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Two-Link Flexible Manipulator Modeling and Tip Trajectory Tracking Based on Absolute Nodal Coordinate Method

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Abstract

It has been demonstrated that the absolute nodal coordinate formulation (ANCF) proposed recently in literature can be used to exactly describe the flexible multibody system unlike traditional methods such as the floating coordinate method and assumed mode method. Therefore, in this paper a new dynamic modeling technique for a two-link flexible manipulator based on absolute nodal coordinate method is proposed. The link shear effect was taken into account by using the 2D ANCF shear beam element. The resulting state equation can be explicitly described by generalized coordinate since the system mass matrix is constant in the ANCF framework. The proposed method is validated through the two-link flexible manipulator tip circle and square trajectory tracking control simulations by using a simple PD controller. To improve computational efficiency, the invariant matrix method and the Broyden quasi-Newton method are introduced. To improve the tracking accuracy, different PD parameters in different simulation periods are used. The simulation results indicate that modeling and controlling the flexible manipulator based on the ANCF is effective.

Keywords: flexible manipulator, ANCF, shear beam element, Broyden quasi-Newton method and PD controller

1 Introduction

Research on flexible manipulators has being carried out for the last two decades, because Flexible robot manipulators exhibit many advantages over rigid robots: they require less material, are lighter in weight, consume less power, require smaller actuators, are more maneuverable and transportable, have less overall cost and higher payload to robot weight ratio. Despite these advantages, modeling and control of flexible manipulators is difficult, especially for those with large deformation and rotation subsystems. The main reason is that the conventional modeling method can not lead to exact results for those subsystems [1]. Furthermore, the number of control inputs is less than the number of variables to be controlled since the actuators are located at the joints. This means that the link deflections can be suppressed only indirectly, which make the control for flexible manipulator much more difficult than the rigid manipulator. Hence, to achieve greater tip trajectory tracking accuracy, one has to start with very accurate mathematical models for the flexible link system.

In the past, many kinds of flexible manipulator modeling schemes have been proposed. The dynamics model for flexible manipulators was generally derived by using Lagrangian formulation, the Newton-Euler formulation, Hamilton's principle or Kane's method [2]. The robotic systems with flexible links are continuous dynamical systems which are usually discretized by using assumed mode method (AMM), finite elements method (FEM) or lumped parameter methods. These methods have been widely used by many researchers. Buffinton and Kane [2] developed equations of motion for flexible robots containing translational motion of elastic members. The specific system investigated is a two-degree of freedom manipulator. The assumed mode method and an alternative form of Kane's method are used in the formulation of equation of motion. The assumed mode expansion method is also used by Green and Sasiadek [3] for two-link manipulators. Morris and Madani [4] developed the equation of motion for a large single-link manipulator including shear deformation. Lee [5] showed that the conventional Lagrangian modeling of flexible link robots does not fully incorporate the bending mechanism of flexible link as it allows free link elongation in addition to link deflection. This elongation causes modeling inaccuracy for links with rotation. To correct this he proposed a new dynamic model. Kalra and Sharan [6] extended the work of model the flexible manipulator using the finite element method, where a lumped parameter FEM model was developed. Meghdari and Fahimi [7] used Kane's method of multibody systems to decouple the dynamic equation of motion of the two-link flexible manipulator. Han and Mao [8] used Hamilton's principles, finite element method and symplectic integration to develop equation of motion for two-link manipulator. Zhang et al [9] derived a partial differential equation model for a flexible two-link manipulator using Hamilton's principle and then transform this to a form suitable for the development of stable controllers.

In AMM, the link flexibility is usually represented by a truncated finite modal series in terms of spatial mode functions and time-varying mode amplitudes. The main drawback of this method is the difficulty in finding modes for links with non-regular cross sections and multi-link manipulators. While traditional FEM are based on small deformation and small rotation assumption, the beam elements are not isoparametric elements and will lead to a nonzero strain energy when the flexible multibody system have a rigid motion. Hence, the traditional FEM will lead to inexact dynamics model for the flexible manipulator, especially for those with large deformation and rotation systems [10]. In the lumped parameter method, which is the simplest one for analysis purpose, the manipulator is modeled as spring and mass system, which does not often yield sufficiently accurate results [2]. Furthermore by using all methods mentioned above, the resulting state equations for two-link flexible manipulator can not be described by generalized coordinate explicitly because of the inertial coupling effects.

The coupling of the large deformation, large rotation and system's rigid motion brings us new challenges for modeling and control these systems. In 1996, Shabana [11] proposed an absolute nodal coordinate method for exactly describing the flexible multibody systems, which can overcome the drawbacks of traditional FEM、AMM and lumped parameter method. In the absolute nodal coordinate formulation, no infinitesimal or finite rotations parameters are used as nodal coordinates, instead absolute displacements and global slopes are used as the element nodal coordinates [12]. As the locations and the deformations of the material points on the finite element are defined in the global coordinate system, using the element shape function and the nodal coordinates, the system governing equations' mass matrix remains constant and the centrifugal and Coriolis forces are identically equal to zero. Using the absolute nodal coordinate formulation, the beam, plate and shell elements are all isoparametric elements, can also be used to obtain exact modeling of the rigid body dynamics. The ANCF has been used in many multibody dynamics research field especially for large deformation flexible multibody systems, such as Pantograph/Catenary system [13], Belt-Drives system [14] etc. The work in this paper is the pioneer work on modeling the flexible manipulators by using the ANCF.

