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Prism-Based Sunlight Concentrator Layout: A Genetic Algorithm Solution

Solar energy is a promising source of energy because it is abundant and harmless to the environment. One of the critical issues involving solar energy is the layout design of sunlight concentrators. This study presents a computational method for reforming the layout of a special sunlight concentrator that consists of a set of prisms to significantly enhance its light intensity. Sunlight movement toward the prisms is modeled, and a genetic algorithm is applied to find a good concentrator layout. Experiments under various light transmission rates validate the performance of our proposed method. The proposed optimal sunlight concentrator layout improves the intensity of light by 55%. [DOI: 10.1115/1.4025845]

Keywords: genetic algorithm, prisms layout, sunlight concentrator

1 Introduction

Sunlight offers significant energy conservation. Besides, sunlight for illumination has many benefits, such as visual connection with the outdoors as well as good quality and healthy illumination that promote worker productivity [1]. The sunlight concentrator is a key component that introduces sunlight into a solar energy collection system. Although many studies on trough or lens are available, including those conducted by [2–7], only a few studies have been focused on prism layouts. Prismatic elements (prisms) are typical devices for natural light illumination systems used for redirecting and collecting daylight. A prism plays the role of light guide elements, comprising transparent plastic that transmits and reflects converging and reflecting light beams. Prisms possess two functions: guiding the movement of sunlight toward the inside of the concentrator and redirecting light to the guiding fibers to concentrate it with the least light intensity loss through appropriate prism arrangement. Thus, an appropriate prism arrangement can concentrate light efficiently.

A sunlight concentrator comprises four types of successive receiving prisms, and four types of guiding fibers. The receiving prisms reflect sunlight into the concentrator by creating paths that allow the light to penetrate in any direction. The guiding fibers reflect light within the concentrator to facilitate light convergence (details are discussed in Sec. 3.3). This study focuses on the layout design of a sunlight concentrator by developing a special genetic algorithm that considers light movement and exhaustion.

The rest of this paper is organized as follows: Sec. 2 discusses sunlight concentrator problems; Sec. 3 describes the prism layout modeling with light transmission and movement; Sec. 4 illustrates a prism layout genetic algorithm (PLGA) and the corresponding light movement and intensity evaluation modules for solving the sunlight concentrator layout optimization problem; Sec. 5 demonstrates a rapid PLGA convergence to a near-optimal prism layout that outperforms the existing patented layout against different light transmission loss rates; and Sec. 6 provides the conclusion and suggestion for future research.

2 Literature Review

To optimize the design of a sunlight concentrator, several proposals have been made by Ref. [8]. A sunlight concentrator mainly focuses on optimizing a single-component design, such as in the studies of [9,10]. However, the integration of single prisms to create an efficient concentrator is challenging. A certain patented light concentrator has been designed but it involves time-consuming procedures and lacks options for guiding fiber settings [1,11].

The manual concentrator layout design cannot completely resolve the efficiency issue because many variables are involved in such a multidimensional nonlinear problem. Some methods used for continuous functions, such as the method of steepest descent and Newton's method, cannot be used either in such a problem because integer variables need to be considered. Furthermore, the branch and bound method were originally employed for discrete variables, but its determination of the global optimum is guaranteed only in linear or convex problems [12].

Soft-computing is a promising solution for sunlight concentrator layout problems. Notably, the genetic algorithm (GA) is a near-optimization algorithm and computerized search based on the mechanics of natural genetics and natural selection. GA and traditional methods of optimization are distinctly different. First, GA is less likely to be trapped at a local optimum. Second, GA corresponds to the chromosomes in natural genetics. Thus, this search method is naturally applicable for solving discrete, integer programming problems and for avoiding computationally expensive solutions [13]. In addition, GA has a sufficient probability of finding global optimization by using mutations that are not time consuming [14,15]. Therefore, GA can be employed to improve the light intensity in the current sunlight concentrator layout.

3 Modeling of Optimal Sunlight Concentrator Layout

The angle of departure of a light beam that passes through the prism relies on the angle of arrival. Light intensity losses are assessed by air gaps. The light intensity of a beam that passes through prisms is reduced before it approaches the exit [16]. Such problem can be modeled by considering each prism location in the concentrator as an integer variable that contains prism position information. Thus, the layout, which is defined as an integer variable matrix with a size of $[M \times N]$ (M rows and N columns of prisms in a concentrator layout), needs to be optimized.

