PARAMETERS ESTIMATION OF FRACTIONAL ORDER SYSTEM WITH DOMINANT POLE USING CO-EVOLUTIONARY PARTICLE SWARM OPTIMIZZATION (CPSO) ALGORITHM

MAHMOOD GHANBARI[a1], AMIR AHMADIAN[b], ALI MOTALEBI SARAJI[c]

[ab]Department of Electrical Engineering, AliAbad Katoul Branch, Islamic Azad University, AliAbad Katoul, Iran.
[c]Young Researchers and Elite Club, AliAbad Katoul Branch, Islamic Azad University, AliAbad Katoul, Iran

ABSTRACT

This paper deals with fractional order systems parameters estimation by use of Co-evolutionary Particle Swarm Optimization (CPSO) method. in some cases such as fractional order systems identification in spite of existing different methods, it is difficult to obtain estimation of model structure parameters and generally it leads to solving the with constrained complex non-linear optimization problems and this topic is one of the identification challenges of these systems. Since some of systems are inherently fractional order and because of having special behavior in these systems which in its similar integer order systems are not found. There for necessity of fractional modeling is double for such systems. In this paper, at first, we assume that the measured out-input data exists and for approximation to reality is considered that these data has been corrupted with noise. Then considering model structure as the linear combination of fractional orthogonal basis functions by use of CPSO suitable algorithm leads to estimation of fractional order system parameters and related to the complexity level of master system, suitable or acceptable approximation is obtained. In finally, by simulating of physical-typical sample system in noisy conditions leads to system identification which gained results shows the effectiveness of presented method.

KEYWORDS: Fractional Order Systems, Parameter Estimation, System Identification, Co-Evolutionary Particle Swarm Optimization (Cpso) Algorithms

Although the mathematics of fractional calculations has a few hundred years old, but in the two decades ago, it has been attracted in research and applicable fields of various sciences. Also, it was seen that some of the real systems have inherent fractional order behaviour and for example we can refer to real systems such as: viscoelastic materials, cell diffusion processes, transmission of signals via strong magnetic fields and some systems with disturbance characteristics that they have inherent fractional order behaviour (Benchellal et al., 2006; Chen, 2006; Feliu-Batlle et al., 2007; Rossikhin and Shitikova, 1997; Tavazoei et al., 2008).

One of the features of behaviour of fractional order systems is presence of non-periodic modes that they are decay in polynomial form and also a behaviour that it is called long memory that we can't find its similarity in integer order rational systems (Aoun et al., 2004). So, if modelling, identification, controlling and other studies on these systems want to be accurate and close to reality, it should be based on fractional order model of these systems. Even in integer order systems, modelling in the form of fractional order mode or controller design with fractional model is also more effective, because of its more degrees of freedom and also the systems with integer order are special state of fractional order systems. This topic has been shown in several researches, therefore, the importance of fractional models and their synthesis is clear in practice (Chen, 2006).

Suitable estimation parameters of the processes for fractional or integer is the challenge that we are facing or confronting with in the field of systems identification (Ljung, 1999; Das, 2008). Problem of parameters estimations by getting help from time domain or frequency domain data is more difficult for a fractional order systems by comparison with the integer order systems, due to be highest degree of non-linearity

that it makes a new parameter that called fractional order (Das, 2008; Aoun et al., 2007; Malti et al., 2004; Ghanbari and Haeri, 2010; Dorcak, 2002; Hartley and Lorenzo, 2003; Poinot.and Trigeassou, 2004; Malti et al., 2009; Valério and da Costa, 2009, Ghanbari and Haeri, 2011).

