Optimization method for the determination of material parameters in damaged composite structures

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Abstract

An optimization method to identify the material parameters of composite structures using an inverse method is proposed. This methodology compares experimental results with their numerical reproduction using the finite element method in order to obtain an estimation of the error between the results. This error estimation is then used by an evolutionary optimizer to determine, in an iterative process, the value of the material parameters which result in the best numerical fit. The novelty of the method is in the coupling between the simple genetic algorithm and the mixing theory used to numerically reproduce the composite behavior. The methodology proposed has been validated through a simple example which illustrates the exploitability of the method in relation to the modeling of damaged composite structures.

Keywords: composites, composite structures, parameter identification, inverse method, finite element method, mixing theory, optimization, genetic algorithm

1. Introduction

The structural use of composite materials is widespread in many fields, including civil infrastructures and the aerospace, automotive and marine industries [1]. The high strength-to-weight and stiffness-to-weight ratios of these materials, in addition to their corrosion resistance and thermal stability, make them well suited for structural applications in which weight reduction is a priority.

Composite materials are made of two or more simple materials or components, typically exhibiting the best qualities of these components and, often, superior properties to those of the individual components alone [2]. In general, composites are designed to meet certain structural needs. The determination of the overall behavior of the composite material is key to the design process. Representing the composite as a single orthotropic material with the averaged properties of the whole set has proven unsatisfactory. The main drawback of this approach is that it cannot capture correctly the behavior of the composite if one or more of its components exceeds the elastic limits [3]. Hence, composite materials need to be modeled using theories that allow taking into account the behavior of the simple materials, which can be quite diverse and include anisotropy, plasticity and damage, among other characteristics. One of the most commonly used is the mixing theory [4], whose general theoretical framework was initially developed by Truesdell and Toupin [5].

The classical mixing theory explains the behavior of a composite material according to the interaction between the components of the composite. It is based on the hypotheses that all components suffer the same strains and each component contributes to the behavior of the composite in the same proportion as their volumetric participation. Each component is a material in itself whose individual behavior can be represented by its own constitutive law. Thus, the mixing theory can be considered a constitutive equation manager. This behavior combination technique allows preserving the original constitutive law of each component, which is especially useful when studying composite structures with the Finite Element Method (FEM).

Finite element analysis has proven to be an extremely useful tool in the design process of composite materials. The use of FEM for the structural analysis and characterization of composites offers an insight into its internal behavior in addition to reducing physical testing and its associated costs. However, the reliability of the numerical result is heavily dependent on the adequacy of the input data, with the material parameters of the simple materials playing an important role. Composite manufacturers tend to report the composite properties as a whole rather than specify the component's properties separately [6]. For this reason, correct parameter identification is an issue which is being addressed more and more in this field.

An optimization method for the determination of material parameters in damaged composite structures is presented in this paper. The proposed methodology faces an inverse problem from a numerical point of view. It adjusts the material parameters of a composite test specimen with unknown properties but available experimental results.

The experimental set-up is reproduced in FEM using the in-house code PLCd [7], with the material properties taking the

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values assigned by the in-house optimizer Optimate [8]. The numerical result is compared with the experimental data to obtain an error value for the objective function, which is fed back to Optimate. An l_∞–norm is used to estimate the error. Then, by means of a genetic algorithm, Optimate adjusts the material properties until the numerical result is as close as possible to the experimental one.

Several authors have presented inverse methods for the determination of material parameters based on the same fundamental idea as the methodology proposed here. Markiewicz et al. [9] and Geers et al. [10] first used the inverse approach to determine parameters for material models of an aluminum alloy and a glass-fiber reinforced polypropylene composite, respectively. Since then, several variations and improvements on this method have been presented, with different authors putting more focus on particular aspects of the methodology. These include the type of optimization algorithm used [11, 12, 13], the objective function defined [14, 15] and the material parameters to identify in the context of its applications [11, 16, 17, 18, 19].

The work presented here provides a novel way of identifying the material parameters which define the components of a composite material. Even with limited experimental data available, the methodology proposed manages to correctly reproduce numerically its behavior. The coupling between the constitutive model and the optimization algorithm is highly flexible and can be easily adapted to different experimental contexts. In addition, the mixing theory used to formulate the constitutive model of the composite is versatile enough to be capable of numerically representing distinct types of composites as long as the simple materials that compose them are correctly characterized.

In the following section, the mixing theory used to numerically model the composite behavior is detailed, including a brief description of the constitutive models considered for the component materials. Section 3 describes the optimization method developed to determine the material parameters of the simple materials which form a composite. An example which illustrates the utility of the coupling between the optimization algorithm and the mixing theory in the proposed methodology is provided in section 4 to validate the method. Finally, the conclusions of the work are presented.

