Proposal for Calculations of Flat Oval Pipes Under Internal Pressure Under Consideration of Material Plastification

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In the wall of an oval pipe, additional to the circumferential forces, shear forces and bending moments occur under internal pressure load. Under this condition, the bending stresses in certain cross sections reach a figure many times that of normal stresses so that yield strength of the material can be exceeded. The usual stress calculation method is based on the calculation of the bending moments with the use of the elastic bending equation. The use of the part-plastic equation presented in the paper gives more accurate values for the bending moments sought in the cross sections being checked. This paper shows that even though the new calculation method leads to a smaller wall thickness of the flat oval pipe, the design of the flat oval pipe is nevertheless safe.

Introduction

In the technology of heat exchanger construction and in boilermaking, pressure components such as collectors, chambers, etc., pipes with oval, square, or other noncircular sections are often taken. A simple and often used form is the flat oval pipe (Fig. 1). It consists of two flat wall sections and two bent into round shape. The geometry of the pipe is exactly defined by the specification of the wall thickness s and two ratios r/a and s/a. It is assumed that the wall thickness s is constant about the complete circumference, the pipe being open at its ends.

The advantages of the flat oval pipe lies in the easy of manufacturing from round pipe and also in the high load capacity. Internal pressure load is resulting in transverse forces and bending moments in addition to the hook forces. Stresses which are caused by the bending moments are usually several times higher than normal and shear stresses, and thus are the deciding factor for the wall thickness. In a flat oval pipe, the maximum bending moment occurs either at the vertex A or in the middle of the flat plate.

In ductile material, the bending moment stress of the wall can be increased to the fully plastic state across the whole wall section. The part-plastic bending occuring due to inner load, leads to greater deformations of the pipe as elastic bending. The bending moment at the highest stressed position will be reduced and the course of the bending moment being altered.

The part-plastic bending of a wall section differs from elastic bending due to the nonlinear behavior of the material. It can be described by an equation based on an ideal plastic behavior of the material.

In the present paper, both elastic and part-plastic bending equations for the determination of the bending moment in the wall is described. The reason for the explanation of both methods is the fact that in the current technical rules the elastic method has been used exclusively. The advantage of the new method is evident only in comparison of both methods.

Part-Plastic Bending of a Pipe Wall

The material used in boilermaking and heat exchanger construction has, as a rule, a high notch impact strength and a good ductility. According to the load beam theory, a wall made of ductile material can be loaded beyond the yield point of the material on the wall surface until a plastic tension state occurs throughout the cross section of the wall.

The decisive factor for part-plastic bending is the behavior of the material after reaching the yield point. The real stress/elongation characteristic of the material points to the initial proportionality of a more or less marked yield point, which through constant deformation and stabilization of the material is characterized by a gradual rise in stress (Kussmaul, 1985).

For practical calculations, it is convenient to assume the stress/elongation characteristic as an ideal plastic approximation (Fig. 2). This model makes it possible to derive an integrable bending equation.

With the bending load of a plate beyond the yield point, the wall cross section is divided into two plastic zones and an elastic one (Fig. 3). The result is a nonlinear curve of stress, in which the plate is subjected to a greater bending than could be ascertained according to the equation for elastic bending.

$$\frac{\partial \varphi}{\partial x} = \frac{(1-\mu^2)\cdot M}{EJ} \quad (1)$$

In part-plastic bending, the bending moment is dependent on the ratio of the elastic zone to the wall thickness $K = l/s$

$$M = 1.5 \cdot M_{\text{EL}}(1 - \tfrac{1}{3}K^2) \quad (2)$$

The parameter M_{El} designates the bending moment on reaching the yield point on the surface of the plate.

An equation for part-plastic bending is already documented in technical literature (Reckling, 1967). For application to the strength calculation of flat oval pipes, it is useful to describe the bending moment occuring in the equation as a function of the bending moment aimed at M_B.

