Reduced Complexity MIMO Concatenated Code in Fading Channels

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***Abstract*—In conventional concatenations of Convolutional Codes (CC) and Space-Time Block Codes (STBC), the CC and STBC are utilized to provide coding gain and spatial diversity, respectively. We propose a concatenated code that achieves the full system diversity by appropriately selecting the outer CC with an inner *reduced-rank* STBC. The advantage of the lower rank STBC is that the number of RF chains can be reduced. For any number of RF chains, R, we show that a desirable diversity order Y can be easily achieved i.e., $1 \leq R \leq Y$. Using trellis diagram, we formalize the method to determine the maximum diversity.**

***Index Terms*—Convolutional codes, diversity, multiple-input multiple-output.**

I. INTRODUCTION

IN Multiple-Input Multiple-Output (MIMO) systems, Space-Time Block Codes (STBCs) [1] are known to achieve full diversity with low complexity. In order to obtain coding gain, some concatenated coding schemes and relevant analyses have been presented in the literature ([2],[3] and references therein). In [2], the authors have provided a thorough analysis under temporally and spatially correlated block fading channels using the uniform interleaver concept. A concatenated STBC with Trellis Coded Modulation (TCM) has been proposed in [3] where it has been shown that the achievable temporal diversity in the absence of Channel State Information (CSI) is half of that for CSI aware system. Serially concatenated space time Convolutional Codes (CCs) and continuous phase modulation have been analyzed in [4], where design criteria are derived for an arbitrary number of transmit antennas. In all works [2]-[4] mentioned above, the spatial freedom is exploited using a full-rank STBC (i.e., the rank of the STBC equals the degree of spatial freedom), and the channel code is simply utilized to obtain coding gain.

Here we propose a reduced-complexity concatenated CC and STBC where antenna selection is utilized to reduce the number of RF chains while guaranteeing full system diversity. Our contribution is in the framework presented by the coding technique where it utilizes the inherent memory of CC to reduce the complexity of the system while maintaining the full diversity without affecting the rate. That is by simplifying the inner STBC (i.e., lowering the rank of the code) through the *memory* of the outer CC we can achieve the full diversity. In that sense, we utilize the CC to provide both diversity and coding gains. The diversity gain is achieved through the spatial freedom transferred by the spatial interleaver (i.e., switch) to the time domain, and the coding gain is inherently achieved. We regard one extreme case where no STBC is used as the concatenation of CCs with a trivial rank-one STBC. The other extreme case corresponds to the conventional concatenation of CC and full-rank STBC where the number of RF chains is equal to the number of available antennas. Using trellis diagram, we formalize the method to determine the maximum achievable diversity for any concatenation of CCs and reduced-rank STBCs.

Manuscript received November 11, 2011. The associate editor coordinating the review of this letter and approving it for publication was K. K. Wong.

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Digital Object Identifier 10.1109/LCOMM.2011.060111.102186

II. CONCATENATION SCHEME

The underlying MIMO system consists of N-transmit and M-receive antennas connected, through *switch* enabling antenna selection, to R transmit and F receive RF chains, where $R \leq N$ and $F \leq M$. For the case where no STBC is employed, the STBC encoder/decoder are ignored. While for the other extreme case where a full-rank STBC is utilized (i.e., conventional concatenation of CCs and STBCs), the *switch* is discarded.

The rate of the concatenated scheme is equal $R_{CC}R_{STBC}$, where R_{CC} and R_{STBC} are the rates of the CC and STBC, respectively. Let us consider R selected antennas for transmission and F receive RF chains. Unless otherwise specified, we consider the size of the ST codeword as $R \times R$. Also for ease of description, we assume that $M = 1$ and $N = Y$, where Y is the desired diversity order.

At the transmitter side, the incoming data stream is first encoded by a rate R_{CC} CC and then the coded stream (e.g., s_1, s_2, s_3, s_4, s_5, s_6, ...etc.) is fed into a ST encoder. The output codewords of the ST encoder are labeled as z_1, z_2, z_3, z_4, ...etc. When $R = 1$, no STBC is employed and the convolutionally coded symbols are transmitted on a symbol basis using different antenna at each time slot. Note that the switch between the single RF chain and N transmit antennas operates in a cyclic manner.

