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Restoring the Symmetry Between Space Domain and Time Domain in the Channel Capacity of MIMO Communication Systems

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Introduction

For sake of simplicity, let us consider a constant channel not known at the transmitter (TX,) and a number of TX antennas equal to the number of receiving (RX) antennas, let M be. In this case, the channel capacity is given by:

$$C = B \log_2 \det\left(\mathbf{I} + \frac{P}{M\,BN_0} \mathbf{H}\mathbf{H}^H \right) = B \sum_{m=1}^{r} \log_2 \left(1 + \frac{P}{M} \frac{\sigma_m^2}{BN_0} \right) \text{ bits/s} \qquad (1)$$

wherein B is the bandwidth, P is the transmitted power, N_0 is noise spectral density at any of the RX antennas, $\mathbf{I}$ is the unit matrix, $\mathbf{H}$ is the channel matrix, the apex H stands for Hermitian (complex conjugate), r is the rank of the channel matrix, and σ_m are the singular values of the channel matrix. In standard MIMO literature this formula is discussed considering a number r of SISO subchannels.

Let us consider now the maximum number of bits that we can send in an temporal interval T using a bandwidth B without equivocation. The supremum of such a number, that will be call $n(\cdot)$, is $2BTC$, wherein C is the channel capacity, e.g.:

$$n(T,B;\mathbf{H}) = \frac{2BT}{2} \sum_{m=1}^{r} \log_2 \left(1 + \frac{2E_T}{M[2BT]} \frac{\sigma_m^2}{N_0} \right) \qquad \text{bits} \qquad (2)$$

wherein E_T is the transmitted energy.

The formula clearly shows an asymmetry between how the time-domain and the space domain are handled. The spatial information is completely associated to the channel matrix, that is the input-output operator for the spatial "variation" of the signal. Also in the time domain we have an input-output operator, that is the band-limited operator, but its transfer function does not appear in the formula. Instead, in the formula the basic limitations on the bit rate due to this operator, expressed by the time-bandwidth product $2BT$, appear.

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Restoring the symmetry

Clearly, in order to restore the symmetry, we must treat the input-output spatial operator like the input-output temporal operator, introducing a quantity equivalent to the time-bandwidth product.

In order to reach this result, let us discuss the classical SISO AWGN bandlimited channel [1]. The Shannon approach to the capacity of the *continuous* bandlimited AWGN channel consists of introducing the differential entropy *per degree of freedom* (*or per channel use*). Then, he considered the maximum number of times in a second that the continuous channel can be used to send information. In case of bandlimited channel with bandwidth B this number turns out to be $2B$ times each second, e.g. $2BT$ times in a time interval T, obtaining that

$$n(T,B)=\frac{[2BT]}{2}\log_2\left(1+\frac{2E}{[2BT]N_0}\right) \qquad \text{bits} \qquad (3)$$

wherein N_0 is the noise spectral density, and E is the energy of the received signal.

An interesting observation for our discussion is that the time-bandwidth product is not related to probabilistic processes, like the Shannon entropy, but is strictly related to the *optimal* representation of the set of signals at the output of the input-output operator (in this case the bandlimited operator). Indeed, a more rigorous evaluation of such a number can be obtained by means of the proper basis expansion for almost time-domain frequency-domain limited signals, that are the prolate spheroidal functions (PSF) [2]. The PSF are an *optimal* basis in L_2, e.g. a basis whose number of elements are the minimum one to represent the set of functions at the output of the banlimitation operator within a given approximation. This observation is the key to extend the classical Shannon capacity of time-domain bandlimited AWGN channel to the space domain.

An optimal representation of the field radiated by space-limited sources can be obtained by Singular Value Decomposition of the field [3], [4]. However, paralleling the approach followed in the framework of classic time-domain signal processing, in practice the adoption of a slightly less efficient representation based on bandlimited signals is more convenient. Accordingly, the signal in the spatial domain is represented by a function whose Fourier transform has compact support, let us call W such a spatial bandwidth [5], [4] (in this paper we will suppose a flat spatial bandwidth for sake of simplicity). It turns out that the spatial bandwidth W depends on the size and shape of the surface enclosing the sources and the scattering objects [5], [4].

