Optimal Rotor Structure Design of Interior Permanent Magnet Synchronous Machine based on Efficient Genetic Algorithm Using Kriging Model

Dong-Kyun Woo[†], Il-Woo Kim* and Hyun-Kyo Jung*

Abstract – In the recent past, genetic algorithm (GA) and evolutionary optimization scheme have become increasingly popular for the design of electromagnetic (EM) devices. However, the conventional GA suffers from computational drawback and parameter dependency when applied to a computationally expensive problem, such as practical EM optimization design. To overcome these issues, a hybrid optimization scheme using GA in conjunction with Kriging is proposed. The algorithm is validated by using two mathematical problems and by optimizing rotor structure of interior permanent magnet synchronous machine.

Keywords: Interior permanent magnet synchronous machine, Kriging, Metamodel, Optimization, Rotor design

1. Introduction

Optimization is the skill of getting the best solution among all feasible solutions. The objective of electromagnetic (EM) device optimization is to determine physical parameters satisfying design specifications against certain constraints [1]. Previous work in EM design (motor, transformer, etc.) has primarily focused on optimizing specific problems and involved the use of evolutionary schemes. Many EM optimization problems have been solved by the use of highly accurate models (e.g., the finite element method) with the genetic algorithm (GA) [2]. However, like other population based algorithms, the GA requires a very high number of fitness function evaluations.

The cost of optimizing an expensive solution for EM device optimization is dominated by the number of function calls in order to reach a global optimum with an acceptable solution. Therefore, the GA is not practical for such an application [1]. Moreover, the difficulty of finding suitable evolutionary operator is a weakness in the application of the GA to practical EM optimization problems [3].

The GA using a computationally expensive evaluation may reduce the cost through a metamodel. The metamodel should be understood as a surrogate evaluation model that is built by exploiting information about evaluated individuals [4]. From the various modelisation methods, the Kriging is chosen since it is more suitable for the design and analysis of computer experiments [5]. In traditional optimization methods, the model and algorithm were mostly discussed separately. However, in this paper,

the optimization process involves simultaneous update of the model and algorithm.

2. Genetic Algorithm

The GA is a search and optimization algorithm based on the principle of natural evolution to produce the best-fit design. The GA is composed of a population of strings and evolutionary operators (selection, crossover and mutation). Each string is an encoding of a solution to the problem and each individual has an associated fitness that depends on the problem. The initial populations are randomly generated typically or may be characterized by the user. A population having highly suitable fitness is evolved through the generations by the use of selection, crossover and mutation operators with given probabilities [6].

In application of a GA using a computationally expensive evaluation, one of the most frequent difficulties encountered is the selection of the most suitable evolutionary parameters in the optimization process. In addition, the tuning parameters cause the GA to yield possibly inconclusive solutions and the convergence characteristics may vary due to such parameter changes. Also, different runs with a GA would yield different results. Consequently, a measure of luck may be involved in producing the optimal solution. However, that is not necessarily bad, because it may allow the GA to find an acceptable solution.

The parameter tuning includes the restriction of the selection pressure and the change of mating probability. However, all these methods are heuristic in nature. Their effects are problem dependent and consequently different

situations require distinct strategies. Therefore, the difficulty of finding suitable parameter values is a weakness of the GA.

In this paper, the relationship between the convergence ratio and GA parameters such as population size, crossover probability and mutation probability is investigated. Moreover, under the condition that either the mutation or the crossover probability is zero, the search ability of the GA is investigated. These results are compared with the proposed algorithm to show that the proposed method can effectively reduce the number of function calls and its performance is robust in spite of the parameter changes.

3. Kriging Model

Several interpolation methods can be accomplished using a polynomial fitting such as

$$y(x) = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} x_i x_j \qquad (1)$$

where the β's are regression coefficients, n is the number of design variables and x's are the design variables based on n sampling points $[x_1 \ x_2 \ ... \ x_n]$. However, for a higher order polynomial, these methods show a highly oscillatory characteristic at some locations between the sampled points. However, the Kriging model exhibits much less oscillation [7].

