Does Manufacturer Advertising Crowd-In or Crowd-Out Retailer Advertising? An Application of an Endogenous Prize Contest with Asymmetric Players

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Abstract

When a manufacturer advertises, what is the impact on retailer advertising? I analyze a contest model of advertising where total advertising by the manufacturer and by retailers determines market size, and the relative level of advertising by each retailer determines market share. If retailers are symmetric I show that there is a crowding-in effect so increased manufacturer advertising increases retail advertising. But if one retailer is stronger, then marginal increases in manufacturer advertising have a crowding-out effect on retailer advertising, while sufficiently large increases have a crowding-in effect by "jump-starting" competition between retailers for the larger market. Furthermore, asymmetric abilities in such contests can lead the weaker player to effectively drop out of the contest, thereby undermining the ability of increased prizes to increase effort by intensifying competition. More generally the model can be applied to other contests such as patent races or promotion tournaments where not just the probability of winning but also the value of winning depends on contest effort levels.

JEL: C72, D72

Key Words: asymmetric contests; endogenous prize; game theory

1 Introduction

Because retailer advertising has both market share and market size effects, retailers face conflicting incentives. The market share effect provokes each retailer to out-compete his rivals by advertising more, but the market size effect provides an incentive to free ride on his rival's advertising and advertise less. And if retailers differ in their ability to effectively attract customers to their store, how does this asymmetry affect advertising incentives?

Furthermore, how a manufacturer's use of advertising that increases market size affects the trade-off between these conflicting incentives, and thereby affects total retailer advertising is also of interest. By "sweetening the pot" does manufacturer advertising intensify market share competition and lead to higher advertising by retailers? Or does it reduce the incentive for retailers to build market size themselves and lead them to free-ride on the manufacturer as well as on each other?

For example, Lowes and Home Depot are retailers who compete in branded hand tools. Marketing effort by Lowes steals market share away from Home Depot while also expanding the overall branded hand tool market. If Home Depot responds with higher marketing effort to regain market share, Lowes' market share declines, but the size of the market still increases. If the manufacturer of branded hand tools, Black&Decker, promotes their new product through advertising, thereby increasing market demand, will the downstream competing retailers, Home Depot and Lowes, increase their own advertising to take advantage of expanded consumer demand, or will they instead reduce their own advertising and free-ride on Black&Decker? Should Black&Decker's decision and magnitude of advertising depend on the relative asymmetry between the rival retailers? (*Advertising Age* 2007)

To address these questions and more easily apply the results to other competitive environments, I adopt a contest theory approach. I focus on the contestable market for a product of a single manufacturer. Since retailers consider all elements of the market mix, I use the term advertising interchangeably with marketing effort. Contests usually involve two or more players competing for a prize, where the player exerting the highest effort usually wins the prize. In the advertising game, the market size (i.e., the prize) depends on the total advertising of the retailers and manufacturer, whereas the market share depends only on the relative advertising of the

retailers. This increased market size benefits both the retailers and the manufacturer through increased unit sales. However, as retailers become asymmetric in advertising effectiveness, retailer competition might weaken, reducing overall advertising and unit sales (Krishnamurthy 2000; Espinosa and Mariel 2001; Bass et al. 2005; Geylani et al. 2007).

I build on a standard Tullock (1975, 1980) ratio-form contest model of competition between players. Previous research using contest models of advertising include Friedman (1958), Bell, Keeny, and Little (1975), Monahan and Sobel (1994), and Krishnamurthy (2000). Following Rosen (1986), Baik (1994, 2004) and Nitzan (1994), I depart from the original symmetry assumption in such contests and allow for one player to be stronger than the other, i.e., one of the retailers is more effective in using advertising to attract customers. Furthermore, I follow Chung (1996) and Morgan (2000), and depart from the assumption that the prize is exogenous and instead assume that the prize is increasing in the effort levels of the players, i.e., the size of the market is increasing in the amount of advertising.

I show that the combination of asymmetric ability and an endogenous prize change two basic results from contests. First, in a standard ratio-form contests for a common-value prize, both players always exert the same effort even for arbitrarily large ability differences. So a weak player will still put forth effort, albeit very small, even though his probability of winning is very small. The stronger player also competes even though a victory is almost assured. This surprising result is sometimes difficult to accept in application. For example, many elections go unchallenged, markets seem to lack sufficient competition, and requests for proposals may attract only one bidder. If instead, the prize is endogenous to effort, I find that a sufficiently weak player will exert zero effort. Because the prize is increasing in total effort, the stronger player will always want to exert some effort even if the weaker player does not. Anticipating this, and knowing that substantial effort is needed to fight the stronger player, the weaker player finds it best to exert no effort. In the advertising application this means that sufficiently asymmetric advertising effectiveness of the retailer marketing effort can completely eliminate the competition between retailers.

Second, a marginal increase in the prize offered in a contest normally leads to a proportional increase in effort by contestants, but with asymmetric abilities and an endogenous prize this effect

can be reversed. Since the prize is increasing in total effort by the contestants, an exogenous addition to the prize allows the contestants to free-ride on this increase and reduce their own efforts. This tradeoff between increasing efforts in response to an increased prize and decreasing efforts in response to free-riding on the additional prize creates tension for the contestants. Similarly, the contest designer must consider the magnitude and intensity of the prize increase. In the context of advertising, advertising by the manufacturer first leads to a decrease rather than increase in retailer advertising if retailers are sufficiently asymmetric. Only if manufacturer advertising is sufficiently large to "jump start" competition between the retailers does it lead to an increase in total retailer advertising.

While the model is presented in the context of retailers and a manufacturer, it is easily applicable to other contest environments where players have asymmetric abilities and the prize is an increasing function of total effort. In R&D contests, the probability of an innovation and the value of the innovation is increasing in research effort. The government should consider how asymmetries in research institutions eliminate competition between researchers if prizes are not sufficiently high. (Fullerton McAfee 1999; Baye and Hoppe 2003; Fu and Lu 2010) In career promotion contests the probability of advancement is increasing in investment in one's skill, and the value of being promoted is also a positive function of how much skill one has acquired. Here, too, a firm must consider how asymmetric skill levels can induce some contestants to completely drop out if promotion incentives are not sufficiently strong. (Lazear and Rosen 1981; Nalebuff and Stiglitz 1983) In CEO selection, Tsoulouhas, Knoeber, and Agrawal (2007) show that the effort of the internal candidates contributes to the future profitability of the firm, increasing the value of the position. Similarly, the probability of winning a primary election in political contests is increasing in campaign expenditures, and such expenditures also build name recognition of the political party for the general election. The model supports the simple intuition that if primary contestants are sufficiently asymmetric then the primary might not be contested at all.