Using such a representation we have a space-domain bandlimited time-domain bandlimited (SBTB) signal than can be represented by a double Whittaker-Kotel'nikov-Shannon (WKS) sampling series with a number of temporal samples equal to the time-bandwidth product $2BT$ and a number of spatial samples whose number is equal to the *space-bandwidth* product $2W\Xi$, wherein Ξ is related to the length of the observation spatial interval [4].

According to this representation, the channel is *both temporal and spatial bandlimited*. By supposing that the signal is affected by a spatial-temporal AWG noise, we have that the supremum of the number of bits that we can send observing the signal in an temporal interval T and a spatial interval Ξ, and using a temporal signal bandwidth B and a spatial signal bandwidth W, is:

$$n(T,B;\Xi,W) = \frac{[4BTW\Xi]}{2}\ \log_2\left(1+\frac{2}{[4BTW\Xi]}\frac{E}{N_0'}\right) \quad \text{bits} \tag{4}$$

wherein N_0' has the same role of the noise spectral density defined in the classic time-domain communication channel, E is the energy of the reveived signal, and $4BTW\Xi$ is the number of degrees of freedom of our space-time bandlimited channel, or equivalently the number of times that the space-time channel can be "used" in a temporal interval T and a spatial interval Ξ. Note that in the formula the space domain and the time domain are treated exactly in the same way. For example, in the SBTB AWGN channel the concept of SISO subchannels is substituted by the concept of *spatial channel use*, completely analogous to the concept of *temporal channel use* of the classic bandlimited AWGN channels.

Discussion and conclusions

The space-time bandlimited channel is used *2BT* times in the temporal interval *T*. This is the most efficient way to use the temporal resource. Let us suppose that we undersample the signal in the time domain. This means that the channel is used *less* than *2BT* times in the temporal interval *T*. This is obviously an inefficient way to use the temporal possibilities of the channel (and of course any technician would avoid this!). Furthermore, if we try to use the channel more than *2BT* times in the temporal interval *T* we simply oversample the signal, *without any advantage* in terms of number of bits received (of course requiring a vanishing error block probability).

Let us consider now the spatial domain. Paralleling the discussion regarding the use of the channel in the time domain, the space-time bandlimited channel is used $2W\Xi$ times in the spatial (observation) interval Ξ. As in the temporal domain, also in the spatial domain we can undersample, or oversample, the signal. In the case of undersampling, the channel is used *less* than $2W\Xi$ times in the spatial observation interval Ξ, while in case of oversampling the channel *cannot be used* more than $2W\Xi$ times.

Let us now consider the classic MIMO channel capacity. The maximum channel capacity for a given number M of TX and RX antennas is obtained when the matrix $\boldsymbol{H}$ coincides with the unit matrix $\boldsymbol{I}$. In this case we have that:

$$n(T, B; \mathbf{I}) = \frac{[2BTM]}{2} \log_2 \left(1 + \frac{2E}{[2BTM] N_0} \right) \qquad \text{bits} \tag{5}$$

wherein E is the energy of the transmitted signal, that in this case is also equal to the energy of the received signal due to the particular channel matrix considered.

If we compare (4) to (5) it is clear that the role of the space-bandwidth product is substituted by the number of antennas M. If the number of antennas is smaller than the space-bandwidth product, the channel is used *only M* times in the spatial observation interval. This is what usually happens, and represents an *inefficient* way of using the spatial resource from the point of view of the channel capacity potentiality. On the contrary, if the number of TX/RX antennas is greater than the space-bandwidth product, we are *oversampling* the spatial signal, and the channel matrix will have a rank not larger than the space-bandwidth product [3], [4]. We are using too many antennas obtaining no advantage (in terms of channel capacity) than using $2W\Xi$ antennas.

The spatial-temporal bandlimited channel model not only clearly shows that the space is a *limited*, and consequently *precious*, resource, but permits us also to *quantify* this resource. The efficient use of such a resource is a challenging but mandatory task for electromagnetic and communication engineers.

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