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Interferencing intensity in two Bose–Einstein condensates with Josephson-like coupling

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Abstract

We derive a general expression for interferencing intensity in two Bose–Einstein condensates with Josephson-like coupling when the two Bose–Einstein condensates are initially in arbitrary quantum pure states. The expression can be used to numerically calculate the intensity. As examples, we study the time behaviors of the intensity when the two Bose–Einstein condensates are initially in coherent states, Fock states, and squeezed states, respectively. ©1999 Elsevier Science B.V. All rights reserved.

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The experimental realization of Bose–Einstein condensation of trapped rubidium [1], sodium [2], and lithium [3] atoms has initiated new areas of atomic, molecular and optical physics. Many of these new areas are based on the analogy between the matter waves and electromagnetic waves, or between bosonic atoms and photons.

Recent developments include reports of a new trap capable of holding larger number of atoms and measurements of condensate fraction and mean-field energy [4], non-direct observation of the development of the condensate [5], measurements of the collective oscillations of the condensate [6–8] and an output coupler for an atomic Bose–Einstein condensate (BEC) [9]. The measurements of the collective excitations have been found

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to be in excellent agreement with the theoretical predictions from mean-field theory [10–13].

Wright et al. investigated the interference of two BECs in small samples [14,15]. Kuang and Zeng studied the interferencing intensity in two BECs with a Josephson-like coupling when the two BECs are initially in coherent states [16] and found that there exist quantum collapses and revivals in the time evolution of the intensity.

In this paper, we consider the interference of two BECs with a Josephson-like coupling when the two BECs are initially in arbitrary quantum pure states. As examples, we study the time behaviors of the intensity when the two BECs are initially in coherent states, Fock states and squeezed states, respectively.

We consider a system which consists of atoms trapped in two identical magnetic optical traps a and b. In the formalism of the second quantization, such a system is described by the Hamiltonian

$$H = H_a + H_b + H_I \, ,$$

$$H_i = \int \mathrm{d}\vec{x}\, \hat{\psi}_i^\dagger(\vec{x}) \left[-\frac{\hbar^2 \nabla^2}{2m} + V_i(\vec{x}) + U_i \hat{\psi}_i^\dagger(\vec{x}) \hat{\psi}_i(\vec{x}) \right] \hat{\psi}_i(\vec{x}) \quad (i = a, b) \, ,$$

$$H_I = \frac{1}{2}\hbar\lambda \int \mathrm{d}\vec{x} [\hat{\psi}_a^\dagger(\vec{x}) \hat{\psi}_b(\vec{x}) + \hat{\psi}_a(\vec{x}) \hat{\psi}_b^\dagger(\vec{x})] \, , \tag{1}$$

where $\hat{\psi}_i(\vec{x})$ and $\hat{\psi}_i^\dagger(\vec{x})$ are the atomic field operators which annihilate and create atoms at position $\vec{x}$ and satisfy the standard bosonic commutation relations $[\hat{\psi}_i(\vec{x}), \hat{\psi}_j^\dagger(\vec{x}')] = \delta_{ij}\delta(\vec{x} - \vec{x}')$. H_a and H_b describe the atoms in traps a and b in the absence of interaction between atoms in traps a and b. In Eq. (1) $U_i = 4\pi\hbar^2 a_i^{sc}/m$, and a_i^{sc} are $s-$wave scattering lengths for collisions. For the sake of simplicity, we assume that $a_a^{sc} = a_b^{sc} = a^{sc}$, $V_a(\vec{x}) = V_b(\vec{x})$, and $U_a = U_b = U$. H_I describes Josephson-like coupling and term $U_i \hat{\psi}_i^\dagger(\vec{x}) \hat{\psi}_i(\vec{x})$ in H_i describes the elastic collisions in trap i.

