Failure of compressible/dilatant geomaterials

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The paper presents a general constitutive equation for geomaterials allowing to describe dilatancy and/or compressibility during transient and stationary creep. The constitutive equation also describes the instantaneous response of the geomaterial, work-hardening during transient creep, instantaneous failure and creep failure. The damage produced by dilatancy is used to formulate a criterion for creep failure. Thus ultimate failure may be involved in various ways, depending on the initial and boundary conditions and certainly on the constitutive equation. Typical mining engineering examples are given. First is discussed the creep closure of a deep vertical cylindrical cavern, various possible instantaneous failures, creep failure, and spreading of damage by dilatancy into the rock mass. Second example discusses the instantaneous failure and creep failure around a horizontal tunnel, and the location where damage by dilatancy is more pronounced. The third example presents the case of a rectangular-like shaped cavern.

1. INTRODUCTION

In this paper a general constitutive equation for geomaterials is presented. This constitutive equation was formulated starting from the desire to describe volumetric compressibility and/or dilatancy during both transient and stationary creep. Dilatancy has long been recognized as being related to failure and even predicting an imminent possible failure. Dilatancy is also related to an increase of permeability, to the stability of underground excavations, boreholes, wells, etc., and to a possible creep failure.

Two approaches are possible. Either we assume the yield function and viscoplastic potential to be of a certain form in order to be able to describe dilatancy (several authors have followed this approach, as for instance Desai and Zhang, 1987, Desai and Varadarajan, 1987, Desai *et al.*, 1986), or we can find procedures to determine these constitutive functions from the experimental data without making any a prori assumption concerning their forms. This is the approach followed by the present author (see also Aubertin *et al.*,1991a, 1991b, Chan *et al*, 1992, Dragon and Mroz, 1979). However, it was assumed that geomaterials exhibit "instantaneous" responses and "non-instantaneous" ones, as for instance creep and relaxation. Irreversible volumetric deformation has also been considered for hydrostatic loadings. "Instantaneous" failure is incorporated in the constitutive equation, which can describe also creep failure.

2. CONSTITUTIVE EQUATION

Starting from the assumptions that in most geomaterials both longitudinal and transverse extended body seismic waves can propagate, that the material displacements and rotations are small, the initial yield stress of the rock can be assumed to be zero or very close to it, the constitutive equation is written in the form (Cristescu, 1987, 1989, 1991, 1993, 1994a)

$$\dot{\boldsymbol{\epsilon}} = \frac{\dot{\boldsymbol{\sigma}}}{2G} + \left(\frac{1}{3K} - \frac{1}{2G}\right)\dot{\sigma}\mathbf{1} + k_T\left\langle 1 - \frac{W_T(t)}{H(\boldsymbol{\sigma})}\right\rangle \frac{\partial F}{\partial \boldsymbol{\sigma}} + k_S \frac{\partial S}{\partial \boldsymbol{\sigma}} \tag{1}$$

where G and K are the elastic parameters, generally not constant, $H(\sigma)=W_T(T)$ is the stabilization boundary for the transient creep, with < > the positive part of the function shown, and $H(\sigma)$ the "yield" function, and

$$W_T(T) - W^P = \int_0^T \sigma(t)\,\dot{\epsilon}_V^I(t)\,dt + \int_0^T \boldsymbol{\sigma}'(t)\cdot\dot{\boldsymbol{\epsilon}}'^I(t)\,dt = W_V(T) + W_D(T) \tag{2}$$

the irreversible stress power per unit volume used as a work-hardening parameter or internal state variable; W^P is a constant, the "primary" value of $W_T(T)$ at a specific location. Further $F(\sigma)$ is the potential for transient creep, $S(\sigma)$ the potential for stationary creep, k_T and k_S are the corresponding viscosity coefficients. It is assumed that all constitutive functions depend on mean stress σ and on equivalent stress $\bar{\sigma}$ or octahedral shear stress $\tau=(\sqrt{2/3})\,\bar{\sigma}$.

