpha_{1_i}},\epsilon_{\alpha_{2_{ij}}}}{\textit{Maximize}} \quad \tau \textit{ subject to } (19),\ (6),\ \textit{and } (7).$$

Step 3: For α_1, α_2, and M_i given in the previous step, find $\epsilon_{\alpha_{1_i}}$, $\epsilon_{\alpha_{2_{ij}}}$, and Q such that the following GEVP has solutions

$$\underset{Q,\epsilon_{\alpha_{1_i}},\epsilon_{\alpha_{2_{ij}}}}{\textit{Maximize}} \ \tau \textit{ subject to } (19),\ (6),\ \textit{and } (7).$$

Step 4: Return to Step 2 until the convergence of τ is attained with a desired accuracy.

4 An Example

An example is presented to visualize the proposed design technique. Consider the following nonlinear mass-spring-damper mechanical system illustrated in Fig. 1:

$$M\ddot{\theta}(t) + D(\dot{\theta}(t))\,\dot{\theta}(t) + k\theta(t) = \phi F(t - d(t))$$

where $\theta(t)$ is the relative position of the mass; $F(t-d(t))$ the delayed external force; $M=1$ the mass of this system; $k=0.1$ the stiffness of the spring. $\phi=1$ the input coefficient. The damping coefficient of the nonlinear damper is assumed to be $D(\dot{\theta}(t)) = 0.5+0.75\dot{\theta}^2(t)$. Furthermore, we assume that k is unknown but bounded within 10% of its nominal values.

Choosing the state as $x(t)=[\dot{\theta}(t),\theta(t)]^T$ and the input variable $u(t)$ as $F(t)$ yields the following state-space representation.

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix} -0.75x_1^3(t) - 0.5x_1(t) + 0.1(1+0.1\zeta(t))x_2(t) + u(t-d(t)) \\ x_1(t)\end{bmatrix} \tag{20}$$

where $|\zeta(t)|^2 \leqslant 1$. The system (20) has one nonlinear term, $-0.75x_1^3(t)$. Assume $x_1(t)\in[-\Omega,\Omega]$ and if this nonlinear term can be represented as a convex sum, the *T--S* fuzzy system of (20) can be constructed. Consider the following equations:

$$-0.75x_1^3(t) = \Gamma_1(x(t))\cdot 0 + \Gamma_2(x(t))\cdot(-0.75\Omega^2)x_1(t)$$

$$1 = \Gamma_1(x(t)) + \Gamma_2(x(t)). \tag{21}$$

Solving (21) yields

$$\Gamma_1(x(t)) = 1 - \frac{x_1^2(t)}{\Omega^2}$$

$$\Gamma_2(x(t)) = \frac{x_1^2(t)}{\Omega^2}.$$

Now, by adopting these as fuzzy sets, the *T--S* fuzzy system of (20) can be constructed as follows under the assumption of $\Omega = 0.8165$:

$$R^1: \begin{cases}\text{IF } x_1(t) \text{ is about } \Gamma_1\\ \text{THEN } \dot{x}(t) = (A_1+\Delta A_1)x(t) + (B_1+\Delta B_1)u(t-d(t))\end{cases}$$

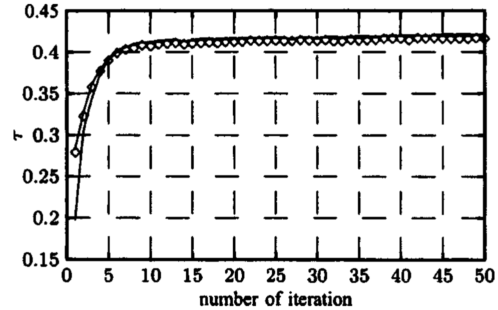

Fig. 2 Behavior of the maximal bound of the time delay via the proposed algorithm: τ computed from Step 1 (solid); τ obtained from Step 2 (solid-diamond)

$$R^2: \begin{cases}\text{IF } x_1(t) \text{ is about } \Gamma_2\\ \text{THEN } \dot{x}(t) = (A_2+\Delta A_2)x(t) + (B_2+\Delta B_2)u(t-d(t))\end{cases}$$

where the associated matrices are given by

$$A_1 = \begin{bmatrix}-0.5 & 0.1\\ 1 & 0\end{bmatrix},\quad A_2 = \begin{bmatrix}-1 & 0.1\\ 1 & 0\end{bmatrix},$$

