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ELECTRON MOBILITY NOISE SPECTRAL DENSITY AND VARIANCE IN A SEMICONDUCTOR

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Abstract – The influence of external uniform electric field on the electron mobility variance in a non-degenerate n-type semiconductor is considered. In the course of analysis of results of mobility fluctuation theory, according to which electron mobility variance in equilibrium semiconductor equals infinity, it is shown that in the presence of uniform electric field the mobility variance becomes finite. The effect is explained in terms of the so-called electron-phonon FIT (field-induced tunnel) scattering. The results of numerical computations of mobility variance dependence on the electric field for n-Si and n-Ge at 300 K are presented. It is revealed that mobility variance decreases by the logarithmic law with the electric field increase. The consideration of a mobility noise reciprocal problem established that the frequency dependence of mobility noise spectral density has a range of low-frequency plateau as well as ranges of 1/f and approximately 1/f dependencies. Low-frequency limit of 1/f dependence decreases to zero when the electric field tends to zero. A good agreement between mobility noise and current 1/f-noise in single crystal n-silicon is observed.

Keywords: semiconductor; electron mobility; fluctuation; variance; FIT scattering

1. Electron mobility variance

The development of the electron mobility fluctuation theory and the establishment of main peculiarities of mobility fluctuations can have an important meaning for the physics of semiconductors and semiconductor devices. Spectral density and variance are the basic parameters, which describe fluctuation processes. Study of current, carrier concentration and mobility fluctuations by the variance of those quantities was called as variance approach [1, 2]. It

is an effective method, which allows detecting and investigating the microscopic mechanisms of fluctuations in equilibrium as well as in non-equilibrium semiconductors. So, in Ref.[2], on the basis of variance approach, the variance of electron current in thermal equilibrium semiconductor is calculated and analyzed. It is established that not only the electron intraband random scattering (Nyquist's thermal noise) but also generation-recombination random transitions and the shot effect can be current equilibrium noise sources in a semiconductor. It should be noted that these three components of current equilibrium fluctuations have the same variance [2] equal to variance of Nyquist's thermal noise. In Ref.[1] for the electron mobility variance σ_μ^2 in non-degenerate n-type semiconductor the following expression was obtained:

$$\sigma_\mu^2 = \frac{\overline{\mu}^2}{\overline{N}}\rho, \tag{1}$$

where

$$\rho \equiv \frac{\left\langle \overline{\tau}_{\mathbf{k},char}^2 \right\rangle}{\left\langle \overline{\tau}_{\mathbf{k},char} \right\rangle^2} - 1, \tag{2}$$

$$\overline{\tau}_{\mathbf{k},char} \equiv \overline{\tau}_{\mathbf{k}} + \frac{k}{3}\frac{d\overline{\tau}_{\mathbf{k}}}{dk}, \tag{3}$$

where μ is the electron mobility, N is the number of conduction electrons, $\tau_{\mathbf{k}}$ is the electron quasi-momentum relaxation time, the bolded symbol $\mathbf{k}$ is the electron quasi-wave vector, and the non-bolded symbol k is the magnitude of $\mathbf{k}$, "$\bar{}$" is the symbol of averaging over statistical ensemble and indicates the non-fluctuating dc component of the corresponding value, $\langle\cdots\rangle$ is the symbol of $\mathbf{k}$-averaging (averaging via quantum states of the conduction band):

$$\langle(\cdots)\rangle = \frac{2}{\sqrt{\pi}}\int_0^\infty dx x^{1/2} e^{-x}(\cdots), \tag{4}$$

$x = \varepsilon/k_B T$, $\varepsilon = \hbar^2 k^2/2m$ is the conduction electron energy, m is the electron density-of-states effective mass, T is the temperature, k_B is the Boltzmann's constant.

Eq.(1) clearly shows that mobility variance depends on the electron scattering mechanisms through electron quasi-momentum relaxation time $\tau_{\mathbf{k}}$. Analyzing the contributions of different types of electron scattering processes in Ref.[3] it is established that in equilibrium flat-bands semiconductors the mobility variance equals infinity ($\sigma_\mu^2 = \infty$). On the other hand, from the well-known relation between mobility fluctuation spectral density $S_\mu(f)$ and mobility variance

$$\sigma_\mu^2 = \int_0^\infty df S_\mu(f) \tag{5}$$

it follows that mobility variance is equal to the square of the surface limited by the curve $S_\mu(f)$ and frequency f axis. From this simple geometrical interpretation of relation (5) it follows that mobility variance infinity is directly related to the peculiarities of frequency dependence of mobility fluctuations spectral density $S_\mu(f)$ in a low-frequency range.

Investigating the physical background of the mobility variance infinity in Ref.[3] electron-phonon interaction theory in the presence of an electric field F is developed and a new mechanism of electron-phonon scattering, called as electron-phonon field-induced tunnel (FIT) scattering, is observed. The results of Ref.[3] show that in non-equilibrium titled-bands semiconductor mobility variance becomes finite ($\sigma_\mu^2 \neq \infty$). It is concluded that mobility variance depends on the electric field. In the present paper, details of the mobility variance dependence on the uniform constant electric field is studied for n-Si and n-Ge.

2. Electron relaxation time

As follows from Eqs.(1)-(3), for the evaluation of the mobility variance magnitude it is necessary to determine the scattering mechanisms which act in the given semiconductor sample. It is well known that in 77 K - 400 K wide temperature range in n-Ge and n-Si electron mobility is mostly determined by lattice scatterings (inter-valley and intra-valley scatterings induced by acoustic and/or optical phonons) [4, 5]. Therefore, in that temperature range the total relaxation time of the electron quasi-momentum can be presented as

$$\frac{1}{\bar{\tau}_{\mathbf{k}}} = \frac{1}{\bar{\tau}_{\mathbf{k},ac}} + \frac{1}{\bar{\tau}_{\mathbf{k},opt}}, \tag{6}$$

where $\bar{\tau}_{\mathbf{k},ac}$ and $\bar{\tau}_{\mathbf{k},opt}$ are the electron quasi-momentum relaxation times related to electron-acoustic phonon and electron-non-polar (intra-valley or inter-valley) optical phonon scatterings, respectively.