In order to identify fractional order systems, an effective ideas is taking model structure into consideration as fractional order orthogonal basis functions linear combination (Aoun et al., 2007; Malti et al., 2004; Ghanbari and Haeri, 2010; Ghanbari and Haeri, 2011; Heuberger et al., 2005; Ninness et al., 2000; Akçay, 2008; Nazari et al., 2008) that use of it has been attracted much attention recently. Since in structure of each fractional orthogonal basis function. The adjustment parameters must be determined, so, some efforts have been done to estimate adjustment parameters that we can refer to some cases such as: error and trial method used in (Aoun et al., 2007; Malti et al., 2004; Ghanbari and Haeri, 2010;) for determining parameters despite of non-optimal method, a great deal of calculations are needed, Parameters estimation method from system impulse response which is needed to have systems precise impulse response after a long time that under real condition with noise and disturbance, its application is ineffective (Nazari et al., 2008), assistance method from bode diagram which is a creative and relatively graphical method (Ghanbari and Haeri, 2011) and use of intelligent algorithm method which has been considered (Deepyaman and Amit, 2008), for special systems.

Many of engineering problems such as systems identification can be formulated as a constrained optimization problem (He Q. and Wang, 2007). The penalty function method is one of the method which has many fans in solving of constrained optimization problem that the principal advantages of this method are principles ease and performance disadvantages

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[1]Corresponding author

of this method are setting and suitable preparing of penalty factors.

Many of constraints management techniques for optimization algorithms have been presented so far (He Q. and Wang, 2007)one of the suitable and comfortable method is CPSO method by use of PSO and evolutionary thinking which penalty factors in that method is adapted by use of self-adjustment method (He Q. and Wang, 2007). In this paper, considering linear combination model structure of fractional order orthogonal basis functions, we consider model structure with assumption being dominant pole of studied systems as the first terms of this linear combination and by use of CPSO suitable algorithm, we give for estimating all or part of free parameters of model structure.

The paper organized as follows: part 2, it consists of some introductions about fractional order system concept. In part 3, CPSO and optimization algorithm is described. In part 4, fractional order system parameters estimation method with dominant pole is presented. In part5, typical sample and applicable system is introduced and their simulation to show effectiveness of expressed method is done and in the end, this paper will end with part 6 which its conclusion and deduction.

FRACTIONAL ORDER SYSTEMS

Evidently, the first description of fractional order derivative is presented by Riemann and Liouville in 19th century. While a function $f(t)$ is placed at $t=0$ means that the function and all its derivatives are equal to 0 for all $t<0$, the Laplace transform of the described derivative $D^{\alpha}f(t)$, $\forall \alpha \in \mathbf{R}_{+}$ is gained as follows (Das S, 2008; Podlubny, 1999; Cafagna, 2007):

$$L(D^{\alpha}f(t)) = s^{\alpha}F(s) \qquad (1)$$

The Grunwald-Letnikov description (Das, 2008; Podlubny, 1999; Cafagna, 2007).

Resulted from Grunwald-Lentnikov description, the numerical calculation formula of fractional derivative could be obtained as follows:

$$_{t-L}D_{t}^{\alpha}f(t) \approx h^{-\alpha}\sum_{j=0}^{[L/T]}b_{j}f(t-jh) \qquad (2)$$

In which L is the length of memory. T, the sampling time usually substitutes while increase h during approximation. Weighting factor bj could be measured in return by:

$$b_{0}=1, b_{j}=\left(1-\frac{1+\alpha}{j}\right)b_{j-1}, (j \geq 1). \quad (3)$$

A linear time same fractional order system could be described by a fractional order differential equation as below:

$$y(t)+b_{1}D^{\beta_{1}}y(t)+...+b_{m_{B}}D^{\beta_{m_{B}}}y(t)=a_{0}D^{\alpha_{0}}u(t)+...+a_{m_{A}}D^{\alpha_{m_{A}}}u(t)$$

(4)

On the base of description for fractional derivative and its Laplace change in (1), transfer function of a system proposed by (4) would be:

$$H(s) = \frac{\sum_{i=0}^{m_{A}}a_{i}s^{\alpha_{i}}}{1+\sum_{j=1}^{m_{B}}b_{j}s^{\beta_{j}}} \qquad , \quad (5)$$

Where $(a_{i},b_{j}) \in \mathbb{R}^{2}, (\alpha_{i},\beta_{j}) \in \mathbb{R}^{+2}, \alpha_{0} < \cdots < \alpha_{m_{A}}$, and $\beta_{1} < \cdots < \beta_{m_{B}}$. Transfer function $H(s)$ is named a jointing transfer function at order $\alpha \in \mathbb{R}^{+}$ while all the distinct orders were precisely divisible by similar number (α) (the greatest number is usually selected).

