vection. The main conclusion from this study is that the negative concentration gradient of the surface tension is a trigger for inducement of Marangoni convection before the additive solubility, while the imbalance of the surface tension and the interfacial tension is a trigger after the solubility limit.

Acknowledgment

The authors thank Mr. K. Iizuka, Tokyo University of Agriculture and Technology, for his experimental assistance. The authors acknowledge that this work has been partially funded by the Japan Science and Technology Corporation (JST).

References

Beutler, A., Greiter, I., Wagner, A., Hohhmann, L., Schreier, S., and Alefeld, G., 1996, "Surfactants and Fluid Properties," *Int. J. Refrigeration,* Vol. 19, No. 5, pp. 342–346.

Chavepeyer, G., Salajan, M., Platten, J. K., and Smet, P., 1995, "Interfacial Tension and Surface Adsorption in j-Heptanol/Water Systems," *Journal of Colloid and Interface Science,* Vol. 174, pp. 112–116.

Daiguji, H., Hihara, E., and Saito, T., 1997, "Mechanism of Absorption Enhancement by Surfactant," *Int. J. Heat and Mass Transfer,* Vol. 40, No. 8, pp. 1743–1752.

Fujita, T., 1993, "Falling Liquid Films in Absorption Machines," *Int. J. Refrigeration,* Vol. 16, No. 4, pp. 282–294.

Hihara, E., and Saito, T., 1993, "Effects of Surfactant on Falling Film Absorption," *Int. J. Refrigeration,* Vol. 16, No. 5, pp. 339–346.

Hozawa, M., Inoue, M., Sato, J., and Tsukada, T., 1991, "Marangoni Convection During Steam Absorption into Aqueous LiBr Solution With Surfactant," *J. of Chemical Engineering of Japan,* Vol. 24, No. 2, pp. 209–214.

Kang, Y. T., Akisawa, A., and Kashiwagi, T., 1999, "Experimental Correlation of Combined Heat and Mass Transfer for NH_3–H_2O falling film absorption," *International Journal of Refrigeration,* Vol. 22, No. 4, pp. 250–262.

Kashiwagi, T., 1988, "Basic Mechanism of Absorption Heat and Mass Transfer Enhancement by the Marangoni Effect," *Newsletter, IEA Heat Pump Center,* Vol. 6, No. 4, pp. 2–6.

Kim, K. J., Berman, N. S., and Wood, B. D., 1996, "The Interfacial Turbulence in Falling Film Absorption: Effects of Additives," *Int. J. Refrigeration,* Vol. 19, No. 5, pp. 322–330.

Uemura, T., and Hasaba, S., 1964, "Studies on the Lithium Bromide-Water Absorption Refrigeration Machine," Technical Report of Kansai University, Vol. 6, pp. 31–55.

Yao, W., Bjurstrom, H., and Setterwall, F., 1991, "Surface Tension of Lithium Bromide Solutions With Heat Transfer Additives," *J. Chem. Eng. Data,* Vol. 36, pp. 96–98.

Transient Free Convection Flow Past an Infinite Vertical Plate With Periodic Temperature Variation

U. N. Das,[1] R. K. Deka,[1] and V. M. Soundalgekar[2]

Nomenclature

C_p = specific heat at constant pressure
g = gravitational acceleration
K = thermal conductivity
L_R = reference length
Pr = Prandtl number
T' = temperature of the fluid near the plate
T'_∞ = temperature of the fluid far away from the plate
t' = time
t_R = reference time
u = velocity of the fluid
U_R = reference velocity
ω' = frequency
X_p = distance of the transition point from the leading edge
β = coefficient of volume expansion
ρ = density
ϵ = amplitude (constant)
θ = nondimensional temperature
u = nondimensional velocity
$i = \sqrt{-1}$