The Kriging has been known under different names such as Gaussian process and Gaussian random function method [8]. The term Kriging points directly to the origin of prediction method which was first conceived of in the late fifties by D. G. Krige, a South African mining engineer [9]. It was originally developed as a technique for estimating yields of ore deposits from sparsely distributed core samples. However, it has been widely applied to many different fields now.

In this paper, the Kriging model is used to construct a surrogate of the real fitness function. Although the Kriging has been widely used and discussed, a brief review of this algorithm is included in Appendix.

4. Proposed Algorithm

The proposed algorithm is based on the GA assisted by the Kriging model. Total process of the proposed algorithm is as follows :
Step 0 - Initialization
a) Initialize evolutionary operator.
P_c : The crossover probability.
P_m : The mutation probability.
S_p : The population size.
Since the diversity of the solution is decisively dependent upon the population size, an adequate selection of the population size is essential.
b) Create initial population
Step 1 - Generation of initial metamodel
From initial population, a metamodel is generated. The candidate solution on the surrogate model is selected as members of the initial parent set. These are determined by following rule.
a) Set the current candidate solution to the optimum and evaluate the position of the candidate one.
b) An evaluated solution is compared with estimated one. Then, it can be determined whether the estimated value qualifies as optimum or not. If so-computed solution is not real optimum, the mutation operator is applied to the checked position.
Step 2 – Generation of children set
Make children set using crossover and selection operator.
Step 3 - Mutation
In conventional GA, since a high mutation ratio decreases the convergence rate, the mutation ratio is dropped generally according to convergence degree. However, in the proposed method, since the randomly generated solution improves overall quality of the surrogate model, the mutation ratio is increased in proportion to optimization iteration.
Step 4 - Reshaping
The metamodel is continually updated using populations obtained during iteration that does not satisfy the convergence criterion.
Step 5 - Convergence check
Repeat *Step 1 - Step 5* until the terminal condition is satisfied.

In order to compare it with the performance of the GA, the proposed algorithm was applied to the optimization of two mathematical functions and a practical EM optimization problem.

5. Numerical Test and Result

5.1 A Tests for mathematical functions

Two mathematical functions were tested. The formula for the test function 1 is

$$f = \sin \sum_{i=1}^{2} |x_i - 10| \Big/ \sum_{i=1}^{2} |x_i - 10| \qquad (2)$$

where $0 \le x_i \le 20$. The formula for test function 2 is

$$f = \sum_{i=1}^{2} (x_i - \cos(18 \times x_i)) \qquad (3)$$

where $-1 \le x_i \le 1$. Table 1 shows the basic condition needed to execute the test functions.

Table 1. Conditions to execute the test functions

	Test function 1	Test function 2
Number of variables	2	2
Number of population	10	10
Population size S_p	10	12
mutation probability p_m	0.01	0.1
crossover probability p_c	0.8	0.8

The optimization using only a metamodel without an evaluation causes the risk of an incorrect convergence. The optimal value computed in this way is not a real optimum because it is not evaluated by means of a function call. So, for each iteration, an exact solution needs to be computed through an exact cost tool to determine the next elite solution.

Table 2 shows that the proposed algorithm can effectively reduce the number of function calls. Fig. 1 and Fig. 2 show the shapes of the test functions and the optimization process of the proposed method. It demonstrates that the number of data sets has significantly more effect on the quality of the metamodel than sample position. It means that Kriging model cannot compensate for lack of data.