$H(s)$ May be proposed by a model form gained by partial fraction expansion:

$$H(s) = \sum_{l=1}^{L}\frac{A_{l}}{(s^{\alpha}-\lambda_{l})^{q_{l}}}, \qquad (6)$$

Where q_{l} is the multiplicity of eigenvalues (pole locator) λ_{l} and α (a real number) is identified as common differentiation order.

Transfer functions, for example $1/s^{\alpha}$ are not easily used for calculating objective. Simulations are always conducted by software created just to work with integer powers of s.Several methods used for finding such approximations.

Permanent situation for commensurable fractional order transfer function is proposed by Matignon in (Matignon, 1998). Here, we present a reviewed version of the theorem (Aoun et al., 2007):

Stability theorem: A commensurate transfer function of order α,

$$H(s)=N(s^{\alpha})/D(s^{\alpha})=N(w)/D(w)=\mathrm{H}(w) \quad (7)$$

Where N and D are two coprime polynomials, is BIBO stable iff:

$$0 < \alpha < 2, \qquad (8)$$

And for every $w \in \mathbb{C}$ such that $D(w)=0$,

$$|arg(w)| > 0.5\alpha\pi . \quad (9)$$

For a rational system, the stability condition guarantees that the related transfer function would belong to $H_{2}(\mathbb{R}^{+})$. However, in fractional order systems it is not so. In (Malti et al., 2003), it is demonstrated that a stable fractional transfer function as defined in (5), belongs to $H_{2}(\mathbb{R}^{+})$ iff its relative degree is greater than 0.5:

$$\beta_{m_B} - \alpha_{m_A} > 0.5 \,. \tag{10}$$

CPSO ALGORITHM

Within two recent decades PSO method which is a evolutionary calculation technique with individual improvement mechanism, teamwork and competition, has attracted much attention and successfully in several ways has been used specially in unconstraint continuous optimization problems (Malti et al., 2003).

The CPSO method for with constraints optimization by making PSO and evolutionary thinking was presented in reference (He Q. and Wang, 2007) paper, which its short definition can be expressed as follows:
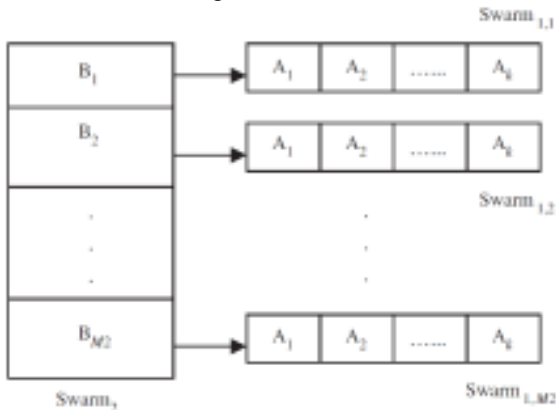

Figure1. Schematic shows for CPSO.

In CPSO two varieties of swarm is obtained with the mutual effect (according to figure 1) that an individual swarm, $swarm_2$ with size M2 for suitable compatibility of penalty factors is used and the other variety of multiple swarms, $swarm_{1,j}$ that each is used with size M1 to search of decision results parallelly.

Each particle (B_j) in $swarm_2$ is including the complex of penalty factors for particles into swarm1,j that its each particles is expressing a decision result.

In each production the co-evolutionary process of each $swarm_{1,j}$ with particle in (B_j) in $swarm_2$ as penalty factors by making use of PSO is obtained for the definite number of productions (G_1) for estimating the result, obtaining to the news $swarm_{1,j}$ .then the cost function, of each particle (B_j) in $swarm_2$ is obtained. After that all particles is $swarm_2$ is estimated, $swarm_2$ by use of PSO with a production to obtain new $swarm_2$ which is reagent of new penalty factor.