2. Constitutive modeling

The classical mixing theory assumes strain compatibility as the closing equation [20]:

$$\epsilon_{ij} = \left(\epsilon_{ij}\right)_1 = \left(\epsilon_{ij}\right)_2 = ... = \left(\epsilon_{ij}\right)_c \tag{1}$$

where ϵ_{ij} is the strain of the composite material and the subscript $(\bullet)_c$ refers to the c-component of the composite material.

The hypothesis that the contribution of each component is proportional to its volumetric participation is enforced through the specific Helmholtz free energy:

$$\Psi = \sum_{c=1}^{n} k_c \Psi_c \; ; \quad \sum_{c=1}^{n} k_c = 1 \tag{2}$$

where n is the total number of components and k is the volume fraction, which must fulfill the mass conservation principle.

By means of the Clausius-Planck inequality, the secant constitutive equation for the whole composite is obtained in the standard manner [20, 21, 22]:

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \epsilon_{ij}} = \sum_{c=1}^{n} k_c \frac{\partial \Psi_c}{\partial \epsilon_{ij}} = \sum_{c=1}^{n} k_c \left(\sigma_{ij}\right)_c \tag{3}$$

where σ_{ij} is the Cauchy stress tensor. The expression for the free energy of each component Ψ_c will depend on the type of constitutive model chosen for each simple material. In this work, the composite being modeled is a carbon fiber reinforced epoxy matrix so an anisotropic elasto-plastic constitutive model is proposed for the fibers and an isotropic scalar damage for the matrix [23]. However, other constitutive equations could be easily introduced if required for different type of composites.

2.1. Anisotropic elasto-plasticity

The anisotropic elasto-plastic constitutive model is based on the generalization of the classical plasticity theory [20, 24]. The anisotropic theory used to derive this model [23, 25] is based on the concept of *mapped stress tensor* first introduced by Betten [26].

2.1.1. Plastic Damage Model

The specific Helmholtz free energy of an elasto-plastic material is:

$$\Psi = \Psi^{\,e} + \Psi^p = \frac{1}{2}\epsilon_{ij}^e \mathbb{C}_{ijkl} \epsilon_{kl}^e + \Psi^p \tag{4}$$

where Ψ^e is the specific elastic free energy, Ψ^p is the specific plastic free energy, $\mathbb{C}_{ijkl}$ is the constitutive tensor of the material and ϵ_{ij}^e is the elastic strain. The total strain is split into an elastic and a plastic part, following the Prandtl-Reus hypothesis:

$$\epsilon_{ij} = \epsilon_{ij}^e + \epsilon_{ij}^p \tag{5}$$

Then, the constitutive equation of an isotropic elasto-plastic material is:

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \epsilon_{ij}} = \mathbb{C}_{ijkl}\left(\epsilon_{kl} - \epsilon_{kl}^p\right) \tag{6}$$

The plastic strain is obtained by means of the flow rule:

$$\dot{\epsilon}_{ij}^p = \dot{\lambda} \frac{\partial G^\sigma}{\partial \sigma_{ij}} \tag{7}$$

where λ is the plastic consistency factor as derived by Simo and Ju [27] and G^σ is the plastic potential function.

To fully characterize the plastic response, the yield function F^σ must satisfy the yield condition and a plastic hardening law must be defined. In this case, the expression proposed by Oller [28] is used:

$$\dot{\kappa}^p = \mathbf{h}_{ij} \dot{\epsilon}_{ij}^p \tag{8}$$

where κ^p is the plastic damage internal variable and $\mathbf{h}_{ij}$ is a second-order tensor defined in [28] which requires the definition of a scalar hardening parameter, H.

2.1.2. Anisotropy theory

Anisotropy is modeled by transporting all the constitutive parameters of the material and its stress and strain states from a *real anisotropic space* to a *fictitious isotropic space*. This mapping technique allows reproducing the behavior of the real anisotropic material by means of a well known and developed constitutive model of an isotropic material. The two spaces are related through a linear transformation, using a fourth-order tensor which contains all the information regarding the anisotropy of the real material. It is assumed that both spaces have the same elastic strains, which are related through the *strain transformation tensor* a^e_{ijkl}:

$$\overline{\epsilon}^e_{ij} = a^e_{ijkl}\epsilon^e_{kl} \tag{9}$$

where $\overline{\epsilon}^e_{ij}$ and ϵ^e_{kl} are the strains in the fictitious isotropic and real anisotropic spaces, respectively. The strain transformation tensor is defined as:

$$a^e_{ijkl} = \left(\overline{\mathbb{C}}_{ijmn}\right)^{-1} a^\sigma_{ijkl}\mathbb{C}_{klrs} \tag{10}$$

where $\overline{\mathbb{C}}_{ijmn}$ and $\mathbb{C}_{klrs}$ are the fictitious isotropic and real anisotropic constitutive tensors of the material, respectively. The *stress transformation tensor* a^σ_{ijkl} relates the Cauchy stresses of the isotropic and anisotropic spaces, $\overline{\sigma}_{ij}$ and σ_{kl}, respectively, as follows:

$$\overline{\sigma}_{ij} = a^\sigma_{ijkl}\sigma_{kl} \tag{11}$$

Then, the relation between the constitutive tensors in the real and fictitious spaces $\mathbb{C}_{jlmn}$ and $\overline{\mathbb{C}}_{ikrs}$, respectively, can be derived:

$$\mathbb{C}_{jlmn} = \left(a^\sigma_{ijkl}\right)^{-1} \overline{\mathbb{C}}_{ikrs}\; a^e_{rsmn} \tag{12}$$

Note that the anisotropic constitutive tensor is typically given in a local reference system and it must be transformed into the global reference system by means of a rotation tensor as shown in [23, 25].

The stress transformation tensor is defined such that it can be exactly adjusted to the desired isotropic or orthotropic yield criterion:

$$a^\sigma_{ijkl} = \overline{f}_{ij}\,(f_{kl})^{-1} \tag{13}$$

where $\overline{f}_{ij}$ and f_{kl} are the yield strengths in the isotropic and anisotropic spaces, respectively.

The yield and potential functions in the anisotropic space, $F^{\overline{\sigma}}$ and $G^{\overline{\sigma}}$, respectively, can be defined in terms of the irreducible basis of the invariants of $\overline{\sigma}_{ij}$, as detailed in [25]:

$$F^\sigma\left(\sigma_{ij}, \kappa^p\right) = F^{\overline{\sigma}}\left(\overline{\sigma}_{ij}, \kappa^p\right) = 0 \tag{14}$$

$$G^\sigma\left(\sigma_{ij}, \kappa^p\right) = G^{\overline{\sigma}}\left(\overline{\sigma}_{ij}, \kappa^p\right) = K \tag{15}$$

where K is a constant. Therefore, the known functions based on the invariants formulated for isotropic materials can be used as yield and potential functions in anisotropic materials.

2.2. Isotropic scalar damage

The damage model used was first proposed by Simo and Ju [27, 29] in the context of continuum damage mechanics, which is based on the *effective stress* concept initially introduced by Kachanov [30]. The constitutive equation defines the Cauchy stress σ_{ij} through a scalar damage parameter d which is a measure of the loss of rigidity in the material and must be within the limits $d \in [0, 1]$:

$$\sigma_{ij} = (1 - d)\,\mathbb{C}_{ijkl}\epsilon_{kl} \tag{16}$$

where $\mathbb{C}_{ijkl}$ is the constitutive tensor of the undamaged material and ϵ_{kl} is the strain. In this model, damage is understood as a loss of stiffness, which starts once the damage threshold of the material, σ^d, is reached.

Oller et al. [20] formulated a generalized damage model, translating the yield criteria used in plasticity to damage. However, the evolution of the internal damage variable d is different to that of the plastic parameter κ^p. Here, an explicit exponential softening model has been used:

$$d = 1 - \frac{\sigma^d}{(F^\sigma)^d}\mathrm{e}^{\left[A\left(1-\frac{F^\sigma}{\sigma^d}\right)\right]} \tag{17}$$

where $(F^\sigma)^d$ is the yield function chosen for the generalized damage model and A is a constant given by:

$$A^{-1} = \frac{E\; g^d}{\left(\sigma^d\right)^2} - \frac{1}{2} \tag{18}$$

Here E is the Young's modulus and g^d is the dissipated energy.

2.3. Composite constitutive model

Finally, the constitutive equation of the whole composite results is:

$$\sigma_{ij} = k_m\left(\sigma_{ij}\right)_m + k_f\left(\sigma_{ij}\right)_f \tag{19}$$

where the subscripts $(\cdot)_m$ and $(\cdot)_f$ indicate matrix and fiber, respectively. The stress value for each component is calculated according to the constitutive model chosen and developed above:

$$\left(\sigma_{ij}\right)_m = (1 - d)\left(\mathbb{C}_{ijkl}\right)_m\epsilon_{kl} \tag{20}$$

$$\left(\sigma_{ij}\right)_f = \left(a^\sigma_{mnij}\right)^{-1}\left(\mathbb{C}_{mnkl}\right)_f\left(\epsilon_{kl} - (\epsilon_{kl})^p_f\right) \tag{21}$$

where the damage variable d is obtained from (17), the stress transformation tensor a^σ_{mnij} is calculated as in (13) and the plastic strain $(\epsilon_{kl})^p$ is given by (7).