1 Introduction

Transient laminar-free convection flow past an infinite vertical plate under different plate conditions was studied by many researchers. The first closed-form solutions for Prandtl number Pr = 1.0 in case of a step change in wall temperature with time was derived by Illingworth (1950) and for Pr $\neq$ 1.0, he derived the solution in integral form. Siegel (1958) studied the unsteady free-convection flow past a semi-infinite vertical plate under step-change in wall temperature or surface heat flux by employing the momentum integral method. Experimental evidence for such a situation was presented by Goldstein and Eckert (1960). For a semi-infinite vertical plate, unsteady free-convection flow was studied analytically by Chung and Anderson (1961), Sparrow and Gregg (1960), and Hellums and Churchill (1961, 1962) under boundary layer assumptions. Siegel (1958) first pointed out that the initial behavior of the temperature and velocity fields for a semi-infinite vertical flat plate is the same for the doubly infinite vertical flat plate, i.e., in the case of a temperature field it is the same as a solution of an unsteady one-dimensional heat conduction problem. The transition from conduction to convection begins only when some effect from the leading edge has propagated up the plate as a wave, to the particular point in question. Before this time, the fluid in this region effectively does not know that the plate has a leading edge. Hence, Goldstein and Briggs (1964) studied analytically the problem of transient free-convection flow past an infinite isothermal plate and introduced the idea of leading edge effect. Because of the importance of these problems, Gebhart et al. (1988) discussed this problem in their book. Jaluria and Gebhart (1974) also studied the transition mechanisms of vertical natural convection flow.

In all these papers, the plate was assumed to be maintained at a constant temperature T'_∞, which is also the temperature of the surrounding stationary fluid. But in industrial applications, quite often the plate temperature starts oscillating about a nonzero mean temperature. Under these physical conditions, how transient free-convection past an infinite vertical plate is affected? This has not been studied in the literature. Hence, it is now proposed to study the behavior of transient free-convective flow when the mean plate temperature is superposed by periodic plate temperature with frequency ω'. In many engineering applications, transient free-convection flow occurs as such a flow acts as a cooling device. However, free-convection flow is enhanced by superimposing oscillatory temperature on the mean plate temperature. Again transient natural convection is of interest in the early stages of melting adjacent to a heated surface or in transient heating of insulating air gaps by heat input at the start-up of furnaces. In Section 2 the mathematical formulation of the problem is presented, along with its solution by the Laplace transform technique and in Section 3 the conclusions are summarized.

2 Mathematical Analysis

Consider the infinite vertical plate held in infinite mass of fluid, the temperature of both being assumed at T'_∞ initially. At time $t' > 0$, the plate temperature is raised to T'_w and a periodic temperature

[1] Department of Mathematics, Gauhati University, Guwahati 781 014, India.
[2] 31A-12 Brindavan Society, Thane 400 601, India.

Contributed by the Heat Transfer Division for publication in the JOURNAL OF HEAT TRANSFER. Manuscript received by the Heat Transfer Division, June 9, 1998; revision received, July 23, 1999. Keywords: Computational, Finite Element, Heat Transfer, Inverse, Nonintrusive Diagnostics, Stress. Associate Technical Editor: P. Ayyaswamy.

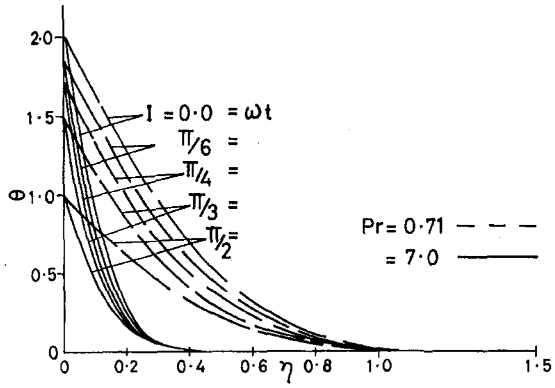

Fig. 1 Temperature profiles

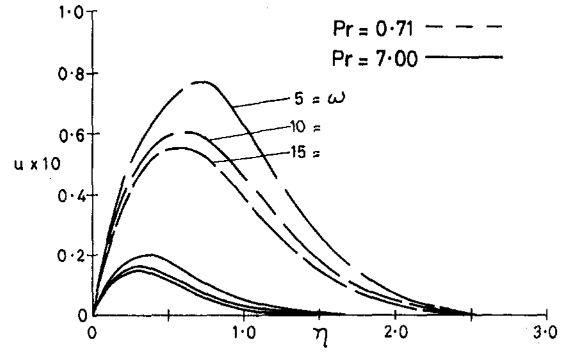

Fig. 3 Velocity profiles $\omega t = \pi/6$, $t = 0.2$

is assumed to be superimposed on this mean constant temperature of the plate. Then neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation), the problem can be shown to be governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = v \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

with following initial and boundary conditions:

$$u' = 0, \quad T' = T'_\infty \quad \text{for all} \quad y', t' \le 0$$

$$u' = 0, \; T' = T'_\infty + \epsilon(T'_w - T'_\infty) \cos \omega' t' \;\; \text{at} \;\; y' = 0, \; t' > 0$$

$$u' \to 0, \quad T' \to T'_\infty \quad \text{as} \quad y' \to \infty, \quad t' > 0. \tag{3}$$

The temperature distribution is independent of the flow and heat transfer is by conduction alone. This is true for fluids in the initial stage due to absence of convective heat transfer or at small Grashof number Gr flow (Gr $\ll$ 1).