This paper is organized as follows. In Section 2, the 2D shear beam element based on ANCF is introduced. In Sections 3, the explicit state equations of flexible manipulators are deduced. In Section 4, the proposed trajectory planning method and a PD control scheme are explained. Section 5 illustrates simulation results. Finally, Section 6 gives conclusions.

2 Two Dimensional Shear Beam Element Based on Absolute Nodal Coordinate Formulation

In this paper, in order to take the shear deformation effect of the flexible manipulator into account, the ANCF 2D shear beam element [15] was adopted to discretize the manipulator flexible link, as shown in Figure 1.

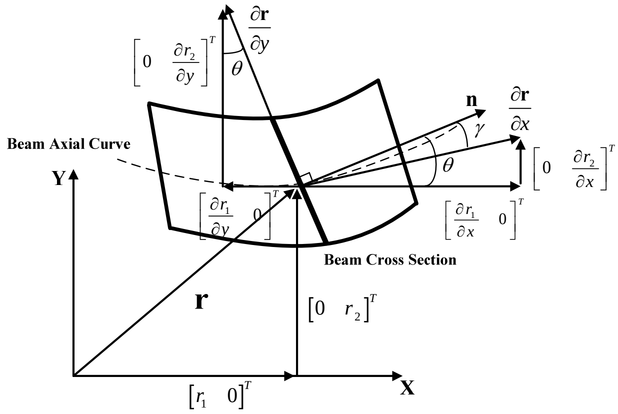

Figure 1: The beam element with shear deformation

According to ANCF method, for the two dimensional shear beam element, the displacement field can be defined in the global coordinate system as:

$$\mathbf{r} = \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 x^3 \\ b_0 + b_1 x + b_2 y + b_3 xy + b_4 x^2 + b_5 x^3 \end{bmatrix} = \mathbf{Se} \qquad (1)$$

Where X and Y denote the nodal coordinate in global coordinate system, x and y denote the coordinate in element local coordinate system, $\mathbf{S}$ is the shape function, a matrix with 2 rows and 12 columns. The absolute nodal coordinates for nodal i and nodal j can be expressed as:

$$e = [e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}]^T$$
$$= [r_{i1}, r_{i2}, \frac{\partial r_{i1}}{\partial x}, \frac{\partial r_{i2}}{\partial x}, \frac{\partial r_{i1}}{\partial y}, \frac{\partial r_{i2}}{\partial y}, r_{j1}, r_{j2}, \frac{\partial r_{j1}}{\partial x}, \frac{\partial r_{j2}}{\partial x}, \frac{\partial r_{j1}}{\partial y}, \frac{\partial r_{j2}}{\partial y}]^T \qquad (2)$$

The vector, $\frac{\partial \mathbf{r}}{\partial x} = [\frac{\partial r_1}{\partial x}, \frac{\partial r_2}{\partial x}]^T$, does not remain perpendicular to the beam section, The angle θ indicates the beam section orientation, which can be expressed by vector, $\frac{\partial \mathbf{r}}{\partial y} = [\frac{\partial r_1}{\partial y}, \frac{\partial r_2}{\partial y}]^T$. $\mathbf{n}$ denotes the beam section normal vector. The angle γ indicates the element shear deformation. Therefore, this new beam element model relaxes the assumptions of Euler-Bernoulli beam models. According to the conventional finite element method, the element shape function can be obtained. By using Newton-Euler formulation, the element equations of motion can be obtained:

$$\mathbf{M}_e \ddot{\mathbf{e}} = \mathbf{Q}_e - \mathbf{F}_e \qquad (3)$$

Where $\mathbf{M}_e$ denotes the element constant mass matrix, $\mathbf{F}_e$ denotes the element elastic force vector, which can be deducted by continuum mechanics approach [15]. $\mathbf{Q}_e$ denotes the element generalized force vector. Suppose a constant torque τ was acted on the nodal i, then generalized moment vector can be expressed as:

$$\mathbf{Q}_e = \begin{bmatrix} 0 & 0 & 0 & 0 & \dfrac{\tau e_6}{f_i^2} & \dfrac{-\tau e_5}{f_i^2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (5)$$