The up co-evolutionary process repeats so much until the predefined stop criterion satisfies (for example the maximum value of productions $swarm_2$.)

So in CPSO, not only the decision results is found as the evolutionary by $swarm_{1,j}$ but also the penalty factors with self-adjustment method has been adapted by $swarm_2$.

PARAMETERS ESTIMATION OF FRACTIONAL ORDER SYSTEMS WITH DOMAINANT POLE

An effective method which has a lot of advantages and usage in approximating and identifying usual systems and recently generalized in to fractional order systems, is using orthogonal basis functions (Aoun et al., 2007; Malti et al., 2004; Ghanbari and Haeri, 2010; Ghanbari and Haeri, 2011; Akçay, 2008). In this method, model structure considered as linear combination of orthogonal basis functions (11 relation). That finally to identify, by assuming existence of time domain output-input data leads to a linear regression and a convex optimization problem.

$$H(s) \approx \hat{H}(s) = \sum_{n=n_0}^{N} \hat{a}_n \mathrm{P}_n(s) = \theta^T \mathrm{P}(s) \,, \tag{11}$$

Classical Laguerre basis functions, Classical Legendre basis functions and Kautz basis functions is considered as the most applicable and popular integer orthogonal basis functions and fractional Legendre orthogonal basis functions and fractional order Laguerre orthogonal basis functions is implied as the most applicable and popular fractional orthogonal basis functions (Aoun et al., 2007; Malti et al., 2004; Ghanbari and Haeri, 2010; Ghanbari and Haeri, 2011; Heuberger et al., 2005; Ninness et al., 2000; Akçay, 2008; Nazari et al., 2008).

In this study, regarding to suitable features and sometimes orthogonal parameterizing unique, this method selected for system identification and in order to cover model structure fractional dimension systems as linear combination of fractional order orthogonal basis functions.

The model structure of $H(s)$ is described as a linear combination of the fractional order ORFs as below:

$$H(s) = \sum_{i=n_0}^{\infty} \kappa_i \mathrm{P}_i(s) \,, \tag{12}$$

At $\mathrm{P}_i(s)$ are fractional order Laguerre or Legendre orthogonal basis function that are linear combination of their equal generating functions (F_n) described as below:

Laguerre OBFs :

$$\mathrm{P}_n(s) = \sum_{l=1}^{n} B_{n,l} F_l(s), \quad F_n(s) = \left(\frac{1}{s^\alpha + a} \right)^n \tag{13}$$

Legendre OBFs:

$$\mathrm{P}_n(s) = \sum_{l=1}^{n} A_{n,l} F_l(s), \quad F_n(s) = \prod_{j=1}^{n} \frac{1}{s^\alpha + a(2j-1)}$$

$A_{n,l}$ And $B_{n,l}$ are computed in a way defined in (Aoun et al., 2007; Malti et al., 2004; Ghanbari and Haeri, 2010).

We consider model structure for fractional order systems with fractional order dominant pole as $G_1(s) = \dfrac{k}{s^{\alpha} + a}$, $0 \prec \alpha \prec 2$, $a \succ 0$, $k \in \Re$ which prepares (8, 9) relation stability condition and the term of belonging to H_2. We follow three parameters of above model structure such as: α commensurate fractional order, a pole position, and k coefficient. In order to identify these parameters two procedure to a problem procedures followed by the help of CPSO method.

Procedure 1: we obtain all three parameters by CPSO method.

Procedure 2: at first all three parameters by CPSO method. Then we obtain k coefficient regarding to the existence of true system output-input data which is mixed with noise by the help of LS (the Least Square) method.