All the physical variables are defined in the Nomenclature. Because the plate is infinite in nature, the physical variables are functions of y' and t' only as the flow is fully developed.

We now introduce the following nondimensional quantities in Eqs. (1)–(3).

$$\mathrm{Pr} = \mu C_p/K, \quad t = t'/t_R, \quad \omega = t_R\omega', \quad y = y'/L_R,$$

$$u = u'/U_R, \quad \theta = (T' - T'_\infty)/(T'_w - T'_\infty), \quad \Delta T = T'_w - T'_\infty,$$

$$U_R = (\nu g\beta\Delta T)^{1/3}, \quad L_R = (g\beta\Delta T/\nu^2)^{1/3},$$

$$t_R = (g\beta\Delta T)^{-2/3}\nu^{1/3} \tag{4}$$

Then Eqs. (1)–(3) reduce to following nondimensional form:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta \tag{5}$$

$$\mathrm{Pr}\, \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{6}$$

with following initial and boundary conditions:

$$u = 0, \quad \theta = 0 \quad \text{for all} \quad y, t \le 0$$

$$u = 0, \quad \theta = 1 + \epsilon \cos \omega t \quad \text{at} \quad y = 0, \quad t > 0$$

$$u \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty, \quad t > 0. \tag{7}$$

The solutions of these coupled linear Eqs. (5) and (6) satisfying the initial and boundary conditions (7) are derived by the Laplace transform technique and are as follows:

$$\theta = \mathrm{erfc}\,(\eta\sqrt{\mathrm{Pr}}) + \frac{\epsilon}{2}\{g(\eta\sqrt{\mathrm{Pr}}, \omega i) + g(\eta\sqrt{\mathrm{Pr}}, -\omega i)\} \tag{8}$$

$$u = \frac{1}{(\mathrm{Pr}-1)}\{f(\eta) - f(\eta\sqrt{\mathrm{Pr}})\} - \frac{i\epsilon}{2\omega(\mathrm{Pr}-1)}\{g(\eta, \omega i) - g(\eta, -\omega i)\} + \frac{i\epsilon}{2\omega(\mathrm{Pr}-1)}\{g(\eta\sqrt{\mathrm{Pr}}, \omega i) - g(\eta\sqrt{\mathrm{Pr}}, -\omega i)\}. \tag{9}$$

When $t = \pi/2$, the plate is isothermal and the solutions reduce to

$$\theta = \mathrm{erfc}\,(\eta\sqrt{\mathrm{Pr}}) \tag{10}$$

$$u = \frac{1}{\mathrm{Pr}-1}\{f(\eta) - f(\eta\sqrt{\mathrm{Pr}})\}. \tag{11}$$

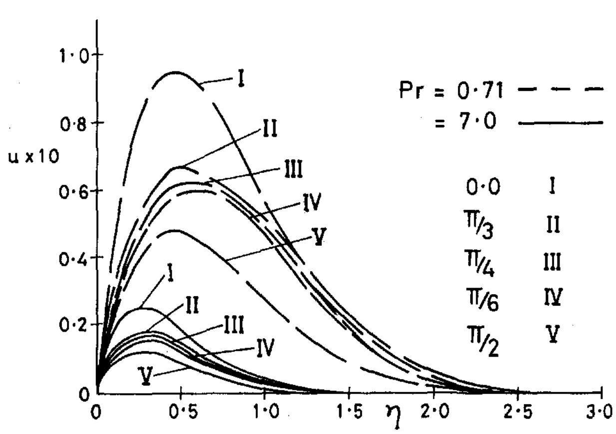

Fig. 2 Velocity profiles $\omega = 10.0$, $6 = 0.2$

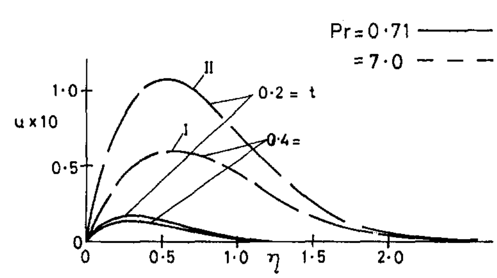

Fig. 4 Velocity profiles $\omega t = \pi/6$, $w = 10$

In order to gain physical insight into the problem, we have computed numerical values of θ and u for $\epsilon = 1.0$ and these are shown graphically. However, the argument of the complementary error function is imaginary. So during computation, we have separated the real and imaginary parts by using the formula

