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Pieter Heres[1] and Wil Schilders[2]

[1] Eindhoven University of Technology, Department of Mathematics and Computer Science, p.j.heres@tue.nl
[2] Philips Research Laboratories, Eindhoven

Summary. Krylov subspace methods are well-known for their nice properties, but they have to be implemented with care. In this article the mathematical consequences encountered during implementation of Krylov subspace methods in an existing layout-simulator are discussed. Briefly, the representation in a circuit is visited and two methods to avoid parts of the redundancy are drawn.

Key words: Model Order Reduction, Krylov subspace methods, orthogonalisation, circuit simulation

1 Introduction

Wireless applications are gaining interest in the electronic industry nowadays. Integration plays a more and more important role into the design of these applications. Technologies like SoC and RF-SiP are needed to meet the demands set by the consumer market. All this makes an accurate and fast modelling of the electromagnetic (EM) effects of passive electronic structures needed.

The EM analysis of arbitrary shaped layouts can be calculated with existing tools. One specific example of such a tool was the drive for our research. The Boundary Element Models initially generated by this tool can be simply too large to be handled. Several reduction methods can be applied to make the treatment of these models feasible.

In stead of the already implemented reduction method, Krylov subspace methods, like the methods presented in [3] and [4], were proposed to be implemented in the layout simulator. These methods were chosen because of their well-known properties with respect to preservation of stability and passivity. In this article mathematical consequences encountered during implementation are discussed.

2 Equation setting

We consider the following set of equations:

$$\begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & -\mathbf{L} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix} + \begin{bmatrix} \mathbf{G} & \mathbf{P}^T \\ \mathbf{P} & -\mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \mathbf{u} \tag{1}$$

In this system the values for the capacitive elements are in the matrix $\mathbf{C} \in \mathbb{R}^{n \times n}$, the inductive values are in $\mathbf{L} \in \mathbb{R}^{m \times m}$. The matrices $\mathbf{G} \in \mathbb{R}^{n \times n}$ and $\mathbf{R} \in \mathbb{R}^{m \times m}$ represent the resistive values. $\mathbf{P} \in \mathbb{R}^{m \times n}$ is an incidence matrix consisting of 1's, -1's and 0's. $\mathbf{u}$ is the input signal. The state space vector consists of voltages $\mathbf{v}$ and currents $\mathbf{i}$. In this way the system represents an RCL-circuit. Despite the precise formulation in (1), the methods mentioned in this paper are generally applicable to systems of this form:

$$\begin{aligned} \mathbf{C}\frac{d}{dt}\mathbf{x}(t) &= -\mathbf{G}\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{B}_o^T\mathbf{x}(t); \end{aligned} \tag{2}$$

they are not specific for circuits.

The latter system is a Linear Time Invariant system. Because the matrix $\mathbf{C}$ can be singular, this can be a Differential Algebraic Equation (DAE). A common way to solve these systems is to transform them to the frequency domain with a Laplace transform:

$$\begin{aligned} (\mathbf{G} + s\mathbf{C})\mathbf{X}(s) &= \mathbf{B}_i\mathbf{U}(s) \\ \mathbf{Y}(s) &= \mathbf{B}_o^T\mathbf{X}(s); \end{aligned} \tag{3}$$

After elimination of the state space $\mathbf{X}(s)$ a transfer function is obtained:

$$\mathbf{H}(s) = \mathbf{B}_o^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B}_i \tag{4}$$

This function gives a direct relation between the input and the output of the system and is therefore representative for the behaviour of the system in frequency domain. If the system has more than one inputs and outputs, the transfer function is a matrix representing the transfers from one port to the other. Typically, one tries to approximate the behaviour of this transfer function.

3 Model Order Reduction

The aim of Model Order Reduction is to capture the essential features of a large model into a much smaller approximation. Thus, the large system is replaced by a smaller approximation, with the same amount of input signals, i.e. ports in terms of a circuit and a comparable behaviour.

The idea behind Krylov subspace methods is to generate a (basis for a) Krylov space. A Krylov space is defined as:

$$\mathcal{K}_n(\mathbf{b}, \mathbf{A}) = [\mathbf{b}, \mathbf{AR}, \dots \mathbf{A}^n\mathbf{b}] \tag{5}$$

Next, an orthonormal basis of this small space is calculated and the system matrices are projected onto this basis. Due to space limitations for the publication, we refer the reader to [5] for the issues induced by the orthogonalisation of the Krylov space.

If a system has more than one port, $\mathbf{B}$ is a matrix; the number of the columns in $\mathbf{B}$ is equal to the number of ports, say p. In that case the Krylov space consists of blocks: with every iteration a block of p columns in added to the Krylov spac. This makes our approximation p columns and rows larger.