Then, the material parameters needed to define such model are: (1) the volumetric participations of matrix and fiber, k_m and k_f; (2) the Poisson ratio and Young's modulus of matrix and fiber, ν_m, ν_f, E_m and E_f, which define their elastic behavior; (3) the damage threshold stress and the dissipated fracture energy of the matrix material, σ^d_m and g^d_m; and (4) the yield stress and hardening parameter of the fiber material, σ^y_f and H_f. In addition, the damage criterion of the matrix material and the yield criterion of the fiber material must be established, as well as the evolution law of the plastic and damage variables.

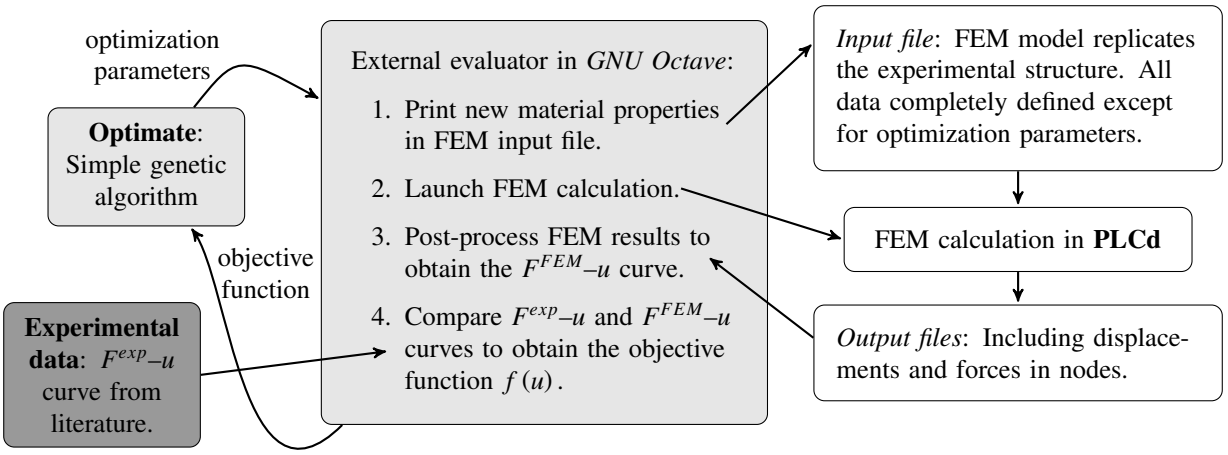

Figure 1. Scheme of the proposed optimization method for the determination of material parameters.

3. Optimization method for the determination of material parameters

The proposed optimization method for the determination of material parameters, from here on simply *optimization method*, is divided into three different blocks: (1) the optimizer or optimization algorithm *per se*; (2) the FEM calculations launched for each evaluation of the optimizer's objective function; and (3) the experimental data used to calculate the optimizer's objective function. These three blocks can be understood as independent units which interact with each other to form the complete method, as schematized in Figure 1.

The optimizer used in block (1) is the in-house simple genetic algorithm Optimate [8], described in section 3.3.1. Optimate is coupled to an external objective function evaluator written in GNU Octave [31]. The external evaluator also acts as the interface with the in-house FEM code PLCd [7] used in block (2). It launches the FEM calculation for each set of parameters proposed by Optimate and uses the results obtained to calculate the objective function which is fed back to Optimate. The calculation of the objective function requires the experimental data in block (3), as will be detailed in section 3.3.2. The experimental data also dictates the FEM model to be used in block (2).

3.1. Experimental data

The optimization method requires adequate experimental data with which to compare the numerical results in order to identify the correct material parameters. Since the experimental set-up must be reproduced in FEM, it is essential that the geometrical details of the specimen used as well as the imposed boundary conditions are known. To calculate the objective function required by the optimizer, a simple load vs. displacement curve such as those obtained by standardized tensile tests suffices.

For the purpose of illustrating how the method works, an example is presented in section 4, based on the numerical data used by Car et al. [23]. In this way, the numerical result obtained can be validated. However, the methodology only requires the experimental data mentioned above to identify the material parameters of the FEM model.