It is known [4, 5] that

$$\frac{1}{\tau_{\mathbf{k},ac}} = \frac{(mk_BT)^{3/2}\varepsilon_{ac}^2\sqrt{2x}}{\pi\hbar^4\rho_r v_0^2}, \tag{7}$$

where ρ_r is the reduced mass density of the crystal, v_0 is the sound velocity of the longitudinal acoustic mode, ε_{ac} is the acoustic-phonon deformation potential.

In the framework of FIT scattering mechanism in Ref.[3] the following more general expression for $\bar{\tau}_{\mathbf{k},opt}$ in a low electric field region is obtained

$$\frac{1}{\overline{\tau}_{\mathbf{k},opt}} = \frac{D^2 m^{3/2}}{2^{1/2}\pi\rho_r\omega_0\hbar^3}\left[\overline{N}_0\sqrt{\varepsilon+\hbar\omega_0} + \left(1+\overline{N}_0\right)\pi\sqrt{\hbar\omega_F}\int_{-\beta_e}^{\infty} dsAi^2(s)\right]. \tag{8}$$

Here D is the non-polar optical deformation potential constant, $\hbar\omega_0$ is the non-polar optical phonon energy, $\overline{N}_0$ is the non-polar optical phonon equilibrium distribution according to the Bose-Einstein statistics, $Ai(s)$ is the Airy function [6],

$$\beta_e \equiv (\varepsilon-\hbar\omega_0)/\hbar\omega_F\,, \quad \omega_F \equiv \frac{(eF)^{2/3}}{(2m\hbar)^{1/3}}. \tag{9}$$

Parameter β_e can be represented as

$$\beta_e = \left(\frac{\varepsilon}{\hbar\omega_0}-1\right)(F_c/F)^{2/3}, \quad F_c \equiv \omega_0^{3/2}\sqrt{2m\hbar}/e\,. \tag{10}$$

Note that Eqs.(7) and (8) describe the intra-valley scattering processes. In many-valley semiconductors such as Ge and Si the inter-valley scattering, when electrons are scattered between different valleys plays a very important role in electron transport [4, 5]. Electron transitions between states in two different equivalent[1] or non-equivalent valleys can be induced by electron scattering with both acoustic and optical modes. Inter-valley scattering is very similar to the intra-valley non-polar optical phonon scattering [4, 5]. It can be treated as a deformation-potential interaction in the same way as intra-valley scattering is treated by non-optical phonons [4, 5]. The relaxation time for inter-valley phonon scattering is evaluated with a relation similar to that for non-polar optical phonon scattering [4, 5]. Therefore, Eq.(8) can be modified for equivalent inter-valley electron-phonon scattering by simply replacing the values D^2 and $\hbar\omega_0$ by the $z_i D_i^2$ and $\hbar\omega_i$, respectively, where z_i is the number of possible equivalent final valleys in the inter-valley scattering, D_i and $\hbar\omega_i$ are the inter-valley deformation potential and the inter-valley phonon energy, respectively. As a result of such replacement the relaxation time of electron-phonon equivalent inter-valley scattering can be presented as

$$\frac{1}{\overline{\tau}_{\mathbf{k},opt,i}} = \frac{z_i D_i^2 m^{3/2}}{2^{1/2}\pi\rho_r\omega_i\hbar^3}\left[\overline{N}_{0,i}\sqrt{\varepsilon+\hbar\omega_i} + \left(1+\overline{N}_{0,i}\right)\pi\sqrt{\hbar\omega_F}\int_{-\beta_{e,i}}^{\infty} dsAi^2(s)\right], \tag{11}$$

where

$$\overline{N}_{0,i} = \frac{1}{e^{\hbar\omega_i/k_BT}-1}, \quad \beta_{e,i} = \left(\frac{\varepsilon}{\hbar\omega_i}-1\right)(F_{c,i}/F)^{2/3}, \quad F_{c,i} \equiv \omega_i^{3/2}\sqrt{2m\hbar}/e\,. \tag{12}$$

[1] Equivalent inter-valley scattering process is subdivided into f-type and g-type processes (more detail, see [4, 5]).

3. Numerical computation results

Substituting Eq.(6) into Eq.(3) and taking into account expressions (7)-(11) and (2), the dependence of electron mobility relative variance $\rho = \sigma_\mu^2 \overline{N} / \overline{\mu}^2$ on electric field F can be established. However, the obtaining of an exact analytical expression for $\rho(F)$ dependence is related to difficulties of integral computations. For revelation of $\rho(F)$ function peculiarities here numerical methods of integration are used. Numerical computations of $\rho(F)$ are carried out on the basis of Eq.(2) for n-Si and n-Ge at $T = 300$ K using "Wolfram Mathematica" PC program packet. Numerical values of the intra-valley and equivalent inter-valley deformation potentials and corresponding TA, LA, TO, LO phonon energies relative to n-Si (in n-Si the optical intra-valley scattering is negligible [4, 7]) and n-Ge are presented in Tables I and II from [4, 7]. At numerical calculations following additional parameters of conduction band principal valley are used [7]:

Si: $m/m_0 = 0.32$, $\varepsilon_{ac} = 9$ eV, $v_0 = 8.43 \cdot 10^3$ m/s, (in the direction [100]), $\rho_r = 2329$ kg/m^3;

Ge: $m/m_0 = 0.217$, $\varepsilon_{ac} = 11$ eV, $v_0 = 5.4 \cdot 10^3$ m/s (in the direction [111]), $\rho_r = 5320$ kg/m^3.

Table I. Deformation potentials and phonon energies for electron lattice scatterings in Si [4, 7].