Table 2. Simulation results

	Number of iteration in test function 1	Number of iteration in test function 2
Conventional GA	45	180
Proposed algorithm	5	12
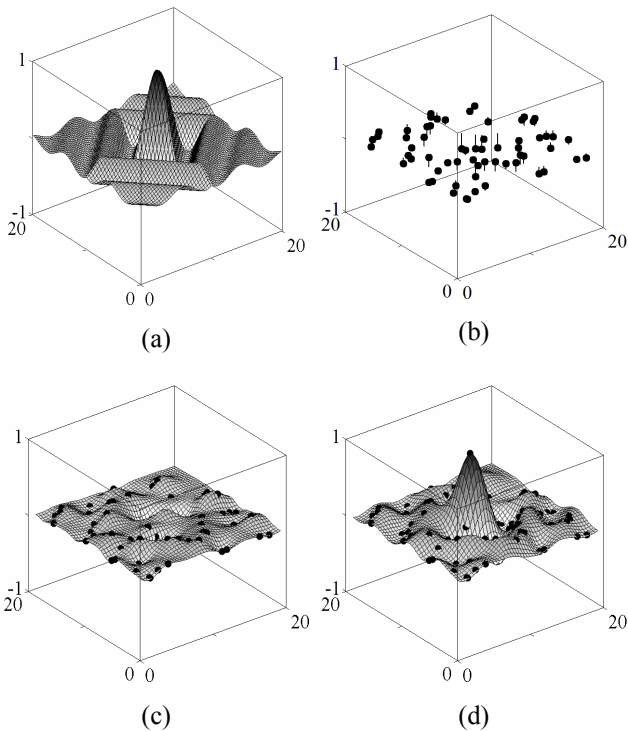
Fig. 1. Optimization process for a test function 1: (a) The shape of test function 1; (b) Initial samples; (c) Iteration 0; (d) Iteration 2.
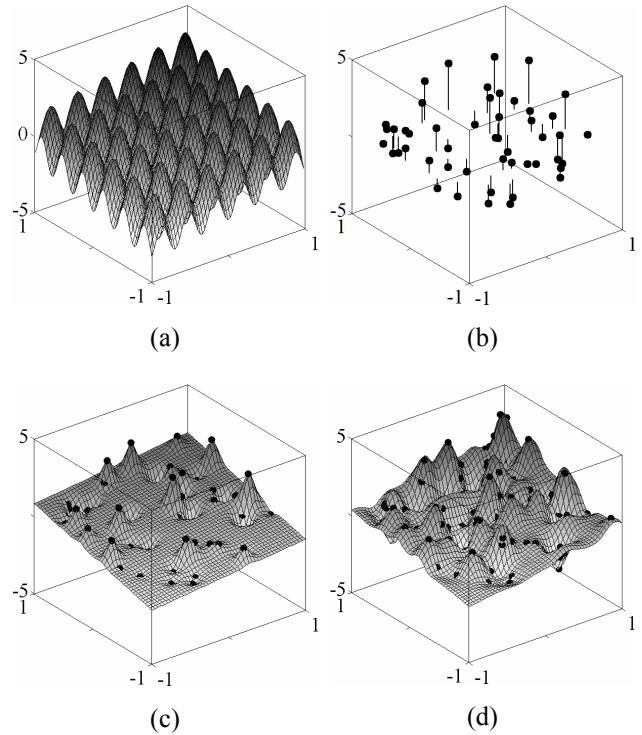

Fig. 2. Optimization process for a test function 2: (a) The shape of test function 2; (b) Initial samples; (c) Iteration 0; (d) Iteration 3.

Also, the feature of the proposed algorithm is that its performance is robust in spite of parameter changes. The parameter tuning is heuristic in nature and its effect varies with different problems. Therefore, the difficulty of finding suitable parameters values is a weakness of the GA. In order to overcome this difficulty, the proposed method provides a robust solution against parameter changes with a computational cost which is more outstanding than that of GA.

By varying the crossover and mutation probability, the trends in changes of convergence were obtained. The search ability of the GA was also investigated under the condition that either the mutation or the crossover probability is zero. This means that the degree of population diversity was dropped and the search ability of the GA was decreased. The same simulation was repeated for different population sizes in order to examine if there were significant differences. Finally, the performance of the proposed algorithm was evaluated as well.

The results of the proposed method are presented from Fig. 3 to Fig. 8. A moderate mutation probability contributed to good performance while a zero mutation rate degraded the performance. Also, the smaller the population size, the more notable was the effect of the mutation probability on the performance.