2 Model

2.1 Symmetric Retailers

I first consider the case of two symmetric retailers who compete in a standard imperfectly discriminating Tullock contest for market share with advertising. Advertising expenditures (general marketing effort) of retailer W, retailer S, and the upstream manufacturer M, are denoted a_W, a_S, and a_M, respectively. The size of the market value is endogenous to the sum total advertising expenditures, $v\left(\sum a_i\right)$. The functional form of market size, v, has the properties $v' > 0$, $v'' < 0$, and $v(0) = 0$ in all advertising expenditures a_W, a_S, $a_M \geq 0$.[1] To ensure an interior solution, $v'(0) > 1$ and $v'(\infty) \leq 0$. In other words, the manufacturer's advertising and both retailers' advertising have the same marginal impact on the size of the market. It also means that an increase in total advertising increases market size (v), but at a decreasing rate. The market is defined as the contestable market between the two retailers of the particular manufacturer's brand. The market share of retailer W is based on its own advertising relative to total retailer advertising and is given by $f = a_W/(a_W + a_S)$. Similarly, the market share of retailer S is simply $1 - a_W/(a_W + a_S)$.[2] In this way, an increase in a_W will increase retailer W's market share and an increase in a_S reduces its market share. Retailer W's profit function is written as

$$\pi_W = \frac{a_W}{a_W + a_S}v - a_W. \tag{1}$$

Equation (1) defines retailer W's profit where the market size, v, and its market share, $a_W/(a_W + a_S)$, are dependent on its own advertising. The average cost-per-view of advertising is assumed to be linear and equal to one. Retailer W's first-order condition for profit maximization

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[1] This assumption contrasts with Hirshleifer (1991) where the value of the common prize is increasing in production resources, but decreasing in fighting resources. Each player in his paper is therefore restricted by a resource endowment budget constraint. Krishnamurthy (2000) is similar in the use of a budget constraint on marketing expenses allocation. Both models examine a tradeoff of expenses whereas the current paper only has one expense that is compared to an expected payoff, and therefore does not have a budget constraint.

[2] This us/(us+them) is commonly referred to as the Tullock ratio form specification in contests (but has been utilized as early as Friedman, 1958) and is frequently used in the literature since a pure strategy equilibrium is found as the solution and contains certain axiomatic properties relevant to competitive environments.

is

$$\frac{\partial \pi_W}{\partial a_W} = \frac{a_S}{(a_W + a_S)^2} v + \frac{a_W}{a_W + a_S} v' - 1 = 0. \tag{2}$$

Equation (2) shows that a change in the retailer's advertising has two effects on its profit. The first term increases retailer W's market share relative to its rival. This market share effect is positive, but diminishes as the amount of its own advertising increases. The second term increases the value of the market through the market size function. By assumptions of v, this market size effect is positive and diminishing as well. When $a_S = 0$, W's best response satisfies the condition $v' = 1$, and is positive. The third term is simply the marginal cost of advertising. Profits are maximized when the sum of the two effects equal marginal cost.[3] The second-order condition for profit maximization is

$$\frac{\partial^2 \pi_W}{\partial a_W^2} = -\frac{2a_S}{(a_S + a_W)^3} v + \frac{2a_S}{(a_W + a_S)^2} v' + \frac{a_W}{a_W + a_S} v'' < 0. \tag{3}$$

When retailer W advertises, v expands due to an increase in demand for the product. This market size effect attracts retailer S to advertise more and steal market share away from retailer W. Recall that retailer S's advertising is also increasing the market size, albeit at a decreasing rate. This increase in market size, and simultaneous loss of market share, causes retailer W to increase its own advertising even further, and so on. Since v is concave in total advertising, the incremental returns from advertising to both retailers gets smaller. This diminishing marginal benefit eventually equals the marginal cost of advertising to each retailer in equilibrium. The cross-partial of retailer W's profit functions is

$$\frac{\partial^2 \pi_W}{\partial a_W \partial a_S} = \frac{a_W - a_S}{(a_W + a_S)^3} v + \frac{a_S - a_W}{(a_W + a_S)^2} v' + \frac{a_W}{a_W + a_S} v''. \tag{4}$$

When their advertising expenditures are equal, as in the symmetric equilibrium, only the third term remains. Using the assumptions of v the third term is negative, implying that advertising efforts are strategic substitutes, and that an increase in retailer S's advertising will decrease the profit maximizing advertising level of retailer W. Since they are symmetric, an increase in retailer W's advertising will decrease the profit maximizing advertising level of retailer S and vice versa. Note that when v is exogenous, only the first term remains.

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[3] The second-order condition is also met for maximization of the profit function and similarly for retailer S.

It is standard to find the symmetric equilibrium (Chung 1996) where the equilibrium advertising for each retailer in the endogenous prize contest is denoted as $a^* = a_W = a_S$. The size of the market is $v(2a^*)$ and is split evenly between the retailers. Recall that retailers advertise to increase market share and market size, but suppose market size was not endogenous to advertising and instead was fixed at $v(2a^*)$. The retailers would then only compete for market share, with total advertising of $\frac{v(2a^*)}{2}$.[4] Because the marginal benefit of increasing the market size is eliminated (the second term in Equation 2), retailers spend less on advertising and earn higher profits than if the market size were endogenous, shown as $\frac{v(2a^*)}{2} < a^*$. By allowing advertising to also increase the size of the market both retailers will advertise more.

Suppose there were only one retailer in the market, a monopolist, then denote a_μ as the profit maximizing advertising amount, such that $v'(a_\mu) = 1$. Then, it standard to assert that $a_\mu < 2a^*$. Hence, over-advertising by the retailers is a result of the competition for the market they create.[5] So the manufacturer benefits from the retailers competing since more advertising is being spent than if there was just one retailer. Note that under a monopoly specification, the only function advertising serves is generic, i.e., it expands the market. By introducing competition, the function of advertising also includes brand advertising, which increases total expenditures, but has a second-order effect of expanding the market further.

2.2 Asymmetric Retailers

We now examine when the retailers differ in their effectiveness of advertising and each retailer knows the other retailer's ability difference. This asymmetry may arise from spatial differences between stores or from operating in a different sales environment.[6] That is, the retailer's ad-

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[4]Baik (2004) showed that in a ratio form contests the proportion of players' efforts to the prize was not dependent on the size of the prize, but on the asymmetry between the players. In a standard ratio form contest of two symmetric players who have same valuation of the prize, each player exerts effort equal to one fourth of the prize.