The atomic field operators in the above Hamiltonian can be expressed as a mode expansion over single-particle states: $\hat{\psi}_a(\vec{x}) = a\phi_{aN}(\vec{x}) + \tilde{\psi}_a(\vec{x})$ and $\hat{\psi}_b(\vec{x}) = b\phi_{bN}(\vec{x}) + \tilde{\psi}_b(\vec{x})$, where $a^\dagger = \int \mathrm{d}\vec{x}\, \phi_{aN}(\vec{x}) \hat{\psi}_a^\dagger(\vec{x})$ and $b^\dagger = \int \mathrm{d}\vec{x}\, \phi_{bN}(\vec{x}) \hat{\psi}_b^\dagger(\vec{x})$ create particles with distributions $\phi_{aN}(\vec{x})$ and $\phi_{bN}(\vec{x})$ with $[a, a^\dagger] = 1$ and $[b, b^\dagger] = 1$, respectively. The first term in the expansion of $\hat{\psi}_i(\vec{x})$ acts only on the condensate state vector, whereas the second term $\tilde{\psi}_i(\vec{x})$ accounts for non-condensate atoms.

For the two condensates a and b, substituting the mode expansion into the second quantized Hamiltonian (1), retaining only the first term representing the condensates, we obtain the following Hamiltonian

$$H = \hbar\omega(a^\dagger a + b^\dagger b) + \hbar q(a^{\dagger 2} a^2 + b^{\dagger 2} b^2) + \hbar g(a^\dagger b + b^\dagger a) \, , \tag{2}$$

where parameters ω, q and g are given by

$$\hbar\omega = \int d\vec{x}\left[\frac{\hbar^2}{2m}(|\nabla\phi_{aN}(\vec{x})|^2 + |\nabla\phi_{bN}(\vec{x})|^2) + V(\vec{x})(|\phi_{aN}(\vec{x})|^2 + |\phi_{bN}(\vec{x})|^2)\right],$$

$$\hbar q = U_0\int d\vec{x}(|\phi_{aN}(\vec{x})|^4 + |\phi_{bN}(\vec{x})|^4),$$

$$\hbar g = \frac{\lambda}{2}\int d\vec{x}[\phi_{aN}^\dagger(\vec{x})\phi_{bN}(\vec{x}) + \phi_{bN}^\dagger(\vec{x})\phi_{aN}(\vec{x})].$$

In the above Hamiltonian, the second term describes interatomic collisions in each condensate. The third term is the Josephson-like tunneling Hamiltonian, in which $ab^\dagger$ describes the annihilation of a BEC atom in trap a and the creation of a BEC atom in trap b and $a^\dagger b$ describes reverse process.

Now, we introduce the unitary transformation $V(\theta) = \exp[\theta(a^\dagger b - b^\dagger a)]$. It is easy to obtain the following property:

$$V^\dagger(\theta)\begin{pmatrix} a \\ b \end{pmatrix} V(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}. \tag{3}$$

Under the transformation $V(\pi/4)$, the Hamiltonian H is transformed to

$$H' = V(-\pi/4)HV(\pi/4) = \hbar\omega(a^\dagger a + b^\dagger b) + \hbar g(b^\dagger b - a^\dagger a)$$
$$+\tfrac{1}{4}\hbar q[3(a^\dagger a + b^\dagger b)^2 - 2(a^\dagger a + b^\dagger b) - (a^\dagger a - b^\dagger b)^2 + 2a^{\dagger 2}b^2 + 2b^{\dagger 2}a^2]. \tag{4}$$

Now, we drop the term $a^{\dagger 2}b^2$ as well as its Hermitian conjugate $b^{\dagger 2}a^2$, i.e., neglect the physical process of annihilation and creation of two BEC atoms. Finally, the Hamiltonian H' is approximated as

$$H_1 = \hbar\omega(a^\dagger a + b^\dagger b) + \hbar g(b^\dagger b - a^\dagger a) + \tfrac{1}{4}\hbar q[3(a^\dagger a + b^\dagger b)^2$$
$$-2(a^\dagger a + b^\dagger b) - (a^\dagger a - b^\dagger b)^2]. \tag{5}$$

From Eq. (4), the unitary evolution operator corresponding to Hamiltonian H is given by

$$U(t) = \exp(-iHt/\hbar) = V(\pi/4)U_1(t)V(-\pi/4), \tag{6}$$

where $U_1(t) = \exp(-iH_1t/\hbar)$. The state vector $|\psi(t)\rangle$ at time t is formally written as