A large population size presented that the degree of population diversity increased at the cost of a smaller mutation probability. Furthermore, the simulation results show that the impact of population size is more favourable
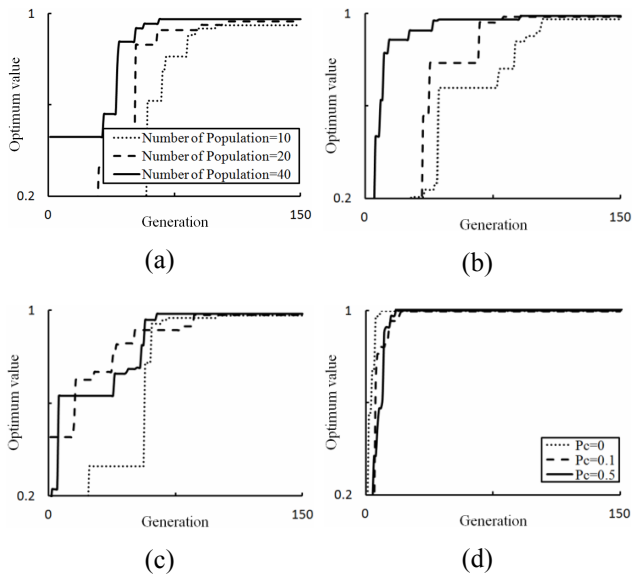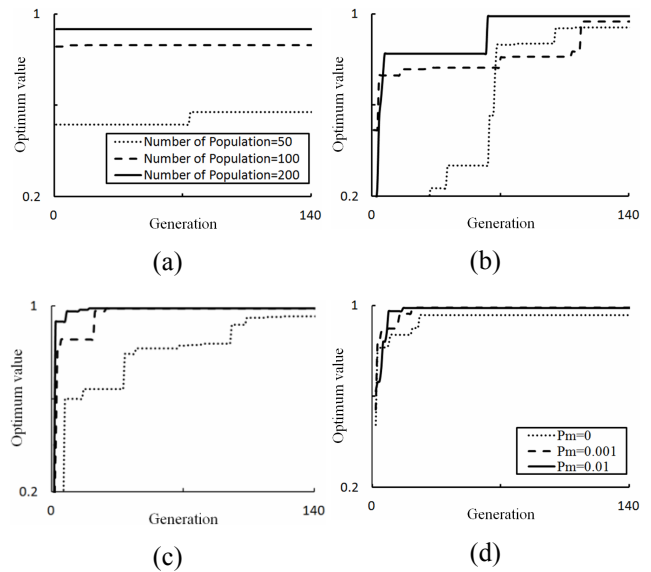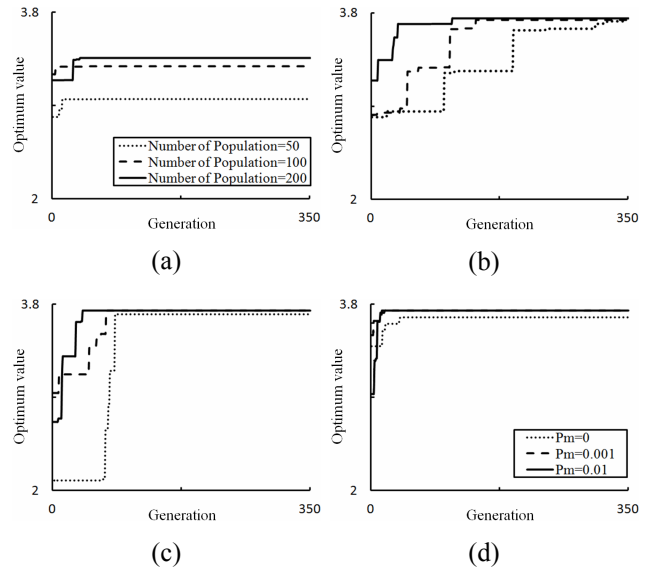
Fig. 3. The convergence curves with different crossover probability P_c and the number of populations N in test function 1: (a) $P_c = 0$; (b) $P_c = 0.1$; (c) $P_c = 0.5$; (d) Result in the proposed method (N =10).
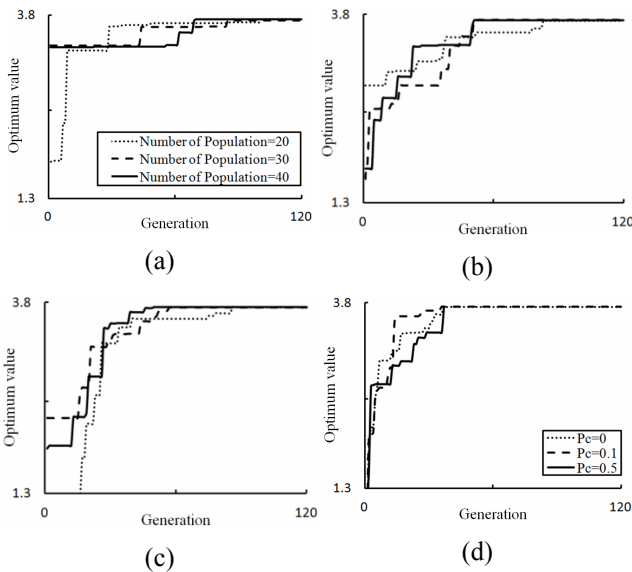
Fig. 4. The convergence curves with different crossover probability P_c and the number of populations N in test function 2: (a) $P_c = 0$; (b) $P_c = 0.1$; (c) $P_c = 0.5$; (d) Result in the proposed method (N =20).