3.2. FEM module – PLCd

The FEM code PLCd is not directly accessed by the user while the optimization process is working. However, previous to launching this process, the FEM model must have been prepared to run in PLCd. This model must include a complete test specimen with the geometry, loads and boundary conditions completely defined, as well as the desired output results which will be used to calculate the FEM load vs. displacement curve.

The type of constitutive model to be used must be fixed for all materials, with all parameters defined except for the ones for which the optimum value is sought. That is, the type of constitutive models used to represent the behavior of the component materials in the composite must be established *a priori*.

3.3. Optimizer module – Optimate

The optimizer module includes the optimization program Optimate which seeks to minimize an objective function through evolutionary methods. This objective function is evaluated externally by the Optimate–PLCd interface written in GNU Octave.

This interface receives the optimization parameter values for the present evaluation of the objective function which correspond to the selected material parameters of the FEM model. Then, the previously prepared PLCd input file is accessed, the new material values are written in the adequate positions of the file and the calculation is launched. Once the calculation is completed, GNU Octave accesses the output result file and post-processes the data to obtain the FEM curve which is then compared to the experimental data. By means of an l_∞–norm estimation of the error between the two curves, the objective function is evaluated and the value obtained is fed back to Optimate.

3.3.1. Optimization algorithm

The algorithm developed in Optimate is a genetic algorithm with SBX [32] crossover, polynomial mutation, and binary tournament selection. These operators have been widely used in the well known NSGA-II [33] and omnioptimizer [34] algorithms for multiobjective optimization.

The main algorithm is as follows:

1. A random population $P(t)$ is generated inside the search limits. The population size is denoted as n_p.
2. The objective function is evaluated for $P(t)$, obtaining the objective function vector $F(t)$.
3. Using the binary tournament selection, a set of parents of size n_p is selected as follows:
   - Two individuals, i and j, are randomly chosen from $P(t)$ and the objective function is evauated for these individuals, F_i and F_j, respectively.
   - If $F_i(t) < F_j(t)$, then i is selected for minimization; if $F_j(t) < F_i(t)$, then j is selected; otherwise, one of the two is selected randomly.
   - The procedure is repeated until n_p individuals have been selected.
   - The selected individuals are denoted as $P_a(t)$.
4. The crossover is applied. Using a Bernoulli experiment, the algorithm determines if the two parents are used for reproduction (with the crossover operator) or for cloning (a simple copy). If the parents are suitable for reproduction, another Bernoulli experiment is used per variable in order to decide if the operator is applied to it or not. Two consecutive parents in $P_a(t)$ generate two children. This procedure is repeated until n_p children are generated. The children population is denoted as $\hat{P}(t)$. Two user-given parameters (probabilities) are required for this operation:
   - `pcroind`: probability of crossover for individuals.
   - `pcrovar`: for two individuals selected for crossover, probability of applying the crossover operator to a variable of these individuals.
5. The mutation is applied. The mutated population is denoted as as $\bar{P}(t)$.Similar to the crossover, a Bernoulli experiment decides whether an individual is mutated or not, and another is used for each variable of an individual suited for mutation. Hence, two additional user-given parameters are required for this operation:
   - `pcmutind`: probability of mutation for individuals.
   - `pmutvar`: for two individuals selected for mutation, probability of applying the mutation operator to a variable of these individuals.
6. $\bar{P}(t)$ is evaluated and the new objective function values are denoted as $\bar{F}(t)$.
7. The old population is replaced with the n_p best individuals obtained from the union of $\bar{P}(t)$ and $P(t)$.
8. The procedure is repeated from step 3 until the stopping criterion is reached.

Three possible stopping criteria are defined for the algorithm. If one of these criteria is satisfied, the algorithm stops:

- *A minimum objective function value to reach*. If the best individual in the population has an objective function value less than a user-given value (`minObj`), the algorithm stops.
- *A minimum allowed variance of the objective function*. The variance of the objective function is computed for each generation (iteration) and, for a given generation, if it is less than a user-given value (`maxVar`), the algorithm stops.
- *A number of iterations for which the minimum variance does not change*. The minimum variance of the objective function values of the population from the beginning of the generations is computed and stored for each generation, if it does not change in a number of generations (user-given value, `minVarCount`), the algorithm stops.

3.3.2. Objective function

An l_∞–norm is used to estimate the error between the experimental and FEM curves in order to obtain a value for the objective function:

$$f(x) = f(u_i) = \max\left[\frac{F^{exp}(u_i) - F^{FEM}(u_i)}{F^{exp}(u_i)}\right] \tag{22}$$

where x represents the optimization parameters, u_i are the displacement values of the curves, and $F^{exp}(u_i)$ and $FEM(u_i)$ are the load values of the experimental and FEM curves for each u_i, respectively.

