

# CALIBRATION OF 2-DOF PARALLEL MECHANISM

O. Sato, M. Hiraki, K. Takamasu and S. Ozono

*Department of Precision Engineering, The University of Tokyo,  
7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-8656, Japan  
Tel: +81-3-5841-6472, FAX: +81-3-5841-8556  
e-mail: ri@nano.pe.u-tokyo.ac.jp*

## Abstract

In order to avoid some drawbacks of traditional Coordinate measuring machine (CMM) based on serial mechanism, we are developing Parallel-CMM based on a parallel mechanism. We have already built the prototype of 3-DOF Parallel CMM and now we are researching about the calibration of our Parallel CMM. When we use a CMM to measure the objects, we need to calibrate the geometrical parameters of the CMM to evaluate the uncertainty of measurement. In this paper, we discuss the details of calibration for 2-DOF parallel mechanism instead of 3-DOF parallel mechanism.

## Keywords

Parallel mechanism, Calibration, Coordinate Measuring Machine

## 1. INTRODUCTION

We are developing Parallel-CMM (Coordinate Measuring Machine) based on a parallel mechanism [HIRAKI 1997]. In parallel mechanism, the base unit and end-effector are connected by many links in parallel. The advantages of parallel mechanism are its robustness against external force and error accumulation. We have already built the prototype of 3-DOF Parallel CMM shown in Figure 1 [HIRAKI 1999], and now we are researching about the calibration of our Parallel CMM.

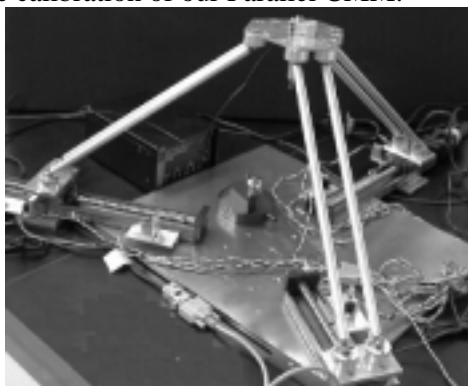


Figure 1 Prototype of Parallel CMM

If the end-effector move in parallel to the base unit, each pair of the rods can be replaced by one rod and three virtual rods are regarded as sides of trigonal pyramid, which upper vertex is the center of end-effector. Figure 2 shows the virtual link model. When we use a CMM to measure objects, we need to calibrate geometrical parameters of the CMM to evaluate the uncertainty of measurement. For this prototype, we need to identify parameters shown in Fig.2.

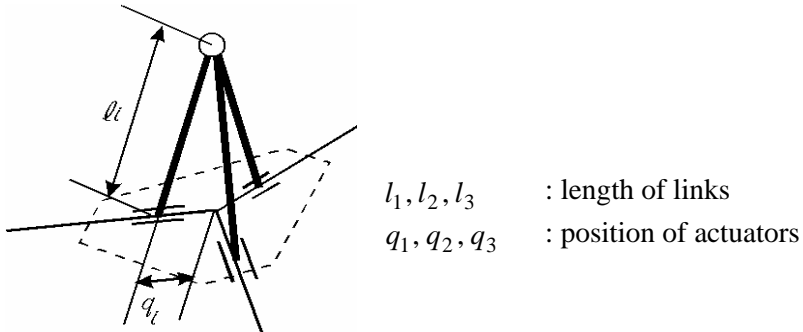


Figure 2 Geometrical model of Parallel CMM

There are some methods for identifying both the geometrical and kinematical parameters. One is calculating the parameters by measuring artifacts whose size is known. To identify the parameters efficiently with high accuracy, what artifact we should measure i.e. what the sets of measured points we should get? Now we think about the calibration of 2-DOF parallel mechanism like Figure 3 to discuss about the above questions. This mechanism has two actuators ( $q_1, q_2$ ), two links ( $l_1, l_2$ ), and one end-effector ( $P$ ).

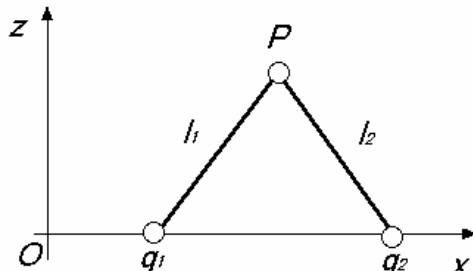


Figure 3 2-DOF parallel mechanism

Through the simulation to calibrate the geometrical parameters for the parallel mechanism, we got the basic knowledge about the way to calibrate the Parallel CMM.

## 2. KINEMATICS AND PARAMETER IDENTIFICATION

Forward kinematics of the model in Figure 3 can be solved analytically as follow equations.

$$x = \frac{q_2^2 - q_1^2 - l_2^2 + l_1^2}{2(q_2 - q_1)} = \frac{q_1 + q_2}{2} - \frac{2l\Delta l}{q_2 - q_1}$$

$$z = \frac{1}{2} \sqrt{-\frac{16l^2 \Delta l^2 - 4(l^2 + \Delta l^2)(q_2 - q_1)^2 + (q_2 - q_1)^4}{(q_2 - q_1)^2}} \quad (1)$$

$$(l_1 = l + \Delta l, l_2 = l - \Delta l)$$

Here, the kinematical parameters are calculated using the result of measure by other measuring system. The parameter identification model is shown in Figure 4. Two coordinate planes  $x_M - z_M$  and  $x_P - z_P$  are the measuring machine's and the parallel mechanism's respectively.