[5]This is similar to Chung (1996) where social welfare is reduced when there is competition for a public good.

[6]This assumption should not be confused with a poor marketing mix strategy, in which case, the retailer could simply find a better marketing firm. The asymmetry is assumed to come strictly from a natural endowment of the retailers that is impossible to change in the short-term. This is contrasted with Geylani et al. (2007) where retailers are asymmetric based on bargaining power. Banerjee and Bandyopadhyay (2003) define retailer

vertising increases the total market, or awareness, of the product, but is not as effective in attracting that market to their own store. We define the relative effectiveness, $r = \frac{e_W}{e_S}$, where e_W and e_S are advertising abilities of retailer W and S, respectively. And $0 \leq r \leq 1$ since $e_W \leq e_S$, without loss of generality. As such, denote W and S as the weak retailer and strong retailer, respectively. Therefore, when $r < 1$ then retailer W's advertising is less effective than the advertising of retailer S. For example, if $r = 1/2$ then retailer S's advertising is twice as effective as retailer a_W's advertising. And if $r = 1$, then the retailers are identical in their advertising effectiveness. The parameter r, to some extent, measures the relative market power between retailers. This could also be driven by an historically developed characteristic such as differences in brand equity between the retailers.[7] Because the effectiveness parameter directly impacts the ability to capture market share, market share of the weaker retailer is is now given by $ra_W/(ra_W + a_S)$, and its profit function is

$$\pi_W = \frac{ra_W}{ra_W + a_S} v - a_W. \tag{5}$$

Retailer W's first-order condition for profit maximization is

$$\frac{\partial \pi_W}{\partial a_W} = \frac{ra_S}{(ra_W + a_S)^2} v + \frac{ra_W}{ra_W + a_S} v' - 1 = 0. \tag{6}$$

Notice the first term, the market share effect, and the second term, the market size effect, are both decreasing as r decreases. This decline in the marginal benefit to the weaker retailer leads to a reduction in his own advertising. The second-order condition for profit maximization is

$$\frac{\partial^2 \pi_W}{\partial a_W^2} = -\frac{2r^2 a_S}{(ra_W + a_S)^3} v + \frac{2ra_S}{(ra_W + a_S)^2} v' + \frac{ra_W}{ra_W + a_S} v'' < 0, \tag{7}$$

and is satisified only when $r > \frac{a_\mu}{v(a_\mu)}$.

Note that if both retailers advertise the same amount the market share effect of retailer a_W becomes smaller as the asymmetry between the two retailers increases, (i.e., as r decreases).

Retailer S's profit function is

$$\pi_S = \frac{a_S}{ra_W + a_S} v - a_S, \tag{8}$$

and for first-order condition for profit maximization is

$$\frac{\partial \pi_S}{\partial a_S} = \frac{ra_W}{(ra_W + a_S)^2} v + \frac{a_S}{ra_W + a_S} v' - 1 = 0. \tag{9}$$

Here, the market share effect is still decreasing as r gets smaller, but the market size effect is increasing. When $a_W = 0$, the solution that satisfies Equation (9) is the single-retailer advertising amount shown in Section 1.1, $a_S = a_\mu$. Only a corner solution exists for $r < \frac{a_\mu}{v(a_\mu)}$ leading to the following proposition.

Proposition 1 *(i) For sufficiently symmetric retailers, both the weak and strong retailers advertise in the unique equilibrium. (ii) For sufficiently asymmetric retailers, the weaker retailer does not advertise, and the stronger retailer advertises at the monopoly level in the unique equilibrium.*

Proof of Proposition **1** is in the appendix. The results of Proposition **1** indicate that when the asymmetry between the two retailers becomes sufficiently large the weaker retailer does not compete, and the stronger retailer, being the only retailer in the market, will advertise at the monopoly level $a_S = a_\mu$ as illustrated in Figure 1.[8] This is consistent with Bass et al. (2005) where, as retailers become more asymmetric, total generic advertising increases converging to the most profitable amount for the industry (i.e., only one retailer advertises).[9] This asymmetry increases total retailer profits due to less competition. From the manufacturer's perspective, however, total advertising is suboptimal due to lower sales and smaller profits.

Note that when $r \leq \frac{a_\mu}{v(a_\mu)}$ the weaker retailer no longer advertises. That is, when the advertising effectiveness of the strong retailer is sufficiently greater than the weaker retailer's effectiveness, his best response is to advertise $a_S = a_\mu$. Retailer W's equilibrium advertising

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[8]This does not mean that the weaker retailer goes bankrupt. Recall that the market is defined as the contestable market between the retailers. We assume they already have their own loyal customer base or even other brands with which they generate sufficient profits for business.

[9]Clearly, as the retailers become more asymmetric their total advertising decreases, but the function of advertising shifts strictly to generic.
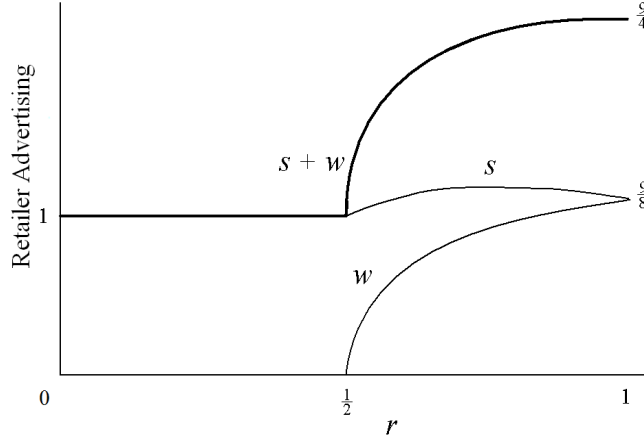
Figure 1: Retailer Advertising as a Function of Retailer Symmetry for $v = 2\left(a_S + a_W\right)^{\frac{1}{2}}$.

decreases as the retailers become more asymmetric. The advertising of the strong retailer, on the other hand, initially increases, then decreases as the asymmetry becomes great. The net effect of these two changes is a decrease in total advertising by the retailers.

The decrease in total advertising clearly lowers the overall demand for the product. However, profits to the strong retailer are increasing faster than the weaker retailer's profits are decreasing, and total profits converge to the monopoly case. This is attributed to the stronger retailer's market share strictly increasing as the retailers become more asymmetric. As a result, the stronger retailer's advertising functions mainly to increase market size (i.e., generic advertising) rather than competing for market share. The weaker retailer's incentive to compete diminishes as its own advertising becomes less effective in gaining market share. Any market expansion from its advertising is extracted by the stronger retailer overwhelming the weaker retailer by its dominant marketing effectiveness.