Fig. 5. The convergence curves with different mutation probability P_m and the number of populations N in test function 1: (a) $P_m = 0$; (b) $P_m = 0.001$; (c) $P_m = 0.1$; (d) Result in the proposed method ($N = 50$).

Fig. 6. The convergence curves with different mutation probability P_m and the number of populations N in test function 2: (a) $P_m = 0$; (b) $P_m = 0.001$; (c) $P_m = 0.1$; (d) Result in the proposed method ($N = 50$).

than that of the mutation probability.

From Fig. 3 to Fig. 8, all simulation results show that the proposed method can be applied to the problems considered without the need for empirical selection of the evolutionary operator. Although the operator has small population size, low selection pressure and mating probability, the proposed method showed that the degree of population diversity was guaranteed through the metamodel.

5.2 Application to a practical problem

As a practical optimization example, the design model in this paper is the interior permanent magnet synchronous motor (IPMSM) for a fuel cell electric vehicle (FCEV) which requires a wide constant power range [10, 11]. Since the magnets are embedded in the rotor core, the dispersion of the magnet is obviously minimized. Moreover, the IPMSM has the advantage of additional reluctance torque.

(a)   (b)

(c)   (d)
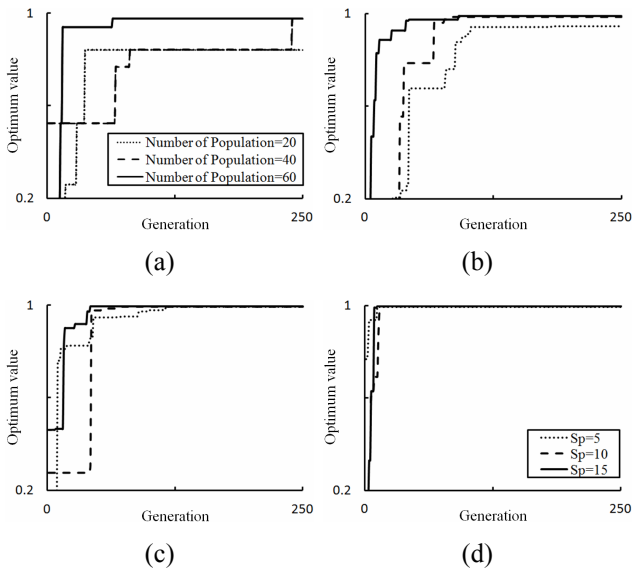
Fig. 7. The convergence curves with different population size S_p and the number of populations N in test function 1: (a) $S_p = 5$; (b) $S_p = 10$; (c) $S_p = 15$; (d) Result in the proposed method ($N = 20$).