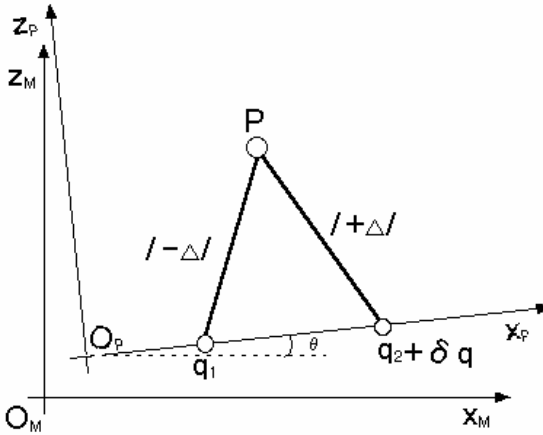


Figure 4 Parameter identification model for 2-DOF mechanism

Kinematical parameters we need to identify are the length of the links ( $l_1, l_2$ ), the offset value between encoders ( $\delta q$ ), and the parameters by the coordinates transforming from  $x_P - z_P$  to  $x_M - z_M$  ( $x_m, z_m$  and  $\theta$ ).

We identified these parameters in simulation. The condition was follows.

|                                |          |
|--------------------------------|----------|
| The number of measured points  | :20      |
| The error of measuring machine | :0.01 mm |
| The error of encoder           | :0.02 mm |

Under this condition, we arranged the measured points under some rules.

The good way to identify parameters with high accuracy is arranging as  $q_1 + q_2$  is constants (Figure 5).

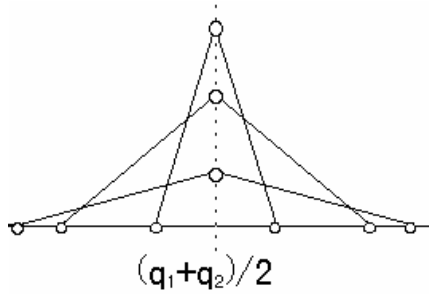


Figure 5 Measured points arranged as  $q_1 + q_2$  is constants.

We got worse result of simulations under other rules by arranging measured points than above rule. The reason  $q_1 + q_2$  constant arrangement gives better result than others is it gives smaller condition number of the pseudo-inverse matrix of Jacobian than other arrangements give.

Table 1 shows the identified parameters when the measured points were arranged as  $q_1 + q_2$  is constants. It shows kinematic parameters are identified with high accuracy.

Table 1 Result of simulation: The measured points were arranged as  $q_1 + q_2$  is constants. The measuring error is regarded as  $20 \mu m$ .

|            | Set values | Identified value |
|------------|------------|------------------|
| $l$        | 100.5550   | 100.5709         |
| $\Delta l$ | 0.6130     | 0.6090           |
| $\delta q$ | 0.9480     | 0.9568           |
| $x_m$      | -0.1720    | -0.1511          |
| $z_m$      | -1.1130    | -1.1314          |
| $\theta$   | -0.00822   | -0.00770         |

( unit : mm and rad )

After the calibration of the kinematic parameters, we evaluated uncertainty of the position of end-effector. Figure 6 and Table 2 show estimated error of positions of the end-effector. The error of positions after calibration is at most  $10 \mu m$ . The minimum error can be achieved when this system uses the sets of measured points arranged as  $q_1 + q_2$  is constants.

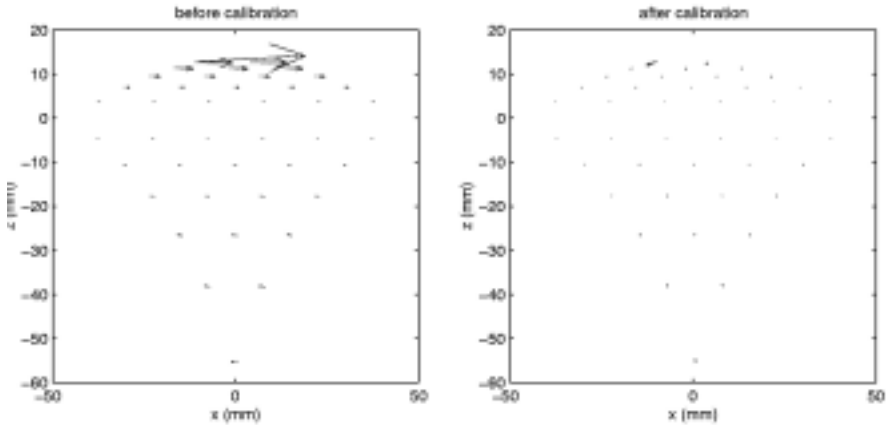


Figure 6 Uncertainty of the position of end-effector

Table 2 Error of positions: The maximum error of positions estimated before calibration and after calibration.

| Direction | Before calibration | After calibration |
|-----------|--------------------|-------------------|
| $x$       | 0.574              | 0.016             |
| $z$       | 0.466              | 0.006             |

(unit : mm )

### 3. CONCLUSION AND FURTHER STYDY

Through the simulation of calibration of 2-DOF parallel mechanism, we got the knowledge about what the sets of measured points should be got to identify kinematic parameters with high accuracy. The best one is to get the measured points arranged as  $q_1 + q_2$  is constants, in other words, make the condition number of the pseudo-inverse matrix of Jacobian small.

Next, we need to try the identification of the kinematical parameters of 3-DOF Parallel CMM with the sets of measured points that gives small condition number of the pseudo-inverse matrix of Jacobian.

### 4. REFERENCES

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