3.1 Snell's Law. Snell's law, also known as the Snell–Descartes law and the law of refraction, describes the

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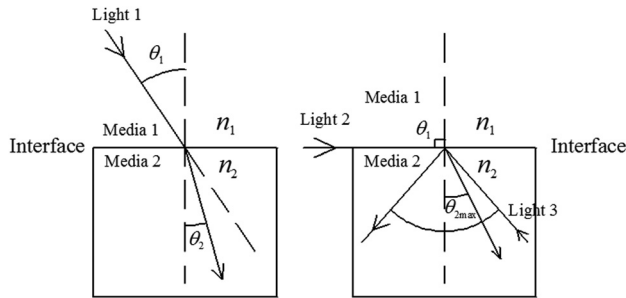


Fig. 1 Refraction of light at the interface between two different media ($n_2 > n_1$)

relationship between the angles of incidence and refraction when referring to light or other waves passing through a boundary between two different isotropic media [17]. The law states that the ratio of the sines of the angles of incidence and refraction is equivalent to the reciprocal of the ratio of the indices of refraction, as shown below (Fig. 1).

$$\text{Rule1: } \frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} \quad (1)$$

θ corresponds to the angles of incidence and departure, whereas n represents the refractive index of the respective media. Notably, if the angle of incident light is greater than $\theta_{2\max}$, the light will be reflected at the interface.

Sunlight is transmitted from the air through the glass or plastic prism in accordance with Snell's law because the sunlight passes through two different media. Therefore, with the appropriate prism arrangement, the concentrator can receive, orient, and converge sunlight (Fig. 2).

Figure 2 illustrates that the sunlight received by prism 1 is oriented by prisms 2–5. Then, the sunlight received by prism 2 is oriented by prisms 3–5. Therefore, the sunlight received by prisms 1 and 2 is converged at the outlet of prism 5. Hence, this prism arrangement receives, orients, and converges sunlight.

3.2 Transmission Loss. This study does not consider absorption coefficient (i.e., extinction coefficient) of the prism material in the analysis. When sunlight passes through the prisms, its light intensity is reduced. A theoretical study on the disturbance of transmission intensity in the prism has proven that the distribution of intensity in the reflected and transmitted beams depends on the refraction index, frequency, polarization, and angle of the sunlight [18]. Sunlight intensity decreases moderately when the beam passes through different materials. Thus, air gap is considered to have a significant effect on sunlight intensity loss, given the polarization and refraction. Figure 2 shows an example of an air gap. When sunlight is received by prism 1, and it is transmitted to prism 2, an air gap exists between the two prisms. Thus, sunlight intensity is reduced as the beam goes through different media. Meanwhile, sunlight is transmitted from prism 2 to 3, 4, and 5, in sequence, and its intensity is maintained as it travels inside the prism material only. Thus, an appropriate prism arrangement should not allow air gaps. To be more specific, the rate of sunlight intensity loss in each air gap is defined as the ratio of sunlight intensity lost and sunlight intensity received. For example, in Fig. 2, the sunlight intensity received by prism 1 is r_1 , and the sunlight intensity received by prism 2 from prism 1 is r_2 . Thus, the loss rate of sunlight intensity in the air gap is defined below.

$$\begin{aligned} \text{Rule2: Loss rate(\%)} &= \frac{r_1 - r_2}{r_1} \times 100\% = 100 - \frac{r_2}{r_1} \times 100 \\ &= 100 - \text{LTR} \end{aligned} \quad (2)$$

where the light transmission rate (LTR) is $(r_2/r_1) \times 100$.

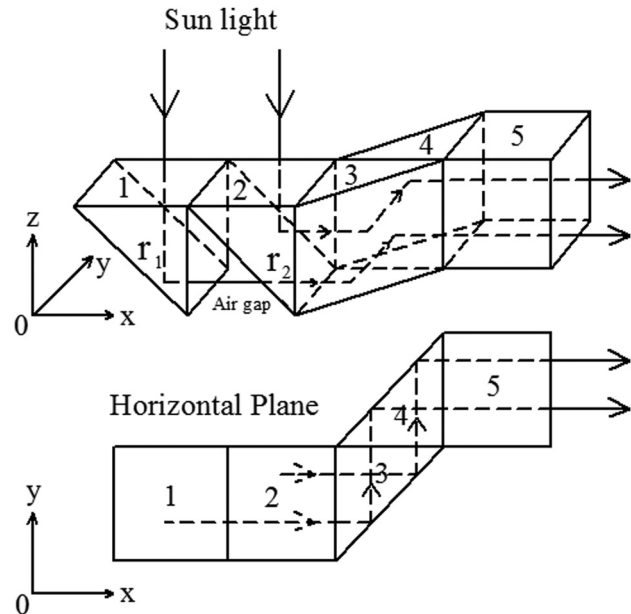


Fig. 2 An example of prisms arrangement to receive, orient and converge sunlight

The loss rate is typically in the range of 20–50%, depending on prism design and air gap thickness. Thus, LTR is 80% to 50%.