From (1) the irreversible transient volumetric rate of deformation is

$$(\dot{\epsilon}_V^I)_T = k_T\left\langle 1 - \frac{W(t)}{H(\boldsymbol{\sigma})}\right\rangle \frac{\partial F}{\partial \boldsymbol{\sigma}}\cdot\mathbf{1}\ . \tag{3}$$

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Any stress change from $\sigma(t_o)$ to $\sigma(t)$ with $t>t_o$ produced by an excavation, say, is a *loading* if $H(\sigma(t))>W_T(t_o)$, with three possible cases (see Fig.1): $\partial F/\partial\sigma>0$ means compressibility, $\partial F/\partial\sigma<0$ means dilatancy, and $\partial F/\partial\sigma=0$ is the equation of the compressibility/dilatancy boundary. If we have $H(\sigma(t))<W_T(t_o)$, an *unloading* with respect to the transient creep is taking place. Thus, dilatancy and compressibility are governed by the function F. For $F \equiv H$ the constitutive equation is associated; it was found that for most geomaterials the constitutive equation is rather nonassociated.

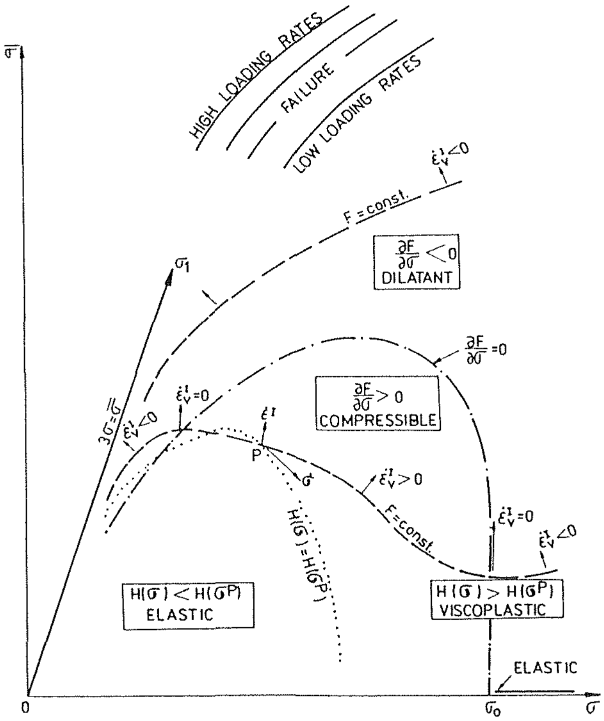

FIG.1 Domains of compressibility, dilatancy and elasticity in the constitutive domain: dash-dot line is the compressibility/dilatancy boundary; failure depends on the loading rates.

Fig.1 is typical and thus does not correspond to a specific material, but in Fig.2 is shown the constitutive domain for rock salt. In both figures the dash-dot line is the compressibility/dilatancy boundary X=0, and solid line in Fig.2 shows the instantaneous failure surface. Since for lower loading rates the failure surface is distinct (failure takes place at smaller τ), that is shown in Fig.1 by several solid lines. As described by (3) the dilatancy or compressibility depends on the orientation of the normal to the surface F = const. The compressibility/dilatancy boundary is to intersect somewhere the 0σ-axis, at a pressure σ_o, which is the smallest pressure closing all microcracks and pores. The compressibility/dilatancy boundary can be determined quite accurately for small and moderate σ, but with some difficulty for high σ. For associated constitutive equation, the compressibility/dilatancy boundary would be $\partial H/\partial\sigma= 0$, and in many cases it is quite far from $\partial F/\partial\sigma=0$. Thus associated constitutive equation can describe dilatancy and compressibility but imperfect. For higher pressures, i.e. $\sigma>\sigma_o$, the behavior is close to elastic.