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle. \tag{7}$$

Each of the two BECs is assumed consisting of N atoms with momenta k_a and k_b directed along the x-axis, respectively. Atoms are detected on a screen placed below the two BECs. A detection at position x is represented by the field operator for the sum of two condensates, $\hat{\Phi}(x) = [a + b\exp(i\phi(x))]/\sqrt{2}$, where $\phi(x) = (k_b - k_a)x$. Then the operator for the intensity of atoms is given by [15,16]

$$\hat{\Phi}^\dagger(x)\hat{\Phi}(x) = \tfrac{1}{2}[a^\dagger a + b^\dagger b + \cos\phi(x)(a^\dagger b + b^\dagger a) + i\sin\phi(x)(a^\dagger b - b^\dagger a)]. \tag{8}$$

From Eqs. (6) and (7), the expectation value of the operator $\hat{\Phi}^\dagger(x)\hat{\Phi}(x)$ on the state vector $|\psi(t)\rangle$, i.e., the interferencing intensity $I(t)$ is evaluated to be

$$I(t) = \tfrac{1}{2}[\langle\psi(0)|a^\dagger a + b^\dagger b|\psi(0)\rangle + \cos\phi(x)\langle\psi(0)|a^\dagger b + b^\dagger a|\psi(0)\rangle$$
$$+\mathrm{i}\sin\phi(x)\langle\psi'(t)|a^\dagger b - b^\dagger a|\psi'(t)\rangle]\,, \tag{9}$$

where

$$|\psi'(t)\rangle = U_1(t)V(-\pi/4)|\psi(0)\rangle\,. \tag{10}$$

Only the last term in Eq. (9) is dependent on time. We make a choice $\phi(x) = \pi/2$ to suppress the spatial dependence of the intensity. In this case, the intensity becomes

$$I(t) = \tfrac{1}{2}\langle\psi(0)|a^\dagger a + b^\dagger b|\psi(0)\rangle + \mathrm{i}\tfrac{1}{2}\langle\psi'(t)|a^\dagger b - b^\dagger a|\psi'(t)\rangle\,. \tag{11}$$

Let the two BECs be initially in arbitrary quantum pure state $|\psi(0)\rangle_a$ and $|\psi(0)\rangle_b$, which are expanded as

$$|\psi(0)\rangle_a = \sum_{n'=0}^{\infty} C_{n'}|n'\rangle_a\,,$$
$$|\psi(0)\rangle_b = \sum_{m'=0}^{\infty} D_{m'}|m'\rangle_b\,, \tag{12}$$

where $|n'\rangle_a$ and $|m'\rangle_b$ are the usual Fock states. Then the initial state can be written as

$$|\psi(0)\rangle = \sum_{n'=m'=0}^{\infty} C_{n'}D_{m'}|n',m'\rangle\,, \tag{13}$$

where $|n',m'\rangle = |n'\rangle_a \otimes |m'\rangle_b$.

From Eqs. (7), (10) and (13), the state vector $|\psi'(t)\rangle$ is obtained as

$$|\psi'(t)\rangle = \sum_{n=m=0}^{\infty} \exp[-\mathrm{i}\Omega(n,m)t]A(n,m)|n,m\rangle\,, \tag{14}$$

where

$$A(n,m) = \sum_{n'=m'=0}^{\infty} C_{n'}D_{m'}V_{nm,n'm'}(-\pi/4)\,,$$
$$V_{nm,n'm'}(\theta) = \langle n,m|V(\theta)|n',m'\rangle\,,$$
$$\Omega(n,m) = \omega(n+m) + \frac{q}{2}[(n+m)(n+m-1) + 2nm] + g(m-n)\,. \tag{15}$$

The matrix elements $V_{nm,n'm'}(\theta)$ are given by [17]

$$V_{nm,n'm'}(\theta) = \sum_{k=0}^{\min(n,n')} (-1)^{n'-k}\left(\binom{n'}{k}\binom{n}{k}\binom{m}{n'-k}\binom{m'}{n-k}\right)^{1/2}$$
$$\cos\theta^{\,m'-n+2k}\sin\theta^{\,n+n'-2k}\,, \tag{16}$$

where $\binom{n}{k}$ denotes the binomial coefficient $n!/[(n-k)!k!]$.