3   Manufacturer Advertising

What are the strategic responses of the retailers when the manufacturer engages in its own ad campaign? In standard contests, there is a fixed prize that is competed for through effort. I consider an initially fixed prize and is still endogenous to the players' efforts. The manufacturer chooses an advertising amount a_M in the first stage. The retailers, observing a_M, simultaneously

choose their own advertising expenditures.[10] Since yhe market size follows the same production function in Section 2.1, clear benefit to the manufacturer is that an increase in her own advertising increases sales. However, it is not clear how the retailers would change their own advertising policies. By intuition, the manufacturer's increase in advertising would cause the retailers to cut back on their own advertising, essentially free-riding on the success of her ad campaign. On the other hand, the retailers may increase their advertising to gain a larger market share of a larger market. Using a symmetric Nash Equilibrium solution concept, the retailers' advertising expenditures would increase due to an increase in the valuation of the market, and leads to the following proposition.

Proposition 2 *For symmetric retailers, an increase in manufacturer advertising increases retailer advertising.*

Proof of Proposition 2 is in the appendix. The implication of Proposition 2 is that for any amount of manufacturer advertising, retailers do not reduce their advertising expenditures. Instead, retailers step up their own advertising, expanding the market size further. The manufacturer increases the market size thereby inducing the retailers to higher equilibrium aggregate advertising. This crowding-in effect gives the manufacturer a double benefit. She directly benefits from the increase in sales from her own advertising, and indirectly benefits from the increase in sales from increased retailer advertising. In other words, the manufacturer intensifies competition by increasing the market size.

Note that as the manufacturer's advertising gets large, $v' \to 0$, reducing the marginal effect of retailers' advertising on market size. However, the marginal effect of a retailer's advertising on his market share increases. In other words, the manufacturer's advertising crowds-out the generic function of the retailers' advertising while still increasing total retailer advertising. Hence, the manufacturer amplifies the over-advertising result.

Proposition 2 also shows that the retailer's market share effect dominates the market size effect. Since v' is less than marginal cost in equilibrium, the net benefit must be negative from any additional advertising expended by the retailers in response to the manufacturer's

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[10]See McGuire and Staelin (1983) for the characterization of a manufacturer Stackelberg model.

expenditures. It follows that any increase in retailer advertising is driven by maintaining market share of a larger market size. Clearly, the competition between the retailers prevents them from free-riding on the manufacturer's advertising.

What then is the manufacturer's advertising decision when retailers are asymmetric? It becomes problematic for the manufacturer as asymmetry increases, total retailer advertising decreases, since this leads to less sales. I have shown that total retailer advertising decreases as relative advertising effectiveness decreases. The proactive response by the manufacturer might be to engage in its own advertising campaign, but retailer asymmetry affects their response.

At $r \leq \frac{a_\mu}{v(a_\mu)}$ only the strong retailer advertises at $a_S = a_\mu$, but both retailers' equilibrium advertising expenditures increase with an increase in manufacturer's advertising. This follows directly from Proposition 2 and is shown analytically in the appendix. So it is clear that at the asymmetry threshold, when the stronger retailer is advertising and the weaker retailer is indifferent, equilibrium efforts for both retailers are increasing in the manufacturer's advertising. Increasing the size of the market increases the retailers' incentives to advertise more to gain market share. Since, both retailers are already advertising, any manufacturer advertising will lead to increased total retailer advertising in equilibrium. The following proposition highlights the impact of the manufacturer's advertising decision on retailer advertising.

Proposition 3 *i) For sufficiently symmetric retailers, manufacturer advertising crowds-in total retailer advertising. (ii) For sufficiently asymmetric retailers, manufacturer advertising first crowds-out then crowds-in total retailer advertising.*

Proof of Proposition 3 is in the appendix. It is therefore in the interest of the manufacturer to advertise when retailers are not sufficiently different in advertising effectiveness for $r > \frac{a_\mu}{v(a_\mu)}$. This crowding-in effect enhances the return on advertising to the manufacturer in the form of increased sales. When retailers are sufficiently asymmetric (i.e., $r \leq \frac{a_\mu}{v(a_\mu)}$), however, manufacturer advertising crowds-out the stronger retailer's advertising. Since the stronger retailer is a monopolist, the gains from extra advertising would be unrivaled. In fact, the stronger retailer already advertises at the profit maximizing level. Therefore, any increase in manufacturer advertising would cause the stronger retailer to respond by decreasing its own advertising dollar

for dollar, that is, their advertising would be perfect substitutes. From Equation (9), $v' < 1$ when $a_S = a_\mu$, $a_W = 0$, and $a_M > 0$. Since this is less than the marginal cost of advertising for retailer a_S, the monopoly spending $a_S = a_\mu$ is no longer optimal for $a_M > 0$.

However, only small levels of manufacturer advertising crowds-out retailer advertising. Suppose the manufacturer decides to advertise more than the monopoly allocation of advertising. Then, the optimal amount of advertising for the dominant retailer decreases to zero, leading to full free-riding. But then the weaker retailer only has to advertise a small amount to grab the entire market. The stronger retailer, anticipating this, also advertises, etc. The stronger retailer still wants to advertise so as not to lose market share to the weaker retailer. In a competitive setting, high manufacturer advertising does not completely crowd-out the retailer's advertising. Therefore, the crowding-out effect reverses when the manufacturer's advertising is sufficiently large.

The intuition behind Proposition 3 is that the manufacturer reduces the marginal benefit of advertising to the retailer below marginal cost, thereby causing him to reduce total advertising. As a result, the manufacturer's advertising crowds-out the stronger retailer's advertising and leads to complete free-riding on the manufacturer.There are two effects at work. The first effect results from the strong retailer cutting back on advertising to free ride on the manufacturer's advertising as previously described. This free-riding effect increases the profits of retailer S. The second effect stems from the stronger retailer wanting to maintain a profitable market share. The manufacturer's advertising eventually becomes large enough such that both retailers' advertising expenditures are increasing in a_M. The threshold amount of manufacturer advertising that reverses the crowding-out effect is

$$\bar{a}_M(r) = a_\mu - rv(a_\mu), \tag{10}$$

and is shown in the proof of Proposition 3. The optimal advertising expenditures of the retailers for any $a_M > \bar{a}_M$, therefore, is critical to the manufacturer's decision to advertise. At this point, retailers will increase their advertising to compete for market share of the expanded market just as was shown in the symmetric case. Figure 2 shows that for a given r when $a_M < \bar{a}_M$, the marginal impact of advertising crowds-out retailer advertising and when $a_M > \bar{a}_M$ the marginal impact of advertising crowds-in retailer advertising.
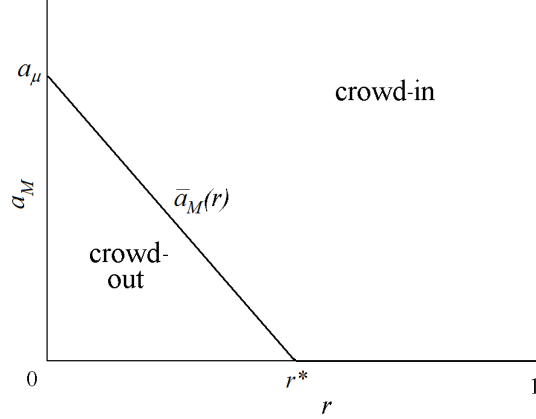
Figure 2: Marginal Effect of Manufacturer Advertising on Total Retailer Advertising as a Function of Retailer Symmetry