$$\Delta A_1 = \Delta A_2 = \begin{bmatrix}0 & 0.1\zeta(t)\\ 0 & 0\end{bmatrix},\quad B_1 = B_2 = \begin{bmatrix}1\\ 0\end{bmatrix}$$

and $\Delta B_1=\Delta B_2=[0]_{2\times 1}$. By applying Theorem 1 and the iterative optimization technique, we get $K_1=[-0.7623 -0.5212]$, $K_2=[-0.3690 -0.5362]$, and $\tau=0.4223$. Figure 2 shows the behavior of τ obtained by the proposed algorithm. It means that the designed *T--S* fuzzy-model-based controller can robustly stabilize (20) in the presence of any time-varying input delay satisfying $d(t)\leqslant\tau=0.4223$ and the parametric uncertainties.

During the simulation process, the system parameter k is randomly varied within 10% of its nominal value. Figure 3 shows $d(t)$ applied to the simulation. Indeed, the assumed time delay does not exceed τ and its maximal time-derivative is much larger than one. Notice that, accordingly, the Lyapunov–Krasovskii functional approach cannot be applied. Compared to that, the proposed approach seems to be suitable, since it allows for the delay to be time-varying with an arbitrary fast rate of change.

The initial value is $x(0)=x_0=[0,1]^T$. For comparison purpose, a conventional fuzzy-model-based controller design technique without consideration of the input-delay [9] is simulated. The simulation result is shown in Fig. 4. The control input is activated at $t=3$ (s). After $t=3$ (s), the conventional method produces oscillatory trajectories, as is expected. On the other hand, the trajectories controlled by the proposed method are quickly guided to the origin without oscillation.

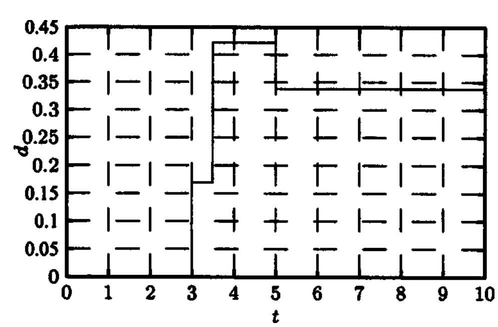

Fig. 3 The applied input delay: $d(t)=0$, $t\in[0,3)$ (s); 0.1689, $t\in[3,3.5)$ (s); 0.4223, $t\in[3.5,5)$ (s); 0.3379 otherwise

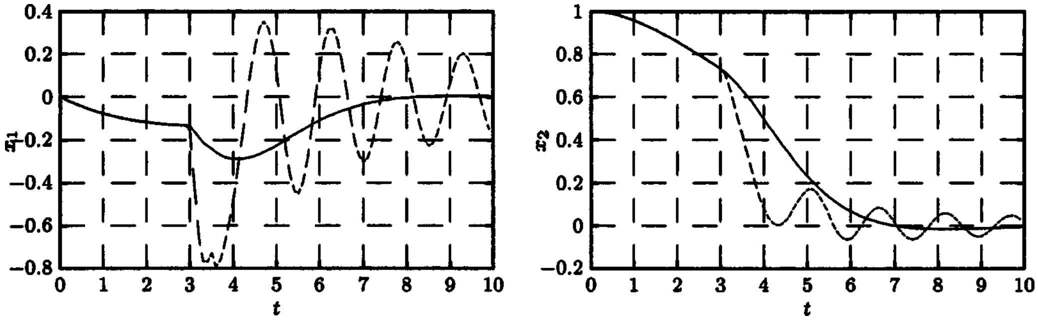

Fig. 4 Time responses of $x(t)$ by: Ref. [9] without consideration of the input delay (dotted); the proposed method with consideration of the input delay (solid)

5 Conclusions

In this paper, we have discussed the robust control of *T--S* fuzzy systems containing uncertainties and input delays. The Lyapunov–Razumikhin stability theorem has been utilized as a synthesis tool. The sufficient condition for the existence of the stabilizing controller has been given in terms of matrix inequalities. The maximal bound of the input delay is searched in an iterative manner. The effectiveness of the proposed design methodology has been thoroughly verified in the simulation example. This means a great potential for industrial applications.

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