(a)   (b)

(c)   (d)
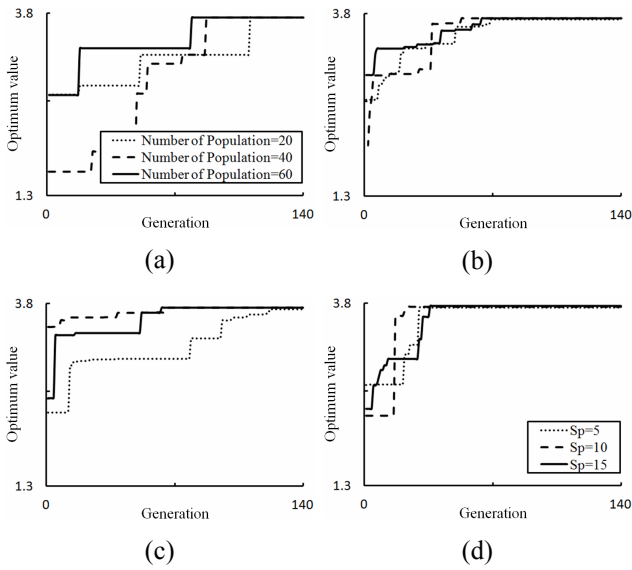
Fig. 8. The convergence curves with different population size S_p and the number of populations N in test function 2: (a) $S_p = 5$; (b) $S_p = 10$; (c) $S_p = 15$; (d) Result in the proposed method ($N = 20$).

The initial structure and design variables of the design model are presented in Fig. 9. The objective of the optimal design to be maximized is defined by

$$Power = \frac{3}{2}\omega_r (\lambda_d i_q - \lambda_q i_d) \tag{4}$$

where i_d and i_q are d- and q-axis currents, while λ_d and λ_q are d- and q-axis flux linkages. There are several design variables related to the power. It is essential to reduce
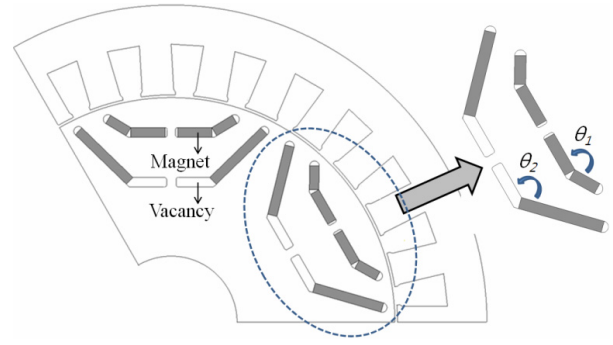
Fig. 9. Design model and variables.

design variables in order to obtain feasible optimal design in a reasonable time. Therefore, we selected two parameter concerned with magnet shape which has an important role in increasing power.

The back-EMF should be constrained in order to protect the inverter devices in the case of a fault condition at the maximum speed. Therefore, the amplitude of the back-EMF is restricted below the maximum voltage level of fuel cell. Moreover, the magnet quantity is constrained. Due to the limit condition, (θ_1, θ_2) were selected as design variables.

Fig. 10 shows the optimization process based on the proposed method. Table 3 and 4 show the basic conditions needed to execute the proposed algorithm and the specifications of the IPMSM, respectively. Because the motor is driven from the fuel cell, the DC link voltage is variable. In order to guarantee the stable performance of the motor, the required voltage is set to the minimum voltage level of fuel cell. Table 5 shows results before and after optimization. Fig. 11 represents magnetic flux density distribution of the optimal model in the no-load condition. Fig. 12 shows characteristic results in the optimal model. From Fig. 12, we can see that about 4 % power is increased compared with the initial model.

**Table 3.** Conditions to execute the proposed algorithm

Number of variables	2
Number of iterations	100
Number of populations	8
Population size S_p	16
crossover probability P_c	0.5
mutation probability P_m	0.7
θ_1 [°]	From 117 to 149
θ_2 [°]	From 115 to 155

**Table 4.** Specifications of the objective model

| Parameters	Value
Pole number/ No. of slot	6 / 27
Outer diameter of the rotor [mm]	80
Outer diameter of the stator [mm]	235
Stacking depth [mm]	117
Maximum power [kW]	225
DC link voltage [V]	240~400
Permanent magnet (NdFeB) [T]	1.05