A direct method to identify specifies this coefficient is measurement of parameters in model structure $\hat{H}(s)$ that is described as below:

$$\hat{H}(s) = \sum_{i=n_0}^{N} \hat{\kappa}_i \mathrm{P}_i(s),\qquad (14)$$

The coefficients $\hat{\kappa}_i$ are computed, therefore the L_2 norm of the prediction error $\varepsilon(t)$ is reduced regarding to the following model structure:

$$y(t) = \sum_{i=n_0}^{N} \hat{\kappa}_i p_i(t) \otimes u(t) + \varepsilon(t)\qquad (15)$$

This model structure is of linear regression kind that is a desirable feature of parameterization with ORFs. If one explains:

$$\theta = \left[\hat{\kappa}_{n_0}\ \hat{\kappa}_{n_0+1} \dots \hat{\kappa}_N\right]^T$$
$$\phi(t) = \left[p_{n_0}(t) \otimes u(t)\ \ p_{n_0+1}(t) \otimes u(t)\ \ \dots\ \ p_N(t) \otimes u(t)\right]^T \qquad (16)$$

Could be written as:

$$y(t) = \phi(t)^T \theta + \varepsilon(t),\qquad (17)$$

The L_2 norm of the prediction error $\varepsilon(t)$ is computed by the equation as below:

$$\sum_{t=1}^{M} |\varepsilon(t)|^2 = \sum_{t=1}^{M} \left|y(t) - \phi(t)^T \theta\right|^2 \qquad (18)$$

When reduction of this norm, leads to a facilitated least squares problem, the reduction quantity of parameters ($\hat{\theta}$) will be normal and usual solution of the equation as below:

$$R(M)\hat{\theta} = F(M)\qquad (19)$$

In which:

$$R(M) = \frac{1}{M}\sum_{t=1}^{M}\phi(t)\phi(t)^T,\ \ F(M) = \frac{1}{M}\sum_{t=1}^{M}\phi(t)y(t)\qquad (20)$$

Above equation solution could be computed by standard numerical method that results leads to (Ljung, 1999).

$$\hat{\theta} = \left(R^T R\right)^{-1} R^T F\qquad (21)$$

In all above procedures, for intelligent algorithm initial adjustments, we determine minimum and maximum limit for parameters and randomly we generate in this range number of determined particles in procedure 1 and 2 in a three dimensional space

Also, in all above procedures, optimization algorithm is used for minimizing output error signal energy according to the following relation ((Ljung, 1999; Aoun et al., 2007; Malti et al., 2004; Ghanbari and Haeri, 2010):

$$F = \sum\left[y_{actual}(t) - y_{identified\ model}(t)\right]^2\qquad (22)$$

Where $y_{actual}(t)$ is actual system output data and $y_{identified\ model}(t)$ is obtained model output data from estimation parameters. F Is a criterion of identification process success and in position $F = 0$ the best identification that's coinciding with identified model output on actual system output has been occurred.

When identified actual model is not complicated, above optimization leads to actual values with a very good accuracy. Otherwise, it is obvious for complicated actual systems, considered model structure must have more terms (in the last position one term was considered) and number of increased parameters and the possibility to obtain parameter actual values could be a good accuracy but obtained values could be a good starting point for determining a range for parameters to reach free optimal parameters by the help of different methods such as trial and error.

SIMULATION RESULTS

In this part, to show proposed method effectiveness for physical system related to heat transfer and measurement by thermocouple (Das, 2008), complete process of estimating its model structure parameters will be brought.

In this system, as a permanent stimulating input signal, (PRBS) pseudo random binary sequence will be used position.

Then output signal which corrupted by standing Gaussian white noise with zero average and the ratio of signal to noise SNR=13db used for estimation.

In this sample, a fractional model is gained for a signal thermocouple junction system (Fig.2) (Das, 2008). A general heat flow equation relating the heat flux conducted through a semi-infinite conductor of heat to the temperature at the origin can be written as:

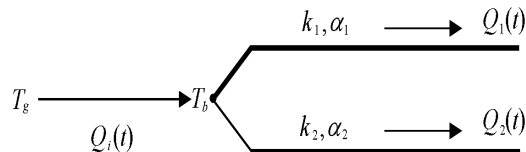$$Q(t) = \frac{k}{\sqrt{\alpha}} \,_a D_t^{0.5} T_b(t) \qquad (23)$$

Figure2. Thermocouple junction for temperature (heat flux) measurement.