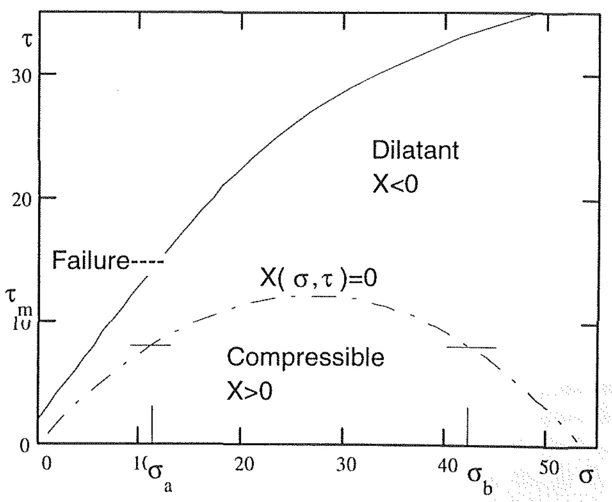

FIG.2 Constitutive domain showing instantaneous failure surface and compressibility/dilatancy boundary X=0.

The procedure to determine the function H is described in detail by Cristescu (1987 and 1989) and it was applied for a variety of rocks: for granite, andesite, various kinds of coal, cement-concrete by Cristescu (1989), for both dry and saturated sand by Cristescu (1991), for rock salt by Cristescu and Hunsche (1992 and 1993) and Cristescu (1994a), for bituminous concrete by Florea (1994a), for bio-calcarenite by Brignoli and Sartori (1993), and for chalk by Dahou *et al.*(1994). The procedure to determine the viscoplastic potential F is described by Cristescu (1991 and 1994a) and was applied to dry and saturated sand and rock salt, and also applied to bituminous concrete (Florea, 1994b), and chalk (Dahou *et al.*, 1994).

While transient creep must describe both volumetric compressibility and/or dilatancy, the stationary creep describes dilatancy only. In the domain where the rock is compressible, we have to expect for stationary creep volumetric incompressibility in addition to a change in shape. For this reason it has been shown (Cristescu, 1993) that the constitutive function $S(\sigma)$ must satisfy certain conditions. The dominant term in the S function is either a power function or a polynom, or a power function multiplying a trigonometric function as (Carter and Hansen, 1983):

$$p_o\left(\frac{\tau}{\sigma_\star}\right)^{q_o} \quad or \quad p_1\left(\frac{\tau}{\sigma_\star}\right)^{q_1}+p_2\left(\frac{\tau}{\sigma_\star}\right)^{q_2} \quad or \quad p\left(\frac{\tau}{\sigma_\star}\right)^{r}\sinh\left(q\frac{\tau}{\sigma_\star}\right) \;. \qquad (4)$$

With this model, stationary creep describes change in shape in the whole constitutive domain (up to failure), in the dilatancy domain describes dilatancy, while in the compressibility domain describes incompressibility. The expressions (4) for the function $Q'(\tau)$ show that there where around the excavation we get high values for τ, one has to expect very high values of the stationary irreversible components of the rate of strains. It is in the same domains that creep failure may be expected after a certain time interval. Examples will be given below.

3. DAMAGE AND FAILURE

Failure of rocks has always been expressed by a relationship between stress invariants, as for instance

$$\tau = f(\sigma) \qquad (5)$$

for compressive stress states, and limit tension cut-off planes

$$\sigma_i = -\sigma_{ti} \quad (i = 1,2,3) \qquad (6)$$

for tensile stress states. These failure conditions describe "instantaneous" failure or "nearly instantaneous" one. Such failure condition is incorporated in the constitutive equation (1) (in fact in the function F). Besides this kind of failure, rocks may fail after a very long time interval elapsed from the moment of excavation. In Cristescu (1986) (see also Cristescu, 1989 and 1993) it was suggested that the energy released by microcracking in the dilatancy domain, can be a measure of damage and ultimate damage. That can be expressed by

$$W_V(T) = \int_{T_m}^{T} \sigma(t)\,\dot{\epsilon}_V^I(t)\,dt \qquad (7)$$

where T_m corresponds to the moment when W_V reaches its maximum. This damage measure was used primarily for rocks whose porosity is due mainly to microcracks and are very dilatant. For such geomaterials W_V increases if the loading path (increasing σ) is in the compressibility domain, but decreases in the dilatant domain (for increasing τ surpassing a certain limit, corresponding to the compressibility/dilatancy boundary). This decrease is due either to the stress variation (τ increases), or to creep (σ =const.). Thus (7) can be used to describe the progressive damage during creep in the dilatancy domain. It can be shown that ultimate creep failure is expected after a relatively short time interval if τ has a value close to the instantaneous failure surface. For smaller values of τ, but still in the dilatancy domain, the time to ultimate creep failure increases exponentially as τ approaches the compressibility/dilatancy boundary. For τ close to this boundary, the time up to failure increases to infinity. Therefore, for practical purposes, creep failure is possible for values of τ close to the instantaneous failure surface, only.