Table 5. Optimization result

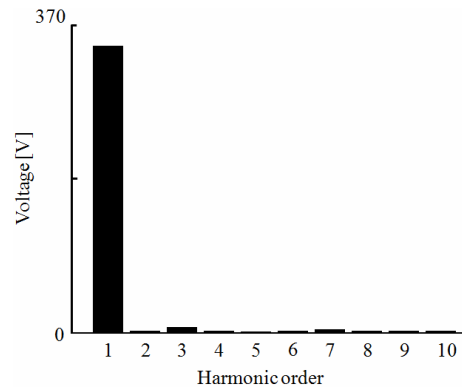|	Initial model	Optimal model
Power [kW] @ 3,000[rpm]	246.2	249.0
Power [kW] @ 9,000[rpm]	242.7	252.7
Back-EMF [V_{peak}] @ 9000 [rpm]	358.6	362.1
θ_1 [deg]	148.4	149.5
θ_2 [deg]	130.2	132.4
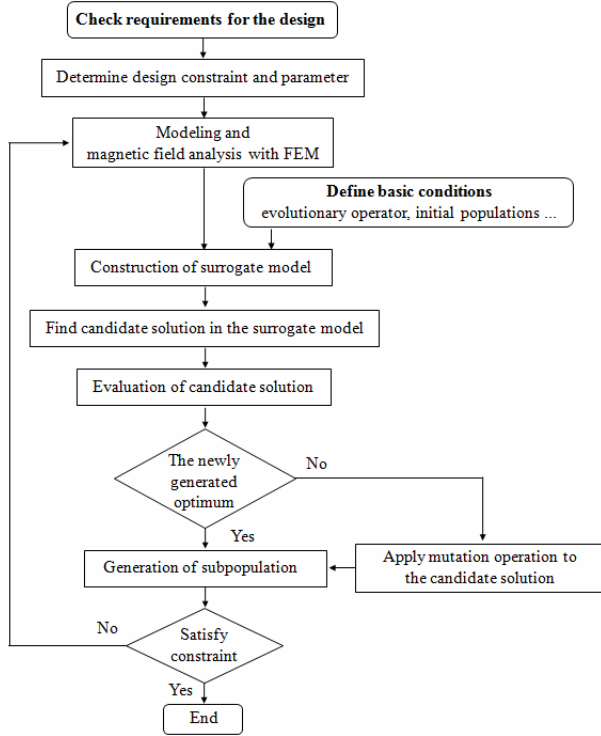
Fig. 10. Flow chart of the proposed method applied to the optimal rotor design of IPMSM.
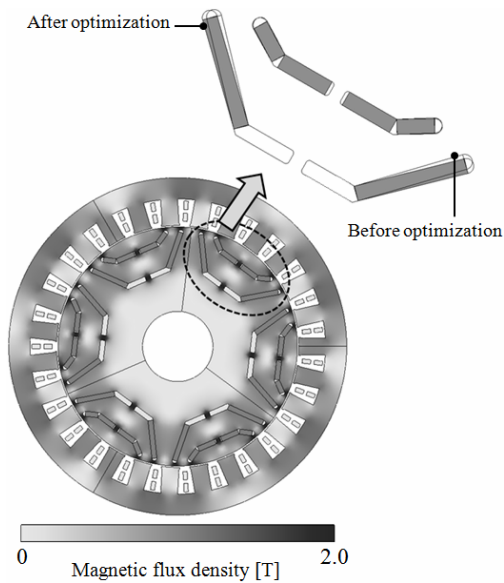
Fig. 11. The magnetic flux density distribution of the optimal model in the no-load condition and the comparison of the model before and after optimization.
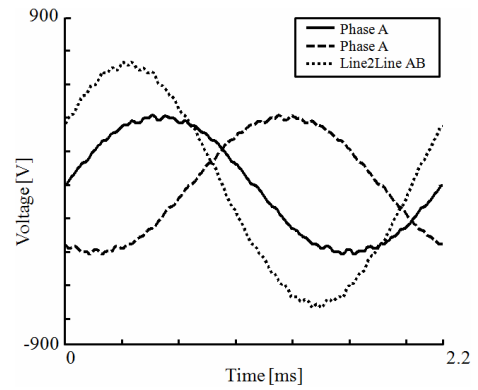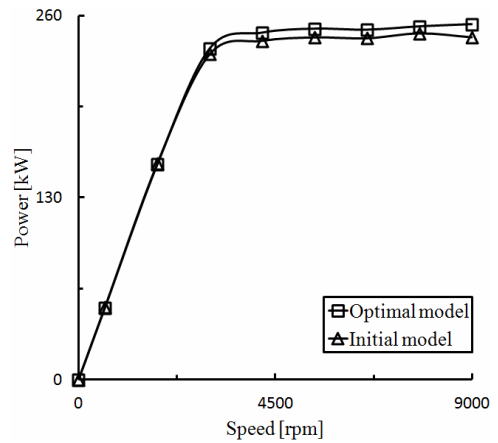
Fig. 12. Characteristic result of the optimal model: (a) The back-EMF wave; (b) Harmonic components in (a); (c) Comparison of generating power between initial and optimal model.