When manufacturer advertising exceeds a_M, the marginal impact of crowding-out total retailer advertising is reversed. The weaker retailer advertises to capture the larger market and the stronger retailer increases its own advertising to compete with the weaker retailer. Simultaneously solving for both retailers' optimal advertising strategies over a range of manufacturer advertising levels I find evidence for the conditional marginal impacts of manufacturer advertising on total retailer advertising. Figure 3 shows the total advertising of retailers dependent on manufacturer advertising for selected values of r. It is easy to see that for $r < \frac{a_\mu}{v(a_\mu)}$, total retailer advertising first decreases then increases as a_M increases.

Recall when the retailers are symmetric in advertising effectiveness and the manufacturer advertises nothing, retailer advertising is $a^* = a_W = a_S$. As shown in this section, when the manufacturer does advertise total retailer advertising increases. On the other hand, when the retailers are perfectly asymmetric, $r = 0$, total retailer advertising is crowded-out for all manufacturer advertising. Since the weak retailer is no longer competing in the market, retailer S free-rides on the market expansion. As noted in Proposition 3, when retailers are sufficiently asymmetric, the weaker retailer may or may not advertise depending on the value of a_M.
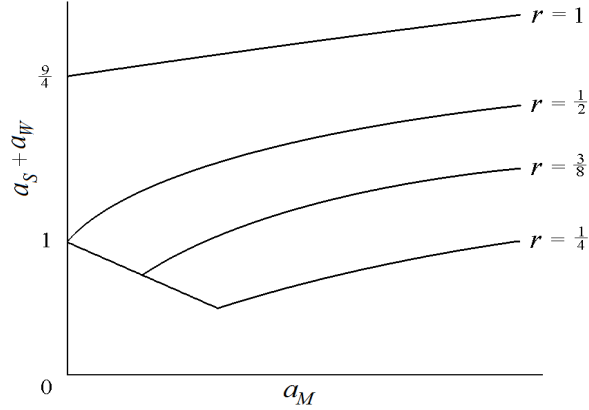
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Figure 3: Total Retailer Advertising as a Function of Manufacturer Advertising for various r values, $v = 2\left(a_W + a_S + a_M\right)^{\frac{1}{2}}$

4 Conclusion

I examine a model where a retailer's advertising both expands the market and increases their market share. However, as the disparity between retailers' relative marketing effectiveness increases, competition decreases, lowering total retailer advertising and causing market demand to be much smaller. When retailers are sufficiently different in marketing effectiveness, the weaker retailer does not advertise in the market and only the stronger retailer remains in the market. This under-advertising is less profitable to the manufacturer. If the manufacturer commits to only a modest marketing campaign, the stronger retailer's advertising is crowded-out. In this case, the manufacturer is better off doing nothing, relying only on the dominant retailer's advertising. On the other hand, for a significantly large marketing campaign by the manufacturer, the crowding-out effect is reversed. By sweetening the pot the manufacturer entices both the strong and weak retailers to compete for market share, leading to crowding-in effect of total retailer advertising.

The manufacturer must decide the most cost-effective method of increasing market demand given retailer asymmetry in marketing effectiveness. If retailers are sufficiently similar in their marketing effectiveness, I find that the main effect of manufacturer advertising intensifies market share competition, leading to higher total retailer advertising. From the perspective of retailers, such advertising aggravates the problem of over-advertising as each retailer tries to grab a larger

market share. From the perspective of the manufacturer, such advertising has a doubly positive effect on market size; both a direct effect from its own advertising and a crowding-in effect as retailers respond to the larger market by increasing their own advertising.

Modeling the contest prize as endogenous to asymmetric player efforts is currently not addressed in the contest literature and extends Chung's (1996) endogenous prize analysis with symmetric players. Since the weaker player drops out when the contest is sufficiently asymmetric, I also introduce a contest designer's effort in the first stage that establishes an initial prize. Even with a boost to productive effort, it is still possible where the weaker player does not compete. A significant increase in the value of the prize, however, induces both players to increase effort. This changes the players' equilibrium efforts in the second stage. Solving for the unique pure strategy Nash equilibrium over a range of retailer asymmetry for given manufacturer marketing effort contributes to the contest literature more generally.

These results can be applied to several other competitive environments more generally. For example, in patent races, firms collective research leads to more valuable products, request for proposals only have one bidder, only one candidate runs for political office, and a single employee is considered for promotion. These can partially be explained by the asymmetric nature of the contest and an endogenously determined prize. Furthermore, a possible policy of outside funding, similar to Fu and Lu (2010), is brought forth as a viable solution to the underfunding of the competition in terms of overall prize value.

There are a few natural extensions to this paper. First, other types of retailer asymmetry can also be considered, e.g., differing advertising costs, or even different profit margins. Although costs are just a variation of ability asymmetry, differences in valuation can also arise based on different profit margins and information about product demand. This should not qualitatively affect the results, but rather increase the threshold where the weaker player stops competing. Adding asymmetry in the market size function would have a similar effect. Secondly, identifying parameter conditions of the manufacturer's payoff function for advertising goes beyond the scope of this paper, but would provide guidelines for more precise decision making. This would make full use of a Stackelberg Equilibrium solution concept, but would be more sensitive the payoff function parameters. Generally, we would expect the manufacturer to advertise more if relative

costs are lower, regardless of symmetry, and advertise less if relative costs are higher than the retailers. Finally, cooperation between the retailers through the formation of ad groups could be explored in the presence of retailer asymmetry. In addition, the decision to use cooperative advertising by the manufacturer in this model structure would give further insight into its use.