$$\operatorname{erf}(a+ib) = \operatorname{erf}(a) + \frac{e^{-a^2}}{2\pi a}\{1 - \cos 2ab + i \sin 2ab\}$$

$$+ \frac{2e^{-a^2}}{\pi} \sum_{n=1}^{\infty} \frac{e^{-n^2/4}}{n^2 + 4a^2}\{f_n(a, b) + i g_n(a, b)\} + \epsilon(a, b)$$

where

$$f_n(a, b) = 2a - 2a \cosh(nb) \cos 2ab + n \sinh(nb) \sin 2ab$$

$$g_n(a, b) = 2a \cosh(nb) \sin 2ab + n \sinh(nb) \cos 2ab$$

$$|\epsilon(a, b)| \ll 10^{-16} |\operatorname{erf}(a + ib)|. \quad (12)$$

The temperature profiles for Pr = 0.71 (air) and 7.0 (water) are shown on Fig. 1 and we observe that there is a fall in temperature due to an increase in the Prandtl number of the fluid. Temperature also falls owing to an increase in the value of ωt. The effect of ωt on the velocity profiles for $\omega = 10.0$ are shown on Fig. 2 for both air and water. In case of both air and water, the velocity is found to decrease with increasing ωt and the Prandtl number Pr. Velocity profiles are shown for different values of the frequency ω on Fig. 3 for both air and water when $\omega t = \pi/6$ and $t = 0.2$. It is interesting to note that the transient velocity is observed to decrease with increasing the frequency ω. The effect of time on the transient velocity can be seen on Fig. 4 and it is found that the transient velocity increases with increasing time.

Initially, the heat is transferred through the plate by conduction. But at a little later stage, convection currents start flowing near the plate. Hence it is necessary to know the position of a point on the plate where conduction mechanism changes to convection mechanism. In scientific literature, this is studied as a leading edge effect. The distance of this point of transition from conduction to convection is given by

$$X_p = \int_0^t u(y, t) dt \quad (13)$$

or in terms of the Laplace transform and its inverse, we have

$$X_p = L^{-1}\left[\frac{1}{s} L\{u(y, t)\}\right]. \quad (14)$$

Substituting for $L\{u(y, t)\}$ from (9) and carrying out the simplification we have

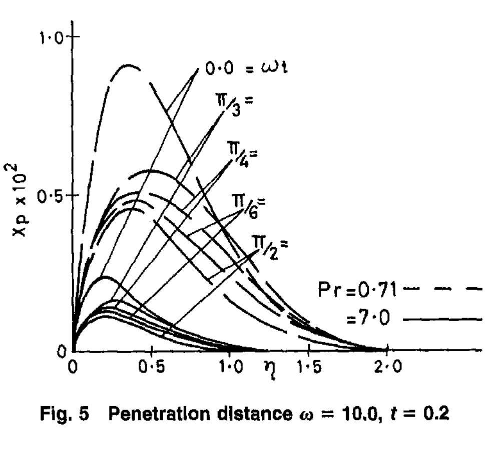

Fig. 5 Penetration distance $\omega = 10.0$, $t = 0.2$

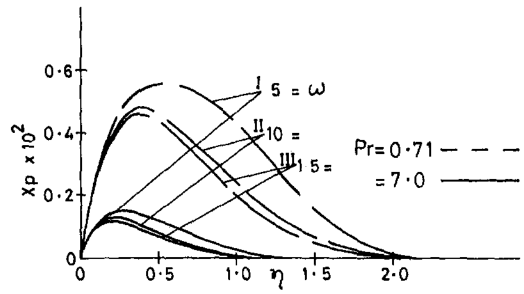

Fig. 6 Penetration distance $\omega t = \pi/6$, $t = 0.2$

$$X_p = \frac{1}{\Pr - 1}\left[h(\eta) - \frac{\epsilon}{2\omega^2}\{g(\eta, i\omega) + g(\eta, -i\omega)\}\right.$$

$$- h(\eta\sqrt{\Pr}) + \frac{\epsilon}{2\omega^2}\{g(\eta\sqrt{\Pr}, i\omega) + g(\eta\sqrt{\Pr}, -i\omega)\}$$

$$\left. - \frac{\epsilon}{2\omega^2} \operatorname{erfc}(\eta\sqrt{\Pr})\right] \quad (15)$$

