

Analysis on Data Collection with Multiple Mobile Elements in Wireless Sensor Networks

Liang He^{1,2}, Jianping Pan¹, and Jingdong Xu²

¹University of Victoria, Victoria, BC, Canada

²Nankai University, Tianjin, China

Abstract—Exploring mobile elements to conduct data collection in wireless sensor networks offers a new approach to reducing and balancing the energy consumption of sensor nodes; however, the resultant data collection latency may be large due to the limited travel speed. Many research efforts have been made on reducing the data collection latency with the scenario where a single mobile element is available. A potential problem with this approach is the scalability, and a straightforward solution is to employ multiple mobile elements to collect data collaboratively. In this paper, the network where multiple homogeneous mobile elements are available is modeled as an $M/G/c$ queuing system, and insights on the data collection performance are obtained through theoretically analyzing the measures of the queue. In addition, a heuristic formula to determine the optimal number of mobile elements is proposed based on this model. The accuracy of our modeling and analysis, along with the performance evaluation of the proposed heuristic formula, is verified through extensive simulation.

Keywords—Wireless sensor networks, mobile elements, data collection latency, $M/G/c$ queue

I. INTRODUCTION

Data collection is one of the most important applications of wireless sensor networks [1]. Typically, data collection only relies on wireless communications between sensor nodes and the sink node, which may excessively consume the limited energy supply of sensor nodes due to super-linear path loss exponents. Furthermore, sensor nodes near the sink tend to consume energy much faster than others since the data aggregation towards the sink imposes them much heavier volumes of traffic to transmit, which results in a very unbalanced energy usage in the network and degrades the overall network lifetime.

Another data collection approach in wireless sensor networks utilizes the controlled mobility of certain devices, referred to as *mobile elements* (MEs) in this paper [2], [3]. By utilizing mobile elements, not only more energy can be conserved and balanced on sensor nodes, but also the communications and networking become possible in very sparse networks with the “store-carry-forward” approach. The main challenge for this mobility-assisted approach is the relatively lower speed of mobile elements, which may result in a much higher data collection latency. Many efforts, focusing on the case where a single mobile element is available to carry out the data collection, have been made to address this problem [4], [5]. A critical bottleneck of this approach is its scalability: the single mobile element may not be enough for a given latency requirement when the number of sensor nodes is large; furthermore, the energy supply for the mobile element itself

may impose another issue if sensor nodes are deployed in a wide area [6].

Employing more mobile elements is a straightforward approach to improving the scalability. An intuitive idea is to divide the sensing field to several subfields with similar areas, according to the number of available mobile elements, and treat them as several smaller-scale networks where a single mobile element exists [7]. However, this approach assumes the number of mobile elements is given, which in turn is a critical to-be-determined factor for the network planner in practice: a smaller number of mobile elements may be not enough to guarantee a given performance requirement, while increasing it will introduce more costs in both network deployment and operation. Determining the optimal number of mobile elements is an important issue on this topic [5], which is proved to be NP-hard [8]. Another limitation with this approach is that all these mobile elements have to be active all the time, even if the task of data collection in the network is light and they will be idle for most of the time.

In this paper, we tackle this problem with a different approach. Observing the fact that the traveling time of mobile elements is usually the dominating one when compared with the actual data transmission time with sensor nodes, we first model the network as an $M/G/c$ queuing system with a given number of mobile elements, and obtain its system measures to gain insights on the performance of data collection, which in turn provide us guidelines to determine whether these mobile elements are enough for a specific performance requirement. Since evaluating the performance of an $M/G/c$ queue is analytically intractable [9], approximation approaches are adopted to obtain the system measures of the queue, which are of both low computation complexity and high approximation accuracy. Furthermore, we propose a heuristic formula to determine the optimal number of mobile elements based on the analytical results of the queuing model, which is of great practical value since the parameters involved can be easily estimated in real applications and for the devices adopted.

The contributions of this paper are threefold. We model the network where multiple homogeneous mobile elements exist as an $M/G/c$ queuing system, and present its system measures to shed light on the network performance. Second, a heuristic formula to determine the optimal number of mobile elements is proposed as well. Third, the correctness of our modeling and analysis, along with the performance of the proposed heuristic formula, is verified through extensive simulation.