Si (300 K)	z_i	b	c	$F_{c,i}$, V/cm	$\alpha_M = c/2$	γ
Si-1 (● - cyan**)** $D_{i,XX}^{g_1(TA)} = 0.5 \times 10^{10}$ eV/m, $\hbar\omega_{i,XX}^{g_1(TA)} = 12.1$ meV	1	0.533	$1.6 \cdot 10^{-4}$	$3.9 \cdot 10^4$	$8 \cdot 10^{-5}$	$1.5 \cdot 10^{-4}$
Si-2 (■ - magenta**)** $D_{i,XX}^{g_2(LA)} = 0.8 \times 10^{10}$ eV/m, $\hbar\omega_{i,XX}^{g_2(LA)} = 18.5$ meV	1	0.528	$2.19 \cdot 10^{-4}$	$7.3 \cdot 10^4$	$1.1 \cdot 10^{-4}$	$2.1 \cdot 10^{-4}$
Si-3 (▲ - green**)** $D_{i,XX}^{g_3(LO)} = 11 \times 10^{10}$ eV/m, $\hbar\omega_{i,XX}^{g_3(LO)} = 62$ meV	1	0.310	$6.64 \cdot 10^{-2}$	$4.5 \cdot 10^5$	$3.3 \cdot 10^{-2}$	0.11
Si-4 (♦ - orange**)** $D_{i,XX}^{f_1(TA)} = 0.3 \times 10^{10}$ eV/m, $\hbar\omega_{i,XX}^{f_1(TA)} = 19$ meV	4	0.538	$7.3 \cdot 10^{-5}$	$7.6 \cdot 10^4$	$3.7 \cdot 10^{-5}$	$6.8 \cdot 10^{-5}$
Si-5 (▼ - blue**)** $D_{i,XX}^{f_2(LA)} = 2 \times 10^{10}$ eV/m, $\hbar\omega_{i,XX}^{f_2(LA)} = 47.4$ meV	4	0.506	$4.1 \cdot 10^{-3}$	$3 \cdot 10^5$	$2 \cdot 10^{-3}$	0.0041
Si-6 (○ - red**)** $D_{i,XX}^{f_3(TO)} = 2 \times 10^{10}$ eV/m, $\hbar\omega_{i,XX}^{f_3(TO)} = 59$ meV	4	0.511	$1.68 \cdot 10^{-3}$	$4.2 \cdot 10^5$	$8.4 \cdot 10^{-4}$	0.0016

Table II. Deformation potentials and phonon energies for electron lattice scatterings in Ge [7].

Ge (300 K)	z_i	b	c	$F_{c,i}$, V/cm	$\alpha_M = c/2$	γ
$D = 5.5\times10^{10}$ eV/m, $\hbar\omega_0 = 37$ meV	-	0.511	$2.95\cdot10^{-3}$	$1.7\cdot10^{5}$	$1.5\cdot10^{-3}$	0.0029
$D_{i,LL}^{f(TA)} = 0.2\times10^{10}$ eV/m, $\hbar\omega_{i,LL}^{f(TA)} = 10.3$ meV	3	0.556	$3\cdot10^{-6}$	$2.5\cdot10^{4}$	$1.5\cdot10^{-6}$	$2.7\cdot10^{-6}$
$D_{i,LL}^{f(LA,LO)} = 3\times10^{10}$ eV/m, $\hbar\omega_{i,LL}^{f(LA,LO)} = 27.6$ meV	3	0.522	$6.4\cdot10^{-3}$	$1.1\cdot10^{5}$	$3.2\cdot10^{-3}$	0.0061

The results of numerical calculations carried out for low electric field ($F < 600\,V/cm$, Ohmic) regime show that the $\rho(F)$ dependences for n-Si and n-Ge have the same forms. In Figs.1 and 2 typical curves of $\rho(F)$ dependences are shown for n-Si and n-Ge, respectively. The cause of the week expressed peaks is the presence of the Airy function in Eqs.(8) and (11). Curves are well described by the logarithmic function

$$\rho(F) = b + c\ln(F_{c,i}/F), \tag{13}$$

where b, c and $F_{c,i}$ are the constants, numerical values of which for different type of electron-phonon scattering presented in Tables I and II. Characteristic field $F_{c,i}$ depends on lattice scattering parameters (see Eq.(12)) and it order is $\sim 10^4 \div 10^5$ V/cm.

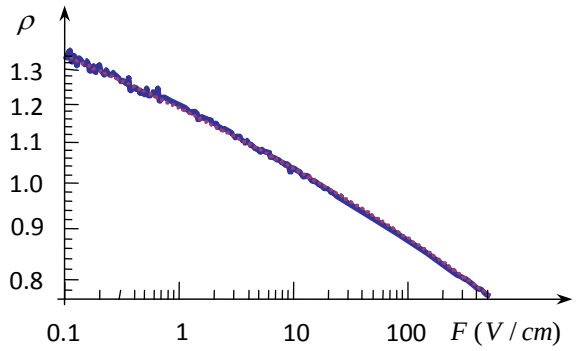

Fig.1. $\rho(F)$ dependence for n-Si in loglog scale, when electron-phonon g-type XX equivalent inter-valley scattering parameters are: $D_{i,XX}^{g_3(LO)} = 11\cdot10^{10}$ eV/m, $\hbar\omega_{i,XX}^{g_3(LO)} = 62$ meV; doted curve refers to $1.48 - 6.64\cdot10^{-2}\ln F$.

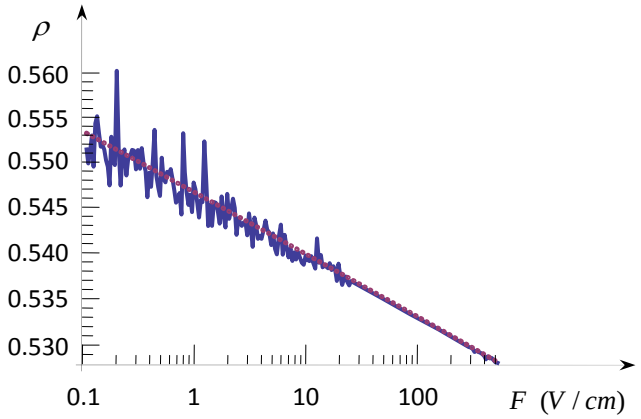

Fig.2. $\rho(F)$ dependence for n-Ge in loglog scale, when electron optical phonon L intra-valley scattering parameters are: $D = 5.5 \cdot 10^{10}$ eV/m, $\hbar\omega_0 = 37$ meV; doted curve refers to $0.56 - 2.95 \cdot 10^{-3} \ln F$.

4. Mobility noise reciprocal problem

Thus, the following peculiarities of electron mobility variance, determined by the integral expression (1), are established:

p.1) in non-equilibrium semiconductor under external electric field F mobility variance is finite, $\sigma_\mu^2(F) < \infty$;

p.2) in thermal equilibrium semiconductor the mobility variance is infinite, $\sigma_\mu^2(F = 0) = \infty$;

p.3) mobility variance depends on electric field F; the dependence described by the $\rho(F) = b + c \ln(F_{c,i} / F)$ logarithmic law with $b >> c$.