In the derivation of the part-plastic bending equation, it is assumed, as indeed it is also the case with elastic bending equations, that the Bernoulli-Navier hypothesis about the cross sections remaining level is still valid. The hypothesis requires a further assumption; namely, that transverse strain is the same in the elastic zone as in the plastic one and no additional transverse tensions occur on the boundary between the two zones with bending stress. Physically, this assumption is only tenable when Poisson's ratio is the same for the plastic zone as for the elastic

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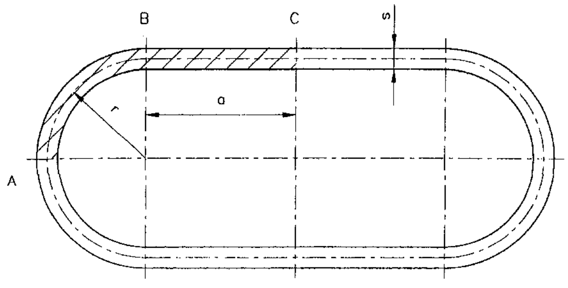

Fig. 1 Geometry of the flat oval pipe

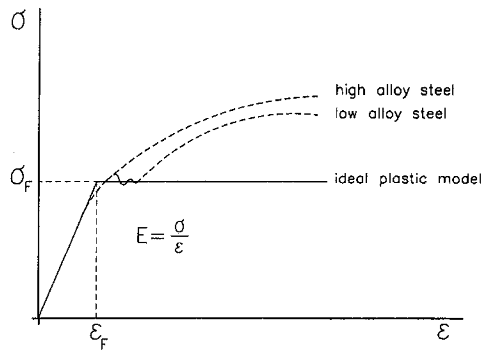

Fig. 2 Stress/strain characteristic

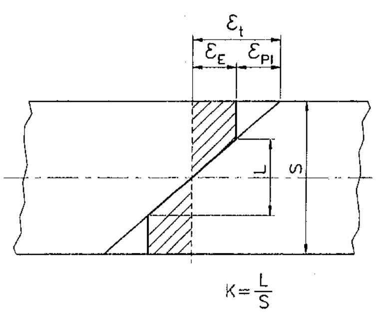

Fig. 3 Part-plastic tension state in a bending beam

one, which in reality is not the case. In spite of the fact that part-plastic bending, owing to the foregoing contradiction, cannot be mathematically described exactly, the part-plastic bending equation is quite usable in practice, particularly for thin-walled plates.

From Fig. 3, it can be seen that the parameter K also shows the relationship of the strain on the surface to the elastic expansion of the material

$$K = \frac{(1 - \mu^2) \cdot \delta_F}{E \cdot \epsilon_t} \quad (3)$$

With small deformations, we can, as is known, write for the curvature of the bending line

$$\frac{1}{\rho} = \frac{2(1 - \mu^2) \cdot \delta_F}{E \cdot K \cdot s} \quad (4)$$

Inserted from the Eq. (2) and through elimination of the tension yield limit, we obtain the part-plastic bending equation in the form

$$\frac{\partial \varphi}{\partial x} = \text{sign}\,(M) \frac{(1 - \mu^2) \cdot M_{\text{EL}}}{EJ} \frac{1}{\sqrt{3 - 2\left(\frac{M}{M_{\text{EL}}}\right)}} \quad (5)$$

The validity of this equation is limited to a bending moment whose absolute value is found between the limit values M_{EL} and 1.5 M_{EL}, therefore, between the maximum elastic and the fully plastic bending moment. As the maximum elastic bending moment is calculated from the moment of inertia and the wall thickness is reached with

$$M_{\text{EL}} = \frac{2J\delta_F}{s}$$

and thus is always positive, the direction of bending depends on the prefix of the bending moment.

From Eq. (5), it can be seen that in the vicinity of the fully plastic bending moment, the angle of bending increases to infinity. If the plastic zone covers the whole cross section, a plastic hinge occurs at which the curvature of the plate can theoretically reach any desired value.

Equation (5) is only valid for a pure bending moment stress without an overlay with normal or transverse forces. An additional normal force would cause greater twisting of the cross section and bending of the plate; but as the normal force acting in the pipe wall is small in comparison with the bending moment, the influence of the normal force on the deformation of the flat oval pipe can be, in principle, ignored.

