Research Article

Stability of Stochastic Reaction-Diffusion Systems with Markovian Switching and Impulsive Perturbations

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This paper is devoted to investigating mean square stability of a class of stochastic reaction-diffusion systems with Markovian switching and impulsive perturbations. Based on Lyapunov functions and stochastic analysis method, some new criteria are established. Moreover, a class of semilinear stochastic impulsive reaction-diffusion differential equations with Markovian switching is discussed and a numerical example is presented to show the effectiveness of the obtained results.

1. Introduction

Markovian jump systems, introduced by Krasovskiĭ and Lidskiĭ [1] in 1961, have received increasing attention, see [2–15] and references therein. Shi and Boukas [3] have probed H_∞ control for Markovian jumping linear systems with parametric uncertainty. Zhang et al. [4–6] have discussed markovian jump linear systems with partly unknown transition probability. Mao et al. [7–13] have established a number of stability criteria for stochastic differential equations with Markovian switching. However, impulsive perturbations have not been included in the above results.

In fact, impulsive effects widely exist in many fields, such as medicine and biology, economics, mechanics, electronics, and telecommunications [16–19]. Recently, impulsive stochastic differential equations have attracted more and more researchers [20–27]. L. Xu and D. Xu [20] have investigated mean square exponential stability of impulsive control stochastic systems with time-varying delay. Li [23] has obtained the attracting set for impulsive stochastic difference equations with continuous time. Pan and Cao [24] have considered

exponential stability of impulsive stochastic functional differential equations. Zhang et al. [25] have studied stability of impulsive stochastic differential equations in terms of two measures via perturbing Lyapunov functions. Moreover, Markovian Jump Systems with impulsive perturbations have been investigated [28–31]. Zhang et al. [28] have established several criteria for stochastic stability analysis of neutral-type impulsive neural networks with mixed time-varying delays and Markovian jumping. Zhu and Cao [30] have obtained several sufficient conditions on stability of Markovian jump neural networks with impulse control and time-varying delays.

Besides impulsive and stochastic effects, reaction diffusion phenomena cannot be ignored in real systems [32–42]. Kao et al. [34] have discussed exponential stability of impulsive stochastic fuzzy reaction-diffusion Cohen-Grossberg neural networks with mixed delays. Wang et al. [40] have probed stochastic exponential stability of the delayed reaction-diffusion recurrent neural networks with Markovian jumping parameters. However, to the best of our knowledge, there are few considering the Markovian jump systems with impulsive perturbations and reaction-diffusion effects.

Motivated by the above discussions, in this paper, we consider mean square stability of a class of impulsive stochastic reaction-diffusion differential systems with Markovian switching. In Section 2, model description and preliminaries are presented. In Section 3, by utilizing Lyapunov function and stochastic analysis, we obtain some new conditions ensuring mean square stability of impulsive stochastic reaction-diffusion differential equations with Markovian switching. Moreover, mean square stability of a class of semilinear stochastic impulsive reaction-diffusion systems has also been discussed. In Section 4, an example is provided. Section 5 is conclusions.

2. Model Description and Preliminaries

In this section, we investigate the impulsive stochastic reaction diffusion equations with Markovian switching described by

$$
\begin{aligned}
du(t,x) = \big[& D(t,x,u)\Delta u + f(t,x,u,\gamma(t)) \big] dt \\
& + \sigma(t,x,u,\gamma(t)) dw(t), \quad t \geq 0, \ t \neq t_k, \ x \in G,
\end{aligned}
\tag{2.1}
$$

$$
u(t_k,x) = H_k\big(u(t_k^-,x),\gamma(t_k)\big), \quad x \in G,
\tag{2.2}
$$

with boundary condition

$$
\left. \frac{\partial u}{\partial N} \right|_{\partial G} = 0, \quad t \geq 0
\tag{2.3}
$$

and initial condition

$$
u(0,x,i_0) = \varphi(x), \quad x \in G, \ i_0 \in \mathbb{S},
\tag{2.4}
$$

where $u(t,x) = (u_1(t,x),\ldots,u_n(t,x))^{\mathsf{T}}$, $x = (x_1,\ldots,x_m)^{\mathsf{T}} \in G \subset R^m$, G is a bounded set with smooth boundary ∂G, $\partial/\partial N$ is the outward normal derivative. t_k is the impulsive