Spectral density and variance are inter-related characteristics of fluctuation processes. If frequency dependence of noise spectral density is known, the noise variance is easy to determine by simple integration of spectral density over frequency (see Eq.(5)). This method of variance determination is widely used in noise theory. In the given case under consideration one has the reciprocal problem: mobility variance peculiarities are known (see p.1)-p.3)); it is necessary to reveal peculiarities of the mobility noise spectrum.

Let us consider the mobility noise reciprocal problem. Mobility fluctuations are equilibrium and stationary fluctuations [1, 3], which exist in equilibrium ($F = 0$) as well as in non-equilibrium ($F \neq 0$) semiconductors. Mobility variance depends on electric field, $\sigma_\mu^2(F)$. Therefore, spectral density of mobility fluctuation depends on electric field also, $S_\mu(f,F)$. According to the well-known Wiener-Khintchine theorem, $S_\mu(f,F) \underset{f\to\infty}{\to} 0$ and derivative $dS_\mu(f,F)/df\big|_{f=0} = 0$. Noise spectral density has a maximum at zero frequency $f = 0$. Therefore, $S_\mu(f,F) \underset{f\to 0}{\to} const(F)$. Around the maximum the noise spectrum is independent of frequency, $S_\mu(f,F) \approx const(F)$. One can assume that near the zero frequency there is characteristic cut off frequency f_0 so that in the range $0 \le f \le f_0$ spectrum $S_\mu(f,F) \approx const(F)$, i.e. near the zero frequency there is low-frequency plateau.

As established in Ref.[3] the cause of electron mobility variance infinity in an equilibrium semiconductor ($\sigma_\mu^2(F=0) = \infty$) is the fracture of the curve $\bar{\tau}(\varepsilon)$; The fracture point ($d\bar{\tau}_{\mathbf{k}}/\varepsilon = \infty$ at $\varepsilon = \hbar\omega_0$) is located in electron long-wavelength (low energy) region. At presence of an electric field the electron-phonon FIT scatterings originate and the fracture disappears [3]. Low-energy electrons with $\varepsilon < \hbar\omega_0$ are characterize by extremely high FIT scattering relaxation times [3]. It can be state that infinity of the mobility variance at $F = 0$ related to divergence of integral in Eq.(5) on lower limit $f = 0$. Therefore, at $F = 0$ in low-frequency region frequency dependence of mobility noise spectrum must have $S_\mu(f, F=0) \sim 1/f^{1+\beta}$ form with $\beta \ge 0$. On the other hand, integral in Eq.(5) must converge on the upper limit $f = \infty$. This means that in high-frequency region noise spectrum must have $S_\mu(f, F=0) \sim 1/f^{1+\gamma}$ form with $\gamma > 0$.

On the basis of the above mentioned let us consider mobility noise spectrum which is given by

$$S_\mu(f,F) = \begin{cases} C_0 = \text{const}, & \text{at } f \le f_0 \\ C/f^{1+\beta}, & \text{at } f \ge f_0 \end{cases}. \qquad (14)$$

Here f_0, C_0, C and β are the parameters, where exponent β is not a negative quantity ($\beta \ge 0$).

From Eq.(14) function two branches suturing condition $C_0 = C/f^{1+\beta}\big|_{f=f_0}$ at $f = f_0$ follows: $C_0 = C/f_0^{1+\beta}$. Coefficients C_0 and C are interrelated quantities. Parameters f_0, C_0, C and β

in general case can be functions of an electric field. At the same time Eq.(14) it is shown that the function $C(F)$ cannot tend to zero or infinity at $F \to 0$. It can be supposed that $C(F) \underset{F\to 0}{\to} const$. Then, around the point $F = 0$ function $C(F)$ can be expanded into the Taylor series and limited by the first term at week electric fields. In low-field region one can assume that $C(F) \cong C(0) = const$. In contrary to C coefficient C_0 can depend on F. That dependence is determined by the possible field dependencies of $f_0(F)$ and/or $\beta(F)$. Coefficient $C(0)$ is more convenient to represent as $C(0) = \alpha_0 \overline{\mu}^2 / \overline{N}$, where α_0 is another field independent coefficient. Then, Eq.(14) is presented as

$$S_\mu(f,F) = \frac{\overline{\mu}^2}{\overline{N}} \times \begin{cases} \alpha_0 / f_0^{1+\beta}, & \text{at } f \le f_{0,}, \\ \alpha_0 / f^{1+\beta}, & \text{at } f \ge f_0. \end{cases} \tag{15}$$

For calculation of electron mobility relative variance on the basis of Eq.(15) it is necessary to evaluate the following integral:

$$\rho(F) = \frac{\overline{N}}{\overline{\mu}^2} \left\{ \int_0^{f_0} df S_\mu(f,F) + \int_{f_0}^{\infty} df S_\mu(f,F) \right\}. \tag{16}$$

To carry out integration the cases $\beta(F) > 0$ and $\beta(F) = 0$ should be considered separately because at $\beta(F) = 0$ the second integral in the r.h.s. of Eq.(16) is expressed by the logarithmic function and at $\beta(F) > 0$ it is expressed by the not logarithmic function. First, let us discusses case when $\beta(F) \neq 0$ and $\beta(F) > 0$. Then, substituting Eq.(15) into Eq.(16), we obtain

$$\rho(F) = \alpha_0 f_0^{-\beta} \left(1 + \frac{1}{\beta} \right). \tag{17}$$

For mobility relative variance one has two expressions: Eq.(17) and Eq.(13). Those expressions must be equal identically:

$$\alpha_0 f_0^{-\beta} + \frac{\alpha_0 f_0^{-\beta}}{\beta} \equiv b + c \ln(F_{c,i} / F). \tag{18}$$

Identity means that the l.h.s and r.h.s. of Eq.(18) must be equal term by term. Below the possible two sub-cases of Eq.(18) identity are considered.

Sub-case a):

$$\begin{cases} \alpha_0 f_0^{-\beta} = c \ln(F_{c,i} / F), \\ \alpha_0 f_0^{-\beta} / \beta = b. \end{cases} . \tag{19}$$

These relations can be presented as

$$\begin{cases} \beta(F) = \dfrac{c}{b}\ln(F_{c,i}/F), \\ f_0 = \dfrac{1}{\alpha_0}\left\{c\ln(F_{c,i}/F)\right\}^{-1/\beta}. \end{cases} \tag{20}$$

Eqs.(20) show that exponent β and the cut off frequency f_0 depend on F. In particular, $\beta \underset{F\to 0}{\to} \infty$, $f_0 \underset{F\to 0}{\to} 1/\alpha_0$ because

$$\lim_{F\to 0} f_0 = \lim_{F\to 0} \frac{1}{\alpha_0}\left\{c\ln(F_{c,i}/F)\right\}^{-\frac{b}{c\ln(F_{c,i}/F)}} = 1/\alpha_0 . \tag{21}$$

It is obvious that $\beta(F=0)=\infty$ is physically incorrect result. The noise spectrum of a physical quantity cannot have infinite exponent β. The sub-case under consideration must be excluded.