Since the displacement values at which the curves are compared must the same, the experimental curve is linearly interpolated to the displacement values of the FEM curve. Obviously, the FEM calculation must be set up to obtain a certain number of displacement values such that the number of curve points used to determine the objective function is sufficient. Also, to avoid an indetermination for the first point of the curves, which is always zero, a very low value is added to the denominator.

4. Validation Example

Experimental data to validate the proposed optimization method has been taken from Car et al. [23]. The geometric details of the specimen used are presented in Figure 2. This specimen is composed of carbon-epoxy T2300/914C, with the fibers oriented at 10° with respect to the longitudinal axis of the sample. The specimen is subjected to a tensile test by fully-fixing the bottom side and imposing displacements on the top side. A load vs. displacement curve is obtained for the top side of the specimen. This is the experimental curve F^{exp} used in the optimization method.

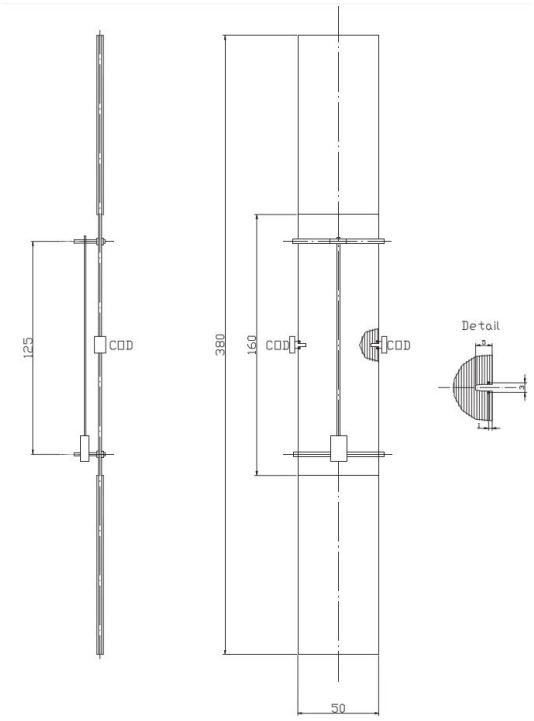

Figure 2. Geometry of the composite material specimen used for the validation example, from [23]. Values given in mm.

4.1. Numerical model

The experimental set-up has been reproduced in FEM using standard 8-noded hexahedral solid elements. The model has been meshed with 897 elements and 1944 nodes, resulting in 5832 degrees of freedom and 7176 Gauss integration points. A displacement of 0.295mm has been imposed on the top nodes of the specimen in 25 equal increments, with the bottom nodes fully-fixed.

The material behavior of the carbon fibers is modeled with an anisotropic elasto-plastic model with the properties shown in Table 1, where *TBD* indicates the value of the properties which have been selected for the optimization method to determine. Note that the Young's Modulus and Poisson coefficient indicated are for the longitudinal direction of the fibers, in the transversal direction the values assigned are those of the epoxy matrix. The anisotropy stress transformation tensor is implemented in Voigt notation into the FEM code and has been defined as a diagonal matrix with ones, except for $a^{\sigma}_{11} = 5$, following the criterion used in the reference model [23]. The material behavior of the epoxy matrix is modeled with an isotropic explicit scalar damage model with the properties shown in Table 1.

The material parameters to be determined are normalized and introduced as optimization parameters of the optimizer. Since these parameters have a physical meaning, reasonable upper and lower limits have been imposed for each. This information is summarized in Table 2, together with the user-given parameters required by Optimate, already described in section 3.3.1.

Material parameter	Normalization value	Lower limit	Upper limit
E_f	1.00E+11	1.0	5.0
σ^y_f	1.00E+08	1.0	30.0
H_f	1.00E+10	1.0	10.0
E_m	1.00E+10	1.0	5.0
σ^d_m	1.00E+08	0.2	0.6
g^d_m	1.00E+04	0.1	1.0

n_p	100
`pcroind`	0.9
`pcrovar`	0.85
`pmutind`	0.8
`pmutvar`	0.5
`minObj`	0.002
`maxVar`	1.0E-09
`minVarCount`	20

Table 2. Normalization values and imposed limits of the optimization parameters (above) and user-given parameters (below) introduced into Optimate.

Material parameter	Normalized value	Real value
E_f	3.982	3.98E+11 Pa
σ^y_f	13.343	1.33E+09 Pa
H_f	9.428	9.43E+10 Pa
E_m	2.124	2.12E+10 Pa
σ^d_m	0.374	3.74E+07 Pa
g^d_m	0.760	7.60E+03 N/m

Table 3. Parameter values identified by the optimization method.