moment satisfying $0 = t_0 < t_1 < t_2 < \cdots$, and $\lim_{k \to \infty} t_k = +\infty$, $u(t_k^+, x)$ and $u(t_k^-, x)$ denote the right-hand limit and left-hand limit of $u(t, x)$ at t_k, respectively. $u(t, x) \in PC[[0, +\infty) \times G, R^n] = \{u(t, x) : [0, +\infty) \times G \to R^n \mid u(t, x)$ is continuous for all $t \geq 0$ but points $t_k, u(t_k^+, x)$ and $u(t_k^-, x)$ exist, furthermore, $u(t_k^+, x) = u(t_k, x), k = \{1, 2, \ldots\}$. $D(t, x, u)$ is a $n \times n$ matrix, $\Delta u = \{\Delta u_1, \ldots, \Delta u_n\}^\mathsf{T}, \Delta u_i = \sum_{l=1}^m (\partial^2 u_i / \partial x_i^2), i = 1, 2, \ldots, n$. f and σ are continuous, in addition, $f(t, x, u, \gamma(t)) = \{f_1(t, x, u, \gamma(t)), \ldots, f_n(t, x, u, \gamma(t))\}^\mathsf{T}$, $\sigma(t, x, u, \gamma(t)) = \{\sigma_1(t, x, u, \gamma(t)), \ldots, \sigma_n(t, x, u, \gamma(t))\}^\mathsf{T}$. $H_k(u(t_k^-, x), \gamma(t)) = \{H_{1k}(u(t_k^-, x), \gamma(t)), \ldots, H_{nk}(u(t_k^-, x), \gamma(t))\}^\mathsf{T}$ represents the impulsive perturbation of u at time t_k. $w(t)$ is a one-dimensional standard Brownian motion on a complete probability space $(\Omega, F, \{F_t\}_{t \geq 0}, P)$ with a natural filtration $\{F_t\}_{t \geq 0}$. $\{\gamma(t), t \geq 0\}$ is a left-continuous Markov process on the probability space $(\Omega, F, (F_t)_{t \in I}, P)$ and takes values in the finite space $\mathbb{S} = \{1, 2, \ldots, \widetilde{N}\}$ with generator $\Lambda = (\pi_{ij})$ $(i, j \in \mathbb{S})$ given by

$$P\{\gamma(t + \Delta) = j \mid \gamma(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta), & \text{if } i = j, \end{cases} \tag{2.5}$$

where $\Delta > 0$ and $\lim_{\delta \to 0} o(\Delta)/\Delta = 0$, $\pi_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ and $\pi_{ii} = -\sum_{j \neq i} \pi_{ij}$. We suppose that the Markov chain $\gamma(\cdot)$ is independent of the Brownian motion $W(\cdot)$. Moreover, we assume that $H_k(0, \gamma_0) = f(t, x, 0, \gamma_0) = \sigma(t, x, 0, \gamma_0) = 0$, then system (2.1) admits a trivial solution $u = 0$. For $u(t, x) = (u_1(t, x), \ldots, u_n(t, x))^\mathsf{T} \in R^n$, we define $\|u(t, x)\|_G = (\int_G |u(t, x)|^2 dx)^{1/2}$ where $|u(t, x)|^2 = \sum_{i=1}^n u_i^2(t, x)$. For simplicity, we denote $\|u(t, x)\|_G$ by $\overline{u}(t)$ throughout this paper.

Let $u(t, x; 0, \varphi, i_0)$ stand for the solution of system (2.1)–(2.4) through $(0, \varphi, i_0)$.

Definition 2.1. The trivial solution $u = 0$ is said to be mean square stable if for any $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon)$ such that for all $i_0 \in \mathbb{S}$, we have

$$E\left\{\|u(t, x; 0, \varphi, i_0)\|_G^2\right\} < \varepsilon, \quad t \geq 0, \tag{2.6}$$

when φ satisfies $E\{\|\varphi\|_G^2\} \leq \delta$.