where

$$f(a) = t\left\{(1 + 2a^2)\operatorname{erfc}(a) - 2\frac{ae^{-a^2}}{\sqrt{\pi}}\right\}$$

$$g(a, b) = \frac{e^{bt}}{2}\{e^{2a\sqrt{bt}} \operatorname{erfc}(a + \sqrt{bt}) + e^{-2a\sqrt{bt}} \operatorname{erfc}(a - \sqrt{bt})\}$$

$$h(a) = \frac{t}{2} f(a) - \frac{at^{3/2}}{3}\left\{\frac{a^2 e^{-a^2}}{\sqrt{\pi}} - a(1 + 2a^2) + \frac{2e^{-a^2}}{\sqrt{\pi}} - 2a \operatorname{erfc}(a)\right\},$$

where $a = \eta$ or $a = \eta\sqrt{\Pr}$ and $b = i\omega$ or $b = -i\omega$.

Numerical values of X_p are computed by using (15) and are plotted on Figs. 5 to 7. On Fig. 5 the effects of ωt on X_p are shown

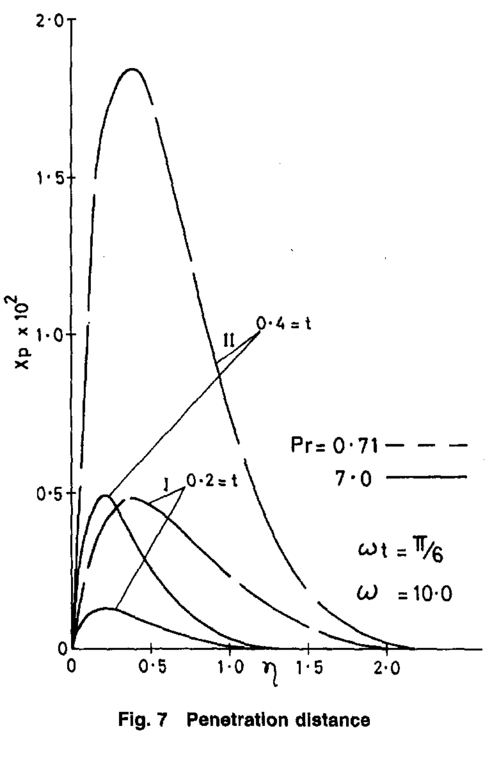

Fig. 7 Penetration distance

and we observe that the penetration distance is more for Pr = 0.71 (air) as compared to that in case for Pr = 7.0 (water), but it decreases with an increase in ωt. On Fig. 6, the penetration distance is shown for different values of the frequency ω. It is seen from this figure that the penetration distance decreases with increasing the frequency ω for both air and water. However, the decrease in X_p is more prominent when ω increases from 5 to 10 and X_p is less affected when ω increases from 10 onwards or at large values of ω. Hence we conclude that the penetration distance is more influenced at small values of the frequency ω and less influenced at large values of frequency ω. On Fig. 7, the penetration distance is shown for different values of time t when ωt and ω are held constant and it is seen that the penetration distance decreases with increasing the Prandtl number Pr.

3 Conclusion

1. Transient velocity decreases with increasing ωt, ω, or Pr.
2. The penetration distance is also found to decrease with increasing ωt, ω, or Pr, but the penetration distance is more affected in case of fluids with small Prandtl number.

References

Chung, P. M., and Anderson, A. D., 1961, "Unsteady Laminar Free Convection," ASME Journal of Heat Transfer, Vol. 83, pp. 474–478.

Gebhart, B., Jaluria, Y., Mahajan, R. L., and Sammakia, B., 1988, *Buoyancy induced flows and transport*, Hemisphere Publishing Corporation, New York, p. 731.

Goldstein, R. J., and Briggs, D. G., 1964, "Transient free convection about vertical plates and circular cylinders," ASME Journal of Heat Transfer, Vol. 86, pp. 490–500.

Hellums, J. D., and Churchill, S. W., 1962, "Transient and steady state free and natural convection," *Numerical solutions, Pt. 1, Amer. Inst. Chem. Engg. Journal*, Vol. 8, pp. 690–695.

Hellums, J. D., and Churchill, S. W., 1961, "Computation of natural convection by finite difference methods," *International Development in Heat Transfer*, Part V, Sec. B, ASME, New York, pp. 985–994.

Illingworth, C. R., 1950, "Unsteady laminar flow of gas near an infinite plate," *Proc. Camb. Phil. Soc.*, Vol. 46, pp. 603–613.