When $R = N$, the rank N orthogonal STBC is utilized, and no switching is needed between RF chains and transmit antennas. The convolutionally coded symbols are transmitted N by N through all antennas in every N time slots. This case corresponds to the conventional concatenated CC and STBC where the rank of the STBC is equal to the degree of spatial freedom. On the other hand when $1 < R < N$, a rank R orthogonal STBC is employed, and each ST codeword is transmitted by R antennas in R time slots. In the first R time slots, the R transmit RF chains are connected to transmit antennas A_1-A_R via switch, and z_1 is transmitted. In the second R time slots (e.g., $R+1$ to $2R$), the transmit RF chains are switched to transmit antennas A_{R+1}-A_{2R} (here we assume $N \geq 2R$), and z_2 is transmitted. The following codewords are transmitted in the same manner. In general, in the g_{th} R time slots (i.e., $(g-1)R+1$ to gR) in which

z_g is transmitted, the transmit RF chains are connected to transmit antennas A_U-A_V, where $U = mod((g-1)R+1, N)$, $V = mod(gR, N)$, and mod represents modulo operation. The ST decoder is then used to decouple the corrupted R symbols in each ST codeword (i.e., decouple $\hat{s}_1$, $\hat{s}_2$, $\hat{s}_3$, ... $\hat{s}_R$ from $\hat{z}_1$). These decoupled symbols are then fed into the Viterbi decoder for soft-decision decoding. Note that, the same switching technique can be applied to receive RF chains and antennas. When both the transmitter and receiver simultaneously employ spatial interleaving, their interleaving (spatial switching) should be coordinated in such a way that all independent channel realizations are contained in all error events of the outer CC.

A. *Criterion on Selecting Convolutional Codes*

For the concatenated code with $R = N$, since each CC symbol is transmitted over all available channels, it is guaranteed that all channel realizations are contained in all error events regardless of the CC used. In this case, there is no restriction on choosing CCs; and the CC with the maximum free distance and highest rate results in the maximum coding gain and rate. For ease of comparison, we consider single-input binary CCs. Therefore, rate 1/2 CCs with maximum free distance are the best choice for conventional concatenation.

It should be noted that CC by itself is known to provide diversity gain when employed in time-selective or fast-fading channels and only coding gain in block-fading channels. Here, using spatial interleaving (switch) and by appropriately choosing the CC, the desirable diversity can also be achieved with any number of transmit RF chains when $1 \leq R < N$. *The general criterion on choosing the CC is that the number of independent channel realizations included in all error events should be equal to the desired diversity order*. However the number of trellis branches of the CC's shortest error event, which is equal to the constraint length, κ, imposes a limit on the maximum diversity. One should note that there is no constructive method for designing a CC of specific distance properties, but for a given CC it can be analyzed to find its distance properties. For the concatenation of rank-R STBC with CC of rate R_{CC} and constraint length κ, the maximum diversity is given by

$$Y = ceil(\kappa/(R_{CC}R))R. \tag{1}$$

Note that in (1), κ/R_{CC} represents the number of convolutionally coded symbols in the shortest error event, and $ceil(\kappa/(R_{CC}R))$ is the maximum number of ST codewords which can be included in the shortest error event. Hence, the maximum number of independent channel realizations within the shortest error event is $ceil(\kappa/(R_{CC}R))R$. Therefore, a necessary condition on the constraint length of the CC is,

$$ceil(\kappa/(R_{CC}R)) \geq N/R \tag{2}$$

for the concatenation to achieve full diversity N. Given that the necessary condition is met, there must be a rate $1/ceil(N/R)$ CC that when concatenated with rank R orthogonal STBC achieves full diversity N.