Definition 2.2. The function $V(t, y, \gamma(t)) : [0, +\infty) \times R^+ \times \mathbb{S} \to R^+$ belongs to class $v_0^{1,2}$ if

(1) for $k = 1, 2, \ldots$, the function V is once continuously differentiable in t and twice in y on $(t_{k-1}, t_k) \times R^+ \times \mathbb{S}$, and, in addition, $V(t, 0, \gamma_0) = 0$ holds for $t \geq 0$;

(2) $V(t, y, \gamma(t))$ is locally Lipschitzian in y;

(3) for each $k = 1, 2, \ldots$, there exist finite limits

$$\begin{aligned} V(t, q, \gamma(t)) &\longrightarrow V(t_k^-, y, \gamma(t_k)), \quad \text{if } (t, q, \gamma(t)) \longrightarrow (t_k^-, y, \gamma(t_k)), \\ V(t, q, \gamma(t)) &\longrightarrow V(t_k^+, y, \gamma(t_k)), \quad \text{if } (t, q, \gamma(t)) \longrightarrow (t_k^+, y, \gamma(t_k)). \end{aligned} \tag{2.7}$$

3. Main Results

In this section, we will discuss mean square stability of the trivial solution of system (2.1)–(2.4). Assume $H_k(u(t_k^-, x), \gamma(t_k))$ satisfies $\|H_k(u(t_k^-, x), \gamma(t_k))\|_G^2 \leq \Gamma_k^2 \|u(t_k^-, x)\|_G^2$, $\Gamma_k \geq 0$, $\gamma(t) = i \in \mathbb{S}$, $k = 1, 2, \ldots$.

Theorem 3.1. *If there exist constants $\alpha > 0$, $\beta > 0$, $\kappa > 0$ and a Lyapunov function $V(t, \overline{u}(t), i)$ such that for $\gamma(t) = i$, $i \in \mathbb{S}$, we have the following.*

(A1) $\alpha \overline{u}(t) \leq V(t, \overline{u}(t), i) \leq \kappa \overline{u}(t)$.

(A2) $\mathcal{L}V(t, \overline{u}(t), i) \leq \beta V(t, \overline{u}(t), i)$, $t \in [t_{k-1}, t_k)$.

Here the operator $\mathcal{L}V(t, \overline{u}(t), i)$ is defined as

$$
\begin{aligned}
\mathcal{L}V(t, \overline{u}(t), i) &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \overline{u}} \left[\iint_G 2u^\mathsf{T} D(t, x, u) \Delta u\, dx + \int_G 2u^\mathsf{T} f(t, x, u, i) dx \right] \\
&\quad + 2\operatorname{trace}\left\{ \left(\int_G u^\mathsf{T} \mathfrak{e}(t, x, u, i) dx \right) \frac{\partial^2 V}{\partial \overline{u}^2} \left(\int_G u^\mathsf{T} \mathfrak{e}(t, x, u, i) dx \right) \right\} \\
&\quad + \sum_{j=1}^{\widetilde{N}} \pi_{ij} V(t, x, j).
\end{aligned}
\tag{3.1}
$$

(A3) $\lambda > 1$, *where $\lambda = \inf\{\lambda_k \mid \lambda_k = \alpha/\kappa\gamma^2, k = 1, 2, \ldots\}$.*

(A4) $\beta(t_k - t_{k-1}) < \ln \lambda$, $k = 1, 2, \ldots$.

Then, the trivial solution $u = 0$ of system (2.1)–(2.4) is stable in mean square.

Proof. For any $\varepsilon > 0$, there must exist a scalar $\delta = \delta(\varepsilon) > 0$ such that $\delta < (\alpha/\kappa\lambda)\varepsilon$. Next we will prove that $E\{\|u(t, x; 0, \varphi, i_0)\|_G^2\} < \varepsilon$ if φ satisfies $E\{\|\varphi\|_G^2\} \leq \delta$.