Appendix

Lemma 1 *For profit functions π_W and π_S, Equations (5) and (8), respectively, there exists a unique pure strategy equilibrium for all $a_M \geq 0$ and $r \geq \frac{1}{2}$.*

Proof of Lemma 1   For analytical tractibility, I borrow from Chung (1996) the specific functional form characterizing the endogenous prize takes the form

$$v = \delta \left(a_W + a_S + a_M \right)^{\frac{1}{2}}, \tag{11}$$

where $\delta > 0$.

Part 1: I first show existence. Let v take the functional form as in Equation 11. The first derivative with respect to a_i is written as:

$$v' = \frac{\delta^2}{2v} > 0. \tag{12}$$

The second derivative with respect to a_i is simply:

$$v'' = -\frac{\delta^4}{4v^3} < 0. \tag{13}$$

An equilibrium exists if the payoff function is concave. Retailer W's profit function from Equation (5):

$$\pi_W = \frac{ra_W}{ra_W + a_S} v - a_W, \tag{14}$$

where a_W, a_S, and a_M are the actions taken by retailer W, retailer S, and the manufacturer, respectively and are in the set $a_W, a_S, a_M \in [0, \infty]$. Recall that $r \in [0, 1]$ is the relative marketing effectiveness of the weaker retailer. The first partial derivative for retailer W can be written as:

$$\frac{\partial \pi_W}{\partial a_W} = \frac{ra_S}{(ra_W + a_S)^2} v + \frac{ra_W}{ra_W + a_S} \frac{\delta^2}{2v} - 1, \tag{15}$$

and is increasing. The second partial derivative is written as:

$$\frac{\partial^2 \pi_W}{\partial a_W^2} = -\frac{2r^2 a_S}{(ra_W + a_S)^3} v + \frac{2ra_S}{(ra_W + a_S)^2} \frac{\delta^2}{2v} - \frac{ra_W}{(ra_W + a_S)} \frac{\delta^4}{4v^3}. \tag{16}$$

Since the third term is always negative, the first two terms are shown to be negative when

$$0 < ra_W + a_S \left(2r - 1 \right) + a_M, \tag{17}$$

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which is true when $r > \frac{1}{2}$. Note that it can still be negative for $r < 1/2$ when a_M is sufficiently large. Retailer W's cross partial is:

$$\frac{\partial \pi_W^2}{\partial a_W \partial a_S} = -\frac{r(a_S - ra_W)}{(a_S + ra_W)^3} v + \frac{r(a_S - a_W)}{(ra_W + a_S)^2} \frac{\delta^2}{2v} - \frac{ra_W}{(ra_W + a_S)} \frac{\delta^4}{4v^3}. \tag{18}$$

Evaluating retailers S's profit function:

$$\pi_S = \frac{a_S}{ra_W + a_S} v - a_S, \tag{19}$$

and its first partial derivative:

$$\frac{\partial \pi_S}{\partial a_S} = \frac{ra_W}{(ra_W + a_S)^2} v + \frac{a_S}{ra_W + a_S} \frac{\delta^2}{2v} - 1, \tag{20}$$

and its second partial derivative:

$$\frac{\partial^2 \pi_S}{\partial a_S^2} = -\frac{2ra_W}{(a_S + ra_W)^3} v + \frac{2ra_W}{(ra_W + a_S)^2} \frac{\delta^2}{2v} - \frac{a_S}{(ra_W + a_S)} \frac{\delta^4}{4v^3}. \tag{21}$$

Looking only at the first two terms to determine sign since third term is always negative we get:

$$0 < a_W (2 - r) + a_S + a_M,$$

which is true for all $r \leq 1$. Retailer S's cross partial is:

$$\frac{\partial^2 \pi_S}{\partial a_S \partial a_W} = \frac{r(a_S - ra_W)}{(a_S + ra_W)^3} v + \frac{r(a_W - a_S)}{(ra_W + a_S)^2} \frac{\delta^2}{2v} - \frac{a_S}{(ra_W + a_S)} \frac{\delta^4}{4v^3}. \tag{22}$$

$\square$

Part 2: Next I show the uniqueness of an equilibrium under retailer asymmetry with manufacturer intervention. According to the Gale-Nikaido Theorem (1965), and similar to Rosen (1965), if the Jacobian of first-order conditions of the payoff functions is negative quasi-definite for all actions, then the equilibrium that simultaneously solves the first order conditions, if one exists, is the unique equilibrium. The Jacobian matrix is written as:

$$J = \begin{bmatrix} \frac{\partial^2 \pi_W}{\partial a_W \partial a_W} & \frac{\partial^2 \pi_W}{\partial a_W \partial a_S} \\ \frac{\partial^2 \pi_S}{\partial a_W \partial a_S} & \frac{\partial^2 \pi_S}{\partial a_S \partial a_S} \end{bmatrix}. \tag{23}$$

If $\frac{\partial^2 \pi_W}{\partial a_W \partial a_W} \leq 0$ and $|J| \geq 0$ for all a_W, a_S, and a_M, then J is negative quasi-definite. Since $\frac{\partial^2 \pi_W}{\partial a_W \partial a_W} \leq 0$ from Equation (17), the condition:

$$|J| = \begin{vmatrix} \frac{\partial^2 \pi_W}{\partial a_W \partial a_W} & \frac{\partial^2 \pi_W}{\partial a_W \partial a_S} \\ \frac{\partial^2 \pi_S}{\partial a_W \partial a_S} & \frac{\partial^2 \pi_S}{\partial a_S \partial a_S} \end{vmatrix} \geq 0, \tag{24}$$

is satisfied for all $r \geq \frac{1}{2}$, $\delta \geq 0$, and for all $a_W, a_S, a_M \geq 0$. Hence, there exists a unique equilibrium. $\square$

Lemma 2 For any given functional form, there exists an ability symmetry, r, such that the weaker player does not exert effort and strong player exert positive effort.