4.2. Results and Discussion

The material parameters identified by the optimization method are reported in Table 3, which correspond to an objective function value of 2.23E-03. The use of these parameters results in a load vs. displacement curve which matches the experimental one, as shown in Figure 3. The optimizer required 2860 evaluations of the objective function, with each evaluation requiring about half a minute of CPU time in a personal computer which uses an OpenSuse 12.3 operative system and is equipped with a 3.4GHz Intel(R) Core(TM) processor and 16GB RAM.

The material parameters identified using the proposed optimization method agree with the expected margin of values for these properties, as provided by manufacturers and seen in literature. Examination of the numerical result obtained also reveals expected behavior of the specimen under loading, as shown in Figure 4. The displacement pattern obtained as well

Carbon fibers	
Young's modulus, E_f	*TBD*
Poisson coefficient, ν_f	0.0
Yield stress, σ_f^y	*TBD*
Post yield behavior law	Linear with hardening
Yield criterion	Von Mises
Hardening parameter, H_f	*TBD*
Volumetric participation, k_f	47.5 %

Epoxy matrix	
Young's modulus, E_m	*TBD*
Poisson coefficient, ν_m	0.325
Damage threshold stress, σ_m^d	*TBD*
Damage behavior law	Exponential with softening
Damage (yield) criterion	Von Mises
Fracture energy, g_m^d	*TBD*
Volumetric participation, k_m	52.5 %

Table 1. Material properties defined in the FEM model.

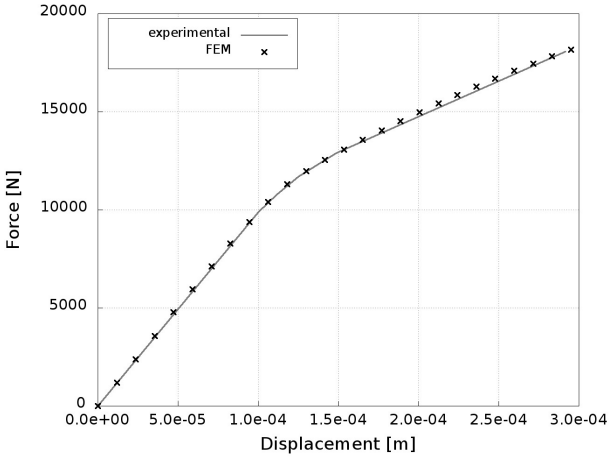

Figure 3. Load vs. displacement curve for the FEM model with the identified material parameters.

as the stress distributions closely match those of the reference model [23].

5. Conclusions

An optimization methodology which uses an inverse method for the identification of material parameters in composite structures has been presented and discussed. The method numerically reproduces the experimental set-up of a test using mixing theory and compares the numerical results obtained to the experimental ones. By means of a population-based evolutionary optimizer it then determines the value of the material parameters which result in a closest approximation of the FEM model to the experimental test.

The example chosen for the numerical validation of the optimization method shows that the procedure developed is capable of correctly identifying the material parameters of the component materials of a composite based solely on an experimental load vs. displacement curve. Yet, the constitutive model used in each material must be defined *a priori* and certain parameters of the models must be fixed, such as the yield and damage criteria and evolution laws. The user can then select which parameters with numerical values require identification. These can include basic material properties such as the Young's Modulus.

The bottleneck of the optimization method proposed is the computational cost of each FEM computation, which hinders the computational cost of the overall procedure. Since a complete FEM computation is required for each evaluation of the objective function, and the optimization method requires a significant number of evaluations, certain FEM models might result prohibitive to work with. Thus, a compromise must be reached between accuracy and computational efficiency when it comes to numerically reproducing the experimental set-up.

Moreover, the methodology is highly flexible due to its modular structure: the in-house codes both for the FEM calculation (PLCd) and optimization algorithm (Optimate) could be easily replaced by other in-house or commercial equivalent softwares and the experimental data and equivalent numerical model could also be changed, requiring only minor changes in the interface code.

However, the novelty of the approach is precisely in the simple but functional coupling of the FEM mixing theory with the genetic optimization algorithm. The mixing theory is a powerful tool for modeling the behavior of composite structures in FEM. It allows combining several constitutive models which represent the individual behavior of the component materials to obtain the global behavior of the composite.

As to the optimizer, it is highly parallelizable in the most expensive part of the code, the evaluation of the objective function. Also, it allows using l_∞–norms which cannot be used in gradient-based algorithms since they are not differentiable in all the domain. In fact, practically any function can be defined as objective function in the genetic algorithm used because there is no need for it to be derivable, continuous and convex in all the domain. This makes the optimizer especially well-suited for purposes such as the identification of material parameters through comparison of experimental and numerically-obtained curves. In addition, the multiobjective capability of the optimizer code could be easily exploited in the future for the identification of material parameters when additional result curves, for example, torsion-rotation curves, must be satisfied.