Where $f_i = \sqrt{e_5^2 + e_6^2}$

3 State Equation Derivation and Numerical Solution Method

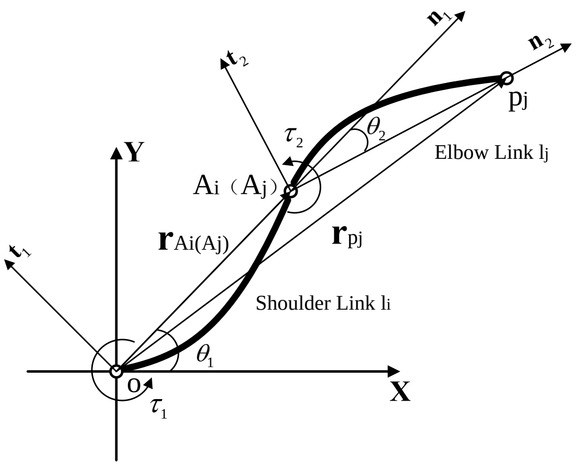

Figure 2: Two-Link Flexible Manipulator Defined in the Global Coordinate System

In order to control the tip point P_j trajectory, two sets of local coordinate systems, $\mathbf{t}_1$-$\mathbf{n}_1$ and $\mathbf{t}_2$-$\mathbf{n}_2$, are defined. The coordinate system, $\mathbf{t}_1$-$\mathbf{n}_1$, is decided by the position vector of point Ai, $\mathbf{r}_{Ai}$. The unit vector $\mathbf{n}_1$ can be expressed as:

$$\mathbf{n}_1 = \frac{\mathbf{r}_{Ai}}{\left\|\mathbf{r}_{Ai}\right\|} \quad (6)$$

While the coordinate system, $\mathbf{t}_2$-$\mathbf{n}_2$, is decided by the position vector of point A_j and point P_j, they are $\mathbf{r}_{Aj}$ and $\mathbf{r}_{Pj}$ respectively. Then the unit vector $\mathbf{n}_2$ can be expressed as:

$$\mathbf{n}_2 = \frac{\mathbf{r}_{Pj} - \mathbf{r}_{Aj}}{\left\|\mathbf{r}_{Pj} - \mathbf{r}_{Aj}\right\|} \quad (7)$$

Where $\|\cdot\|$ indicates the mold of a vector. The vectors $\mathbf{t}_1$, $\mathbf{t}_2$ can be obtained by rotating $\mathbf{n}_1$ and $\mathbf{n}_2$ counter clockwisely. The 2D shear beam element described in above section is used to discretize the flexible manipulator shoulder link and elbow link and the constraint equations at the revolve joints can be simply expressed as:

$$\mathbf{r}_{Ai} = \mathbf{r}_{Aj} \quad (8)$$

In the absolute nodal coordinate framework, the assembly of the element mass matrix and stiff matrix can be carried out by conventional finite element method. By using Lagrangian method, the equations of motion for two-link flexible manipulator were obtained.

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_\mathbf{q}^T \\ \mathbf{C}_\mathbf{q} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \tau - \mathbf{F}_\mathbf{q} \\ \mathbf{Q}_d \end{bmatrix} \quad (9)$$

Where $\mathbf{M}$ denotes the system constant mass matrix, $\mathbf{q}$ is system generalized coordinate, $\mathbf{C}_\mathbf{q}$ is the derivative matrix of system constraint equations to generalized coordinate. As for the two link flexible manipulator, it is also a constant matrix. τ is system generalized external control torques, which can be evaluated by using of equation (5). $\boldsymbol{\lambda}$ is Lagrangian multiplier, $\mathbf{F}_\mathbf{q}$ is elastic forces .

$$\mathbf{Q}_d = \mathbf{C}_\mathbf{q} \ddot{\mathbf{q}} = -\mathbf{C}_{tt} - 2\mathbf{C}_{\mathbf{q}t} \dot{\mathbf{q}} - (\mathbf{C}_\mathbf{q} \dot{\mathbf{q}})_\mathbf{q} \dot{\mathbf{q}} \quad (10)$$

For numerical integration stability, the Baumgarte's stability method [16] was introduced into the above equations. Then the above equations can be expressed as

$$\begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{C}_\mathbf{q}^\mathrm{T} \\ \mathbf{C}_\mathbf{q} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \tau - \mathbf{F}_\mathbf{q} \\ \boldsymbol{\gamma} \end{bmatrix} \quad (11)$$

Where $\boldsymbol{\gamma} = \mathbf{Q}_d - 2\partial(\mathbf{C}_\mathbf{q} \dot{\mathbf{q}} + \mathbf{C}_t) - \beta^2 \mathbf{C}$

In the absolute nodal coordinate framework, because the system mass matrix is constant and the $\mathbf{C}_\mathbf{q}$ is also constant for the manipulator, so

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_\mathbf{q}^\mathrm{T} \\ \mathbf{C}_\mathbf{q} & \mathbf{0} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{H}_{\mathbf{qq}} & \mathbf{H}_{\mathbf{q}\lambda} \\ \mathbf{H}_{\lambda\mathbf{q}} & \mathbf{H}_{\lambda\lambda} \end{bmatrix},$$

Where
$$\mathbf{H}_{\lambda\lambda} = (\mathbf{C}_\mathbf{q} \mathbf{M}^{-1} \mathbf{C}_\mathbf{q}^\mathrm{T})^{-1},$$
$$\mathbf{H}_{\mathbf{qq}} = \mathbf{M}^{-1} + \mathbf{M}^{-1} \mathbf{C}_\mathbf{q}^\mathrm{T} \mathbf{H}_{\lambda\lambda} \mathbf{C}_\mathbf{q} \mathbf{M}^{-1},$$
$$\mathbf{H}_{\mathbf{q}\lambda} = \mathbf{H}_{\lambda\mathbf{q}}^\mathrm{T} = -\mathbf{M}^{-1} \mathbf{C}_\mathbf{q}^\mathrm{T} \mathbf{H}_{\lambda\lambda}$$