3.3 Layout Modeling. Although prism arrangement can help receive, orient, and converge optimum sunlight intensity, layout arrangement cannot be designed arbitrarily because of manufacturability. Hence, the prisms should be shaped like a “brick” after they are arranged together. To be more specific, a sunlight concentrator is a rectangular plate with a size of $M \times N$ square units, in which the prisms are scattered. The concentrator comprises various components, such as guiding fibers, successive receiving prisms, and paths. These components, along with their identification numbers, and the redirection angle response of the angle of incident sunlight, which is considered as the third rule of sunlight intensity movement, are shown in Table 1. First, if the sunlight moves vertically toward the guiding fibers, which orient the sunlight toward the sunlight concentrator for convergence, it will be lost because it is not oriented inside the concentrator. If the sunlight comes from other directions, such as Forward (F), Backward (B), Left (L), or Right (R), it will be reflected or will just pass through to any direction depending on the incidence angle (Rule 1) and the type of prism (identifier). Second, if the sunlight moves vertically toward the receiving fiber, which receives the sunlight and orients it toward the concentrator, it will be reflected to any direction; the sunlight will be lost if it comes from another direction. The columns “Direction of Incident Light” and “Direction of Departure Light” provide brief information regarding the performance of the components.

Sunlight that passes through each receiving prism can exit the concentrator after passing through various prisms with transmission losses. In this study, the target concentrator contained two particular exits oriented at positions (1,1) (Exit 1) with forward movement and $(M \times N)$ (Exit 2) with backward movement.

Light intensity is calculated after the sunlight passes through the receiving fiber and goes into the guiding fibers with regulated directions, as shown in Table 1. The light is then subjected to the LTR as defined in Eq. (2), ending at the particular exit with the oriented direction. To be more specific, the function returns a value of zero when the light beam is unable to arrive at either exit 1 with forward direction or exit 2 with backward direction. This function is described by $BR(x(i, j))$, where $x(i, j)$ is the component number (identifier) at a checked location (i, j) . Therefore, the total

Table 1 Rule 3: Redirection angle response of sunlight to angle of incident sunlight

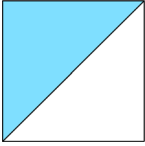
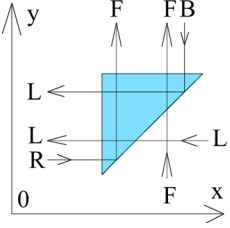
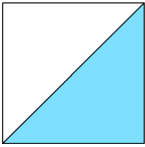
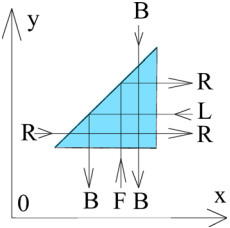
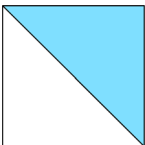
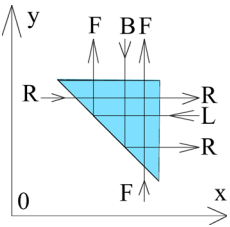
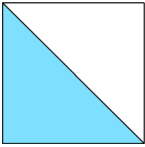
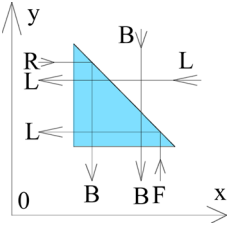
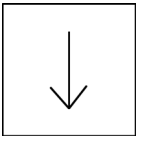
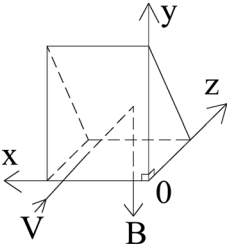
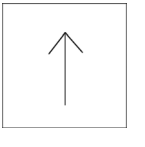
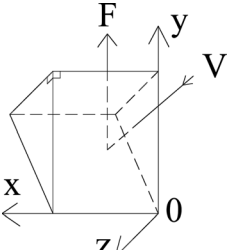
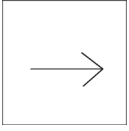
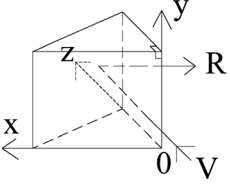
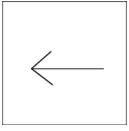
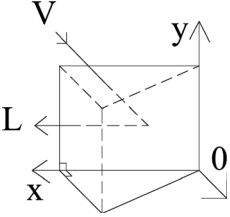