Bending Moment in the Wall of a Flat Oval Pipe

The symmetry of the flat oval pipe around the x and y-axes makes it possible to set the limiting conditions for forces and deformations (Schwaigerer, 1983). Thus, the normal and transverse forces in the intersections A and C can be determined from the equilibrium conditions of a quarter-section (Fig. 4).

The bending moment is calculated from the deformation equation, while the angle of bending in the cross sections A and C has to be zero.

In the elastic zone, the following relationships result for the moments in the intersections:

Nomenclature

a = half-width of flat wall
E = modulus of elasticity
F = coefficient of bending equation
J = 2nd moment of area
K = relationship of strain to elastic expansion of material
K_B, K_{EL} = dimensionless parameters of deformation equation
I = height of elastic zone
M = hoop moment
M_{EL} = max. elastic bending moment
M_{VP} = fully plastic bending moment
N = normal force
N_{VP} = normal force on reaching yield point
P = pressure
r = radius of curved round wall
s = wall thickness
x = longitudinal coordinates
x_0 = start of material plastifying in flat wall
α_0 = start of material plastifying in curved round wall
β = bending angle
ϵ = strain
φ = angle coordinates
μ = Poisson's ratio
ρ = radius of curvature
δ_F = tension on reaching yield point

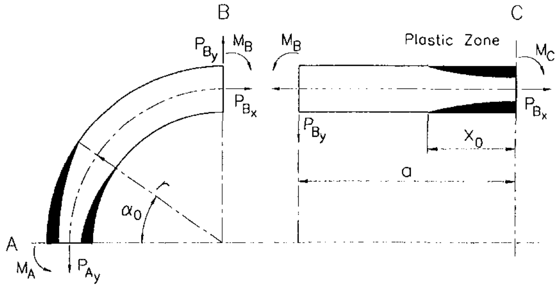

Fig. 4 Quarter-section of a flat oval pipe

$$M_A = p \cdot a^2 \frac{\left(\frac{r}{a}\right)^2 - \frac{\pi}{2}\left(\frac{r}{a}\right)^2 - \left(\frac{r}{a}\right) - \frac{1}{3}}{\frac{\pi}{2}\left(\frac{r}{a}\right) + 1} \qquad (6)$$

$$M_C = p \cdot a^2 \frac{\left(\frac{r}{a}\right)^2 + \frac{\pi}{4}\left(\frac{r}{a}\right) + \frac{1}{6}}{\frac{\pi}{2}\left(\frac{r}{a}\right) + 1} \qquad (7)$$

The bending moment between points A and C is a monotone function with two extreme values in intersections A and C. Which of the two bending moments reaches the higher absolute value depends, as can be seen from Eqs. (6) and (7), solely on the relationship (r/a). The limit value, at which the bending moment in both sections is the same, can be achieved from direct comparison of 6 and 7. For (r/a), the quadratic equation is valid

$$\left(\frac{r}{a}\right)^2\left(\frac{\pi}{2} - 2\right) + \left(\frac{r}{a}\right)\left(1 - \frac{\pi}{4}\right) + \frac{1}{6} = 0 \qquad (8)$$

The positive root of this equation is the sought after limit value $(r/a)_G = 0.921435$.

If the geometrical ratio (r/a) is larger than the limit value $(r/a)_G$, the bending moment with the larger absolute value is situated in cross section C, with the smaller in cross section A. With a rise in pressure, the yield point on the wall surface is similarly first reached in the cross section with the greater bending moment. The part-plastic bending, which sets in after reaching the yield point, can first only occur in one cross section, but with a further increase in pressure, in both sections. Depending on the geometrical ratio (r/a) and the pressure p, besides the elastic state, three cases with part-plastic bending are possible, which can occur either in cross section A or C or both simultaneously.