The rest of this paper is organized as follows. The related work on mobility-assisted data collection in wireless sensor networks is given in Section II. We formulate the problem and highlight our approaches in Section III. In Section IV, we present the $M/G/c$ queue-based analytical model, from which we derive the performance metrics of the queuing system, and we present the evaluation results in Section V. The heuristic formula to calculate the optimal number of mobile elements is presented in Section VI, and we conclude this paper in Section VII.

II. RELATED WORK

Recent research has shown that mobility-assisted data collection has many advantages over the traditional approaches, e.g., direct communication and multihop forwarding. A lot of efforts have been made to explore the scenario where a single mobile element is available to conduct the data collection, in terms of both scheme design [5] and performance analysis [4].

A path selection algorithm for the mobile element was proposed in [5], which starts with a *connected dominating set* of the network, then gets a *minimum spanning tree* based on it, and finally generates a *Hamiltonian circuit* for the mobile element. The resultant data collection path frees sensor nodes from their routing obligations and data aggregation becomes more secure. [4] modeled the network where a single mobile element is available as an $M/G/1$ queue, and several critical performance metrics, e.g., the average service time, and the average and the distribution of the queue length, were derived and verified.

A potential bottleneck of a single mobile element is its scalability, and employ more mobile elements for data collection is a straightforward approach to addressing it. [6] was an early work that focused on the multiple mobile elements scenario, where the mobile elements can only travel along fixed straight tracks to collect data from sensor nodes. Load balancing among mobile elements was also considered in terms of the number of sensor nodes served by each mobile element. A proactive data reporting protocol, SinkTrail, was proposed in [10], which established a logical coordinate system for predicting and tracking mobile sinks. It has been shown that SinkTrail achieves energy efficient data forwarding to multiple mobile sinks, and effectively reduces the number of sink-location broadcast messages. A tour design algorithm for the mobile elements was proposed in [3], which minimizes the number of mobile elements according to the constraints in both distance and time. In [2], an adaptive data gathering protocol that employs multiple mobile elements to help wireless sensor networks achieve both energy efficiency and low data collection latency was presented, which adopted a virtual elastic-force model to adjust the travel speed and direction of mobile elements.

It was proved in [8] that determining the minimum number of required mobile elements with a given latency requirement is NP-hard and cannot be approximated within a factor of 2. Although that paper originally dealt with the problem of sweep coverage with mobile sensors, the results can also be applied

to the problem of mobility-assisted data collection with simple transformation.

Most existing work on data collection with multiple mobile elements focuses on scheme design, i.e., how the mobile elements should collaborate to collect data in the network, in order to guarantee an acceptable QoS of data collection, in terms of energy efficiency of sensor nodes, data collection latency, load balancing among mobile elements, etc. Our approach in this paper is to theoretically analyze the QoS of data collection through a queue-based model, and observing its NP-hardness, we also propose a heuristic formula to determine the optimal number of mobile elements based on this model. To the best of our knowledge, this is the first attempt in the literature to analytically evaluate the QoS of data collection with multiple mobile elements, and to determine the optimal number of mobile elements as well.

III. PRELIMINARIES

Aiming to achieve a balance between the data collection latency and the deployment/operation costs of *mobile elements* (MEs), we consider the scenario where multiple homogeneous MEs are available to collect data from static sensor nodes. Sensor nodes initiate data collection requests when they have enough data to report, which are forwarded to the MEs by adopting certain existing ME-tracking protocols [11]. The MEs maintain a service queue for received data collection requests, and serve them with the *first-come-first-serve* (FCFS) discipline. By serving a request, we mean that one of the MEs moves to the sensor node that sends the request, and collects data from that node through short-range wireless communications.

Since the typical data relay speed in sensor networks is much faster than the travel speed of MEs [12], and efficient protocols for ME-tracking exist in the literature, we assume the time since a request is sent by a sensor node till it is received by the MEs is small and negligible. Note that usually these ME-tracking protocols rely on the multi-hop forwarding among sensor nodes. Thus instead of using these protocols to carry the sensory data to the MEs directly, which usually are of much larger size, only the requests for data collection is forwarded to reduce the communication overhead of sensor nodes.