6. Acknowledgments

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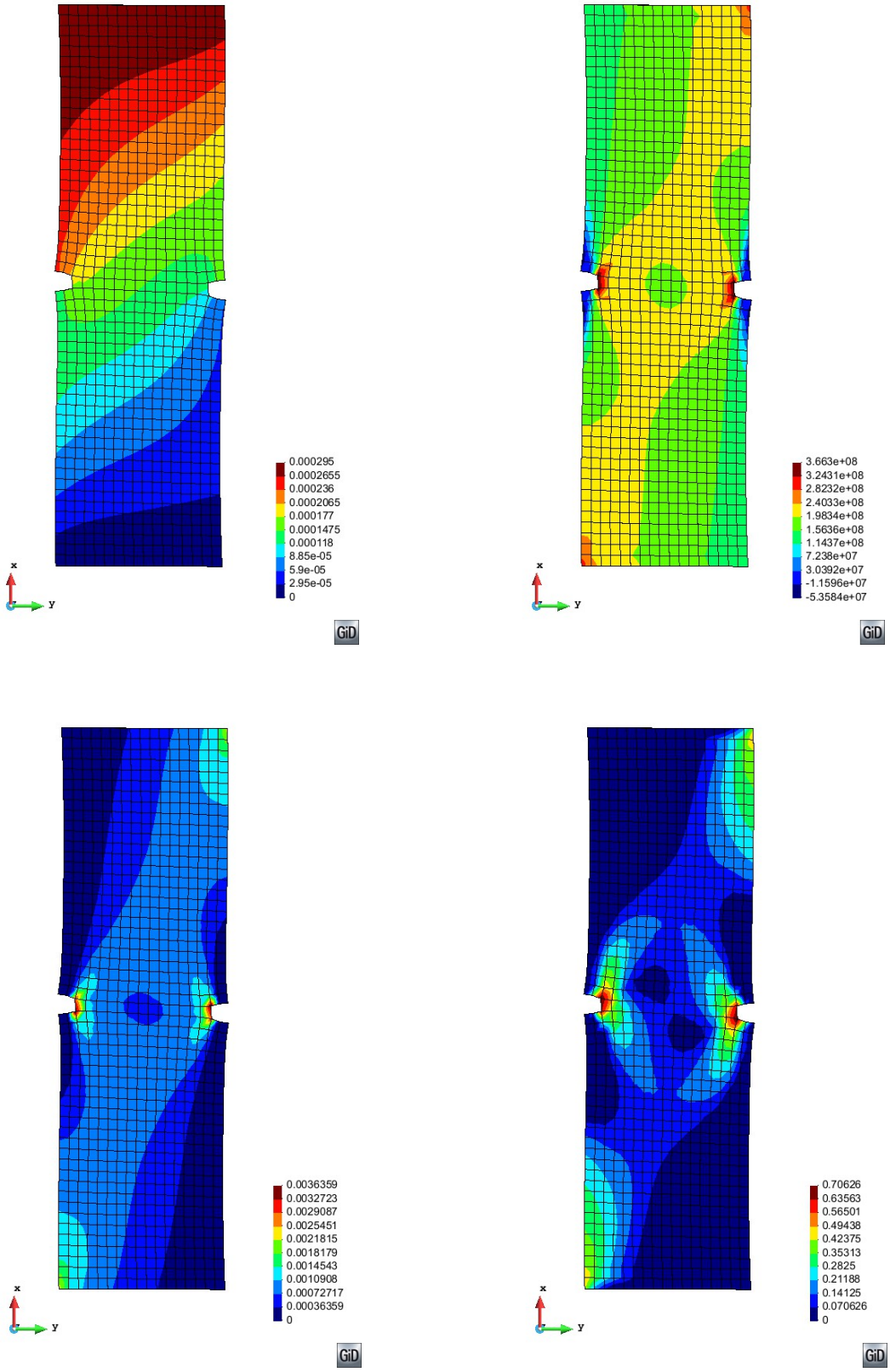

Figure 4. Top left: Contours of total displacement in the FEM specimen with the identified material parameters (values in m). Top right: Contours of stress in the longitudinal direction (values in Pa). Bottom left: Contours of the principal plastic strain in the fibers (adimensional). Bottom right: Contours of the internal damage variable in the matrix (adimensional). Deformation amplified x30.

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