For the setting up of the deformation equation, it is necessary to integrate the part-plastic bending equation for the flat plate and for the curved round one over the part-plastic zones x_0 and a_0 (Fig. 4), respectively, and to ascertain the bending angle of the corresponding wall section. For this purpose, the course of the bending moment is described in the flat wall with the aid of the bending moment in intersection B, which is at first unknown and of the coordinate x with the equation

$$M = M_B + 0,5p(a^2 - x^2) \qquad (9)$$

The integration of the Eq. (5) in the limits between $x = 0$ and $x = x_0$ gives for the bending angle

$$\beta = \frac{(1-\mu^2)\cdot M_{EL}}{EJ}\sqrt{\frac{M_{EL}}{p}}\ln\left|\frac{x_0}{F_1} + \sqrt{1 + \left(\frac{x_0}{F_1}\right)^2}\right| \qquad (10)$$

where

$$F_1 = \sqrt{\frac{3M_{EL} - 2M_B}{p} - a^2} \quad x_0 = \sqrt{\frac{2(M_B - M_{EL})}{p} + a^2}$$

The deformation equation can now be set up from the bending equation for the curved round wall and for the elastic and the part-plastic section of the flat wall

$$K_B\left(\frac{r}{a}\right)\frac{\pi}{2} - \left(\frac{r}{a}\right)^2 = \frac{1}{6}X^3 - \frac{1}{2}X^2 - K_B X$$

$$- K_{EL}^{3/2}\ln\cdot\left|\frac{x_0}{F_1} + \sqrt{1 + \left(\frac{x_0}{F_1}\right)^2}\right| \qquad (11)$$

The variables sought, x_0 and M_B, can conveniently be shown with the aid of the dimensionless parameters

$$K_B = \frac{M_B}{pa^2}, \quad K_{EL} = \frac{M_{EL}}{pa^2} \quad X = 1 - \frac{x_0}{a}$$

From the equilibrium of the bending moments in sections B and C, we get a second equation

$$K_B = 0.5\left(\left(\frac{x_0}{a}\right)^2 - 1\right) + K_{EL} \qquad (12)$$

which makes it possible to determine the bending moment M_B and the start of the plastic deformation at x_0.

The solving of the equation system can either be done through a substitution of the logarithmic function (for example, a series development) and a subsequent solving of the polynomes, or else iteratively.

One proceeds similarly with the part-plastic bending of the curved round wall. The course of the bending moment here has the form

$$M = M_B - p\cdot r\cdot a\cdot\cos\varphi \qquad (13)$$

After the substitution in the part-plastic equation, we get

$$\beta = \int_0^{\varphi_0}\frac{r(1-\mu^2)M_{EL}}{EJ}\cdot\frac{1}{\sqrt{3 + 2\frac{M_B - pra\cos\varphi}{M_{EL}}}}d\varphi \qquad (14)$$

For small angles, the function $\cos\varphi$ can be replaced by the first two members of a Taylor series. After an integration between the limits $\varphi = 0$ and $\varphi = \varphi_0$, we get for the bending angle of the part-plastic section of the curved round wall

$$\beta = \frac{r(1-\mu^2)M_{EL}}{EJ}\sqrt{\frac{M_{EL}}{pra}}\ln\left|\frac{\alpha_0}{F_2} + \sqrt{1 + \left(\frac{\alpha_0}{F_2}\right)^2}\right| \qquad (15)$$

where

$$F_2 = \sqrt{\frac{M_{EL}}{pra} + 2(\cos\alpha_0 - 1)}$$

If the part-plastic bending occurs first in section A, the deformation equation has the form

$$-K_{EL}\sqrt{\left(\frac{r}{a}\right)K_{EL}}\ln\left|\frac{\alpha_0}{F_2} + \sqrt{1 + \frac{\alpha_0}{F_2}}\right| + \left(\frac{r}{a}\right)^2$$

$$\times\left(\left(\frac{\pi}{2} - 1\right)\cos\alpha_0 + \sin\alpha_0 - 1\right)$$

$$+ K_{EL}\left(\frac{r}{a}\right)\left(\alpha_0 - \frac{\pi}{2}\right) = K_{EL} - \left(\frac{r}{a}\right)\cos\alpha_0 - \frac{1}{3} \qquad (16)$$