Sub-case b):

$$\begin{cases} \alpha_0 f_0^{-\beta} = b, \\ \alpha_0 f_0^{-\beta}/\beta = c\ln(F_{c,i}/F). \end{cases} \tag{22}$$

These relations can be presented as

$$\begin{cases} f_0 = \left(b/\alpha_0\right)^{-1/\beta}, \\ \beta(F) = b/\left[c\ln(F_{c,i}/F)\right]. \end{cases} \tag{23}$$

From Eq.(23) it follows that exponent β is positive because $F < F_{c,i}$. Field dependence of the frequency f_0 is given by $f_0 = \left(b/\alpha_0\right)^{-\left[c\ln(F_{c,i}/F)\right]/b}$. Exponent β tends to zero when electric field tends to zero, $\beta(F) \underset{F\to 0}{\to} 0$. Then, for the cut off frequency $f_0(F)$ one should have the following limits:

$$f_0(F) \underset{F\to 0}{\to} \begin{cases} \infty, & \text{at} \quad b/\alpha_0 < 1 \\ 0, & \text{at} \quad b/\alpha_0 > 1 \\ 1, & \text{at} \quad b/\alpha_0 = 1. \end{cases} \tag{24}$$

Case $f_0(F) \underset{F\to 0}{\to} \infty$ corresponds to ideal white noise. It is well known that a real physical quantity fluctuation cannot have ideal white noise spectrum. This case must be ignored. On the other hand, condition $\beta(F=0)=0$ means that in an equilibrium semiconductor frequency dependence of mobility noise spectrum is described by the $1/f$ law in the range $f > 0$ (if $f_0(F) \underset{F\to 0}{\to} 0$) or $f > 1$ (if $f_0(F) \underset{F\to 0}{\to} 1$):

$$S_\mu(f, F=0) = \frac{\overline{\mu}^2 \alpha_0}{\overline{N} f}. \tag{25}$$

At spectrum Eq.(25) the integral in Eq.(5) diverges on the upper limit $f = \infty$. This circumstance and condition $\beta(F = 0) = 0$ are contrary to the above presented initial assumptions according to which the integral in Eq.(5) must converge on the upper limit $f = \infty$ and exponent β should not equal zero ($\beta(F) \neq 0$). Therefore, the sub-case under consideration must be excluded as well. Thus, one can state that mobility noise spectrum cannot have the form of Eq.(15) with $\beta > 0$.

Now case $\beta = 0$ is considered. For convergence of integral in Eq.(5) on the upper limit $f = \infty$ it is necessary to modify the spectrum Eq.(14). It is assumed that in the low-frequency range $f_0 \le f \le f_1$ mobility noise spectrum $S_\mu(f, F)$ varies by the $1/f$ law and in high-frequency range $f \ge f_1$ it varies by the $1/f^{1+\gamma}$ law with $\gamma > 0$. As a result one obtains the following frequency dependence for mobility noise spectrum

$$S_\mu(f, F) = \begin{cases} C_0, \text{ at } f \le f_0 \\ C/f, \text{ at } f_0 \le f \le f_1, \\ C_1/f^{1+\gamma}, \text{ at } f \ge f_1. \end{cases} \tag{26}$$

Here f_0, f_1, C_0, C, C_1 and γ ($\gamma > 0$) are the parameters which can be functions of an electric field F.

Obviously the coefficients C and C_1 can not tend to zero or infinity at $F \to 0$. It can be supposed that $C(F) \underset{F \to 0}{\to} const$, $C_1(F) \underset{F \to 0}{\to} const$. As in the above case of Eq.(14), here also one can use Taylor series near the point $F = 0$ and limit by the first term at weak electric fields. As a result one can assume that in low-field region coefficients C and C_1 are independent of F: $C(F) \cong C(0) = const$, $C_1(F) \cong C_1(0) = const$. From suturing condition of Eq.(26) function branches at $f = f_0$ and $f = f_1$ one has

$$C_0 = C/f_0, \quad C_1 = Cf_1^{\gamma}. \tag{27}$$

From second relation of Eq.(27) follows that exponent γ and frequency f_1 are field independent quantities. Using relation $C = \alpha_M \overline{\mu}^2 / \overline{N}$, Eq.(26) can be presented as

$$S_\mu(f, F) = \frac{\overline{\mu}^2}{\overline{N}} \times \begin{cases} \alpha_M / f_0, \text{ at } f \le f_0 \\ \alpha_M / f, \text{ at } f_0 \le f \le f_1 \\ \alpha_M f_1^{\gamma} / f^{1+\gamma}, \text{ at } f \ge f_1, \end{cases} \tag{28}$$

where α_M is another field independent coefficient which can be called as mobility noise coefficient.

The evaluation of integral

$$\rho(F)=\frac{\overline{N}}{\overline{\mu}^2}\left\{\int_0^{f_0} dfS_\mu(f,F)+\int_{f_0}^{f_1} dfS_\mu(f,F)+\int_{f_1}^{\infty} dfS_\mu(f,F)\right\} \tag{29}$$

on the basis of Eq.(28) gives the following expression for the mobility relative variance:

$$\rho(F)=\alpha_M\left[1+\frac{1}{\gamma}+\ln\left(\frac{f_1}{f_0}\right)\right]. \tag{30}$$

Now let us take into account peculiarity p.2) according to which at the absence of an electric field ($F=0$) mobility variance equals infinity, $\sigma_\mu^2(F=0)=\infty$ (or $\rho(F=0)=\infty$). It means that the r.h.s. of Eq.(30) must tend to infinity at $F\to 0$. On the other hand, coefficient α_M, exponent γ and frequency f_1 are independent of F quantities. It can be concluded that the cut off frequency f_0 depends on electric field F. With decrease in F the frequency f_0 must decrease so that $f_0(F=0)=0$. Then, around the point $F=0$ function $f_0(F)$ can be expanded by the Taylor series. Taking into account that $f_0(F)$ is the even function one obtains

$$f_0(F)=\frac{1}{2}f_0^{''}(F=0)F^2+\frac{1}{6}f_0^{'''}(F=0)F^3+\cdots. \tag{31}$$

Limited by the first term of Taylor series one can assume that in the low field region

$$f_0(F)\cong\chi_0F^2, \tag{32}$$

where $\chi_0=f_0^{''}(F=0)/2$.