Let $u = u(t, x; 0, \varphi, i_0)$. Multiplying both sides of (2.1) by u^T, we obtain

$$
\frac{1}{2} d|u|^2 = \left[u^\mathsf{T} D(t, x, u) \Delta u + u^\mathsf{T} f(t, x, u, \gamma(t)) \right] dt + u^\mathsf{T} \sigma(t, x, u, \gamma(t)) dw(t).
\tag{3.2}
$$

By integrating the above equality with respect to x on G, we then have

$$
\begin{aligned}
\frac{1}{2} d \int_G |u|^2 dx &= \left[\iint_G u^\mathsf{T} D(t, x, u) \Delta u\, dx + \int_G u^\mathsf{T} f(t, x, u, \gamma(t)) dx \right] dt \\
&\quad + \int_G u^\mathsf{T} \sigma(t, x, u, \gamma(t)) dx\, dw(t).
\end{aligned}
\tag{3.3}
$$

Namely,

$$
\begin{aligned}
d\overline{u}(t) &= \left[\iint_G 2u^\mathsf{T} D(t, x, u) \Delta u\, dx + \int_G u^\mathsf{T} f(t, x, u, \gamma(t)) dx \right] dt \\
&\quad + \int_G 2u^\mathsf{T} \sigma(t, x, u, \gamma(t)) dx\, dw(t).
\end{aligned}
\tag{3.4}
$$

Applying Itô formula, we further compute, when $t \neq t_k$,

$$dV(t, \overline{u}(t), i) = \mathcal{L}V(t, \overline{u}(t), i) + \frac{\partial V}{\partial \overline{u}} \int_G 2u^\mathsf{T} \sigma(t, x, u, i) dx \, dw(t), \tag{3.5}$$

where $\gamma(t) = i$ and

$$\mathcal{L}V(t, \overline{u}(t), i)$$
$$= \frac{\partial V(t, \overline{u}(t), i)}{\partial t} + \frac{\partial V(t, \overline{u}(t), i)}{\partial \overline{u}} \left[\int_G 2u^\mathsf{T} D(t, x, u) \Delta u \, dx + \int_G 2u^\mathsf{T} f(t, x, u, i) dx \right]$$
$$+ 2 \operatorname{trace} \left\{ \left(\int_G u^\mathsf{T} \sigma(t, x, u, i) dx \right) \frac{\partial^2 V}{\partial \overline{u}^2} \left(\int_G u^\mathsf{T} \sigma(t, x, u, i) dx \right) \right\} \tag{3.6}$$
$$+ \sum_{j=1}^{\widetilde{N}} \pi_{ij} V(t, x, j).$$

For $t \in [t_k, t_{k+1})$, integrating (3.5) with respect to t from t_k to t, one has

$$V(t, \overline{u}(t), \gamma(t)) = V(t_k, \overline{u}(t_k), \gamma(t_k)) + \int_{t_k}^t \mathcal{L}V(s, \overline{u}(s), \gamma(s)) ds$$
$$+ \int_{t_k}^t \frac{\partial V}{\partial \overline{u}} \int_G 2u^\mathsf{T} \sigma(t, x, u, \gamma(t)) dx \, dw(t). \tag{3.7}$$

Taking the mathematical expectation of both sides of (3.7), we obtain

$$EV(t, \overline{u}(t), \gamma(t)) = EV(t_k, \overline{u}(t_k), \gamma(t_k)) + \int_{t_k}^t E\mathcal{L}V(s, \overline{u}(s), \gamma(s)) ds. \tag{3.8}$$

Choosing small enough $\Delta t > 0$ such that $t + \Delta t \in [t_k, t_{k+1})$, it is easy to see that

$$EV(t + \Delta t, \overline{u}(t + \Delta t), \gamma(t + \Delta t)) = EV(t_k, \overline{u}(t_k), \gamma(t_k))$$
$$+ \int_{t_k}^{t+\Delta t} E\mathcal{L}V(s, \overline{u}(s), \gamma(s)) ds. \tag{3.9}$$

We thus derive from (3.8) and (3.9) that

$$EV(t + \Delta t, \overline{u}(t + \Delta t), \gamma(t + \Delta t)) - EV(t, \overline{u}(t), \gamma(t)) = \int_t^{t+\Delta t} E\mathcal{L}V(s, \overline{u}(s), \gamma(s)) ds$$
$$\leq \int_t^{t+\Delta t} \beta EV(s, \overline{u}(s), \gamma(s)) ds. \tag{3.10}$$

So

$$D^+EV(t, \overline{u}(t), \gamma(t)) \leq \beta EV(t, \overline{u}(t), \gamma(t)). \tag{3.11}$$