We assume a Poisson arrival process for the data collection requests. This assumption holds since that first, the number of sensor nodes in the sensing field is relatively large, and second, the probability for a sensor node to initiate a data collection request is relatively small. Theoretically, if the size of client population of a queuing system is relatively large and the probability by which clients arrive at the queue is relatively low at certain time, the arrival process can be adequately modeled as a Poisson process [27]. This assumption is further confirmed in Section V. Our approach is to model the network as an $M/G/c$ queuing system, i.e., the homogeneous MEs as servers and data collection requests as clients. With this modeling, the data collection latency of our focus is equivalently the response time of clients in the queue-based model, and we derive

analytical results on the latter to shed light on the former, based on which, we also propose a simple heuristic formula to determine the optimal number of MEs given a specific performance requirement, to achieve the desired balance.

IV. QUEUE-BASED MODELING AND ANALYSIS

A. Modeling as an $M/G/c$ Queue

Observing the fact that usually the travel speed of the MEs is much lower than that of the data relayed in the network [13], we consider the service time of a request as the time from the completion of serving the current request, to the time the ME moves to the sensor node that sends the next to-be-served request. From the existing results on the distance distribution $f_D(d)$ of two random points in a unit grid [14], we know that

$$f_D(d) = \begin{cases} 2d(\pi - 4d + d^2) & 0 \leq d \leq 1 \\ 2d[2\sin^{-1}(\frac{1}{d}) - 2\sin^{-1}\sqrt{1 - \frac{1}{d^2}}] & 1 \leq d \leq \sqrt{2} \\ +4\sqrt{d^2 - 1} - d^2 - 2 & \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Based on (1), and with a constant travel speed v of the MEs, the service time distribution can be derived as $\mathcal{S}(t) = \mathbb{P}\{D \leq vt\}$, and $s(t) = \partial\mathcal{S}(t)/\partial t$. Its expectation and variance can be calculated as $1/\mu = E[D]/v$ and $\sigma^2 = V[D]/v^2$.

With the Poisson arrival process of data collection requests and the service time distribution obtained above, we can model the network as an $M/G/c$ queuing system with G characterized by $\mathcal{S}(t)$, and c is the number of MEs.

Evaluating the performance of an $M/G/c$ queue is analytically intractable, and even when formulas can be obtained, often they are complicated and dependent on particular probability distributions [15], [16]. Thus, instead of pursuing the exact analytical results on the system measures, we adopt the approximation approach. The basic idea is to combine the analytical results on simple queuing systems such as $M/M/c$, $M/D/c$, etc, to approximate the measures of the $M/G/c$ queue, i.e., the so-called *system interpolations* [9]. These approximations require heuristics depending very much on intuition and creativity.

B. Expected Response Time

We first explore the expected latency of data collection requests in the network, or equivalently, the expected response time in the $M/G/c$ queue. A simple two-moment approximation formula for the mean queuing time in an $M/G/c$ queue can be derived from [17], and its approximation quality is verified by comparing with the known solutions in particular cases,

$$\mathbb{E}[\mathcal{W}_{q,M/G/c}] \approx \frac{1 + \gamma}{\frac{2\gamma}{\mathbb{E}[\mathcal{W}_{q,M/M/c}] + \frac{1-\gamma}{\mathbb{E}[\mathcal{W}_{q,M/D/c}]}} \quad (2)$$

where $\gamma = \mu^2\sigma^2$ is the squared coefficient of variance.

The above approximation is actually a weighted combination of $\mathbb{E}[\mathcal{W}_{q,M/M/c}]$ and $\mathbb{E}[\mathcal{W}_{q,M/D/c}]$. The former is well

studied and can be easily calculated by [18]

$$\mathbb{E}[\mathcal{W}_{q,M/M/c}] = \frac{(c\rho)^c}{c!\mu(1-\rho)^2} \cdot \left[\sum_{i=0}^{c-1} \frac{(c\rho)^i}{i!} + \frac{(c\rho)^c}{c!(1-\rho)} \right]^{-1} \quad (3)$$

where λ is the arrival rate of data collection requests, and $\rho = \lambda/\mu$. The latter can be obtained by Crommelin's formula [19]

$$\mathbb{E}[\mathcal{W}_{q,M/D/c}] = \frac{1}{\mu} \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \left[\frac{(ic\rho)^{j-1}}{(j-1)!} - \frac{(ic\rho)^j}{\rho j!} \right] e^{-ic\rho} \quad (4)$$