Then we obtain the Lagrangian multiplier and the generalized acceleration expression:

$$\boldsymbol{\lambda} = \mathbf{H}_{\lambda\mathbf{q}} (\boldsymbol{\tau} - \mathbf{F}_\mathbf{q}) + \mathbf{H}_{\lambda\lambda} \mathbf{Q}_\mathbf{d} \quad (12)$$

$$\ddot{\mathbf{q}} = \mathbf{H}_{\mathbf{qq}} \boldsymbol{\tau} - \mathbf{H}_{\mathbf{qq}} \mathbf{F}_\mathbf{q} + \mathbf{H}_{\mathbf{q}\lambda} \boldsymbol{\gamma} \quad (13)$$

So the state equations for flexible manipulators can be expressed explicitly by the generalized coordinate as:

$$\frac{d}{dt}\left(\begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}\right) = \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{H}_{\mathbf{qq}} \tau(\mathbf{q}) - \mathbf{H}_{\mathbf{qq}} \mathbf{F}_\mathbf{q}(\mathbf{q}) + \mathbf{H}_{\mathbf{q}\lambda} \boldsymbol{\gamma}(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix} \quad (14)$$

For the numerical solution of the above state equation, the explicit algorithm will converge conditionally, so the integration time step can not change arbitrarily and then it is not suit to long time simulation. While the implicit method can converge unconditionally, but the Jacobian matrix should be calculated. Evaluation of the Jacobian matrix is the most cumbersome task when solving equation (14). As can be seen from equation (14), to evaluate the Jacobian matrix, the most complex computation is to calculate the partial derivative matrix of the elastic forces. To improve computation efficiency, García-Vallejo[17] proposed an invariant matrix method.. According to this method, the tangent matrix of elastic forces can be expressed as:

$$\left[\frac{\partial \mathbf{F}_{\mathbf{q}}}{\partial \mathbf{q}}\right]_{ik} = [\mathbf{K}_1]_{ik} - \sum_{j=1}^{n} \mathbf{q}^T([\mathbf{C}^{ij}_{k,}]^T + \mathbf{C}^{ij}_{,k})\mathbf{q}_j - \mathbf{q}^T\mathbf{C}^{ik}_{K_2}\mathbf{q}$$

Where $\mathbf{C}^{ij}_{,k}$, $\mathbf{C}^{ij}_{k,}$ indicates the *kth* collolum and *kth* row of the invariant matrix $\mathbf{C}^{ij}_{K_2}$, respectively. To solve the state equation (14), set:

$$\mathbf{Y} = \begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix}, \quad \mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix}\dot{\mathbf{q}}\\ \mathbf{H}_{\mathbf{qq}}\tau(\mathbf{q}) - \mathbf{H}_{\mathbf{qq}}\mathbf{F}_{\mathbf{q}}(\mathbf{q}) + \mathbf{H}_{\mathbf{q}\lambda}\gamma(\mathbf{q},\dot{\mathbf{q}})\end{bmatrix}$$

According to the trapezoidal method:

$$\mathbf{Y}_{m+1} = \mathbf{Y}_m + \frac{h}{2}(\mathbf{G}_1 + \mathbf{G}_2) \tag{15}$$

Where h is the integration step time, m denotes the *mth* iteration. $\mathbf{G}_1 = \mathbf{f}(t, \mathbf{Y}_m)$, $\mathbf{G}_2 = \mathbf{f}(t, \mathbf{Y}_{m+1})$. So, to obtain $\mathbf{Y}_{m+1}$, the following nonlinear equations should be solved.

$$\mathbf{Y}_{m+1} - \mathbf{Y}_m - \frac{h}{2}(\mathbf{G}_1 + \mathbf{G}_2) = \mathbf{0} \tag{16}$$

Where $\mathbf{Y}_{m+1}$ is unknown. To improve computational efficiency further, the Broyden qusi-Newton method [18] was used to solve the above nonlinear equations, which can avoid calculating the Jacobian Matrix in each numerical iteration of traditional method.

$$\begin{cases} \mathbf{Y}^{(n+1)}_{m+1} = \mathbf{Y}^{(n)}_{m+1} - \mathbf{J}^{-1}_m \mathbf{F}(\mathbf{Y}^{(n)}_{m+1}) \\ \mathbf{y}_m = \mathbf{F}(\mathbf{Y}^{(n+1)}_{m+1}) - \mathbf{F}(\mathbf{Y}^{(n)}_{m+1}) \\ \mathbf{s}_m = \mathbf{Y}^{(n+1)}_{m+1} - \mathbf{Y}^{(n)}_{m+1} \\ \mathbf{J}_{m+1} = \mathbf{J}_m + \dfrac{(\mathbf{y}_m - \mathbf{J}_m\mathbf{s}_m)\mathbf{s}^T_m}{\mathbf{s}^T_m\mathbf{s}_m} \end{cases} \tag{17}$$