It is more comfortable the characteristic frequency f_1 presented as $f_1=\chi_1F_{c,i}^2$, where χ_1 is the coefficient of proportionality. Then, Eq.(30) presented as

$$\rho(F)=\alpha_M\left[1+\frac{1}{\gamma}+\ln\left(\frac{\chi_1F_{c,i}^2}{\chi_0F^2}\right)\right]. \tag{33}$$

Thus for mobility relative variance one has two expressions: Eq.(33) and Eq.(13). Those expressions must be equal identically:

$$\alpha_M\left[1+\frac{1}{\gamma}+\ln\left(\frac{\chi_1}{\chi_0}\right)\right]+2\alpha_M\ln\left(\frac{F_{c,i}}{F}\right)\equiv b+c\ln\left(\frac{F_{c,i}}{F}\right). \tag{34}$$

Here it is possible the following two sub-cases of Eq.(34) identity.

Sub-case c):

$$\begin{cases}\alpha_M\left[1+\frac{1}{\gamma}+\ln\left(\frac{\chi_1}{\chi_0}\right)\right]=c\ln\left(F_{c,i}/F\right),\\ 2\alpha_M\ln\left(F_{c,i}/F\right)=b.\end{cases} \tag{35}$$

Eq.(35) can be presented as

$$\begin{cases} 2\alpha_M^2\left(1+\dfrac{1}{\gamma}+\ln\left(\dfrac{\chi_1}{\chi_0}\right)\right)=cb, \\ 2\alpha_M \ln\left(F_{c,i}/F\right)=b. \end{cases} \tag{36}$$

As it is mentioned above, parameters α_M, b, c, χ_0, χ_1 and γ are independent of F. Second equation of Eq.(36) cannot be satisfied because its l.h.s. depends on F at the same time its r.h.s. is independent of F. This sub-case must be excluded obviously.

Sub-case d):

$$\begin{cases} \alpha_M\left[1+\dfrac{1}{\gamma}+\ln\left(\dfrac{\chi_1}{\chi_0}\right)\right]=b, \\ 2\alpha_M \ln\left(F_{c,i}/F\right)=c\ln\left(F_{c,i}/F\right). \end{cases} \tag{37}$$

Eq.(37) can be presented as

$$\left[\frac{1}{\gamma}+\ln\left(\frac{\chi_1}{\chi_0}\right)\right]=2b/c-1. \tag{38}$$

$$\alpha_M = c/2. \tag{39}$$

It is easy to make sure that Eq.(28) with relations Eqs.(38) and (39) perfectly meet the above presented requirements. There is not any contrast between these relations and peculiarities p.1)-p.3). It can be stated that the frequency dependence of mobility noise spectrum $S_\mu(f,F)$ is described by Eqs.(28), (38) and (39).

Now let us analyze relations Eq.(38) on the basis of numerical values of b and c parameters presented in Tables I and II. As the data in Tables I and II show, there are essentially big differences between the magnitudes of b and c. Order of the ratio $2b/c$ is $\sim 10^2 \div 10^5$. Then, Eq.(38) with good approximation can be presented as

$$\left[\frac{1}{\gamma}+\ln\left(\frac{\chi_1}{\chi_0}\right)\right]=2b/c. \tag{40}$$

Now consider the two limiting cases. First assume that $1/\gamma << \ln(\chi_1/\chi_0)$. Then, in l.h.s. of Eq.(40), ignoring term $1/\gamma$, one obtains $\chi_1/\chi_0 \approx e^{2b/c}$. From numerical estimation $\chi_1/\chi_0 \approx \exp(10^2 \div 10^5)$ follows that $\chi_1 >> \chi_0$. There is a very big difference between the magnitudes of characteristic frequencies f_0 and f_1 because $f_0 \sim \chi_0$, $f_1 \sim \chi_1$. Taking into account condition $f_1 > f_0$ it can conclude that the either frequency f_1 is very large quantity or frequency f_0 is near zero ($f_0 \sim 0$). Mobility noise spectrum varies by the $1/f$-law in very large frequency range $f_0 < f < f_1$. At second limiting case when $1/\gamma >> \ln(\chi_1/\chi_0)$ from Eq.(38) one

has $\gamma = c/2b$. The magnitude of exponent γ is determined by the ratio of constants c and b. Practically analogous results are obtained if we assume that $1/\gamma$ and $\ln(\chi_1/\chi_0)$ are the same order quantities ($1/\gamma \sim \ln(\chi_1/\chi_0) \approx 2b/c$). Numerical values of coefficient α_M and exponent γ evaluated on the basis of relations $\alpha_M = c/2$ and $\gamma = c/2b$ at different scattering parameters of n-Si and n-Ge are presented in Tables I and II.

Mobility noise, like the well-known generation-recombination, thermal, shot or 1/f noises, is a real physical phenomenon [1,3] and must be observed in electric current fluctuations. According to the above-presented peculiarities there are significant similarities between the mobility noise and 1/f-noise. The Tables' data seem very interesting if Hooge's dimensionless coefficient α_H experimental data relative to current 1/f-noise in n-Si and n-Ge are taken into account. So, in Refs. [8,9] (where some experimental results from single crystal semiconductors and metals are summarized) for different samples of n-Si and n-Ge the following values or ranges of α_H at 300 K are presented:

n-Si: $3\cdot10^{-5} \div 4\cdot10^{-4}, 10^{-5} \div 3\cdot10^{-3}, 2\cdot10^{-6} \div 10^{-4}, 2\cdot10^{-4} \div 10^{-3}$ [8]; $6\cdot10^{-5}, 2\cdot10^{-5}, 3\cdot10^{-6}$ [9];

n-Ge: $2\cdot10^{-3}$, $4\cdot10^{-3}$ [8].