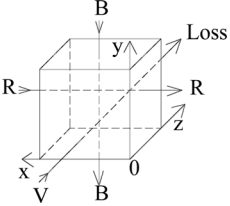
Components	Identifier	Symbol	Visual performance	Direction of incident light	Direction of departure light
Guiding fiber	1			Forward (F) Backward (B) Left (L) Right (R) Vertical (V)	Forward (F) Left (L) Left (L) Forward (F) Loss
	2			Forward Backward Left Right Vertical	Right Backward Backward Right Loss
	3			Forward Backward Left Right Vertical	Forward Right Forward Right Loss
	4			Forward Backward Left Right Vertical	Left Backward Left Backward Loss
Receiving prism	5			Forward Backward Left Right Vertical	Loss Loss Loss Loss Backward
	6			Forward Backward Left Right Vertical	Loss Loss Loss Loss Forward

Table 1 Continued

Components	Identifier	Symbol	Visual performance	Direction of incident light	Direction of departure light
	7			Forward Backward Left Right Vertical	Loss Loss Loss Loss Right
	8			Forward Backward Left Right Vertical	Loss Loss Loss Loss Left
Path	9			Forward Backward Left Right Vertical	Forward Backward Left Right Loss

```

function calculate the total light intensity of a sunlight
concentrator
Input x;
% x is matrix size [M x N] of component number of prism ...
of sunlight concentrator
Round x;
% x is rounded because x is integer variable matrix (Identifier)
Set total light intensity to 0;
for i equal 1 to M
    for j equal 1 to N
        if (x(i,j) is receiving fiber)
            % x(i,j)=5,6,7 or 8.
            Starting with the moving of vertical sunlight going
            from solar into a sunlight concentrator via receiving
            fiber at location (i,j);
            Set ef equal to col;
            % ef is the light intensity after passing the each
            prism
            % col is the light intensity after passing an initial
            receiving fiber.
            while (light cannot get out concentrator)and(there is
            not a looped beam inside the sunlight
            concentrator)and(light can go to next prism)do
                Update location of next prism which sunlight is
                going to;
                Update direction which sunlight is going to the
                prism above;
                % the next location and direction of sunlight
                are complied by the rule 1 and 3.
                Update light intensity (ef) which is going to
                the prism above;
                % the light intensity is complied by rule 2.
            end while
            if (sunlight can get out at exit 1 and exit 2
            with forward and backward direction
            respectively)Adding the light intensity(ef) to
            total light intensity;
        end if
    end if
end for
end for
Return value of total light intensity to the value of function;
end function

```

Fig. 3 Logic of building the total light intensity function

light intensity that can be obtained from a sunlight concentrator is the total of the individual light beams determined by Eq. (3). The model for the optimal sunlight concentrator is defined as follows:

```

function genetic algorithm of prism layout
Creating randomly an initial population;
while (there is not termination criteria)do
    evaluate each individual's fitness;
    %Score each member of the current population by computing its
    fitness value
    prune population;
    %Scales the raw fitness scores to convert them into a more
    usable range of values
    select pairs to mate from best fitness(by the logic described
    in Figure 3) of individuals;
    %Selecting the parents relied on their fitness. The
    individual that have lower fitness, are passed to the next
    population
    replenish population;
    % employing selected pairs by mutation or crossover
    replaces the current population with children to form the
    next generation;
    check for termination criteria;
    % generations, time limit, fitness limit, stall generations,
    stall time limit, function tolerance
end while
return best children and fitness value to function;
end function

```

Fig. 4 PLGA structure of prism layout in pseudo-code

$$\text{Maximize } \sum_{i=1}^M \sum_{j=1}^N BR(x(i,j)) \quad (3)$$

Subject to

$$1 \leq x(i,j) \leq 9, x(i,j), \text{ is integer variable} \quad (4)$$

$$1 \leq i \leq M, \quad 1 \leq j \leq N \quad (5)$$

All light paths follow the rules 1, 2, and 3 as shown in Eqs. (1), (2), and defined in Table 1, respectively.