Jaluria, Y., and Gebhart, B., 1974, "On transition mechanisms in vertical natural convection flow," *Journal of Fluid Mechanics*, Vol. 66, pp. 309–337.

Siegel, R., 1958, "Transient free convection from a vertical flat plate," *Transactions of the ASME*, Vol. 30, pp. 347–359.

Sparrow, E. M., and Gregg, J. L., 1960, "Nearly Quasi-Steady Free Convection Heat Transfer in Gases," ASME Journal of Heat Transfer, Vol. 82, pp. 258–260.

Onset of Convection in a Fluid Saturated Porous Layer Overlying a Solid Layer Which is Heated by Constant Flux

C. Y. Wang[1]

The thermoconvective stability of a porous layer overlying a solid layer is important in seafloor hydrothermal systems and thermal insulation problems. The case for constant flux bottom heating is considered. The critical Rayleigh number for the porous layer is found to increase with the thickness of the solid layer, a result opposite to constant temperature heating.

[1] Departments of Mathematics and Mechanical Engineering, Michigan State University, East Lansing, MI 48824. Mem. ASME.

Contributed by the Heat Transfer Division for publication in the Journal of Heat Transfer. Manuscript received by the Heat Transfer Division, Dec. 1, 1998; revision received, May 11, 1999. Keywords: Geophysical, Heat Transfer, Instability, Natural Convection, Porous Media. Associate Technical Editor: T. Tong.

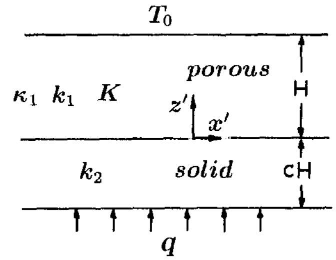

Fig. 1 Constant flux heating of a porous layer overlying a solid layer

1 Introduction

Thermoconvective stability of a porous medium is important in thermal insulation problems and geothermal energy problems. For a porous layer heated from below, heat transfer is through conduction only until a critical Rayleigh number R_c is reached. Above this critical value convection starts and the heat transfer is increased dramatically. See, e.g., Cheng (1978) and Gebhart et al. (1988) for a review.

For a homogeneous unbounded porous layer under isothermal conditions, Horton and Rogers (1945) found R_c to be $4\pi^2$. The case of two different porous layers was studied by Gheorghitza (1961) and Masuoka et al. (1978) who found incipient convection may be confined to a specific layer. The multiple porous layers case was considered by McKibbin and O'Sullivan (1980). They concluded that large permeability differences are required to alter the basic homogeneous incipient mode. The case of one porous layer overlying a solid layer is important in seafloor hydrothermal systems (Rosenberg et al., 1993). The first analysis of the effect of the solid layer was done by Donaldson (1962) and extended by McKibbin (1983).

All the above references considered constant temperature bottom heating. Few papers addressed the constant flux bottom heating. This situation occurs, for example, when the heat source is an exothermic chemical reaction. In thermal insulation problems, the heat source may be electric heating elements. Nield (1968) and Ribando and Torrance (1976) considered the constant flux heating of an infinite porous layer, and concluded the critical Rayleigh number is lowered to 27.1. Wang (1998) studied case where the porous layer is constrained by a cylindrical enclosure, showing R_c is increased from 27.0976, but not monotonically, as the radius is decreased. The present paper considers the case of a porous layer overlying a solid layer which is bottom heated by constant flux. The results would complement those of McKibbin (1983) which is bottom heated by constant temperature.

2 Formulation

Figure 1 shows a porous layer of thickness H overlying a solid layer of thickness cH. Let the bottom of the system be heated with constant flux q and the top be kept at constant temperature T_0. The Darcy-Boussinesq equation for the porous layer is

$$u' = -\frac{K}{\mu} p'_{x'} \tag{1}$$

$$w' = -\frac{K}{\mu}\{p'_{z'} + \rho_0 g[1 - \delta(T'_1 - T_0)]\}. \tag{2}$$

Here (u', w') are velocity components in the directions (x', z'), respectively, K is the permeability, μ is the viscosity, p' is the pressure, g is the gravitational acceleration, δ is the fluid coefficient of thermal expansion, T'_1 is the temperature of the porous layer, and ρ_0 is the fluid density at the top temperature T_0. The energy equation is

$$u'T'_{1x'} + w'T'_{1z'} = \kappa_1(T'_{1x'x'} + T'_{1z'z'}) \tag{3}$$