1/f-noise parameter α_H is between $\sim 10^{-6}$ and $\sim 10^{-3}$ at 300 K. As it is seen from the Tables I and II the numerical values of the mobility noise coefficient α_M, which depends on electron-phonon scattering parameters[2], is between $\sim 10^{-6}$ and $\sim 10^{-2}$. The wide range of variation of α_M involves the above mentioned experimental data of Hooge's coefficient α_H. There is a quite good agreement between the Hooge's coefficient α_H and the mobility noise coefficient α_M numerical values.

Second interesting circumstance relates to the numerical values of the exponent γ, which are very small positive quantities. Its order is between 10^{-1} and 10^{-4} for n-Si; 10^{-3}, 10^{-6} for n-Ge at 300 K. The wide range of variation of γ is meant that it is very sensitive to electron both lattice and impurity scattering parameters. Mobility noise coefficient α_M and exponent γ are interrelated quantities. The ratio α_M/γ equals b. It is nearly a constant quantity equal to ~ 0.5 for n-Si and n-Ge at 300 K. Note that observing and distinguishing the $1/f$ and $1/f^{1+\gamma}$ dependency ranges is a very difficult experimental task because $0 < \gamma << 1$.

[2] In scientific literature data there are some deviations among the numerical values of electron-phonon scattering

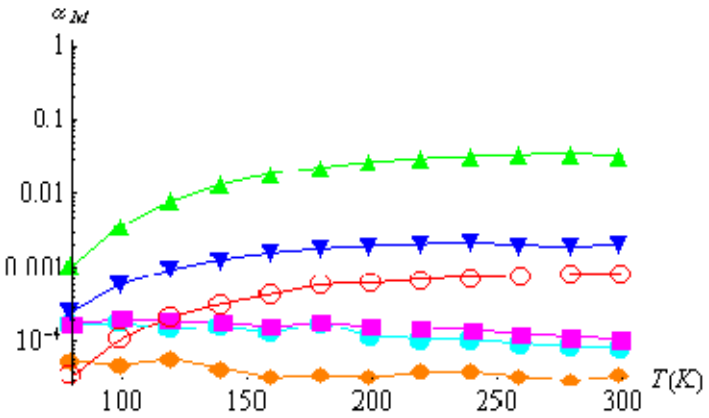

Fig.3. $\alpha_M(T)$ dependence for n-Si at electron-phonon scattering parameters corresponding to Si-1 (● -cyan), Si-2 (■ - magenta), Si-3 (▲ - green), Si-4 (♦ - orange), Si-5 (▼ - blue) and Si-6 (○ - red) (see, first column of Table I).

Coefficients b, c and γ are temperature dependent quantities. In Table I the results of numerical calculation for silicon carried out for $T = 300$ K are presented. Analogous calculation we carried out for temperatures: 80 K, 100 K, 120 K,…, 300 K. As a result we obtained a set of Tables for those temperatures. On the base of the Tables set the temperature dependencies of $c(T)$, $b(T)$ and $\gamma(T)$ can be established. In particular, in Fig.3 plotted $\alpha_M(T) = c(T)/2$ dependences for n-silicon, calculated at electron-phonon scattering parameters corresponding to Si-1, Si-2, …, Si-6 (see, first column of Table I). Figs. 4-7 illustrate the experimental data of temperature dependencies of Hooge's coefficient $\alpha_H(T)$ for n-Si from the well-known Refs. [9-12] and theoretical curves which are chosen from Fig.3. As it is seen, there are good quantitative and qualitative agreements between the theoretical and experimental data.

parameters (e.g., deformation potentials and phonon energies) in Si and Ge [4, 7].

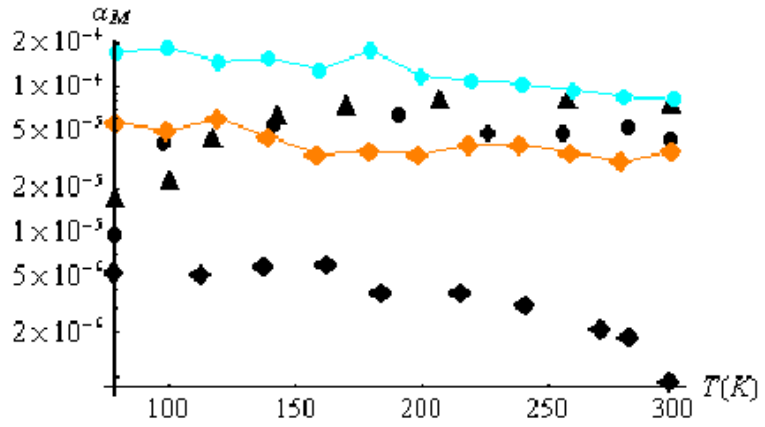

Fig.4. Temperature dependences of α_H and α_M for n-Si; experimental points of α_H are taken from Ref.[9]; theoretical curves of α_M refers to Si-1 (● -cyan) and Si-4 (♦ - orange).

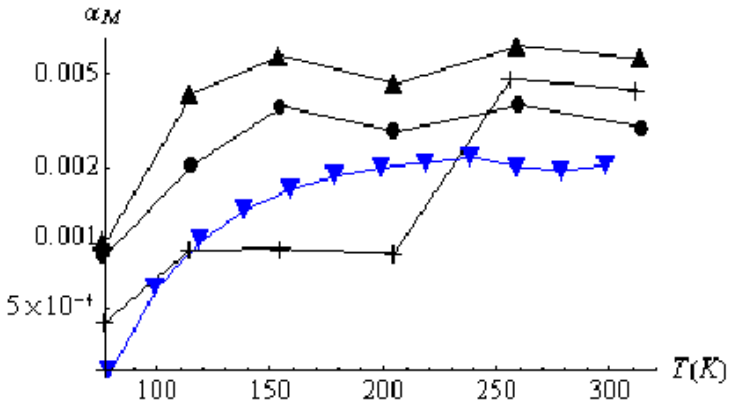

Fig.5. Temperature dependences of α_H and α_M for n-Si; experimental points of α_H are taken from Ref.[10]; theoretical curve of α_M refers to Si-5 (▼ - blue).