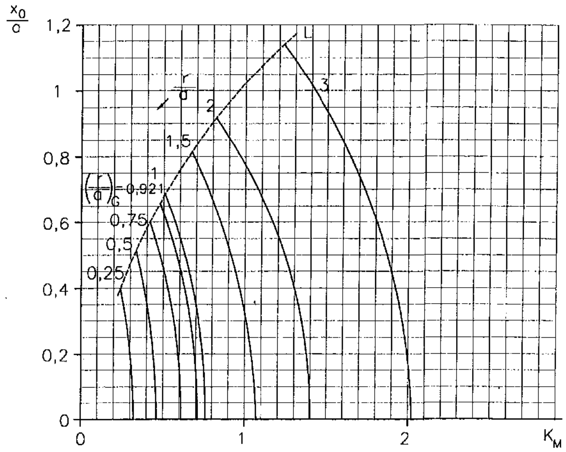

Fig. 5 Characteristic factor x_0/a as function of the parameters r/a and $K_M = M_{EL}/pa^2$

The attendant equilibrium condition for the bending moment is

$$K_B = -K_{EL} + \left(\frac{r}{a}\right)\cos\alpha_0 \qquad (17)$$

Finally, if both cross sections A and C are in the part-plastic section, the deformation equation assumes the general form

$$-K_{EL}\sqrt{\frac{r}{a}K_{EL}}\ \ln\left|\frac{\alpha_0}{F_2} + \sqrt{1 + \frac{\alpha_0}{F_2}}\right| + \left(\frac{r}{a}\right)^2$$

$$\times\left(\left(\frac{\pi}{2} - 1\right)\cos\alpha_0 + \sin\alpha_0 - 1\right)$$

$$+ K_{EL}\left(\frac{r}{a}\right)\left(\alpha_0 - \frac{\pi}{2}\right) = \frac{1}{6}X^3 - \frac{1}{2}X^2K_BX$$

$$- K_{EL}\sqrt{K_{EL}}\ \ln\left|\frac{x_0}{F_1} + \sqrt{1 + \left(\frac{x_0}{F_1}\right)^2}\right| \qquad (18)$$

The solving of Eqs. (11), (12), and (16)–(18) analytically is really difficult due to the logarithmic functions. On the other hand, an iterative solution can only usefully be carried out with the aid of a computer.

For a practical application, it is therefore useful to make out diagrams from which the sought variables α_0 and x_0 can be ascertained as functions of the dimensionless parameters (r/a) and $K_{EL} = M_{EL}/pa^2$ (Figs. 5 and 6). The upper lines L show

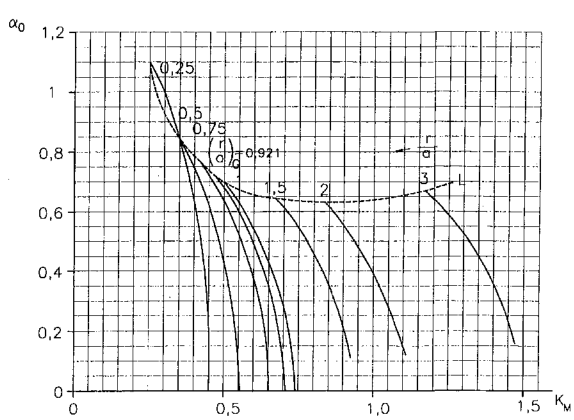

Fig. 6 Characteristic factor α_0 as function of the parameters r/a and $K_M = M_{EL}/pa^2$

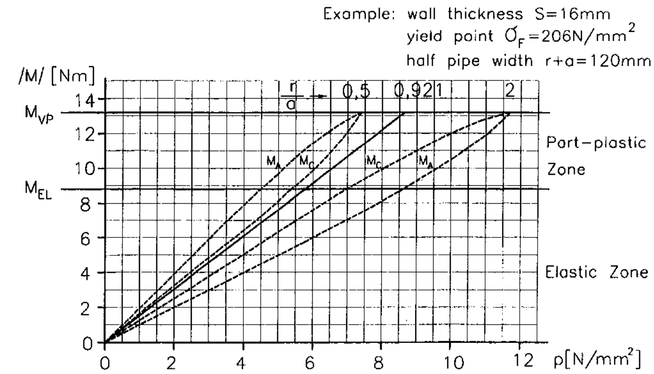

Fig. 7 Bending moment in cross sections A and C, depending on internal pressure

the maximum values of the variables α_0 and X_0/a in the fully plastic bending state in cross sections A and C.

