

An approximate general model for multipactor in curved geometries

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Abstract

In order to formulate practical guidelines to avoid multipactor in a quadri-filar helix antenna we have developed a simplified statistical treatment of the electron dynamics between two opposing conducting surfaces of arbitrary curvature. The model allows us to judge the effect of the curvature and the inhomogeneous electric field on the electron density between the conductors. It is found that the parameter regions in these types of systems where multipactor will be possible are quite restricted. In particular it was found that multipactor will not be possible in any realistic helix antenna configuration.

Introduction

Multipactor is a serious failure mechanism for microwave equipment operating under vacuum conditions [1,2], where the most relevant applications are space vehicle rf hardware and waveguides for particle accelerators. The resulting electron avalanche will generate noise, detune the equipment, and may even lead to gas breakdown and system damage. Therefore multipactor prediction is an important technical problem, and when new rf equipment is designed it is standard practice to take into account the multipactor limits. Analytical treatment of the multipactor dynamics is generally limited to the geometry of parallel metal plates, and even approximate models have only been developed for the simple geometries of coaxial, rectangular, wedge-shaped, and circular waveguides, as well as the waveguide iris. To treat other geometries one is typically restricted to extensive numerical simulations, which can be time-consuming and hard to interpret. Therefore it is always desirable to develop simplified theoretical models able to take account of the main physical mechanisms. In order to treat the complicated problem of multipactor in a quadri-filar helix antenna for satellites we have developed a model which treats the average dynamics of electrons moving between two conductors having arbitrary surface curvature [3]. The model is based on an approximate statistical treatment, where the random electron emission velocity is taken into account. It is found that certain geometries will lead to focusing of the electrons,

while others lead to defocusing. The case when electrons are focused during successive passages is most dangerous from a multipactor point of view, and depending on the magnitude of the emission velocity, and the size of the conductors this focusing can be balanced by electron losses. Taking all these factors into account we find general criteria for when multipactor is possible, and what the limiting value of the secondary emission coefficient will be for different arrangements of electrodes.

Statistical treatment of electron dynamics

As the secondary electrons are launched from the lower surface the main part of their motion will be in the direction normal to the surface, with a small addition from the thermal velocity. The emission velocity of an electron can then be written [2,3]

$$v_{y0} = v_{\omega} \cos \theta + v_{T,n} \cos \theta + v_{T,t} \sin \theta, \quad v_{x0} = v_{\omega} \sin \theta + v_{T,n} \sin \theta + v_{T,t} \cos \theta \quad (1)$$

Where $v_{\omega} = eE/m\omega$ is the oscillatory velocity, e the electron charge, E the electric field amplitude at the surface, m the electron mass, ω the field radian frequency, v_T the thermal velocity, and x, y, n, t signifies the directions indicated in Fig. 1, and the normal and tangential directions with respect to the emission surface.

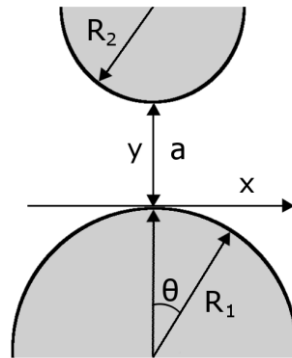


Fig. 1: The fundamental geometry for the simplified analysis. If the surfaces would be concave, the curvature radii, $R_{1,2}$, would be negative.

Under the assumptions that $v_{\omega} \gg v_T$, and $x \ll R$ we get

$$v_{y0} \approx v_{\omega}, \quad v_{x0} = \frac{x}{R} v_{\omega} + v_{T,t} \quad (2)$$

As the electrons cross the gap they will oscillate along the field lines, and as the field is inhomogeneous they will experience acceleration due to the ponderomotive force [3] amounting to $\dot{v}_x = \Lambda^2 x$, where

$$\Lambda^2 = \frac{1}{2} x \frac{e^2 E}{m^2 \omega^2} \frac{\partial^2 E}{\partial x^2} \quad (3)$$

And it has been assumed that the electrons move close to the symmetry line between the conductors, and that the field has a maximum on this line. Assuming $x(0) = x_0$, and $v_x(0) = v_{x0}$, the equation of motion for the electrons in the x -direction becomes

$$x(t) = x_0 \cosh \Lambda t + \frac{v_{x0}}{\Lambda} \sinh \Lambda t \quad (4)$$