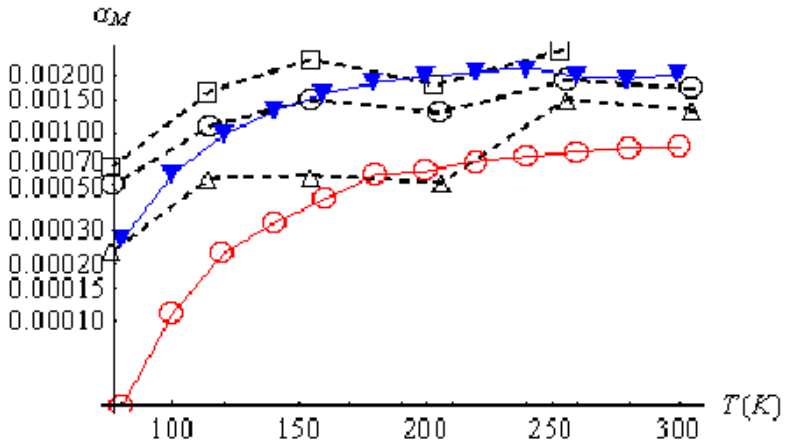

Fig.6. Temperature dependences of α_H and α_M for n-Si; experimental points of α_H are taken from Ref.[11]; theoretical curves of α_M refers to Si-5 (▼ - blue) and Si-6 (○ - red).

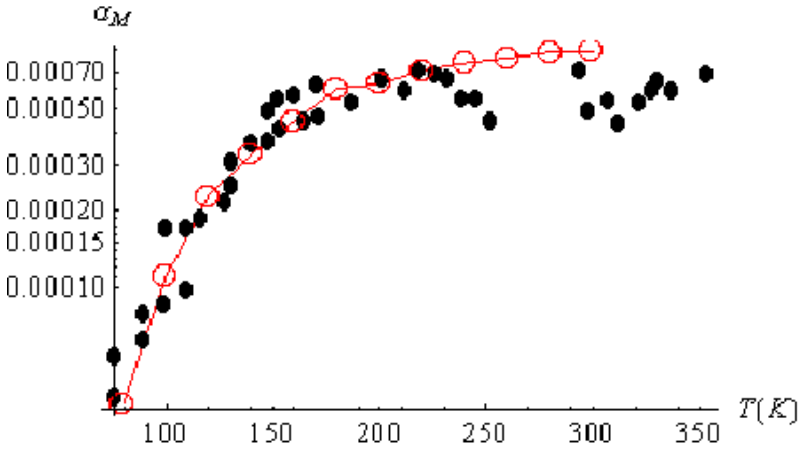

Fig.7. Temperature dependences of α_H and α_M for n-Si; experimental points of α_H are taken from Ref.[12]; theoretical curve of α_M refers to Si-6 (○ - red).

5. Conclusions

Thus discussing the mobility noise characteristics in low field region $F < F_{c,i}$ one can state that frequency dependence of mobility noise spectrum has the form Eq.(28). That dependence is plotted in Fig.8. On the curve of $S_\mu(f,F)$ there is a range of low-frequency plateau ($0 < f < f_0$) and ranges of $1/f$ (at $f_0 < f < f_1$) and $1/f^{1+\gamma}$ (at $f > f_1$) dependencies.

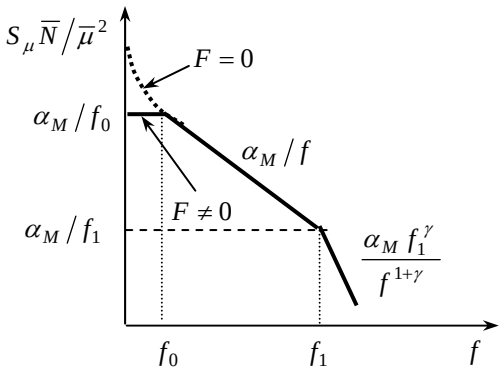

Fig.8. Frequency dependence of mobility noise spectrum.

Dependences $1/f$ and $1/f^{1+\gamma}$ coincide practically because $0 < \gamma << 1$ (see Tables I and II). There is a very big difference between the magnitudes of characteristic frequencies f_0 and f_1. Mobility noise coefficient α_M, exponent γ and frequency f_1 are independent on electric field. The cut off frequency f_0, which determines the length of low-frequency plateau, is located near zero and it depends on the electric field. Dependence $f_0(F)$ is given by the parabolic law $f_0(F) \cong \gamma F^2$. Low-frequency plateau disappears when electric field tends to zero, $f_0(F) \underset{F\to 0}{\to} 0$. For a thermal equilibrium semiconductor ($F = 0$) from Eq.(28) one obtains

$$S_\mu(f, F=0) = \frac{\overline{\mu}^2}{\overline{N}} \times \begin{cases} \alpha_M / f, & \text{at } 0 \le f \le f_1, \\ \alpha_M f_1^\gamma / f^{1+\gamma}, & \text{at } f \ge f_1. \end{cases} \tag{41}$$

For mobility variance study on the basis of Eq.(1) a numerical method integration is used above. However, Eq.(1) gives principal possibility to obtains logarithmic dependence Eq.(13) in the analytical form by using approximate methods of integration. It is a very important task because in that case one will have analytical expressions for exponent γ, coefficient α_M and frequencies f_0, f_1 or ratio f_1/f_0.

It should be noted that the background of the present study are the peculiarities of electron-non-polar optical phonon FIT scattering, which are observed and considered in Ref.[3]. On the

basis of electron FIT scattering is the phenomena of semiconductor bands titled anther electric field. Bands of real semiconductor with finite sizes can be tilted under external as well as under internal electric fields such as, e.g., a semiconductor surface, semiconductor-metal or p-n junctions fields etc. Mobility noise spectrum (and variance) depends on external as well as internal fields. That dependence reveals as $f_0(F)$ dependence. The length of low-frequency plateau of mobility noise spectrum depends on the electric field.

The above-developed theory of electron mobility noise in non-polar semiconductors (Si, Ge) can be employed for polar (compound) semiconductors such as GaAs, InSb, GaSb, etc. Non-polar optical phonon scattering may exist in non-polar as well as in polar crystals. However, in polar semiconductors electron scattering by polar optical phonons have played an important role [4, 5]. Here mobility fluctuations in polar semiconductors are not analyzed because the theory of electron FIT scattering by polar optical phonons is not developed and a corresponding expression for relaxation time is not obtained, yet.

Generalizing, it can be stated that FIT scattering ontology and the concept itself provide handy theoretical toolkit for a clear explanation of the finiteness and infiniteness switching effect of the mobility variance; it also reveals that mobility variance decreases with the electric field increase by logarithmic law. The developed theory has perspectives to become an instrumental element in modeling advanced semiconductor devices where noise issues are of key importance.

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