Next, we will first prove

$$EV(t, \overline{u}(t), \gamma(t)) \leq \kappa\lambda\delta, \quad 0 \leq t < t_1. \tag{3.12}$$

Obviously,

$$EV(0, \overline{u}(0), \gamma_0) \leq E\kappa\overline{u}(0) = \kappa E\|\varphi\|_G^2 \leq \kappa\delta < \kappa\lambda\delta. \tag{3.13}$$

If inequality (3.12) does not hold, there must exist some $s \in (0, t_1)$ such that

$$EV(s, \overline{u}(s), \gamma(s)) > \kappa\lambda\delta > \kappa\delta \geq EV(0, \overline{u}(0), \gamma_0). \tag{3.14}$$

Let $s_1 = \inf\{s \in (0, t_1) \mid EV(s, \overline{u}(s), \gamma(s) > \kappa\lambda\delta\}$. Since $EV(t, \overline{u}(t), \gamma(t))$ is continuous on $[0, s_1]$, there exist $\widehat{s} \in (0, s_1)$ such that

$$EV(\widehat{s}, \overline{u}(\widehat{s}), \gamma(\widehat{s})) = \kappa\lambda\delta,$$

$$EV(t, \overline{u}(t), \gamma(t)) \leq \kappa\lambda\delta, \quad t \in [0, \widehat{s}), \tag{3.15}$$

$$D^+EV(\widehat{s}, \overline{u}(\widehat{s}), \gamma(\widehat{s})) > 0.$$

From $EV(\widehat{s}, \overline{u}(\widehat{s}), \gamma(\widehat{s})) = \kappa\lambda\delta > \kappa\delta$, if $EV(0, \overline{u}(0), \gamma_0) \leq \kappa\delta$, we know that there is $s_2 \in [0, \widehat{s})$ such that

$$EV(s_2, \overline{u}(s_2), \gamma(s_2)) = \kappa\delta,$$

$$EV(t, \overline{u}(t), \gamma(t)) \geq \kappa\delta, \quad t \in [s_2, \widehat{s}], \tag{3.16}$$

$$D^+EV(s_2, \overline{u}(s_2), \gamma(s_2)) > 0.$$

On the other hand, noticing $D^+EV(t, \overline{u}(t), \gamma(t)) \leq \beta EV(t, \overline{u}(t), \gamma(t))$, we obtain

$$\frac{D^+EV(t, \overline{u}(t), \gamma(t))}{EV(t, \overline{u}(t), \gamma(t))} \leq \beta. \tag{3.17}$$

Integrating both sides of (3.17) on $t \in [s_2, \widehat{s}]$ gives

$$\int_{s_2}^{\widehat{s}} \frac{D^+EV(s, \overline{u}(s), \gamma(s))}{EV(s, \overline{u}(s), \gamma(s))} ds \leq \int_{s_2}^{\widehat{s}} \beta ds < \beta t_1 < \ln\lambda. \tag{3.18}$$

However,

$$\int_{s_2}^{\hat{s}} \frac{D^+ EV(s,\overline{u}(s),\gamma(s))}{EV(s,\overline{u}(s),\gamma(s))} ds = \int_{EV(s_2,\overline{u}(s_2),\gamma(s_2))}^{EV(\hat{s},\overline{u}(\hat{s}),\gamma(\hat{s}))} \frac{d\eta}{\eta} = \int_{\kappa\delta}^{\kappa\lambda\delta} \frac{d\eta}{\eta} = \ln(\kappa\lambda\delta) - \ln(\kappa\delta) = \ln\lambda,$$

(3.19)

which is a contradiction. Therefore,

$$EV(t,\overline{u}(t),\gamma(t)) \leq \kappa\lambda\delta, \quad 0 \leq t < t_1.$$

(3.20)

Furthermore,

$$EV(t_1,\overline{u}(t_1),\gamma(t_1)) \leq E\kappa\overline{u}(t_1) = \kappa E\|H_1(u(t_1^-,x))\|_G^2 \leq \kappa\Gamma_1^2 E\|u(t_1^-,x)\|_\Omega^2$$

$$\leq \frac{\kappa\Gamma_1^2}{\alpha} EV(t_1,\overline{u}(t_1^-,\gamma(t_1))) \leq \frac{\kappa\lambda\delta}{\lambda_1} \leq \kappa\delta.$$