Where $\mathbf{F}(\mathbf{Y}_{m+1}) = \mathbf{Y}_{m+1} - \mathbf{Y}_m - \frac{h}{2}(\mathbf{G}_1 + \mathbf{G}_2)$, $\mathbf{Y}^{(n+1)}_{m+1}$ represents the *(n+1)th* iteration for calculating $\mathbf{Y}_{m+1}$. $\mathbf{J}$ is the jacobian matrix of $\mathbf{F}$ .The partial derivative of $\mathbf{Y}_{m+1}$ to generalized coordinate $\mathbf{q}_{m+1}$ and generalized velocity $\dot{\mathbf{q}}_{m+1}$ is a constant unit matrix, the partial derivative of $\mathbf{G}_2$ to generalized coordinate and generalized velocity can be easily calculated by using invariant matrix method, so the computational efficiency improved again by using the Broyden qusi-Newton method.

4 Tip PD Tracking Control Scheme

We have chosen a PD controller, since it is the simplest type of controller, and it is used most often in practice and also in industry. For a PD controller, only the errors in joint angle and joint angular velocity are needed to calculate the controller outputs, Figure 3. shows the simple control scheme for the two-link manipulator tip trajectory tracking simulation.

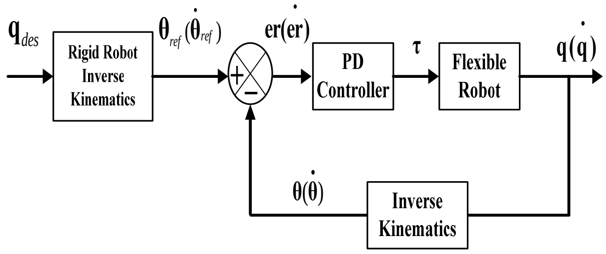

Figure 3: The PD control scheme

In Figure 3, $\mathbf{q}_{des}$ denotes the desired trajectory for the tip of the manipulator in the operational space, $\boldsymbol{\theta}_{ref}$ and $\dot{\boldsymbol{\theta}}_{ref}$ is calculated from $\mathbf{q}_{des}$ using inverse kinematic equations of a two-link, planar rigid robot. The joint angle $\boldsymbol{\theta}$ and angular velocity $\dot{\boldsymbol{\theta}}$ are calculated from ANCF based simulation results by using inverse kinematics. $\mathbf{er}$ and $\dot{\mathbf{er}}$ represent the tracking errors and can be expressed as:

$$\mathbf{er} = \boldsymbol{\theta}_{ref} - \boldsymbol{\theta} \qquad \dot{\mathbf{er}} = \dot{\boldsymbol{\theta}}_{ref} - \dot{\boldsymbol{\theta}} \tag{18}$$

Where $\boldsymbol{\theta} = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}$, $\dot{\boldsymbol{\theta}} = \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$

As shown in Figure 2., all the element nodal coordinates are defined in global coordinate system, Although the direct position can be easily used as the control feed back based on the ANCF, but in practice, it is very difficult to measure the manipulator tip poison. While the shoulder angle θ_1 and elbow angle θ_2 can be easily measured by angle sensor, so some transformation of global position results into joint angles was needed to realize to trajectory tracking control. The shoulder angle θ_1 and elbow angle θ_2 can be expressed as:

$$\cos(\theta_1) = \mathbf{n}^T_1 \times \begin{bmatrix}1\\0\end{bmatrix} \qquad \cos(\theta_2) = \mathbf{n}^T_2 \times \mathbf{n}_1 \tag{19}$$

The feedback shoulder angular velocity $\dot{\theta}_1$ and elbow angular velocity $\dot{\theta}_2$ can be obtained through some simple mathematic operation:

$$\dot{\theta}_1 = \frac{\mathbf{V}_{Ai}^T \times \mathbf{t}_1}{D_{OAi}} \qquad \dot{\theta}_2 = \frac{\mathbf{V}_{PAj}^T \times \mathbf{t}_2}{D_{PAj}} \tag{20}$$

Where $\mathbf{V}_{Ai}$ is the absolute nodal velocity of the point A_i . D_{OAi} indicates the distance between the point O and point A_i, D_{PAj} indicate the distance point P_j and point A_j, respectively. $\mathbf{V}_{PAj}$ is the relative velocity and can be expressed as:

$$\mathbf{V}_{PA} = \mathbf{V}_{pj} - \mathbf{V}_{Aj} \tag{21}$$

The absolute velocity vectors $\mathbf{V}_{Aj}$, $\mathbf{V}_{pj}$ can be directly obtained based on the ANCF. So, the control torque can be expressed as:

$$\boldsymbol{\tau} = \mathbf{K}_p \mathbf{er} + \mathbf{K}_v \dot{\mathbf{er}} \tag{22}$$

Where $\mathbf{K}_p$ is the proportional gain, $\mathbf{K}_v$ is the derivative gain. $\mathbf{K}_p$ and $\mathbf{K}_v$ can be tuned easily, even by trial and error. In order to improve tracking accuracy, in this paper In order to improve tracking accuracy, different value for $\mathbf{K}_p$ and $\mathbf{K}_v$ was used in different simulation period.