Well-known Krylov subspace methods in chronological order of publication are PVL [2], PRIMA [4] and Laguerre-SVD [3]. PVL and PRIMA make use of the fact that the transfer function can be written as:

$$\mathbf{H}(s) = \mathbf{B}_o^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B}_i = \mathbf{B}_o^T(\mathbf{I} - (s - s_0)\mathbf{A})^{-1}\mathbf{R} \tag{6}$$

with $\mathbf{A} = -(\mathbf{G} + s_0\mathbf{C})^{-1}\mathbf{C}$ and $\mathbf{R} = (\mathbf{G} + s_0\mathbf{C})^{-1}\mathbf{B}_i$. With this formulation a Krylov space is generated, which represents the moments of the transfer function:

$$\mathcal{K}_q(\mathbf{R}, \mathbf{A}) = [\mathbf{R}, \mathbf{AR}, \dots \mathbf{A}^q\mathbf{R}] \tag{7}$$

Laguerre-SVD is based on the fact that the transfer function can be expanded into scaled Laguerre functions in frequency domain:

$$\mathbf{H}(s) = \mathbf{L}^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B} = \frac{2\alpha}{s+\alpha}\mathbf{L}^T \sum_{n=0}^{\infty} \left((\mathbf{G} + \alpha\mathbf{C})^{-1}(\mathbf{G} - \alpha\mathbf{C})\right)^n (\mathbf{G} + \alpha\mathbf{C})^{-1}\mathbf{B}_i \left(\frac{s-\alpha}{s+\alpha}\right)^n \tag{8}$$

From this expansion very naturally a definition for a Krylov subspace araises. The starting vector is then $\mathbf{R} = (\mathbf{G} + \alpha\mathbf{C})^{-1}\mathbf{B}_i$ and the generating matrix $\mathbf{A} = (\mathbf{G} + \alpha\mathbf{C})^{-1}(\mathbf{G} - \alpha\mathbf{C})$. Then the Krylov space is defined as in (7).

Advantages of Krylov subspace methods are that they are very generally applicable, because $\mathbf{C}$ and $\mathbf{G}$ do not need to be regular. Furthermore, they are relatively cheap. Because it can be proven that the moments in the moment expansion of the transfer function are preserved, the methods are accurate. For PRIMA and Laguerre-SVD it is proven that stability and passivity of the system are preserved during reduction. Especially this last property is important in the implementation of Model Order Reduction methods into the layout simulator. PVL convergences faster than PRIMA, but stability can be lost in this methods. Therefore, in this setting PVL is left out of consideration.

In spite of these advantages, there are some severe disadvantages known for Krylov subspace methods. First of all no error bound is known in general. For PVL a bound is known and published in [1]. In PRIMA and SVD-Laguerre it is not known when to stop. Hence, easily an unnecessarily large approximation is generated.

4 Validation of results

Because the original model can be represented as a circuit, namely an RCL-circuit, and because the EM layout simulator uses a representation of its reduced model in a circuit, it is desired to represent our reduced model in terms of a circuit. This circuit representation enables us to use the speed of existing circuit simulators, in evaluating the behaviour of the reduced model.

In the original model, see (1) the state space vector consisted of voltages and currents. After projecting the system onto a smaller space, these voltages and currents are mixed and therefore the physical meaning of the reduced model is lost. Hence, it is not possible to represent the system without making use of controlled sources or controlled components. Nevertheless, we obtained a circuit representing the reduced model and this representation is tested and compared to the output of the layout simulator. The results in frequency domain can be made as accurate as wanted, together with the increasing size of the reduced system. More important is that the results for a transient analysis is stable. This was not the case for the existing reduction method, which gave a good approximation in frequency domain, but could be instable in time domain.

5 Redundancy

Next to the already mentioned disadvantages of Krylov subspace methods, there is another drawback to Krylov subspace methods. Because they do not carefully choose the needed information, a lot of information is incorporated in the smaller model which is not needed for a good approximation. So, even if we stopped the iterative process in time, the models are redundant. In our research we found two ways to avoid parts of this redundancy, without too much computational expenses.

The first proposal is a deflation of converged columns. Sometimes it can happen that a column is generated which already existed in the space. At that moment we want to stop iterating with this direction and want to be able to proceed with the other columns in the block. This convergence should be treated with care, because if we violate the basic property of Krylov spaces, the small approximation can become really cumbersome. In the Block Arnoldi Algorithm, used to generate the Block Krylov space a specialized QR, i.e. a rank-revealing QR step is substituted. In this way smaller approximations with the same transfer function can be generated.

Our second proposal is to remove insignifant poles, via an eigendecomposition of the reduced system. Because the reduced system is small,a full eigendecomposition can be calculated cheaply:

$$\mathbf{CV} = \mathbf{GV}\mathbf{\Lambda} \tag{9}$$

Here the diagonal matrix $\mathbf{\Lambda} \in \mathbb{C}^{q \times q}$ consists of the eigenvalues, where q is the size of the reduced system. The associated eigenvectors are in $\mathbf{V} \in \mathbb{C}^{q \times q}$. Once this decomposition is obtained, the transfer function can be written in a pole-residue expansion:

$$\mathbf{H}(s) = c + \sum_{j=1}^{q} \frac{r_j}{s - p_j}, \tag{10}$$

with r and $p \in \mathbb{C}$.

We saw that in this sum there are terms which do not contribute to the transfer function. This can be either because r_j is very small or p_j is very large. These poles are removed, which comes down to removing the associated columns from $\mathbf{V}$. Complex poles are always removed in conjugate pairs. Naxt a real basis is generated for the eigenvector matrix. This is finally used to project our reduced system on.

6 Conclusions

In this article we presented the mathematical challenges of implementing Krylov subspace methods in an existing layout simulator. We showed that Krylov subspace methods are efficient for the given examples, but have to implemented with care. Several adjustments can be implemented to the existing methods, to make them more efficient. There is an obvious need for realization. Realization enables the application of Model Order Reduction in time domain simulations of the EM behaviour.

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