For geomaterials which up to failure are compressible only, things are slightly different. For such materials the integral (7) is strictly increasing during standard loading paths used in triaxial tests. Creep failure is again possible, though such materials stay compressible up to failure. These are mainly high porosity materials with the porosity due mainly to pores.

Another kind of failure is the one which takes place in saturated sand subjected to repeated small loadings (sand liquefaction).Consider for instance Fig.3 showing the surfaces H=const and F=const for saturated sand (Cristescu, 1991), and domain comprised between $\partial H/\partial \sigma=0$ and $\partial F/\partial \sigma=0$.

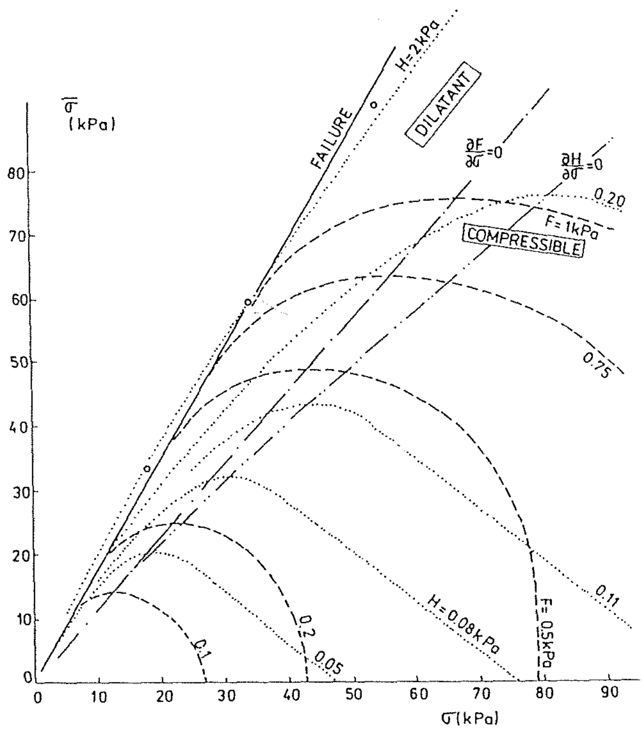

FIG.3 Surfaces H=const. and F=const. for saturated sand. Domain situated between $\partial F/\partial \sigma=0$ and $\partial H/\partial \sigma=0$ is a domain of possible instability.

4. MINING EXAMPLES

The first example is the creep closure and failure around a deep well or a circular cylindrical cavern of radius a. A constitutive equation for rock salt is used. For details concerning the formulation of the problem and the mathematical procedure used see Cristescu (1994b). We consider the case of a very deep cavern with an internal pressure (in the example it is taken constant and equal to 10 MPa). After an "instantaneous" excavation the stress distribution along the cavern wall is shown in Fig.4. Thus

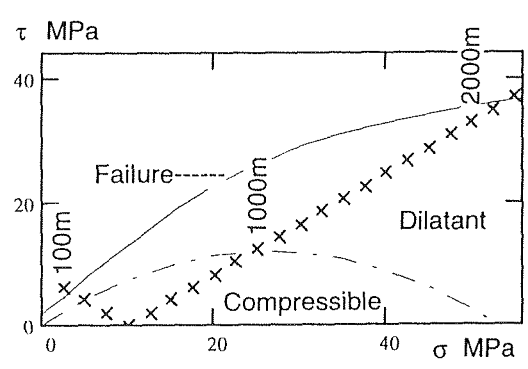

FIG.4 Stress distribution along a deep cavern.

both at shallow depths and at deep depths (under 2170 m, say) instantaneous failure is taking place. The amount of rock involved in failure is shown in Fig.5 and Fig.6. The failure domain labelled F1 is obtained with a condition (6), i.e. failure in tensile stresses, while the domain F2 is obtained