The selection of a candidate CC consists of two steps. First, the necessary condition in (2) must be satisfied. Second, all channel realizations should be included in the error events of the CC under consideration. Let

$$W = ceil(N/R), \quad B = ceil(WRR_{CC}). \tag{3}$$

Here W represents the minimum number of R-tuple allowing for N channel realizations (i.e., $WR \geq N$), and B is the minimum number of branches containing W R-tuple (i.e., $(B/R_{CC}) \geq WR$). To determine whether N channel realizations are contained in all error events of a CC, one should check for any all-zero R-tuple among the first W R-tuple in the first B branches of all possible paths on the trellis. If there is no such all-zero R-tuple, full diversity is guaranteed and the underlying CC is a candidate. If there exists a single error event that contains less than N channels, the underlying CC is not a candidate.

B. *Diversity Order Eight using R=1,2,3,4,8 RF Chains*

Here we show how a given diversity order can be achieved using different number of RF chains. We resort to the trellis diagram to determine the number of channel realizations included in error events. Without loss of generality, we consider the antenna configuration $N = 8$ and $M = 1$. In this case, the degree of spatial freedom is eight. To achieve full diversity of eight, one has to ensure that all eight channel realizations are contained in the error events of the selected CC.

When $R = 1$, we choose rate $1/8$ CC with generator polynomials $(7, 7, 5, 5, 5, 7, 7, 7)$ to achieve the full diversity order of eight. The necessary condition is obviously met since $ceil(3/(0.125 \times 1) > 8$. Also since the first not-all-zero branch of all error events of the CC is an all-one branch, it is guaranteed that all eight channel realizations are included in all error events.

For $R = 2$, we use rate $1/4$ CC $(7, 5, 7, 5)$ with Alamouti scheme to exploit the spatial freedom. Let the output symbols of the convolutional encoder be denoted by $s_1, s_2, s_3, s_4, .., s_j$. For the 2x2 STC, every two consecutive convolutional codewords (e.g., s_1, s_2) are transmitted according to Alamouti scheme and represented by codeword z_i. That is at the transmitter side, the coded stream is coded pair by pair using Alamouti scheme. The first (2x2) codeword z_1 out of the ST encoder is transmitted using transmit antennas A_1 and A_2 in the first two time slots, the second codeword z_2 using antenna A_3 and A_4 in the second two time slots, z_3 using A_5 and A_6 in the third two time slots, and so forth. We first check the necessary condition, where $ceil(3/(0.25 \times 2) > 4$. Then we determine if all eight channels are included in all error events. As clear from Fig. 1 that in the first two branches (i.e., $B = ceil(ceil(N/R)RR_{CC}) = 2$) there is no all-zero pairs among the first four pairs of coded bits ($W = ceil(N/R) = 4$), marked on the trellis as "11111010" and "11110101". Hence the full diversity order of eight is guaranteed.

With three RF chains, we employ rate $1/3$ CC $(4, 7, 5)$ with the STBC in [eq. (27), [1]] to achieve full diversity. Next, we show that the rank four STBC [eq. (4), [1]] with rate $1/2$ CC $(7, 5)$ achieves diversity order of eight using four RF chains. From the corresponding trellis, there are three branches in the shortest error event. Hence, $ceil(3/(0.5 \times 4)) = 2$ and

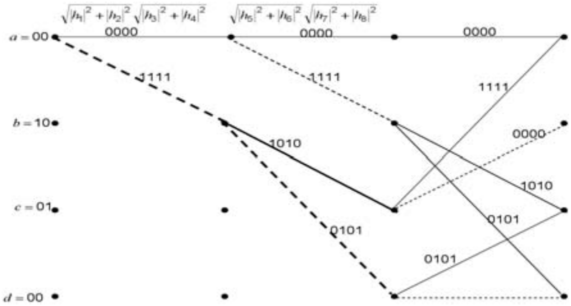

Fig. 1. Trellis for rate 1/4 CC (7,5,7,5), h_i is the channel corresponding to the channel from transmit antenna i to the single receive antenna, M=1.

the necessary condition in (2) is satisfied. In this case, $W = ceil(N/R) = 2$ and $B = ceil(ceil(N/R)RR_{CC}) = 4$. From the trellis diagram, we can note there is no all-zero 4-tuple in the first four branches of all error events. As a result, the full diversity order of eight is achieved. When $R = 8$, rank-eight STBC [eq.(5),[1]] is combined with rate $1/2$ CC $(7, 5)$ to achieve both diversity and coding gains. Note that all CCs discussed have the same constraint length, $\kappa = 3$, and hence same decoding complexity. In general, the lower the rank of the STBC, the less number of required transmit RF chains, and the lower the system rate. On the contrary, the higher the rank of the STBC, the higher the rate of the concatenation, and the lower the probability of experiencing deep fades. Therefore, one should expect some coding gain loss when employing lower rank STBC for $R < N$. Also, it is also easier to find a candidate CC with maximum free distance when a higher rank STBC is employed.