Criteria for the Dimensioning of Wall Thickness

If we calculate the bending moments in cross sections A and C with deformation Eqs. (11), (16), and (18), we find that with increasing pressure, the initially different-sized bending moments equalize as a result of the part-plastic bending until the fully plastic bending moment is reached (Fig. 7). The fact that the fully plastic bending moment occurs in both cross sections at the same pressure and regardless of the geometric ratio can be explained by the fact that, in the vicinity of the fully plastic bending state, the bending in the wall section concerned increases sharply. The result is an unloading of the heavily loaded and a loading of the little loaded cross section of the pipe wall until an equalization is reached.

Setting out from the assumption that the plastic hinge in cross sections A and C represents the limit of load capacity, the criterion for the dimensioning of the wall thickness can be described with two equations applying to the two cross sections

$$M_{VP} = M_B + 0,5pa^2 \qquad (19)$$

$$-M_{VP} = M_B - pra \qquad (20)$$

After the resolution of both equations, the maximum pressure for which the plastic hinge will be reached can be calculated as follows:

$$p = \frac{2M_{VP}}{a^2\left(0.5 + \left(\frac{r}{a}\right)\right)} \qquad (21)$$

From the fully plastic bending moment, we get for the wall thickness

$$\frac{s}{a} = \sqrt{\frac{p}{\delta_F}\left(1 + 2\left(\frac{r}{a}\right)\right)} \qquad (22)$$

Equation (22) applies with greater accuracy only for flat oval pipes with a (r/a) ratio smaller than 3. With a greater geometric ratio (r/a), the normal force is not negligibly small in comparison with the bending moment and must thus be taken into account in the ascertainment of the plastic hinge.

According to the theory of beams, the combined bending moment-normal force stress can be increased until the plastic hinge criterion

$$\left(\frac{M}{M_{VP}}\right) + \left(\frac{N}{N_{VP}}\right)^2 = 1 \qquad (23)$$

is reached.

Strictly taken, Eq. (23) is valid only for one dimension strain state (Tresca yielding criterion). As in the controlled cross sections, hoop strain is much higher than the strain in the radial and axial direction; Eq. (23) can be accepted for the real case.

Considering the equilibrium conditions for forces and moments from Eq. (23) for cross sections A and C, two equations are evaluated

$$\delta_F(4M_B + 2pa^2) + r^2p^2 = s^2\delta_F^2 \quad (24)$$

$$\delta_F(-4M_B + 4pra) + (r + a)^2p^2 = s^2\delta_F^2 \quad (25)$$

From both equations, finally, the formula

$$\frac{s}{a} = \sqrt{\frac{p}{\delta_F}}\sqrt{\frac{p}{\delta_F}\left(\left(\frac{r}{a}\right)^2 + \left(\frac{r}{a}\right) + \frac{1}{2}\right) + \left(1 + 2\left(\frac{r}{a}\right)\right)} \quad (26)$$

can be derived for wall thickness.

Equation (26) is valid for any geometric ratio (r/a). For smaller values of the parameters (p/δ_F) and (r/a), the results of Eq. (26) are very similar to those of Eq. (22) (Fig. 8). On the other hand, for a small dimension a, Eq. (26) goes over into the strength equation of a thin-walled pipe. Setting out from the geometry of a flat oval pipe, Eq. (26) thus represents a general formula for the strength calculation of cylindrical shells under internal pressure load.

The internal pressure for which the plastic hinge in the tube wall will be achieved can be calculated according to the following formula:

$$p = \delta_F \frac{\sqrt{\left(1 + \left(\frac{s}{a}\right)^2\right)\left(\left(1 + 2\left(\frac{r}{a}\right)\right)^2 + \left(\frac{s}{a}\right)^2\right)} - \left(1 + 2\left(\frac{r}{a}\right)\right)}{2\left(\frac{r}{a}\right)^2 + 2\left(\frac{r}{a}\right) + 1} \quad (27)$$

In the practical application of Eqs. (22) and (26), we must take into account that both equations do not contain any safety supplement. It is a question of safety philosophy to consider corresponding safety requirements, for example, such as are prescribed in the individual sets of rules.