6. Conclusion

In this paper, a hybrid optimization scheme to reduce the number of fitness function evaluations is proposed. The algorithm is based upon combination of GA and the Kriging model. It has a robust convergence characteristic

in spite of parameter changes. The usefulness of the proposed method was verified by test functions and the optimal rotor design of the IPMSM. The results show that the proposed method is appropriate for optimizing an expensive solution such as EM device optimization.

Appendix

Given a set of m sample points $X=[x_1...x_m]$ with $x_i \in \Omega^j$ and responses $Y=[y_1...y_m]$ with $y_i \in \Omega^k$. In the Kriging method, the estimated value is a weighted linear combination of the m sampling points. Thus

$$\tilde{y} = \sum_{i=1}^{m} \omega_i y_i \qquad (5)$$

where $\tilde{y}$ is the estimated value at a specific location and the variables $[\omega_1\ \omega_2\ ...\ \omega_m]^T$ are the weights. So, in order to compute the estimate value, it is necessary to evaluate the value of all weights ω_i. The estimated value $\tilde{y}$ must be a good estimator of y, so to evaluate ω_i, two conditions are imposed:

(1) The expectation of error between $\tilde{y}$ and y must be zero.
(2) The error between $\tilde{y}$ and y must be minimized.

When the mean is supposed as unknown for every x_i's, the first condition allows us to write

$$\sum_{i=1}^{m} \omega_i = 1 \ . \qquad (6)$$

Moreover, in order to ensure that the estimated value will be unbiased, the above the constraint should be also defined. The second condition is equivalent to minimization of the estimation of the variance of the error. If the true value at a location x_0 is y_0, the expected error e will be

$$e = E[\bar{y} - y_0]$$
$$= E\left[\sum_{i=1}^{m} \omega_i y_i - y_0 \right] . \qquad (7)$$

The estimation variance of the error is defined as

$$\sigma_e^2 = E\left[\left(\sum_{i=1}^{m} \omega_i y_i - y_0 \right)^2 \right]$$
$$= \sum_{i=1}^{m}\sum_{j=1}^{m} \omega_i \omega_j c_{ij} - 2\sum_{i=1}^{m} \omega_i c_{i0} + c_{00} \qquad (8)$$

When (8) is minimized with respect to ω_i, the simple Kriging equation is obtained and when (8) is minimized

with respect to ω_i constrained by (6), the ordinary Kriging equation is obtained as follows

$$\begin{bmatrix} C & F \\ F^T & 0 \end{bmatrix}\begin{bmatrix} w \\ \lambda \end{bmatrix} = \begin{bmatrix} B \\ 1 \end{bmatrix} . \qquad (9)$$

The general form of C is the covariance matrix $cov(x_i,x_j)$ and λ is a Lagrange multiplier. The general form of the matrix B is $cov(x,x_j)$. If the covariance matrix is defined, the problem can be solved. The covariance matrix c_{ij} can be defined as

$$c_{ij} = \sigma^2 \Re\left[R\left(x_i, x_j \right) \right] \qquad i,j = 1 \cdots m \qquad (10)$$

where $\Re$ is the correlation matrix and R is the user-defined correlation function. There are several correlation functions, but, in this paper, a Gaussian correlation function is adopted i.e.

$$R\left(x_i, x_j \right) = \exp\left[-\sum_{k=1}^{m} \alpha_k \left| x_i - x_j \right|^2 \right] \qquad (11)$$

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