III. Simulation Results

In our simulations, we consider BPSK transmission, and a receiver with $M = 1$ where the channels are modeled as *i.i.d* quasi-static Rayleigh flat-fading channels.

In Fig. 2, we plot the BER performance for different concatenations achieving diversity order eight. Confirming our previous conjuncture, the full diversity order eight is achieved using one, two, three, four, and eight RF chains. The spectrum efficiency of these concatenated schemes is 1/8, 1/4, 1/3, 1/2, and 1/2 bit/s/Hz, respectively. The coding gain is largest when the full rank STBC is employed, while it is smallest when no STBC is involved. As seen, all concatenations achieve the same diversity order which is clear from the equal slope of their corresponding BER curves. Note that the spectrum efficiency of the last two concatenated schemes is the same, and the coding gain is larger with a higher rank STBC. There is no clear difference in coding gain for the cases where two, three, and four RF chains are utilized, which is reasonable since the comparison based on different rates is not fair. From a practical point of view, the concatenation of rank four STBC and rate 1/2 CC offers the best tradeoff since it requires only half the number of RF chains with a small coding gain loss.

We present the BER performance of four different combinations achieving the same diversity order of four in Fig. 3. In this case, the spectrum efficiency is 1/4, 1/2, 1/2, and 1/2 bit/s/Hz, respectively. It is clear that the coding gain increases as the rank of the STBC gets higher. We remark that the

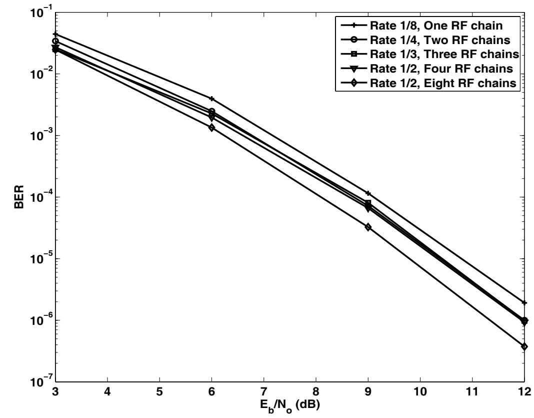

Fig. 2. BER performance of concatenations achieving diversity order eight.

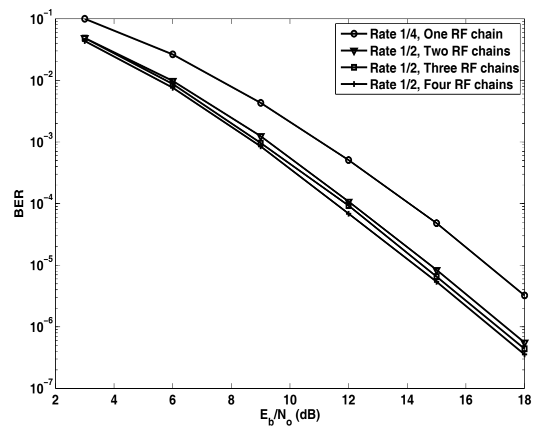

Fig. 3. BER performance of concatenations achieving diversity order four.

concatenation of rate 1/2 CC and Alamouti scheme offers the best tradeoff between coding gain and system complexity (i.e., number of RF chains). Also, such a concatenation is favorable since it is applicable to both real and complex constellations.

IV. Conclusion

A reduced complexity concatenated CC and STBC scheme was presented. Using *only* half the number of RF chains, the concatenation of half-rank STBC with the same CC used in conventional concatenated coding is shown to achieve full diversity with small coding gain loss.

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