4 A Genetic Algorithm Solution

4.1 Prism Layout Genetic Algorithm. The proposed PLGA for generating a good sunlight concentrator layout extends a regular evolution procedure and employs the notion of light

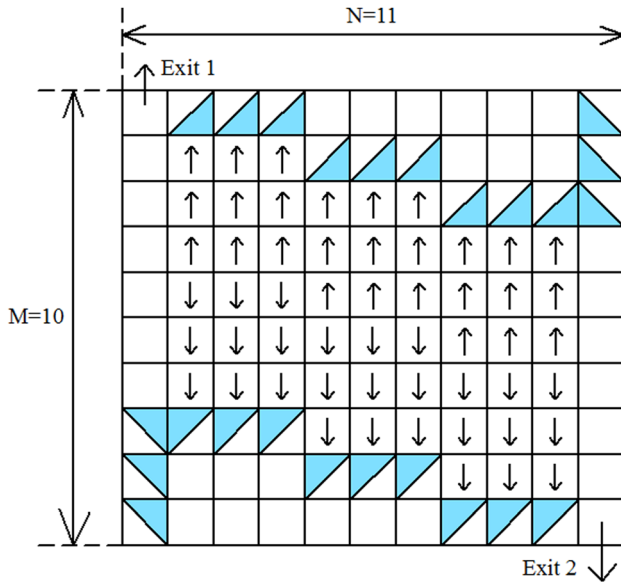


Fig. 5 The existing patent of sunlight concentrator layout

transmission. The PLGA is composed of two components: the procedures and the chromosome operators. Figure 3 shows how the total light intensity obtained from a sunlight concentrator is computed. The PLGA for the prism layout is shown in Fig. 4.

4.2 The Operators of PLGA. The PLGA population is operated by the three operators: reproduction, crossover, and mutation, to produce a new population that is further evaluated to determine the fitness value and to check for convergence [19]. The proposed PLGA is implemented using MATLAB. A random initial population with a uniform distribution is created. In addition, reproduction is the operation applied to the population in selecting the good chromosomes of the population to form a mating pool. Specifically, the stochastic uniform method is chosen as the selection function because it is very fast [20]. The crossover operator is implemented after reproduction to produce new chromosomes by exchanging information among the chromosomes in the mating pool. The PLGA applies single-point, two-point, and uniform crossovers [21,22]. Then, the mutation operator is employed to the new chromosomes with a specific and small mutation probability.

5 Experiments

5.1 Result by PLGA. The existing layout patent of the sunlight concentrator in Fig. 5 developed by Whang et al. [11] has

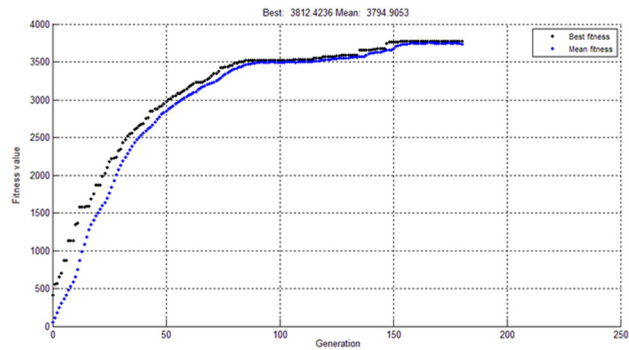


Fig. 6 Performance convergence of PLGA

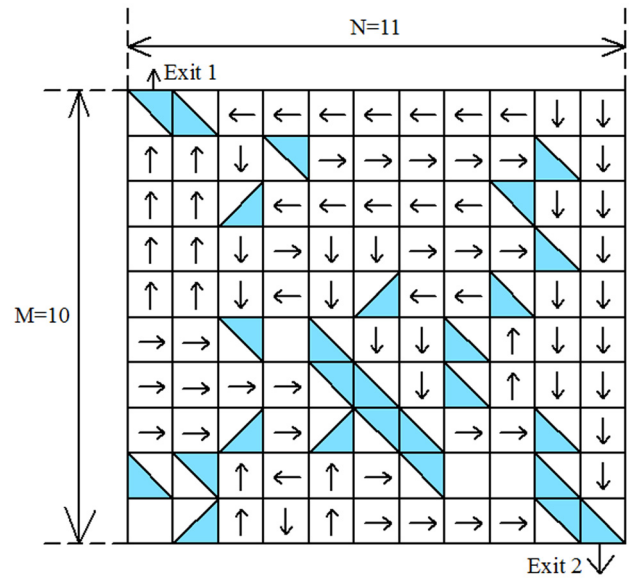


Fig. 7 The resulting layout by PLGA (LTR = 0.8, BFV = 3812)

been adopted in practice. Computational results of PLGA are obtained to develop an optimal sunlight concentrator layout that maximizes the light intensity of the concentrator.