When initial forcing conditions are zero, $_a D_t^{0.5}$ would be replaced by $d^{0.5}/dt^{0.5}$. Here $\alpha = k/c\rho$ (c is the heat capacity ($Jkg^{-1}K^{-1}$), ρ is density (kgm^{-3}), k is the coefficient of heat conduction, ($Wm^{-1}K^{-1}$)) is thermal diffusivity, and T_b (K) is the body temperature at the junction point. Input heat flux to the thermocouple from the source temperature to the tip of the thermocouple junction is determined by $Q_i(t) = hA(T_g(t) - T_b(t))$ where h ($Wm^{-2}K^{-1}$) is the convective heat transfer coefficient and A (m^2) is the surface area. The input heat flux flows into two thermocouple wires (Q_1 and Q_2) as shown in Fig. 2. Therefore:

$$m c \frac{dT_b(t)}{dt} = Q_i(t) - Q_1(t) - Q_2(t) \qquad (24)$$

m (kg) is mass of the thermocouple.

Combining the given equations, the relation between input (T_g) and output (T_b) of this system can be represented by a commensurate fractional order transfer function (Das, 2008):

$$\frac{T_b(s)}{T_g(s)} = \frac{1}{\left(\frac{mc}{hA}\right)s + \frac{1}{hA}\left(\frac{k_1}{\alpha_1^{-0.5}} + \frac{k_2}{\alpha_2^{-0.5}}\right)s^{0.5} + 1} \qquad (25)$$
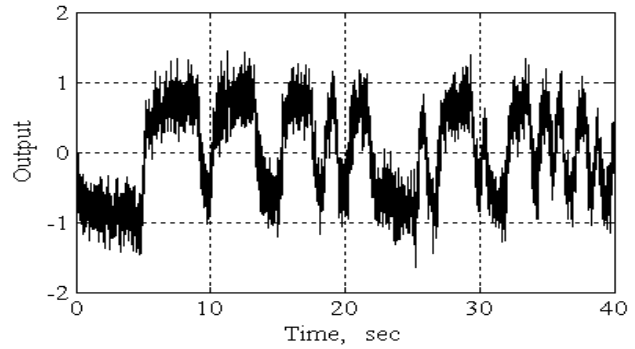
Figure3. Output signal of the system in (28) for a pseudorandom input.

As a practical case, the coefficients are taken values such that the following transfer function is obtained:

$$\frac{mc}{hA} = 0.005, \quad \frac{1}{hA}\left(\frac{k_1}{\alpha_1^{-0.5}} + \frac{k_2}{\alpha_2^{-0.5}}\right) = 0.5$$

$$H_1(s) = \frac{T_b(s)}{T_g(s)} = \frac{200}{s + 100s^{0.5} + 200} = \frac{2.08514}{s^{0.5} + 2.04168} \frac{2.08514}{s^{0.5} + 97.9583} \qquad (26)$$

This system has two pole locators far from each other.

Fig. 3 shows the output signal of system H_1 for a PRBS input signal with magnitude $\{-1, 1\}$. A white Gaussian noise with SNR=13 dB has been added to the output signal.

By having the cost function as error signal according to relation 22, we act for its minimization by use of described CPSO algorithm.

Regarding to described steps, H_1 modeled as $G_1(s) = \frac{k}{s^\alpha + a}$, $0 \prec \alpha \prec 2$, $a \succ 0$, $k \in \Re$.

According to described procedure 2 in part 4 (two parameters estimation of fractional order (α) and estimation of pole condition (a) based on CPSO algorithm and estimation of gain coefficient (k) based on Ls method) to obtain $\hat{H}_{11}(s)$ model and according to described procedure 1 in part 4 (three parameters estimation based on CPSO algorithm) to obtain $\hat{H}_{12}(s)$ model and for showing effectiveness of new method we compare the obtained results by $\hat{H}_{13}(s)$ that in paper (Ghanbari and Haeri, 2011) the author gained within a creative by use of system bode diagram.