5 Numerical example and Simulation Results

Simulations are performed on the two-link, planar, flexible manipulator. The elbow tip of the manipulator is required to track a circle trajectory, see Figure 4 and an square trajectory, see Figure 5, respectively.

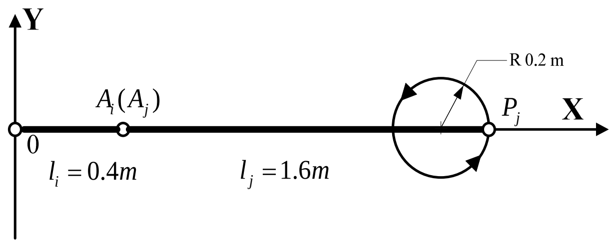

Figure 4: Manipulator Initial configuration for circle trajectory tracking

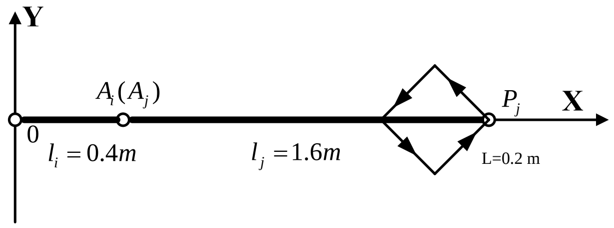

Figure 5: Manipulator Initial configuration for square trajectory tracking

The shoulder link length(l_i) is equal to 0.4*m* and the elbow link length(l_j) is 1.6*m*, the shoulder link was discretized by two elements, while the elbow link is disretized by four elements. The beam element section is square with 0.05*m* side-edge length. For the circle tracking example, the material Young's modulus for the two-link flexible manipulator is 3e10*Pa*, the Poisson's ratio is 0.25 and the material density is 6000Kg/m^3. While for the square tracking example, the material Young's modulus for the two-link flexible manipulator is 8e10*Pa*, the Poisson's ratio is 0.3 and the material density is 3500 Kg/m^3.

Different proportional gain and derivative gain value was used in different simulation period and are obtained through numerous simulations. For example, the whole simulation period is 25*s* for circle tracking, to improve tracking accuracy, using two groups of PD parameters, when the time is less than 12.5*s*, one group of parameters is determined by numerous simulations, after that, another group of PD parameters is determined when the simulation time is over 12.5*s* , also by numerous simulations. Using different PD parameters according to trajectory characteristics is very useful to improve the tracking accuracy, especially for enhancing the tracking accuracy on the square trajectory corners, as for the velocity change is discontinuous on the square trajectory corners.

Figures 6 to Figure 9, illustrate the results of the simulations of circle trajectory tracking and Figures 10 to Figure 13, illustrate the results of the simulations of square trajectory tracking. As can be seen from Figure 9 and Figure 13, the shoulder tip transverse deflection is much smaller than the elbow tip transverse deflection, that's mainly because the shoulder is much shorter than the elbow. Furthermore, as for the material for the square trajectory tracking simulation is more rigid than the one for the circle trajectory tracking simulation (the material Young's modulus is different), so the transverse deflections in the square trajectory tracking simulation are smaller than the transverse deflections in the other example. Except for some initial big errors, the actual trajectory matches the desired trajectory well. The comparison the simulation results, Figure 11a, to Figure 11b, the results of [19], show that the tracking accuracy is improved greatly, especially on the square corners. All the simulation results demonstrate that the simple PD controller with variable PD parameters in different simulation period can also obtain high accurate results based on the ANCF manipulator model. The tracking error was also obtained in two examples, see Figure 8 and Figure 12, respectively. As we can seen from the two Figures, the elbow tip trajectory tracking error in Y-direction is much larger than the errors in X-direction.

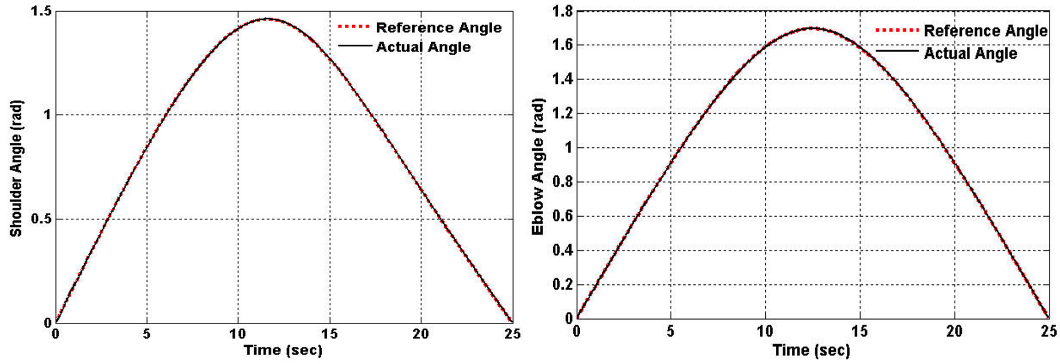

Figure 6: Reference (dotted lines) and actual (solid lines) angles for the shoulder and elbow angles