However, the series in (4) converges very slowly, especially when the traffic intensity ρ is high [20]. Again, approximation approaches are adopted to address this convergence issue. An approximation on $E[\mathcal{W}_{q,M/D/c}]$ with simple computation complexity and good accuracy is presented in [9]

$$\mathbb{E}[\mathcal{W}_{q,M/D/c}] \approx [1 + \mathcal{F}(\theta)g(\rho)(1 - e^{-\frac{\theta}{\mathcal{F}(\theta)g(\rho)}})] \cdot \mathbb{E}[\mathcal{W}_{q,M/M/c}] \quad (5)$$

where $\theta = (c-1)/(c+1)$, $\mathcal{F}(\theta) = (\theta[(\frac{9+\theta}{1-\theta})^{\frac{1}{2}} - 2])/(8(1+\theta))$, and $g(\rho) = (1-\rho)/\rho$.

Substituting (3) and (5) into (2), we can easily calculate $\mathbb{E}[\mathcal{W}_{q,M/G/c}]$, with which the expected response time of the requests in the $M/G/c$ queue can be calculated by

$$\mathbb{E}[\mathcal{W}_{r,M/G/c}] = \mathbb{E}[\mathcal{W}_{q,M/G/c}] + 1/\mu \quad (6)$$

C. Queue Length Distribution

The above results on the expected response time is important since it not only offers us insights on the average data collection latency, but also helps to obtain the measures on the length of the $M/G/c$ queue. Denote X as the number of requests either waiting or being served at arbitrary time in the $M/G/c$ queue. Let $\mathcal{P}_i = \mathbb{P}\{X = i\}$ ($i \geq 0$), and define

$$\mathcal{R} = \mathbb{E}[\mathcal{W}_{q,M/G/c}]/\mathbb{E}[\mathcal{W}_{q,M/M/c}]. \quad (7)$$

A geometric-form approximation for the queue length probability is proposed in [21]

$$\mathcal{P}_{i,M/G/c} = \begin{cases} [(c\rho)^i/i!]\mathcal{P}_{0,M/M/c} & i = 0, \dots, c-1 \\ (1-\zeta)\zeta^{i-c}\mathcal{D}_{M/M/c} & i \geq c \end{cases} \quad (8)$$

where $\zeta = \rho\mathcal{R}/(1-\rho + \rho\mathcal{R})$, $\mathcal{P}_{0,M/M/c} = [\sum_{i=0}^{c-1} (c\rho)^c/i! + (c\rho)^c/(c!(1-\rho))]^{-1}$, and $\mathcal{D}_{M/M/c} = (c\rho)^c\mathcal{P}_{0,M/M/c}/(c!(1-\rho_c))$ is the probability that a newly arrived request has to wait before being served in an $M/M/c$ queue.

We can easily calculate the expected queue length based on its approximated distribution, $\mathbb{E}[\mathcal{L}] = \sum_{i=0}^{\infty} i \cdot \mathcal{P}_{i,M/G/c}$, and the probability that a newly arrived request has to wait before being served, i.e., the equivalent of $\mathcal{D}_{M/M/c}$ in $M/G/c$ queue, can be calculated by $\mathcal{D}_{M/G/c} = 1 - \sum_{i=0}^{c-1} \mathcal{P}_{i,M/G/c}$.

D. Response Time Distribution

By *distributional Little's law* [22], the number of customers in the queue has the same distribution as the number of arrivals during the waiting time. Based on this and the above approximation results on the queue length distribution, [23] proposes an approximation for the queuing time distribution in the $M/G/c$ queuing system

$$\mathcal{W}_{q,M/G/c}(t) \approx 1 - \mathcal{D}_{M/M/c} e^{-c\mu(1-\rho)t/\mathcal{R}} \quad (9)$$

and $w_{q,M/G/c}(t) = \partial \mathcal{W}_{q,M/G/c}(t) / \partial t$.