(3.21)

Now we assume that

$$EV(t,\overline{u}(t),\gamma(t)) \leq \kappa\lambda\delta, \quad t_{m-1} \leq t < t_m$$

$$EV(t_m,\overline{u}(t_m),\gamma(t_m),\gamma(t_m)) \leq \kappa\delta$$

(3.22)

and then prove

$$EV(t,\overline{u}(t),\gamma(t)) \leq \kappa\lambda\delta, \quad t_m \leq t < t_{m+1},$$

$$EV(t_m+1,\overline{u}(t_m+1),\gamma(t_m+1)) \leq \kappa\delta.$$

(3.23)

If not, there must exist some $\tau \in (t_m, t_{m+1})$ such that

$$EV(\tau,\overline{u}(\tau),\gamma(\tau)) > \kappa\lambda\delta > \kappa\delta \geq EV(t_m,\overline{u}(t_m),\gamma(t_m),\gamma(t_m)).$$

(3.24)

Let

$$\tau_1 = \inf\{\tau \in (t_m, t_{m+1}) \mid EV(\tau,\overline{u}(\tau),\gamma(\tau)) > \kappa\lambda\delta\}.$$

(3.25)

Since $EV(t,\overline{u}(t),\gamma(t))$ is continuous in $[t_m,\tau_1]$, there exists $\tau_2 \in (t_m,\tau_1)$ satisfying

$$EV(\tau_2,\overline{u}(\tau_2),\gamma(\tau_2)) = \kappa\lambda\delta,$$

$$EV(t,\overline{u}(t),\gamma(t)) \leq \kappa\lambda\delta, \quad t \in [t_m,\tau_2],$$

$$D^+ EV(\tau_2,\overline{u}(\tau_2),\gamma(\tau_2)) > 0.$$

(3.26)

Because of $EV(\tau_2, \overline{u}(\tau_2), \gamma(\tau_2)) = \kappa\lambda\delta > \kappa\delta$ and $EV(t_m, \overline{u}(t_m), \gamma(t_m)) \leq \kappa\varepsilon$, there is $\tau_3 \in [t_m, \tau_2)$ such that

$$EV(\tau_3, \overline{u}(\tau_3), \gamma(\tau_3)) = \kappa\delta,$$

$$EV(t, \overline{u}(t), \gamma(t)) \leq \kappa\delta, \quad t \in [\tau_3, \tau_2]. \tag{3.27}$$

$$D^+EV(\tau_3, \overline{u}(\tau_3), \gamma(\tau_3)) > 0.$$

Noticing $D^+EV(t, \overline{u}(t), \gamma(t)) \leq \beta EV(t, \overline{u}(t), \gamma(t))$, we obtain

$$\frac{D^+EV(t, \overline{u}(t), \gamma(t))}{EV(t, \overline{u}(t), \gamma(t))} \leq \beta. \tag{3.28}$$

Integrating both sides of (3.28) on $t \in [\tau_3, \tau_2]$, we claim that

$$\int_{\tau_3}^{\tau_2} \frac{D^+EV(s, \overline{u}(s), \gamma(s))}{EV(s, \overline{u}(s), \gamma(s))} ds \leq \int_{\tau_3}^{\tau_2} \beta \, ds < \beta(t_{m+1} - t_m) < \ln\lambda. \tag{3.29}$$

However,

$$\int_{\tau_3}^{\tau_2} \frac{D^+EV(s, \overline{u}(s), \gamma(s))}{EV(s, \overline{u}(s), \gamma(s))} ds = \int_{EV(\tau_3, \overline{u}(\tau_3), \gamma(\tau_3))}^{EV(\tau_2, \overline{u}(\tau_2), \gamma(\tau_2))} \frac{d\eta}{\eta} = \int_{\kappa\delta}^{\kappa\lambda\delta} \frac{d\eta}{\eta} = \ln(\kappa\lambda\delta) - \ln(\kappa\delta) = \ln\lambda.$$
$$\tag{3.30}$$

This leads to a contradiction. Then, we have

$$EV(t, \overline{u}(t), \gamma(t)) \leq \kappa\lambda\delta, \quad t_m \leq t < t_{m+1}. \tag{3.31}$$