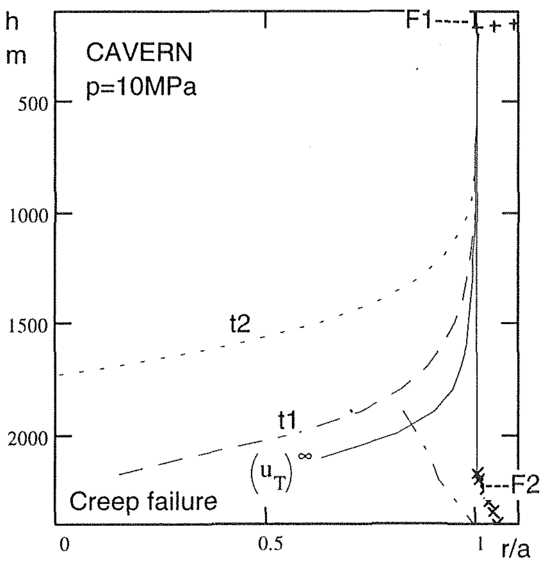

FIG.5 Creep closure of a vertical cavern at two successive times.

with a condition of the form (5). The creep closure of the cavern up to an arbitrary dimensionless time t1 is shown as interrupted line (Fig.5), while the contribution of the transient creep alone by a full line: u_T^∞ is the radial displacement due to transient creep after an infinite time. The position of the walls at a subsequent time t2>t1 is shown as a dotted line. At the moment t1 the domain where due to excessive dilatancy a creep failure has taken place is bounded by the dash-dot line (the bottom of cavern). The state of the rock in the

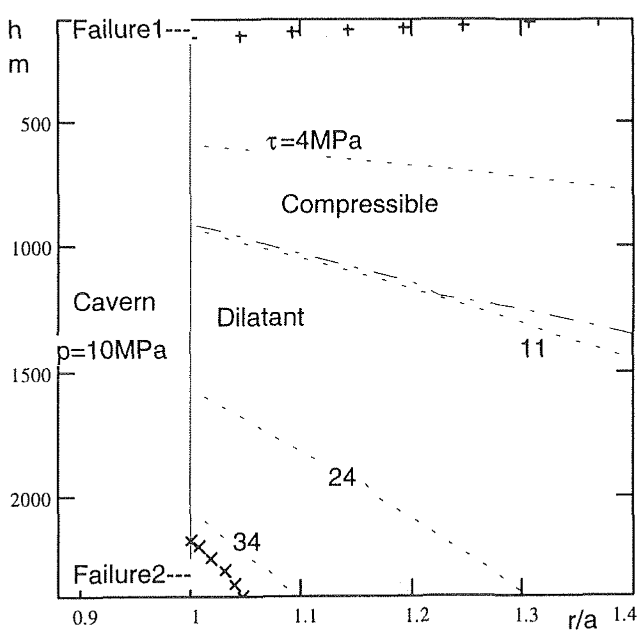

FIG.6 Failure around the cavern, and compressibility/dilatancy boundary

neighborhood of the cavern is shown in Fig.6. The dash-dot line shows the boundary between the domain where due to the excavation the rock becomes dilatant and the domain where it becomes compressible. There where the rock becomes dilatant, an increase of permeability will take place, which is more pronounced there where τ has high values. Fig.6 also shows why the cavern closes much faster at great depths, but very slowly at shallow depths. This is shown by the dotted lines marking the values taken by τ just after the excavation. At shallow depths τ is much smaller than at great depths. Recalling that the stationary creep functions are of the form (4) and that the exponent q_o, say, is usually equal to 5, one can conclude that $\dot{\varepsilon}_S^I$ is of the order of magnitude 10 at shallow depths, and 10^7 at great depths. At shallow depths one can obtain a very small expansion of the cavity due to the existing internal pressure.