From Eqs. (11) and (14), the intensity is evaluated to be

$$I(t)=\frac{1}{2}\left(\sum_{n=0}^{\infty} n|C_n|^2+\sum_{m=0}^{\infty} m|D_m|^2\right)$$
$$+\sum_{n=m=0}^{\infty} A(n+1,m)A(n,m+1)\sqrt{(n+1)(m+1)}\sin[2gt-q(m-n)t]\,. \tag{17}$$

The intensity obtained above is general for arbitrary initial states of the two BECs. Next, we consider several special cases of initial states of the BECs.

Case 1: $|\psi(0)\rangle_a=|\alpha\rangle_a, |\psi(0)\rangle_b=|\beta\rangle_b$ (α and β are real). Here $|\alpha\rangle_a=D_a(\alpha)|0\rangle_a$ and $|\beta\rangle_b=D_b(\beta)|0\rangle_b$ are coherent states. $D_a(\alpha)$ and $D_b(\beta)$ are displacement operators defined as

$$D_a(\alpha)=\exp(\alpha a^\dagger-\alpha^* a)\,,$$
$$D_b(\beta)=\exp(\beta b^\dagger-\beta^* b)\,. \tag{18}$$

The two BECs are initially in coherent states. Using the relation

$$V(-\pi/4)D_a(\alpha)D_b(\beta)V(\pi/4)=D_a[(\alpha-\beta)/\sqrt{2}]D_b[(\alpha+\beta)/\sqrt{2}]\,, \tag{19}$$

we can obtain the quantity $A(n,m)$ in Eq. (17) as

$$A(n,m)=\exp\left[\frac{1}{2}(\alpha^2+\beta^2)\right]\left(\frac{\alpha-\beta}{\sqrt{2}}\right)^n\left(\frac{\alpha+\beta}{\sqrt{2}}\right)^m\Big/\sqrt{n!m!}\,. \tag{20}$$

Substituting the above equation into Eq. (17), we get

$$I(t)=\tfrac{1}{2}(\alpha^2+\beta^2)+\tfrac{1}{2}(\alpha^2-\beta^2)\sin[2gt-2\alpha\beta\sin(qt)]\exp[-2(\alpha^2+\beta^2)\sin^2\tfrac{qt}{2}]\,. \tag{21}$$

The above equation is slightly different from that obtained in Ref. [16]. We have numerically tested that Eqs. (21) and (17) are identical. Time behaviors of the intensity have been discussed in detail [16] and will not be addressed again.

Case 2: $|\psi(0)\rangle_a=|n\rangle_a, |\psi(0)\rangle_b=|m\rangle_b$. The two BECs are initially in Fock states $|n\rangle_a$ and $|m\rangle_b$.

From Eqs. (16) and (17), we can numerically calculate the interferencing intensity. Fig. 1 gives the time evolution of the intensity for various cases of inter-atomic collisions and atomic tunneling. In the strong tunneling regime, $g>q$, we can clearly see the quantum collapses and revivals. When atomic tunneling effect becomes comparable to inter-atomic collision effect (i.e., $g\sim q$), the intensity exhibits a simple periodical oscillation and does not show the quantum collapses and revivals. In the weak coupling regime, $g<q$, the intensity shows neither periodical oscillations nor quantum collapses and revivals. The times behaviors of the intensity in the weak tunneling region is quite different from those in the strong tunneling region. We also find that the intensity is independent of time t when $n=m$.