Moreover,

$$EV(t_{m+1}, \overline{u}(t_{m+1}), \gamma(t_{m+1})) \leq E\kappa\overline{u}(t_{m+1}) = \kappa E\|H_{m+1}(u(t_{m+1}^-, x))\|_G^2 \leq \kappa\Gamma_{m+1}^2 E\|u(t_{m+1}^-, x)\|_G^2$$

$$\leq \frac{\kappa\Gamma_{m+1}^2}{\alpha} EV(t_{m+1}, \overline{u}(t_{m+1}^-), \gamma(t_{m+1})) \leq \frac{\kappa\lambda\delta}{\lambda_{m+1}} \leq \kappa\delta. \tag{3.32}$$

Therefore,

$$EV(t, \overline{u}(t), \gamma(t)) \leq \kappa\lambda\delta, \quad t \geq 0, \tag{3.33}$$

which results in

$$\alpha E\overline{u}(t) \leq EV(t, \overline{u}(t), \gamma(t)) \leq \kappa\lambda\delta, \quad t \geq 0 \tag{3.34}$$

Namely,

$$E\left\{\left\|u(t,x;0,\varphi)\right\|_G^2\right\} < \varepsilon, \quad t \geq 0. \tag{3.35}$$

This ends the proof of Theorem 3.1. □

As an application, we consider a class of semilinear impulsive stochastic reaction-diffusion equations with Markovian switching as follows:

$$du(t,x) = \left[C(\gamma(t))\Delta u(t,x) + A(\gamma(t))u(t,x)\right]dt$$
$$+ \sigma(t,x,u,\gamma(t))dw(t), \quad t \geq 0,\ t \neq t_k,\ x \in G, \tag{3.36}$$

$$u(t_k,x) = H_k\left(u(t_k^-,x),\gamma(t_k)\right), \quad x \in G, \tag{3.37}$$

with boundary condition

$$\left.\frac{\partial u}{\partial N}\right|_{\partial G} = 0, \quad t \geq 0 \tag{3.38}$$

and initial condition

$$u(0,x,i_0) = \varphi(x), \quad x \in G,\ i_0 \in \mathbb{S} \tag{3.39}$$

where $C(\gamma(t)) = \mathrm{diag}\{c_1(\gamma(t)),\ldots,c_n(\gamma(t))\} = \mathrm{diag}\{c_{i1},\ldots,c_{in}\}$ with $c_{ij} \geq 0$ for $i \in \mathbb{S}$, $j = 1,2,\ldots,n$. $A(\gamma(t)) = A_i = (a_{lj}(\gamma(t)))_{n\times n} = (a_{lj}^{(i)})_{n\times n}$, $i \in \mathbb{S}$, is matrices. The remainder of system (3.36)–(3.39) is the same as that defined in system (2.1)–(2.4).

Theorem 3.2. *Assume that*

(A5) $\lambda > 1$, *where* $\lambda = \inf\{\lambda_k \mid \lambda_k = 1/\Gamma_k^2, k = 1,2,\ldots\}$,

(A6) $2\max_i\{\lambda_{\max}(A_i)\}(t_k - t_{k-1}) < \ln\lambda$, $k = 1,2\ldots,\ i \in \mathbb{S}$.

Then, the trivial solution $u = 0$ of system (3.36)–(3.39) *is stable in mean square.*

Proof. Construct a Lyapunov function $V(t,\bar{u}(t),\gamma(t)) = \bar{u}(t)$, and compute the operator $\mathscr{L}V(t,\bar{u}(t),\gamma(t))$ that

$$\mathscr{L}V(t,\bar{u}(t),\gamma(t)) = \int_G 2u^{\mathsf{T}}C(\gamma(t))\Delta u\,dx + \int_G 2u^{\mathsf{T}}A(\gamma(t))u(t,x)dx. \tag{3.40}$$

By Green formula, we get

$$\int_G 2u^\mathsf{T} C_i \Delta u \, dx = 2 \int_G \sum_{j=1}^n u_j c_{ij} \Delta u_j dx = 2 \sum_{j=1}^n \int_G u_j c_{ij} \Delta u_j dx$$

$$= 2 \sum_{j=1}^n \int_{\partial\Omega} u_j c_{ij} \frac{\partial u_j}{\partial N} ds - 2 \sum_{j=1}^n \int_G c_{ij} \nabla u_j \cdot \nabla u_j dx. \tag{3.41}$$