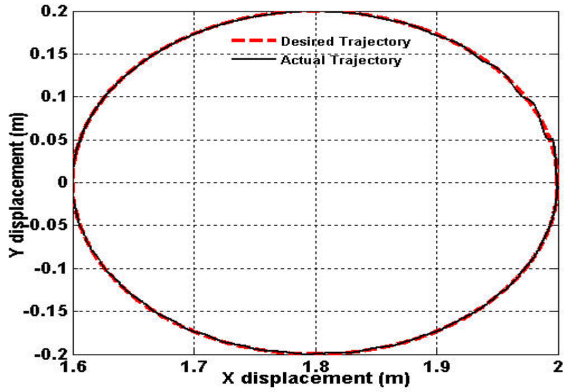

Figure 7: The Actual trajectory(solid lines) and the desired trajectory(dotted lines) of the manipulator tip

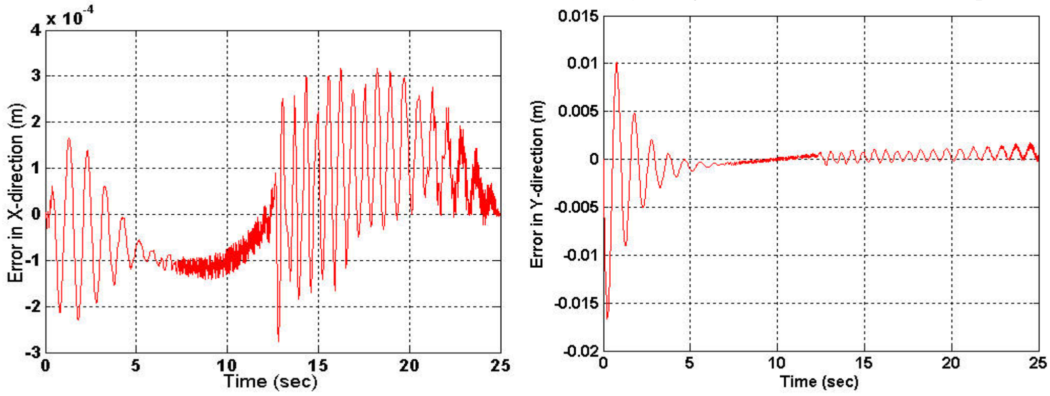

Figure 8: The tracking error in X- and Y-direction of the elbow tip of the manipulator

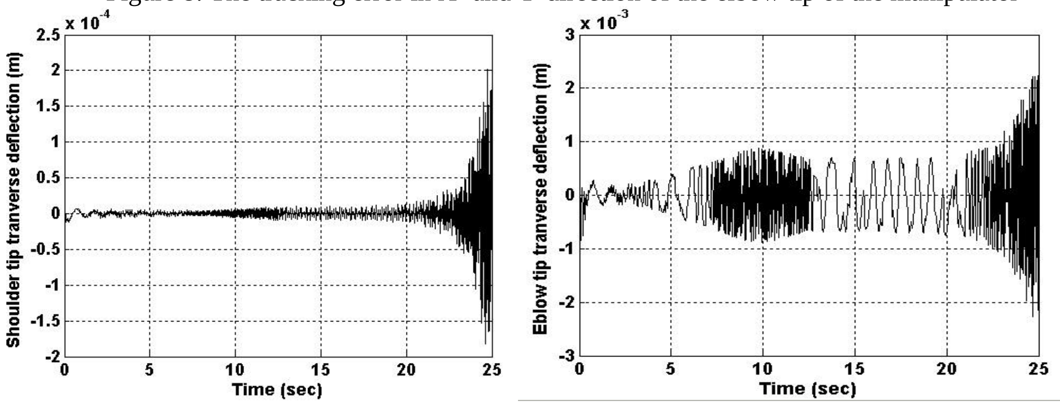

Figure 9: Transverse deflection of the shoulder and the elbow tip

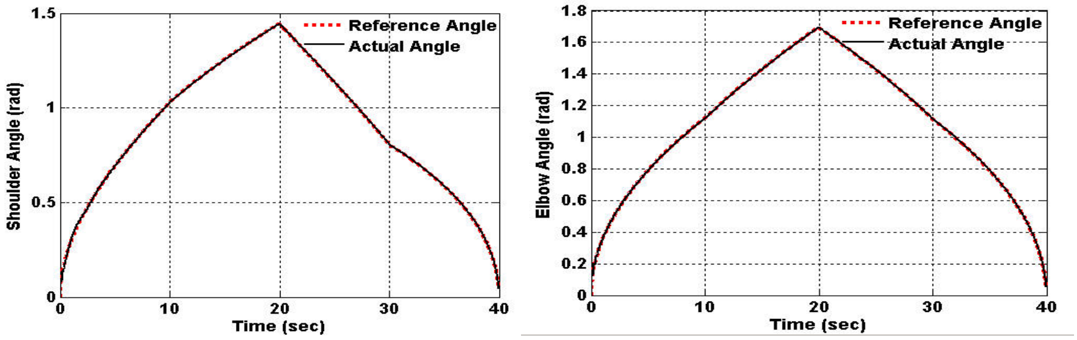

Figure 10: Reference (dotted lines) and actual (solid lines) angles for the shoulder and elbow angles