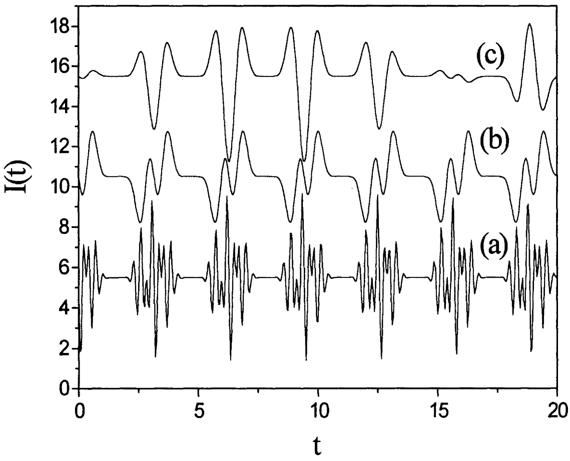

Fig. 1. Time evolution of the interferencing intensity when the two BECs are initially in Fock states for (a) $I(t)$, $g=10$; (b) $I(t)+5$, $g=1$; (c) $I(t)+10$, $g=0.1$. Here we take $q=1$, $n=1$, $m=10$.

Case 3: $|\psi(0)\rangle_a = S_a(r_1)|\alpha\rangle_a, |\psi(0)\rangle_b = S_b(r_2)|\beta\rangle_b$ (r_1 and r_2 are real), i.e., the two BECs are in squeezed states. $S_a(r_1)$ and $S_b(r_2)$ are squeezing operators defined as

$$S_a(r_1) = \exp[r_1(a^{\dagger 2} - a^2)] ,$$
$$S_b(r_2) = \exp[r_2(b^{\dagger 2} - b^2)] . \quad (22)$$

The squeezed state $|r,\alpha\rangle = S(r)D(\alpha)|0\rangle$ (r is real) can be expanded in Fock space as [18]

$$|r,\alpha\rangle = (\cosh r)^{-1/2} \exp[-|\alpha|^2/2 + \alpha^2 \tanh r/2]$$
$$\times \sum_{n=0}^{\infty} \frac{H_n[\alpha/\sqrt{\sin(2r)}]}{\sqrt{n!}} \left(\frac{1}{2}\tanh r\right)^{n/2} |n\rangle . \quad (23)$$

Fig. 2 gives the numerical results of the intensity when the two BECs are initially in squeezed states for different squeezing parameters. We fix $r_2 = 0.05$ and let r_1 vary. For $r_1 = r_2$, the intensity does not depend on time. We find that the intensity for $r_1 = 0.025$ is of opposite phase to that for $r_2 = 0.075$.

In conclusion, we have given the expression for interferencing intensity when the two BECs are initially in arbitrary quantum pure states. The expression can be used to numerically calculate the interferencing intensity. As examples, we studied the time behaviors of the intensity when the two BECs are initially in coherent states, Fock states and squeezed states, respectively. For the initial states which are coherent states, we obtain the analytical expression for the intensity, which is slightly different from that obtained before. We have numerically tested its validity. When the initial states are Fock states, we find that there exists the quantum collapses and revivals. The time behaviors of the intensity in the weak and strong tunneling regime are discussed in

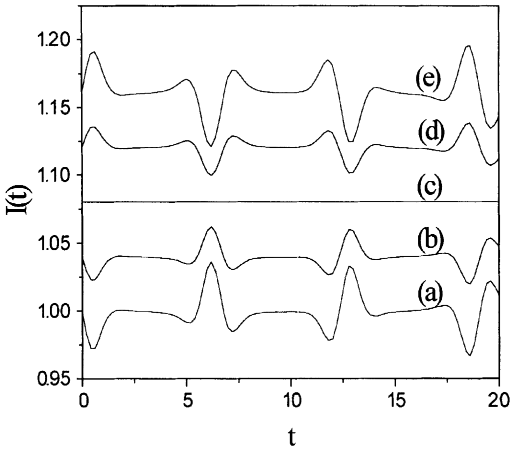

Fig. 2. Time evolution of the interferencing intensity when the two BECs are initially in squeezed states for (a) $I(t)$, $r_1 = 0.01$; (b) $I(t) + 0.04$, $r_1 = 0.025$; (c) $I(t) + 0.08$, $r_1 = 0.05$; (d) $I(t) + 0.12$, $r_1 = 0.075$; (e) $I(t) + 0.16$, $r_1 = 0.1$. Here we take $\alpha = \beta = 1$, $g = 0.1$, $q = 1$, $r_2 = 0.05$.

detail. We also investigated the time behaviors of the intensity when the two BECs are initially in squeezed states.

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