It follows from boundary condition that

$$\sum_{j=1}^n \int_{\partial\Omega} u_j c_{ij} \frac{\partial u_j}{\partial N} ds = 0. \tag{3.42}$$

Thus,

$$\int_G 2u^\mathsf{T} C_i \Delta u \, dx = -2 \sum_{j=1}^n \int_G c_{ij} \nabla u_j \cdot \nabla u_j dx \le 0. \tag{3.43}$$

Therefore,

$$\mathrm{L}V(t, \overline{u}(t), \gamma(t)) \le 2 \int_G u^\mathsf{T} A(\gamma(t)) u(t, x) dx \le 2 \max_i \{\lambda_{\max}(A_i)\} \overline{u}(t). \tag{3.44}$$

According to Theorem 3.1, we find that the trivial solution of system (3.36)–(3.39) is stable in mean square. □

4. Example

Consider the following two dimension Markovian jumping impulsive stochastic reaction diffusion systems with two modes. The parameters are given as follows: Let $|G| = 1/8$, when $r(t) = 1$, we have

$$\begin{pmatrix} du_1(t, x) \\ du_2(t, x) \end{pmatrix} = \begin{pmatrix} 3.4 & 0 \\ 0 & 5.5 \end{pmatrix} \begin{pmatrix} \Delta u_1(t, x) \\ \Delta u_2(t, x) \end{pmatrix} + \begin{pmatrix} 2.6 & 0 \\ 0 & 1.7 \end{pmatrix} \begin{pmatrix} u_1(t, x) \\ u_2(t, x) \end{pmatrix}$$

$$+ \begin{pmatrix} 2u_1(t, x) \\ -u_2(t, x) \end{pmatrix} dw(t), \quad t \ge 0, \ t \ne t_k,$$

$$\begin{pmatrix} u_1(t_k, x) \\ u_2(t_k, x) \end{pmatrix} = e^{-0.1k} \begin{pmatrix} 0.5 & -0.2 \\ 0.3 & 0.6 \end{pmatrix} \begin{pmatrix} u_1(t_k^-, x) \\ u_2(t_k^-, x) \end{pmatrix}, \tag{4.1}$$

when $r(t) = 2$

$$\begin{pmatrix} du_1(t,x) \\ du_2(t,x) \end{pmatrix} = \begin{pmatrix} 2.4 & 0 \\ 0 & 2.5 \end{pmatrix} \begin{pmatrix} \Delta u_1(t,x) \\ \Delta u_2(t,x) \end{pmatrix} + \begin{pmatrix} 1.6 & 0 \\ 0 & 2.7 \end{pmatrix} \begin{pmatrix} u_1(t,x) \\ u_2(t,x) \end{pmatrix}$$
$$+ \begin{pmatrix} 2u_1(t,x) \\ -u_2(t,x) \end{pmatrix} dw(t), \quad t \geq 0, \ t \neq t_k, \tag{4.2}$$
$$\begin{pmatrix} u_1(t_k,x) \\ u_2(t_k,x) \end{pmatrix} = e^{-0.1k} \begin{pmatrix} 0.6 & -0.3 \\ 0.3 & 0.4 \end{pmatrix} \begin{pmatrix} u_1(t_k^-,x) \\ u_2(t_k^-,x) \end{pmatrix},$$

where $t_0 = 0, t_k = t_{k-1} + 0.1, \ (k = 1,2,\ldots)$. By simple calculation, we obtain $\Gamma_k = 0.6e^{-0.1k}, \lambda = 2.6 > 1, 2\lambda_{\max}(A_i)(t_k - t_{k-1}) < \ln\lambda = 0.856$. From Theorem 3.2, the trivial solution of this system is stable in mean square.

5. Conclusion

In this paper, we discuss mean square stability of stochastic reaction diffusion equations with Markovian switching and impulsive perturbations, by means of Lyapunov function and stochastic analysis. As an application, we investigate a class of semilinear impulsive stochastic reaction-diffusion equations with Markovian switching and establish the stability criterion. Finally, we provide an example to demonstrate the effectiveness and efficiency of the obtained results.

Acknowledgments

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