PLGA experiments were conducted to examine the efficiency of our proposed model under different settings of algorithmic parameters in the case of the panel size (10 × 11). Given that different settings generate different results, we searched for the optimal settings of the parameters. Table 2 shows that at 80% LTR (the effect of LTR on result optimization is discussed in Sec. 5.2), the

Table 2 The total light intensity of sensitivity experiments on PLGA parameters (LTR = 80%)

GA parameters	Crossover 0.85			Crossover 0.90			Crossover 0.95		
	Mutation 0.01	Mutation 0.03	Mutation 0.05	Mutation 0.01	Mutation 0.03	Mutation 0.05	Mutation 0.01	Mutation 0.03	Mutation 0.05
Popu-size 600									
BFV—mean:	2874	3270	2784	2854	3206	3032	3009	2872	2896
2977—STD: 167									
CPU time—mean:511—STD: 114	450	484	418	521	754	570	575	469	359
Popu-size 800									
BFV—mean:	3704	3440	3416	2992	3505	3102	2919	2935	2950
3218—STD: 299									
CPU time—mean:660—STD:168	712	759	689	808	942	451	515	617	449
Popu-size 1000									
BFV—mean:	3579	3812	3379	3028	3299	3120	3094	2952	3246
3279—STD: 277									
CPU time—mean:782—STD:175	605	1018	831	747	792	1044	514	667	820

Table 3 Comparison of light intensity of PLGA-produced and an existed patented sunlight concentrators

Light transmission rate (LTR) (%)	Layout by PLGA (a)	Existing patented layout (b)	Light intensity improvement by PLGA ((a - b)/b)
LTR = 100 (no transmission loss)	7400	5400	37%
LTR = 80	3812	2905	31%
LTR = 70	2571	2100	22%
LTR = 60	1897	1506	26%
LTR = 50	1659	1072	55%

highest light intensity was recorded when the population size, mutation, and crossover rate values were 1000, 0.03, and 0.85, respectively. Each experiment was executed on a personal computer with Intel Core i7-2600 3.40 GHz and 4.0 GB RAM. In general, the experiments respectively took 511, 660, and 782 CPU-SEC on the average to perform under various parameter settings. The best fitness values (BFV) were 2977, 3218, and 3279 on the average, respectively. Among the settings, a population size of 1000, a crossover rate of 0.85, and a mutation rate of 0.03 generated the highest BFV.

In addition, Fig. 6 shows the fitness evolution of the best layout in Table 2, which has two indices, the best and the mean fitness. The best fitness plots the best function value, whereas the mean fitness illustrates the average of all fitness values in each generation. As shown in Fig. 6, both indices continuously converge toward steady-state after running PLGA for up to 150 generations. Figure 7 shows the sunlight concentrator layout suggested by the PLGA.

5.2 New Layout Assessment for Light Intensity Improvement. To compare the existing sunlight concentrator layout with the design proposed by PLGA, their light intensities were measured, as shown in Table 3. The light intensity of the proposed concentrator layout by PLGA was improved by up to 55% against different light transmission rates. Such new layout provides a significant improvement in the sunlight concentration efficiency as compared with that of the existing patented sunlight concentrator layout.

6 Conclusion

PLGA, a new algorithm, was designed in this study to enhance the light intensity of a sunlight concentrator. Compared with previous studies that focused on the mechanism design of a single optical device/prism, the current study focused on improving the efficiency of the entire sunlight concentrator layout. Experiments with different settings were performed in this study to find the best algorithmic parameters. The proposed PLGA produced a better sunlight concentrator layout as compared with the existing patented layout against different loss rate scenarios. Experiment results show that the PLGA-produced layout significantly outperforms the existing design by up to 55%. Future research will involve the improvement of the proposed algorithm by using

massive/parallel computing and considering a nonrectangular sunlight concentrator layout.

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