The use of the plastic hinge criterion for the design calculation of the oval flat pipe instead of the elastic calculation method leads to the smaller wall thickness.

For the determination of the wall thickness according to the elastic calculation method—limit of the internal pressure load

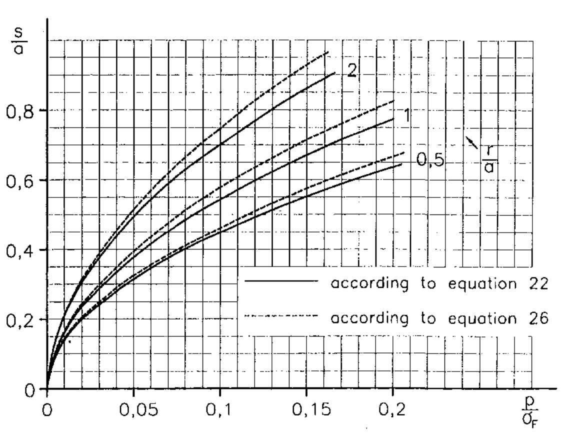

Fig. 8 Dimension of the wall thickness according to the plastic hinge criterion

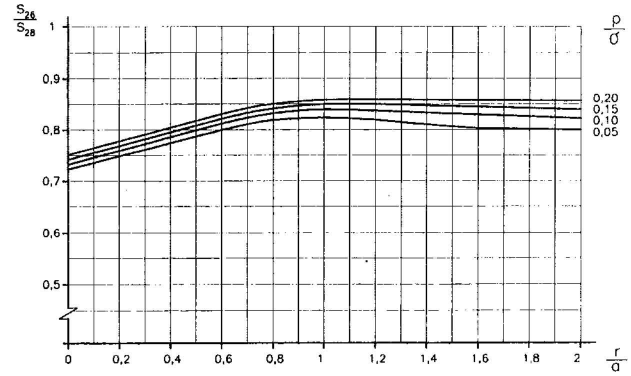

Fig. 9 Relation of the wall thickness calculated according to the plastic hinge criterion s_{26} and elastic calculation method s_{28}

is the achievement of the yield point on the surface of the checked cross section—two formulas from Eqs. (6) and (7) can be achieved

$$s = a\sqrt{6\frac{p}{\sigma_F}\frac{\left(\frac{r}{a}\right)^2\left(1 + \frac{\pi}{2}\right) + \left(\frac{r}{a}\right) + \frac{1}{3}}{\frac{\pi}{2}\left(\frac{r}{a}\right) + 1}} \quad \text{for} \quad \left(\frac{r}{a}\right) < \left(\frac{r}{a}\right)_G$$

$$s = a\sqrt{6\frac{p}{\sigma_F}\frac{\left(\frac{r}{a}\right)^2 + \frac{\pi}{4}\left(\frac{r}{a}\right) + \frac{1}{6}}{\frac{\pi}{2}\left(\frac{r}{a}\right) + 1}} \quad \text{for} \quad \left(\frac{r}{a}\right) > \left(\frac{r}{a}\right)_G \quad (28)$$

The comparison of formulas (26) and (28), (19) shows (Fig. 9) that with the loading of the tube wall until full plastic hinge, the wall thickness can be reduced up to 72–85 percent.

Summary

Flat oval pipes count as special parts which are not covered by most sets of rules. The few sets of rules that deal with oval pipes under internal pressure, such as the ASME Code or the Swedish norm, are restricted to certain ideal forms and the elastic behavior of the material. As the numerical procedures, such as the FE method, are costly and are not suitable as a means for dimensioning, a search was made for a more suitable analytical procedure.

In the present paper, the calculation of the bending moments in a flat oval pipe was presented taking into account a plastifying of the material. From the fully plastic tension state in two cross sections of the pipe, a formula was derived for the determination of the wall thickness. An application of this calculation procedure to other noncircular pipes is possible, but needs for each pipe form an adequate consideration of validity.

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