Another example is failure around a horizontal tunnel. This topic was already presented in previous author's papers starting with Cristescu (1985) (see also Cristescu, 1989), and observed in the laboratory and in the field by several authors (Haimson and Song, 1993, Lee and Haimson, 1993, Aoki *et al.*, 1993, Du and Kemeny, 1993, Young and Martin, 1993). We observe here an extreme case, when again two kinds of failure are possible. This depends mainly on the ratio of the

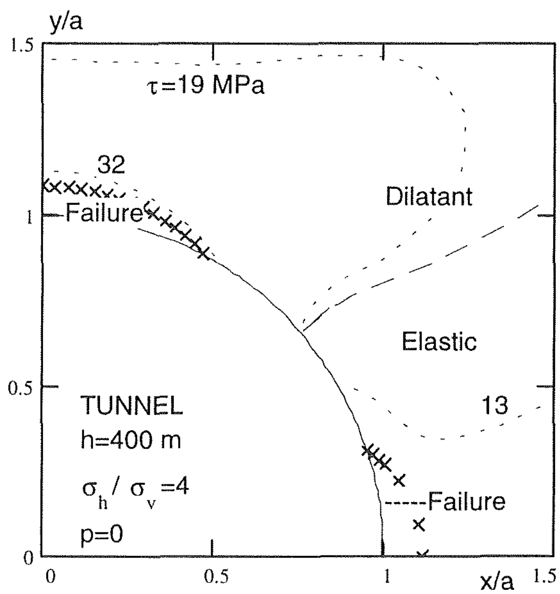

FIG.7 Failure around a horizontal tunnel.

far field stresses σ_h/σ_v and on depth. The example shown in Fig.7 corresponds to an empty tunnel excavated in rock salt at depth h=400m when σ_h/σ_v=4 (Cristescu, 1994c). Since this ratio departs very much from unity, just after excavation failure will take place in the vertical and horizontal directions, i.e. in the direction of maximum $\sigma^P_{max}=\sigma_h$, and minimum $\sigma^P_{min}=\sigma_v$, far field stresses. However, the two cases are quite distinct. While failure in the direction σ^P_{max} takes place just after excavation and involves the amount of rock shown, the failure in the direction σ^P_{min} is also instantaneous, but followed by a progressive creep failure in the same direction which steadily increases the failure region.

Fig.7 also shows that in the vertical directions the values of τ are quite high, which means that in this direction a significant creep closure will take place. Thus a faster closure is in the σ^P_{min} direction. It is in the same direction that a quite wide dilatancy domain spreads quite far from the excavation. It is very important to determine accurately the amount of rock damaged by dilatancy since there, besides the strength problem, a significant increase of permeability will take place.

A third example is failure and damage around a rectangular-like shaped cavern as shown in Fig.8. As for the other examples given above, the stress distribution around such a cavern is obtained by exact formulae (see Massier, 1994, Paraschiv and Cristescu, 1994). The cavern shape

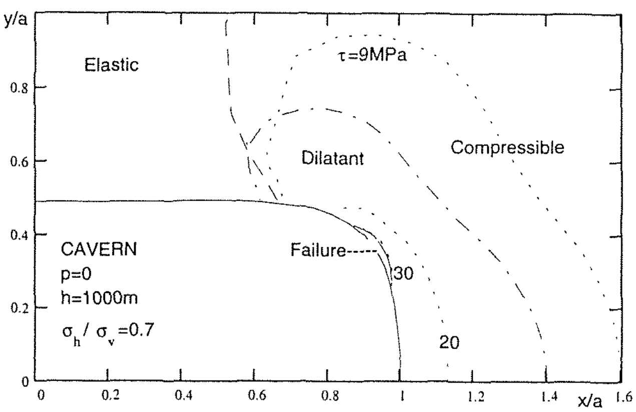

FIG.8 Failure and dilatancy around a rectangular-like shaped cavern

shown is very favorable, in the sense that for such shape no instantaneous failure is obtained at smaller depth (see Paraschiv and Cristescu, 1994). However, at the depth shown a small failure region is appeared at the corner. The dash-dot line shows up to where the rock becomes dilatant. For some other ratios σ_h/σ_v the failure, if present, is always at the corner, but the shape of the dilatant domain changes significantly with this ratio. However, as shown in the Fig.8, the greatest damage (dilatancy) is at the corner, where creep failure is following just after the instantaneous failure, where an important increase of permeability takes place, and where the fastest closure of the walls will take place.

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