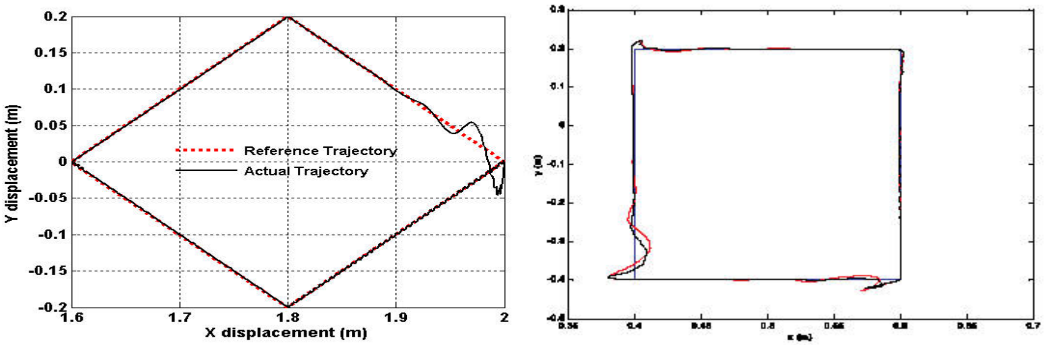

Figure 11a Figure 11b

Figure 11a: The Actual trajectory(solid lines) and the desired trajectory(dotted lines) of the manipulator tip

Figure 11b: Simulation results of [19]

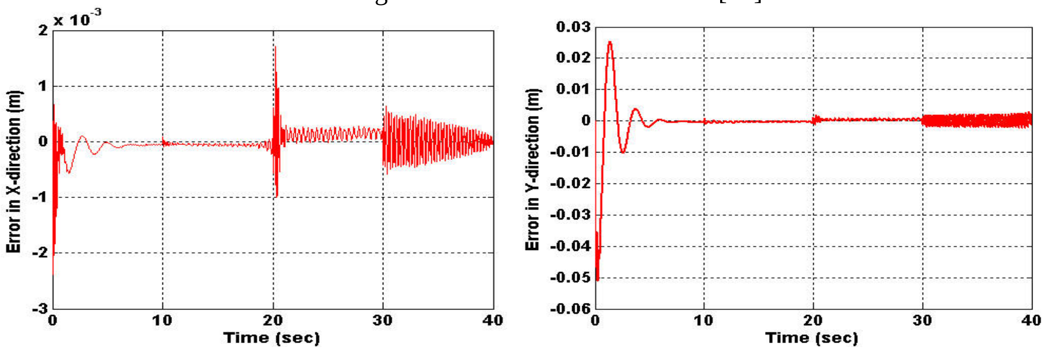

Figure 12: The tracking error in X- and Y-direction of the elbow tip of the manipulator

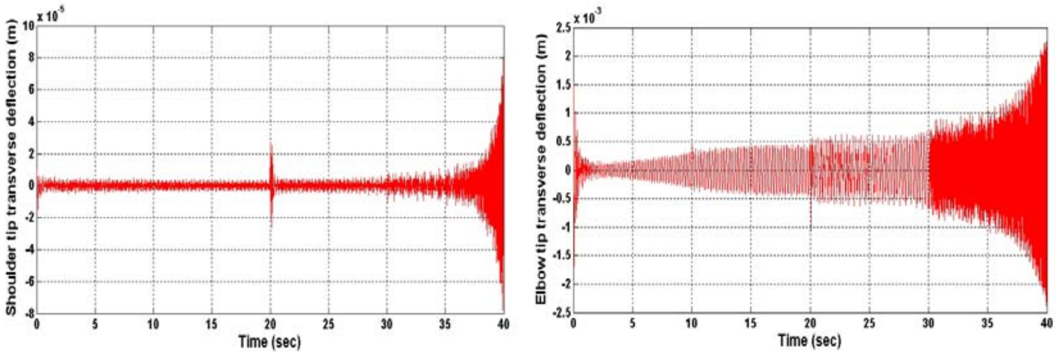

Figure 13: Transverse deflection of the shoulder and the elbow tip

6 Conclusions

For flexible manipulator systems, conventional modeling method can not exactly describe the system. In this paper, based on the absolute nodal coordinate modeling method , by using a simple PD controller with variable PD parameters in the simulation period, a dynamic modeling method was proposed for the planar two-link flexible robot manipulator and tip circle and square trajectories were tracked. Based on the absolute nodal coordinate method, the manipulator state equations can be explicitly expressed by generalized absolute nodal coordinate. The two dimensional shear beam element was used to discrete the flexible links, therefore the shear deformation for the manipulator can be considered. By using the invariant matrix method [17] and the Broyden qusi-Newton method [18], the computational efficiency was significantly improved. The two-link rigid robot inverse kinematic equations are utilized to solve for the reference values of joint angle trajectories. Based on the absolute nodal coordinate method, the joint angle and angular velocity are directly calculated by the simulation results as control feedback. The different PD parameters were used to enhance to tracking accuracy in the simulation period. The accuracy was improved greatly, especially on the square trajectory corners. Although this paper only demonstrated the effectiveness and accuracy of the flexible manipulator control based on the ANCF method. This method can be easily extended to other flexible multibody systems such as space systems, undersea cable mining systems etc.

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