Furthermore, since the response time of a request is the sum of its queuing time and service time, which are independent to each other, by *convolution theorem* [24], we have

$$w_{r,M/G/c}(t) = w_{q,M/G/c}(t) * s(t) \quad (10)$$

V. PERFORMANCE EVALUATION

We evaluate our modeling and analysis results on the system measures in this section. Based on the parameters from the real system [26], we consider a square sensing field with size $100 \times 100 \text{ m}^2$, where a total number of 100 nodes are uniformly deployed at random, and the constant travel speed of MEs is 1 m/s . A total number of 10,000 requests are served during each run of the simulation, which is repeated for 50 times. We explore the cases with $c = 2$ and $c = 3$, respectively, and also present the results with $c = 1$ for comparison, in which case the queuing model regresses to $M/G/1$, and its measures can be obtained by [18].

To verify the $M/G/c$ modeling, we examine the arrival process of the data collection requests by an event-driven simulator, where stochastic events occur randomly in the sensing field. Sensor nodes within a certain distance (i.e., the sensing range) of the event can detect the event, and data are generated to record it. The data size for recording each event varies from 10–100 B . Events happen independently in both spatial and temporal domain, and sensor nodes initiate the data collection requests when their buffers become full. We explore the cases where the sensor node buffer size is 4 and 8 KB , respectively, and record the inter-arrival time of the requests to compare with an exponential distribution with the same mean value. The results shown in Fig. 1(a) indicate that they agree with each other very well, and thus verify the assumption of the Poisson arrival process. Also, a larger node buffer results in a smaller arrival rate, since the sensor nodes can hold the data on-board longer.

To deal with the inconvenience of the piecewise distance probability density function in (1), we use *least squares fitting* to approximate it by a high order polynomial, and adopt the approximation polynomial function to derive the service time distribution in our performance evaluation.

The approximation results on the expected queuing time are verified in Fig. 1(b). Note that no results for $c = 1$, or $c = 2$, are shown when λ is larger than 0.018, or 0.036, since the further increase of λ will result a ρ larger than 1, and no steady-state measures can be obtained.

Figure 1(c) shows the evaluation results of the approximation on the queue length distribution with a λ of 0.018. Besides

the accuracy of the approximation, we can see that increasing c from 1 to 2 can greatly shorten the queue length, which in turn reduces the data collection latency. However, the benefit of increasing c further from 2 to 3 is quite limited, which shows the necessity of an efficient approach to determining the optimal number of MEs. The results on the expected queue length and the probability for requests to wait before being served are shown in Fig. 1(d) and Fig. 1(e), respectively.

Keeping λ as 0.018, the results of the queuing time and response time distributions of data collection requests are shown in Fig. 1(f) and Fig. 1(g), respectively. Again, we observe that the gain in reducing the data collection latency when increasing c from 2 to 3 is not so obvious.

VI. THE OPTIMAL NUMBER OF MEs

As observed above, certain optimal values for the number of MEs exist, and further increasing it cannot gain much for the performance of data collection. In this section, we present a simple heuristic formula to determine this optimal value \hat{c} based on the service demand in the network and the service ability of MEs. Denote \mathcal{B} as the steady-state average number of busy MEs, by definition

$$\mathbb{E}[\mathcal{L}] = \mathbb{E}[\mathcal{L}_q] + \mathcal{B} \quad (11)$$

and by Little's law

$$\mathbb{E}[\mathcal{L}] = \mathbb{E}[\mathcal{L}_q] + \lambda/\mu \quad (12)$$

From (11) and (12), we know

$$\mathcal{B} = \lambda/\mu \quad (13)$$

It is obvious that $\lceil \mathcal{B} \rceil$ is a lower bound for \hat{c} , and we propose the following simple heuristic formula to find \hat{c} based on it

$$\hat{c} = \lceil \mathcal{B} - \eta + 1 \rceil \quad (14)$$

where $\eta \in (0, 1)$ is defined as a boundary parameter that reflects whether the traffic in the $M/G/c$ queue is heavy or not: the traffic is considered heavy if $\rho \geq \eta$, or light otherwise. The heuristic is also of great practical value since the parameters involved, λ and μ , can be easily estimated by the data generation rate of specific applications and the features of the adopted MEs.