Proof of Lemma 2 The profit function of the weaker player in Equation (5) is negative if:

$$\frac{r a_W}{r a_W + a_S} v < a_W,$$

or

$$v < a_W + \frac{a_S}{r}.$$

There exists an $a_S = \bar{a}$ such that $v(\bar{a}) - \bar{a} = 0$. By Equation (11), for any $a_S < \bar{a}$ the inequality $v(a_S) < v(\bar{a})$ holds. Since $a_S < \bar{a}$, then for any a_S there exists some $0 < r^* < 1$ such that $\bar{a} = a_S/r^*$ and:

$$v(a_S) = \frac{a_S}{r^*}, \tag{25}$$

and that for any $r \leq r^*$:

$$v(a_S) \leq \frac{a_S}{r}. \tag{26}$$

Rearranging, we get

$$r^* \leq \frac{a_S}{v(a_S)} = \frac{a_\mu}{v(a_\mu)},$$

Since the $a_S = a_\mu$ in the monopoly case. $\square$

Proof of Proposition 1 *part (i)*

The equilibrium advertising levels that simultaneously solve for the first-order conditions for profit maximization are positive for $r > \frac{1}{2}$ and satisfy the second-order conditions. By Lemma 1 this is the unique equilibrium. $\square$

Proof of Proposition 1 *part (ii)*

To show that the equilibrium is unique, I simultaneously solve for the retailers' first order conditions from Equations (6) and (9). The relationship reduces to:

$$\frac{(2r - 1)}{(2 - r)} = \frac{r a_W^2}{a_S^2}, \tag{27}$$

which has no internal solution when $r < \frac{1}{2}$. The two possible corner solutions are examined. If $a_S = 0$, then the profit maximizing advertising level is $a_W = \delta^2/4$. Plugging these values into

the first order conditions yield $\frac{\partial \pi_W}{\partial a_W} = 0$ and $\frac{\partial \pi_S}{\partial a_S} > 0$ and violates the Kuhn-Tucker condition $\frac{\partial \pi_S}{\partial a_S}(a_S) \le 0$. At the other corner solution, when $a_W = 0$, the profit maximizing advertising level is $a_S = \delta^2/4$. Plugging these values into the first order conditions yield $\frac{\partial \pi_W}{\partial a_W} < 0$ and $\frac{\partial \pi_S}{\partial a_S} = 0$ and satisfy the Kuhn-Tucker conditions $\frac{\partial \pi_S}{\partial a_S}(a_S) \le 0$ and $\frac{\partial \pi_W}{\partial a_W}(a_W) \le 0$. $\square$

Proof of Proposition 2

Part 1 Comparative statics (r = 1): The first part looks at the effect of manufacturer advertising on the symmetric equilibrium expenditures of the retailers. Evaluating at the symmetric equilibrium, $a = a_S^* = a_W^*$, comparative statics for retailer a_W are:

$$\frac{da_W^*}{da_M} = -\frac{|H_{a_W}|}{|H|} = -\frac{\begin{vmatrix} \frac{\partial^2 \pi_W}{\partial a_W \partial a_M} & \frac{\partial^2 \pi_W}{\partial a_W \partial a_S} \\ \frac{\partial^2 \pi_S}{\partial a_S \partial a_M} & \frac{\partial^2 \pi_S}{\partial a_S \partial a_S} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi_W}{\partial a_W \partial a_W} & \frac{\partial^2 \pi_W}{\partial a_W \partial a_S} \\ \frac{\partial^2 \pi_S}{\partial a_W \partial a_S} & \frac{\partial^2 \pi_S}{\partial a_S \partial a_S} \end{vmatrix}}, \tag{28}$$

which is positive if and only if:

$$\frac{1}{2a}v > v' > -2av'' \text{ or } \frac{1}{2a}v < v' < -2av''. \tag{29}$$

By symmetry, this is identical to $\frac{da_S^*}{da_M}$. Next, using Equation (11) as the specification for v and letting $\delta = 2$, we obtain:

$$v' = (a_W + a_S + a_M)^{-\frac{1}{2}}, \tag{30}$$

and

$$v'' = -\frac{1}{2}(a_W + a_S + a_M)^{-\frac{3}{2}}. \tag{31}$$

Then, substituting (30) and (31) into Equation (29) and evaluating at the symmetric equilibrium, $a = a_S = a_W = \frac{9}{8}$, I verify for all $a_M > 0$ that:

$$\frac{1}{2a}v > v' > -2av'', \tag{32}$$

or:

$$\frac{4}{9}\left(a_M + \frac{9}{4}\right)^{\frac{1}{2}} > \frac{1}{\left(a_M + \frac{9}{4}\right)^{\frac{1}{2}}} > -\frac{9}{8\left(a_M + \frac{9}{4}\right)^{\frac{3}{2}}}. \tag{33}$$

Therefore, $\frac{da_W}{da_M} > 0$, and by symmetry $\frac{da_S}{da_M} > 0$, when $v = 2(a_W + a_S + a_M)^{\frac{1}{2}}.\square$

Part 2 Comparative statics $(r > \frac{1}{2})$: For retailer W:

$$\frac{da_W^*}{da_M} = -\frac{|H_{a_W}|}{|H|} = -\frac{\begin{vmatrix} \frac{\partial^2 \pi_W}{\partial a_W \partial a_M} & \frac{\partial^2 \pi_W}{\partial a_W \partial a_S} \\ \frac{\partial^2 \pi_S}{\partial a_S \partial a_M} & \frac{\partial^2 \pi_S}{\partial a_S \partial a_S} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi_W}{\partial a_W \partial a_W} & \frac{\partial^2 \pi_W}{\partial a_W \partial a_S} \\ \frac{\partial^2 \pi_S}{\partial a_W \partial a_S} & \frac{\partial^2 \pi_S}{\partial a_S \partial a_S} \end{vmatrix}} > 0. \tag{34}$$

Similarly for retailer S:

$$\frac{da_S^*}{da_M} = -\frac{|H_{a_W}|}{|H|} = -\frac{\begin{vmatrix} \frac{\partial^2 \pi_S}{\partial a_W \partial a_S} & \frac{\partial^2 \pi_S}{\partial a_W \partial a_S} \\ \frac{\partial^2 \pi_W}{\partial a_W \partial a_M} & \frac{\partial^2 \pi_W}{\partial a_W \partial a_W} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi_W}{\partial a_W \partial a_W} & \frac{\partial^2 \pi_W}{\partial a_W \partial a_S} \\ \frac{\partial^2 \pi_S}{\partial a_W \partial a_S} & \frac{\partial^2 \pi_S}{\partial a_S \partial a_S} \end{vmatrix}} > 0, \tag{35}$$

if and only if $a_S^2 > r a_W^2$ in equilibrium. $\square$

Part 3 Comparative Statics $(r = \frac{1}{2})$: Here I show that an increase in advertising by the manufacturer increases the equilibrium advertising of the retailers when $r = \frac{1}{2}$. Looking at the comparative statics, I examine the partial derivatives. Recall that when $r = \frac{1}{2}$ and $\delta = 2$ the equilibrium advertising expenditures are $a_S^* = 1$ and $a_W^* = 0$. For retailer W, using Equation (6), I find the following partial derivatives:

$$\frac{\partial^2 \pi_W}{\partial a_W^2} = -\frac{1}{2}v + v', \tag{36}$$

$$\frac{\partial^2 \pi_W}{\partial a_W \partial a_S} = -\frac{1}{2}v + \frac{1}{2}v',$$