$$\begin{aligned} \hat{H}_{11}(s) &= \frac{1.980070}{s^{0.50576} + 1.9781} \\ Identified\ Model: \hat{H}_{12}(s) &= \frac{1.9796}{s^{0.50576} + 1.9781} \\ \hat{H}_{13}(s) &= \frac{2.698}{s^{0.58} + 2.85} \end{aligned} \qquad (27)$$

In order to see success level in estimated models frequency domain, figure.4 shows actual system range Bode

diagram and estimated models.

$$J = 10 \log_{10}(\int_0^T \left(y(t) - y_{model}(t)\right)^2 dt / \int_0^T y^2(t)dt) \quad (28)$$

Also, in order to show estimated parameters values optimization with presented method, at first normalized remained described as relation (28) that's a conventional criterion for successful estimation ((Ljung, 1999; Aoun et al., 2007; Malti et al., 2004; Ghanbari and Haeri, 2010). For comparing the estimated model of normalized remainder (J) value (in table 1) is brought. Then by considering $\alpha \in [0.42, 0.72], a \in [0.95, 3.95]$ calculate. Normalized remained isocounters in relation (28) at 49 points. (α, a) Fig.5 shows obtained isocounters from determined fractional model structure. It is seen that estimated parameter values, approached to optimal value very much, this matter confirm presented estimation process for reaching parameters optimal value.
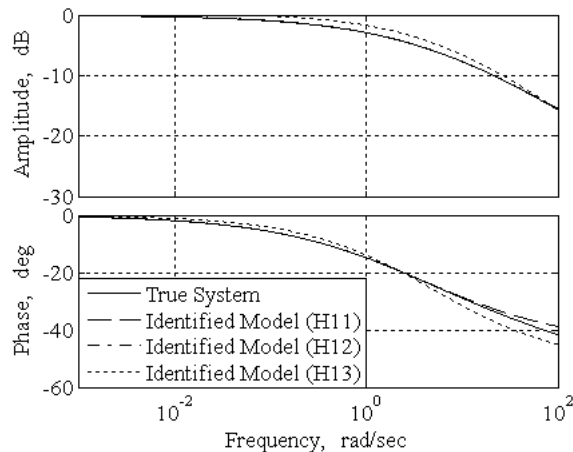

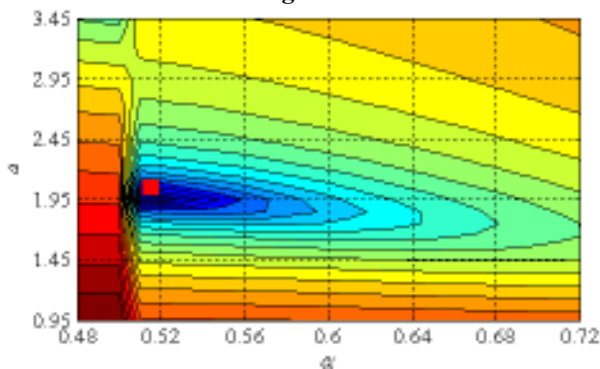**Figure4. Estimated models and H_1 system range Bode diagram.**

Figure5. Obtained isocounters from determined fractional model structure.

Table 1: Normalized Remained (J) of Estimated Model

Normalized remained (J)	$\hat{H}_{11}(s)$ Model	$\hat{H}_{12}(s)$ Model	$\hat{H}_{13}(s)$ Model
	-10.8723622	-10.8723110	-10.0151334

CONCLUSION

Regarding to fractional systems application increase and unique features of this system behavior and necessity of their recognition in different process such as modeling and control, estimation of fractional models structure parameter attracted much attention.

As the systems identification problem is a with constraints optimization problem too. So in this paper considering model structure as the first term, fractional orthonormal parameters by use of CPSO algorithm develops which in two varieties of swarm by use of PSO which has the mutual effect. From the input-output data which corrupted with noise, the suitable estimation obtains from parameters of this model structure.

In order to cover fractional system with complicated and changing behaviors, must consider poles as different and incorporate ones.

We are hopeful that in our future works we can identify more complex systems by use of suitable method of CPSO.

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