(14) is motivated by the idea that the gain of further increasing the number of MEs, in terms of the data collection latency, is quite limited. In the case where other constraints, e.g., the expected delay of requests, exist, we have to take them into account when calculating \hat{c} as well.

The efficiency of the proposed heuristic is verified in Fig.1(h), where η is 0.6. By exploring the resultant average response time with different λ for the cases where c is $\hat{c} - 1$, \hat{c} , and $\hat{c} + 1$, respectively, we observe a large reduction when increasing c from $\hat{c} - 1$ to \hat{c} , while that for increasing from \hat{c} to $\hat{c} + 1$ is much smaller, which verifies the efficiency of the proposed approach. Another observation is that the difference between the gain of increasing c from $\hat{c} - 1$ to \hat{c} and from \hat{c} to $\hat{c} + 1$ is getting smaller when λ increases, since \hat{c} is relatively large in this case, and the difference of increasing or decreasing it by 1 is not so obvious as before.

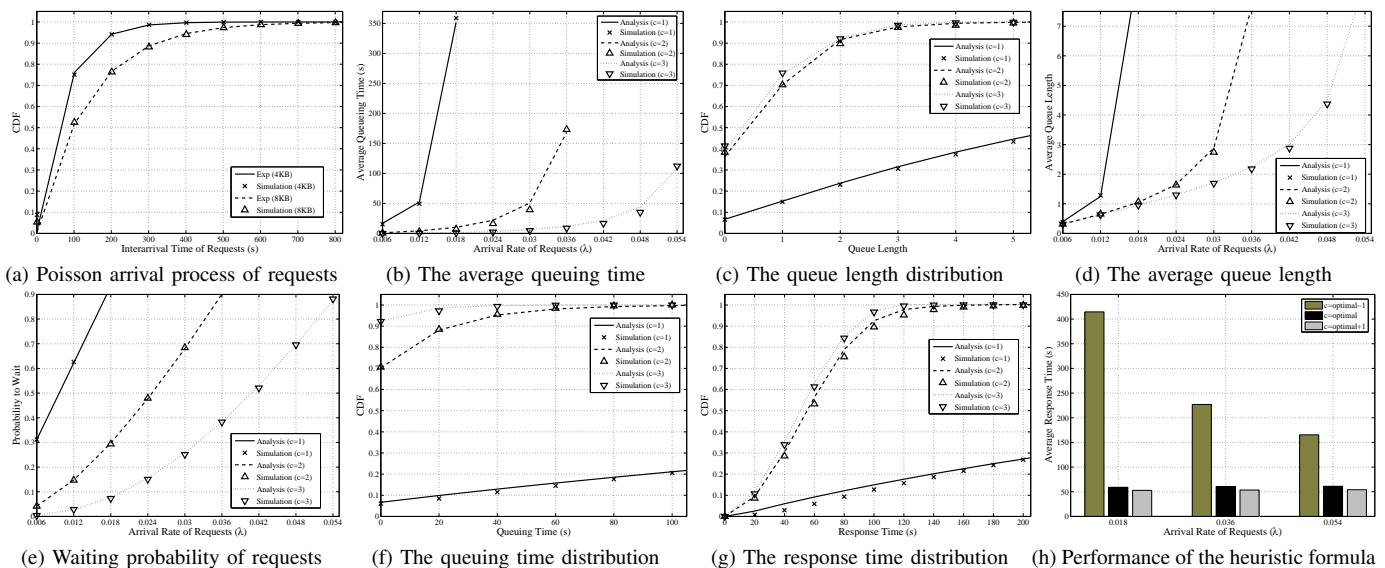


Fig. 1: Evaluation of the queue-based model and analysis.

VII. CONCLUSIONS

In this paper, we have modeled the problem of exploiting multiple homogeneous MEs to carry out the data collection task in wireless sensor networks as an $M/G/c$ queuing system. System measures of the queue, e.g., the expected values and distributions of queue length, queuing time and response time have been explored. The $M/G/c$ queuing model helps us to understand the impact of different parameters on the network performance, and the corresponding analytical results can serve as important guidelines to evaluate whether the number of MEs is enough for a given performance requirement. We have also proposed a simple heuristic formula to determine the optimal number of MEs. Our future work will focus on exploring non-FCFS disciplines to further investigate the resultant performance of data collection.

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