$$\frac{\partial \pi_W}{\partial a_W \partial a_M} = \frac{1}{2}v'. \tag{37}$$

For retailer S, using Equation (9), the following partial derivatives are calculated:

$$\frac{\partial^2 \pi_S}{\partial a_S \partial a_W} = \frac{1}{2}v + v'', \tag{38}$$

$$\frac{\partial^2 \pi_S}{\partial a_S^2} = v'', \tag{39}$$

$$\frac{\partial^2 \pi_S}{\partial a_S \partial a_M} = v''. \tag{40}$$

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Substituting these values into Equation (34) simplifies to:

$$\frac{da_W^*}{da_M} = -\frac{|H_{a_W}|}{|H|} = -\frac{\frac{1}{2}v(v'')}{\left(\frac{1}{4}v^2 - \frac{1}{4}vv' + \frac{1}{2}v'v''\right)}. \tag{41}$$

In the same way, Equation (35) reduces to:

$$\frac{da_S^*}{da_M} = -\frac{|H_{a_S}|}{|H|} = -\frac{\left(\frac{1}{2}v'v'' - \frac{1}{4}vv' - \frac{1}{2}vv''\right)}{\left(\frac{1}{4}v^2 - \frac{1}{4}vv' + \frac{1}{2}v'v''\right)}. \tag{42}$$

So, $\left(\frac{1}{4}v^2 - \frac{1}{4}vv' + \frac{1}{2}v'v''\right) > 0$, or $v'' > \frac{1}{2}v(1-v)$, for $\frac{da_W^*}{da_M} > 0$.

In equilibrium, $v = 2$, $v' = 1$, and $v'' = -\frac{1}{2}$. Inserting these values into Equations (41) and 42), we find that $\frac{da_W^*}{da_M} > 0$ and $\frac{da_S^*}{da_M} > 0$ meaning that an increase in manufacturer advertising will cause an increase in both retailers' advertising at the threshold for $r = \frac{1}{2}$. $\square$

Proof of Proposition 3 *part (i)*

By Lemma 1, there exists a unique equilibrium for all $a_M \geq 0$. $\square$

Proof of Proposition 3 *part (ii)*

Part 1 Sufficiently low manufacturer advertising: For low values of r, only the strong retailer advertises at the monopoly allocation, $a_S = \delta^2/4$. Suppose the manufacturer advertises, then the payoff function to the strong retailer is simply:

$$\pi_S = \delta(a_W + a_M)^{1/2} - a_S, \tag{43}$$

its first order condition for profitability is:

$$\frac{\partial \pi_S}{\partial a_S} = \frac{\delta}{(a_S + a_M)^{1/2}} = 1, \tag{44}$$

yielding its explicit reaction function as:

$$a_S^* = \delta^2/4 - a_M, \tag{45}$$

and the cross partial is:

$$\frac{\partial^2 \pi_S}{\partial a_S \partial a_M} = \frac{\partial^2 v}{\partial a_S \partial a_M} < 0, \tag{46}$$

implying that the manufacturer's advertising crowds-out the stronger retailer's advertising.

Part 2a (existence) Sufficiently high manufacturer advertising: From Equation (45), when the manufacturer advertises $a_M > 0$, retailer S's best response is $a_S = \left(\delta^2/4 - a_M\right)$. However,

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as a_M increases to $\delta^2/4$, $\left(\delta^2/4 - a_M\right)$ goes to 0. From Equation (26) there is a minimum a_S that dissuades the weaker retailer from actively advertising in this market. Inserting a_M into Equation (26) and representing $v\left(a_S\right)$ as in Equation (11):

$$\delta \left(a_S + a_M\right)^{1/2} \leq \frac{a_S}{r}, \tag{47}$$

and solving for a_S:

$$\frac{1}{2} r\delta \left(r\delta + \sqrt{r^2\delta^2 + 4a_M}\right) \leq \underline{a_S}. \tag{48}$$

This condition is met when $a_M = 0$ and $r \leq \frac{1}{2}$, since $a_S^* = \delta^2/4$ at this point. However, as a_M increases two effects occur. First, a_S^* decreases for profitability, and secondly the $\underline{a_S}$ that satisfies Equation (48) increases to "prevent" retailer W from entering and protects retailer S's market control. The maximum a_M, denoted $\overline{a_M}$, that does not crowd-out retailer advertising occurs when Equation (48) equals $\left(\delta^2/4 - a_M\right)$:

$$r\delta\sqrt{\left(\delta^2/4 - a_M\right) + a_M} = \left(\delta^2/4 - a_M\right),$$

and simplifies to

$$\overline{a_M} = \frac{\delta^2}{4}\left(1 - 2r\right). \tag{49}$$

At this point, $a_S > \left(\delta^2/4 - a_M\right)$ for all $a_M > \overline{a_M}$ and $a_S = \left(\delta^2/4 - a_M\right)$ for all $a_M < \overline{a_M}$. That is, any additional manufacturer advertising above $\overline{a_M}$ will not reduce the stronger retailer's advertising any further, but actually increase it. This is shown in part 2c of the proof of Proposition 2. $\square$

Uniqueness:

Setting the first order conditions equal to each other reduces to:

$$\frac{a_S}{ra_W} = \frac{\left(a_W\left(2 - r\right) + 2a_M\right)}{\left(a_M 2r - a_S\left(1 - 2r\right)\right)}. \tag{50}$$

When $a_M = 0$ and $r \leq \frac{1}{2}$, the only solution is $a_S = \frac{\delta^2}{4}$ and $a_W = 0$. From this, in order for there to be an internal solution when $r \leq \frac{1}{2}$, the following condition must be met:

$$a_M > \frac{a_S\left(1 - 2r\right)}{2r}. \tag{51}$$

From Equation (45) that $a_S = \delta^2/4 - a_M$ is optimal when the weaker retailer remains out of the market. Inserting this value into Equation (51) yields:

$$a_M > \frac{\delta^2}{4}\left(1 - 2r\right).$$ (52)

From Equation (49) if $a_M > \overline{a_M}$ then the solution is internal and if $a_M < \overline{a_M}$ then the solution is on the corner. Evaluating the first-order conditions when $a_M < \overline{a_M}$, $a_W = 0$, and $a_S = \delta^2/4 - a_M$, the Kuhn-Tucker conditions $\frac{\partial \pi_S}{\partial a_S}(a_S) \leq 0$ and $\frac{\partial \pi_W}{\partial a_W}(a_W) \leq 0$ are satisfied. $\square$

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