The gap is traversed in the time $\tau \approx a/v_\omega$, which means that the impact coordinate becomes

$$x(\tau) \approx x_0 \left(\cosh \Lambda \tau + \frac{v_\omega}{R} \sinh \Lambda \tau \right) + \frac{v_{T,t}}{\Lambda} \sinh \Lambda \tau \equiv \alpha x_0 + \beta v_{T,t} \quad (5)$$

In this paper we will only look at the case $R_1 = R_2$, see [3] for a full treatment. To describe the avalanche of electrons we want to investigate how the average position, $\langle x \rangle$, and the width, D , of an electron bunch changes during successive passages. We take the spatial averages assuming $\langle v_{T,t} \rangle = 0$, and $\langle v_{T,t}^2 \rangle = V^2$.

$$\langle x \rangle = \alpha \langle x_0 \rangle, \quad D^2 = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2 = \alpha^2 (\langle x_0^2 \rangle - \langle x_0 \rangle^2) + \beta^2 V^2 \quad (6)$$

This means that during for an arbitrary passage, the average position and bunch width is given by

$$\langle x_{k+1} \rangle = \alpha^2 \langle x_k \rangle, \quad D_{k+1}^2 = \alpha^2 D_k^2 + \beta^2 V^2 \quad (7)$$

Hence

$$\langle x_k \rangle = \alpha^k \langle x_0 \rangle, \quad D_k^2 = \alpha^{2k} \left(D_0^2 - \frac{\beta^2 V^2}{1 - \alpha^2} \right) + \frac{\beta^2 V^2}{1 - \alpha^2} \quad (8)$$

In general, the first term will cause the average electron position to deviate from the symmetry line exponentially provided $\alpha > 1$, however, we are mainly interested in the most dangerous cases, when $\alpha \approx 1$, and the dilution of the electron density is the main ‘‘loss’’ source of electrons.

Infinite conductors

If the size of the conductors is large in comparison with the radii of curvature and the conductor separation, no electrons are lost over the edges, but the electron density will grow or decay depending on the value of the secondary emission yield (SEY), and whether D_k decays or grows over successive passages. If a bunch of electrons are emitted with a density n_k and a width D_k , the density will have changed to $n_{k+1} = n_k D_k / D_{k+1}$ as it approaches the opposing surface. After impact and emission the new bunch moving in the opposite direction will have the density $n'_{k+1} = \sigma n_{k+1}$, where σ is the SEY for the impact speed. Multipactor will appear if the density grows, i.e. $n_{k+1}' > n_k$ yielding

$$\sigma > \frac{n_k}{n_{k+1}} = \frac{D_{k+1}}{D_k} \quad (9)$$

If we let k become large we see that the criterion becomes $\sigma \geq 1$ if $1 \leq \alpha$, and $\sigma > \alpha$ if $\alpha > 1$. Physically this means that without taking into account electron losses over the conductor edges all concave surfaces are susceptible to multipactor regardless of the SEY characteristics, and that for convex surfaces the SEY must be larger than a factor set both by the surface curvature and the strength of the ponderomotive spreading, whereas the thermal emission spread is of little consequence.

Finite conductor width

If the conductors are of a finite width, $2b$, we need to take into account electron losses over the edges. Following the reasoning in the previous section and assuming that $b < D_k$ we can infer that the fraction of electrons that are lost at each impact is $(D_{k+1} - b)/D_k$. Furthermore the new electron bunch will have the width $2b$ when it is launched from the surface. The criterion for multipactor to be possible is then

$$\sigma > \frac{D_{k+1}}{b} = \sqrt{\alpha^2 + \frac{\beta^2}{b^2} V^2} \quad (10)$$

In this case the emission velocity spread becomes very important in determining the loss of electrons over the edges, as seen in the ratio V^2/b^2 . In paper [3] this criterion is applied to several structures including the helix antenna, and it is found that multipactor is only possible in realistic helix systems if $\sigma > 6 \times 10^5$, which is not physically realisable.

Conclusions

By applying a statistical method on the electron dynamics between opposing conductors of arbitrary curvature we have been able to formulate multipactor limits for a large class of waveguides. Although these limits will not provide exact predictions for the multipactor power thresholds they do provide strict boundaries outside which multipactor is not possible. These type of guidelines are much more practical than the extensive simulations otherwise necessary to analyse novel structures.

References

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